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# ABSTRACT

The allowance for low-probability disasters, suggested by Rietz (1988), explains a lot of puzzles related to asset returns and consumption. These puzzles include the high equity premium, the low risk-free rate, the volatility of stock returns, and the low values of typical macro-econometric estimates of the intertemporal elasticity of substitution for consumption. Another mystery that may be resolved is why expected real interest rates were low in the United States during major wars, such as World War II. This resolution works even though price-earnings ratios tended also to be low during the wars. This approach achieves these explanations while maintaining the tractable framework of a representative agent, time-additive and iso-elastic preferences, complete markets, and i.i.d. shocks to productivity growth. Perhaps just as puzzling as the high equity premium is why Rietz's framework has not been taken more seriously by researchers in macroeconomics and finance.

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The Mehra-Prescott (1985) article on the equity risk-premium puzzle has received a great deal of attention, as indicated by its 597 citations through 2004. An article published three years later by Rietz (1988) purported to solve the puzzle by bringing in the potential for low-probability disasters. I think that Rietz's basic reasoning is correct, but the profession seems to think differently, as gauged by his much smaller number of citations (49) and the continued attempts to find more and more complicated ways to resolve the equity-premium puzzle.

In this study, I extend Rietz's analysis and argue that it provides a plausible resolution of the equity-premium and related puzzles. Included in these other puzzles are the low risk-free rate, the volatility of stock returns, and the low values of typical macro-econometric estimates of the intertemporal elasticity of substitution for consumption. Another mystery that may be resolved is why expected real interest rates were low in the United States during major wars, such as World War II. This resolution works even though price-earnings ratios tended also to be low during the wars.

## I. Representative-Agent Model of Asset Pricing

### A. Setup of the model

Following Mehra and Prescott (1985), I use a version of Lucas's (1978) representative-agent, fruit-tree model of asset pricing with exogenous, stochastic production. Output of fruit in each period is $A_t$. Since the economy is closed and all output is consumed, consumption, $C_t$, equals $A_t$.

One form of asset in period $t$ is a claim on period $t+1$'s output, $A_{t+1}$. If the period $t$ price of this risky asset in units of period $t$'s fruit is denoted by $P_{t1}$, the one-period gross rate of return on the asset is

$$(1) \qquad\qquad R_{t1}^r = A_{t+1}/P_{t1}.$$

I consider later claims in period $t$ on output in periods $t+2$, $t+3$, and so on. An equity share in the fruit-tree is a claim on all of these future outputs (dividends). I assume in the main analysis that property rights are secure, so that an equity claim ensures ownership over next period's fruit, $A_{t+1}$, with probability one.

There is also a risk-free asset, on which the gross rate of return from period $t$ to period $t+1$ is denoted by $R_{t1}^f$. Risk-free rates of return set in period $t$ for future periods are denoted by $R_{t2}^f$, $R_{t3}^f$, and so on. As with equity claims, I assume in the main analysis that property rights are secure, so that the risk-free asset really is risk-free.

The representative consumer maximizes a time-additive utility function with iso-elastic utility:

$$(2) \qquad\qquad U_t = E_t \sum_{i=0}^{\infty} [e^{-\rho i} \cdot u(C_{t+i})],$$

where

$$(3) \qquad\qquad u(C) = (C^{1-\theta} - 1)/(1 - \theta).$$

In these expressions, $\rho \geq 0$ is the rate of time preference and $\theta > 0$ is the magnitude of the elasticity of marginal utility (and the coefficient of relative risk aversion). The intertemporal elasticity of substitution for consumption is $1/\theta$.

The usual first-order optimization condition implies

$$(4) \qquad\qquad u'(C_t) = e^{-\rho} \cdot E_t[u'(C_{t+1}) \cdot R_{t1}],$$

where $R_{t1}$ is the one-period gross rate of return on any asset traded at date $t$. Using

Eq. (4), substituting C = A for periods $t$ and $t+1$, and replacing $R_{t1}$ by the formula for $R_{t1}^r$

in Eq. (1) gives

$$(5) \qquad (A_t)^{-\theta} = e^{-\rho} \cdot (1/P_{t1}) \cdot E_t[(A_{t+1})^{1-\theta}].$$

Therefore, the price of the one-period risky asset is

$$(6) \qquad P_{t1} = e^{-\rho} \cdot (A_t)^{\theta} \cdot E_t[(A_{t+1})^{1-\theta}].$$

If we instead replace $R_{t1}$ by the one-period risk-free rate, $R_{t1}^f$, we get

$$(7) \qquad R_{t1}^f = e^{\rho} \cdot (A_t)^{-\theta} / E_t[(A_{t+1})^{-\theta}].$$

I assume that the log of output (productivity) evolves as a random walk with drift,

$$(8) \qquad \log(A_{t+1}) = \log(A_t) + \gamma + u_{t+1} + v_{t+1} + w_{t+1}.$$

where $\gamma \geq 0$. The random term $u_{t+1}$ is assumed to be i.i.d. normal with mean 0 and

variance $\sigma^2$. This term will give results similar to those of Mehra and Prescott. I assume

that $\gamma$ and $\sigma$ are known. Weitzman (2005) argues that learning about $\sigma$ is important for

asset pricing—this idea is not pursued here. However, Weitzman's learning model

generates "fat tails" that have effects analogous to the low-probability disasters

considered by Rietz (1988) and in the present model.

The other random terms, $v_{t+1}$ and $w_{t+1}$, pick up low-probability disasters. Two

types of disasters are distinguished. In the first, v-type, output contracts sharply but

property rights are respected and the world goes on. The Great Depression is a prototype

v-event.[1] The second, w-type, can be thought of as the end of the world. Prototypes are

all-out nuclear war and an asteroid collision. In the representative-agent framework,

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[1] In a seminar discussion of his paper, Weitzman (2005) suggested that a tsunami in Japan would be an analogous event.

generalized default associated with loss of property rights on assets is equivalent in terms of asset pricing to the end of the world.

I assume that the probabilities of the two types of disasters are independent and also independent of $u_{t+1}$. The probability of a v-type disaster is the known amount $p \geq 0$ per unit of time. (The probability of more than one disaster in a period is assumed to be small enough to neglect.) If a disaster occurs, the log of output contracts by the known amount $b \geq 0$. The idea is that the probability of disaster in a period is small but b is large. The distribution of $v_{t+1}$ is

$$\text{probability } e^{-p}: \qquad v_{t+1} = 0,$$

$$\text{probability } 1 - e^{-p}: \qquad v_{t+1} = -b.$$

This specification creates negative skewness in the distribution of $A_{t+1}$, because disasters are not offset in a probabilistic sense by bonanzas. However, the asset-pricing results are similar for a symmetric specification in which favorable events of size b also occur with probability p. With diminishing marginal utility of consumption, bonanzas do not count nearly as much as disasters for the pricing of assets.

The probability of a w-type disaster—the end of the world—is the known constant $q \geq 0$ per unit of time. Hence, the world exists after one period with probability $e^{-q}$ and does not exist with probability $1 - e^{-q}$. When viewed in terms of general loss of property rights, the probability q refers to 100% default. However, the model turns out to be linear in the sense that a 1% chance of 100% default has the same effects on asset pricing as a 2% chance of 50% default. (This linearity does not apply to p.) An important assumption is that the probability of default is independent of $A_t$, notably on the occurrence of v-type disasters. In some contexts, such as wartime, it would be

preferable to assume that default is more likely when a v-type disaster occurs. In this case, the "risk-free" asset looks more like the equity asset.

### B. Disasters in the United States and other countries

From a U.S. perspective, a consideration of economic disaster immediately brings to mind the Great Depression. The Depression fits cleanly with v-type events in the sense that the economic decline was large and did not trigger default on assets such as government bills.[2] However, from the standpoint of sizes of economic disaster in many OECD countries in the 20th century, war devastation was more important than the Great Depression. For the United States, at least since 1815 and aside from the Confederacy during the Civil War, wars did not involve massive destruction of domestic production capacity. In fact, the main wars, especially World War II, were times of robust economic activity. The history for many other OECD countries is very different, notably for World Wars I and II and their aftermaths.

Part A of Table 1 shows all episodes of 15% or greater decline in real per capita GDP in the 20th century for 20 advanced countries covered over a long period by Maddison (2003). This group comprises the major economies of Western Europe plus Australia, Japan, New Zealand, and the United States—all members of the OECD since the 1960s. I consider not just one-year changes in real per capita GDP but rather declines that applied to consecutive years, such as 1939-45 for some countries during World War II.[3] Nine of the contractions are associated with World War II, eight with World

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[2] The rise in the gold price and abrogation of gold clauses in bond contracts may be viewed as forms of default—see McCulloch (1980).

[3] Kehoe and Prescott (2002) extend the concept of a great depression to cases where the growth rate of real per capita GDP falls well below the historical average for an extended period. Thus, they classify as

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War I, eight with the Great Depression, and two with the Spanish Civil War.[4]  There are

also four aftermaths of major wars—three following World War I and one after World

War II.  However, these experiences involved demobilizations with substantial declines

in government purchases, work effort, and capital utilization and—with the exception of

Canada after World War I—did not feature substantial decreases in consumption.[5]

Therefore, except for Canada in 1917-21, these cases are not applicable to my analysis.

Although 15% or greater declines in real per capita GDP are rare events, only 2 of

the 20 OECD countries lack any such events in the 20th century, and these came close

(see the notes to Table 1).  The striking observation from part A of Table 1 is the

dramatic decreases in real per capita GDP during the major wars and the Great

Depression.  The falls during World War II ranged between 45% and 64% for Italy,

France, Japan, the Netherlands, Austria, Greece, and Germany.  Moreover, the deviations

from trend real per capita GDP (which would have risen over the several years of war)

were even greater.  In addition, the sharp expansions of government purchases during the

wars suggest that consumption fell proportionately by even more than GDP (although

investment likely declined sharply and net imports may have increased in some cases).

Part B of Table 1 shows declines of 15% or more in real per capita GDP for

additional countries—eight in Latin America and seven in Asia—that have nearly

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depressions the periods of slow economic growth in New Zealand and Switzerland from the 1970s to the 1990s.  Hayashi and Prescott (2002) take a similar approach to Japan in the 1990s.  These experiences could be brought into the present framework by allowing for a small probability of a substantial cutback in the productivity growth parameter, $\gamma$.

[4] My conjecture, thus far unconfirmed, is that the sharp fall in output in Portugal in 1935-36 reflected spillovers from the Spanish Civil War.  Per capita GDP happened also to decline in Portugal in 1934-35 (by 6%).

[5] For the United States, data from Bureau of Economic Analysis show that real consumer expenditure did not decline from 1944 to 1947.  The same holds for real consumer expenditure from 1918 to 1921 in the United Kingdom (see Feinstein [1972]) and Italy (see Rossi, Sorgato, and Toniolo [1993]). Long-term national-accounts data for Canada from Urquhart (1993) do not break down GDP into expenditure components.  However, my estimate from Urquhart's data is that real consumer expenditure per person fell by about 18% from 1917 to 1921, compared to the decline by 30% in real GDP per person in Table 1.

continuous data from Maddison (2003) back at least before World War I.  These data

show ten sharp economic contractions in the post-World War II period (eight in Latin

America), eight during the Great Depression, eight in World War II (six in Asia), and five

around World War I.[6]  Of the 15 countries considered, 3 lack 15% events (see the notes

to the table).

The kinds of episodes shown in Table 1 are v-type events if we can maintain the

assumption of non-default on the risk-free asset.  In fact, outright default does not typify

the group of 20 advanced economies considered in part A of the table—which notably

omits Czarist Russia and, from an earlier time, the American Confederacy.  For example,

France did not default after World War II on debts incurred by the Third Republic or the

Vichy government.  Similarly, Belgium and the Netherlands did not explicitly default

after World War II on government bills and bonds but did have forced conversions into

illiquid instruments.  The most common mechanism for partial default was depreciation

of the real value of nominal debt through (unanticipated?) increases in price levels.

