NBER WORKING PAPER SERIES

THE EVOLUTION OF INCOME AND FERTILITY INEQUALITIES OVER THE COURSE OF ECONOMIC DEVELOPMENT: A HUMAN CAPITAL PERSPECTIVE

Isaac Ehrlich
Jinyoung Kim

Working Paper 10890
http://www.nber.org/papers/w10890

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2004

We are indebted to Chi-wa Yuen and Jianguo Zhong for collaboration on earlier related work, and to Omer Moav, Gary Becker, Kevin Murphy, Stephen Turnovsky, Matthias Doepke, and Francis Lui for comments on previous drafts. We are especially indebted to Thomas Muench for very helpful comments concerning dynamic stability issues in our heterogeneous agent system, and to Jong Kook Shin for valuable research assistance. We alone are responsible for errors. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

The Evolution of Income and Fertility Inequalities over the Course of Economic Development: A Human Capital Perspective
Isaac Ehrlich and Jinyoung Kim
NBER Working Paper No. 10890
November 2004, Revised December 2007
JEL No. D3,J1,O1

ABSTRACT

Using an endogenous-growth, overlapping-generations framework where human capital is the engine of growth, we trace the dynamic evolution of income and fertility distributions and their interdependencies over three endogenous phases of economic development. In our model, heterogeneous families determine fertility and children's human capital, and generations are linked via parental altruism and social interactions. We derive and test discriminating propositions concerning the dynamic behavior of inequalities in fertility, educational attainments, and three endogenous income inequality measures -- family-income inequality, income-group inequality, and the Gini coefficient. In this context, we also reexamine the "Kuznets hypothesis" concerning the relation between income growth and inequality.

Isaac Ehrlich
415 Fronczak Hall
University of Buffalo
Box 601520
Buffalo, NY 14260-1520
and NBER
mgtehrl@buffalo.edu

Jinyoung Kim
Department of Economics
Korea University
5-1, Anam-Dong, Sungbuk-Ku
Seoul, Korea 136-701
jinykim@korea.ac.kr

INTRODUCTION

Using historical data from a number of developing and developed countries, Kuznets (1955, 1963) argued that income inequality first rises and then falls during a transitional development period. Post-Kuznets studies fall into two main groups. The first re-tested his hypothesis against data from many other countries.[1] The second dealt with the development–inequality nexus theoretically as a causal relation going from either growth to inequality or vice versa.[2] Both sets of studies offered conflicting conclusions about the competing theoretical and empirical hypotheses.

One issue that was hardly addressed so far is the relevance of relative fertility choices of different income groups for understanding the evolution of income inequalities over all phases of economic development, including the comparative levels of income inequality across less-developed, stagnant economies and highly developed, persistently growing ones. The relevance of fertility and related population trends to income distributions is two-fold: measures of the "size distribution of income", such as the shares of specific income brackets in total income, or the Gini coefficient are weighted by the population shares of different income brackets, which reflect underlying fertility differences across these brackets. But even individual- or family-based income inequality measures are indirectly affected by parental choices concerning fertility and investment in human capital of offspring. This is especially important over the transitional development period, which typically involves a "demographic transition" as well.

We tackle this issue by developing an overlapping-generations endogenous-growth model with heterogeneous families, where human capital is the engine of growth and parents determine fertility and educational investments in offspring. Our deterministic model offers a dynamic extension of Becker's (1967) model of income distribution, as well as a generalization of more recent work by Ehrlich and Lui (1991), Zhong (1998), and Ehrlich and Yuen (2000). We show that the behavior of income inequality over the transitional development phase can vary in different countries, depending on the factors triggering economic takeoff and their impact on fertility and investment behavior of different family groups.

Our model suggests that the relationship between income growth and income inequality is **associative**, not causal. Three main forces influence this association: a. interactions between overlapping generations within families; b. heterogeneities in endowments and investment efficiencies across families; and c. social interactions across families.[3] The model consists of finitely-lived individuals. Parents optimize on investments in the quantity and quality of children (in an extended version outlined in Appendix A, on savings as well). Persistent heterogeneities and social interactions across families enable us to derive equilibrium paths of income, schooling, and fertility distributions over three phases: a stagnant steady state, characterizing economies in a pre-takeoff stage, a perpetual-growth steady state, more related to economies in a highly developed stage, and a transition phase linking the two.

By this approach we are also able to provide new insights about the "Kuznets hypothesis." The association between income growth and inequality is influenced partly by the parameters determining the inequality levels at the growth vs. stagnant steady state. The association is in a state of flux during the transitional development phase. Specifically, we show that over this phase the paths of alternative income inequality measures can assume a U shape, an inverted-U shape, or combinations of the two, with the inequality level ultimately falling, rising, or staying the same, depending on the way heterogeneity sources are correlated across families, how relative fertility and population shares are affected across groups, and the inequality measure used. We derive three such measures as endogenous variables: family-income inequality, income-group inequality, and the Gini coefficient. A distinct implication of our model is that regardless of the specific shape of the income inequality path over the transitional development phase, fertility inequality can be expected to assume an inverted-U shape, with both tails converging on virtual equality.

Section I introduces the model. In section II we derive equilibrium regimes and comparative dynamic implications, and in section III we present simulated dynamic paths of our basic inequality measures. Section IV presents supportive evidence on the dynamic evolution of fertility, schooling, and income inequality based on international panel data from 1950-98.

I. THE MODEL AND EQUILIBRIUM SOLUTIONS

A. The Economic Environment

To derive income and fertility inequality paths over the entire development process, we extend the deterministic representative-family, OLG model of endogenous growth in Ehrlich and Lui [EL] (1991) to a heterogeneous-family case that recognizes inter-family interactions as well.

The Economy: The economy is comprised of heterogeneous family groups of varying income levels. For simplicity we illustrate the model by recognizing just two family groups indexed by $i = 1,2$: a leading (1) and a following group (2), which in principle could switch places over the process of development. We implicitly rely on positive assortative mating within groups to maintain their separate identity in any stable steady state, because if inter-group mating is allowed, and children inherit the average characteristics of their parents, human capital attainments and income would eventually converge in all families, given our deterministic setup.[4] In the benchmark model agents live over two periods: childhood, t-1, when human capital is formed through parental investments, and parenthood, t (but see Appendix A). All family-based decisions are made by working parents.

Similarly to Becker (1967), we focus on three objective sources of inherited heterogeneity: a. differences in learning or production abilities (A^i); b. differences in income-yielding "endowments" ($\overline{H}^i$), stemming from inherited social status, political power, or other personal assets; c. differences in education-financing costs (θ^i). We abstract from differences in preferences or external production technologies, since these need not be related to idiosyncratic personal differences.

Goods Production and Income: The economy is competitive and human capital is the sole asset. Production or earning capacity of a parent in group i, ($\overline{H}^i+H^i_t$), is composed of a fixed, inherited income-producing endowment ($\overline{H}^i$) measured in units of human capital, and a stock of human capital, (H^i_t), attained through parental inputs. Labor supply is assumed fixed. For convenience, we also assume that consumer goods, including educational services, can be purchased. Under a linear and strongly additive production technology for all goods, aggregate earnings (Y) equals aggregate production capacities in each period (L), and firms' zero-profit condition, $\pi=Y-\varpi L=0$, yields a time-invariant real rental rate per unit of production capacity, $\varpi=1$, which also guarantees full employment. We initially ignore savings, so earnings are identical to income. In **Appendix A**, we allow for savings and thereby for an old-age period of life for retired parents, and show that our inferences concerning earnings inequality hold for income inequality as well.

Human-capital production: The human-capital formation rule is given by:

(1) $H^i_{t+1} = A^i h^i_t(\overline{H}^i+H^i_t)^{1-\gamma}[(\overline{H}^1+H^1_t)(N^1_t/N^i_t)]^\gamma \equiv A^i h^i_t(\overline{H}^i+H^i_t)(S^i_t)^\gamma$,

where h^i_t is the share of earning capacity ($\overline{H}^i+H^i_t$) a parent from family-group i (i = 1, 2) invests in educating each child, $\overline{H}^i$ and H^i_t are the parent's endowed and attained human capital stocks, respectively, and N^1_t/N^i_t is the ratio of the population shares of parents in family groups 1 and i. S^i_t denotes an inter-family, "social interaction" factor, which we define below. Note that for the top income group i=1, equation (1) becomes $H^1_{t+1} = A^1h^1_t(\overline{H}^1+H^1_t)$.

Equation (1) captures two types of interactions in human capital production within and across families: a. Persistent human capital formation can be sustained over time only if parents invest in their children's knowledge; b. Knowledge attained by agents with the higher earning capacity augments the attained production capacity of, and thus has a spillover effect on, agents with lower capacity (see below). Human capital formation is thus a social, as well as a private, process.

