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A TEST OF PORTFOLIO CROWDING-OUT AND RELATED ISSUES IN FINANCE

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ABSTRACT

This paper tests hypotheses regarding the parameters in investors' asset demand functions. Most important is the hypothesis that federal bonds are closer substitutes for equity than for money; it is associated with the hypothesis of "portfolio crowding out" by federal borrowing.

Previous regression studies of asset demand functions have not been able to obtain precise and plausible estimates for the parameters, without the imposition of prior beliefs. The present paper uses a MLE technique that dominates regression in that it makes full use of the constraint that the parameters are not determined arbitrarily, but rather are determined by mean-variance optimization on the part of the investor. The technique also dominates, on the other hand, previous estimates of the optimal portfolio from ex post return data, in that expected returns are not assumed to be constant over time, or to change slowly, but rather are allowed to fluctuate freely. Thus the framework is consistent with questions such as the effects of a sudden increase in federal debt on the expected returns of the various assets.

Some hypotheses are tested where the answer seems clear in advance, such as a negative effect of the supply of money on the expected rate of return on equities. There the results of the MLE technique are much more plausible than the regression results. In the case of greatest controversy, a point estimate shows portfolio crowding <u>in</u>, not portfolio crowding out.

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1. INTRODUCTION

A number of questions regarding the behavior of portfolio-holders in financial markets have recently moved to center stage in the national policy-making arena. These are questions that have a long history of study but that never have been answered satisfactorily. Chief among them is the question whether federal government debt drives up the rate of return on capital and thus crowds out private investment in plant and equipment. (The effect is known as "portfolio crowding out" to distinguish it from the "transactions crowding out" effect of government spending itself that is familiar from the textbooks.) The framework in which to examine the question is well-established, but the crucial parameters elude successful estimation: how close a substitute is government debt for corporate capital in investors' portfolios? This question, and many others, depend on the parameters in investors' asset demand functions.

One obvious way to attempt to estimate the parameters in the asset-demand functions is simply to regress actual portfolios shares held by investors against some measure of expected returns by Ordinary Least Squares (OLS) or by some simultaneous equation method.[1] Alternatively the returns could be regressed against the actual portfolio shares to estimate the system of asset-demand functions in inverted form.[2] In this form the system can be thought of as a market equilibrium condition; it tells what the expected returns on the assets must be for given supplies of them to be willingly held. For example, consider the hypothesis that two particular assets are perfect substitutes, that the asset demands are infinitely sensitive to the relative expected rate of return on those assets. The hypothesis is more testable when the asset-demand functions are estimated in inverted form. It says that supplies of the various assets have no effect on the expected relative return of the two assets. (In the matrix B^{-1} below, the relevant row consists of zeros).

The OLS approach is tried out below. The problem with it is that the estimates tend to be imprecise. Many of the parameter estimates are highly implausible, and we cannot have much confidence in tests of portfolio crowding out. It would be desirable to bring more information to bear on the parameters in the asset-demand functions.

The theory of portfolio optimization constitutes such additional information. If investors maximize expected utility, then the parameters in their asset-demand functions are not determined arbitrarily, but rather are related to the degree of variability of ex post returns and to the degree of risk-aversion. Under certain assumptions made in this paper (one-period maximization of expected utility, constant relative risk-aversion, and normally distributed returns) the relationship is extremely simple: the coefficient matrix, in its inverted form, is proportional to the variance-covariance matrix of returns. Then the proposition that two assets are close substitutes becomes the hypothesis that the covariance of their returns is high, and that they have similar covariances with third assets.[3]

This train of thought is a common one in the literature. Roley (1982, P. 646) sums up the woeful status of the regression studies: "Despite the theoretical plausibility of significant relative asset supply effects on security yields, virtually all empirical research has been unsuccessful in isolating these effects." The responses in the literature, attempts to bring to bear the additional information contained in the theory of portfolio optimization, fall into two distinct categories. The first group maintains the framework of regressions of asset quantities and rates of return, but uses the Theil-Goldberger mixed-estimation technique to bring in a priori beliefs like gross substitutability among the assets. Examples are Smith and Brainard (1976), Backus, Brainard, Smith and Tobin (1980), and Backus and Purvis (1980).[4] One problem with this approach is that it does not use all the information contained in the portfolio optimization theory. But in another sense, it uses too much information: the assumption of gross substitutability among all assets is a strict one that is not particularly likely to be borne out by the true variance-covariance matrix, as Blanchard and Plantes (1977) have argued.

The second category of studies make no use of time-series data on asset quantities, and instead compute the optimal portfolio from data on ex post rates of return. The difficult question is how to measure expected rates of return. The most common method is to assume expected returns constant over the sample period; then they can be estimated by the sample means, and the variance-covariance matrix can be estimated by the second moments around the means. Examples are Roley (1979) and Nordhaus and Durlauf (1982). The problem here is that the assumption that expected returns are constant is inconsistent with the seemingly-evident fact that nominal interest rates, expected inflation, real interest rates, expected returns on equity, etc., are observed to vary over time. At best, expected returns have been allowed to change in an ad hoc manner, by estimating the expected return from a distributed lag or ARIMA process of actual returns, or from a rolling regression of returns against lagged values of an arbitrarily chosen set of variables. For example in a study with similar aims to this one, Bodie, Kane and McDonald (1983) estimate the expected real return on Treasury bills by an AR(1) and similar methods. Such an approach allows expected returns to change, but only slowly from one period to the next. Thus it is still inconsistent with the framework of the macroeconomic questions we are asking, such as the effect of an increase in the supply of government debt on the various expected rates of return.

This paper estimates the parameters in the asset-demand function and tests hypotheses about them, using a technique that imposes the optimization hypothesis. The essence of this technique is the recognition that the variance-covariance matrix of returns is precisely the variance-covariance matrix of the error term in the system of equations, and that the parameters should be estimated subject to this contraint. The technique dominates the regression studies in that it uses all the information in the optimization hypothesis, and it dominates the optimal portfolio studies in that it allows expected returns to vary freely over time.

2. HYPOTHESES ON THE ASSET-DEMAND FUNCTIONS

In this section we present the specification of the asset-demand functions, and discuss in greater detail various hypotheses regarding their parameters.

We specify asset demands as a linear function of expected returns:

$$x_t = A + B(E_t r_{t+1}) . \tag{1}$$

x_t is a vector $[x_t^T\ x_t^F\ x_t^S\ x_t^C\ x_t^K]'$ of the shares in the total portfolio that are allocated to each of five assets: (1) tangible assets, i.e. real estate and consumer durables; (2) long-term federal debt; (3) long-term state and local debt; (4) long-term corporate debt; and (5) equities. There is a sixth asset that we omit as redundant given that the six shares must sum to one. It is: (6) deposits, which are an amalgamation of a monetary aggregate (basically M3) and short-term corporate and government securities. We aggregate the short-term assets together partly because they all have nominal returns known to the investor with certainty (assuming away default risk), which implies that their only risk comes from a common source, inflation; they should in theory be perfect substitutes with respect to risk. We choose a maturity of one year or less as the definition of short-term, not just because that is the accounting definition of short-term, but also because our data on portfolios held by U.S. households is yearly data. $E_t r_{t+1}$ is a vector $E_t[r^T_{t+1}\ r^F_{t+1}\ r^S_{t+1}\ r^C_{t+1}\ r^K_{t+1}]'$ of the expected real returns on the five assets, each measured relative to the expected real return on the numeraire asset, deposits. A is a vector of five constants. B is a five-by-five matrix of coefficients that describes the responsiveness of asset demands to expected returns. Equation (1) is general in form, in that we have not said what determines the parameters in A and B. But it is restricted somewhat in that we have assumed that wealth enters homogeneously, and that the equation is linear in expected returns. Later we will show that equation (1) is precisely the correct form for asset-demand functions to take, with specific values for A and B, if investors maximize a function of the mean and variance of their real wealth. We have excluded any transactions demand for assets, and any tax effects, though both could in theory be subsumed in the rates of return if they could be properly measured.

We will be working with the system of asset demands, equation (1), in inverted form.[5]

$$E_t r_{t+1} = -B^{-1}A + B^{-1}x_t . \tag{2}$$

We assume that the financial markets are always in equilibrium: expected rates of return are whatever they have to be for asset supplies to be willingly held by investors.[6] We will be examining two sorts of hypotheses.

One sort of hypothesis is that two or more particular assets are perfect substitutes for each other. Perhaps the most interesting such hypothesis is that long-term bonds are perfect substitutes for short-term bills and deposits.

This proposition would follow from the "expectations hypothesis" of the term structure of interest rates, which says that the long-term interest rate should equal the average of the present and expected future short term interest rates. It would be contradicted by the observation that often in recent years long-term interest rates have been high relative to what expected future short-term rates seem likely to be. The most obvious explanation that has been given is that holders of long-term debt require a risk premium to compensate them for the risk of capital loss, and that this risk premium has been forced up in recent years. The often-alleged culprit is the increase in the supply of long-term federal debt.