These inflations occurred during and shortly after some of the wars.[7]  To the extent that

wartime tended to feature default on all forms of assets, we can treat wars as partly

[6] Data are available for a few additional countries starting in the 1920s and for many countries after World War II.  In terms of 15% or greater events, this extension adds 6 cases associated with the Great Depression (Costa Rica, Cuba, El Salvador, Guatemala, Honduras, and Nicaragua), 4 during World War II (Costa Rica, Guatemala, Burma, and China), 1 aftermath of World War II (Paraguay), and 30 post-World War II depressions (about half war related) outside of sub-Saharan Africa.  The largest contractions were 75% for Iraq (1987-91), 46% for Burma (1938-50), 45% for Iran (1976-81), and 44% for West Bank/Gaza (1999-2003).  There were also 25 declines of 15% or more in real per capita GDP in the 1990s for transitions of former Communist countries.  Stock-return data seem to be unavailable during any of the events mentioned in this footnote.
[7] Notable are the hyperinflations in the early 1920s in Germany and Austria, likely due to Reparations payments imposed after World War I, rather than to the war directly.  High inflation also occurred during and after World War I in France and during or after World War II in Austria, Belgium, Finland, France, Greece, Italy, and Japan.  In West Germany, suppressed inflation associated with World War II was effectively ratified by a 10:1 currency conversion and the lifting of price controls in 1948.

w-type events. The important assumption in the model is that, conditioned on crises, default is not more likely for the "risk-free" asset than for the risky one.

To get a sense of the validity of this assumption, Table 2 reports realized real rates of return on stocks and government bills during the economic downturns enumerated in Table 1. Not many observations are available, partly because of the limited number of crises and partly because of missing financial data during these crises.

The Great Depression fits the model for the four countries from Table 1, part A with data on asset returns. I consider returns up to the full year before the rebound in the economy: Australia for 1929-30, France for 1929-31, Germany for 1929-31, and the United States for 1929-32. The averages of the arithmetic annual real rates of return for the four countries were -18.0% for stocks and 8.0% for bills.

Similar results apply to the post-World War II depressions shown in part B of Table 1 for countries with data on asset returns. For Argentina in 1998-2001, the average real stock return was -3.6%, compared to 9.0% for bills. For Indonesia in 1997-98, the respective returns were -44.5% and 9.6%. For the Philippines in 1982-84, the numbers were -24.3% and -5.0%. Given the scarcity of financial data during depressions, it seemed worthwhile to add the recent observation for Thailand (for which GDP data before 1950 are available only in scattered years). The contraction of real per capita GDP in 1996-98 was 14%, just short of the criterion used in Table 1. The average rates of return in 1996-97 were -48.9% for stocks and 6.0% for bills—similar to those for Indonesia in 1997-98.

For World War I, data on asset returns are available for only two of the countries with economic contractions in part A of Table 1. For 1914-18, the average real rate of

return on stocks in France was -5.7%, while that on bills was -9.3%. For Germany, the values were -26.4% and -15.6%.[8] Thus, stocks and bills both performed badly in these countries that suffered economically from World War I. For bills, the reason was high inflation. There is no clear pattern of relative performance—stocks did better in France and worse in Germany.[9]

For World War II, data on asset returns are available for three countries with economic contractions: France, Italy, and Japan. The data are problematic for France, partly because the stock market was closed during parts of 1940 and 1941. The Italian data for the early part of the war also seem unreliable. I report information for 1943-45 in each case. All real rates of return were sharply negative—for bills, the reason again was high inflation. Stocks did worse than bills in France, better than bills in Italy, and about the same in Japan.

The overall conclusion is that government bills were clearly superior to stocks during purely economic crises, represented by the Great Depression and post-World War II depressions in Latin America and Asia. However, bills did not perform obviously better than stocks during economic contractions related to major wars, notably World Wars I and II.[10]

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[8] The impact of the German hyperinflation came later, 1920-23. For 1920-22, the average real rate of return on stocks was -50.7%, while that on bills was -56.2%. Thus, surprisingly, stocks did almost as badly in real terms as bills. The data for 1923, the peak year of the hyperinflation, are unreliable, though stocks clearly did far better in real terms than in 1922.

[9] This conclusion is the same for periods that correspond more closely to the years of economic downturn shown in Table 1, part A. For France from 1916 to 1918, the real rate of return on stocks was -0.3% while that on bills was -12.7%. For Germany from 1913 to 1915, the corresponding numbers were -16.6% and -3.5%.

[10] Better performing assets in such circumstances would be precious commodities, such as gold and diamonds, Swiss bank accounts, and human capital.

### C. Solution of the model

Given the probability distributions for $u_{t+1}$, $v_{t+1}$, and $w_{t+1}$, Eqs. (6) and (7)

determine the price of the risky asset, the expected risky return, and the risk-free return.

The results are as follows:

(9) $$P_{t1} = A_t e^{-q-\rho-(\theta-1)\gamma+(1/2)(\theta-1)^2\sigma^2} \cdot [e^{-p} + (1-e^{-p})e^{(\theta-1)b}],$$

(10) $$E_t(R_{t1}^r) = E_t[A_{t+1}]/P_{t1}$$

$$= e^{q+\rho+\theta\gamma-(1/2)\theta^2\sigma^2+\theta\sigma^2} \cdot \frac{[e^{-p} + (1-e^{-p})e^{-b}]}{[e^{-p} + (1-e^{-p})e^{(\theta-1)b}]},$$

(11) $$R_{t1}^f = e^{q+\rho+\theta\gamma-(1/2)\theta^2\sigma^2} /[e^{-p} + (1-e^{-p})e^{\theta b}].$$

The rates of return, $R_{t1}^r$ and $R_{t1}^f$ in Eqs. (10) and (11), have been computed under

the condition that the world has not ended (within sample!). Thus, if q refers to default

probability, rather than literal end of the world, the formulas are conditioned on default

not having occurred within sample. Under this condition, the "risk-free" asset actually is

risk-free. Expected returns for full samples, which include representative numbers of

defaults, are the multiples $e^{-q}$ of the expressions in Eqs. (10) and (11). (The rate $R_{t1}^f$ is

not risk-free in this context.) Thus, these full expected returns end up independent of q.

It is convenient to think of rates of return as logs of the expressions in Eqs. (10)

and (11). Then we have for the risky rate of return:

(12) $\log[E_t(R_{t1}^r)] =$

$$q + \rho + \theta\gamma - (1/2)\theta^2\sigma^2 + \theta\sigma^2 + \log[e^{-p} + (1-e^{-p})e^{-b}] - \log[e^{-p} + (1-e^{-p})e^{(\theta-1)b}].$$

If p << 1, this rate of return can be approximated as[11]

$$(13) \qquad \log[E_t(R_{t1}^r)] \approx q + \rho + \theta\gamma - (1/2)\theta^2\sigma^2 + \theta\sigma^2 - pe^{-b} \cdot (e^{\theta b} - 1).$$

For the risk-free rate,

$$(14) \qquad \log(R_{t1}^f) = q + \rho + \theta\gamma - (1/2)\theta^2\sigma^2 - \log[e^{-p} + (1 - e^{-p})e^{\theta b}].$$

If p<<1, the approximation is

$$(15) \qquad \log(R_{t1}^f) \approx q + \rho + \theta\gamma - (1/2)\theta^2\sigma^2 - p \cdot (e^{\theta b} - 1).$$

The $\rho + \theta\gamma$ part of the rates of return in Eqs. (12)-(15) corresponds to the usual formula for the steady-state real interest rate in the deterministic neoclassical growth model ($\sigma = q = p = 0$). The equity premium—the spread between the risky and risk-free rate, $\log[E_t(R_{t1}^r)]$ - $\log(R_{t1}^f)$—is given from Eqs. (13) and (15) by

$$(16) \qquad \text{spread} \approx \theta\sigma^2 + p \cdot (1 - e^{-b}) \cdot (e^{\theta b} - 1).$$

Note that the spread is increasing in p and independent of q. These effects will be explored later.

Because the shocks $u_t$, $v_t$, and $w_t$ are i.i.d., the results take the same form for all future periods. The price of a risky asset that pays $A_{t+2}$ in period *t+2* looks like Eq. (9), except that the expression in the first brackets is multiplied by 2 and the expression in the second brackets enters as a square. The expected gross rate of return per period between periods *t* and *t+2*, $E_t(R_{t2}^r)$, on this risky claim is again given by Eq. (10). Similarly, for an asset that pays off risk-free in period *t+2*, the gross rate of return per period between periods *t* and *t+2* is given by Eq. (11).

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[11] This and subsequent approximations are not really approximations; they hold exactly as the arbitrary period length approaches zero.

If an equity share is defined to pay the full stream $A_{t+1}$, $A_{t+2}$, …, the price of this asset equals the sum of the prices of claims on each period's output. If we define

$$(17) \qquad \Phi \equiv e^{-q-\rho-(\theta-1)\gamma+(1/2)(\theta-1)^2\sigma^2} \cdot [e^{-p}+(1-e^{-p})e^{(\theta-1)b}],$$

which is the expression that appears in Eq. (9), the result for the share price is[12]

$$(18) \qquad P_t = A_t \cdot \Phi/(1-\Phi).$$

Therefore, the "price-earnings ratio," $P_t/A_t$, is

$$(19) \qquad P_t/A_t = \Phi/(1-\Phi).$$

We can also note for later purposes that, if $p \ll 1$, the log of $\Phi$ from Eq. (17) is given by

$$(20) \quad \log(\Phi) \approx -q - \rho - (\theta-1)\gamma + (1/2)(\theta-1)^2\sigma^2 + p \cdot (e^{(\theta-1)b}-1).$$

## II. Replication of Mehra-Prescott

I show here that the results that ignore disasters, $p = q = 0$, accord with Mehra and Prescott (1985). The remaining parameters to specify are $\gamma$, $\sigma$, $\rho$, and $\theta$. The parameters $\gamma$ and $\rho$ have no effect on the yield spread, given in Eq. (16).

The values of $\gamma$ and $\sigma$ determine the mean and standard deviation of the growth rate of output and consumption in no-disaster periods. For annual U.S. data from 1890 to 2004,[13] the growth rate of consumption per person has a mean of 0.020 and a standard deviation of 0.035.[14] For real per capita GDP, the values are 0.021 and 0.045. For a more recent period, the means are similar but the standard deviations are much smaller.

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[12] Since $\Phi > 0$, the formula in Eq. (18) is valid if $\Phi < 1$. This condition guarantees that expected utility is finite, as in the model of Kocherlakota (1990). When $\sigma = q = p = 0$, the inequality $\Phi < 1$ reduces to the usual transversality condition for the deterministic neoclassical growth model, which is $\rho + \theta\gamma > \gamma$ in Eq. (14)—the real interest rate exceeds the growth rate. The condition $\Phi < 1$ in the stochastic context is analogous. It is equivalent to the condition that $\log[E_t(R_{t1}^r)]$ from Eq. (12) exceed $\log(E_t A_{t+1}/A_t)$.

[13] National-accounts data since 1929 are from Bureau of Economic Analysis. Earlier data are from Kendrick (1961) and Romer (1987, 1988).

[14] Since it makes little quantitative difference, I calculate average growth rates of consumption and real GDP in the usual geometric-average manner, rather than as arithmetic averages.

For example, from 1954 to 2004, the growth rate of per capita consumption has a mean of 0.024 and a standard deviation of 0.017, whereas the values for real per capita GDP are 0.021 and 0.022. One problem (observed by Romer [1987, 1988]) is that the higher volatility in the period before World War I may reflect poorer data.

Table 3 shows statistics for the growth rate of real per capita GDP for the G7 countries for 1890-2004 and 1954-2004. Standard deviations for the post-1954 period are similar to that for the United States, ranging between 0.02 and 0.03. Values for the longer samples are much higher, reflecting the events shown in Table 1 and probably also the lower quality of the earlier data. Mean growth rates of per capita GDP in the post-1954 period are 0.02-0.03, except for Japan, which has 0.04. For the period since 1890, the mean growth rates range from 0.015 for the United Kingdom to 0.027 for Japan.