The intergenerational interaction is captured by the relationship between H^i_{t+1} and H^i_t in equation (1). The inter-family interaction is defined by the term $(S^i_t)^\gamma$ in equation (1), where $S^2_t \equiv [(\overline{H}^1+H^1_t)/(\overline{H}^2+H^2_t)][N^1_t/N^2_t] \equiv E^2_t P^2_t$. The ratios E^2_t and P^2_t reflect the relative earning capacities and family-group sizes of agents in group 1 relative to 2 in generation t, and $\gamma<1$ is a spillover-intensity parameter. This specification is designed to capture social-interaction as a knowledge spillover effect by which agents with lower earning capacity, or effective knowledge (group 2), benefit from interactions with leaders in knowledge (group 1) in various contexts.[5]

The relevance of E^2_t is straightforward: the greater the disparity in knowledge, the greater the potential learning benefit to members of group 2 from knowledge-transfer from members of group 1, given their own knowledge. $P^2_t \equiv N^1_t/N^2_t$ captures the determinant of effective interaction between individual members of groups 1 and 2, which is simply the conventional "teacher-student ratio".[6]

Preferences and Motivating Forces: For the sake of parsimony, we take parental altruism to be the sole force motivating parents' demand for children. The utility function of agent i at period t is:

(2) $U(C^i_{1,t}, W^i_{t+1}) = [1/(1-\sigma)][C^i_{1,t}{}^{1-\sigma} - 1] + \delta[1/(1-\sigma)][W^i_{t+1}{}^{1-\sigma} - 1]$,

where δ denotes the inverse intertemporal discount factor, and σ the inverse intertemporal elasticity of substitution in consumption. In equation (2), $C^i_{1,t}$ denotes consumption of parents:

(3) $C^i_{1,t} = (\overline{H}^i+H^i_t)[1 - v^i n^i_t - \theta^i h^i_t n^i_t]$.

The variable n^i_t represents the number of children per parent, treated as a continuous and certain variable. The endogenous size of group i thus evolves over time as $N^i_{t+1}=N^i_t n^i_t$. The parameters v^i and θ^i are fixed unit costs, as fractions of earning capacity, of raising a child and financing educational investments per child, respectively. The latter may vary significantly across family groups because of capital-market imperfections. The last term in equation (2),

(4) $W^i_{t+1} \equiv B(n^i_t)^{\beta} H^i_{t+1}{}^{\alpha}$, with α=1 and β>1,

represents a parental altruism function in an OLG context, borrowed from EL (1991), which reflects psychic rewards parents obtain vicariously from children's number and attained human capital.[7] The restrictions on α and β are necessary to obtain interior solutions for both h^i_t and n^i_t. To ensure the concavity of equation (2) we must further restrict $\alpha(1-\sigma)<1$ (or, $\sigma>0$) and $\beta(1-\sigma)<1$.

B. Basic Solutions

The objective function (2) is maximized by choosing n^i_t, and h^i_t, subject to (1), (3), and (4), taking $\{H^i_t, H^1_t, N^i_t, N^1_t\}$ as given. The first-order conditions for optimal n^i_t and h^i_t are:

(5) $0 = -\, C^i_{1,t}{}^{-\sigma}\,(v^i + \theta^i h^i_t)\,(\overline{H}^i + H^i_t) + \delta\, W^i_{t+1}{}^{-\sigma}\,\beta\, B(n^i_t)^{\beta-1} H^i_{t+1}{}^{\alpha}$, for $n^i_t \geq 0$, and

(6) $0 = -\, C^i_{1,t}{}^{-\sigma}\,\theta^i n^i_t\,(\overline{H}^i + H^i_t) + \delta\, W^i_{t+1}{}^{-\sigma}\,\alpha\, B(n^i_t)^{\beta}\, H^i_{t+1}{}^{\alpha-1}\, A^i(\overline{H}^i + H^i_t)(S^i_t)^{\gamma}$, for $h^i_t \geq 0$.

Equations (5) and (6) confirm that in order for interior solutions for n^i_t and h^i_t to co-exist over all development phases, we must restrict $\beta>\alpha$, and α=1 (note that a growth equilibrium cannot be sustained if α>1). The optimal solutions for h^i_t and n^i_t are then found to be:

(7) $h^i_t = [v^i/\theta^i(\beta-1)]$, or $\theta^i h^i_t = v^i/(\beta-1) <1$

(8) $[1-\beta v^i n^i_t/(\beta-1)]^{-\sigma} = \delta\,[B(A^i/\theta^i)(S^i_t)^{\gamma}]^{1-\sigma}\,[v^i/(\beta-1)]^{-\sigma}\,(n^i_t)^{\beta(1-\sigma)-1}$.

Note that in equation (8), the left-hand and right-hand sides represent convex, monotonically rising and falling marginal cost and benefits schedules of n^i_t, respectively. Their intersection thus offers unique solutions for h^i_t and n^i_t (the latter having an upper limit of $n^i = [(\beta\text{-}1)/(\beta v^i)]$).

By equation (7), the equilibrium value of h^i_t is independent of the level of human capital, essentially because for altruistic parents a change in H raises proportionally both the marginal benefits and costs of investment H. An interesting feature of equation (8) is that ability, A^i, and unit financing cost of education, θ^i, exert opposite effects on n_t^i. In fact, the solution depends on the **ratio**

$e^i \equiv A^i/\theta^i$, or families' relative **"investment efficiencies"**. Equation (8) indicates that equilibrium fertility, $n^i_t = n(\cdot) = n(v^i, B, \beta, e^i, \delta, S^i_t)$, falls with v^i and rises with the other parameters entering $n(\cdot)$.

C. Income Inequality Measures

Three income inequality indices become endogenous functions of our model's state variables:

a. $E^2_t \equiv (\overline{H}^1+H^1_t)/(\overline{H}^2+H^2_t)$ is a **family-income inequality** index: the ratio of the (full) incomes of individual families in family-group 1 relative to family-group 2. An inequality measure directly related to E^2_t in our model is inequality in attained human capital stocks, H^1_t/H^2_t, which may be captured by the standard deviation of schooling attainments.

b. $S^2_t \equiv [(\overline{H}^1+H^1_t)/(\overline{H}^2+H^2_t)][N^1_t/N^2_t] \equiv E^2_t P^2_t$ is our **income-group, or income-bracket inequality** index – a product of relative income levels and group sizes of family-group 1 relative to 2– which is also a component of the social interaction term in equation (1). It measures the fraction of **aggregate income** going to the top income bracket (above a given dollar value), relative to lower bracket. Note that $P^2_t \equiv N^2_t/N^2_t$ is a related inequality measure – an **income-group-size inequality index**. It measures the proportion of families in the top income bracket relative to those in the lower bracket, or the relative population shares of families in the two income brackets. The latter is not independent of S^2_t and E^2_t since, by definition, $P^2_t \equiv S^2_t/E^2_t$.

c. The **Gini coefficient**, $G_t \equiv (S^2_t - P^2_t)/[(1+ S^2_t)(1+ P^2_t)]$, turns out to be a non-linear function of S^2_t and P^2_t. Specifically, it is an increasing function of S^2_t, but a decreasing function of P^2_t.

An immediate insight from all of these income inequality measures is their inherent dependence on the population shares of different family groups, resulting from their fertility choices.

II. EQUILIBRIUM REGIMES AND COMPARATIVE DYNAMICS

Equations (7) and (8) represent a recursive model, since the leading group 1 arrives at all of its choices independently as a function of its own parameters, while the following group 2's fertility choices are affected by those of family 1 through the social interaction term, $(S^2_t)^\gamma$. From equations (1) and (7), we derive an explicit, linear law of motion of human capital in group 1:

(9) $H^1_{t+1} = [v^1(A^1/\theta^1)/ (\beta-1)]\, H^1_t + [v^1(A^1/\theta^1)/ (\beta-1)]\, \overline{H}^1$.

Since the economy is dictated by family 1, the equation of motion (9) indicates the existence of two equilibrium regimes, depending on the magnitudes of the model's basic parameters: If the slope $dH^1_{t+1}/dH^1_t = v^1(A^1/\theta^1)/ (\beta-1) = A^1h^1_t$, exceeds 1, H^1_t grows exponentially without bound and the economy is in a persistent growth equilibrium regime. If the slope is below 1, H^1_t becomes constant and a stagnant-equilibrium regime ensues. The transitional development phase connecting the regimes' steady states is supported by the same parameter set that sustains the growth regime.

For equilibrium steady states to exist, however, certain outcomes must hold:

Proposition 1. Both fertility rates and marginal rates of change of human capital stocks in different family groups must converge at any stable equilibrium steady state. Formally, we can show that:

(10) $n^1_t = n^2_t$, and $a^1_t \equiv (dH^1_{t+1}/dH^1_t) = A^1h^1_t = a^2_t \equiv (dH^2_{t+1}/dH^2_t) = A^2h^2_t(S^2_t)^\gamma$.