Since we have chosen the short-term asset for the numeraire, the hypothesis is that all the elements of row i and column i are zero, where $i = 2$ for federal bonds, $i = 3$ for state and local bonds, and $i = 4$ for corporate bonds. This says that the demand for long-term bonds is infinitely sensitive to their rate of return relative to short-term bills, and so arbitrage eliminates any fluctuation. In all our econometrics we will leave the vector of constant terms, $-B^{-1}A$ in equation (2), unconstrained. Thus the null hypothesis of perfect substitutability allows for an asset to pay a differential expected rate of return, as long as it is fixed. (This is by analogy with consumer theory, in which two goods are perfect substitutes if their relative price is fixed, whether or not it is fixed at unity.) In the case of the term structure of interest rates, the hypothesis of perfect substitutability thus interpreted does not rule out the Keynes-Hicks hypothesis of "normal backwardation," according to which longer-term interest rates pay a fixed liquidity premium above expected short-term rates, as compensation to the holder for possible capital losses. The alternative hypothesis that we are testing for is that there is a variable risk premium on long-term bonds, one that is forced up when the quantity of bonds supplied to the market goes up. Thus the test follows in the tradition of such papers as De Leeuw (1965), Modigliani and Sutch (1966, (1967), and Masson (1978).

The other sort of hypothesis we will be testing, besides perfect substitutability, concerns the derivatives of particular expected rates of return with regard to particular asset supplies. Presumably the expected rate of return on an asset, say federal bonds, has a positive derivative with respect to the supply of that asset. An increase in the federal debt raises the expected return that must be paid to investors to induce them to hold it. But the effect on the expected returns on alternative assets is not clear a priori.

Here the most interesting question is whether an increase in federal debt drives up the required rate of return on private capital. The "portfolio crowding out" literature of Blinder and Solow (1973) and Tobin and Buiter (1976) assumed in the tradition of Keynes that all forms of long-term debt and capital were perfect substitutes, so that a ceteris paribus increase in federal debt necessarily raised the required rate of return on capital. Those papers traced out over time the "general equilibrium" effects of cumulating government debt; the issue was whether the contractionary effects were outweighed by other expansionary effects. As Tobin (1961), Benjamin Friedman (1978), and Roley (1979) argue, if we relax the unrealistic assumption that bonds are perfect substitutes for real capital, then an increase in the supply of bonds will not necessarily drive up the required rate of return on capital to begin with. It depends on the degree of substitutability. If bonds are relatively close substitutes for capital (the limiting case being that of Blinder-Solow and

Tobin-Buiter), then an increase in the supply of bonds will indeed drive up the required return on capital. But if bonds are relatively close substitutes for money, then it will drive down the required return on capital, as if it were an increase in the supply of money. Friedman calls this possibility "portfolio crowding-in."

To consider effects on the expected rate of return of equity, we pick out the relevant equation from the system of five equations represented by (2):

$$E_t r^K_{t+1} = c_5 + b_{51}(T/W)_t + b_{52}(F/W)_t + b_{53}(S/W)_t + b_{54}(C/W)_t + b_{55}(K/W)_t \qquad (3)$$

where c_5 is the last element of $-B^{-1}A$,

b_{51}-b_{55} is the last row of B^{-1} ,

T is the supply of tangible assets,

F is the supply of federal bonds

S is the supply of state and local bonds

C is the supply of corporate bonds,

K is the supply of equity,

W is wealth, $\equiv T + F + S + C + K + D$, and

D is the supply of short-term bills and deposits.

The effect of an increase in F without a change in the other variables in equation (3) is given by b_{52}/W_t . Since we are holding wealth constant, the increase in F must come at the expense of the omitted asset, deposits D . Thus the experiment we are considering has precisely the interpretation of an open-market sale of bonds by the central bank. Since we have aggregated together money and short-term Treasury securities, the experiment can also be interpreted as a shift in the term-structure composition of the national debt, as in studies of debt management policy by Rolph (1957), Tobin (1963), Friedman (1978), and Roley (1979, 1982). While one might think that a decrease in the money supply would necessarily have the contractionary effects associated with an increase in $E_t r^K_{t+1}$, Tobin points out that a shift from short-term Treasury securities to long-term Treasury securities will have no effect if the two are perfect substitutes, and Rolph argues that the effect could actually be expansionary. To test whether such an operation raises the expected relative return on equity $E_t r^K_{t+1}$, we would test whether b_{52} is significantly greater than zero.

The effect of an increase in W without a change in the other variables in equation (3) is given by

$$\frac{\partial E_t r^K_{t+1}}{\partial W_t} = [- b_{51}(T/W)_t - b_{52}(F/W)_t - b_{53}(S/W)_t - b_{54}(C/W)_t - b_{55}(K/W)_t]W_t \ . \quad (4)$$

Since we are holding the other asset supplies constant, the increase in wealth must come in the form of an increase in deposits D . Thus equation (4) has precisely the interpretation of the effect of a "helicopter-drop" of money (or of the comparative statics effect of a money-financed government deficit). To test whether it has the negative effect on $E_t r^K_{t+1}$ that one would expect, we test whether $-b_5 x_t$ is significantly less than zero, where b_5 is the last row of B^{-1} .

The main event is portfolio crowding out: the effect of an increase in F , including the wealth effect on $E_t r^K_{t+1}$.

$$\frac{\partial E_t r^K_{t+1}}{\partial F_t} = [-b_{51}(T/W)_t + (1-b_{52})(F/W)_t - b_{53}(S/W)_t - b_{54}(C/W)_t - b_{55}(K/W)_t]W_t \ . \quad (5)$$

Notice that equation (5) is the sum of the effect of open-market substitution of bonds for money, b_{52}/W_t , plus the effect of an increase in money, the wealth effect of equation (4). An expression greater than zero represents portfolio crowding-out; an expression less than zero represents portfolio crowding in. The condition is stated in vector form in Appendix 3 (as are the other hypotheses to be examined).

The rate of return on equity cannot be identified perfectly with the cost of capital relevant to a firm's decision to undertake investment in plant and equipment.[7] Perhaps if we had data on the asset holdings of the aggregated private sector, including real capital held by the corporate sector, we could use the profit rate as an unambiguous measure of the return on real capital. But it seems desirable to avoid such an extreme degree of aggregation, and to work with the holdings of the household sector alone. We will look at the effect on the rate of return on equity because that is what is held by households. We will also look at the effect of an increase in federal bonds F on the expected rate of return on corporate bonds $E_t r^C_{t+1}$. Since corporate bonds are an alternative to equity as a way of financing investment by firms, this effect is also relevant to the question of portfolio crowding-out. Note however that our goal here is nothing more than to examine the effect of the supply of government debt on the expected relative rates of return that various assets must pay to private investors. To answer the more ambitious question of whether an increase in debt has an expansionary or contractionary effect on real activity, we would need to know not just the asset-holding preferences of households, but also those of firms, pension funds, banks and other financial intermediaries, not to mention what we would need to know about saving behavior, goods markets and the supply side. The term "portfolio crowding out" is here used merely as shorthand for certain partial derivatives.

3. ESTIMATION OF THE ASSET-DEMAND FUNCTIONS, WITH AND WITHOUT THE CONSTRAINTS OF MEAN-VARIANCE OPTIMIZATION

The frequent stumbling-block to the estimation of asset-demand functions like equation (1), or the inverted form (2), is the measurement of expected returns, which are not directly observable. As discussed in the Introduction, we do not want to measure expected returns by the sample average or by an ad hoc ARIMA function of lagged returns, because this would not allow them to fluctuate freely. But the way we have set up equation (2), all that is necessary is to assume that expectations are rational: expectational errors are random, where "random" means uncorrelated with the information set I_t available at time t. Then actual ex post returns are given by:

$$r_{t+1} = E_t r_{t+1} + \varepsilon_{t+1} \qquad (E_t \varepsilon_{t+1} | I_t) = 0 \quad . \tag{5}$$

Substituting (2) into (5) we get:

$$r_{t+1} = -B^{-1}A + B^{-1}x_t + \varepsilon_{t+1} \tag{6}$$

This system of equations can be estimated by Ordinary Least Squares (OLS) because the lefthand-side variable is now observable and the error term, by the assumption of rational expectations, is uncorrelated with the righthand-side variables x_t .[8]

The system was estimated on yearly observations from 1954 to 1980. The data are described in Appendix 4. The OLS results are reported in Table 1. As previous studies have found, e.g. Smith and Brainard (1976) and Friedman (1978, p. 638), simple OLS estimation of such a system does not yield very satisfactory results. The implausibility of some of the estimates in Table 1 will become clearer when we turn to our hypothesis-testing in the next sections.

Table 1: Unconstrained Estimation of Inverted Asset-Demand Function

Equation-by-equation OLS. Sample: 1954-1980

Dependent variable: Real rate of return on asset relative to short-term bills:	Constant	β^{-1}: coefficients on shares of portfolios allocated to: Tangible assets	Long-term federal debt	State and local debt	Corporate bonds	Equities	D.W.	SSR	R^2	log likelihood	F(5,21)
Tangible assets	-.103 (.409)	.251 (.594)	.279 (.562)	-4.755 (2.750)	3.005 (2.838)	.003 (.445)	2.06	.00648	.52	74.21	4.62*
Long-term federal debt	1.15 (1.19)	-2.071 (1.736)	-2.260 (1.642)	-7.756 (8.040)	22.329* (8.299)	-1.711 (1.300)	2.18	.005538	.52	45.25	4.51*
State and local debt	.979 (1.637)	-2.090 (2.378)	-5.163* (2.250)	-17.020 (11.017)	40.202* (11.372)	-1.635 (1.782)	1.70	.10399	.54	36.74	5.01*
Corporate bonds	.780 (.931)	-1.565 (1.352)	-3.038* (1.279)	-14.240* (6.262)	30.394* (6.464)	-1.365 (1.013)	1.97	.03359	.67	51.99	8.45*
Equities	.110 (2.600)	-.612 (3.776)	6.412 (3.573)	-19.562 (17.491)	3.991 (18.054)	.500 (2.828)	2.08	.26209	.32	24.26	1.95

*Significant at the 95% level. (Standard errors in parentheses.)