Table 3 also shows the kurtosis for growth rates in each country and period. For the 1954-2004 samples, the numbers are close to three, the value for a normal density. Standard tests, such as the Anderson-Darling test, accept the hypothesis of normality with p-values above 0.05. Hence, for these tranquil periods—in which crises of the sort shown in Table 1 did not occur—the growth-rate data seem reasonably described as normal. The situation is very different for the 1890-2004 samples, where the kurtosis always exceeds five and reaches astronomical levels for Germany and Japan. These high values—indications of fat tails—reflect the kinds of disasters shown in Table 1, especially during World War II. Standard tests, including Anderson-Darling, reject normality at low p-values. That is, normality does not accord with samples such as 1890-2004 that include occurrences of disasters.[15] The reasoning in this paper is that the

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[15] With respect to skewness, the data are consistent with the specification that emphasizes disasters over bonanzas. Negative skewness applies to 6 of the 7 countries for the 1890-2004 samples. The skewness is

potential for these disasters also affects behavior in tranquil periods, such as 1954-2004, where disasters happened not to materialize.

Based on the information in Table 3 for the post-1954 periods, I calibrate the baseline specification with $\gamma = 0.025$ and $\sigma = 0.02$. The spread between risky and risk-free yields does not depend on $\gamma$ and is not very sensitive to $\sigma$ in the plausible range (see Eq. [16]).

For the rate of time preference, $\rho$, I use 0.02 in the baseline specification. The spread between risky and risk-free yields does not depend on $\rho$ (see Eq. [16]).

The elasticity of marginal utility or coefficient of relative risk aversion, $\theta$, matters more for the results. From the perspective of risk aversion, the usual view in the finance literature is that $\theta$ should lie in a range of something like 2 to 4. From the standpoint of intertemporal substitution of consumption, Barro and Sala-i-Martin (2004, Ch. 2) argue that a similar range for $\theta$ is needed to accord with country-level observations on levels and transitional behavior of saving rates.[16] I use $\theta = 3$ in the initial specification.

Table 4 has results for the baseline specification and some perturbations. The risky rate of return comes from Eq. (12) with $p = q = 0$. The risk-free rate comes from Eq. (14) with $p = q = 0$. "Expected consumption growth," $\Delta C/C$, is the log of $E_t A_{t+1}/A_t$. The price-earnings ratio is computed from Eqs. (19) and (17) with $p = q = 0$. The results involving leverage and debt-equity ratios are discussed later.

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particularly large in magnitude for Germany and Japan (-5.2 and -5.6, respectively) and is positive only for France (0.5, because of a dramatic rise in per capita GDP in 1946). In contrast, for the 1954-2004 samples, skewness is negative for 4 countries and positive for 3 countries.

[16] Of course, in the current specification, the coefficient of relative risk aversion, $\theta$, equals the reciprocal of the intertemporal elasticity of substitution for consumption. My view is that this restriction may, in fact, be satisfactory for investigating asset pricing and economic growth. Kocherlakota (1990) explores an asset-pricing model in which the two forces can be distinguished.

The model has no problem in matching average real rates of return on stock markets for G7 countries. Table 5 shows average real rates of return, based on arithmetic annual rates of return. Part 1 has long samples back as far as 1880 for six countries (excluding Germany, which has missing data[17]), and part 2 is for the seven countries from 1954 to 2004. For the long samples, the average real rate of return on stocks for the six countries is 0.074, whereas, from 1954 to 2004, the average for the seven countries is 0.087. In comparison, the expected rate of return on the risky asset in the benchmark model (column 1 of Table 4) is 0.094.

The main difficulty—the central point of Mehra and Prescott—is that the risk-free rate of return in the model is far too high—0.093 in the baseline specification—so that the spread between the risky and risk-free rate is way too low, 0.001. For the long samples in Table 5, the average real rate of return on government bills or analogous paper was 0.000. From 1954 to 2004, the average was 0.017. The average spread was 0.073 for the long samples and 0.070 for the post-1954 data. Thus, empirically, the spread between the average real stock-market return and the risk-free (or, more accurately, less risky) real rate was around 0.07, compared to 0.001 in the model. This spectacular failure has been often remarked on in the asset-pricing literature; for discussions, see Campbell (2000), Mehra and Prescott (2003), and Weitzman (2005).

The remainder of Table 4 shows that the problem with the high risk-free rate remains intact for reasonable variations in the underlying parameters. The source of the

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[17] See the notes to Table 5 for a discussion of Germany. Clearly, the data problems that result in the exclusion of German data for the long samples are not exogenous with respect to events such as the German hyperinflation and World War II. This exclusion biases upward the average real rates of return on bills and stocks and biases downward the standard deviations of these returns. For a general discussion of this kind of sample selection problem in the context of stock returns, see Jorion and Goetzmann (1999).

problem is clear from Eqs. (12) and (14). When p = 0, the difference between the logs of

these two expressions gives the equity premium as

(21)                                                  $\text{spread} = \theta\sigma^2$.

For reasonable values of $\theta$ and $\sigma$, this spread is a small number. For example, for the

baseline setting of $\theta = 3$ and $\sigma = 0.02$, the spread is 0.001. To get a spread of 0.07 when

$\theta = 3$, $\sigma$ would have to be 0.15, way above observed standard deviations for annual

growth rates of real per capita GDP and consumption. Alternatively, if $\sigma = 0.02$, $\theta$ would

have to be 175 to get a spread of 0.07.

## III.  Labor Income, Taxes, Leverage

Mehra and Prescott (1985) considered how their results change when equity

shares represent a claim on only a part of real GDP. Three ways to modify the model in

this respect are to allow for labor income and taxes and to have a financial structure that

includes fixed claims (bonds) as well as equities. I extend the model to incorporate these

features, but the bottom-line is that, at least within a representative-agent framework,

these extensions do not seem to resolve the problems.

### A.  Labor income

Let real GDP be $Y_t$.  I continue to assume that there is no investment or

government consumption, so that the representative agent's consumption is $C_t = Y_t$, as

before.[18]  To allow for labor income, suppose that output of fruit is given by the

production function,

$$(22) \qquad\qquad Y_t = A_t \cdot F(K, L_t),$$

where $A_t$ is determined as before, K is the fixed number of trees, and $L_t$ is labor input.

$L_t$ may have deterministic or stochastic variation over time.  Total output, $Y_t$, is divided

as income to tree owners (holders of equity) and income to labor:

$$(23) \qquad\qquad Y_t = \text{capital income} + \text{labor income}.$$

      We can write

$$(24) \qquad\qquad \text{capital income} = \alpha_t Y_t,$$

where $\alpha_t$ is the share of capital income in total income.  If the economy is competitive but

the production function is not Cobb Douglas, changes in $L_t$ would affect $\alpha_t$.  However, if

the probability distribution of future capital-income shares, $\alpha_{t+1}$, …, is independent of the

distribution of future productivity shocks, $A_{t+1}$, …, the basic results from before go

through.  For example, the formula for the price of a one-period equity claim, $P_{t1}$, is the

right-hand side of Eq. (9) multiplied by $E_t \alpha_{tt1}$.  The expected gross risky return, $E_t(R_{t1}^r)$, is

still given by Eq. (10), and the risk-free rate, $R_{t1}^f$, is again given by Eq. (11).  The results

on risky rates of return would change materially only if the capital-income share, $\alpha_t$, co-

varied substantially with the business cycle, that is, with $A_t$.  Similar results would apply

if the model were extended to include other forms of capital that contribute to "output"

but are not owned through corporate equity—for example, residential housing, consumer

durables, privately held businesses, and so on.

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[18] The model, extended now to labor income, still has a representative agent.  The setting can incorporate idiosyncratic shocks to individual labor income if there are complete markets in labor-income insurance. Constantinides and Duffie (1996) analyze a model with individual labor-income shocks and incomplete markets.

## B. Taxes

Return to the initial setting without labor income but suppose that the government

taxes fruit production at the rate $\tau_t \geq 0$ to collect real revenue in the amount

(25) $$T_t = \tau_t A_t.$$

I assume that the government returns this revenue to households as lump-sum transfers

(or as government consumption services that are viewed by households as equivalent to

private consumer expenditure). Therefore, $C_t = A_t$ still applies.[19]

The only difference in the model is that equity holders receive the after-tax part of

fruit production, $(1-\tau_t) \cdot A_t$, rather than $A_t$. If $\tau_{t+1}$ is distributed independently of $A_{t+1}$, the

basic results from before continue to apply. For example, the formula for the price of a

one-period equity claim, $P_{t1}$, is the right-hand side of Eq. (9) multiplied by $(1-E_t\tau_{tt1})$. The

expected gross risky return, $E_t(R_{t1}^r)$, is still given by Eq. (10)—this rate of return should

be interpreted here in an after-tax sense. The risk-free rate, $R_{t1}^f$, is again given by

Eq. (11). The results on risky rates of return would change materially only if the tax rate,

$\tau_t$, co-varied substantially with the business cycle, represented by $A_t$.[20]


## C. Leverage and debt-equity ratios

Suppose that tree owners have a capital structure that involves partly equity and

partly fixed claims—one-period, "risk-free" bonds. I assume that owners issue or retire

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[19] Thus, the model again has a representative agent. The model can incorporate idiosyncratic shocks to individual tax rates if there are complete markets in tax insurance.
[20] The results also change if taxes are used to finance government purchases that do not interact with private consumption in the utility function. A possibility would be wartime expenditure. In this case, high $\tau_t$ would be associated with low $C_t$ for given $A_t$.

enough stock and debt at the start of period $t$ so that the amount of fixed obligations, $B_{t+1}$, due in period $t+1$ is given by

(26) $$B_{t+1} = \lambda A_t e^{\gamma + (1/2)\sigma^2}.$$

That is, the ratio of debt obligation to the expected fruit output in period $t+1$, $A_t e^{\gamma + (1/2)\sigma^2}$, is given by the leverage coefficient, $\lambda \geq 0$. If period $t+1$'s output, $A_{t+1}$, falls short of the fixed obligation, $B_{t+1}$, the payout to bondholders would be limited to $A_{t+1}$. However, the chance of this "bankruptcy" is significant only if the debt exceeds the fruit available in the event of a v-type crisis. (Bondholders also do not get paid if the world ends of if there is a general default.) The condition that rules out a problem with v-type events is $\lambda < e^{-b}$, which I assume holds.

Although leverage does not affect the overall market value of fruit claims (in accordance with Modigliani-Miller), the coefficient $\lambda$ affects the market value and expected risky rate of return on the equity part. The following formulas can be derived as generalizations of Eqs. (9) and (10):

(27) $$P_{t1} = A_t e^{-q-\rho-(\theta-1)\gamma+(1/2)(\theta-1)^2\sigma^2} \cdot \left[ e^{-p} + (1-e^{-p})e^{(\theta-1)b} - \lambda e^{\theta\sigma^2} \cdot [e^{-p} + (1-e^{-p})e^{\theta b}] \right],$$

(28) $$E_t R_{t1}^r = \left[ E_t A_{t+1} - \lambda A_t e^{\gamma + (1/2)\sigma^2} \right] / P_{t1}$$

$$= e^{q+\rho+\theta\gamma-(1/2)\theta^2\sigma^2} \cdot \left[ \frac{e^{-p} + (1-e^{-p})e^{-b} - \lambda}{e^{-\theta\sigma^2} \cdot [e^{-p} + (1-e^{-p})e^{(\theta-1)b}] - \lambda \cdot [e^{-p} + (1-e^{-p})e^{\theta b}]} \right].$$

Using Eq. (28), the formula for the risky rate of return is given as an extension of Eq. (13) by

(29) $$\log[E_t(R_{t1}^r)] \approx q + \rho + \theta\gamma - (1/2)\theta^2\sigma^2 + \theta\sigma^2 + \frac{\theta\sigma^2 - p \cdot (e^{\theta b} - 1)(e^{-b} - \lambda)}{1 - \lambda}.$$

Note on the right-hand side that $e^{-b} > \lambda$ holds.