Proof: In any equilibrium steady state which preserves the heterogeneous family groups, their relative population shares, $P^2_t \equiv N^1_t/N^2_t$ must be constant over time, requiring fertility rates to equalize across families.[8] Suppose there is an exogenous shock which initially lowers n^2 below n^1. This will increase P^2_t and the social interaction term S^2_t in equation (8), raising the marginal benefit from children in family 2 but not in family 1, which is unaffected by S^2_t. The rise in n^2_t subsequently depresses P^2_t and S^2_t, and these adjustments continue until desired fertility rates equalize (although not necessarily actual rates if the latter were subject to purely stochastic deviations from desired fertility rates; see footnote 4). This result is consistent with optimal fertility choices, since in a stable steady state, the impact of lower income is offset by a proportionally lower shadow price of fertility.[9]

Similarly, in a balanced growth steady state, H^1_t/H^2_t, e.g., must be constant over time. This requires the growth rate of human capital, which is the same as the latter's marginal rate of change or **total** derivative (dH^i_{t+1}/dH^i_t) at the growth steady state, to equalize across groups. This result will be shown to apply in a stagnant steady state as well (see section II.A).

Given our assumed uniformity of preferences and external production parameters, $M \equiv \{B, \beta, \gamma, \delta, \sigma\}$, dynamic stability also requires a parameter restriction: the unit cost of raising a child as a fraction of earning capacity must be identical across families, or $v^1=v^2$. Put differently, only initial endowments ($\overline{H}^i$), abilities (A^i), and unit investment-financing costs (θ^i) are allowed to vary across families. This **heterogeneity restriction** necessarily holds in the log utility case.[10] More generally, suppose that $v^1 > v^2$. Equation (7) implies that the income share invested in educating a child would then be higher in family 1 relative to 2, $\theta^1h^1_t > \theta^2h^2_t$. By proposition 1 the rate of growth of human capital must be identical across families in a stable growth steady state, $A^1h^1=A^2h^2 (S^2_t)^\gamma$. These equations imply that $(A^1/\theta^2) < (A^2/\theta^2)(S^2_t)^\gamma$, i.e., family 2's marginal cost of fertility schedule would locate below that of family 1, while its marginal benefit schedule would locate above 1's. Optimal fertility would then be strictly higher in family 2 relative to 1, negating a stable equilibrium. To assure consistent parameter restrictions we must $v^1=v^2=v$ in all development phases.

A. Stagnant Equilibrium Steady State (s)

In a stagnant equilibrium (SE) steady state, the control and state variables and all inequality measures are constant over time, given the parameters affecting the rate of return on human capital, A^i/θ^i and v. If the latter are sufficiently low, so $(dH^1_{t+1}/dH^1_t) = A^1h^1_t = v(A^1/\theta^1)/(\beta-1) <1$ in equation (9), equation (1) for agent 1 would necessarily converge on SE from any arbitrary value of H^1_0. Local stability conditions for agent 1 require, in addition, that $dn^1_t/dN^1_t <1$. For agent 2, and thus the full system, the necessary and sufficient conditions for local stability can be shown to require that the elasticity of fertility $n^2(s)$ with respect to the social-interaction term $S^2(s)$ would lie within the following range: $0<e(s)<2\{1-[\gamma K/(K+1)]\}$, where e(s) is the elasticity of $n^2(s)$ with respect to the social-interaction term $S^2(s)$, or $e(s) \equiv \partial\ln[n^2(s)]/ \partial\ln[S^2(s)]$, and $K \equiv \{H^2(s)/[\overline{H}^2+H^2(s)]\} <1$. In our numerical simulations using the baseline parameters of Table 1 part 1, this condition is satisfied for any permissible values of the social-interaction intensity $0\leq\gamma\leq1$ (see Appendix B.1).

Proposition 2. In a stable stagnant-equilibrium steady state, family-income inequality, $E^2(s)$, and families' relative human capital attainments equal their relative inherited income endowments:

(11) $E^2(s) = [H^1_t/H^2_t](s)= \overline{H}^1/\overline{H}^2$.

Proof: Equation (11) is obtained utilizing equation (1), the stagnancy of human capital attainments in the SE, and the expectation that $a^1(s)\equiv A^1h^1(s)=a^2(s)\equiv A^2h^2(s)[S^2(s)]^\gamma$ (proposition 1). Indeed, the latter must hold in the SE as well as the GE: By the heterogeneity condition $v^1=v^2$, equation (7) implies that

(12) $\theta^1h^1(s) = \theta^2h^2(s)$.

Inserting the condition $a^1(s)=a^2(s)$ in equation (8) is thus seen necessary to guarantees that $n^1(s)=n^2(s)$.

Proposition 2 implies that status differences are the key factor determining family income inequality in economies that are stagnant over long periods, an inference which seems compatible with historical evidence, such as pre-industrial revolution Europe. Equation (11) is also dynamically stable: suppose we start from a stable SE. If a parameter shock lowers a^2, raising H^1_t/H^2_t above $\overline{H}^1/\overline{H}^2$, then E^2_t and S^2_t would also rise initially. This would raise n^2_t and depress P^2_t and S^2_t, which in turn would increase a^2_t and lower H^1_t/H^2_t until the initial equilibrium is restored.

Note that by equations (10) and (12), the income shares spent on both raising (vn) and educating (θh) children equalize across families. Combining these with equation (8) allows us to derive the stagnant-equilibrium value of our income-group inequality measure:

(13) $S^2(s) \equiv E^2(s)\, P^2(s) = [(A^1/\theta^1)/(A^2/\theta^2)]^{(1/\gamma)} \equiv (e^1/e^2)^{(1/\gamma)}$.

Proposition 3. In a stagnant steady state the shares of earning capacity devoted to human capital investments per child, $\theta^i h^i(s)$ are equalized across all family groups, and income-group inequality, $S^2(s)$ depends strictly on the relative "investment efficiencies" of family 1 relative to 2, $(e^1/e^2)^{(1/\gamma)}$. Unlike $S^2(s)$, however, the size distribution of families across income brackets $P^2(s)$ and the Gini coefficient G(s) depend on both relative family endowments and investment efficiencies.

Equation (13) has an intuitive interpretation: The only way the social interaction term $S^2(s) \equiv E^2(s)P^2(s)$ can adjust to satisfy equation (10) is through adjustments in fertility choices and their impact on $P^2(s)$. Fertility choices are strictly a function of relative investment efficiencies (equation 8). Therefore, adjustments in $S^2(s)$ must be a function of relative investment efficiencies as well.

This analysis yields a set of comparative-dynamic implications in the SE steady state. By Proposition 2, family-income inequality, $E^2(s)$, is strictly a function of families' relative endowments. By proposition 3, any changes in relative income-group inequality, $S^2(s) \equiv E^2(s)P^2(s)$, thus occur via the groups' relative population shares, $P^2(s)=[N^1/N^2](s)$. For example, an increase in group 1's relative investment efficiency e^1/e^2 would increase $S^2(s)$ by increasing its population share, $P^2(s)$. A higher intensity of knowledge spillover, γ, leaves family choices intact but lowers $P^2(s)$ hence, $S^2(s)$. Changes in other common parameters can affect, $n^i(s)$, and $h^i(s)$, but not any income inequality measures: by equations (7) and (8), higher fertility unit costs (v) increase h^i and lower n^i, while stronger altruistic preferences (B) raise n^i in all families (See **Table 1 part 1**.)

Does income inequality change systematically at different income levels? Note that while income levels are stagnant in a SE, they can vary with parameter shifts: A skill-biased technological advance raising family 1's relative investment efficiency, e^1/e^2, ultimately raises income levels in all families. By proposition 2, however, family-income inequality, $E^2(s)$, remains unchanged, while by proposition 3, income-group inequality, $S^2(s)$, rises because the wealth effect generated by a higher e^1 **temporarily** increases group 1's relative fertility, and ultimately its population share $P^2(s)$. The impact on Gini is **ambiguous**, since G(s) rises with $S^2(s)$, but falls with $P^2(s)$.[11] The association between income levels and inequality at the SE thus depends on the **inequality measure** used.

B. Growth Equilibrium Steady State (g)

In a steady state of a balanced-growth-equilibrium (GE), human capital stocks H^i_t grow exponentially without bound, while the long-run values of $n^i(g)$, $h^i(g)$, and all inequality measures are constant. Since the relative impact of the endowments, $\overline{H}^i$, vanishes, proposition 1 implies that the long-run growth rate of human capital in all groups converges on its marginal value in family group 1, $\lim_{t\to\infty} (H^i_{t+1}/H^i_t) \equiv a^1(g)=A^1h^1(g) = a^i(g) =A^ih^i(g)S^i(g)^\gamma$. Local stability for group 1 is assured if $a^1(g)>1$, or $v(A^1/\theta^1) > (\beta-1)$, since $dh^1_t/dH^1_t=0$ and $dn^1_t/dN^1_t \leq 0$ as in the SE. The necessary and sufficient conditions for the local stability of the full system, which guaranties a unique solution for income-group inequality, $S^2(g)=E^2(g)P^2(g)$ but not for its component parts (see the discussion below), are shown in Appendix B.2 to require that $\{-\gamma < e(g) < 2-\gamma\}$, where e(g) is the elasticity of $n^2(g)$ with respect to $S^2(g)$. Again, our numerical illustration using the baseline parameters in Table 1, part 2 indicate that this condition is satisfied for all permissible values of $0\leq\gamma\leq1$ (see Appendix B.2).

