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CRISES AND PRICES:
INFORMATION AGGREGATION, MULTIPLICITY AND VOLATILITY

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ABSTRACT

Many argue that crises – such as currency attacks, bank runs and riots – can be described as times of non-fundamental volatility. We argue that crises are also times when endogenous sources of information are closely monitored and thus an important part of the phenomena. We study the role of endogenous information in generating volatility by introducing a financial market in a coordination game where agents have heterogeneous information about the fundamentals. The equilibrium price aggregates information without restoring common knowledge. In contrast to the case with exogenous information, we find that uniqueness may not be obtained as a perturbation from common knowledge: multiplicity is ensured when individuals observe fundamentals with small idiosyncratic noise. Multiplicity may emerge also in the financial price. When the equilibrium is unique, it becomes more sensitive to non-fundamental shocks as private noise is reduced.

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1 Introduction

It's a love-hate relationship, economists are at once fascinated and uncomfortable with multiple equilibria. On the one hand, economic and political crises involve large and abrupt changes in outcomes, but often lack obvious comparable changes in fundamentals. Commentators attribute an important role to more or less arbitrary shifts in 'market sentiments' or 'animal spirits', and models with multiple equilibria formalize these ideas. On the other hand, models with multiple equilibria can also be viewed as incomplete theories that should ultimately be extended along some dimension to resolve the indeterminacy.

The first view is represented by a large empirical and theoretical literature. On the empirical side, Kaminsky (1999), for example, documents that the likelihood of economic crises is affected by observable fundamentals, but that a significant amount of volatility remains unexplained. On the theoretical side, models featuring multiple equilibria attempt to address such non-fundamental volatility. Bank runs, currency attacks, debt crises, financial crashes, riots and political regime changes are modeled as a coordination game: attacking a regime – such as a currency peg or the banking system – is worthwhile if and only if enough agents are also expected to attack.[1]

Morris and Shin (1998, 2000, 2001) contribute to the second view by showing that a unique equilibrium survives in such coordination games when individuals observe fundamentals with small enough private noise.[2] The result is most striking when seen as a perturbation around the original common-knowledge model, which is ridden with equilibria. Most importantly, their contribution highlights the role of the information structure in determining and characterizing the equilibrium set.

The aim of this paper is to understand the role of information in crises. We focus on two distinct forms of non-fundamental volatility: the existence of multiple equilibria and the sensitivity of a unique equilibrium to non-fundamental disturbances. We argue that considering endogenous information is crucial for these questions.

Information is typically treated exogenously in coordination models, but is largely endogenous in most situations of interest. Financial prices and macroeconomic indicators convey information regarding others actions and their information. During times of crises such indicators are monitored intensely and appear to be an important part of the phenomena. As an example, consider the Argentine 2001-2002 crisis, which included devaluation of the peso, sovereign-debt default, and suspension of bank payments. Leading up to the crisis throughout 2001, bank deposits and the peso-forward rate deteriorated steadily. Such

[1] See, for example, Diamond and Dybvig (1983), Obstfeld (1986, 1996), Velasco (1996), Calvo (1988), Cooper and John (1988), Cole and Kehoe (1996).

[2] See also Carlson and van Damme (1993) for the pioneering contribution to global games.

variables were widely reported by news media and investor reports, and were closely watched by people making crucial economic decisions.

These observations lead us to introduce endogenous sources of public information in a coordination game. In our baseline model, individuals observe their private signals and the price of a financial asset. The rational-expectations equilibrium price aggregates disperse private information, but avoids perfect revelation by introducing enough noise in the aggregation process, as in Grossman and Stiglitz (1976). Thus, none of our results are driven by restoring common-knowledge.[3]

The main insight to emerge is that the precision of the *endogenous* public information increases with the precision of the *exogenous* private information. When private information is more precise, individuals' asset demands are more sensitive to their information. As a result, the equilibrium price reacts more sensitively to fundamental variables, thus conveying more precise information.

This result has important implications for the determinacy of equilibria, as a horse-race between private and public information emerges. An increase in the precision of private information makes coordination more difficult as individuals rely more on their own distinct information. However, the consequent increase in the precision of endogenous public information facilitates coordination. This indirect effect typically dominates, reversing the limiting result: multiplicity is *ensured* when individuals observe fundamentals with small enough private noise.

Uniqueness therefore can not be attained as a small perturbation around common-knowledge. To illustrate this point Figure 1 displays the regions of uniqueness and multiplicity in the space of exogenous levels of public and private noise (σ_ε and σ_x, respectively). Multiplicity is ensured when either noise is sufficiently small. In this sense, public and private noise act symmetrically. In contrast, when information is exogenous, less public noise facilitates multiplicity, but less private noise contributes towards uniqueness. Moreover, with endogenous information, equilibria outcomes are continuous with respect to information parameters. In contrast, with exogenous information, the limiting result discussed above illustrates a sharp discontinuity.

Interestingly, multiplicity may emerge, not only with respect to the probability of regime change, but also in the asset price. This occurs when the asset's dividend depends on the endogenous aggregate actions in the coordination game. Different equilibrium prices are then sustained by different self-fulfilling expectations about the others actions, and therefore about the dividend; and multiple such expectations are in turn possible only thanks to the coordinating role prices play in the first place.

[3] Atkeson (2000) has already pointed out that perfectly revealing asset markets could restore multiplicity.

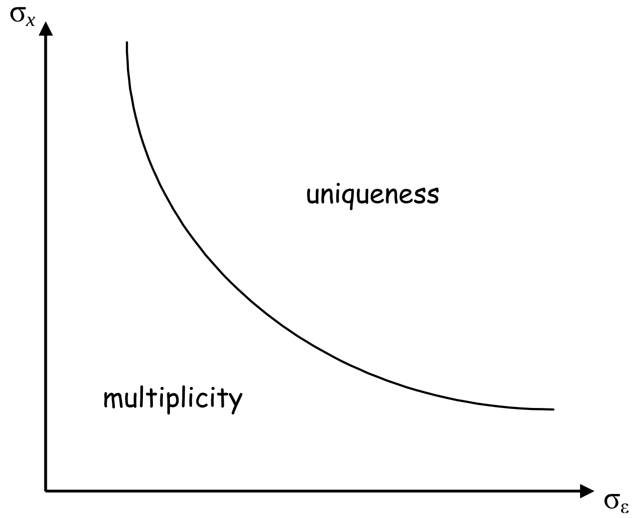

Figure 1: σ_x measures the exogenous noise in private information and σ_ε the exogenous public noise in the aggregation of information.

In regions where the equilibrium is unique, we are able to perform comparative statics. We find that a reduction in exogenous noise increases the sensitivity of the regime outcome to non-fundamental disturbances. When the asset's dividend is endogenous, a reduction in noise may also increase the non-fundamental volatility in the financial price. Since these results parallel the determinants of equilibrium multiplicity, we conclude that lower noise increases volatility for both forms of volatility.

We also study a separate model motivated by the bank run and riots applications. In this model there is no financial market. However, information is endogenous because individuals observe a public noisy signal of others actions. A virtue of this model is that it allows us to study endogenous information with minimal modifications to the original Morris-Shin framework. The model shares the main insights and results for equilibrium multiplicity obtained with financial prices.

Related Literature. The paper contributes to the literature on financial crises[4] by examining the role of information and coordination in generating high-non fundamental volatility, either by introducing multiple equilibria, or with a unique equilibrium. Chari and Kehoe (2003) also study the role of information for non-fundmental volatility, but within the context of a herding model. Our paper can be viewed as bringing coordination games and herding models closer together: by allowing endogenous public signals about others actions, we incorporate an aspect of social learning into a coordination game.

[4]See the references in footnote 1.

Our analysis builds on the work of Morris and Shin and underscores their general theme that the information structure is crucial in coordination games. Within such a framework, our aim is to understand the effect of endogenous sources of public information, such as financial prices and other indicators of aggregate activity. Angeletos, Hellwig and Pavan (2003, 2004) also endogenize the information structure, but not through information aggregation as here. Instead, they examine, respectively, the signaling effects of policy interventions in a static global game and the role of information in a dynamic global game.

Related is Hellwig, Mukherji and Tsyvinski (2004), who consider a currency-crises model in which financial prices directly affect the coordination outcome. In particular, they focus on how multiplicity or uniqueness depends on whether the central bank's decision to devalue is triggered by large reserve losses or high interest rates. Like in our case, they find multiple equilibria for small levels of noise.

Tarashev (2003) also endogenizes interest rates in a currency-crises model, but does not investigate the possibility of equilibrium multiplicity. Dasgupta (2002), on the other hand, introduces signals about past activity in an investment game, but assumes that these signals are purely private, thus also bypassing the possibility of multiple equilibria.

Finally, the paper contributes to the rational-expectations literature by examining the coordinating role of financial prices and its implications for volatility. In Grossman and Stiglitz (1981), the payoff of an agent is independent of the actions of other agents for any given price, and the equilibrium price only aggregates information about the exogenous dividend of the asset. In our environment, instead, the price serves also a coordinating role: agents use the price to predict each others' actions in the coordination game. It is this coordinating role that explains why multiple equilibria and high volatility may emerge in financial prices when the dividend is endogenous.[5]

The rest of the paper is organized as follows. We describe the basic model in Section 2 and review the exogenous information benchmark. Section 3 incorporates the financial asset and examines multiplicity of equilibria. Section 4 examines the determinants of volatility as a comparative static along a unique equilibrium. Section 5 studies the model with a direct signal of aggregate activity. Section 6 concludes.

[5] Barlevy and Veronessi (2004) consider a Grossman-Stiglitz-like economy which admits multiple rational-expectations equilibria, but where the source of multiplicity is very different. In their model, the dividend is exogenous and the price does not play any coordinating role. Multiplicity instead originates from the non-linearity of the inference problem faced by uninformed traders when they interact with informed and less risk-averse agents.

2 The Basic Model: Exogenous Information

Before introducing financial prices or other endogenous signals, we review the basic model with exogenous information, as in Morris-Shin.

Actions and Payoffs. There is a status quo and a measure-one continuum of agents, indexed by $i \in [0,1]$. Each agent i can choose between two actions, either attack the status quo ($a_i = 1$) or not attack ($a_i = 1$). The payoff from not attacking is normalized to zero. The payoff from attacking is $1 - c > 0$ if the status quo is abandoned and $-c$ otherwise, where $c \in (0,1)$ parametrizes the cost of attacking. The status quo is in turn abandoned if and only if $A > \theta$, where A denotes the mass of agents attacking and θ represents the exogenous fundamentals, namely the strength of the status quo. It follows that the payoff of agent i is

$$U(a_i, A, \theta) = a_i(R(A,\theta) - c) \tag{1}$$

where $R(A,\theta)$ denotes the regime outcome, with $R = 1$ if $A > \theta$ and $R = 0$ otherwise.

Interpretations. In models of self-fulfilling currency crises (e.g., Obstfeld, 1986, 1996; Morris and Shin, 1998), a "regime change" occurs when a sufficiently large mass of speculators attacks the currency, forcing the central bank to abandon the peg; θ then parametrizes the amount of foreign reserves or more generally the ability and willingness of the central bank to maintain the peg. In models of self-fulling bank runs, on the other hand, a "regime change" occurs once a sufficiently large number of depositors decide to withdraw their deposits, forcing the bank to suspend its payments; θ then parametrizes the liquid resources available to the banking system.[6]

Throughout the paper, we focus on crises as the main interpretation of the model. Other applications, however, are also possible. For example, one could interpret the model as an economy with investment complementarities, in which $a_i = 1$ represents undertake an investment, A the aggregate level of investment, and θ the exogenous productivity.

In short, the key property of the payoff structure we consider is that it introduces a coordination motive: $U(1,A,\theta) - U(0,A,\theta)$ increases with A, meaning that the individual incentive to take one action increases with the mass of other agents taking the same action. Indeed, when θ is common knowledge, both $A = 1$ and $A = 0$ is an equilibrium whenever $\theta \in (\underline{\theta}, \overline{\theta}] \equiv (0,1]$. The interval $(\underline{\theta}, \overline{\theta}]$ thus represents the set of "critical fundamentals" for which agents can coordinate on multiple courses of action under common knowledge.

Information. Following Morris-Shin, we assume θ is not common knowledge. In the

[6] Other related interpretations include debt crises and financial crashes (Morris and Shin, 2003, 2004; Goldstein and Pauzner; 2000; Corsetti, Guimaraes, and Roubini; 2004; Rochet and Vives 2004). Atkeson (2000), on the other hand, interprets the model as describing riots.

beginning of the game, nature draws θ from a given distribution, which constitutes the agents' common prior about θ. For simplicity, this prior is assumed to be an (improper) uniform over the entire real line.[7] Agent i then receives a private signal $x_i = \theta + \sigma_x \xi_i$, where σ_x is a positive scalar and $\xi_i \sim \mathcal{N}(0,1)$ is idiosyncratic noise, i.i.d. across agents and independent of θ. Agents also observe an *exogenous* public signal $z = \theta + \sigma_z v$, where σ_z is a positive scalar and $v \sim \mathcal{N}(0,1)$ is common noise, distributed independently of both θ and ξ.