The risk-free rate of return, $\log(R_{t1}^f)$, is still given by Eq. (15). The market value of debt is $B_{t+1}/R_{t1}^f$. Therefore, the debt-equity ratio, $(B_{t+1}/R_{t1}^f)/P_{t1}$, can be determined as

(30)    debt-equity ratio = $\dfrac{\lambda \cdot [e^{-p} + (1-e^{-p})e^{\theta b}]}{e^{-\theta \sigma^2} \cdot [e^{-p} + (1-e^{-p})e^{(\theta-1)b}] - \lambda \cdot [e^{-p} + (1-e^{-p})e^{\theta b}]}$.

Table 4 shows, consistent with Mehra and Prescott (1985), that usual leverage coefficients, $\lambda$, do not affect the general nature of the conclusions about the equity premium. For example, a coefficient $\lambda = 0.2$ generates a debt-equity ratio of 0.25 in the baseline specification (column 1). The risky rate of return, $\log[E_t(R_{t1}^r)]$, rises negligibly, from 0.094 to 0.095. A higher coefficient, $\lambda = 0.4$, generates a debt-equity ratio of 0.067 in the baseline specification and raises $\log[E_t(R_{t1}^r)]$ only a little more, still 0.095. According to the Federal Reserve's Flow-of-Funds Accounts, recent debt-equity ratios for the U.S. non-financial corporate sector are around 0.5. Therefore, for realistic values of $\lambda$, the basic conclusions do not change.

**IV. Allowing for a Chance of Disaster**

Table 6 brings in the probability, p, of a v-type disaster, analogous to Rietz (1988). I continue to assume that the probability of a w-type disaster, q, is zero. (However, q enters the asset-pricing formulas additively with $\rho$, and Eq. [16] shows that the spread between the risky and risk-free rates does not depend on q.) If the elasticity of marginal utility, $\theta$, is well above one—for example, if $\theta \approx 3$—what matters most for the results is the probability of a major collapse. That is, a 1% probability, p, of a 50% contraction in real per capita GDP is much more consequential than a 2% probability of a 25% event. Based loosely on the tabulation of events in Table 1, I assume that a 50%

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decline in output is realistic, albeit rare. I begin with a baseline specification of $p = 0.01$ per year for this event.

The specification for $\log(A_t)$ in Eq. (8) implies, since $u_t$ and $v_t$ are assumed to be independent, that the variance of the (geometric) growth rate, $\log(A_{t+1}/A_t)$, is given by

$$(31) \qquad VAR[\log(A_{t+1}/A_t)] \approx \sigma^2 + p \cdot (1-p) \cdot b^2.$$

If we continue to assume $\sigma = 0.02$ and use $p = 0.01$ and $b = \log(2)$, we get that the standard deviation of the growth rate is 0.072. This value accords with the average of 0.061 for the standard deviation of the growth rates of real per capita GDP for the G7 countries from 1890 to 2004 (Table 3). These long samples can be viewed as containing the representative number of disaster events. In contrast, the tranquil period from 1954 to 2004, also shown in Table 3, has an average standard deviation for the growth rate of real per capita GDP in the G7 countries of only 0.023. This value can be thought of as the standard deviation when the samples are conditioned on observing no disasters. Hence, this value corresponds to $\sigma$, which is still set at 0.02.

The baseline specification in Table 6, column 1 shows that an allowance for a small probability of v-type disaster, $p = 0.01$, generates empirically reasonable spreads between the risky and risk-free rates. One consequence of raising $p$ from 0 to 0.01 is that the risk-free rate falls dramatically—from 0.093 to 0.026. The inverse relation between $p$ and $\log(R_{t1}^f)$ applies generally in Eq. (14).

Less intuitively, a rise in $p$ also lowers the rate of return on the risky asset, $\log[E_t(R_{t1}^r)]$, given in Eq. (12). If $\theta > 1$, this change reflects partly an *increase* in the price of equity, $P_{t1}$, in Eq. (9)—that is, the price-earnings ratio of 24.7 in Table 6, column 1 exceeds the value 14.0 in Table 4, column 1. Intuitively, a rise in $p$ motivates a

shift toward the risk-free asset and away from the risky one—this force would lower the equity price. However, households are also motivated to hold more assets overall because of greater uncertainty about the future. If $\theta > 1$, this second force dominates, leading to a net increase in the equity price—see Eq. (20). Even if $\theta < 1$, the negative effect of p on $\log[E_t( R_{t1}^r )]$ applies. The reason is that a rise in p also lowers the expected growth rate of dividends, $(E_tA_{t+1})/A_t$, and this force makes the overall effect negative as long as $\theta > 0$. In any event, the risk-free rate falls by more than the risky rate, so that the spread increases. This property can be seen in Eq. (16).

With no leverage, the spread between the risky and risk-free rates in the baseline specification in Table 6, column 1 is 0.034. With a leverage coefficient, $\lambda$, of 0.2, which generates a debt-equity ratio of 0.26, the spread becomes 0.043. These values are in the ballpark of the range of empirical observations on spreads shown in Table 5.

The results are sensitive to the value of the disaster probability, p. In fact, since the term $\theta\sigma^2$ is small, the spread is nearly proportional to p in Eq. (16). (With the baseline parameters, the coefficient on p in this formula is 3.5.) For example, if p = 0.015, the risk-free rate in Table 6, column 2 becomes negative, -0.007, and the no-leverage spread rises to 0.050. In contrast, if p = 0.005 in column 3, the no-leverage spread is only 0.018.

The results depend a lot on how bad a disaster is, as gauged by the parameter b. This sensitivity can be seen in the formula for the spread in Eq. (16). For example, with p = 0.01, if a disaster reduces output to 40% of its starting value (perhaps the worst of World War II), the risk-free rate in column 4 becomes -0.043 and the spread 0.081. In contrast, if a disaster means only a decline to 75% of the starting value (like the Great

Depression in many countries), the risk-free rate in column 5 is 0.080 and the spread is only 0.005.

The results are also sensitive to the size of the elasticity of marginal utility, $\theta$. Again, the effects on the spread can be seen in Eq. (16). If $\theta = 4$, as in column 6 (when $p = 0.01$ and a disaster event is 50%), the risk-free rate is -0.023 and the spread is 0.069. In contrast, if $\theta = 2$, the risk-free rate in column 7 is 0.040 and the spread is only 0.015.

Campbell (2000) and Weitzman (2005) observe that Rietz's low-probability disasters create a "peso problem" when disasters are not observed within sample. (Indeed, data availability tends to select no-disaster samples.) However, this consideration turns out not to be quantitatively important in the model. In the baseline specification in Table 6, column 1, the risky rate of 0.060 can be compared with the rate of 0.065 that applies to a sample conditioned on no disasters. Thus, the spread from the risk-free rate would be 0.034 in a full sample (which contains the representative number of disasters) versus 0.039 in a selected no-disaster sample. Similarly, the average growth rate of consumption in the no-disaster sample, 0.025, is only moderately above that, 0.020, in the full sample. In other words, a low probability of a v-type disaster, $p = 0.01$, has a major effect on stock prices, risk-free rates, and the spread between risky and risk-free yields even though the disasters that occur have only moderate effects on long-run averages of consumption growth rates and rates of return on equity.

## V. Disaster Probability and the Risk-Free Rate

The results in Table 6 apply when p and q (and the other model parameters) are fixed permanently at designated values; for example, $p = 0.01$ and $q = 0$ in column 1.

However, the results also show the effects from permanent changes in any of the

parameters, such as the disaster probabilities, p and q.  In this and the following sections,

I use the model to assess the effects from changes in p and q.  However, in a full analysis,

stochastic variations in p and q—possibly persisting movements around stationary

means—would be part of the model.

A fall in p raises the risk-free rate, $\log(R_{t1}^f)$, in Eqs. (14) and (15).  The results in

Table 6, columns 2 and 3, suggest a substantial impact:  the risk-free rate rises from

-0.007 to 0.059 when p falls (permanently) from 0.015 to 0.005.  Mehra and Prescott

(1988, p. 135) criticized the analogous prediction from Rietz's (1988) analysis:[21]

> "Perhaps the implication of the Rietz theory that the real interest rate and
>
> the probability of the extreme event move inversely would be useful in
>
> rationalizing movements in the real interest rate during the last 100 years.
>
> For example, the perceived probability of a recurrence of a depression was
>
> probably high just after World War II and then declined.  If real interest
>
> rates rose significantly as the war years receded, that would support the
>
> Rietz hypothesis.  But they did not. … Similarly, if the low-probability
>
> event precipitating the large decline in consumption were a nuclear war,
>
> the perceived probability of such an event surely has varied in the last 100
>
> years.  It must have been low before 1945, the first and only year the atom
>
> bomb was used.  And it must have been higher before the Cuban Missile
>
> Crisis than after it.  If real interest rates moved as predicted, that would
>
> support Rietz's disaster scenario.  But again, they did not."

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[21] Mehra and Prescott (2003, p. 920) essentially repeat this criticism.

The point about the probability of depression makes sense, although I am skeptical that this probability varied over time in the way suggested by Mehra and Prescott. The observations about the probability of nuclear war confuse, using my terminology, the v- and w-type disasters. In Rietz's and my analysis, the probability, p, of a v-type disaster refers to something like a decline in real per capita GDP and consumption by 50%. The analysis is different for a w-type event—such as the end of the world—if that is what a nuclear conflagration entails. Equations (12) and (14) show that an increase in q *raises* the rate of return on equity and the risk-free rate by the same amount and has no effect on the spread. (Recall that these rates of return are conditioned on the w-disaster not materializing during the sample.)

The intuition for why the effect from higher q differs from that from higher p involves incentives to hold risky versus risk-free assets and incentives to save. If p increases, the expected marginal utility of future consumption rises because marginal utility is particularly high after a 50% disaster. This change motivates people to hold more of the risk-free asset, partly because they want to shift from the risky to the risk-free asset and partly because they want to save more. Thus, in equilibrium, the risk-free rate falls. The risky rate also declines, but the spread between the risky and risk-free rate increases.

The end of the world is different because the marginal utility of consumption is not high in this state. Moreover, the risky and risk-free assets are equally good in this situation—that is, both are useless. For this reason, a rise in q does not motivate a shift from risky to risk-free assets. Furthermore, the incentive to hold all assets—that is, to

save—declines.[22]  As is clear from Eqs. (12) and (14), an increase in q has the same

effect as a rise in the pure rate of time preference, ρ.  In equilibrium, the risky and

risk-free rates increase by equal amounts, and the spread does not change.  For the risk-

free rate, the important conclusion is that a rise in q—perhaps identified with the

probability of nuclear war—raises the rate.  As already mentioned, a probability of 100%

default on all assets—a general loss of individual property rights—can also be

represented by a higher q.[23]  We can also allow for partial default, in the sense that a

higher q represents (in a linear way) either a greater chance of default or a larger share of

assets lost when a default occurs.

Empirically, to assess the connection between disaster probability and the risk-

free rate, we have to ascertain whether an event reflects more the potential v-type crisis

(in which the risk-free asset does relatively well) or the w-type crisis (for which all assets

are unattractive).  Changing probabilities of a depression would likely isolate the effect of

changing p, but the analysis depends on identifying the variations in depression

probability that occurred over time or across countries.  From a U.S. perspective, the

onset of the Great Depression in the early 1930s likely raised p (for the future).  The

recovery from 1934 to 1937 probably reduced p, but the recurrence of sharp economic

contraction in 1937-38 likely increased p again.  Less clear is whether the end of World

War II had an effect on future probability of depression.

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[22] This argument accords with Slemrod (1986) and Russett and Slemrod (1993).

[23] Since the model has a representative agent, the proceeds from default have to be remitted in some way—perhaps involving the government as intermediary—back to the agent.  In effect, there is a "tree reform" in which all asset claims are randomly redistributed.  Since $C_t = A_t$ still applies, general default differs from the end of the world.  Nevertheless, the implications for asset pricing are the same.  Note, however, that expected rates of return are computed conditional on the w-event—in this case, default—not occurring during the sample.  Over a sample that includes the representative number of defaults, expected returns would be independent of q.