Proposition 4. Proposition 3 remains valid at the **growth equilibrium** steady state as well. Moreover, if the relative distribution of the heterogeneous parameters A^i and θ^i remains the same in the SE and GE, income-group inequality S^2 would converge on the **same level** in both steady states:

(14) $S^2(g) \equiv E^2(g)P^2(g) = [(A^1/\theta^1)/(A^2/\theta^2)]^{(1/\gamma)} \equiv (e^1/e^2)^{(1/\gamma)} = S^2(s)$.

The proof is the same as for proposition 3. The comparative-dynamic implications of equation (14) are also similar to those of equation (13): $S^2(g)$ rises with the relative investment efficiency (e^1/e^2) and falls with the spillover coefficient γ, as is the case in the SE steady state. No unique solutions exist, however, for the growth-equilibrium values of $E^2(g)$ or $P^2(g)$ (see Appendix B.2). Unlike the stagnant steady-state case, where family-income inequality $E^2(s)$ was determined strictly by the relative family endowments, the endowments' influence vanishes with persistent growth. The comparative values of E^2 in the GE vs. SE steady states thus depend on the **evolution** of E^2_t along the transitional phase. The same holds for the income-group-size inequality index, P^2_t.

Comparative-dynamics effects of parameter shifts on inequalities in family-income, $E^2(g)$, and income-group-size, $P^2(g)$, are thus ambiguous. A **skill-biased** technological or institutional advance favoring the leading family, which by proposition 4 unambiguously raises $S^2(g) \equiv E^2(g)P^2(g)$, must raise either $E^2(g)$, or $P^2(g)$, or both. $E^2(g)$ necessarily rises if the upward shift in A^1 raises the growth rate of human capital and income in family 1, $A^1h^1_t$, above that in family 2 over the entire transitional-adjustment path. As in the SE case, fertility rises as well, as shown by our simulations in **Table 1 part 2**, because an increase in A^1 generates a wealth effect, which favors fertility over educational investment. Over the transitional dynamics adjustment, family 1's higher income growth rate ultimately lifts the income growth rate and fertility in family 2 as well, due to the rising social interaction effect $(S^2_t)^\gamma$. Thus, unlike the SE case, both family-income inequality $E^2(g)$ and income-group-size inequality $P^2(g)$ rise while fertility inequality increases over the initial part of the adjustment period, but ultimately falls and vanishes at a new GE steady state. The change in Gini is ambiguous, since G rises with S^2 and falls with P^2, but our simulations indicate that G(g) rises as well, as $S^2(g)$ rises more sharply.

These results appear to be consistent with the US experience following the "Information Technology revolution": empirical studies have shown that the Gini coefficient rose in the 1980s (e.g., Katz and Murphy, 1992; Acemoglu, 2002). As we predict, Census data also show that over the same period total fertility levels reversed a historic downward trend since the baby boom and started rising from 1.74 in 1977 to a peak of 2.08 in 1990, remaining stable thereafter at about 2.02, while the coefficient of variation of fertility rose from 0.619 in 1983 to 0.710 in 1994, but with the rate of increase diminishing since then (see Figure A).[12]

Table 1, part 2 also confirms the implications of equations (7) and (8) that a rise in v lowers $n^i(g)$ and raises $h^i(g)$, thus the growth rate, while a rise in B raises only $n^i(g)$. Neither affects any income inequality measure. This is in contrast to a skill-biased technical advance, which raises all income inequality measures and the growth rate as well. The association between income growth rate and income inequality thus depends on the **parameter changes** responsible for their co-movements.

C. Takeoff Triggers.

As equation (9) predicts, starting at a stagnant equilibrium, an upward shock in investment efficiency, A^i/θ^i, or in the common unit cost of raising children, v, even one affecting just group 1, can generate a takeoff and a transitional development phase for all groups. Under the altruistic specification of our model, the "demographic transition" typically accompanying a takeoff, whereby fertility generally declines, can be generated by a sufficient upward shift in the unit cost share of raising children, v, but not by a technological advance: the latter generates a higher growth rate and thus a wealth effect favoring fertility (see **Table 1 part 3**).[13] By propositions 3 and 4, however, shifts in v, thus in fertility **levels** have no bearing on the dynamic evolution of our income inequality measures over the transitional development phase. Also, the shapes of the evolution paths of all our income inequality measures, i.e., whether income inequality is monotonically rising or falling or has a U-shape, or an inverted U-shape, or any combinations therein, is dictated just by shifts in the parameters affecting investment efficiency (A^i and θ^i). The paths of income inequality we derive below are thus independent of the evolution of fertility levels, but are affected by the evolution of **relative** fertility levels across groups and their relative population shares, P^2_t, as our analysis in the following section demonstrates.

III. INEQUALITY PATHS OVER THE TRANSITIONAL DEVELOPMENT PHASE

A. Paths of Income Inequality Measures

The preceding analysis indicates that the behavior of inequalities over the development phase partly depends on the type of shock that produces a takeoff. An equally important issue is how fast any given shock reaches different family groups: a skill-biased technological advance, e.g., is likely to first reach the group with the highest investment efficiency, or affect it proportionally more than others. While this group need not necessarily be the one with the highest income – this depends on the correlation between ability and initial endowments across family groups – a positive correlation is likely (see Becker, 1967). To contain the possible scenarios we focus on three that are neither exhaustive nor necessarily of equal empirical plausibility:

a. Synchronous and uniform shocks: These shocks affect all takeoff-triggering parameters (A^i/θ^i, v) simultaneously and by the same proportion. This case can be dubbed "the neutral equilibrium path"; we can show that over the transitional phase:

(15) $S^2_t = S^2(s) = S^2(g) = [(A^1/\theta^1)/(A^2/\theta^2)]^{(1/\gamma)}$, and $E^2_t = E^2(s) = E^2(g) = \overline{H}^1/\overline{H}^2$.

Put differently, our basic earnings inequality measures chart a horizontal path all along the development process. This is because a uniform proportional increase in a takeoff-triggering parameter affects all optimality conditions symmetrically, leaving constant the spillover effect. Since the Gini coefficient is a function of S^2 and P^2, it also exhibits a flat transition path.

b. Shocks favorable to family 1: Such a shock affects family 1 proportionally more than, or ahead of, family 2. An example would be a technical advance or a market reform that enhances just the productivity of especially skilled workers (A^1), such as a shift from a command to a market economy, or one that ultimately enhances the productivity of all agents proportionally but is first integrated by family 1, such as the IT revolution. We implicitly assume a positive correlation between income-generating endowments and efficiency at human capital investments, or COV($\overline{H}^i$, A^i/θ^i) > 0, so the higher-income family 1 is a leading group at both the stagnant and growth steady states.

If family 1 is affected ahead of family 2, the transitional development phase would be characterized by the co-existence of family groups in different stages of transition: Family 1 would initially become a **“growth family”** while the other remains a **“stagnant family”.** But the persistent growth in family 1’s income ultimately produces a takeoff for all, and by proposition 1 all will eventually grow at an equal rate. The time paths of all income inequality measures (S^2, E^2, and G) will exhibit an **inverted-U shape**, consistent with the “Kuznets hypothesis” (see Figure 1).

The shapes of the income inequality paths in all scenarios, including the comparative income inequality **levels** in the growth, relative to the stagnant steady states, depend strictly on whether the technology shift ultimately affects all families “uniformly”, i.e., equi-proportionally, or non-uniformly. If a skill-biased technology shift ultimately becomes uniform, it does not affect the GE income-class inequality $S^2 \equiv E^2 P^2$ by equation (14), but it **lowers** the GE family-income inequality $E^2(g)$, because the wealth effect triggered by the jump in A^1 initially raises the relative fertility level of group 1, and ultimately its relative population share, $P^2(g)$. If the shock raises A^1 proportionally more for family 1, income inequality would then be **monotonically increasing** over the development phase for all our three income inequality measures.

c. Shocks favorable to family 2: A family-2 friendly shock could occur, e.g., when a less segmented capital market lowers the education financing cost to all families, but especially to family 2, thus lowering (e^1/e^2), or when the shock first benefits family 2, which could not initially finance private schooling. In this case, the takeoff-triggering shocks will produce transition paths just opposite to those in case b. The time paths of all inequality measures will assume a **U shape** if family 2 experiences a takeoff shock ahead of family 1 (see **Figure 2**). Whether the inequality **level** rises or falls at the GE, relative to the SE steady state depends on whether the non-synchronized shock ultimately becomes equi-proportional, in which case the income-bracket inequality, S^2, is constant and $P^2(g)$ falls, so family-income inequality, $E^2(g)$, rises. If investment efficiency rises proportionally more for family 2, all income inequality levels would be **monotonously decreasing**, or family 2 may overtake family 1.[14] Our simulations of cases b and c also reveal negative associations between income **growth** rate and income **inequality** over the transitional development phase. In case b, income inequality and per-capita income growth rate are negatively associated at an early stage of the transition but become positively associated at a more advanced stage, as Barro (2000) finds, while in case c they are negatively associated, which is what Forbes (2000) finds. Our analysis thus shows that the dynamic association between income growth and income inequality can vary by the specific takeoff triggers, or at different stages of the transitional development phase.