The information structure is thus parametrized parsimoniously by σ_x and σ_z, the standard deviations of the two noises; or equivalently by $\alpha_x = \sigma_x^{-2}$ and $\alpha_z = \sigma_z^{-2}$, the *precisions* of private and public information.

Equilibrium Analysis. We focus on *monotone equilibria*, that is, equilibria in which an agent attacks if and only if his private signal is sufficiently low.[8] Thus suppose that, given a realization z of the public signal, an agent attacks if and only if the realization x of his private signal is less than a threshold $x^*(z)$. The size of the attack is then $A(\theta, z) = \Phi\left(\sqrt{\alpha_x}\,(x^*(z) - \theta)\right)$ and is decreasing in θ. It follows that regime change occurs if and only if $\theta \leq \theta^*(z)$, where $\theta^*(z)$ is the unique solution to $A(\theta, z) = \theta$, or equivalently

$$x^*(z) = \theta^*(z) + \tfrac{1}{\sqrt{\alpha_x}} \Phi^{-1}(\theta^*(z)). \tag{2}$$

The posterior of the agent about θ is Normal with mean $\frac{\alpha_x}{\alpha_x + \alpha_z} x + \frac{\alpha_z}{\alpha_x + \alpha_z} z$ and precision $\alpha_x + \alpha_z$, where $\alpha_x = \sigma_x^{-2}$ and $\alpha_z = \sigma_z^{-2}$ denote, respectively, the precision of private and public information. It follows that the agent finds it optimal to attack if and only if $x \leq x^*(z)$, where $x^*(z)$ solve the indifference condition $\Pr\left[\theta \leq \theta^*(z) | x, z\right] = c$, or equivalently

$$\Phi\left(\sqrt{\alpha_x + \alpha_z}\left(\theta^*(z) - \tfrac{\alpha_x}{\alpha_x + \alpha_z} x^*(z) - \tfrac{\alpha_z}{\alpha_x + \alpha_z} z\right)\right) = c. \tag{3}$$

Substituting (2) into (3) gives a single equation in $\theta^*(z)$:

$$\Phi^{-1}\left(\theta^*\left(z\right)\right) - \tfrac{\alpha_z}{\sqrt{\alpha_x}} \theta^*\left(z\right) = \sqrt{1 + \tfrac{\alpha_z}{\alpha_x}}\, \Phi^{-1}\left(1 - c\right) - \tfrac{\alpha_z}{\sqrt{\alpha_x}} z \tag{4}$$

Hence, an equilibrium is simply identified with a solution to (4).

A solution to (4) always exist and is unique for all z if and only if $\alpha_z / \sqrt{\alpha_x} \leq \sqrt{2\pi}$, or equivalently $\sigma_x \leq \sigma_z^2 \sqrt{2\pi}$. (See Appendix for a detailed derivation.) We conclude that the

[7] This assumption is without any serious loss of generality: by letting the common prior be uninformative, we bias the results *against* multiplicity.

[8] This is without serious loss of generality for two reasons. First, when the monotone equilibrium is unique and the information is exogenous, a standard argument of iterrated deletion of strongly dominated strategies can be used to show that there are no other non-monotone equilibria either. Second, to prove the existence of multiple equilibria and the convergence to common-knowledge outcomes with either exogenous or endogenous information, it suffices to look at monotone equilibria.

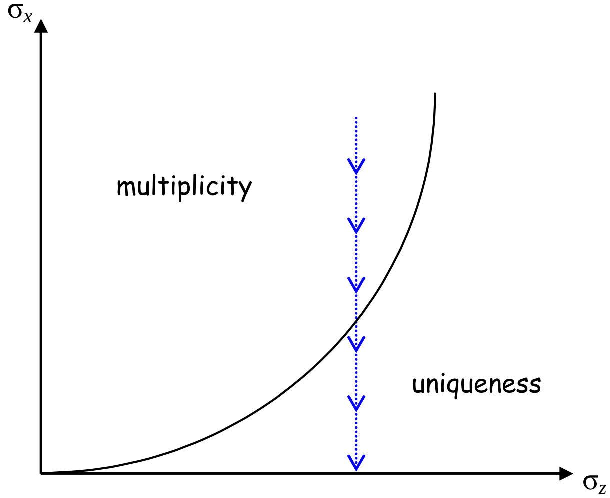

Figure 2: σ_x and σ_z parametrize the noise in private and public information; uniqueness is ensured for σ_x small enough.

equilibrium is unique if and only if the private noise is sufficiently small.

Proposition 1 (Morris-Shin) *In the game with exogenous information, the equilibrium is unique if and only if* $\sigma_x \leq \sigma_z^2 \sqrt{2\pi}$.

This result is illustrated in Figure 2, which depicts the regions of (σ_x, σ_z) for which the equilibrium is unique. For any $\sigma_z > 0$, uniqueness in ensured by letting $\sigma_x > 0$ sufficiently small. In the limit as $\sigma_x \to 0$, the incomplete-information game approaches the common-knowledge game, in which there are multiple equilibria; yet, the equilibrium of the incomplete-information game remains unique and actually asymptotes to a situation where the regime outcome becomes independent of ε. In other words, not only sunspots cease to matter, but all non-fundamental volatility – the volatility of A or R conditional on θ – vanishes.

Corollary 1 *In the limit as* $\sigma_x \to 0$, *there is a unique equilibrium in which* $A(\theta, z) \to 1$ *if* $\theta < \hat{\theta}$ *and* $A(\theta, z) \to 0$ *if* $\theta > \hat{\theta}$, *where* $\hat{\theta} = 1 - c$.

This result is intriguing, as it manifests a sharp discontinuity of the equilibrium set around $\sigma_x = 0$: a *tiny* perturbation away from perfect information obtains a unique equilibrium. The key intuition is that disperse private information serves as an *anchor* for individual behavior: it limits the ability to forecast each others' actions and thereby to coordinate on multiple equilibria.

Indeed, when all agents share the same information about the underlying fundamentals, they can perfectly forecast each others' actions in equilibrium and can therefore perfectly coordinate on either everybody or nobody attacking when $\theta \in (\underline{\theta}, \overline{\theta}]$. But when agents have heterogeneous information about the fundamentals, each agent faces uncertainty regarding other agents' beliefs about θ and their actions. For any given precision of public information, the higher the precision of the agents' private information, the more heavily agents condition their actions on their own private information, and thus the harder it is for other agents to predict their actions. When σ_x is sufficiently small relative to σ_z, this anchoring effect of private information is strong enough that the ability to coordinate on multiple courses of action totally brakes down and a unique equilibrium survives. Finally, as $\sigma_x \to 0$, the private signal become infinitely precise relative to the public signal, in which case agents cease to condition on the public signal and by implication the sensitivity of equilibrium outcomes to the shock ε vanishes.

3 Endogenous Information I: Financial Prices

The results reviewed above presume that the precision of public information remains invariant while changing the precision of private information. This, however, may not be possible when the public information consists of financial prices that endogenously aggregate the disperse private information in the population.

To capture the role of prices as a channel of information aggregation, we modify the environment as follows. There are now two stages. In stage 1, agents trade a financial asset whose dividend depends on the underlying exogenous fundamentals and/or the endogenous outcome of Stage 2. Stage 2 in turn is like the benchmark regime-change game of the previous section, except for that the public signal is now the price that cleared the asset market in stage 1.

This framework opens up new modeling choices regarding the specification of the asset's dividend and the preferences over risky payoffs. We guide our choices with an eye towards tractability: to isolate the effects of information aggregation from any direct payoff linkages between the two stages, we assume that preferences are separable between the first-stage portfolio choice and the second-stage attacking decision;[9] and to preserve normality of the information structure, we model stage 1 along the CARA-normal framework of Grossman and Stiglitz (1976, 1980) and Hellwig (1980).

[9]However, we do allow the payoff of the asset to depend on the aggregate activity of stage 2.

3.1 Model Setup

The game starts again with nature drawing θ from an improper uniform over $\mathbb{R}$. Each agent i has a given endowment of wealth w_i and receives an exogenous private signal $x_i = \theta + \sigma_x \xi_i$. The distribution of w_i is arbitrary and the noise ξ_i is $\mathcal{N}(0,1)$, i.i.d. across agents, and independent of θ.

In stage 1, agents can invest their wealth either in a risk-less asset or a risky asset. The riskless asset is in infinitely elastic supply, it costs 1 in the first stage, and it delivers 1 in the second stage. The risky asset, on the other hand, costs p in the first stage and delivers f in the second.

The agent enjoys utility from final-stage consumption and has constant absolute risk aversion (CARA). The payoff from the portfolio choice is thus $V(c) = -\exp(-\gamma c)/\gamma$, where $c = w - pk + fk$ is consumption, k denotes investment in the risky asset, and $\gamma > 0$ is the coefficient of absolute risk aversion. The net supply of the risky asset is given by an unobserved random variable $K^s(\varepsilon) = \sigma_\varepsilon \varepsilon,$ where $\varepsilon \sim \mathcal{N}(0,1)$ is independent of both the fundamentals and the private noise and σ_ε parametrizes the exogenous noise in the aggregation process. The unobserved shock ε can also be interpreted as the demand of 'noise traders' and its role is to introduce noise in the information revealed by financial prices about fundamentals.

In stage 2, agents chose whether to attack or not. The the status quo is abandoned $(R = 1)$ if and only if $A > \theta$ and the agent's payoff from stage 2 is $U(a, A, \theta) = a(R(A, \theta) - c)$, as in the benchmark model. The regime outcome, the asset's dividend, and the agents' payoffs are realized at the end of stage 2.[10]

For the specification of the asset's dividend f, we consider two alternatives: the case that f is determined merely by the underlying economic fundamentals θ; and the case that f is a function of the endogenous activity A of the coordination stage.

Because of the CARA-normal specification, the demand for the risky asset is independent of wealth. The individual's demand k in stage 1 and action a in stage 2 are thus functions of (x, p) alone, the realizations of the private signal and the equilibrium price. Since there is a continuum of agents, the corresponding aggregates are functions of (θ, p), the underlying fundamental and the price. The equilibrium price, on the other hand, is a function of the exogenous state (θ, ε). We thus define

Definition 1 *A rational-expectations equilibrium is a price function $P(\theta, \varepsilon)$, individual strategies $k(x, p)$ and $a(x, p)$ for investment and attacking, and corresponding aggregates $K(\theta, p)$*

[10] Given the separability of payoff between the two stages, one could reinterpret the model as if the agents who move in stage 1 were different than the agents who move in stage 2, provided of course that the latter observe the price that cleared the asset market in stage 1.

and $A(\theta, p)$, *such that:* (**i**) *in stage* 1,

$$k(x,p) = \arg\max_{k\in\mathbb{R}} \mathbb{E}\left[\ V((f-p)\,k)\ \mid x,p\ \right] \tag{5}$$

$$K(\theta,p) = \int_x k(x,p)d\Phi\left(\frac{x-\theta}{\sigma_x}\right) \tag{6}$$

$$K\left(\theta, P(\theta,\varepsilon)\right) = K^s\left(\varepsilon\right) \tag{7}$$

(**ii**) *in stage* 2,

$$a(x,p) = \arg\max_{a\in[0,1]} \mathbb{E}\left[\ U(a, A(\theta,p), \theta)\ \mid x,p\ \right] \tag{8}$$

$$A(\theta,p) = \int_x a(x,p)d\Phi\left(\frac{x-\theta}{\sigma_x}\right) \tag{9}$$

The interpretation of these conditions is straightforward. Condition (5) requires that an agent's investment take into account the information contained in their private information and prices, whereas condition (7) imposes market clearing in the asset market. Note that (5)-(7) define a standard rational-expectations equilibrium for stage 1, whereas (8)-(9) define a standard Perfect Bayesian equilibrium in stage 2. Finally, with some abuse of notation, we let $R\left(\theta,\varepsilon\right) = R\left(\theta, A\left(\theta, P\left(\theta,\varepsilon\right)\right)\right)$ denote the *equilibrium* regime outcome as a function of the exogenous state $\left(\theta,\varepsilon\right)$.

3.2 Exogenous dividend

We consider first the case that the dividend f depends only on the exogenous economic fundamentals θ. To preserve Normality, we let $f = f\left(\theta\right) = \theta$.