Changing probabilities of nuclear war are unlikely to work—they would involve a mixture of v- and w-type effects, and the net impact on the risk-free rate is ambiguous.[24] From the perspective of the events shown in Table 1, a natural variable to consider is changing probability of the types of wars seen in history—notably World Wars I and II, which were massive but not the end of the world (for most people). My working assumption is that the occurrence of this type of major war raised the probabilities p and q, that is, increased the perceived likelihood of future disasters. The q effect likely refers more to the prospect of general default than to the end of the world.

An increase in p lowers the risk-free rate, whereas an increase in q raises the rate—see Eq. (14) and n. 23. However, the effect from higher p is likely to dominate. For the baseline parameters considered before (Table 6, column 1), the coefficient on p in the formula for $\log(R_{t1}^f)$ in Eq. (15) is seven, whereas that for q is one. Therefore, the risk-free rate falls when disaster probabilities rise unless q increases by more than seven times as much as p.

Figure 1 shows an estimated time series since 1859 of the expected real interest rate on U.S. Treasury Bills or analogous short-term paper.[25] The source of data on nominal returns is Global Financial Data, the same as in Table 5. Before the introduction of T-Bills in 1922, the data refer to high-grade commercial paper.

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[24] I considered using the famous "doomsday clock," discussed by Slemrod (1986), to assess empirically the changing probability of nuclear war. The clock is available online from the *Bulletin of the Atomic Scientists*. I decided not to use these "data" because the settings are heavily influenced by an ideology that always identifies toughness with higher probability of nuclear war and disarmament with lower probability. For example, the clock was nearly at its worst point—three minutes to midnight—in 1984 shortly after President Reagan began his successful confrontation of the "evil empire" of the Soviet Union.
[25] It would be preferable to look at yields on indexed bonds, but these instruments exist only in recent years.

To compute the expected real interest rate, I subtracted an estimate of the expected inflation rate for the CPI. Since 1947, my measure of expected inflation is based on the Livingston Survey. From 1859 to 1946, I measured the "expected inflation rate" as the fitted value from an auto-regression of annual CPI inflation on a single lag.[26] Additional lags lack explanatory power, although there may be a long-run tendency over this period for the price level to adjust toward a stationary target.

One striking observation from Figure 1 is that the expected real interest rate tended to be low during wars—especially the Civil War, World War I, and World War II. The main exception is the Vietnam War. Table 7 shows the nominal interest rate, expected inflation rate, and expected real interest rate during each war and the Great Depression. The typical wartime pattern—applicable to the Civil War, World Wars I and II, and the first part of the Korean War—is that the nominal interest rate changed little, while actual and expected inflation rates increased. Therefore, expected real interest rates declined, often becoming negative. Moreover, the price controls imposed during World War II and the Korean War likely led to an understatement of inflation; therefore, the expected real interest rate probably declined even more than shown for these cases.

Figure 1 and Table 7 show that expected real interest rates fell in 2001-03 during the most recent war—a combination of the September 11th attacks and the conflicts in Afghanistan and Iraq. For this period, we can also observe real yields on U.S. Treasury indexed bonds, first issued in 1997. The 10-year real rate fell from an average of 3.8%

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[26] The inflation rate is the January-to-January value from 1913 to 1946. Before 1913, the CPI data are something like annual averages. The estimated lag coefficient is 0.62 (s.e. = 0.09). The $R^2$ for this regression is 0.35. In this context, I measured the inflation rate as the usual geometric value, $\log(P_{t+1}/P_t)$.

for 1/97-8/01 to 2.3% for 10/01-2/05.[27]  Similarly, the 5-year real rate declined from an

average of 3.2% for 12/00-8/01 to 1.7% for 10/01-2/05.  These real rate reductions on

indexed bonds accord with those shown for the short-term real rate in Table 7.[28]

The tendency for expected real interest rates to be low during U.S. wars has been

a mystery, as described in Barro (1997, Ch. 12).[29]  Most macroeconomic models predict

that a massive, temporary expansion of government purchases would raise expected real

interest rates.  In previous work, I conjectured that military conscription and mandated

production might explain part of the puzzle for some of the wars.  Mulligan (1997)

attempted to explain the puzzle for World War II by invoking a large increase of labor

supply due to patriotism.  A complementary idea is that patriotism and rationing

motivated declines in consumption and increases in saving, perhaps concentrated on war

bonds.  The patriotism explanation does have the virtue of explaining why the real

interest rate would not be low in an unpopular war, such as Vietnam.  However, the low

real interest rate in wartime seems to be too pervasive a phenomenon to be explained by

these kinds of special factors.  The present model offers a more promising explanation:

expected real interest rates tend to fall in wartime because of increases in the perceived

probability, p, of (future) v-type economic disasters.

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[27] The indexed bonds data show that risk-free real interest rates are not close to constant.  For 10-year U.S. indexed bonds, the mean for 1/97-2/05 was 3.1%, with a standard deviation of 0.8% and a range from 1.5% to 4.3%.  For the United Kingdom from 2/83-2/05, the mean real rate on ten-year indexed bonds was also 3.1%, with a standard deviation of 0.8% and a range from 1.7% to 3.7%.

[28] The real rate on 10-year indexed bonds peaked at 4.2% in May 2000 then fell to 3.3% in August 2001— perhaps because of the end of the Internet boom in the stock market but obviously not because of September 11 or the Afghanistan-Iraq wars.  However, the rate then fell to 3.0% in October 2001 and, subsequently, to 1.8% in February 2003.  The lowest level was 1.5% in March 2004.

[29] Barro (1987) finds that interest rates were high during U.K. wars from 1701 to 1918.  However, this evidence pertains to nominal, long-term yields on consols.  Short-term interest rates are unavailable for the United Kingdom over the long history.  Realized short-term real interest rates in the United Kingdom were very low during World Wars I and II.

Table 7 also shows the behavior of the expected real interest rate in the United States during the Great Depression. According to the theory, the expected real rate should have declined if the probability of v-type disaster, p, increased. Matching this prediction to the data is difficult because of uncertainty about how to gauge expected inflation during a time of substantial deflation.

The nominal return on Treasury Bills fell from over 4% in 1929 to 2% in 1930, 1% in 1931, and less than 1% from 1932 on. However, the inflation rate became substantially negative (-2% in 1930, -9% in 1931, -11% in 1932, -5% in 1933), and the constructed expected inflation rate also became negative: -4% in 1931 and -6% in 1932 and 1933. Therefore, the measured expected real interest rate was high during the worst of the depression, 1931-33. However, this construction is likely to be erroneous because the persisting deflation in 1930-33 depended on a series of monetary/financial shocks, each of which was unpredictable from year to year. Hence, rational agents likely did not anticipate much of the deflation in 1931-33, and expected real interest rates were probably much lower in those years than the values reported in the table. From 1934 on, the inflation rate became positive. The combination of positive expected inflation with nominal interest rates close to zero generated low expected real interest rates for 1934-38. This period includes the sharp recession—and possible fears of a return to depression—in 1937-38.

## VI. Disaster Probability and the Price-Earnings Ratio

Campbell and Shiller (2001, Figure 4) observe that price-earnings ratios fell in the United States during some wars, notably the early parts of World Wars I and II and the

Korean War. Figure 2 plots the P-E ratios (annual averages from Global Financial Data) from 1871 to 2004. Some prominent observations are as follows:[30]

- The U.S. P-E ratio fell sharply from the start of World War I in Europe in 1914 until 1916, then recovered through 1919.

- The P-E ratio fell sharply from the lead-in to World War II in Europe in 1938 until 1941, then recovered through 1946.

- The P-E ratio was very low in the first year of the Korean War, 1950 (though slightly higher than that in 1949), then recovered to 1952-53, when the war concluded.

- The P-E ratio rose to a high level during the worst of the Great Depression, 1930-34, fell during the recovery period of 1935-37, then rose again in 1938 during the 1937-38 recession.

To see whether the model can account for these observations, recall first that, if $\theta > 1$, as I assume, an increase in the probability, p, of a v-type disaster raises the price-earnings ratio (see Eq. [20]). However, a rise in the probability, q, of a w-type disaster lowers the ratio. For the baseline parameters used in Table 6, column 1, the effect of a change in p on the P-E ratio is about three times as large as the magnitude of the effect from a change in q—this result comes from the coefficient on p in Eq. (20). In contrast,

[30] For the other G7 countries, the only other long-term series on P-E ratios from Global Financial Data is for the United Kingdom, back to 1927. During World War II, the U.K. P-E ratio fell (using December values) from 10.0 in 1938 to 8.3 in 1941, then recovered to 12.8 in 1945. This pattern is similar to that in the United States.

for the risk-free rate, the effect from a change in p was around seven times as large as that from a change in q. Therefore, the ranges of possible outcomes are as shown in Table 8.

If $\Delta q < 3\Delta p$, the risk-free rate falls—consistent with the argument in the previous section—but the P-E ratio rises, inconsistent with the data in Figure 2 for the early parts of World Wars I and II and the Korean War. If $\Delta q > 7\Delta p$, the P-E ratio falls—consistent with the data—but the risk-free rate rises, inconsistent with the wartime data in Figure 1. However, there is an interval in the middle, $3\Delta p < \Delta q < 7\Delta p$, that is consistent with both "facts"—low risk-free rate and low P-E ratio during some wars. The reason this outcome is possible is that the rise in p has a major effect on the equity premium; consequently, the risk-free rate can move in one direction (downward), while the earnings-price ratio, a kind of rate of return,[31] moves in the opposite direction (upward).

As an example, starting from the baseline specification in Table 6, column 1, suppose that a war raises p from 0.01 to 0.015 (as in column 2) and also raises q from 0 to 0.025. (This rise in q is probably best interpreted as a heightened chance of default, rather than a larger probability of the end of the world.) In this case, the risk-free rate falls from 0.026 to 0.018, the risky rate *rises* from 0.060 to 0.068, the equity premium increases from 0.034 to 0.050, and the P-E ratio *falls* from 24.7 to 19.4. Thus, reasonable specifications of changes in the disaster probabilities, p and q, are consistent with the broad patterns for risk-free rates and P-E ratios in the early parts of World Wars I and II and the Korean War.

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[31] An additional effect is that the expected risky return, $E_t(R_{t1}^r)$ in Eq. (10), involves the expected future dividend, $E_t A_{t+1}$, as well as the stock price. Increases in p and q lower the expected growth rate of dividends.

For the latter parts of these wars, the natural interpretation is that increasing allied military successes (involving partly the U.S. entrances into the world wars) lowered p and, perhaps especially, q. These changes can explain the recoveries of the P-E ratios in the United States while the wars were still going on.

For the Great Depression, a reasonable view is that p increased but q did not change. This configuration is consistent with rising U.S. P-E ratios during the worst of the Depression, 1930-33, falling P-E ratios (along with reductions in p) during the recovery, 1934-37, and an increase in the P-E ratio (along with an increased p) in the 1937-38 contraction.

Clearly, to have more discipline on these kinds of exercises, one would want time-series estimates of p and q. Then these probabilities could be related in a systematic way to the behavior of expected real interest rates and P-E ratios.

## VII.  Volatility of Stock Returns

The variance of the growth rate of $A_t$ is given in Eq. (31).  In the baseline model in Table 6, the price-earnings ratio is constant.  Therefore, the standard deviation of stock returns equals the standard deviation of the growth rate of $A_t$, which equals 0.072 for the baseline parameters in column 1.  This value would apply to a sample that contains the representative number of disasters, such as the long samples displayed in Table 5, part 1. However, the average standard deviation of stock returns over these periods was 0.23, way above the value predicted by the model.  Similarly, the tranquil periods since 1954 displayed in Table 5, part 2 should correspond to the model conditioned on the realization of no disasters.  In this case, the model standard deviation of stock returns is 0.02 (the

value for σ in the baseline specification), whereas the average standard deviation was again 0.23. These discrepancies correspond to the well-known excess-volatility puzzle for stock returns.[32]

A natural way to resolve this puzzle is to allow for variation in underlying parameters of the model, notably the probabilities of disaster, p and q. The results in Table 6 show that the price-earnings ratio is highly sensitive to changes in p. In particular, variations of p between 0.005 and 0.015 (columns 3 and 2) shift the price-earnings ratio between 17.9 and 39.2. A change in q amounts to a change in ρ. Hence, the effect of a rise in q from 0 to 0.01 can be seen by comparing column 8 with column 1. The price-earnings ratio falls from 24.7 to 19.6. As already noted, the variations in p and q shown in Table 6 relate to once-and-for-all, permanent differences in probabilities of disaster. However, an extension of the model to allow for stochastic, persisting variations in $p_t$ and $q_t$ could likely account for the observed volatility of stock returns.