But for the moment, two anomalies stand out. First, many of the coefficients that we might a priori expect to be positive appear negative. While some are not statistically significant, three are: the coefficient of federal bonds in the equation for the return on state and local bonds, and the coefficients of federal bonds and state and local bonds in the equation for the return on corporate bonds. It seems a priori that the three bonds should be substitutes. Second, even when the coefficients are of the sign we would expect a priori, and even when they are statistically significant, the magnitudes are implausibly high. For example, it appears to take a 30.39 per cent increase (3039 basis points!) in the expected rate of return on corporate bonds to induce investors to accept an increase in their holdings of corporate debt equal to 1 per cent of their portfolios (at the expense of money). It is small wonder that previous authors have considered it necessary to adopt techniques that combine the data with their a priori beliefs.

The innovation of this paper is that it estimates the parameters of the asset-demand system, equation (1), using the constraints that come from the hypothesis that investors choose their portfolios so as to maximize a function of the mean and variance of their real wealth. This hypothesis has a distinguished history, consisting notably of the Tobin-Markowitz model (e.g. Tobin (1958)) and the Capital Asset Pricing Model (CAPM). Much of the large literature on CAPM is devoted to testing the model, and the results are often not favorable.[9] However this approach remains the most attractive way of bringing more structure to bear on simple asset-demand functions. Appendix 1 derives the optimal parameters for the asset demand function,[10] under four assumptions:

(A1) perfect capital markets

(A2) maximization of end-of-period expected utility

(A3) normal distribution of returns

(A4) constant relative risk-aversion.[11]

The optimal portfolio turns out to be equation (1), with

$$B = [\rho\Omega]^{-1} \tag{7}$$

where ρ is the constant of relative risk-aversion and Ω is the 5×5 variance-covariance matrix of returns, conditional on information available at time t.[12]

A simple way to estimate the system that would be in keeping with the traditional CAPM literature is as follows. First, expected returns $E_t r_{t+1}$ are assumed constant over time, and are estimated from the averages of ex post returns realized during the sample period. Second, the variance-covariance matrix Ω is assumed constant over time, and is estimated from the squared deviations of realized returns around those constant expected values. The problem with assuming expected returns constant has already been pointed out: it is inconsistent with the framework of changes in asset supplies and consequent changes in expected returns in which we are interested. The solution is to recognize that the variance-covariance matrix Ω is precisely the variance-covariance matrix of the expectational error, the ε_{t+1} term in equation (2), and that the equation should be estimated subject to that constraint:

$$r_{t+1} = -B^{-1}A + \rho\Omega x_t + \varepsilon_{t+1} \qquad \Omega = E\varepsilon\varepsilon' . \tag{8}$$

Imposing a constraint between the coefficient matrix and the error variance-covariance matrix is unusual in econometrics. It requires nonlinear Maximum Likelihood Estimation (MLE). Appendix 2 shows the log likelihood function and its first derivatives, and briefly describes the program used to find the parameter estimates that maximize it.

Table 2 reports the MLE results for the parameters in equation (8). The estimates are far more plausible than those in Table 1. The expected returns on the three assets that one would a priori expect to be the closest substitutes, the three bonds, indeed turn out to depend positively on each others' asset supplies.[13] Furthermore, the magnitudes are far more reasonable. For example, it now appears to take a .404 per cent increase (40.4 basis points) in the expected rate of return on corporate bonds to induce investors to accept an increase in their holdings of corporate debt equal to 1 per cent of their portfolios.

Although we have set up our hypothesis-testing on the matrix in inverted form $B^{-1} = \rho\Omega$, the pre-inverted form $B = (\rho\Omega)^{-1}$ is of interest because it represents investors' original asset-demand functions. Table 3 inverts the 5×5 matrix from Table 2. Two assets are defined to be substitutes if their off-diagonal entry is negative. For example a 1.00 per cent increase (100 basis points) in the expected return on corporate bonds raises the investor's demand for corporate bonds by .205 per cent of his portfolio and lowers his demand for the substitutes, the other four long-term assets. However several pairs of assets are complements. For example the increase in the expected return on corporate bonds raises the demand for deposits, as one can tell by adding the five coefficients. Thus the assumption of gross substitutability among all assets, imposed a priori by some previous studies, does not appear to be borne out.

The one parameter in Tables 2 and 3 that is not at all reasonable is the coefficient of relative risk-aversion ρ. The point estimate is 110.31 , but this

Table 2: Constrained Estimation of Inverted Asset-Demand Function

M.L.E. Sample: 1954-1980

(ρ unconstrained)

Dependent variable Real rate of return on asset relative to short-term bills:	Constant	B^{-1}: coefficients (constrained to $\rho\Omega$) on shares of portfolios allocated to:				
		Real estate and consumer durables	Long-term federal debt	State and local debt	Corporate bonds	Equities
Real estate and consumer variables	-.019 (.068)	.052 (.040)	-.027 (.086)	-.003 (.144)	.010 (.080)	-.023 (.168)
Long-term federal debt	-.032 (.162)	-.027 (.086)	.462 (.370)	.545 (.518)	.373 (.345)	-.021 (.429)
State and local debt	-.043 (.253)	-.003 (.144)	.545 (.518)	.915 (.672)	.545 (.519)	-.032 (.524)
Corporate bonds	-.070 (.151)	.010 (.080)	.373 (.345)	.545 (.519)	.404 (.359)	.048 (.294)
Equities	-.420 (.587)	-.023 (.168)	-.021 (.429)	-.032 (.524)	.048 (.294)	1.194 (.557)

ρ: coefficient of risk-aversion
110.31 (152.16)

Table 3: Constrained Estimate of $(\rho\Omega)^{-1}$, Pre-inverted Asset Demand Function

B^{-1} in Table 2 inverted. ρ unconstrained.

The demand for the assets listed below	depends on the expected real return (relative to the real return on bills) of the following assets				
	Tangible assets	Long-term federal debt	State and local debt	Corporate bonds	Equities
Tangible Assets	25.64	8.05	1.06	-9.61	1.06
Long-term federal debt	8.05	11.95	-1.72	-8.98	0.69
State and local debt	1.06	-1.72	6.29	-6.96	0.44
Corporate bonds	-9.01	-8.98	-6.96	20.54	-1.36
Equities	1.06	0.69	0.44	-1.36	0.94
Short-term bills and deposits (= -sum of other rows)	-26.80	-9.99	+.89	+6.37	-1.77

Table 4: Constrained Estimation of $\rho\Omega$, Inverted Asset-Demand Function

MLE. Sample: 1954-1980

ρ constrained to 2.0

Dependent variable Real rate of return on asset relative to short-term bills:	Constant	B^{-1}: coefficients (constrained to $\rho\Omega$) on shares of portfolios allocated to:				
		Tangible assets	Long-term federal debt	State and local debt	Corporate bonds	Equities
Tangible assets	-.00985	.00099		(symmetric)		
	(.01246)	(.00082)				
Long-term federal debt	-.02728	-.00056	.00841			
	(.03647)	(.00169)	(.00493)			
State and local debt	-.02546	-.00015	.00998	.01689		
	(.04884)	(.00232)	(.00649)	(.00686)		
Corporate bonds	-.02523	.00013	.00682	.01007	.00748	
	(.02882)	(.00146)	(.00404)	(.00566)	(.00436)	
Equities	.05384	-.00086	.00050	.00060	.00185	.02889
	(.05644)	(.00436)	(.01002)	(.00910)	(.00631)	(.01346)

Table 5: Test Results

Hypothesis	t	$R\hat{\beta}$	OLS: Coefficients not constrained to be optimizing (Table 1) — Test Statistic $(R\hat{\beta})'[RV(\hat{\beta})R']^{-1}(R\hat{\beta})$	q	χ^2_q (95%)	MLE: Coefficients constrained to be optimizing (Table 4) — $R\hat{\Sigma}$	Test Statistic $(R\hat{\Sigma})'(RV(\hat{\Sigma})R'^{-1}(R\hat{\Sigma})$	q	χ^2_q (95%)
Perfect substitutes									
1. All assets			93.4*	25	37.7				
2. Three bonds			26.34*	16	26.30				
3. Corporate, state & local bonds			19.8*	9	16.9				
4. Three bonds and deposits			58.9*	21	32.7				
"Crowding out"									
5. $\partial Er^K/\partial K$	1954	+.36	.37	1	3.8	+8.2	4.4*	1	3.8
	1980	+.58	1.17			+9.6	3.9*		
6. $\partial Er^K/\partial D$	1954	-.14	.13			-6.3	4.7*		
	1980	+.08	.05			-4.9	5.3*		
7. $\partial Er^K/\partial F$	1954	+6.3	1.52			-6.0	1.6		
	1980	+6.5	1.59			-4.6	0.8		
8. $\partial Er^F/\partial F$	1954	-1.1	.24			+3.8	1.6		
	1980	-0.9	.14			+4.2	2.2		
9. $\partial Er^C/\partial F$	1954	-2.3	1.6			+2.6	1.7		
	1980	-2.1	1.3			+3.0	2.3		

*significant at the 95 per cent level

number is normally considered to be far lower. Given a priori beliefs about the coefficient of relative risk-aversion, it makes sense to impose them in order to get the most efficient estimates of the other parameters. The literature appears to have settled roughly on 2.0 as a value for the coefficient. (See, for example, the evidence in Friend and Blume (1975).) Table 4 reports the results of using the MLE technique to impose not only the constraint of mean-variance optimization, but also the further constraint $\rho = 2$. The magnitude of the coefficients is smaller than those in Table 2, reflecting the smaller ρ. But their relative values, which are all that matters for the portfolio crowding-out tests, are very similar. The theme of this paper is the imposition of a priori constraints to obtain more efficient estimates; accordingly we use the estimates from Table 4 in the tests that follow.