We start the characterization of the equilibrium with stage 1 (the financial market). Thanks to the CARA-Normal specification, the expected utility of the agent reduces to

$$\mathbb{E}\left[V\left(c\right)|x,p\right] = V\left(\left(\mathbb{E}\left[f|x,p\right]-p\right)k - \tfrac{\gamma}{2}\mathrm{Var}[f|x,p]k^2\right)$$

The FOC, which is both necessary and sufficient, implies that the optimal demand for the asset is

$$k(x,p) = \frac{\mathbb{E}[f|x,p]-p}{\gamma\mathrm{Var}[f|x,p]}, \tag{10}$$

where $f=\theta$. We propose that the posterior of θ conditional on x,p has mean $\delta x + (1-\delta)p$ and precision α, for some $\delta \in (0,1)$ and $\alpha > 0$. It follows that $k(x,p) = \left(\delta\alpha/\gamma\right)(x-p)$ and therefore $K(\theta,p) = \left(\delta\alpha/\gamma\right)(\theta-p)$. In equilibrium, $K(\theta,p) = K^s\left(\varepsilon\right)$, which gives the market-clearing price as $p = \theta - (\delta\alpha/\gamma)^{-1}\sigma_\varepsilon\varepsilon$, or equivalently

$$P(\theta, \varepsilon) = \theta - \sigma_p\varepsilon \tag{11}$$

where $\sigma_p = (\delta\alpha/\gamma)^{-1}\sigma_\varepsilon$ is the standard deviation of the price conditional on θ. The observation of p is therefore equivalent to the observation of a Normal signal about θ with precision $\alpha_p = \sigma_p^{-2}$. Moreover, x and p are independent and therefore $\theta|x, p \sim N(\delta x + (1-\delta)p, \alpha)$, $\delta = \alpha_x/\alpha$, and $\alpha = \alpha_x + \alpha_p$, which verifies our initial guess. Solving for α, δ, and α_p, we get $\alpha = \alpha_x(1 + \alpha_x\alpha_\varepsilon/\gamma^2)$, $\delta = 1/(1 + \alpha_x\alpha_\varepsilon/\gamma^2)$, and $\alpha_p = \alpha_\varepsilon\alpha_x^2/\gamma^2$. That is, the precision of the public information revealed by the price is an increasing function of both the precision the aggregation process and the agents' primitive private information. Equivalently,

$$\sigma_p = \gamma\sigma_\varepsilon\sigma_x^2, \tag{12}$$

which together with (11) completes the characterization of stage 1.

Next, consider stage 2 (the coordination stage). Given (11), the continuation game in stage 2 is isomorphic to the benchmark game of Section 2, except for the fact that the public signal is $z = p = P(\theta, \varepsilon)$ and hence $\alpha_z = \alpha_p$. The following are thus immediate implications of our earlier analysis in Section 2: first, the agent's strategy is $a(x, p) = 1$ if and only if $x < x^*(p)$, the aggregate attack is $A(\theta, p) = \Phi\left(\sqrt{\alpha_x}(x^*(p) - \theta)\right)$, and the regime is abandoned if and only if $\theta < \theta^*(p)$; second, the thresholds $x^*(p)$ and $\theta^*(p)$ are given by the solution to (2) and (4) once we replace z with p and α_z with α_p; third, the solution is unique if and only if and only if $\sigma_x \leq \sigma_p^2\sqrt{2\pi}$. Using then (11), we conclude that the equilibrium is unique if and only if $\sigma_\varepsilon^2\sigma_x^3 \geq \gamma^2(2\pi)^{-1/2}$.

Proposition 2 *In the economy with $f = f(\theta)$, there are multiple equilibria if and only if $\sigma_\varepsilon^2\sigma_x^3 < \gamma^2(2\pi)^{-1/2}$. There are then multiple strategies, $a(x, p)$, attacks $A(\theta, p)$ and regime outcomes, $R(\theta, p)$, for the coordination stage, but the asset demands, $k(x, p)$ and $K(\theta, p)$, and the price function, $P(\theta, \varepsilon)$, are always uniquely determined.*

Note how this result contrasts with Proposition 1: when public information is exogenous, a sufficiently low σ_x ensures uniqueness; but when the public information is endogenous as in the present model, a sufficiently low σ_x ensures multiplicity. This is because more precise private information now implies more precise public information, indeed at a rate high enough that the agents' ability to predict each others' actions increases as their information improves. Figure 3 illustrates the result in the (σ_x, σ_z) space: unlike Figure 2, as σ_x decreases, σ_z also decreases and fast enough that the economy necessarily enters the region of multiplicity.

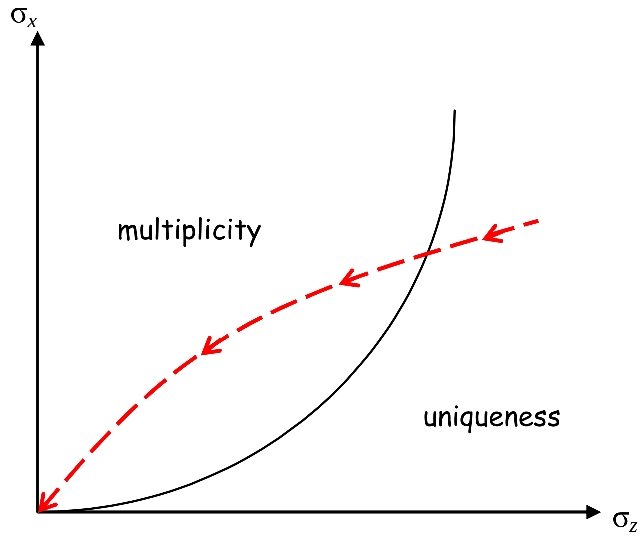

Figure 3: With endogenous public information, as σ_x decreases, σ_z also decreases; multiplicity is therefore ensured for sufficiently small σ_x.

Uniqueness can therefore not be seen as a small perturbation away from common knowledge. As it was earlier illustrated in Figure 1, when either σ_x or σ_ε is small enough, the precision of the endogenous public information is sufficiently high that multiple courses of action can be sustained in the coordination stage. Moreover, the common-knowledge outcomes can actually be recovered as as either type of noise vanishes.

Corollary 2 *Consider the limit as* $\sigma_x \to 0$ *for given* σ_ε*, or the limit as* $\sigma_\varepsilon \to 0$ *for given* σ_x*. There exists an equilibrium in which* $R(\theta,\varepsilon) \to 0$ *whenever* $\theta \in (\underline{\theta}, \overline{\theta})$*, as well as an equilibrium in which* $R(\theta,\varepsilon) \to 1$ *whenever* $\theta \in (\underline{\theta}, \overline{\theta})$*. In every equilibrium,* $P(\theta,\varepsilon) \to \theta$ *for all* (θ,ε)*.*

3.3 Endogenous dividend

We now consider the case that the payoff of the asset is determined by the outcome of the coordination stage. To preserve Normality, we now let $f = f(A) = -\Phi^{-1}(A)$. The asset thus pays more the lower the size of the attack, which we could interpret as a situation where "attack" means refraining from some short of investment.

We start again the equilibrium analysis with stage 1. Guessing (and later verifying) that agents use monotone strategies in stage 2 such that $a(x,p) = 1$ if and only if $x < x^*(p)$ for some threshold $x^*(p)$, we have that $A(\theta,p) = \Phi\left(\sqrt{\alpha_x}[x^*(p) - \theta]\right)$ and therefore $f = \sqrt{\alpha_x}[\theta - x^*(p)]$. Hence, if the agents posterior about θ is Normal (which we verify later), so is f. It follows that the individual optimal demand for the asset is again as in (10). Substituting $f=\sqrt{\alpha_x}[\theta - x^*(p)]$ into the latter and letting $\widetilde{\gamma}=\gamma\sqrt{\alpha_x}$ and

$$\widetilde{p}=\frac{1}{\sqrt{\alpha_x}}p+x^*(p) \tag{13}$$

we can rewrite the optimal demand as $k=\left(\mathbb{E}[\theta|x,p]-\widetilde{p}\right)/\left(\widetilde{\gamma}\text{Var}[\theta|x,p]\right)$. Like in the previous section, we then propose $\mathbb{E}[\theta|x,p]=\delta x+(1-\delta)\,\widetilde{p}$ and $\text{Var}[\theta|x,p]=\alpha$. It follows that $K(\theta,p)=(\delta\alpha/\widetilde{\gamma})(\theta-\widetilde{p})$ and therefore market clearing implies

$$\widetilde{p}=\theta-\frac{\widetilde{\gamma}}{\delta\alpha\sqrt{\alpha_\varepsilon}}\varepsilon. \tag{14}$$

Hence, provided that there is a one-to-one mapping between p and $\widetilde{p}$, the observation of p is equivalent to the observation of a Normal signal about θ with precision $\alpha_p=(\delta\alpha/\widetilde{\gamma})^2\alpha_\varepsilon$ and therefore $\theta|x,p\,\tilde{}\,\mathcal{N}\left(\delta x+(1-\delta)\,\widetilde{p},\alpha\right)$ with $\delta=\alpha_x/\left(\alpha_x+\alpha_p\right)$ and $\alpha=\alpha_x+\alpha_p$. Solving for δ,α, and α_p gives $\alpha_p=\alpha_\varepsilon\alpha_x^2/\widetilde{\gamma}^2=\alpha_\varepsilon\alpha_x^3/\gamma^2$, or equivalently

$$\sigma_p=\gamma\sigma_\varepsilon\sigma_x^3. \tag{15}$$

Next, consider stage 2. The threshold $\theta^*(p)$ solves $\theta=A(\theta,p)$; equivalently,

$$x^*(p)=\theta^*(p)+\tfrac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(p)). \tag{16}$$

The threshold $x^*(p)$, on the other hand, solves $\Pr\left[\theta\leq\theta^*(p)|x,p\right]=c$; equivalently,

$$\Phi\left(\sqrt{\alpha_x+\alpha_p}\left(\theta^*(p)-\tfrac{\alpha_x}{\alpha_x+\alpha_p}x^*(p)-\tfrac{\alpha_x}{\alpha_x+\alpha_p}\widetilde{p}\right)\right)=c. \tag{17}$$

These equations are the analogues of (2) and (4) in the benchmark game. However, once we substitute (13) and (16) into (17), we get a unique solution for $\theta^*(p)$:

$$\theta^*(p)=\Phi\left(\sqrt{\tfrac{\alpha_x}{\alpha_x+\alpha_p}}\Phi^{-1}(1-c)-\tfrac{\alpha_p}{\alpha_x+\alpha_p}p\right), \tag{18}$$

in which case (16) implies also a unique $x^*(p)$. Hence, unlike either the benchmark game or the case of the previous section, the strategy of the agents in the coordination game and the regime outcome are *always* uniquely determined as functions of the realized price.

To complete the analysis, we need to determine the mapping between p and $\widetilde{p}$, that is, the equilibrium price mapping $p=P(\theta,\varepsilon)$. Using (13), the aggregate demand for the asset is

$$K(\theta,p)=\frac{\delta\alpha}{\widetilde{\gamma}}\left(\theta-\frac{1}{\sqrt{\alpha_x}}p-x^*(p)\right). \tag{19}$$

Since $x^*(p)$ is uniquely determined, $K(\theta,p)$ is also uniquely determined (and similarly for $k(x,p)$). Moreover, for any θ, $K(\theta,p)$ is continuous in p, $K(\theta,p)\to\infty$ as $p\to-\infty$, and $K(\theta,p)\to-\infty$ as $p\to+\infty$. It follows that $K(\theta,p)=\varepsilon$, or equivalently (13), always admits a solution; that is, a market-clearing price $p=P(\theta,\varepsilon)$ exists for any θ and ε. However, the market-clearing price need not always be unique. Note that $\theta^*(p)$ and $x^*(p)$ are decreasing in p, which in turn implies the asset's dividend, $f=\sqrt{\alpha_x}[\theta-x^*(p)]$, is itself increasing in p. As a result, the demand for the asset can be non-monotonic. Indeed, (16) and (19) imply

$$sign\left\{\frac{\partial K(\theta,p)}{\partial p}\right\}=-sign\left\{\frac{\sqrt{\alpha_x}}{\alpha_p}-\phi\left(\Phi^{-1}(\theta^*)\right)\right\}$$

and therefore $K(\theta,p)$ is globally decreasing in p if and only if $\sqrt{\alpha_x}/\alpha_p\geq\sqrt{2\pi}$, or equivalently $\sigma_\varepsilon^2\sigma_x^3\geq\gamma^2/\sqrt{2\pi}$. If instead $\sigma_\varepsilon^2\sigma_x^3<\gamma^2/\sqrt{2\pi}$, the aggregate demand for the asset is non-monotonic and there is a non-empty interval $(\widetilde{p}_1,\widetilde{p}_2)$ such that (13) admits a single solution whenever $\widetilde{p}\notin(\widetilde{p}_1,\widetilde{p}_2)$ but three solutions whenever $\widetilde{p}\in(\widetilde{p}_1,\widetilde{p}_2)$. Different selections from these solutions then sustain different mappings between $\widetilde{p}$ and p, that is, different equilibrium price functions $P(\theta,\varepsilon)$.