## VIII. Regression Estimates of θ (the coefficient of relative risk aversion)

Suppose that the baseline model in Table 6 generates macroeconomic data on rates of return and consumption growth rates. What would an econometrician estimate for θ—the elasticity of marginal utility and the coefficient of relative risk aversion—from these data with standard regression techniques? As a background, the results on macroeconomic data in Hall (1988) suggest a tremendous range for estimates, $\hat{\theta}$, as well as a tendency to find implausibly high values, that is, surprisingly low intertemporal elasticities of substitution in consumption.

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[32] Since the model has a constant risk-free rate, it also fails to explain the variability in real bill returns shown in Table 5. However, this discrepancy arises partly because the real bill returns are not risk-free in practice. In particular, uncertainty about inflation is substantial around major wars.

As the model stands, with fixed parameters, the only variations in the data come from realizations of the productivity shock, $A_t$. Since the shocks are i.i.d., a number of variables are constant—the risk-free rate, the expected risky rate, the price-earnings ratio, and the expected growth rate of consumption. Thus, it is clear immediately that regressions involving the risk-free rate could not even be calculated from data generated by the model.

The realized growth rate of consumption is determined from

(32)                               $C_{t+1}/C_t = A_{t+1}/A_t.$

The realized return on equity comes from the formula for the equity price, $P_t$, given in Eqs. (19) and (17). This realized return is a combination of dividends and price appreciation:

(33)    realized gross return on stocks $= (A_{t+1} + P_{t+1})/P_t = (1/\Phi)\cdot(A_{t+1}/A_t),$

where $\Phi$ is given in Eq. (17). Thus, the realizations of consumption growth rates and returns on stock are perfectly correlated.

The usual regression (aimed at retrieving an estimate of $1/\theta$) relates the log of $C_{t+1}/C_t$ from Eq. (32) to the log of the gross return in Eq. (33). In the model, this regression has an intercept of $-\log(\Phi)$ and a slope of one. Thus, the slope reveals nothing about $\theta$. The intercept, based on Eq. (17), is

$$q + \rho + \gamma\cdot(\theta-1) - (1/2)\cdot(\theta-1)^2\sigma^2 + \log[1 - p + pe^{(\theta-1)\cdot b}].$$

The intercept also does not reveal much about $\theta$. If $\sigma = p = q = 0$, the intercept is $\rho + \gamma\cdot(\theta-1)$.

In order to identify $\theta$, the model needs variation in the parameters that have, thus far, been treated as fixed. Two possibilities that illustrate the general issues are variations

in the productivity growth rate, γ, and the probability of v-type disaster, p. As already

noted, a richer analysis would have stochastic variation in these parameters as part of the

model.[33] The present model allows for a consideration of different data sets (e.g.

countries or time periods), each of which is generated from a different (but then fixed)

value of each parameter. Then I can evaluate regression estimates that come from

variations in means across the data sets. The results of this exercise are in Table 9.

Consider first variations in γ. An increase in γ raises the average growth rate of

real GDP and consumption, along with the growth rate of $A_t$ in Eq. (8). The expected

risky rate, given by Eq. (12), and the risk-free rate, from Eq. (14), each rise by θγ.

Therefore, a cross-sample regression of mean rates of return (either on risky or risk-free

assets) on mean growth rates of consumption yields the coefficient θ. In accordance with

this result, the first line of Table 9 shows for all cases that the regression estimate of θ for

the baseline specification in Table 6, column 1 is the true value, 3.0.

In the present setting, the variations in γ pertain to differences in long-run growth

rates of productivity and real GDP. However, in an extended model, the variations in

productivity growth rates might refer to predictable differences over the business cycle.

The differences in γ might also reflect systematic variations of growth rates that arise

during the transition to the steady state in the standard neoclassical growth model, where

the productivity of capital declines with capital accumulation.

Now consider variations in p. As noted before (Table 6, columns 1 and 2), a

higher p goes along with lower risk-free and risky rates. However, conditional on no

disaster, a higher p has no effect on the average growth rate of $A_t$ in Eq. (8) and,

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[33] One way to generate movements in expected productivity growth is to allow for serial correlation in the shocks $u_t$.

therefore, no effect on the average growth rates of real GDP and consumption. Thus, if one considers samples conditioned on no disasters, regression estimates of $\theta$ would be $\infty$, as shown in columns 2 and 4 of Table 9. Put alternatively, the estimate of the intertemporal elasticity of substitution, $1/\theta$, would be zero.

For samples that include representative numbers of disasters, an increase in p reduces the expected growth rate of $A_t$. The relevant term is $\log(1-p + pe^{-b}) \approx -p\cdot(1-e^{-b})$—the effect of p on this term is negative. For example, an increase in p from 0.01 to 0.015 in going in Table 6 from column 1 to column 2 lowers the average growth rate of consumption from 0.020 to 0.018. Since a rise in p also reduces the risky and risk-free rates of return, usual regression estimates would get the right sign—positive—for $\theta$. However, the estimated coefficients bear little relation to $\theta$.[34] Table 9 shows that, if one uses the risk-free rate in the regression, the estimate is $\hat{\theta} = 13.0$ (column 1), whereas, with the risky rate, the estimate is $\hat{\theta} = 6.8$ (column 3).

Table 9 shows the regression estimates that correspond to cross-sample variations in the other parameters: b, $\sigma$, $\theta$, and $\rho$. The results are $\infty$ for $\theta$ and $\rho$ because these parameters do not affect the growth rate of $A_t$ and, hence, the growth rate of consumption. Variations in $\sigma$ generate the wrong sign—negative—for $\hat{\theta}$. These results follow because higher $\sigma$ raises the average growth rate of consumption but lowers the risky and risk-free rates.

Given the findings in Table 9, it is not surprising that empirical estimates of $\theta$ from macroeconomic data, exemplified by Hall (1988), have a broad range, tend to be

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[34] For the risk-free rate, the coefficient is approximately $(e^{\theta b} - 1)/(1 - e^{-b})$. For the risky rate, the coefficient is approximately equal to the same value multiplied by $e^{-b}$, which is 0.5 in the baseline specification.

very high (so that $1/\theta$ is often indistinguishable from zero), and sometimes have a negative sign. The empirical estimates often use instrumental variables, but the instruments are typically lagged values of variables such as rates of return, GDP growth rates, and consumption growth rates. These instruments would not necessarily isolate the underlying variation—in the productivity growth rate, $\gamma$—that would reveal the true $\theta$. For example, the lagged variables could pick up persisting variations in p. To be successful, the instrumental variables would have to select out exogenous variations in productivity growth (in a long-run sense or in the contexts of business fluctuations or transitional dynamics).

## IX. Concluding Observations

The allowance for low-probability disasters, suggested by Rietz (1988), explains a lot of puzzles related to asset returns and consumption. Moreover, this approach achieves these explanations while maintaining the tractable framework of a representative agent, time-additive and iso-elastic preferences, complete markets, and i.i.d. shocks to productivity growth. Perhaps just as puzzling as the high equity premium is why Rietz's framework has not been taken more seriously by researchers in macroeconomics and finance.

A natural next step is to extend the model to incorporate stochastic, persisting variations in the disaster probabilities, $p_t$, and $q_t$. Then the empirical analysis could be extended to measure $p_t$ and $q_t$ more accurately and to relate these time-varying probabilities to asset returns and consumption. Far out-of-the-money options prices

might help in the measurement of disaster probabilities.[35]  Other possibilities include insurance premia and contract prices in betting markets.

The model can be extended to include capital accumulation; however, preliminary analysis suggests that this extension does not have important effects on the results.  The asset menu could be expanded to include precious commodities, which are likely to be important as hedges against disasters.  It would also be useful to distinguish local disasters from global ones.  Finally, the structure could allow for variations in the growth-rate parameter, $\gamma$.  Some of this variation could involve business-cycle movements—then the model might have implications for cyclical variations in rates of return and the equity premium.

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[35] Xavier Gabaix made this suggestion.

| Table 1  Declines of 15% or More in Real Per Capita GDP in the 20th Century | | | |
|---|---|---|---|
| Part A:  20 OECD Countries in Maddison (2003) | | | |
| | | | |
| **Event** | **Country** | **Years** | **% fall in real per capita GDP** |
| **World War I** | Austria | 1913-15 | 23 |
| | Belgium | 1916-18 | 30 |
| | Denmark | 1914-18 | 16 |
| | Finland | 1914-18 | 32 |
| | France | 1916-18 | 31 |
| | Germany | 1913-15 | 21 |
| | Netherlands | 1916-18 | 15 |
| | Sweden | 1914-18 | 17 |
| | | | |
| **Great Depression** | Australia | 1929-31 | 17 |
| | Austria | 1929-33 | 23 |
| | Canada | 1929-33 | 33 |
| | France | 1929-32 | 16 |
| | Germany | 1929-32 | 17 |
| | Netherlands | 1929-34 | 16 |
| | New Zealand | 1929-32 | 18 |
| | United States | 1929-33 | 31 |
| | | | |
| **Spanish Civil War** | Portugal? | 1934-36 | 15 |
| | Spain | 1935-38 | 31 |
| | | | |
| **World War II** | Austria | 1944-45 | 58 |
| | Belgium | 1939-43 | 24 |
| | Denmark | 1939-41 | 24 |
| | France | 1939-44 | 49 |
| | Germany | 1944-46 | 64 |
| | Greece | 1939-45 | 64 |
| | Italy | 1940-45 | 45 |
| | Japan | 1943-45 | 52 |
| | Netherlands | 1939-45 | 52 |
| | | | |
| **Aftermaths of wars** | Canada | 1917-21 | 30 |
| | Italy | 1918-21 | 25 |
| | United Kingdom | 1918-21 | 19 |
| | United States | 1944-47 | 28 |

| Part B:  Eight Latin American & Seven Asian Countries in Maddison (2003) | | | |
|---|---|---|---|
| **Event** | **Country** | **Years** | **% fall in real per capita GDP** |
| **World War I** | Argentina | 1912-17 | 29 |
| | Chile | 1912-15 | 16 |
| | Chile | 1917-19 | 23 |
| | Uruguay | 1912-15 | 30 |
| | Venezuela | 1913-16 | 17 |
| | | | |
| **Great Depression** | Argentina | 1929-32 | 19 |
| | Chile | 1929-32 | 33 |
| | Mexico | 1926-32 | 31 |
| | Peru | 1929-32 | 29 |
| | Uruguay | 1930-33 | 36 |
| | Venezuela | 1929-32 | 24 |
| | Malaysia | 1929-32 | 17 |
| | Sri Lanka | 1929-32 | 15 |
| | | | |
| **World War II** | Peru | 1941-43 | 18 |
| | Venezuela | 1939-42 | 22 |
| | Indonesia* | 1941-49 | 36 |
| | Malaysia** | 1942-47 | 36 |
| | Philippines*** | 1940-46 | 59 |
| | South Korea | 1938-45 | 59 |
| | Sri Lanka | 1943-46 | 21 |
| | Taiwan | 1942-45 | 51 |
| | | | |
| **Post-WWII Depressions** | Argentina | 1979-85 | 17 |
| | Argentina | 1998-02 | 21 |
| | Chile | 1971-75 | 24 |
| | Chile | 1981-83 | 18 |
| | Peru | 1981-83 | 17 |
| | Peru | 1987-92 | 30 |
| | Uruguay | 1998-02 | 20 |
| | Venezuela | 1977-85 | 24 |
| | Indonesia | 1997-99 | 15 |
| | Philippines | 1982-85 | 18 |

**Notes to Table 1**

Part A covers 20 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States. Those with no 15% events are Norway (-14% in 1916-18, -13% in 1941-44) and Switzerland (-11% in 1915-18). Satisfactory data for Ireland are unavailable until after World War II. Data for Greece are missing around World War I, 1914-20.