4. RESULTS OF TESTS OF PERFECT SUBSTITUTABILITY

We now perform Wald tests on hypotheses of the type discussed above. The results are reported in Table 5. In this section we discuss the tests of perfect substitutability, applied to the unconstrained parameter estimates of B^{-1} in Table 1. (The perfect substitutability hypotheses cannot be properly nested within the MLE framework of mean-variance optimization because they will imply zero or identical rows in the variance-covariance matrix, which is on the edge of the parameter space.) Each hypothesis consists of a number q (between 1 and 25) of linear constraints on the 25 estimated parameters. The test-statistic is distributed asymptotically χ^2 with q degrees of freedom. Appendix 3 contains the algebraic details of the test-statistics.

HYPOTHESIS 1: ALL ASSETS ARE PERFECT SUBSTITUTES.

We start with the hypothesis that all coefficients are zero: asset supplies have no effect on expected returns. We would certainly expect to reject the null hypothesis. The test-statistic can be thought of as a generalized F test of the significance of the entire system. As expected, Table 5 shows that we easily reject Hypothesis 1.

HYPOTHESIS 2: THE THREE KINDS OF BONDS ARE PERFECT SUBSTITUTES

In looking at only six assets, we have presumed a high degree of aggregation to begin with. Perhaps a still higher degree of aggregation is possible. Of course state and local bonds are exempt from income taxes, and federal bonds have a lower probability of default than corporate bonds or state and local bonds. But these advantages could conceivably be worth a fixed premium in the expected rate of return, as opposed to the variable premium that occurs when investors wish to balance their portfolios among the different assets.

Table 5 shows that we again reject the hypothesis. Our finding is the same as that of Fair and Malkiel (1971), that federal bonds are not perfect substitutes for other bonds. We can also reject the hypothesis that the three bonds are perfect substitutes for equity (test statistic not reported), which is not surprising given Hypothesis 2. This confirms the argument of Tobin (1961) and Friedman (1978) that the traditional Keynesian aggregation is misleading, and that portfolio crowding out is not a foregone conclusion.

HYPOTHESIS 3: CORPORATE BONDS ARE PERFECT SUBSTITUTES FOR STATE AND LOCAL BONDS.

This is an attempt to see whether <u>any</u> degree of aggregation beyond six assets is possible. Corporate bonds and state and local bonds seem the most likely pair to be close substitutes because their interest rates are highly correlated, and they share an element of default risk. But we reject this hypothesis as well.

HYPOTHESIS 4: SHORT-TERM DEPOSITS ARE PERFECT SUBSTITUTES FOR LONG-TERM BONDS.

We have been examining aggregation across issuer. Here we move to aggregation across the term structure; we are testing the expectations hypothesis of the term structure of interest rates. Each F statistic reported in Table 1 tests the hypothesis that the five coefficients in its row are zero, i.e. that the asset is a perfect substitute for deposits. We reject the hypothesis for each of the three long-term bonds - federal, state and local, and corporate - considered individually. As indicated in Table 5, we also reject the hypothesis that the three bonds <u>jointly</u> are perfect substitutes for deposits. This is an important finding, given the absence of significant effects on the term structure in OLS studies like Modigliani and Sutch (1966, 1967).

5. RESULTS OF TESTS OF PORTFOLIO CROWDING OUT

We now turn to the tests of hypotheses regarding "portfolio crowding out," or the derivatives of expected rates of return with respect to asset supplies.

In each case, after testing the hypothesis on the unconstrained estimates, we then test it on $B^{-1} \equiv \rho\Omega$, the parameter estimates of Table 4 that are constrained to come from mean-variance optimization by the investor. Since the matrix is symmetric, there are in effect only 15 estimated parameters (not counting the intercept terms). Each hypothesis consists of a single linear constraint, and the test-statistic is again χ^2.

HYPOTHESIS 5: THE SUPPLY OF EQUITIES HAS A POSITIVE EFFECT ON THE EXPECTED RELATIVE RETURN ON EQUITIES.

This is a fairly unexceptionable proposition. We would expect an increase in any of the asset supplies to drive up the own rate of return, in order to induce investors to hold the increased supply willingly. Indeed the diagonal terms of the matrix estimated by MLE are positive by construction. But we begin our testing of the derivatives or "crowding out" effects with this example in order to get some idea of the power of our tests. In other words, if we cannot find a significant effect here, then the practical usefulness of the technique is in some doubt.

Recall that the wealth effect of an increase in any asset supply depends not only on the coefficients but also on the x_t, the shares of the portfolio already allocated to the various assets. As shown in Appendix 3, the constraints tested take the form that a linear combination $R\beta$ of the coefficients β is zero, where the weights in R come from the elements of x_t. Since the x_t vary over time, the test-statistic will vary somewhat over time. Table 6 prints out the portfolio-share data. There is a pronounced upward trend in the share allocated to tangible assets (real estate and consumer durables), and there are corresponding downward trends in the other assets. In the case of each derivative, to ensure that our results are not sensitive to the point in time that we pick, we will try the test once using portfolio shares at the beginning of the sample period, x_{54}, and once using shares at the end of the sample period, x_{80}. It is of course the second test that is more relevant for any possible policy prescriptions in the 1980's.

Under the unconstrained OLS estimates, the point estimate of the own-effect for equities is indeed seen to be positive, in 1954 as well as 1980. However the test statistic is not significant, not even at the 75 per cent level should one choose to go that low. Under the constrained MLE estimates,

Table 6: Portfolio Shares x_t , t = 1954-1980

	x^T	x^F	x^S	x^C	x^K
10	0.328570	0.534155e-01	0.163424e-01	0.249475e-01	0.440811
11	0.329761	0.482825e-01	0.172253e-01	0.234889e-01	0.449080
12	0.338335	0.424069e-01	0.162025e-01	0.219424e-01	0.448076
13	0.354365	0.418538e-01	0.171062e-01	0.241885e-01	0.421993
14	0.335525	0.343121e-01	0.151716e-01	0.227541e-01	0.457613
15	0.337421	0.308486e-01	0.163654e-01	0.215222e-01	0.455924
16	0.338708	0.324603e-01	0.188927e-01	0.234931e-01	0.444444
17	0.323320	0.272117e-01	0.179402e-01	0.229321e-01	0.469024
18	0.335957	0.294345e-01	0.184446e-01	0.242863e-01	0.438986
19	0.325358	0.249860e-01	0.168922e-01	0.228709e-01	0.450385
20	0.323831	0.242935e-01	0.175273e-01	0.217561e-01	0.449625
21	0.319007	0.216757e-01	0.161306e-01	0.195559e-01	0.456441
22	0.338244	0.226245e-01	0.168247e-01	0.187930e-01	0.430738
23	0.327976	0.191632e-01	0.131307e-01	0.171457e-01	0.449569
24	0.327560	0.162945e-01	0.109490e-01	0.164106e-01	0.457141
25	0.354732	0.158699e-01	0.105148e-01	0.155830e-01	0.426247
26	0.364097	0.182886e-01	0.114417e-01	0.180124e-01	0.408380
27	0.358475	0.174891e-01	0.109503e-01	0.191291e-01	0.411745
28	0.359642	0.159613e-01	0.107802e-01	0.191755e-01	0.408504
29	0.388405	0.147505e-01	0.111256e-01	0.179620e-01	0.368055
30	0.422225	0.149372e-01	0.969381e-02	0.164569e-01	0.327967
31	0.410246	0.127177e-01	0.944685e-02	0.163844e-01	0.345025
32	0.407128	0.146014e-01	0.102608e-01	0.181577e-01	0.346376
33	0.422477	0.121059e-01	0.934536e-02	0.160447e-01	0.328258
34	0.437006	0.110662e-01	0.744927e-02	0.135246e-01	0.321738
35	0.432993	0.101875e-01	0.625305e-02	0.117113e-01	0.331031
36	0.428551	0.768905e-02	0.406782e-02	0.917385e-02	0.347983
	1	2	3	4	5

on the other hand, the positive effect is highly significant in either year. We accept Hypothesis 5, i.e. we reject a zero effect. This case appears to be a good illustration of the benefit gained by using the extra information embodied in the constraint of mean-variance optimization.