Proposition 3 *In the economy with $f=f(A)$, there are multiple equilibria if and only if $\sigma_\varepsilon^2\sigma_x^5<\gamma^2(2\pi)^{-1/2}$. There are then multiple price functions $P(\theta,\varepsilon)$, but the asset demands $k(x,p)$, $K(\theta,p)$, the strategy $a(x,p)$, and the attack $A(\theta,p)$ are always uniquely determined.*

In the model of the previous section, the dividend was a function of θ alone and therefore the price played the role of a signal about the exogenous fundamental. As a result, indeterminacy emerged for the strategies and the size of the attack in the coordination stage, but not for the price clearing the asset market. Here, instead, the dividend depends on A and therefore the price plays the role of an *anticipatory* signal about the endogenous size of the attack. A particular price realization coupled with an agent's private information then pins down a unique posterior about the size of the attack, in which case the agent's best response is of course unique. This explains why the strategies in stage 2 are uniquely determined.

In stage 1, on the other hand, different levels of the price are associated with different expectations about the size of the attack and therefore about the dividend of the asset: in equilibrium, a higher price signals a higher dividend. When this effect is strong enough, it can offset the usual negative payoff effect of the price – namely that a higher price means a higher cost of obtaining the asset – and may therefore induce the demand for the asset to increase with the price. This *non-monotonicity* of the asset demand introduces the possibility for multiple market-clearing prices. Finally, the positive effect of the price is stronger when α_p is higher, for a higher α_p increases the sensitivity of the agents' actions in stage 2 to the price and therefore of the dividend to the price. This explains why multiple equilibrium prices exist for for small levels of noise.

To further understand why multiplicity emerges in the price and not in the strategy of the agents, it helps to consider for a moment the no-noise game ($\sigma_x = \sigma_\varepsilon = 0$). The exogenous signal is then $x = \theta$ and the equilibrium price is $p = -\Phi^{-1}(A)$ and therefore an agent learns perfectly θ by observing x and A by observing p. Clearly, the agent then finds it optimal to attack if and only if $A \geq \theta$, or equivalently $x \leq \Phi(-p)$, which means that his strategy is uniquely determined as a function of x and p. However, the equilibrium values of p and A are not uniquely determined. Instead, for every $\theta \in (\underline{\theta}, \overline{\theta}]$, both $(p, A) = (\infty, 0)$ and $(p, A) = (-\infty, 1)$ can be sustained in equilibrium.

In conclusion, small noise now ensures multiplicity in both the asset price and the coordination outcome: different equilibrium price functions result to different realizations of the price and the size of the attack for the same θ and ε and therefore to different regime outcomes as well. What is more, the no-noise (or common-knowledge) outcomes are once again obtained as the noise vanishes.

Corollary 3 *Consider the limit as $\sigma_x \to 0$ for given σ_ε, or the limit as $\sigma_\varepsilon \to 0$ for given σ_x. There is an equilibrium in which $R(\theta, \varepsilon) \to 1$ and $P(\theta, \varepsilon) \to -\infty$ whenever $\theta \in (\underline{\theta}, \overline{\theta})$, as well as an equilibrium in which $R(\theta, \varepsilon) \to 1$ and $P(\theta, \varepsilon) \to -\infty$ whenever $\theta \in (\underline{\theta}, \overline{\theta})$.*

3.4 Discussion

Although the property that the precision of endogenous public information increases with the precision of exogenous private information is likely to be very robust, how strong this effect is and whether it can overturn the direct effect of private information may depend on the details of the channel through which private information is aggregated. Indeed, the result that multiplicity is ensured for small σ_x relies on the precision of public information increasing at a rate higher than the square root of the precision of public information. That was the case in the two specifications we consider above (as evident in (12) and (15)) but need not be true in general. We now provide an example which illustrates this possibility.

To this aim, we modify stage 1 as follows. The agent is assumed to be risk neutral and face a quadratic liquidity/transaction cost for investing in the risky asset. The indirect utility from the portfolio choice is thus given by

$$V(c) = c = w - pk - \frac{\kappa}{2}k^2 + fk \tag{20}$$

where the scalar $\kappa > 0$ parametrizes the liquidity/transaction cost. We consider again two cases for the dividend, exogenous and endogenous.

Proposition 4 *Suppose V is given by (20).*

(i) When $f = \theta$, the equilibrium price function P is always unique, whereas the equilibrium regime outcome R is unique if and only if σ_x is either sufficiently small or sufficiently high relative to σ_ε.

(ii) When $f = -\Phi^{-1}(A)$, there are multiple equilibria if and only if σ_x and/or σ_ε are sufficiently small. Multiplicity then emerges in both the regime outcome R and the price function P.

Here, unlike the earlier CARA-normal cases, the increase in the precision of public information generated by an increase in the precision of private information is not always strong enough to offset the direct effect of the private information when the payoff of the asset is exogenous. This is because the slope of individual demand on the expected dividend is independent of the risk in the dividend, which implies that the precision of public information increase less with private information than when the slope itself increases with more precise information (less risk). When the dividend depends only on θ, the anchoring effect of private information dominates, thus ensuring uniqueness for σ_x small enough.

When, however, the dividend of the asset depends on the size of the attack, the fact that the sensitivity of the size of the attack and therefore of the dividend itself to the fundamental increases as the agents' information improves compensates for the lack of such a sensitivity in the demand for the asset. As a result, coordination once again becomes easier as private information becomes more precise and multiplicity reemerges for small enough noise.

These two examples highlight that the details of the channel through which information is aggregated matters: whether it takes a small or a large perturbation away from common knowledge for uniqueness to prevail – or, more gererally, what is the region of σ_x and σ_ε for which there are multiple equilibria – may depend on the set of financial assets traded, the preferences of the agents, and other institutional details of the particular application one examines.

In Section 5, we consider a different model where there is no financial market to aggregate information, but agents observe a direct signal about others' activity when they decide whether to attack. We find that the coordinating effect of this endogenous signal is once again strong to deliver multiplicity when σ_x or σ_z are small enough.

Hellwig, Mukherji, and Tsyvinski (2004), on the other hand, consider a currency-crises model in which the coordination game is embedded in the financial market: they assume that the dividend of the asset (i.e., a peso bond in their model) is itself low when the price of the asset is low (i.e., when interest rates are high). They also find that multiple equilibria exist when the private noise is small enough.

In the next section, which examines the implications of our analysis for non-fundamental volatility, we continue to focus on cases in which the coordinating effect of prices is sufficiently strong. Although, as we have just shown, this is not always true, we beleive this is an interesting benchmark.

4 Noise and Volatility

We now investigate the role of the information structure for *non-fundamental* volatility, that is, the volatility of equilibrium outcomes conditional on θ. We consider in turn two sources of non-fundamental volatility: volatility generated by payoff-irrelevant variables (sunspots) when there are multiple equilibria and agents use these sunspots to coordinate their behavior; and volatility generated by the shock ε when the equilibrium is unique.

With exogenous information, multiplicity disappears when agents observe the fundamentals with small idiosyncratic noise. By implication, there is no sunspot volatility when σ_x is small enough. What is more, as $\sigma_x \to 0$, the size of the attack and by implication the regime outcome become independent of ε. Hence, all non-fundamental volatility vanishes as $\sigma_x \to 0$. On the other hand, non-fundamental volatility is maximized when $\sigma_z \to 0$ for given σ_x, as in this case the common-knowledge outcomes are obtained.

With endogenous information, the impact of private noise on volatility is quite different. A sufficiently large reduction in either σ_x or σ_ε can *increase* volatility by ensuring multiplicity and therefore introducing more sunspot volatility. Corollaries 2 and 3 indeed imply that sunspot volatility is maximized when either noise vanishes: as $\sigma_x \to 0$ or $\sigma_\varepsilon \to 0$, the regime can either collapse or survive for any given $\theta \in (\underline{\theta}, \overline{\theta}]$, purely as a function of the sunspot. What is more, sunspot volatility can show up in prices as well: when the dividend is endogenous, the equilibrium price can be arbitrarily low or arbitrarily high for any given $\theta \in (\underline{\theta}, \overline{\theta}]$.

The property that, with endogenous information, less noise may increase volatility does not rely on the existence of multiple equilibria. As we show next, when the equilibrium is unique, a reduction in either σ_x or σ_ε may increase the sensitivity of the regime outcome and the asset price to the exogenous shock ε and may therefore result to higher non-fundamental volatility.

4.1 Regime Volatility

In equilibrium, the regime is abandoned ($R = 1$) if and only if $\theta \leq \theta^* (p)$, but p in turn depends on θ, since $p = P(\theta, \varepsilon)$. To express the equilibrium regime outcome as a function of the exogenous variables θ and ε, note that, as long as the equilibrium is unique, $\theta^* (p)$ is continuously decreasing in p and $P(\theta, \varepsilon)$ is continuously increasing in θ. It follows that $R(\theta, \varepsilon) = 1$ if and only if $\theta \leq \hat{\theta}(\varepsilon)$, where $\hat{\theta}(\varepsilon)$ is the unique solution to

$$\hat{\theta} = \theta^*(P(\hat{\theta}, \varepsilon)).$$

We can thus examine the non-fundamental volatility of the regime outcome by examining the sensitivity of $\hat{\theta}(\varepsilon)$ to ε.

Consider first the case the dividend is exogenous ($f = f(\theta)$). Using the results of Section 3.2, we get

$$\hat{\theta}(\varepsilon) = \Phi\left(\psi + \frac{1}{\gamma \sigma_\varepsilon \sigma_x^2}\varepsilon\right) \tag{21}$$

where $\psi = \sqrt{1 + 1/(\gamma^2 \sigma_\varepsilon^2 \sigma_x^4)}\Phi^{-1}(1 - c)$. It follows that

$$\frac{\partial \hat{\theta}}{\partial \varepsilon} = \frac{\phi(\Phi^{-1}(\hat{\theta}))}{\gamma \sigma_\varepsilon \sigma_x^2} \tag{22}$$

and therefore, for any given $\hat{\theta}$, a reduction in σ_ε or σ_x increases the slope of $\hat{\theta}(\varepsilon)$ with respect to ε. By implication, $\hat{\theta}$ satisfies a single-crossing property with respect to σ_x or σ_ε: let ε_0 be the unique value of ε for which $\partial \hat{\theta}/\partial \sigma_\varepsilon = 0$ or equivalently $\partial \hat{\theta}/\partial \sigma_x = 0$; for any ε_1 and ε_2 such that $\varepsilon_1 < \varepsilon_0 < \varepsilon_2$, we still have that $\partial |\hat{\theta}(\varepsilon_2) - \hat{\theta}(\varepsilon_1)|/\sigma_\varepsilon < 0$ and similarly $\partial |\hat{\theta}(\varepsilon_2) - \hat{\theta}(\varepsilon_1)|/\sigma_\varepsilon < 0$. In this sense, a reduction in either type of noise increases non-fundamental volatility.

The result is illustrated in Figure 4. The solid line depicts the threshold $\hat{\theta}(\varepsilon)$ as a function of ε for a relatively high σ_x, whereas the dashed line corresponds to a relatively low σ_x. A similar single-crossing property holds when the dividend is endogenous ($f = f(A)$). Indeed, (21) and (22) continue to hold if we replace γ with $\widetilde{\gamma} = \gamma \sigma_x$. We conclude that

Proposition 5 *Less noise implies more non-fundamental volatility even when the equilibrium is unique: for any given $\hat{\theta}$, a reduction in σ_ε or σ_x increases the slope of $\hat{\theta}(\varepsilon)$ with respect to ε.*

Our earlier results regarding equilibrium multiplicity can thus be viewed as an extreme reincarnation of the above result. When the noise is sufficiently small, volatility can be high,

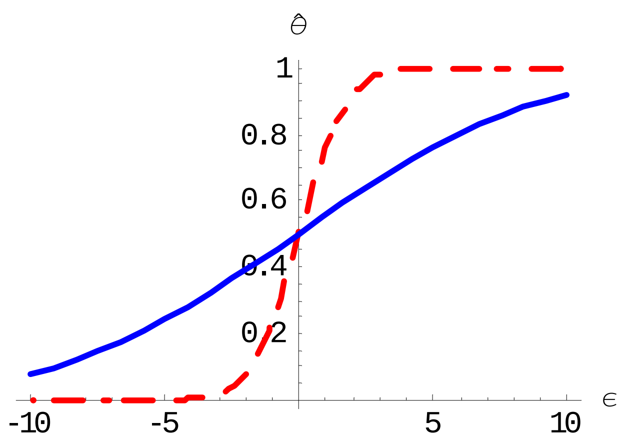

Figure 4: The regime-change threshold $\hat{\theta}(\varepsilon)$ as a function of the shock ε.

not only because the outcome is very sensitive to the exogenous noise, but also because the outcome can depend on arbitrary sunspots.

4.2 Price Volatility

We next examine the comparative statics of the volatility of prices. To emphasize the implications for volatility that do not derive from multiplicity, we first focus again on case that the equilibrium is unique or that there are no sunspots.