Regardless of the way a takeoff-generating shock affects different families, a fundamental implication of our model is that the shape of the family-income inequality path over the transitional development period, $E^2_t \equiv (\overline{H}^1+H^1_t)/(\overline{H}^2+H^2_t)$, would always be congruent to that of human capital attainments, H^1_t/H^2_t, regardless of the specific shape of the paths. Our simulations indicate that this result applies to our other measures of income inequality as well.

B. Paths of Inequality in Fertility and Human Capital Investment

Since by proposition 1, desired fertility is equalized across families in the SE and GE steady states, while generally deviating across families over the transitional phase linking the two, we have:

Proposition 5. Except in the "neutral equilibrium" case, the inequality path of desired fertility will exhibit an **inverted-U shape**, but tend toward equality in the two steady states framing the transitional phase (see Figures 1d and 2d). In the neutral-equilibrium case, inequality in desired fertility assumes a flat time path.

If income inequality measures assume an inverted-U shape due to a skill-oriented technology advance, as in case b of the preceding section, family 1's relative fertility will initially rise above that of family 2 and fall below it in subsequent periods, but ultimately result in a higher population share of group 1, $P^2(g)$. This pattern of evolution in **relative** fertility and income inequality, would not be altered at all if the technology shock is followed by a rise in the cost of housing or opportunity costs of female's time, which raises the unit cost-share of bearing and raising children, v, and generates a demographic transition for both families. This association between fertility rankings and income inequality is consistent with the findings in Kremer and Chen (2002) and De la Croix and Doepke (2003). If the income inequality path assumes a U shape as a result of lower financing costs benefiting lower-income groups, as in case c of the preceding section, family 1's fertility will initially fall short of, but then exceed, that of family 2 during the transition phase. Again, this shape of the income inequality path would not be altered if a subsequent shock in v generates a demographic transition with family 2's fertility level falling ahead of 1's. In our GE framework, however, such associations do not indicate causality, nor can they persist, as desired fertility differences ultimately vanish in a steady state.

IV. EMPIRICAL ANALYSIS

A. Basic Tests

We test empirically two basic implications of the model: a. By Proposition 5, we expect fertility inequality to display an inverted-U shape with flat tails over the development phase; b. Since schooling levels may approximate human capital attainments, we can test another basic prediction: We expect the shapes of schooling and family-income inequality paths to be similar over the transition phase.[15] By our simulation analysis, this expectation applies to all our income-inequality measures as well. To test these propositions we use panel data from different countries over periods of varying length. Since all countries in our sample exhibit positive growth rates over the period, they represent economies in transition toward a steady state of growth.

B. Data and Variables Used

a. *Completed fertility*. Data on the distribution of surviving children per woman are available from the World Fertility Surveys and the Demographic and Health Surveys. Our sample is based on 72 surveys of 29 developing countries in various years between 1974 and 2000. From the individual-level data in each survey, we derive the distribution of surviving children of women age 40 an over. We make this restriction to insure that our measures relate to women who completed childbearing. We then use the **standard deviation** of the distribution of surviving children per woman age 40 and over [SD-FERT] as our fertility inequality measure. But since the [SD-FERT] is subject to a secular drift, we enter the average **level** of completed fertility as a control variable, [AV-FERT].

b. *Human capital*. Our data are taken from Barro and Lee (2000).[16] We use the average number of years of schooling in the population age 15 and over as a proxy for human capital stock. As a measure of inequality in educational attainments we use the standard deviation of the distribution of schooling years [SD-SCHYR] in the population age 15 and over. As in the fertility inequality regressions, we also add the mean schooling years as a control variable [AV-SCHYR].

c. *Income inequality*. The data are taken from Dollar and Kraay (2001). These data cover 86 countries over the period 1950-1998. No data are available about income-group relative inequality, S^i. We proxy our family-income relative inequality measure, E^i, however, by an inter-quintile income inequality ratio [QUINT] (each 'quintile' representing, by definition, equal number of households), and Gini (G) by the conventional Gini coefficient [GINI]. To be consistent with our model, we use only observations that are calculated from **household income** data, excluding observations based on personal income and expenditure data.

d. *Regressors*. We use real per capita GDP level [RGDPn], reported in Heston, Summers, and Aten

[HSA] (2001) as a measure of the economy's development level. Note that since RGDPn is an endogenous variable in our model, its level summarizes the impact of the model's basic parameters which affect our dependent variables as well. We use government's share of GDP [GOV] as an additional regressor, however, as a proxy for government re-distributional policies which may have an exogenous effect on our inequality measures. As a robustness check, we also enter the time trend as (T) to account for other possible missing trended indicators of the development phase. Summary statistics for all variables are given in Appendix C.

C. Regression Models

Our basic regression specification links our inequality measures as dependent variables with regressors introduced in the preceding paragraph plus AV-FERT and AV-SCHYR in the fertility and schooling inequality regressions, all in linear form, but RGDPn is entered in **cubic** or **higher-order** polynomial forms. This is because we predict the **flattening** of the income inequality, educational attainments and fertility paths as the economy converges on a growth steady state.

Our regression specification is not intended to identify a causal effect going from RGDPn to the inequality measures, as these variables are simultaneously determined by our model. However, the use of RGDPn as an indicator of an economy's stage of development is likely to expose the estimated regression coefficients, which we use to depict the shape of the inequality paths over the development process, to simultaneity biases. To derive unbiased and consistent estimated regression coefficients, we use a 2SLS method where RGDPn is treated as an endogenous variable. As instrumental variables (IVs) we include in the first-stage regression the inflation rate in log form ln(INFLA) and one-year-lagged RGDPn. INFLA is used to capture the impact of inflation on the real economy, which is shown to affect RGDPn adversely in the first stage regression. Alternatively, we also estimated the structural 2SLS regression equation using 2-year-lagged or 3-year-lagged RGDPn, as well as all three lagged RGDPn variables jointly as IV's, which produced very similar coefficient estimates. Basmann's test indicates that INFLA and each of the lagged RGDPn variables can serve as an IV in the first-stage regression, and when introduced as additional regressors in the second stage, each of these variables had insignificant and inconsistent effects, validating their use as IVs.

As robustness tests, models 1-3 in Tables 2-5 present OLS regression results based on a few model modifications. In model 1 we enter only the basic regressors accounting for behavior of the relative inequality measures over the development phase. Model 2 allows for fixed country effects, which capture just within-country variability in all variables, and in model 3 we add also GOV and T as regressors. Models 4-6 in Tables 2-5 repeat the specification of models 1-3 using the 2SLS method, which we use to derive Figure 3. In the fertility regressions of Table 2, we employ country-specific **random-effects,** instead of fixed-effects, models to increase the regressions' degrees of freedom, because the number of observations per country is small (2.6 on average). To test for serial correlation, we applied an AR(1) specification using models 2 and 5 of each table. The Durbin-Watson test cannot reject the null hypothesis of no autocorrelation in all cases.

D. Results

The fertility results are reported in Table 2. All models in the table produce an inverted-U-shaped association between fertility inequality and real income. The estimated regression line we chose to depict in **Figure 3a** is based on the 2SLS results from model 4 (with no random effects), because this allows for regression estimates based on variability in regressors both within and across the countries in our relatively small sample. The shape of the inequality path remains virtually the same, however, if we base it on model 5 (with random effects). Note that the sample is dominated by developing countries (In Figure 3a, fertility inequality peaks at an RGDPn level of $3,354 which means that 67% of the observations lie below this real GDP level). Therefore, if we extrapolate the regression line to RGDPn typical of developed economies, fertility inequality would drop sharply, as we predict theoretically. As for the effects of other regressors, the standard deviation of the fertility distribution is monotonically related to the distribution's mean, as one would expect for any distribution. GOV has a negative but insignificant estimated coefficient in the fertility regression (it has, however, a significant effect in the Gini regressions). The time-trend regressor is related inversely to fertility inequality, but directly to educational attainments inequality.