HYPOTHESIS 6: THE SUPPLY OF DEPOSITS HAS A NEGATIVE EFFECT ON THE EXPECTED RELATIVE RETURN ON EQUITIES.

As a matter of economics, this proposition is of interest because it says that an increase in the money supply has a stimulating effect, to the extent that business fixed investment responds to the required rate of return on capital. As a technical matter, the derivative with respect to deposits is of interest because they are the numeraire asset. The total effect of an increase in, say, federal bonds (considered in the next hypothesis), is given by a wealth or "Helicopter drop" effect, represented by the derivative considered here, plus a substitution or "Open Market Operations" effect, represented by the single relevant element of B^{-1}.

Under the unconstrained OLS estimates the test-statistic is extremely low in significance. The derivative even changes sign during the sample period. Under the constrained MLE estimates the derivative is negative, as hypothesized, and is highly significant. This finding holds in 1954 as well as 1980. Again we see the benefit of using the information in the constraint of mean-variance optimization.

HYPOTHESIS 7: THE SUPPLY OF FEDERAL BONDS HAS A POSITIVE EFFECT ON THE EXPECTED RELATIVE RETURN ON EQUITIES.

This is the derivative that is most easily associated with the controversy surrounding recent government deficits and the possibility of portfolio crowding out. The unconstrained OLS estimates show positive effects but they are not significant at the 95 or 90 per cent levels. In this case the constrained MLE estimates are quite different. The point estimates of the effect are negative, indicating, not portfolio crowding out, but portfolio crowding in. This finding may appear surprising, but it is consistent with the fact that equities are not substitutes, but complements for government bonds in our estimates. To see this one must look at the preinverted form $B = [\rho\Omega]^{-1}$ reported in Table 3. The demand for government bonds is seen to be a positive function of the expected return on equities. The complementarity of these two assets in turn follows from the fact that the rate of return on government bonds has a much lower correlation with the rate of return on equities than with the rates of return on most of the other assets. After all, why should investors treat long-term obligations of the government so very differently from short-term obligations (money and Treasury bills), which we found in Hypothesis 6 to have a negative effect?

The effect is not significantly less than zero at the 95 or 90 per cent levels. So we should probably describe the finding as a failure to reject the absence of any effect, leaving prominent the possibility of insufficient power in the test. On the other hand, if one wanted to describe it more aggressively as a rejection of portfolio crowding out, one could draw some slight support from the fact that the effect is significantly negative at the 75 per cent level, as of 1954. Perhaps it would be best to describe the finding by arguing that evidently we are not far from the borderline case in which we can ignore any portfolio effects of debt-financed government deficits on the expected return to capital.

HYPOTHESIS 8: THE SUPPLY OF FEDERAL BONDS HAS A POSITIVE EFFECT ON THE EXPECTED RELATIVE RETURN ON FEDERAL BONDS.

As with Hypothesis 5, a positive own-derivative might seem assured a priori.

But it is worth recalling at this juncture the Ricardo-Barro proposition that government debt has no effects because there are implied future tax liabilities that offset it.[14] It is not a true "outside asset." (The same would be true of the debt of state and local governments, but perhaps more so because it is clearer that they have to pay off their debt eventually. Corporate debt is different because, with a given real capital stock, every dollar of corporate debt reduces the equity of the firm by one dollar.) The proposition is usually evaluated in an intergenerational or intertemporal framework. One could try to address it in the present one-period framework. Under the assumption of rational expectations, any irrelevant variable that is thrown in to equation (6), so long as it is known at time t and so is uncorrelated with the expectational error, should show up with a zero coefficient. This is true of lagged expectational errors, any VAR process, the forecasts of any model, the outstanding quantity of IOU's between citizens, and the outstanding number of bottle caps produced. According to the Ricardo-Barro proposition, it is also true of the outstanding value of the national debt. If the debt showed up with significant non-zero coefficients, that would constitute a rejection of the hypothesis. This is not to say that the proposition puts any restrictions on the Ω matrix. Even assuming government bonds are not true outside assets, if their return is for whatever reason correlated with the return on those that <u>are</u> true outside assets, then they will have to pay a risk premium like the others. To make an analogy, if one makes a private bet on the outcome of a football game one can in theory expect to get fair odds, but if one makes a private bet on the fate of the Dow Jones Industrial Average, one cannot in theory expect to get fair odds because the risk is not diversifiable. Only if the risk in federal bonds is completely diversifiable will they pay no risk premium (i.e. their expected return will be the same as the return on a risk-free asset, if there is one, say Treasury bills in the absence of inflation risk).

But even if they pay a risk premium, under the null hypothesis it would not be affected by the supplv of government bonds.

We cannot test the Ricardo-Barro proposition properly from our estimation results because federal bonds F appears in equation (6) not just as an extra, potentially irrelevant, "thrown-in" variable, but also as a component of total wealth W in the denominator of each of the portfolio shares. Its rate of return also appears as an equation of its own. To test the proposition properly we might exclude F from the list of assets. (One could think of it as adding a seventh asset representing the present discounted value of households' future tax liabilities, in theory equal to the negative of F) . Then we could add F as an additional variable in each of the four remaining equations; the null hypothesis would be zero coefficients. But under the alternative hypothesis that government bonds are in reality outside assets, such equations would not be properly specified.

In any case, in the six-asset model the derivative of Er^F with respect to F appears of the wrong sign and insignificant when estimated by unconstrained OLS, a symptom of the negative value estimated for b_{FF} in Table 1. It is positive as hypothesized but insignificant, except at the 75% level, when estimated by constrained MLE. One cannot claim much evidence on the Ricardo-Barro proposition from these results.

HYPOTHESIS 9: THE SUPPLY OF FEDERAL BONDS HAS A POSITIVE EFFECT ON THE EXPECTED RELATIVE RETURN ON CORPORATE BONDS

Since much business fixed investment is financed by the issue of corporate debt rather than equity, this proposition may be as relevant to the crowding out issue as Hypothesis 7. When we use the unconstrained OLS estimates we get an (insignificant) apparent negative effect, attributable to the estimated negative value for b_{CF}. When we impose the optimization constraint the MLE estimate becomes positive as we would expect: federal bonds and corporate bonds are substitutes because their returns are highly correlated. However we cannot reject the hypothesis of a zero effect.

6. CONCLUSION

This paper has introduced a MLE technique to obtain the most efficient estimates of the parameters in investors' asset demand functions of the portfolio-balance type. The technique itself may be as important as the specific results obtained. It dominates previous OLS attempts to relate asset supplies to rates of return because it brings more information to bear on the question: the information that the parameters are not determined arbitrarily but rather depend on the variances and covariances of real returns, assuming investors optimize with respect to the mean and variance of real wealth. The technique dominates previous estimates of the optimal portfolio in that it allows expected returns to vary freely, rather than assuming them measurable by a constant sample mean or by a slowly-moving ARIMA process.

It might be objected that the assumption of constant expected returns (first moments) is no worse than the assumption of constant variances and covariances (second moments) which _is_ maintained in this paper. It is certainly true that parameters like the variances in our asset demand functions can change over time. This is the essence of the famous "Lucas critique." One could split up the sample period to see if the parameters shifted, for example when the Federal Reserve Board switched from a policy of targetting the interest rate to a policy of targetting the money supply. One could even allow the variances to change gradually over time as in Rob Engel's "ARCH" model. But this paper is written under the supposition that fluctuations in expected returns are more of a problem than fluctuations in variances. After all, the former are the variables in the asset demand functions, and the latter are the parameters. If we did not think that the expected returns varied more than the parameters, we would not call them "variables." Allowing expected returns to vary was first priority. Allowing the parameters to vary is a subject for future research.

Some specific results of interest have been obtained. In general, there is little justification for aggregation beyond what has already gone into the six assets used here, i.e. the assets are not perfect substitutes for each other. Indeed some pairs of assets are not substitutes at all, but rather are complements. A particularly important finding is a rejection of perfect substitutability between long-term bonds and short-term bills. Evidently long-term bonds pay an expected return that differs from the expected short-term returns by a risk-premium. Nor is the premium constant, as in the theory of "normal backwardation" in its simplest form. It is indeed possible that recent debt-financed federal deficits, or the fear of future debt-financed deficits, have driven up long-term interest rates.[15]

The benefits of using the MLE technique are seen from the test results for the portfolio crowding out hypotheses. In the cases where the sign of the effect seemed clear a priori, Hypotheses 5, 6, 8 and 9, the results of the MLE technique is much more in conformity than the results of OLS. In the one case in which the sign is a subject of controversy (Hypothesis 7, the effect of government debt on the required expected relative return on equity), the MLE technique changes the point estimate from crowding out to crowding in. While the degree of portfolio crowding-in is not significantly different from zero, a 95 per cent confidence interval would exclude all but a small degree of crowding out (relative to some of the other effects). Federal debt and equities are not close substitutes in investors portfolios because their returns are not highly correlated.