When the dividend is exogenous, we have $f = \theta$, $p = f - \sigma_p \varepsilon$, and $\sigma_p = \gamma \sigma_\varepsilon \sigma_x^2$. When instead the dividend is endogenous, we have $f = \Phi^{-1}(A) = (\theta - x^*)/\sigma_x$, $p = f - (\sigma_p/\sigma_x)\,\varepsilon$, and $\sigma_p = \gamma \sigma_\varepsilon \sigma_x^3$. In either case, we can write the equilibrium price as the sum of the dividend and the supply shock appropriately weighted:

$$p = f - \left(\gamma \sigma_\varepsilon \sigma_x^2\right) \varepsilon \tag{23}$$

Keeping f constant, the volatility of the price clearly decreases with a reduction in either σ_x or σ_ε. But what about the volatility of the dividend itself?

When the dividend is exogenous, f is independent of ε. The impact of noise is then exactly like in Grossman-Stiglitz: a reduction in either σ_x or σ_ε implies lower volatility in equilibrium prices.

When instead the dividend is endogenous, f depends on ε, because agents' actions in the second stage depend on the price, which in turn depends on the shock ε. Moreover, for essentially the same reason that a reduction in noise increases the sensitivity of the regime outcome to the shock ε, a reduction in noise increases the sensitivity of f on ε. As a result,

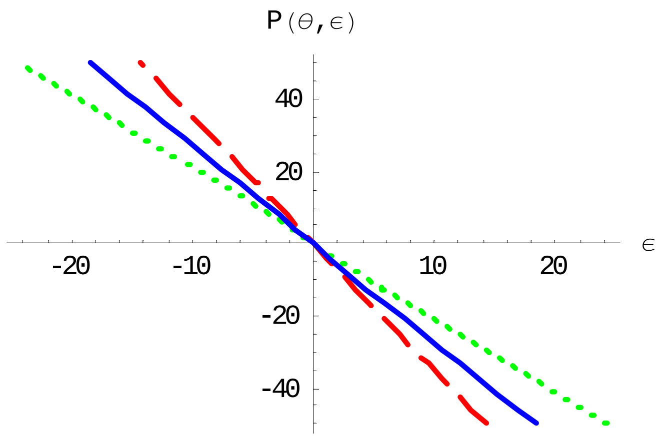

Figure 5: The equilibrium price $P(\theta, \varepsilon)$ as a function of the shock ε.

the overall impact of noise on the volatility of prices is now ambiguous: on the one hand, less noise reduces the volatility of the price for any given volatility of the dividend; on the other hand, less noise increases the volatility of the dividend itself.

The second effect indeed dominates in some cases. An example is illustrated in Figure 5, which depicts $P(\theta, \varepsilon)$ for different values of ε and a given θ. The solid line corresponds to a relatively high σ_x, the dotted one to an intermediate σ_x, and the dashed one to a relatively low σ_x. (In all cases, however, σ_x is high enough that the equilibrium in unique.) The effect of σ_x on the sensitivity of $P(\theta, \varepsilon)$ to ε is non-monotonic: it is highest when σ_x is either high or low. A similar picture emerges if we consider the impact of σ_ε. We thus conclude:

Proposition 6 *When the dividend is exogenous, the volatility of the price conditional on θ necessarily decreases with a reduction in either σ_x or σ_ε. But when the dividend depends on the coordination outcome, a reduction in either noise can increase price volatility.*

5 Endogenous Information II: Observable Actions

In the analysis so far, we have assumed that agents observe a signal generated by a different stage of economic interactions than the coordination game. We now examine the case that the information originates in the coordination game itself. In particular, we assume away the financial market and let agents observe a noisy public signal about the activity of other agents in the coordination game.

We first consider a model where the signal is about others' *contemporaneous* actions. In this case, our equilibrium concept is novel and unavoidably at the crossroads of rational-expectations and game theory. We later show that the same results can be obtained in a dynamic variant of this model, in which the population is divided into two groups: 'early' movers, who have only private information about the fundamentals, and 'late' movers, who can also observe a public signal about the early movers' activity. In the latter case, the equilibrium concept is standard game-theoretic.

5.1 Model set-up

There is no asset market and, except for the endogenous public signal, the game is identical to the benchmark model analyzed in Section 2. All agents move simultaneously after observing private signals about the fundamentals and a public signal about the size of the attack. The private signals are again $x = \theta + \sigma_x \xi$, whereas the public signal is

$$y = s(A, \varepsilon)$$

where $s : [0,1] \times \mathbb{R} \to \mathbb{R}$ and ε is noise, independent of θ and ξ. To preserve Normality of the information structure and obtain closed-form solution, we let $s(A, \varepsilon) = \Phi^{-1}(A) + \sigma_\varepsilon \varepsilon$ and $\varepsilon \sim \mathcal{N}(0,1)$.[11] The exogenous information structure is then parameterized by the pair of standard deviations $(\sigma_x, \sigma_\varepsilon)$.

Since the information of each agent includes a signal about other agents' actions, the equilibrium concept we define is a hybrid of a perfect Bayesian equilibrium and a rational-expectations equilibrium.

Definition 2 *A rational-expectations equilibrium consists of an endogenous signal $y = Y(\theta, \varepsilon)$, an individual attack strategy $a(x, y)$, and an aggregate attack $A(\theta, y)$, that satisfy:*

$$a(x,y) = \arg\max_{a \in [0,1]} \mathbb{E}[\, U(a, R(\theta, y)) \mid x, \ y \,] \tag{24}$$

$$A(\theta, y) = \int_x a(x,y) d\Phi\left(\frac{x-\theta}{\sigma_x}\right) \tag{25}$$

$$y = s(A(\theta, y), \varepsilon) \tag{26}$$

for all $(\theta, \varepsilon, x, y) \in \mathbb{R}^4$, where $R(\theta, y) = 1$ if $A(\theta, y) \geq \theta$ and $R(\theta, y) = 0$ otherwise.

Condition (24) means that $a(x, y)$ is the optimal strategy for the agent, whereas condition (25) means that $A(\theta, y)$ is simply the aggregate across agents. The fixed-point relation introduced by (26) is the rational-expectations feature of our context: the signal y must be generated by individual actions, which in turn are contingent on y.

[11] This convenient specification was introduced by Dasgupta (2002).

5.2 Equilibrium analysis

We focus again on monotone equilibria, in which an agent attacks if and only if $x \leq x^*(y)$ and the status quo is abandoned if and only if $\theta \leq \theta^*(y)$. A monotone equilibrium is thus identifies with a triplet of functions x^*, θ^*, and Y.

Given that an agent attacks if and only if $x \leq x^*(y)$, the aggregate attack is $A(\theta, y) = \Phi\left(\sqrt{\alpha_x}(x^*(y) - \theta)\right)$. It follows that the regime is abandoned if and only if $\theta \leq \theta^*(y)$, where $\theta^*(y)$ solves $A(\theta, y) = \theta$, or equivalently

$$x^*(y) = \theta^*(y) + \tfrac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(y)). \tag{27}$$

Condition (26), on the other hand, implies that the signal must satisfy $y = \sqrt{\alpha_x}\left[x^*(y) - \theta\right] + \sigma_\varepsilon \varepsilon$, or equivalently

$$x^*(y) - \sigma_x y = \theta - \sigma_x \sigma_\varepsilon \varepsilon. \tag{28}$$

Note that (28) is a mapping between y and $z = \theta - \sigma_x \sigma_\varepsilon \varepsilon$. Define the correspondence

$$\mathcal{Y}(z) = \left\{\, y \in \mathbb{R} \mid x^*(y) - \sigma_x y = z \,\right\}. \tag{29}$$

We will later show that $\mathcal{Y}(z)$ is non-empty and examine when it is single- or multi-valued. Now take any function $\tilde{Y}(z)$ that is a selection from this correspondence – that is, such that $\tilde{Y}(z) \in \mathcal{Y}(z)$ for all z – and let $Y(\theta, \varepsilon) = \tilde{Y}(\theta - \sigma_x \sigma_\varepsilon \varepsilon)$. Any such selection preserves normality of the information structure.

Indeed, the observation of $y = Y(\theta, \varepsilon)$ is then equivalent to the observation of $z = \theta - \sigma_x \sigma_\varepsilon \varepsilon = Z(y)$, where $Z(y) = x^*(y) - \sigma_x y$. Therefore, it is as if the agents observe a Normal public signal about θ with precision $\alpha_z = \alpha_x \alpha_\varepsilon$, or equivalently

$$\sigma_z = \sigma_x \sigma_\varepsilon. \tag{30}$$

The precision of endogenous public information is thus once again increasing in the precision of exogenous private information.

For the individual agent in turn to find it optimal to attack if and only if $x \leq x^*(y)$, it must be that $x^*(y)$ solves the indifference condition $\Pr\left(\theta \leq \theta^*(y) \mid x, y\right) = c$, or equivalently

$$\Phi\left(\sqrt{\alpha_x + \alpha_z}\left(\theta^*(y) - \tfrac{\alpha_x}{\alpha_x + \alpha_z} x^*(y) - \tfrac{\alpha_z}{\alpha_x + \alpha_z} Z(y)\right)\right) = c. \tag{31}$$

Using $Z(y) = x^*(y) - \sigma_x y$ and substituting $x^*(y)$ from (27), we get

$$\theta^*(y) = \Phi\left(\frac{\alpha_z}{\alpha_x + \alpha_z}y + \sqrt{\frac{\alpha_x}{\alpha_x + \alpha_z}}\Phi^{-1}(1-c)\right), \tag{32}$$

which together with (27) determines unique $\theta^*(y)$ and $x^*(y)$. Hence, the strategy of the agents $a(x, y)$ and the aggregate attack $A(\theta, y)$ are uniquely determined.

We finally need to examine the equilibrium correspondence $\mathcal{Y}(z)$. Recall that this is given by the set of solutions to $x^*(y) - \sigma_x y = z$. Using (27) and (32), this reduces to

$$F(y) \equiv \Phi\left(\frac{\alpha_z}{\alpha_x + \alpha_z}y + \Lambda\right) + \frac{1}{\sqrt{\alpha_x}}\left(-\frac{\alpha_x}{\alpha_x + \alpha_z}y + \Lambda\right) = z, \tag{33}$$

where $\Lambda \equiv \sqrt{\alpha_x/(\alpha_x + \alpha_z)}\Phi^{-1}(1-c)$. Note that $F(y)$ is continuous in y, and $F(y) \to -\infty$ as $y \to +\infty$, and $F(y) \to +\infty$ as $y \to -\infty$, which ensures that $\mathcal{Y}(z)$ is non-empty and therefore that an equilibrium always exist. Next, note that

$$sign\{F'(y)\} = -sign\left\{1 - \frac{\alpha_z}{\sqrt{\alpha_x}}\phi\left(\tfrac{\alpha_z}{\alpha_x + \alpha_z}y + \Lambda\right)\right\}$$

and therefore $F(y)$ is globally monotonic if and only if $\alpha_z/\sqrt{\alpha_x} \leq \sqrt{2\pi}$, in which case $\mathcal{Y}(z)$ is single valued. If instead $\alpha_z/\sqrt{\alpha_x} > \sqrt{2\pi}$, there is a non-empty interval $(\underline{z}, \overline{z})$ such that $\mathcal{Y}(z)$ takes three values whenever $z \in (\underline{z}, \overline{z})$. Different selections then sustain different equilibrium signal Y.

Using $\alpha_z = \alpha_\varepsilon \alpha_x$, we conclude that multiple equilibria survive as long as either source of noise is sufficiently small.

Proposition 7 *A monotone rational-expectations equilibrium exists for all* $(\sigma_x, \sigma_\varepsilon)$ *and is unique if and only if* $\sigma_\varepsilon^2 \sigma_x \geq 1/\sqrt{2\pi}$. *If* $\sigma_\varepsilon^2 \sigma_x < 1/\sqrt{2\pi}$, *the equilibrium strategy* a *and aggregate attack* A *remain unique, but there are multiple signal functions* Y.

Interestingly, when multiplicity arises, it is with respect to aggregate outcomes but not with respect to individual behavior. To understand this result, consider the no-noise limit ($\sigma_x = \sigma_\varepsilon = 0$), in which case $x = \theta$ and $y = \Phi^{-1}(A)$. Clearly, the agent finds it optimal to attack if and only if $x \leq \Phi(y)$, which uniquely determines the equilibrium strategy $a(x, y)$ for the agent. However, for every $\theta \in (\underline{\theta}, \overline{\theta}]$, both $(y, A) = (-\infty, 0)$ and $(y, A) = (+\infty, 1)$ can be sustained as equilibria. When σ_x and σ_ε are non-zero, the same nature of indeterminacy remains: the behavior of the agent is uniquely determined for any given x and y, but there can be multiple equilibrium value for y and A for any given realization of θ and ε. In this sense, the multiplicity here is similar to the one we encountered in Section 3.3, when agents traded a financial asset whose dividend depended on the size of the attack.