Table 3 reports the results concerning inequality in educational attainments. Regression results without fixed effects indicate an inverted-U-shaped association between income and educational-attainment inequality, as depicted in **Figure 3b** that is based on model 4. Results with fixed effects show a slightly different pattern (U shape at lower RGDPn values and inverted-U shape at higher values), essentially because schooling attainments have much lower variability among developed countries: The association between income and educational-attainment inequality is literally the same with or without allowance for fixed effects when only non-OECD-country data are used. Mean schooling expectedly raises the SD of schooling.

Tables 4 and 5 present the regression results concerning the income inequality paths of G (Gini) and QUINT (a proxy for E^i_t), respectively. All model specifications indicate an inverted-U-shaped association between income inequality and income level. In models 4-6 of both tables we derive this association based on the subset of countries for which both income and educational attainments data are available. We do so because our model and simulation results imply that inequality paths of family-income (E^i_t) and human capital attainments would exhibit an increasingly similar shape as the economy converges on a GE steady state. This prediction is borne out in **Figures 3c** and **3d**, based on model 4 in each table.[17] In fact, the simple correlation coefficient of the predicted values shown in Figures 3b and 3d is .999 and the correlation coefficient of indicator variables of the predicted paths in these figures (indicator variable takes the value of 1 if the slope of the path at each RGDPn value is nonnegative and 0 otherwise) is .992. This lends support to our human capital approach to income distribution. In both figures, income inequality at the highest RGDPn level in our sample is lower than the extrapolated value of income inequality at RGDPn=0, which is consistent with the path derived in case a of our simulations, whereby the income inequality measures E and GINI are lower at the growth steady state than at the stagnant steady state.

Since the estimated regression lines linking both educational attainments and income inequality measures with income levels exhibit an inverted-U shape, the results militate in favor of the Kuznets hypothesis. Note, however, that these results cannot be taken to support the Kuznets hypothesis as a general "law": our analysis indicates that the observed association can be affected by the specific **composition** of countries in our sample and their development stage, as well as by the specific takeoff triggers operating in different countries.

Our results concerning the dynamic behavior of income inequality can be compared to those of Deininger and Squire [DS] (1998). Although DS use the same data, they employ a different fixed-effects regression format with RGDPn and 1/RGDPn entered as regressors. When we add a cubic or higher-order form of RGDPn to the DS specification, however, the plotted relationships between GINI or QUINT and RGDPn exhibit inverted-U shapes in this specification as well, similar to those depicted in Figures 3c and 3d.

CONCLUDING REMARKS

Our deterministic model offers two main messages. The first is that income distribution in the population is linked fundamentally to the corresponding distribution of human capital attainments, not just under static conditions, as in Becker's 1967 paper, but under dynamic conditions as well. Contrary to inferences reached in earlier dynamic models (see footnote 3), neither inequality disappears at an advanced level of development, let alone at a low and stagnant level of development – a conclusion justified by the observed systematic linkage between inequalities in income and educational attainments even in highly developed economies. In this context, our propositions concerning the behavior of income inequality over the entire development process follow from the diverging trends in educational attainments across different income groups as well as the social interaction forces which bind them. The second, and more novel, message is that the dynamic evolution of the level and distribution of income over the entire development process cannot be fully understood without recognizing their linkage to the evolution of the relative fertility rates and population shares of different income classes. Our model shows how income and fertility distributions evolve conjointly over the transitional development phase. It also enables us to derive inferences about the comparative income inequality levels in the growth, relative to the stagnant equilibrium steady states, which frame the transition phase.

Regardless of the dynamic pattern of any of our income inequality measures, a distinct implication of our model is that the time path of fertility inequality will generally exhibit an inverted-U shape with attenuated tails over the transitional development phase. This prediction is supported by our empirical investigation, based on a sample of countries in different stages of development. It is also consistent with historical evidence indicating that the association between relative variances in fertility and income levels across Western European regions exhibited an inverted-U shape between the mid-19th century and 1970, with fertility variances being quite low in the pre-demographic transition phase (see Coale and Treadway, 1986).

Concerning the shape of the dynamic association between income growth and inequality, no “general law” applies. We offer, however, several new insights. First, the shape can vary depending on the parameter(s) that trigger the takeoff and the manner in which they reach different family groups. In ex-command economies, e.g., where the opening of markets benefits more those with greater ability to acquire new knowledge, it is likely that case (b) illustrated in Figure 1 emerges, with income inequality measures (initially) rising. If the takeoff-trigger reflects largely government subsidization of education, case (c) illustrated in Figure 2 may be more likely to occur. Second, the shape depends on whether the economy is in a stagnant or growth steady state, or in a transitional development phase where it becomes sensitive to the specific mix of countries in the sample. This may partly explain why different studies reach different conclusions about it. Third, the shape partly depends on the inequality measure used. We derive three such measures as endogenous variables: family-income inequality (E^2_t); income-group inequality (S^2_t=E^2_t P^2_t), which also depends on the distribution of families across income brackets (P^2_t); and the Gini coefficient (G_t), which is increasing in S^2_t but decreasing in P^2_t. Our model offers some strong predictions concerning the dynamic behavior of these measures.

For example, under given preferences and external production technologies, and a stable distribution of investment efficiencies, we expect the income-group inequality measure, S^2_t=$E^2_tP^2_t$, to converge on equal levels in stagnant and growth steady states, or $S^2(s) = S^2(g)$. The comparative levels of this measure's components, E^2_t and P^2_t, and therefore the Gini coefficient, in contrast, may vary across the two steady states. We expect family-income inequality in a stagnant steady state, $E^2(s)$, to be strictly a function of inequality in inherited family-specific endowments such as social or legal status. No clear-cut predictions can be made, however, about the shape of the family-income inequality path (E^2_t) over the transition phase: An inverted-U-shaped path is likely to emerge as a result of a uniform, skill-biased technological advance, which first reaches the top-skilled family group. The relative population share of that group (P^2) would then rise at the growth, compared to the stagnant, steady state, and in this scenario, family-income inequality would ultimately fall at the highest development level, compared to the lowest, as figure 3d illustrates. No government re-distribution policies are required to achieve such outcome. In contrast, a U-shaped family-income inequality path with P^2 falling and family-income inequality turning **higher** at advanced, relative to pre-takeoff, development phases, can emerge as a result of educational subsidies which lower the financing-cost disadvantage of especially lower-income families.

The link between income and fertility choices also sheds light on the behavior of income and fertility distributions at the stagnant- and growth-equilibrium steady states. In the SE, a skill-biased technology advance raises the steady state income and fertility levels as well as fertility inequality and two income inequality measures - $S^2(s)$ and $G(s)$. In the GE, a similar advance raises income growth and all inequality measures, as well as fertility level and inequality. This prediction is borne out by the evidence we presented for the US following the IT revolution in the 1980s.

Although our analysis is based on a deterministic model of heterogeneous family groups with inherited differences in endowments and investment efficiencies, it allows for a degree of social mobility, especially in the case where leadership in human capital formation can switch from the group with initially highest earning capacity to the initially followers' group over the transitional development phase. Moreover, our key implications hold if we allow also for stochastic variations in ability within groups (see fn. 4). And although our basic model relates to inequality in earning capacity, the propositions we derive may apply to total income as well (see Appendix A).

A critical implication of our model is that the dynamic path of family-income inequality, regardless of its shape, should mirror that of educational attainments, both having flat tails. This is what we find empirically. This finding supports the basic premise of our model, that family-income growth and distribution are directly linked to human capital formation and distribution.

APPENDICES

A. The Model with Savings

Although our model abstracts from capital markets, we can incorporate returns on savings as an outcome of "home production" in which old parents' human capital serves as an input, and the yield is subject to diminishing returns. This is a natural assumption in the context of our closed-economy framework. The extension allows us to recognize inequalities in labor earnings as well as in total income, incorporating both earnings and property income.

Formally, we now assume that each agent lives through three periods: childhood, young adulthood, and old age. Total savings is defined by $K_t \equiv (\overline{H}^i + H^i_t)s^i_t$, where s^i_t is the fraction of productive capacity saved at adulthood, and K_t is assumed to fully depreciate within one generation. Income from savings is generated when old parents combine their accumulated assets, K_t, with their human capital inputs via the production function, $F = D(\overline{H}^i + H^i_t)^{1-\kappa}[(\overline{H}^i + H^i_t)s^i_t]^{\kappa}$, $0<\kappa<1$. The relevant objective is to maximize:

(2′) $U(C^i_{1,t}, C^i_{2,t+1}, W^i_{t+1}) = [1/(1-\sigma)][C^i_{1,t}{}^{1-\sigma} - 1] + \delta[1/(1-\sigma)]\{[C^i_{2,t+1}{}^{1-\sigma} - 1] + [W^i_{t+1}{}^{1-\sigma} - 1]\}$,

where the consumption flows at adulthood and old age are given by

$C^i_{1,t} = (\overline{H}^i + H^i_t)[1 - vn^i_t - \theta^i h^i_t n^i_t - s^i_t]$,

$C^i_{2,t+1} = D(\overline{H}^i + H^i_t)^{1-\kappa}[(\overline{H}^i + H^i_t)s^i_t]^{\kappa}$,

and the altruism function W^i_{t+1} is defined as in equation (4).