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FOOTNOTES

1. The simultaneous equation method has been used by B. Friedman (1977), Roley (1982), and Masson (1978), among others.

2. Examples of regressions in inverted form are Fair and Malkiel (1971) and Modigliani and Sutch (1966, 1967).

3. Note that this is not the same as the stronger proposition that the ex post returns on the two assets are always the same. Their ex post returns can have independent components, as long as these components are uncorrelated with the returns on the market portfolio. If the individual asset is a small proportion of the market portfolio, e.g. an individual equity or bond, then that component of its return that is uncorrelated with the returns on other assets is diversifiable and will have no effect on demand for the asset.

4. Brainard and Tobin (1968) impose a priori values on the parameters outright. All these papers run simulations on the chosen parameter values to see the effects of changes in government debt, etc. They also emphasize a low degree of aggregation: separate asset-demand functions are estimated for households, banks, etc.

5. The choice to express returns relative to a numeraire is not restrictive. We could generalize (1) slightly to

$$x_t = A + \tilde{B}E \begin{bmatrix} r_{t+1} \\ \dots \\ r^d_{t+1} \end{bmatrix}$$

where B is $G - 1$ by G . Then when we invert

Appendix 1

In this appendix we derive in discrete time the correct form for the asset-demands of an investor who maximizes a function of the mean and variance of his end-of-period real wealth.

Let W_t be real wealth. The investor must choose the vector of portfolio shares x_t that he wishes to allocate to the various assets. End-of-period real wealth will be given by:

$$W_{t+1} = W_t[x_t' r_{t+1} + 1 + r^D_{t+1}] \; , \tag{A1}$$

where r_{t+1} is defined as the vector of returns on the 5 assets relative to the numeraire asset (deposits).

The expected value and variance of end-of-period wealth (A1), conditional on current information are as follows:

$$EW_{t+1} = W_t[x_t' Er_{t+1} + 1 + Er^D_{t+1}]$$

$$VW_{t+1} = W_t^2[x_t'\Omega x_t + Vr^D_{t+1} + 2x_t' \operatorname{Cov}(r_{t+1}, r^D_{t+1})] \; ,$$

where we have defined the variance-covariance matrix of relative returns:

$$\Omega \equiv E(r_{t+1} - Er_{t+1})(r_{t+1} - Er_{t+1})' \quad .$$

The hypothesis is that investors maximize a function of the expected value and variance:

$$F[E(W_{t+1}), V(W_{t+1})] \; .$$

We differentiate with respect to x_t :

$$\frac{dF}{dx_t} = F_1 \frac{dEW_{t+1}}{dx_t} + F_2 \frac{dVW_{t+1}}{dx_t} = 0 \ .$$

$$F_1 W_t [Er_{t+1}] + F_2 W_t^2 [2\Omega x_t + 2 \ \mathrm{Cov}(r_{t+1}, \ r^D_{t+1})] = 0 \ .$$

We define the coefficient of relative risk-aversion $\rho \equiv -W_t 2F_2/F_1$, which is assumed constant. Then we have our result:

$$Er_{t+1} = \rho \ \mathrm{Cov}(r_{t+1}, \iota r^D_{t+1}) + \rho\Omega x_t \ . \tag{A2}$$

This is just equation (2) with B^{-1} constrained to be $\rho\Omega$, as claimed by equation (7) in the text. Combining with the rational expectations assumption (5) is another way to get equation (8), the equation estimated in the text. (There is also a constraint imposed on the intercept term A . But it is not convenient to impose this constraint in the econometrics. Nor do we need it, since the constraint on the coefficient matrix already gives us 25 overidentifying restrictions.)

For economic intuition, we can invert (A2) to solve for the portfolio shares, the form analogous to (1):

$$x_t = -\Omega^{-1} \ \mathrm{Cov}(r_{t+1}, \ r^D_{t+1}) + (\rho\Omega)^{-1} Er_{t+1} \ . \tag{A3}$$

The asset demands consist of two parts. The first term represents the "minimum-variance" portfolio, which the investor will hold if he is extremely risk-averse $(\rho = \infty)$. For example, suppose he views deposits as a safe asset, which requires that the inflation rate is nonstochastic. Then his minimum-variance portfolio is entirely in deposits: the 5 entries in x_t are all zero because the Cov in (A3) is zero. The second term represents the "speculative" portfolio. A higher expected return on a given asset induces investors to hold more of that asset than is in the minimum-variance portfolio, to an extent limited only by the degree of risk-aversion and the uncertainty of the return.

APPENDIX 2

Using the assumption of normally-distributed returns, the log likelihood function when no constraint is imposed on the coefficient matrix is

$$L = -\frac{5\tau}{2}\log 2\pi - \frac{\tau}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{\tau} \varepsilon'_{t+1}\Omega^{-1}\varepsilon_{t+1} , \qquad \text{(A4)}$$

where we know from equation (8) that $\varepsilon_{t+1} = r_{t+1} - c - B^{-1}x_t$.
The unconstrained MLE is simply the OLS estimates that we looked at in Table 1.

For the constrained MLE, we substitute $\rho\Omega$ for B^{-1} . Ω now appears in the likelihood function in two ways. To maximize, we differentiate. The derivatives with respect to the coefficient of risk-aversion and the intercept term are easy:

$$\begin{aligned}
\partial L/\partial\rho &= -\sum \varepsilon'_{t+1}\Omega^{-1}(\partial\varepsilon_{t+1}/\partial\rho) \\
&= -\sum \varepsilon'_{t+1}\Omega^{-1}(-\Omega x_t) \\
&= \sum \varepsilon'_{t+1}x_t \\
\partial L/\partial c &= -\sum \varepsilon'_{t+1}\Omega^{-1}(\partial\varepsilon_{t+1}/\partial c) \\
&= \sum \varepsilon'_{t+1}\Omega^{-1} .
\end{aligned}$$

The derivative with respect to the elements of the variance-covariance matrix is trickier. We use the two facts (from Theil (1971, pp. 31-32), equations (6-14) and (6-8), respectively):

$$\frac{\partial \log|\Omega|}{\partial \Omega} = \Omega^{-1} \quad \text{and} \quad \frac{\partial(\varepsilon'\Omega^{-1}\varepsilon)}{\partial \Omega^{-1}} = \varepsilon\varepsilon' \quad .$$

$$\partial L/\partial \Omega = -\frac{T}{2}\,\Omega^{-1'} - \frac{1}{2}\sum\,[-\Omega^{-1'}\varepsilon_{t+1}\varepsilon'_{t+1}\Omega^{-1'} + (\Omega^{-1} + \Omega^{-1'})\varepsilon_{t+1}(\partial\varepsilon_{t+1}/\partial\Omega)]$$

where $(\partial\varepsilon_{t+1}/\partial\Omega) = x'_t$. This much would be true for the derivatives with respect to <u>any</u> matrix "Ω." The fact that Ω is symmetric, and so contains only 15 independent parameters, means that all off-diagonal elements must be doubled. See the appendix to Frankel and Engel (1982). An equivalent approach is to work in terms of the 15 parameters in the Choleski factorization of Ω : the lower triangular S , such that $S'S = \Omega$. (This approach can be worthwhile if the MLE program is reluctant to converge when asked to work in terms of Ω . Note that $\partial\Omega/\partial S = 2S$.)

Setting the derivatives equal to zero gives first order conditions that characterize the MLE. However, due to nonlinearity they cannot be solved explicitly for the estimates of ρ , c , and Ω . The Berndt, Hall, Hall and Hausman (1974) algorithm uses the first derivatives to find the maximum of the likelihood function in non-linear models. For our problem, we modified a program written by Paul Ruud, based on this algorithm. As initial values for $\hat{\Omega}$ in the iteration, we used the simple variance-covariance matrix of the relative rates of return.

Appendix 3: Constructing the Test-Statistics

In this appendix we show the algebra of constructing the test-statistics relevant for the various hypotheses. We do the unconstrained OLS parameters first, and then the parameters that are constrained by MLE to be mean-variance optimizing.

Define β to be the parameters of B^{-1}, estimated unconstrained in Table 1, expressed as a vector of 25 elements:

$$\beta' = [b_{TT}b_{TF}b_{TS}b_{TC}b_{TK} \quad b_{FT}b_{FF}b_{FS}b_{FC}b_{FK} \quad b_{ST}b_{SF}b_{SS}b_{SC}b_{SK} \quad b_{CT}b_{CF}b_{CS}b_{CC}b_{CK}$$
$$b_{KT}b_{KF}b_{KS}b_{KC}b_{KK}]$$

Each of the hypotheses can be expressed as a set of linear constraints on the parameter vector:

$$R\,\beta = 0 \quad .$$

When we want to test the hypothesis that a particular subset of the assets are perfect substitutes, and one of those assets happens to be the numeraire asset, deposits D, the constraints are that each of the relevant elements is zero. For example, to test Hypothesis 1, that <u>all</u> assets are perfect substitutes, R_1 is the 25×25 identity matrix. To test Hypothesis 4, that deposits are perfect substitutes for all three bonds together, R_4 has 21 rows, each consisting only of a single non-zero element: a "one" in, respectively, columns 2-4, 6-20, and 22-24.