Part B covers eight Latin American and seven Asian countries that have nearly continuous data from Maddison (2003) at least from before World War I. The sample is Argentina, Brazil, Chile, Colombia, Mexico, Peru, Uruguay, Venezuela, India, Indonesia, Malaysia, Philippines, South Korea, Sri Lanka, and Taiwan. Data for Argentina and Uruguay after 2001 are from Economist Intelligence Unit, *Country Data*. Countries with no 15% events are Brazil (-13% in 1928-31, -13% in 1980-83), Colombia (-9% in 1913-15), and India (-11% in 1916-20, -12% in 1943-48). Data for Peru appear to be unreliable before the mid 1920s.

Adjustments were made by Maddison to account for changes in country borders.

*No data available for 1942-48.
**No data available for 1941-45.
***No data available for 1943-46.

| Table 2  Stock and Bill Returns during Economic Crises | | |
|---|---|---|
| | | |
| **Event** | **average real rate of return on stocks** | **average real rate of return on bills** |
| **World War I** | | |
| France, 1914-18 | -5.7 | -9.3 |
| Germany, 1914-18 | -26.4 | -15.6 |
| | | |
| **Great Depression** | | |
| Australia, 1929-30 | -13.1 | 9.7 |
| France, 1929-31 | -20.5 | 1.4 |
| Germany, 1929-31 | -22.2 | 11.2 |
| United States, 1929-32 | -16.3 | 9.7 |
| | | |
| **World War II** | | |
| France, 1943-45 | -29.3 | -22.1 |
| Italy, 1943-45 | -33.9 | -52.6 |
| Japan, 1943-45 | -12.9 | -13.7 |
| | | |
| **Post-WWII Depressions** | | |
| Argentina, 1998-01 | -3.6 | 9.0 |
| Indonesia, 1997-98 | -44.5 | 9.6 |
| Philippines, 1982-84 | -24.3 | -5.0 |
| Thailand, 1996-97* | -48.9 | 6.0 |

Note:  The table shows real rates of return on stocks and government bills over periods
with available financial data that correspond to the economic downturns shown in
Table 1.  Rates of return are computed as averages of arithmetic annual rates of return.
Data are from Global Financial Data, except for Indonesia, where the real rate of return
on bills comes from data on money-market interest rates from EIU *Country Data*.
Stock-return data for France and Italy prior to 1943 during World War II appear to be
problematic.  Therefore, I used the periods 1943-45 for these cases, although the
economic downturns began earlier.

*Thailand's contraction of real per capita GDP by 14% for 1996-98 falls just short of the
15% criterion used in Table 1.

| Table 3 Growth Rates of Real Per Capita GDP in G7 Countries | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Canada | France | Germany | Italy | Japan | U.K. | U.S. |
| | growth rate of real per capita GDP, 1890-2004 | | | | | | |
| **mean** | 0.021 | 0.020 | 0.019 | 0.022 | 0.027 | 0.015 | 0.021 |
| **standard deviation** | 0.051 | 0.069 | 0.090 | 0.059 | 0.082 | 0.030 | 0.045 |
| **kurtosis** | 5.4 | 5.4 | 40.6 | 10.4 | 49.0 | 5.8 | 5.8 |
| | growth rate of real per capita GDP, 1954-2004 | | | | | | |
| **mean** | 0.022 | 0.026 | 0.027 | 0.030 | 0.043 | 0.021 | 0.021 |
| **standard deviation** | 0.023 | 0.017 | 0.024 | 0.022 | 0.034 | 0.018 | 0.022 |
| **kurtosis** | 3.4 | 2.5 | 3.9 | 2.8 | 2.4 | 3.1 | 2.6 |

Note:  Except for the U.S., data are from Maddison (2003), updated through 2004 using information from Economist Intelligence Unit, *Country Data*.  For the U.S., the sources are noted in the text.  The GDP series for Germany has a break in 1918; hence, the growth-rate observation for 1918-19 is missing.

| Table 4  Replication of Mehra and Prescott | | | | | | |
|---|---|---|---|---|---|---|
| (these results assume p = q = 0) | | | | | | |
| | **(1)** | **(2)** | **(3)** | **(4)** | **(5)** | **(6)** |
| **parameter/variable** | **baseline** | **low θ** | **high θ** | **high σ** | **high ρ** | **high γ** |
| **θ** | 3 | **2** | **4** | 3 | 3 | 3 |
| **σ** | 0.02 | 0.02 | 0.02 | **0.03** | 0.02 | 0.02 |
| **ρ** | 0.02 | 0.02 | 0.02 | 0.02 | **0.03** | 0.02 |
| **γ** | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | **0.030** |
| **expected risky rate** | 0.094 | 0.070 | 0.118 | 0.094 | 0.104 | 0.109 |
| **risk-free rate** | 0.093 | 0.069 | 0.117 | 0.091 | 0.103 | 0.108 |
| **spread** | 0.001 | 0.001 | 0.002 | 0.003 | 0.001 | 0.001 |
| **expected ΔC/C** | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.030 |
| **price-earnings ratio** | 14.0 | 21.8 | 10.2 | 14.2 | 12.1 | 12.1 |
| **leverage coefficient** | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| **debt-equity ratio** | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| **r: levered equity** | 0.095 | 0.070 | 0.119 | 0.094 | 0.105 | 0.110 |
| **leverage coefficient** | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| **debt-equity ratio** | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 |
| **r: levered equity** | 0.095 | 0.071 | 0.119 | 0.095 | 0.105 | 0.110 |

Note:  These results use p = q = 0.  The expected risky rate is from Eq. (12).  The risk-free rate is from Eq. (14).  The spread is the difference between these two rates. Expected ΔC/C is the expected growth rate of consumption, given by $\log(E_t A_{t+1}/A_t)$.  The price-earnings ratio comes from Eqs. (17) and (19).  The leverage coefficient, λ, takes on the values 0.2 or 0.4.  The expected rate of return on levered equity is the log of the expression in Eq. (28).  The debt-equity ratio is given in Eq. (30).

## Table 5

## Stock and Bill Returns for G7 Countries

**(averages of arithmetic annual returns, standard deviations in parentheses)**

| Country & time period | real stock return | real bill return | spread |
|---|---|---|---|
| **1.  Long samples** | | | |
| **Canada, 1934-2004** | 0.074 (0.160) | 0.010 (0.036) | 0.063 |
| **France, 1896-2004** | 0.070 (0.277) | -0.018 (0.095) | 0.088 |
| **Italy, 1925-2004** | 0.063 (0.296) | -0.009 (0.128) | 0.072 |
| **Japan, 1923-2004** | 0.092 (0.296) | -0.012 (0.138) | 0.104 |
| **U.K., 1880-2004** | 0.063 (0.183) | 0.016 (0.055) | 0.047 |
| **U.S., 1880-2004** | 0.081 (0.189) | 0.015 (0.048) | 0.066 |
| **Means for 6 countries** | 0.074 (0.234) | 0.000 (0.083) | 0.073 |
| | | | |
| **2.  1954-2004** | | | |
| **Canada** | 0.074 (0.165) | 0.024 (0.024) | 0.050 |
| **France** | 0.091 (0.254) | 0.019 (0.029) | 0.072 |
| **Germany** | 0.098 (0.261) | 0.018 (0.015) | 0.080 |
| **Italy** | 0.067 (0.283) | 0.016 (0.034) | 0.051 |
| **Japan** | 0.095 (0.262) | 0.012 (0.037) | 0.083 |
| **U.K.** | 0.097 (0.242) | 0.018 (0.033) | 0.079 |
| **U.S.** | 0.089 (0.180) | 0.014 (0.021) | 0.076 |
| **Means for 7 countries** | 0.087 (0.235) | 0.017 (0.028) | 0.070 |

Note:  Indexes of cumulated total nominal returns on stocks and government bills or analogous paper are from Global Financial Data.  See Taylor (2005) for a discussion. The nominal values for December of each year are converted to real values by dividing by consumer price indexes.  Annual real returns are computed arithmetically based on December-to-December real values.  CPI data since 1970 are available online from Bureau of Labor Statistics and OECD.  Earlier data are from Bureau of Labor Statistics, U.S. Department of Commerce (1975), Mitchell (1980, 1982, 1983), and Mitchell and Deane (1962).  German data for a long sample were omitted because the German CPI has breaks corresponding to the hyperinflation in 1923-24 and the separation into East and West in 1945.  German data on dividend yields are also unavailable for 1942-52.

| parameter/variable | (1) baseline $p = 0.01$ | (2) high p | (3) low p | (4) high b | (5) low b | (6) high θ | (7) low θ | (8) high ρ |
|---|---|---|---|---|---|---|---|---|
| **Table 6  Rates of Return when Disasters Are Possible (p > 0)** | | | | | | | | |
| **θ** | 3 | 3 | 3 | 3 | 3 | **4** | **2** | 3 |
| **σ** | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| **ρ** | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | **0.03** |
| **γ** | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 |
| **p** | 0.01 | **0.015** | **0.005** | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| **exp(-b)** | 0.5 | 0.5 | 0.5 | **0.4** | **0.75** | 0.5 | 0.5 | 0.5 |
| **expected risky rate** | 0.060 | 0.043 | 0.077 | 0.037 | 0.084 | 0.046 | 0.055 | 0.070 |
| **risk-free rate** | 0.026 | -0.007 | 0.059 | -0.043 | 0.080 | -0.023 | 0.040 | 0.036 |
| **spread** | 0.034 | 0.050 | 0.018 | 0.081 | 0.005 | 0.069 | 0.015 | 0.034 |
| **expected risky rate (conditional)** | 0.065 | 0.050 | 0.080 | 0.043 | 0.087 | 0.051 | 0.060 | 0.075 |
| **expected ΔC/C** | 0.020 | 0.018 | 0.023 | 0.019 | 0.023 | 0.020 | 0.020 | 0.020 |
| **expected ΔC/C (conditional)** | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 |
| **price-earnings ratio** | 24.7 | 39.2 | 17.9 | 55.0 | 15.8 | 38.7 | 28.2 | 19.6 |
| **leverage coefficient** | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| **debt-equity ratio** | 0.26 | 0.27 | 0.26 | 0.28 | 0.25 | 0.27 | 0.26 | 0.26 |
| **r: levered equity** | 0.069 | 0.056 | 0.082 | 0.059 | 0.085 | 0.064 | 0.059 | 0.079 |
| **leverage coefficient** | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| **debt-equity ratio** | 0.71 | 0.73 | 0.69 | 0.77 | 0.67 | 0.76 | 0.69 | 0.71 |
| **r: levered equity** | 0.084 | 0.078 | 0.089 | 0.095 | 0.087 | 0.095 | 0.066 | 0.094 |

Note: These results assume $p > 0$ and $q = 0$. The expected risky rate is from Eq. (12). The risk-free rate is from Eq. (14). The spread is the difference between these two rates. The expected risky rate (conditional) is the expected risky rate conditioned on no disasters. Expected $\Delta C/C$ is the expected growth rate of consumption, given by $\log(E_t A_{t+1}/A_t)$. Expected $\Delta C/C$ (conditional) is the expected growth rate of consumption conditioned on no disasters. The price-earnings ratio comes from Eqs. (17) and (19). The leverage coefficient, $\lambda$, takes on the values 0.2 or 0.4. The expected rate of return on levered equity is the log of the expression in Eq. (28). The debt-equity ratio is given in Eq. (30).