Finally, the common-knowledge outcomes are once again obtained as either source of information becomes infinitely precise, and therefore sunspot volatility is maximized when the noise vanishes. On the other, the Morris-Shin limit can be obtained if the endogenous public signal becomes infinitely imprecise, in which case it is as if the endogenous signal were not available.

Corollary 4 *(i) Consider the limit as either $\sigma_x \to 0$ or $\sigma_\varepsilon \to 0$. There exists an equilibrium in which $R(\theta, \varepsilon) \to 0$ whenever $\theta \in (\underline{\theta}, \overline{\theta}]$, as well as an equilibrium in which $R(\theta, \varepsilon) \to 1$ whenever $\theta \in (\underline{\theta}, \overline{\theta}]$.*

(ii) Consider the limit as $\sigma_\varepsilon \to \infty$. There is a unique equilibrium in which $R(\theta, \varepsilon) \to 1$ whenever $\theta < \hat{\theta}$ and $R(\theta, \varepsilon) \to 0$ whenever $\theta > \hat{\theta}$, where $\hat{\theta} \equiv 1 - c$.

5.3 Non-simultaneous signal

The analysis above has assumed that agents can condition their decision to attack on a noisy indicator of *contemporaneous* aggregate behavior. We now show that our results extend to a simple dynamic model in which no agent has information about contemporaneous actions of other agents and therefore a standard game-theoretic equilibrium concept (namely PBE) can be used.

The population is divided into two groups, 'early' and 'late' agents. Early agents move first, on the basis of their private information alone. Late agents move second, on the basis of their private information as well as a noisy public signal about the aggregate activity of early agents. Neither group can observe contemporaneous activity, but late agents can condition their behavior on the activity of early agents.

Let $\mu \in (0, 1)$ denote the fraction of early agents, A_1 the aggregate activity of early agents, and A_2 the aggregate activity of late agents. The signal generated by early agents and observed only by late agents is given by

$$y_1 = \Phi^{-1}(A_1) + \varepsilon, \tag{34}$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is independent of θ and ξ. Early agents can condition their actions only on their private information, whereas late agents can condition their actions also on y_1. Finally, the regime changes if and only if $\mu A_1 + (1 - \mu) A_2 \geq \theta$.

We look for perfect Bayesian equilibria in which the strategy of the agents in monotonic in their private information. Since late agents can condition their behavior on y_1 but early agents not, a monotone equilibrium is a scalar $x_1^* \in \mathbb{R}$ and a pair of functions $x_2^* : \mathbb{R} \to \mathbb{R}$ and $\theta^* : \mathbb{R} \to (0,1)$ such that: an early agent attacks if and only if $x \leq x_1^*$; a late agent attacks if and only if $x \leq x_2^*(y_1)$; and the regime is abandoned if and only if $\theta \leq \theta^*(y_1)$.

In such an equilibrium, the aggregate attack of early agents is $A_1(\theta) = \Phi\left(\sqrt{\alpha_x}\,[x_1^* - \theta]\right)$. It follows that $y_1 = \Phi^{-1}(A_1(\theta)) + \varepsilon = \sqrt{\alpha_x}\,[x_1^* - \theta] + \varepsilon$. Hence, in equilibrium, the observation of y_1 is equivalent to the observation of

$$z = x_1^* - \frac{1}{\sqrt{\alpha_x}} y_1 = \theta - \sigma_x \varepsilon,$$

which is a public signal with precision $\alpha_z = \alpha_\varepsilon \alpha_x$ (equivalently, with standard deviation $\sigma_z = \sigma_\varepsilon \sigma_x$), like in the simultaneous-signal model.

Since y_1 and z have the same informational content, we can equivalently express the strategy of a late agent as a function of z instead of y_1 and replace the functions $x_2^*(y_1)$ and $\theta^*(y_1)$ with $x_2^*(z)$ and $\theta^*(z)$. The aggregate attack of late agents is then $A_2(\theta, z) = \Phi\left(\sqrt{\alpha_x}\,[x_2^*(z) - \theta]\right)$ and the overall attack from both groups is $A(\theta, z) = \mu A_1(\theta) + (1 - \mu) A_2(\theta, z)$. It follows that the regime changes if and only if $\theta \leq \theta^*(z)$, where $\theta^*(z)$ solves $A(\theta^*(z), z) = \theta^*(z)$, or equivalently

$$\mu \Phi\left(\sqrt{\alpha_x}\,[x_1^* - \theta^*(z)]\right) + (1 - \mu) \Phi\left(\sqrt{\alpha_x}\,[x_2^*(z) - \theta^*(z)]\right) = \theta^*(z). \tag{35}$$

Next, consider the optimal behavior of the agents. Since the realization of z is known to late agents, their decision problem is like in the benchmark model: the threshold $x_2^*(z)$ solves $\Pr\left[\theta \leq \theta^*(z) | x_2^*(z), z\right] = c$, or equivalently

$$\Phi\left(\sqrt{\alpha}\left(\delta x_2^*(z) + (1 - \delta) z - \theta^*(z)\right)\right) = 1 - c, \tag{36}$$

where $\delta = \alpha_x / (\alpha_x + \alpha_z)$ and $\alpha = \alpha_x + \alpha_z$. Early agents, on the other hand, do not observe z and therefore face a double forecast problem: they are uncertain about both the fundamental and the signal upon which late agents will condition their behavior. The threshold x_1^* solves $\Pr\left[\theta \leq \theta^*(y) | x_1^*\right] = c$, or equivalently

$$\int \Phi\left(\sqrt{\alpha_x}\,[x_1^* - \theta^*(z)]\right) \sqrt{\alpha_1} \phi\left(\sqrt{\alpha_1}\,[x_1^* - z]\right) dz = 1 - c, \tag{37}$$

where $\alpha_1 = \alpha_x \alpha_\varepsilon / (1 + \alpha_\varepsilon)$.[12]

A monotone equilibrium is therefore a joint solution to (35)-(37). We can reduce the

[12]To see this, note that $z = \theta - \sigma_x \varepsilon = x - \xi - \sigma_x \varepsilon$, so that $z | x \sim \mathcal{N}\left(0, \sigma_x^2 + \sigma_x^2 \sigma_\varepsilon^2\right)$. That is, conditional on x, z is distributed normal with precision $\alpha_1 = \alpha_x \alpha_\varepsilon / (1 + \alpha_\varepsilon)$.

dimensionality of the system by solving (35) for $x_2^*(z)$:

$$x_2^*(z) = \theta^*(z) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}\left(\theta^*(z) + \frac{\mu}{1-\mu}\left\{\theta^*(z) - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta^*(z)\right]\right)\right\}\right).$$

Substituting the above into (36) and using $\delta = \alpha_x/(\alpha_x + \alpha_z)$ and $\alpha = \alpha_x + \alpha_z$, we obtain:

$$\Gamma(\theta^*(z), x_1^*) = g(z), \tag{38}$$

where

$$\Gamma(\theta, x_1) = -\frac{\alpha_z}{\sqrt{\alpha_x}}\theta + \Phi^{-1}\left(\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta\right]\right)\right\}\right),$$

$g(z) = \sqrt{1 + \alpha_z/\alpha_x}\Phi^{-1}(1-c) - \left(\alpha_z/\sqrt{\alpha_x}\right)z$, and $\alpha_z = \alpha_\varepsilon\alpha_x$. We conclude that an equilibrium is a joint solution of (37) and (38) for a threshold $x_1^* \in \mathbb{R}$ and a function $\theta^* : \mathbb{R} \to (0,1)$.

Let $\mathcal{C}$ denote the set of piecewise continuous real functions with range a subset of $[0,1]$. For any given function $\theta^* \in \mathcal{C}$, (37) always defines a unique $x_1^* \in \mathbb{R}$. For given $x_1^* \in \mathbb{R}$, on the other hand, (38) may admit a unique or multiple solutions in $\theta^* \in \mathcal{C}$, depending on $(\alpha_x, \alpha_\varepsilon, \mu)$. Different solutions to (38) for given x_1^* represent different continuation equilibria for the game between late agents defined by a fixed strategy for the early agents. The question of interest, however, is the determinacy of equilibrium in the entire game.

In the appendix we show that, when (38) admits a unique solution for *every* $x_1^* \in \mathbb{R}$, the fixed point of (37)-(38) is also unique. On the other hand, when (38) admits multiple solutions for *every* $x_1^* \in \mathbb{R}$, we prove that (37)-(38) also admits multiple fixed points. This provides us with the following sufficient (but not necessary) conditions for uniqueness and multiplicity:

Proposition 8 (**i**) *There exists a unique equilibrium if*

$$\sigma_\varepsilon^2\sigma_x \geq \frac{1}{\sqrt{2\pi}}(1-\mu)$$

(**ii**) *There exist multiple equilibria if*

$$\sigma_\varepsilon^2\sigma_x < \frac{1}{\sqrt{2\pi}}\left(1 - \mu - \mu\sigma_\varepsilon^2\right)$$

For any σ_ε and σ_x such that $\sigma_\varepsilon^2\sigma_x < 1/\sqrt{2\pi}$, multiplicity in ensured for μ low enough. In this sense, the multiplicity result of the simultaneous-move game survives as long as the fraction of informed (late) agents is high enough. Indeed, the dependence of (38) on x_1^* vanishes as $\mu \to 0$ and therefore it can be shown that the equilibria of the simultaneous-signal model can be approximated by equilibria of this dynamic model as $\mu \to 0$. What is more, for *any* $\mu < 1$, multiple equilibria exist as long as σ_ε and σ_x are sufficiently low. On the other hand, for any σ_x and any σ_ε, uniqueness is ensured by taking $\mu \to 1$, which is also intuitive, since in this case the role of the informed agents vanishes.

We conclude that the insights we derived in the simultaneous-signal model extend to the present framework and do not hinge on the fixed-point nature of the hybrid equilibrium concept we used there.

6 Final Remarks

We view the main theme in Morris-Shin as emphasizing the importance of the details of the information structures for understanding the determinacy of equilibria and the volatility of outcomes. This paper contributes to this same theme by studying the importance of endogenous information aggregation. We model public information by either (i) a financial asset's price that reveals information in equilibrium, or (ii) a direct noisy signal of aggregate activity. An important feature of the equilibrium in all cases is that the precision of public information is endogenous and rises with the precision of private information.

We showed that this effect is typically strong enough to reverse the limiting uniqueness result obtained with exogenous information: multiplicity is now *ensured* when either the idiosyncratic noise in the individuals' observation of fundamentals or the common noise in the aggregation process is small enough; conversely, a unique equilibrium survives when the noise is large enough.

We also showed that less noise may have a destabilizing effect even when the equilibrium is unique: a reduction in either source of noise may increase the sensitivity of the coordination outcome and the price to exogenous shocks, thus leading to an increase in non-fundamental volatility. Our multiplicity result can thus be interpreted as an extreme version of this negative effect of on volatility.

Our results on volatility may help understand crises phenomena such as currency attacks, bank runs, or debt crises. However, we have abstracted from the institutional details of each specific application, which may also be important for the questions of multiplicity and volatility. Extending our analysis to particular applications is thus a promising direction for future research.

7 Appendix

Proof of Proposition 1. Rewrite (4) as

$$G\left(\theta^{*}(z)\right)=g\left(z\right), \tag{39}$$

where $G(\theta)\equiv-\left(\alpha_z/\sqrt{\alpha_x}\right)\theta+\Phi^{-1}\left(\theta\right)$ and $g\left(z\right)=\sqrt{1+\alpha_z/\alpha_x}\Phi^{-1}\left(1-c\right)-\left(\alpha_z/\sqrt{\alpha_x}\right)z$. For every $z\in\mathbb{R}$, $G(\theta)$ is continuous in θ, with $G(0,z)=-\infty$ and $G(1,z)=\infty$, which implies that there necessarily exists a solution and any solution satisfies $\theta^{*}(z)\in(0,1)$. Next, note that

$$G'\left(\theta\right)=-\frac{\alpha_z}{\sqrt{\alpha_x}}+\frac{1}{\phi(\Phi^{-1}\left(\theta\right))}$$

and $\max_{w\in\mathbb{R}}\phi(w)=1/\sqrt{2\pi}$. If $\alpha_z/\sqrt{\alpha_x}\leq\sqrt{2\pi}$ we have that G is strictly increasing in θ, which implies a unique solution to (39). If instead $\alpha_z/\sqrt{\alpha_x}>\sqrt{2\pi}$, then G is non-monotonic in θ and there is an interval $(\underline{z},\overline{z})$ such that (39) admits multiple solutions $\theta^{*}(z)$ whenever $z\in(\underline{z},\overline{z})$ and a unique solution otherwise. We conclude that monotone equilibrium is unique if and only if $\alpha_z/\sqrt{\alpha_x}\leq\sqrt{2\pi}$. **QED**

Proof of Corollary 1. Consider the limits as $\sigma_x\to0$ for given σ_z, or $\sigma_z\to\infty$ for given σ_x. In either case, $\alpha_z/\sqrt{\alpha_x}\to0$ and $\sqrt{\left(\alpha_x+\alpha_z\right)/\alpha_x}\to1$. Condition (39) then implies that $\theta^{*}(z)\to\hat{\theta}=1-c$ for any z, so that the regime-change threshold is unique and independent of z. Similarly, $x^{*}(z)\to\hat{x}$, where $\hat{x}=\hat{\theta}$ if we consider the limit $\sigma_x\to0$, and $\hat{x}=\hat{\theta}+\sigma_x\Phi^{-1}(\hat{\theta})$ if we instead consider the limit $\sigma_z\to\infty$. **QED.**

Proof of Propositions 3 and 4. See main text.