When we want to test the hypothesis that a subset of assets are perfect substitutes <u>not</u> including the numeraire asset, the constraints are that the relevant pairs of parameters are equal. To test Hypothesis 3, that corporate bonds are perfect substitutes for state and local bonds, the constraints are represented as follows:

$$
R_3 \equiv \begin{bmatrix}
00000 & 00000 & 10000 & -10000 & 00000 \\
00000 & 00000 & 01000 & 0-1000 & 00000 \\
00000 & 00000 & 00100 & 00-100 & 00000 \\
00000 & 00000 & 00010 & 000-10 & 00000 \\
00000 & 00000 & 00001 & 0000-1 & 00000 \\
001-10 & 00000 & 00000 & 00000 & 00000 \\
00000 & 001-10 & 00000 & 00000 & 00000 \\
00000 & 00000 & 001-10 & 00000 & 00000 \\
00000 & 00000 & 00000 & 00000 & 001-10
\end{bmatrix}
$$

To test Hypothesis 2, that the three bonds are perfect substitutes, the constraints are represented as follows:

$$
R_2 \equiv \begin{bmatrix}
 & & R_3 & & \\
00000 & 10000 & -10000 & 00000 & 00000 \\
00000 & 01000 & 0-1000 & 00000 & 00000 \\
00000 & 00100 & 00-100 & 00000 & 00000 \\
00000 & 00001 & 0000-1 & 00000 & 00000 \\
01-100 & 00000 & 00000 & 00000 & 00000 \\
00000 & 00000 & 00000 & 00000 & 01-100
\end{bmatrix}
$$

When we want to test whether the derivative of an expected relative return with respect to an asset supply is positive or negative, there is a single constraint on the parameters. The derivatives are given by $R\,\beta$. Three effects on the expected relative return on equity are given first; they are the derivatives, respectively, with respect to equity (hypothesized positive), deposits (hypothesized negative), and federal bonds (hypothesized positive if there is portfolio-crowding out).

Hypothesis 5 $\quad R_5 \equiv [00000 \quad 00000 \quad 00000 \quad 00000 \quad -x^T \; -x^F \; -x^S \; -x^C \; 1-x^K]$

Hypothesis 6 $\quad R_6 \equiv [00000 \quad 00000 \quad 00000 \quad 00000 \quad -x^T \; -x^F \; -x^S \; -x^C \; -x^K]$

Hypothesis 7 $\quad R_7 \equiv [00000 \quad 00000 \quad 00000 \quad 00000 \quad -x^T \; 1-x^F \; -x^S \; -x^C \; -x^K]$

Lastly we do the effects of the supply of federal bonds on the expected relative return of, respectively, federal bonds and corporate bonds.

Hypothesis 8 $\quad R_8 \equiv [00000 \quad -x^T \; 1-x^F \; -x^S \; -x^C \; -x^K \quad 00000 \quad 00000 \quad 00000]$

Hypothesis 9 $\quad R_9 \equiv [00000 \quad 00000 \quad 00000 \quad -x^T \; 1-x^F \; -x^S \; -x^C \; -x^K \quad 00000]$

We turn now to the representation of the last five hypotheses when imposed on parameters that have been estimated by MLE subject to the constraint of mean-variance optimization. The only difference to the R matrices is that because Ω is symmetric, there are fewer free parameters involved. We define Σ to be the parameters of Ω, estimated in Table 4, expressed as a vector of 15 elements:

$$\Sigma' \equiv [\; \Omega_{TT} \;|\; \Omega_{TF} \quad \Omega_{FF} \;|\; \Omega_{TS} \quad \Omega_{FS} \quad \Omega_{SS} \;|$$
$$\Omega_{TC} \quad \Omega_{FC} \quad \Omega_{SC} \quad \Omega_{CC} \;|\; \Omega_{TK} \quad \Omega_{FK} \quad \Omega_{SK} \quad \Omega_{CK} \quad \Omega_{KK}]$$

$$\hat{\Sigma}' = [.00050 \;|\; -.00028 \quad .00421 \;|\; -.00008 \quad .00499 \quad .00845 \;|\; .00007$$
$$.00007 \quad .00341 \quad .00504 \quad .00374 \;|\; -.00043 \quad .00025 \quad .00030 \quad .00093 \quad .01445]$$

The parameter estimates are exactly half of those in Table 4 because $B^{-1} = \rho\Omega$ and $\rho = 2$. We will express the hypotheses as $R\,\Sigma = 0$; the equation is exactly equivalent to $R[\rho\Sigma] = 0$ (for any non-zero ρ) .

The "crowding out" constraints are represented as follows:

Hypothesis 5 $\quad R_5 = \left[\begin{array}{c|cc|ccc|cccc|ccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x^T & -x^F & -x^S & -x^C & 1-x^K \end{array}\right]$

Hypothesis 6 $\quad R_6 = \left[\begin{array}{c|cc|ccc|cccc|ccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x^T & -x^F & -x^S & -x^C & -x^K \end{array}\right]$

Hypothesis 7 $\quad R_7 = \left[\begin{array}{c|cc|ccc|cccc|ccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x^T & 1-x^F & -x^S & -x^C & -x^K \end{array}\right]$

Hypothesis 8 $\quad R_8 = \left[\begin{array}{c|cc|ccc|cccc|ccccc} 0 & -x^T & 1-x^F & 0 & -x^S & 0 & 0 & -x^C & 0 & 0 & 0 & -x^K & 0 & 0 & 0 \end{array}\right]$

Hypothesis 9 $\quad R_9 = \left[\begin{array}{c|cc|ccc|cccc|ccccc} 0 & 0 & 0 & 0 & 0 & 0 & -x^T & 1-x^F & -x^S & -x^C & 0 & 0 & 0 & -x^K & 0 \end{array}\right]$

In the unconstrained OLS case, to test a hypothesis $R\beta = 0$ we compute a Wald test-statistic T that is asymptotically distributed χ^2 with q degrees of freedom, where q is the number of rows in R :

$$T = (R\hat{\beta})' \; [\hat{V}(R\hat{\beta})]^{-1}(R\hat{\beta})$$
$$= (R\hat{\beta})'[R\hat{V}(\hat{\beta})R']^{-1}(R\hat{\beta})$$

where $\hat{V}(\;)$ is an estimated variance-covariance matrix. (Note that when $q = 1$, in Hypotheses 5 through 9, the χ^2 statistic is simply the square of a t-statistic. Although the rejection criterion is the same, we maintain the χ^2 formulation for congruence with the other tests.) The equation-by-equation OLS can be thought of as a single "stacked" OLS regression where the righthand-side variables are repeated five times: $X \equiv I_5 \otimes x$ where I_5 is the 5 by 5 identity matrix, x is the τ by 5 matrix representing the time series observations of x_t , and $\otimes$ is the Kronecker product. In a GLS regression the variance-covariance matrix is given by

$$\hat{V}(\hat{\beta}) = [X'(\hat{\Omega}^{-1} \otimes I_\tau)X]^{-1} \; .$$

$\hat{\Omega}$ is here simply the variance-covariance matrix of the residuals across the five equations. (Recall that we are assuming that the variances and covariances are constant over time, and that there is no serial correlation, the latter an implication of the rational expectations assumption.) Ω has not yet taken on the double significance that it does under MLE. With identical righthand-side variables, this simplifies, as shown by Henri Theil, Principles of Econometrics, 1971 (John Wiley: N.Y.), p. 308-310:

$$\hat{V}(\hat{\beta}) = \hat{\Omega} \otimes (x'x)^{-1} \; .$$

The two components of the matrix $\hat{V}(\hat{\beta})$ are printed out below. The diagonal elements are the squares of the standard errors reported under the parameter estimates in Table 1 (but for the fact that the MLE variances divide by τ observations while the OLS variances divide by τ-6 observations).

Components of Variance-Covariance Matrix
of OLS Parameter Estimates $\hat{V}(\hat{\beta}) = \hat{\Omega} \otimes (x'x)^{-1}$

$\hat{\Omega}$ =

	T	F	S	C	K
T	.000597				
F	-.000041	.004954		(symmetric)	
S	.000153	.005653	.009024		
C	.000293	.004052	.005609	.004310	
K	-.001024	-.001438	-.001292	-.000596	.018195

$(x'x)^{-1}$

	1	T	F	S	C	K
1	541.44					
T	-781.44	1142.22				
F	277.20	-359.08	1022.61		(symmetric)	
S	-172.72	665.78	225.04	24511.7		
C	-1323.56	1562.07	-2946.80	-17506.46	26116.52	
K	-578.21	822.45	-286.81	-150.03	1359.08	640.94

($\log|\hat{\Omega}| = -30.5548$.)

In the constrained MLE case, the χ^2 test-statistic is defined similarly:

$$T = (R\hat{\Sigma})'[R\hat{V}(\hat{\Sigma})R']^{-1}(R\hat{\Sigma}) \quad .$$

The variance-covariance matrix $V(\hat{\Sigma})$ of the parameter estimates cannot be estimated directly as in the OLS case. It is, rather, the inverse of the information matrix, which consists of expected values of the second-order derivatives of the log-likelihood function.

The matrix is printed out below. Note that the diagonal elements are approximately the squares of half of the standard errors of the parameters $(B^{-1} = \rho\Omega$, with $\rho = 2)$ reported in Table 4.