The first order optimality conditions for n^i_t and s^i_t are thus given by,

(A1) $0 = -C^i_{1,t}{}^{-\sigma}(v + \theta^i h^i_t)(\overline{H}^i + H^i_t) + \delta W^i_{t+1}{}^{-\sigma} \beta B(n^i_t)^{\beta-1} H^i_{t+1}{}^{\alpha}$, for $n^i_t \geq 0$, and

(A2) $0 = -C^i_{1,t}{}^{-\sigma}(\overline{H}^i + H^i_t) + \delta C^i_{2,t+1}{}^{-\sigma} \kappa D(\overline{H}^i + H^i_t)(s^i_t)^{\kappa-1}$, for $s^i_t \geq 0$.

Combining these two equations, we can show that n^i_t and s^i_t are inversely related as follows: $1 - \beta v n^i_t/(\beta - 1) = s^i_t + [\delta D^{1-\sigma} \kappa]^{-1/\sigma}(s^i_t)^{[1-(1-\sigma)\kappa]/\sigma}$. It is easy to show that the optimal solution for h^i_t remains the same as in equation (7).

We can now distinguish income inequality from earnings inequality. The measures of the pooled income of a family head – earnings as well as property income from savings – can be defined parallel to our earnings-inequality measures in section I.C. For example, TS^2_t below corresponds to the ratio of **total** income-group inequality (wage earnings of adult parents plus non-wage income of old parents) of group 1 relative to group 2, and the same holds for family income inequality, TE^2_t, and the Gini coefficient, TG_t:

$TS^2_t \equiv [N^1_t(\overline{H}^1 + H^1_t) + N^1_{t-1} D(\overline{H}^1 + H^1_{t-1})(s^1_{t-1})^{\kappa}] / [N^2_t(\overline{H}^2 + H^2_t) + N^2_{t-1} D(\overline{H}^2 + H^2_{t-1})(s^2_{t-1})^{\kappa}]$,

$TE^2_t \equiv TS^2_t / TP^2_t$; $TP^2_t \equiv [(N^1_t + N^1_{t-1})/(N^2_t + N^2_{t-1})]$, and

$TG_t \equiv [TS^2_t - (N^1_t + N^1_{t-1})/(N^2_t + N^2_{t-1})] / (1 + TS^2_t)/[1 + (N^1_t + N^1_{t-1})/(N^2_t + N^2_{t-1})]$.

Under our heterogeneity restriction, we can show that in any steady state, the optimal savings rates (s^i) and shares of income spent on raising and educating a child, (vn^i) and ($\theta^i h^i$), are identical in all family groups, since the first-order optimality conditions governing these control variables become identical for all family groups in all stable steady states. Our total income inequality measures are therefore identical to the corresponding earnings-inequality measure at both the stagnant- and growth-equilibrium steady states. Moreover, we can show that the relative inequality in earnings, and hence in total income in this extended model is the same as that derived in our benchmark model sans savings, as given by equations (11), (13) and (14). All our propositions in sections II and III are also maintained in this extended model, as are the qualitative results of the comparative dynamics reported in Table 1 for both the SE and GE steady states. The time paths of the inequality measures derived in section III are also shown to have the same shape as in the extended model with savings.

Over the transitional development phase, however, the savings rate may differ across families. For example, when a takeoff occurs as a result of a skilled-bias technological advance reaching initially the higher-income family group 1, our income inequality measures assume an inverted-U shape, and the savings rate of family 1 initially falls below that of (stagnant) family 2. In the following stage, however, as family group 2 experiences a takeoff because of the social-interaction effects coming from family-group 1, its savings rate falls below that of family 1. The aggregate savings rate then starts rising while income inequality is falling. The resulting association between income inequality and the aggregate savings rate does not indicate causality, however (as in Keynes, 1920, or Kaldor, 1957).

What would be the effect of changes in D or κ on our income inequality measures? As long as these changes are common to all families, they will affect only the **composition** of family income, but not the total income inequality measures, as our simulations confirm.

B. Local Stability Conditions in the Steady States

1. The stagnant equilibrium

At the stagnant equilibrium, the dynamic system is given by

(B1) $P^2_{t+1} = n^1/[n^2(S^2_t)]\, P^2_t$

(B2) $H^2_{t+1} = A^2\, h^2(\overline{H}^2 + H^2_t)\,(S^2_t)^\gamma$

To check for local stability, we first fully differentiate equations (B1) and (B2). Following some algebraic manipulations and omitting the superscript 2, the linearized system becomes:

(B3) $$\begin{bmatrix}\hat{H}_{t+1}\\ \hat{P}_{t+1}\end{bmatrix} = \begin{bmatrix}(1-\gamma)K & \gamma\\ eK & 1-e\end{bmatrix}\begin{bmatrix}\hat{H}_t\\ \hat{P}_t\end{bmatrix} = M\begin{bmatrix}\hat{H}_t\\ \hat{P}_t\end{bmatrix},\ \text{where,}$$

$\hat{X}_t = d\ln(X_t)$ denote percentage deviations from steady state values of X = H or P,

$K \equiv \{H(s)/[\overline{H} + H(s)]\}$, and $e \equiv [\partial n(s)/\partial S(s)]/[n(s)/S(s)]$.

The characteristic polynomial of M or det(M-XI), is then given by

$F(X) = X^2 + [e - (1-\gamma)K - 1]X + [(1-e)(1-\gamma)K - \gamma e K]$.

The necessary and sufficient condition for general stability (covering both oscillatory and non-oscillatory equilibrium scenarios) requires the two roots of the characteristic equation to be smaller than one in modulus. These conditions are summarized by

(B4) $0 < e < 2\,\{1 - [\gamma K/(K+1)]\}$

2. The growth equilibrium

In the growth equilibrium, the relative effect of $\overline{H}^i$ is negligible since H^i_t is increasing without bound. Thus we can suppress $\overline{H}^i$ to examine the growth equilibrium dynamics. We can then express the system of growth equilibrium in terms of E^2_t and P^2_t as follows:

(B5) $$P^2_{t+1} = \frac{n^1}{n^2(S^2_t)}P^2_t$$

(B6) $$E^2_{t+1} = \frac{H^1_{t+1}}{H^2_{t+1}} = \frac{A^1h^1H^1_t}{A^2h^2H^2_t(S^2_t)^\gamma} = \frac{A^1h^1}{A^2h^2}E^2_t(S^2_t)^{-\gamma} = \frac{A^1h^1}{A^2h^2}(E^2_t)^{1-\gamma}(P^2_t)^{-\gamma}.$$

Taking the total differentials of (B5) and (B6) around the growth steady state, the linearized system can be shown equal to:

(B7) $\begin{bmatrix} \hat{P}_{t+1} \\ \hat{E}_{t+1} \end{bmatrix} = \begin{bmatrix} 1-e(g) & -e(g) \\ -\gamma & (1-\gamma) \end{bmatrix} \begin{bmatrix} \hat{P}_t \\ \hat{E}_t \end{bmatrix} = M \begin{bmatrix} \hat{P}_t \\ \hat{E}_t \end{bmatrix}$,

where $e(g) \equiv [\partial n^2(g)/\partial S^2(g)]/[n^2(g)/S^2(g)]$.

The characteristic polynomial of M is thus given by

$F(X) = X^2 + [\gamma + e(g) - 2]X + [1 - e(g) - \gamma]$,

and the roots of the polynomial equation F(X) are 1 and $[1 - e(g) - \gamma]$.

The unit root indicates that we do not have a unique equilibrium steady state values of $E^2(g)$ and $P^2(g)$, as we note in the text. However we do have a unique equilibrium solution for $S^2(g)$. Since $S^2_t = E^2_t P^2_t$, we obtain by adding the solutions for the rates of change in E^2_t and P^2_t in equation (B7) around the growth steady:

$\hat{E}_{t+1} + \hat{P}_{t+1} = [1 - e(g) - \gamma] \left(\hat{E}_t + \hat{P}_t\right)$ or $\hat{S}_{t+1} = [1 - e(g) - \gamma]\ \hat{S}_t$.

Hence, we have stable and unique equilibrium for S_t as long as $-1<[1 - e(g) - \gamma]<1$, or

(B8) $-\gamma < e(g) < 2 - \gamma$.