Proof of Proposition 4. *Part (i).* Consider an agent who receives a private signal x and observes a price p. His optimal investment k solves

$$u_1'(w-k)=\mathbb{E}\left[\ (f-p)u_2'\left((f-p)k\right)\ \mid x,p\ \right]. \tag{40}$$

We assume that $u_1(c)$ is quadratic and $u_2(c)$ is linear, in which case (40) reduces to a simple linear relation, $k_i=\kappa\left\{\mathbb{E}\left[f|x,p\right]-p\right\}+\lambda$, for some constants $\kappa>0,\ \lambda\in\mathbb{R}$. With out any loss of generality, we normalize $\lambda=0$. Finally, we let $f=f(\theta)=\theta$. That is, the return of the asset depends only on the exogenous fundamental.

The analysis here is similar to that in the first example. The optimal individual demand for the asset is

$$k=\kappa\left\{\mathbb{E}\left[\ f\mid x,p\ \right]-p\right\}=\kappa\left\{\mathbb{E}\left[\ \theta\mid x,p\ \right]-p\right\}.$$

We conjecture

$$\mathbb{E}[\ \theta \mid x,p\ ] = \delta x + (1-\delta)p$$

for some $\delta \in (0,1)$ to be determined. It follows that $k = k(x,p) = \kappa\delta(x-p)$ and therefore $K(\theta,p) = \kappa\delta\left(\theta - p\right)$. In equilibrium, $K = \varepsilon$. Hence, the equilibrium price is

$$p = P(\theta,\varepsilon) = \theta - \frac{1}{\kappa\delta}\varepsilon.$$

By implication, p is a public signal about θ with precision $\kappa^2\delta^2\alpha_\varepsilon$. That is, in this example $Z(p) = p$ and $v = -\frac{1}{\kappa\delta}\varepsilon$. It remains to pin down δ and the function Q.

Note that α_p is bounded above by $\kappa^2\alpha_\varepsilon$ and therefore we immediately have that uniqueness is ensured for α_x high enough. To complete the analysis, note that

$$\delta = \frac{\alpha_x}{\alpha_x + \alpha_p} = \frac{\alpha_x}{\alpha_x + \alpha_\varepsilon\delta^2\kappa^2}.$$

The above uniquely determines $\delta \in (0,1)$ as an increasing function of α_u and a decreasing function of α_ε. To see this, let $\alpha = \alpha_x/\left(\alpha_\varepsilon\kappa^2\right)$ and rewrite the above as $\alpha = \delta^3/(1-\delta)$. Obviously, this gives a monotonic relation between α and δ, with $\delta \to 0$ as $\alpha \to 0$ and $\delta \to 1$ as $\alpha \to \infty$. Using these results, we find

$$\begin{aligned}\frac{\alpha_p}{\sqrt{\alpha_x}} &= \frac{\kappa^2\delta^2\alpha_\varepsilon}{\sqrt{\alpha_x}} = \left(\kappa\sqrt{\alpha_\varepsilon}\right)\frac{\delta^2}{\sqrt{\alpha_x}} \\ &= \left(\kappa\sqrt{\alpha_\varepsilon}\right)\sqrt{\delta\left(1-\delta\right)}.\end{aligned}$$

The fact that $\delta\left(1-\delta\right) \to 0$ as either $\alpha_x \to 0$ or $\alpha_x \to \infty$ then implies that, given α_ε, we have that $\alpha_p/\sqrt{\alpha_x} < \sqrt{2\pi}$ and therefore the equilibrium in unique if and only if α_x is either sufficiently small or sufficiently high. On the other hand, for given α_x, we have $\delta\left(1-\delta\right) \leq 1/4$ necessarily and therefore $\alpha_\varepsilon < 8\pi/\kappa^2$ is sufficient for uniqueness, whereas α_ε sufficiently high is sufficient for multiplicity.

Part (ii). Let $x^*(p)$ denote the threshold agents use in stage 2 in deciding whether to attack. In equilibrium,

$$A = A(\theta,p) = \Phi^{-1}\left(\frac{x^*(p)-\theta}{\sigma_x}\right),$$

so that the asset return is $f = \sqrt{\alpha_x}[\theta - x^*\left(p\right)]$. The demand for the asset is thus

$$k = \kappa\left\{\mathbb{E}[\ f \mid x,p\ ] - p\right\} = \kappa\left\{\sqrt{\alpha_x}\mathbb{E}[\ \theta \mid x,p\ ] - p - \sqrt{\alpha_x}x^*(p)\right\}.$$

Let

$$\widetilde{p} = \frac{1}{\sqrt{\alpha_x}} p + x^*(p) \tag{41}$$

and note that, for every p, the above defines a unique $\widetilde{p}$. We can thus write the demand as

$$k = \widetilde{\kappa} \{\mathbb{E}[\ \theta \mid x, p\ ] - \widetilde{p}\}$$

where $\widetilde{\kappa} = \kappa\sqrt{\alpha_x}$. We now conjecture

$$\mathbb{E}[\ \theta \mid x, p\ ] = \delta x + (1 - \delta)\widetilde{p}.$$

It follows that $K = \widetilde{\kappa}\delta(\theta - \widetilde{p})$ and therefore

$$\widetilde{p} = \theta - \frac{1}{\widetilde{\kappa}\delta}\varepsilon. \tag{42}$$

Hence, the observation of p is equivalent to the observation of $\widetilde{p}$, which is a public signal for θ with precision $\alpha_p = \widetilde{\kappa}^2\delta^2\alpha_\varepsilon$. It follows that

$$\mathbb{E}[\ \theta \mid x, p\ ] = \mathbb{E}[\ \theta \mid x, \widetilde{p}\ ] = \delta x + (1 - \delta)\widetilde{p},$$

where

$$\delta = \frac{\alpha_x}{\alpha_x + \alpha_p} = \frac{\alpha_x}{\alpha_x + \alpha_\varepsilon\delta^2\widetilde{\kappa}^2}.$$

This is the same as in the previous example, with $\widetilde{\kappa}$ replacing κ. Using $\widetilde{\kappa} = \kappa\sqrt{\alpha_x}$, we infer

$$\delta = \frac{1}{1 + \alpha_\varepsilon\delta^2\kappa^2},$$

so that δ is decreasing in α_ε but independent of α_x. This means that α_p is proportional to α_x, like in the benchmark model. Indeed, the critical ratio is now given by

$$\frac{\alpha_p}{\sqrt{\alpha_x}} = \frac{\widetilde{\kappa}^2\delta^2\alpha_\varepsilon}{\sqrt{\alpha_x}} = \left(\kappa^2\delta^2\alpha_\varepsilon\right)\sqrt{\alpha_x}, \tag{43}$$

and is increasing in both α_ε and α_x.

The rest of the analysis is similar to the second example. In particular, the thresholds

$\theta^*(p)$ and $x^*(p)$ are uniquely determined and are given by

$$\theta^*(p) = \Phi\left(\frac{\sqrt{\alpha_x}}{\sqrt{\alpha_x+\alpha_p}}\Phi^{-1}(1-c/b) - \frac{\alpha_p}{\alpha_x+\alpha_p}p\right),$$
$$x^*(p) = \theta^*(p) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}(\theta^*(p)),$$

with α_p as in (43). Next, combining (41) and (42), we infer that the equilibrium price solves

$$p = \sqrt{\alpha_x}\theta - \frac{1}{\kappa\delta}\varepsilon - \sqrt{\alpha_x}x^*(p).$$

If $\alpha_p/\sqrt{\alpha_x} < 2\pi$, the above has a unique solution. If instead $\alpha_p/\sqrt{\alpha_x} > 2\pi$, the above has multiple solutions. **QED**

Proof of Corollary 4. *Part (i).* From conditions (31) and (32) we have that, for every y, $\theta^*(y) \to 1-c = \hat{\theta}$ and $x^*(y) \to \hat{\theta} + \sigma_x\Phi^{-1}(\hat{\theta}) = \hat{x}$ as $\sigma_\varepsilon \to \infty$. Condition (28) then implies $\theta - \sigma_x\varepsilon = x^*(y) - \sigma_x y \to \hat{x} - \sigma_x y$ and therefore the unique signal function in the limit is $Y(\theta,\varepsilon) \to (\hat{x}-\theta)/\sigma_x + \varepsilon$.

Part (ii). First, note that $\underline{y} \to -\infty$ and $\overline{y} \to +\infty$ as $\sigma_x \to 0$. Next, note that both $|\sigma_\varepsilon^2\sigma_x - \phi(\underline{y})|$ and $|\sigma_\varepsilon^2\sigma_x - \phi(\overline{y})|$ vanish. Since $\lim_{y\to-\infty}\phi(y)y = \lim_{y\to+\infty}\phi(y)y = 0$, the latter implies $\sigma_x\underline{y} \to 0$ and $\sigma_x\overline{y} \to 0$. Hence, $\underline{z} \to \Phi(-\infty) = \underline{\theta}$ and $\overline{z} \to \Phi(+\infty) = \overline{\theta}$ as $\sigma_x \to 0$. Moreover, for every θ and ε, $\theta - \sigma_x\varepsilon \to \theta$ as $\sigma_x \to 0$. It follows that

$$\Pr\left[\ \theta - \sigma_x\varepsilon \in (\underline{z},\overline{z}) \mid \theta \in (\underline{\theta},\overline{\theta})\ \right] \to 1 \quad \text{as} \quad \sigma_x \to 0.$$

Next, let $\underline{Y}(\theta,\varepsilon) \equiv \min\mathcal{Y}(\theta-\sigma_x\varepsilon)$ and $\overline{Y}(\theta,\varepsilon) \equiv \max\mathcal{Y}(\theta-\sigma_x\varepsilon)$ and consider (θ,ε) such that $\theta - \sigma_x\varepsilon \in (\underline{z},\overline{z})$. Note that $\underline{Y}(\theta,\varepsilon) < \underline{y} < \overline{y} < \overline{Y}(\theta,\varepsilon)$ and therefore

$$\underline{Y}(\theta,\varepsilon) \to -\infty \quad \text{and} \quad \overline{Y}(\theta,\varepsilon) \to +\infty \quad \text{as} \quad \sigma_x \to 0.$$

From (32), $\theta^*(y)$ is independent of σ_x, $\theta^*(y) \to \Phi(-\infty) = \underline{\theta}$ as $y \to -\infty$, and $\theta^*(y) \to \Phi(+\infty) = \overline{\theta}$ as $y \to +\infty$. It follows that, as long as $\theta \in (\underline{\theta},\overline{\theta})$,

$$\Pr\left[\ \theta \le \theta^*\left(\underline{Y}(\theta,\varepsilon)\right)\ \right] \to 0 \quad \text{and} \quad \Pr\left[\ \theta \le \theta^*\left(\overline{Y}(\theta,\varepsilon)\right)\ \right] \to 1 \quad \text{as} \quad \sigma_x \to 0,$$

which establishes the result. **QED**

Proof of Propositions 6, 7 and 8. See main text.

Proof of Proposition 8. For any $\mu \in (0,1)$ and any $x_1^* \in \mathbb{R}$, $\Gamma(\theta, x_1^*)$ is continuous in

θ, with $\Gamma(\underline{\theta}, x_1^*, \mu) = -\infty$ and $\Gamma(\overline{\theta}, x_1^*, \mu) = \infty$, where $\underline{\theta} = \underline{\theta}\left(x_1^*, \alpha_x, \mu\right)$ and $\overline{\theta} = \overline{\theta}\left(x_1^*, \alpha_x, \mu\right)$ solves, respectively, $\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta\right]\right)\right\} = 0$ and $= 1$, and therefore satisfy $0 < \underline{\theta} < \overline{\theta} < 1$. It follows that (38) always admits a solution $\theta^*(z) \in \left(\underline{\theta}, \overline{\theta}\right)$. That is, for any given $x_1^* \in \mathbb{R}$, (38) defines at least one function $\theta^* : \mathbb{R} \to \left(\underline{\theta}, \overline{\theta}\right)$.