Variance-Covariance Matrix of MLE Parameter Estimates $V(\Omega)$

COVARIANCE MATRIX

0.169838D-06				
-0.214301D-06	0.711924D-06			
-0.266656D-06	0.441387D-06	0.608492D-05		
-0.169568D-06	0.817859D-06	0.678068D-06	0.135559D-05	
-0.432890D-06	0.967018D-06	0.752999D-05	0.957633D-06	0.105450D-04
-0.420395D-06	0.968090D-06	0.725914D-05	0.149076D-05	0.993018D-05
-0.111339D-06	0.570997D-06	0.276450D-06	0.763868D-06	0.658236D-06
-0.164931D-06	0.479911D-06	0.453523D-05	0.732507D-06	0.579967D-05
-0.276289D-06	0.847063D-06	0.479162D-05	0.131797D-05	0.639236D-05
-0.166325D-06	0.673993D-06	0.330134D-05	0.954788D-06	0.446442D-05
-0.479392D-06	0.117704D-05	0.123075D-06	0.134050D-05	0.726013D-06
0.982834D-06	-0.303770D-05	0.700811D-07	-0.330372D-05	-0.152647D-05
0.247568D-06	-0.128062D-05	0.261115D-05	-0.190792D-05	0.346505D-05
0.434222D-06	-0.135999D-05	0.463098D-06	-0.162628D-05	-0.163030D-07
0.114400D-06	-0.575721D-06	0.373625D-05	-0.114949D-05	0.465170D-05
Ω_{TT}	Ω_{TF}	Ω_{FF}	Ω_{TS}	Ω_{FS}

0.117904D-04				
0.691377D-06	0.537974D-06			
0.619683D-05	0.314554D-06	0.409471D-05		
0.829064D-05	0.521225D-06	0.516920D-05	0.801061D-05	
0.567339D-05	0.417754D-06	0.386644D-05	0.600764D-05	0.474732D-05
0.315428D-06	0.112165D-05	-0.408028D-06	-0.723921D-06	-0.700943D-06
-0.558325D-06	-0.276605D-05	0.482410D-06	0.479470D-06	0.373386D-06
0.321919D-05	-0.155114D-05	0.226363D-05	0.303642D-05	0.209598D-05
0.315095D-06	-0.137967D-05	0.896559D-06	0.156545D-05	0.136039D-05
0.464246D-05	-0.121719D-05	0.487653D-05	0.803202D-05	0.666579D-05
Ω_{SS}	Ω_{TC}	Ω_{FC}	Ω_{SC}	Ω_{CC}

0.476700D-05				
-0.991576D-05	0.251411D-04			
-0.623295D-05	0.177973D-04	0.206999D-04		
-0.502188D-05	0.135977D-04	0.127697D-04	0.995820D-05	
-0.679025D-05	0.147557D-04	0.195647D-04	0.145651D-04	0.452727D-04
Ω_{TK}	Ω_{FK}	Ω_{SK}	Ω_{CK}	Ω_{KK}

APPENDIX 4

DATA

The main source for data on supplies of nine assets held by households was the Federal Reserve Board's Balance Sheets for the U.S. Economy (October 1981) Table 702. This source was used in place of the Fed's Flow of Funds Accounts, Assets and Liabilities Outstanding, to which it is closely related, because only the Balance Sheets include data for tangible assets, i.e. real estate and consumer durables (see page iii of the Flow of Funds for an explanation). The variables used in the econometrics are shares of wealth, the supply of the asset in question divided by the sum of all nine asset supplies.

The asset supplies were taken from the Balance Sheets as follows. Real estate is line 1 (total tangible assets) minus line 7 (consumer durables).[1] Consumer durables is line 7.[2] Open market paper is line 25. Short-term U.S. government securities are line 20 [not available before 1951]. Deposits is the sum of lines 13, checkable deposits and currency, 14, small time and savings deposits, 15, money market fund shares, and 16, large time deposits. Long-term federal debt is line 18 (U.S. government securities) minus line 20. State and local debt is line 23. Private bonds are line 24 (corporate and foreign bonds) plus line 26 (mortgages held).[3] Finally, equities are line 27 (corporate equities) plus line 32 (noncorporate business equity).[4]

For three of the asset supplies--long-term federal debt, state and local bonds, and private bonds--the numbers represent book value and must be multiplied by some measure of current market prices to get the correct measure of market value. The very large decline in prices of bonds over the postwar period make this correction a crucial one. (Equities and tangible assets are already measured at market value, while capital gains and losses are irrelevant for the three short-term assets.) Measures of the current market bond prices are reported by Standard and Poor's Trade and Security Statistics Security Price Index Record (1982): page 235 for U.S. government bond prices, 233 for municipal bond prices, and 231 for high grade corporate bond prices. Standard and Poor's computes the price indexes from yield data, assuming a 3% coupon with 15 years to maturity for the federal bonds and a 4% coupon with 20 years to maturity for the other two.[5]

Among the rates of return (all in level form for this paper) the two most problematical are those on real estate and durables, taken here as the percentage change in price indices reported in the Economic Report of the President 1982: the home purchase component of the CPI (p. 292) and the durable goods personal consumption expenditure component of the GNP deflator (p. 236). There exist better measures of house prices, and unpublished estimates of imputed service returns on housing and durables, but they are not available for the entire sample period. When the two tangibles are aggregated, we use real estate appreciation as the return.

The short-term assets are straight-forward. The rate of return on open market paper is the interest rate on commercial paper from the Federal Reserve Board: Banking and Monetary Statistics 1941-1970, table 12.5, Annual Statistical Digest 1970-79, table 22A, and ASD 1980, table 25A. The rate of return on short-term government securities is the treasury bill rate: 9-12 month issues (certificates of indebtedness and selected note and bond issues; the 1-year bill market yield rate is not available before 1960) from BMS 1941-1970, and the 1-year bill secondary market from ASD 1970-1979, table 22A, and ASD 1980, table 25A. The rate of return on deposits is the rate on 90-day bankers' acceptances from BMS 1941-1970, table 12.5, ASD 1970-1979, table 22A, and ASD 1980, table 25A. Alternatives such as the return on money market funds might be theoretically preferable but are not available for the early part of the sample period. Note that in aggregating non-interest paying money together with interest-paying accounts, we are assuming that the former performs an implicit liquidity service that brings its return up to the explicit return of the latter. When the three short-term assets are aggregated, we use the Treasury bill rate as the return.

Each of the long-term assets entails a yield plus capital gains. For each of the three kinds of bonds, capital gains are percentage change in the same bond prices from Standard and Poor's Trade and Securities Statistics that were discussed above. The yields are from the same source: respectively, the median yield to maturity of a number of government bonds restricted to those issues with more than ten years to maturity, p. 234, an arithmetic average of the yield to maturity of fifteen high grade municipal bonds, p. 232, and an average of the AAA Industrial and Utility bonds, p. 219. (The yields are also available from the Fed sources: BMS 1941-1970, table 12.12, ASD 1970-1979, table 22A and ASD 1980, table 25A.) For equities, capital gains are percentage change in Stanford and Poor's index of common stock prices from BMS 1941-1970, table 12.16, ASD 1970-1979, table 22A, and ASD 1980, table 26A. To capital gains we add the dividend price ratio on common stock, from BMS 1941-1970, table 12.19, ASD 1970-79, table 22A, and ASD 1980, table 25A.

The foregoing are all nominal returns. To convert to real returns when computing percentage returns on levels, we use the percentage change in the CPI, from the Economic Report of the President 1982. To be precise we divide one plus the nominal return by one plus the inflation rate. Subtracting the inflation rate from the nominal return would give approximately the same answer, and when we computed real returns relative to the numeraire asset the two inflation rates would conveniently drop out, but this answer would differ from the correct one by a convexity term.

Absent from the calculations is any allowance for differences in tax treatment. In particular, the returns on state and local bonds, and to some extent on tangibles, are here understated relative to the other assets because they are tax-free. The unconstrained constant term that we allow for in the econometrics should capture most of this effect (and any other constant omitted factors such as the service return from tangibles, as well). But it would be desirable to compute after-tax real returns instead.

1. An alternative here is to subtract lines 38 and 39, mortgages owed by households, viewing them as a liability that is institutionally tied to the real estate asset. One cannot explain otherwise households' decision to hold on net a negative quantity of mortgages on risk-return considerations, as the mortgage rate is higher than that on other bonds.
2. An alternative here is to subtract lines 40 and 41, consumer credit, viewing it as a liability that is tied to the durables asset, for the same reason as in the previous footnote.
3. An alternative here is to add in also lines 30 (life insurance reserves), 31 (pension fund reserves) and 34 (miscellaneous assets). These cannot be treated as separate assets because their rates of return are not available, but it is desirable to have all forms of wealth included somewhere, and they fit into the category of private bonds better than anywhere else.
4. An alternative here is to subtract the difference of lines 44 and 33, representing net security credit, viewing it as a liability that is tied to the equity asset.
5. These same bond prices were reported in the Federal Reserve Board's <u>Banking and Monetary Statistics 1941-1970</u>. They have been discontinued apparently because the Capital Markets Section at the Federal Reserve Board feels that dispersion in the coupon rate and shifts in the term structure make the aggregation of all long-term bonds no longer possible. But some correction for the market price is clearly preferable to none.