| Table 7  Interest and Inflation Rates during Wars and the Great Depression in the U.S. | | | |
|---|---|---|---|
| Year | nominal return | expected inflation rate | expected real return |
| **Civil War** | | | |
| 1860 | 0.070 | 0.006 | 0.063 |
| 1861 (start of war) | 0.066 | 0.026 | 0.039 |
| 1862 | 0.058 | 0.063 | -0.005 |
| 1863 | 0.051 | 0.082 | -0.031 |
| 1864 | 0.062 | 0.128 | -0.066 |
| 1865 | 0.079 | 0.050 | 0.029 |
| **Spanish-American War** | | | |
| 1897 | 0.018 | 0.015 | 0.004 |
| 1898 (year of war) | 0.021 | 0.006 | 0.015 |
| **World War I** | | | |
| 1914 | 0.047 | 0.021 | 0.026 |
| 1915 | 0.033 | 0.011 | 0.022 |
| 1916 | 0.033 | 0.026 | 0.007 |
| 1917 (U.S. entrance) | 0.048 | 0.075 | -0.028 |
| 1918 | 0.059 | 0.116 | -0.057 |
| **Great Depression** | | | |
| 1929 | 0.045 | 0.000 | 0.044 |
| 1930 (start of depression) | 0.023 | 0.006 | 0.016 |
| 1931 | 0.012 | -0.038 | 0.050 |
| 1932 | 0.009 | -0.059 | 0.068 |
| 1933 (worst of depression) | 0.005 | -0.057 | 0.062 |
| 1934 | 0.003 | 0.022 | -0.020 |
| 1935 | 0.002 | 0.025 | -0.023 |
| 1936 | 0.002 | 0.015 | -0.014 |
| 1937 (onset of sharp recession) | 0.003 | 0.018 | -0.016 |
| 1938 | 0.001 | 0.012 | -0.012 |
| **World War II** | | | |
| 1939 | 0.000 | -0.005 | 0.006 |
| 1940 | 0.000 | 0.005 | -0.005 |
| 1941 (U.S. entrance) | 0.001 | 0.014 | -0.012 |
| 1942 | 0.003 | 0.072 | -0.068 |
| 1943 | 0.004 | 0.053 | -0.049 |
| 1944 | 0.004 | 0.024 | -0.021 |
| 1945 | 0.004 | 0.021 | -0.017 |

| Table 7, continued | | | |
|---|---|---|---|
| Year | nominal return | expected inflation rate | expected real return |
| **Korean War** | | | |
| 1950 | 0.012 | 0.014 | -0.002 |
| 1951 | 0.016 | 0.026 | -0.010 |
| 1952 | 0.017 | 0.005 | 0.012 |
| 1953 | 0.019 | -0.009 | 0.028 |
| **Vietnam War** | | | |
| 1964 | 0.036 | 0.011 | 0.025 |
| 1965 | 0.041 | 0.012 | 0.029 |
| 1966 | 0.049 | 0.018 | 0.031 |
| 1967 | 0.044 | 0.022 | 0.022 |
| 1968 | 0.055 | 0.029 | 0.026 |
| 1969 | 0.069 | 0.032 | 0.037 |
| 1970 | 0.065 | 0.036 | 0.029 |
| 1971 | 0.044 | 0.035 | 0.008 |
| 1972 | 0.042 | 0.033 | 0.009 |
| **Gulf War** | | | |
| 1990 | 0.077 | 0.039 | 0.038 |
| 1991 (year of war) | 0.054 | 0.035 | 0.020 |
| 1992 | 0.035 | 0.034 | 0.001 |
| **Afghanistan-Iraq War** | | | |
| 2000 | 0.058 | 0.025 | 0.033 |
| 2001 (September 11) | 0.033 | 0.025 | 0.008 |
| 2002 (start of Afghanistan war) | 0.016 | 0.022 | -0.006 |
| 2003 (start of Iraq war) | 0.010 | 0.017 | -0.006 |
| 2004 | 0.014 | 0.018 | -0.004 |

Note: Nominal returns on U.S. Treasury Bills or commercial paper (before 1922) are calculated as in Table 5. The expected inflation rate for the CPI is constructed as described in the notes to Figure 1. The expected real return is the difference between the nominal return and the expected inflation rate.

| Table 8  Effects of Increases in Disaster Probabilities on Risk-Free Rate and P-E Ratio | | |
| --- | --- | --- |
| range for increases in p and q | effect on risk-free rate | effect on P-E ratio |
| | | |
| $\Delta q < 3\Delta p$ | negative | positive |
| $3\Delta p < \Delta q < 7\Delta p$ | negative | negative |
| $7\Delta p < \Delta q$ | positive | negative |

Note:  The table shows the effects on the risk-free rate and the P-E ratio from increases in the disaster probabilities, p and q.  The risk-free rate is given in Eq. (14).  The P-E ratio is determined from Eqs. (17) and (18).  The results assume the baseline parameter values given in Table 6, column 1.

| Table 9  Regression Estimates of θ | | | | |
|---|---|---|---|---|
| **Generated by Variations in Underlying Parameters** | | | | |
| | | | | |
| | **(1)** | **(2)** | **(3)** | **(4)** |
| **variation in parameter:** | **expected risky rate** | **expected risky rate (conditional)** | **risk-free rate** | **risk-free rate (conditional)** |
| | | | | |
| γ | 3.0 | 3.0 | 3.0 | 3.0 |
| p | 6.8 | ∞ | 13.0 | ∞ |
| b | 16.2 | ∞ | 43.9 | ∞ |
| σ | -3.0 | -3.0 | -9.0 | -9.0 |
| θ | ∞ | ∞ | ∞ | ∞ |
| ρ | ∞ | ∞ | ∞ | ∞ |

Note:  The results correspond to the baseline specification in Table 6, column 1, where the true value of θ is 3.  Each cell shows the estimate of θ that would be found from a standard regression when the data are generated from small variations around the baseline value of the parameter shown in the first column.  In each case, the estimate of θ is the ratio of the change in an expected rate of return to the change in the expected growth rate of consumption.  Column 1 uses the expected risky rate.  Column 2 uses the expected risky rate conditioned on no disasters and also uses the growth rate of consumption conditioned on no disasters.  Column 3 uses the risk-free rate.  Column 4 uses the risk-free rate and the growth rate of consumption conditioned on no disasters.

# Figure 1

## Expected Real Interest Rate on
## U.S. T-Bills/Commercial Paper, 1859-2004
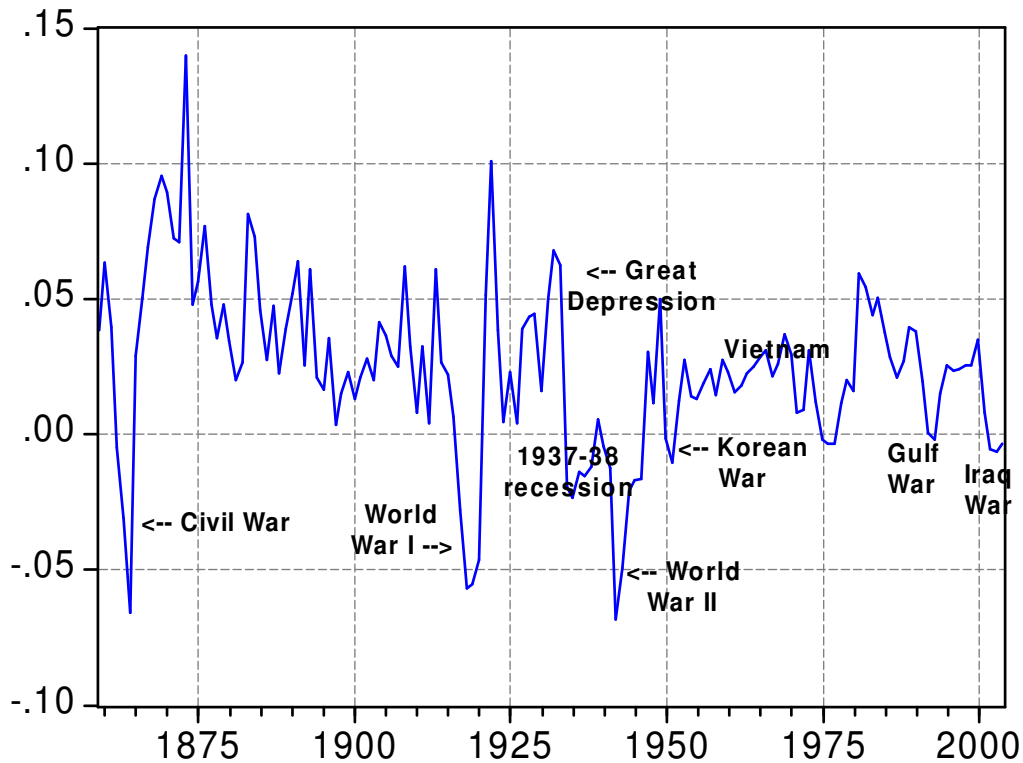


Note: Data on nominal returns on U.S. Treasury Bills (1922-2004) and Commercial Paper (1859-1921) are from Global Financial Data. See the notes to Table 4. From 1947-2004, expected real returns are nominal returns less the Livingston expected inflation rate for the CPI (using six-month-ahead forecasts from June and December). For 1859-1946, the expected real return is the nominal return less a constructed estimate of expected inflation derived from a first-order auto-regression of CPI inflation rates for 1859-1946. The CPI data are from Bureau of Labor Statistics (January values since 1913, annual averages before 1913) and U.S. Department of Commerce (1975).

## Figure 2

## P-E Ratio for U.S. Stock Market, 1871-2004
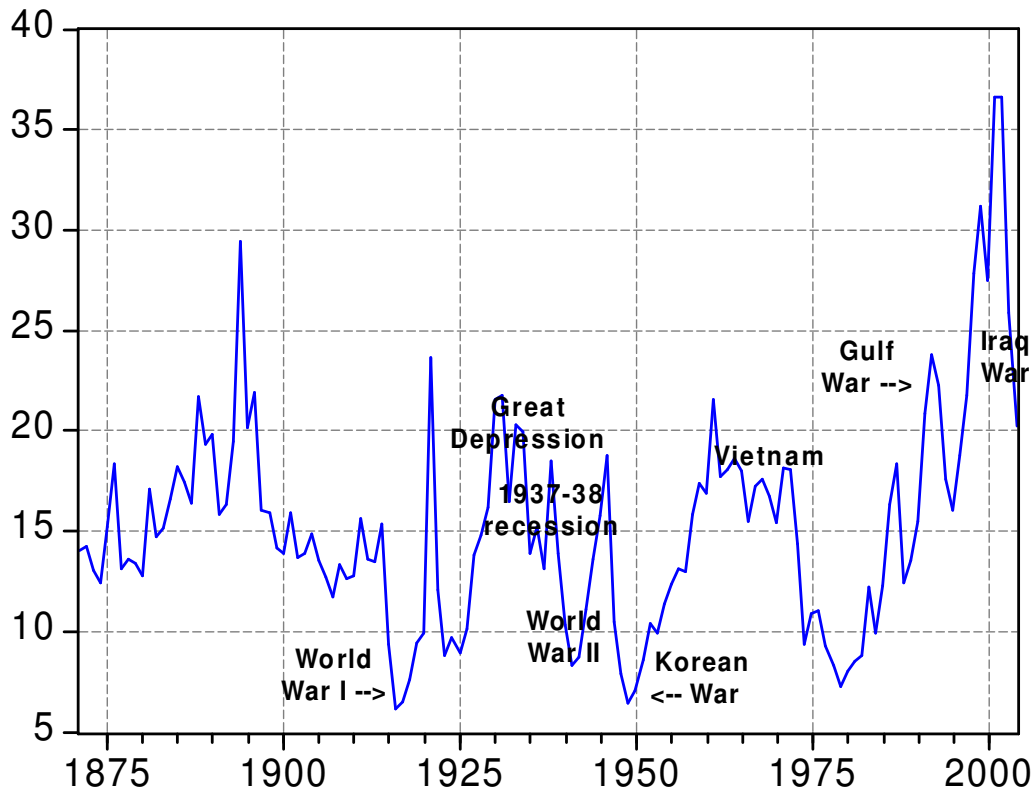


Note:  Data on P-E ratios (annual averages) are from Global Financial Data.  The values correspond to the S& P 500 stock index and analogs computed by S&P and Cowles Commission prior to 1957.

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