We next examine under what conditions the function that solves (38) is unique. Note that

$$\frac{\partial \Gamma}{\partial \theta} = -\frac{\alpha_z}{\sqrt{\alpha_x}} + \Lambda\left(\theta; x_1^*, \alpha_x, \mu\right)$$

where

$$\Lambda\left(\theta; x_1^*, \alpha_x, \mu\right) \equiv \frac{1}{\phi\left(\Phi^{-1}\left(\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta\right]\right)\right\}\right)\right)}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \sqrt{\alpha_x}\phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta\right]\right)\right]\right\}$$

As $\theta \to \underline{\theta}$ or $\overline{\theta}$ (equivalently, $z \to \pm\infty$), $\Lambda\left(\theta; x_1^*, \alpha_x, \mu\right) \to +\infty$. Let

$$K\left(x_1^*, \alpha_x, \mu\right) \equiv \inf_{\theta \in \mathbb{R}} \Lambda\left(\theta; x_1^*, \alpha_x, \mu\right)$$

and note that, since ϕ takes values in $(0, 1/\sqrt{2\pi}]$,

$$K\left(x_1^*, \alpha_x, \mu\right) \geq \frac{1}{1/\sqrt{2\pi}}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \sqrt{\alpha_x}0\right]\right\} = \frac{\sqrt{2\pi}}{1-\mu}.$$

Moreover, letting $\hat{\theta} = \hat{\theta}\left(x_1^*, \alpha_x, \mu\right) \in \left(\underline{\theta}, \overline{\theta}\right)$ be the solution to $\phi\left(\Phi^{-1}\left(\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta\right]\right)\right\}\right)\right) = 1/\sqrt{2\pi}$, or equivalently the solution to $\theta + \frac{\mu}{1-\mu}\left\{\theta - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta\right]\right)\right\} = 1/2$, and using again the fact that the maximal value of ϕ is $1/\sqrt{2\pi}$, we have

$$\begin{aligned} K\left(x_1^*, \alpha_x, \mu\right) &\leq \Lambda\left(\hat{\theta}; x_1^*, \alpha_x, \mu\right) = \frac{1}{1/\sqrt{2\pi}}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \sqrt{\alpha_x}\phi\left(\sqrt{\alpha_x}\left[x_1^* - \hat{\theta}\right]\right)\right]\right\} \\ &\leq \sqrt{2\pi}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \frac{\sqrt{\alpha_x}}{\sqrt{2\pi}}\right]\right\} \end{aligned}$$

Combining, we conclude that, for all $\left(x_1^*, \alpha_x, \mu\right)$,

$$\overline{K}\left(\alpha_x, \mu\right) \geq K\left(x_1^*, \alpha_x, \mu\right) \geq \underline{K}\left(\mu\right)$$

where

$$\overline{K}\left(\alpha_x, \mu\right) \equiv \sqrt{2\pi}\left\{1 + \frac{\mu}{1-\mu}\left[1 + \frac{\sqrt{\alpha_x}}{\sqrt{2\pi}}\right]\right\} \quad \text{and} \quad \underline{K}\left(\mu\right) \equiv \frac{\sqrt{2\pi}}{1-\mu}.$$

Note that, importantly, neither bound is a function of x_1^*.

Case (i): $\frac{\alpha_z}{\sqrt{\alpha_x}} \leq \underline{K}(\mu)$.

In this case, $\frac{\alpha_z}{\sqrt{\alpha_x}} \leq \underline{K}(\mu) \leq \inf_{x_1^*} K(x_1^*, \alpha_x, \mu)$ and therefore Γ is strictly increasing in θ for all θ and all x_1^*. It follows that (38) defines a unique function $\theta^* : \mathbb{R} \to \left(\underline{\theta}, \overline{\theta}\right)$ for any given x_1^*. Moreover, since Γ is decreasing in x_1^* and g is decreasing in z, the function θ^* is decreasing in z and increasing in x_1^*. Finally, θ^* is continuous in both z and x_1^*.

Next, consider (37). For any given $\theta^* : \mathbb{R} \to \left(\underline{\theta}, \overline{\theta}\right)$, (37) admits a unique solution $x_1^* \in \mathbb{R}$. Moreover, this solution is continuous and increasing in θ^*.

Let C be the set of continuous (and bounded) functions $\theta^* : \mathbb{R} \to \left(\underline{\theta}, \overline{\theta}\right)$. Then, (38) is a mapping $\mathbb{R} \to C$ and (37) is a mapping $C \to \mathbb{R}$. Together, they define a continuous and increasing mapping $T : \mathbb{R} \to \mathbb{R}$.

It is easy to check that $T(-\infty) > -\infty$ and $T(+\infty) < \infty$. Hence, a fixed point always exists. Moreover, for arbitrary x_1^* and $a > 0$, let $x_1^{**} = x_1^* + a$ and let θ^* and θ^{**} be the solutions to (38) for x_1^* and x_1^{**}, respectively. Then, (38) and (37) give

$$
\begin{aligned}
-\frac{\alpha_z}{\sqrt{\alpha_x}}\theta^* + \Phi^{-1}\left(\frac{1}{1-\mu}\theta^* - \Phi\left(\sqrt{\alpha_x}\left[x_1^* - \theta^*\right]\right)\right) &= g \\
-\frac{\alpha_z}{\sqrt{\alpha_x}}\theta^{**} + \Phi^{-1}\left(\frac{1}{1-\mu}\theta^{**} - \Phi\left(\sqrt{\alpha_x}\left[x_1^* + a - \theta^{**}\right]\right)\right) &= g
\end{aligned}
$$

$$
\begin{aligned}
\int \Phi\left(\sqrt{\alpha_x}\left[Tx_1^* - \theta^*(z)\right]\right)\sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[Tx_1^* - z\right]\right)dz &= 1 - c \\
\int \Phi\left(\sqrt{\alpha_x}\left[Tx_1^{**} - \theta^{**}(z)\right]\right)\sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[Tx_1^{**} - z\right]\right)dz &= 1 - c
\end{aligned}
$$

Since $\frac{\alpha_z}{\sqrt{\alpha_x}} < \frac{\sqrt{2\pi}}{1-\mu}$, we clearly have $\theta^{**} < \tilde{\theta} = \theta^* + a$ and, by implication, $Tx_1^{**} < \tilde{x}_1$, where $\tilde{x}_1$ solves

$$
\int \Phi\left(\sqrt{\alpha_x}\left[\tilde{x}_1 - \theta^*(z) - a\right]\right)\sqrt{\alpha_1}\phi\left(\sqrt{\alpha_1}\left[\tilde{x}_1 - z\right]\right)dz = 1 - c
$$

If $\tilde{x}_1 = Tx_1^* + a$ ($> Tx_1^*$), the above would have been positive, so it must be that $\tilde{x}_1 < Tx_1^* + a$. Therefore, $Tx_1^{**} < Tx_1^* + a$, which proves that the slope of the mapping T is less than one for every x_1^*. It follows that T has a unique fixed point.

Case (ii): $\sigma_\varepsilon^2\sigma_x < \frac{1}{\sqrt{2\pi}}\left(1 - \mu - \mu\sigma_\varepsilon^2\right)$, or equivalently $\frac{\alpha_z}{\sqrt{\alpha_x}} > \overline{K}(\mu)$.

In this case, $\frac{\alpha_z}{\sqrt{\alpha_x}} > \overline{K}(\mu) \geq \sup_{x_1^*} K(x_1^*, \alpha_x, \mu)$ and therefore Γ necessarily has a non-empty region of non-monotonicity in θ for all x_1^*. It follows that, for any x_1^*, there is a non-empty interval $Z = (\underline{z}, \overline{z}) = Z(x_1^*)$ such that (38) admits three distinct solutions whenever $z \in Z$ and a unique one otherwise. Let θ_L^* (θ_H^*) be the function defined by selecting the lowest (highest) solution whenever $z \in Z$ and the unique one whenever $z \notin Z$. Let T_L

(T_H) be the associated mappings. Each of the mappings T_L and T_H are continuous and satisfy $T(-\infty) > -\infty$ and $T(+\infty) < \infty$. Hence, there is a fixed point (at least one) for each mapping. Moreover, for any given x_1^*, $\theta_L^*(z) < \theta_H^*(z)$ for all $z \in Z$, which in turn implies (because of the monotonicity in (37) and the fact that Z has positive measure) that $T_L(x_1^*) < T_H(x_1^*)$ for any x_1^*. It follows that the fixed point of T_L is lower than the fixed point of T_H, which together with the fact that $\theta_L^* < \theta_H^*$ for any given x_1^* implies that the associated x_2^* and θ^* satisfy the same ordering.

References

[1] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2003) "Coordination and Policy Traps," MIT/UCLA/Northwestern working paper.

[2] Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2004) "Information Dynamics and Equilibrium Multiplicity in Global Games of Regime Change," MIT/UCLA/Northwestern working paper.

[3] Atkeson, Andrew (2000), "Discussion on Morris and Shin," *NBER Macro Annual*.

[4] Barlevy, Gadi, and Pietro Veronesi (2003), "Rational Panics and Stock Market Crashes," *Journal of Economic Theory* 110, 234-263.

[5] Calvo, G. (1988), "Servicing the Public Debt: the Role of Expectations." *American Economic Review,* 78(4), 647-661.

[6] Carlsson, Hans, and Eric van Damme (1993), "Global Games and Equilibrium Selection," *Econometrica* 61, 5, 989-1018.

[7] Chari, V.V., and Patrick Kehoe (2003), "Hot Money," *Journal of Political Economy* 111, 1262-1292.

[8] Cole, Harold, and Timothy Kehoe (1996), "Self-fulfilling Debt Crises," Federal Reserve Bank of Minneapolis staff report 211.

[9] Corsetti, Giancarlo, Bernardo Guimaraes, and Nouriel Roubini, (2004), "International Lending of Last Resort and Moral Hazard: A Model of IMF's Catalytic Finance," working paper, University of Rome.

[10] Dasgupta, Amil (2002), "Coordination, Learning and Delay," working paper, London School of Economics.

[11] Diamond, Douglas, and Philip Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* 91, 401-19.

[12] Goldstein, Itay, and Ady Pauzner (2000), "Demand Deposit Contracts and the Probability of bank Runs," working paper, Duke University and Tel Aviv University.

[13] Green, Jerry "Information, Efficiency and Equilibrium," Harvard Institute of Economic Research Discussion Paper No. 284, Harvard University.

[14] Grossman, Sanford (1977), "The Existence of Futures Markets, Noisy Rational Expectations and Informational Externalities," *Review of Economic Studies* 44, 431-449.

[15] Grossman, Sanford (1981) "An Introduction to the Theory of Rational Expectations under Asymmetric Information," Review of Economic Studies, 48, 541-559.

[16] Grossman, Sanford, and Joseph Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets," *American Economic Review* 70, 393-408.

[17] Grossman, Sanford, and Joseph Stiglitz (1976), "Information and Competitive Price Systems," *American Economic Review* 66, 246-253.

[18] Hellwig, Martin (1980) "On the Aggregation of Information in Competitive Markets" *Journal of Economic Theory*, 22, 477-498.

[19] Hellwig, Christian (2002), "Public Information, Private Information, and the Multiplicity of Equilibria in Coordination Games," *Journal of Economic Theory*, 107, 191-222.

[20] Hellwig, Christian, Arijit Mukherji, and Alej Tsyvinski (2004), "Coordination Failures and Asset Prices," working paper, UCLA.

[21] Morris, Stephen, and Hyun Song Shin (1998), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks", *American Economic Review*, 88, 3, 587-597.

[22] Morris, Stephen, and Hyun Song Shin (1999), "Private versus Public Information in Coordination Problems", Yale/Oxford mimeo.

[23] Morris, Stephen, and Hyun Song Shin (2000), "Rethinking Multiple Equilibria in Macroeconomics," *NBER Macro Annual*.

[24] Morris, Stephen, and Hyun Song Shin (2001), "Global Games - Theory and Applications," in *Advances in Economics and Econometrics*, 8th World Congress of the Econometric Society (M. Dewatripont, L. Hansen, and S. Turnovsky, eds.), Cambridge University Press, Cambridge, UK.

[25] Morris, Stephen, and Hyun Song Shin (2003), "Coordination Risk and the Price of Debt," forthcoming in *European Economic Review*.

[26] Morris, Stephen, and Hyun Song Shin (2004), "Liquidity Black Holes," forthcoming in *Review of Finance.*

[27] Obstfeld, Maurice (1986), "Rational and Self-Fulfilling Balance-of-Payments Crises", *American Economic Review* 76, 1, 72-81.

[28] Obstfeld, Maurice (1996), "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review* 40, 3-5, 1037-47.

[29] Rochet, Jean-Charles, and Xavier Vives (2004), "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?," working paper, University of Toulouse.

[30] Velasco, Andres (1996), "Fixed Exchange Rates: Credibility, Flexibility and Multiplicity," *European Economic Review* 40, 3-5, 1023-36.