3. Numerical sensitivity analysis of the conditions for local stability

To perform sensitivity analyses of our local stability conditions, we evaluate the elasticity e(s) by totally differentiating equation (B4), and then applying the baseline parameters used to derive the SE as well as the GE in Table 1, parts 1 and 2. Equations (B4) and (B8) are found to be satisfied for all values of $\gamma \in (0,1)$.

C. Variables used in the regressions and summary statistics

Variable	Description	Mean [Std. Dev.]
SD-FERT	Standard deviation of the distribution of surviving children per female ≥ 40	2.520 [0.306]
AV-FERT	Average of the distribution of surviving children per female ≥ 40	4.179 [1.161]
SD-SCHYR	Standard deviation of the distribution of schooling years in the population ≥ 15	3.684 [0.806]
AV-SCHYR	Average of the distribution of schooling years in the population ≥ 15	4.888 [2.755]
GINI*	Gini coefficient	37.76 [7.948]
QUINT*	Share of total income received by the top relative to the bottom quintile of families in the population	8.826 [5.259]
RGDPn	Real per-capita income	6340 [5960]
GOV	GDP shares of government spending	19.53 [8.821]
INFLA	Annual inflation rate in GDP deflator (percent)	56.39 [587.1]

* We calculate GINI and QUINT exclusively based on household income data reported in Dollar and Kraay (2001), excluding observations based on personal income, personal expenditures, or household expenditure data.

REFERENCES

Acemoglu, Daron, "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review*, 89, 1259-1278, 1999.

Acemoglu, Daron, "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*, 40, 7-72, 2002.

Aghion, Philippe, Eve Caroli and Cecilia Garcia-Penalosa, "Inequality and economic growth: the perspective of the new growth theories," *Journal of Economic Literature*, 37, 1615-1660, 1999.

Alesina A. and R. Perotti, "Income Distribution, Political Instability and Investment," *European Economic Review,* 81, 1170-1189, 1996.

Alesina, A., and D. Rodrik, "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, 108, 465-490, 1994.

Anand, S., R. Kanbur, "The Kuznets Process and the Inequality-Development Relationship," *Journal of Development Economics*, 40, 25-52, 1993.

Banerjee A. and E. Duflo, "Inequality and growth: What can the data say?" NBER working paper no. 7793, 2000.

Banerjee, A. and Newman, A., "Occupational Choice and the Process of Development," *Journal of Political Economy,* 101, 274-98, 1993.

Barro, Robert, "Inequality and Growth in a Panel of Countries," *Journal of Economic Growth*, 5, 5-32, 2000.

Barro, Robert J. and Jongwha Lee, "International Data on Educational Attainment: Updates and Implications," working paper, 2000.

Becker, Gary, "Human Capital and the Personal Distribution of Income: An Analytical Approach," Woytinski Lecture No. 1, Institute of Public Administration, University of Michigan Press, Ann Arbor, 1967.

Becker, Gary, "A Theory of Marriage: Part I," *Journal of Political Economy*, 81, 1973.

Becker, G.S. and N. Tomes, "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility", *Journal of Political Economy* 87, 1979.

Benhabib, J. and A. Rustichini, "Social Conflict and Growth," *Journal of Economic Growth,* 1, 129 -146, 1996.

Burdett, Ken and Melvyn Coles, "Marriage and Class," *Quarterly Journal of Economics*, 112, 1997.

Coale, Ansley J. and Roy Treadway, “A Summary of the Changing Distribution of Overall Fertility, Marital Fertility, and the Proportion Married in the Provinces of Europe,” in Ansley J. Coale and Susan C. Watkins ed., *The Decline of Fertility in Europe, The Revised Proceedings of a Conference on the Princeton European Fertility Project*, Princeton University Press, Princeton, 1986.

De Gregorio, J. and J. Lee, “Education And Income Distribution: New Evidence From Cross-Country Data,” *Review of Income and Wealth*, 48, 395-416, 2002.

De la Croix, David and Matthias Doepke, “Inequality and Growth: Why Differential Fertility Matters,” *American Economic Review*, 93, 2003.

Deininger, Klaus and Lyn Squire, “New ways of looking at old issues: inequality and growth,” *Journal of Development Economics*, 57, 259-287, 1998.

Dollar, David, and Aart Kraay, “Growth is Good for the Poor,” The World Bank: Washington, D.C. , 2001.

Durlauf, S.N., “A Theory of Persistent Income Inequality,” *Journal of Economic Growth*, 1, 75-94, 1996.

Ehrlich, Isaac and Francis T. Lui, “Intergenerational trade, longevity, and economic growth,” *Journal of Political Economy*, 99, 1029-1059, 1991.

Ehrlich, Isaac and Chi-Wa Yuen, “The Growth-Inequality Nexus in the Absence of Borrowing Restrictions and Government Intervention: Some Implications from a Prototype Model”, *Annals of Economics and Finance*, 1, 283-300, 2000.

Eicher, Theo, “Interaction between Endogenous Human Capital and Technological Change,” *Review of Economic. Studies*, 63, 127–44, 1996

Fields, Gary, “Measuring Inequality Change in an Economy with Income Growth: Reply,” *Journal of Development Economics*, 32, 205-210, 1990.

Fields, Gary, and G.H. Jakubson, “New Evidence on the Kuznets Curve,” mimeograph, Department of Economics, Cornell University, 1994.

Forbes, K., “A Reassessment of the Relationship between Inequality and Growth,” *American Economic Review*, 90, 869-887, 2000.

Galor, Oded and Joseph Zeira, “Income distribution and macroeconomics,” *Review of Economic Studies*, 60, 35-52, 1993.

Galor, Oded and Omer Moav, “From Physical to Human Capital Accumulation: Inequality and the Process of Development,” *Review of Economic Studies*, 71, 1001-10026, 2004

Heston, Alan, Robert Summers, and Bettina Aten, Penn World Table Version 6.0 Center for

International Comparisons at the University of Pennsylvania (CICUP), 2001.

Kaldor, N., "A Model of Economic Growth," *Economic Journal*, 57, 1957.

Katz, Lawrence F. and Kevin M. Murphy, "Changes in Relative Wages, 1963-87: Supply and Demand Factors," *Quarterly Journal of Economics*, 107, 35-78, 1992.

Keynes, J. M., *The Economic Consequences of the Peace*, Macmillan and Co. Limited, 1920.

Kremer, Michael and Daniel Chen, "Income Distribution Dynamics with Endogenous Fertility," *Journal of Economic Growth*, 7, 2002.

Kremer, Michael and Eric Maskin, "Wage Inequality and Segregation by Skill," NBER Working Paper No. 5718, 1996.

Kuznets, Simon, "Economic growth and income inequality," *American Economic Review*, 45, 1-28, 1955.

Kuznets, Simon, "Quantitative Aspects of the Economic Growth of Nations," *Economic Development & Cultural Change*, 12, 1-80, 1963.

Lindbeck, Assar and Dennis Snower, "Reorganization of Firms and Labor-Market Inequality," *Ame rican Economic Review*, 86, 315–21, 1997.

Lindert, P. H., and J. G. Williamson, "Growth, Equality, and History," *Explorations in Economic History*, 22, 341-377, 1985.

Loury, Glenn, "Intergenerational Transfers and the Distribution of Earnings," *Econometrica,* 49, 843-867, 1981.

Lucas, R. E., Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3-42, 1988.

Persson, Torsten and Guido Tabellini, "Is inequality harmful for growth?" *American Economic Review*, 64, 600-621, 1994.

Tamura, Robert, "Income convergence in the endogenous growth model," *Journal of Political Economy*, 99, 522-540, 1991.

Venieris, Y. and D. Gupta, "Income Distribution and Sociopolitical Instability as Determinants of Savings: A Cross Sectional Model," *Journal of Political Economy,* 94, 873-883, 1986.

United Nations Educational, Scientific and Cultural Organization, *Statistical Yearbook*, some issues.

Zhong, Jianguo, "Income Growth and Income Inequality: Theory and Evidence," Ph.D. dissertation, State University of New York at Buffalo, 1998.

ENDNOTES

[1] Some studies favor the Kuznets hypothesis: e.g., Lindert and Williamson (1985) and Barro (2000). Others reject it, or find no systematic relation, e.g., Anand and Kanbur (1993), Fields (1990), Fields and Jakubson (1994), and Deininger and Squire (1998). Studies of the way income **growth rate** relate to income inequality also report mixed results: Persson and Tabellini (1994), Alesina and Rodrik (1994), and Deininger and Squire (1998) find a negative relation; Forbes (2000) finds a positive one; Barro (2000) finds that higher inequality lowers the growth rate in poor countries but raise it in rich ones, while Banerjee and Duflo (2000) find an inverted-U relation between the two.

[2] Models supporting Kuznets' direction of causality rely, e.g., on structural shifts in a two-sector model (Kuznets 1955, 1963, Anand and Kanbur 1993); skill-biased technical progress (Eicher 1996, Aghion et al. 1999); and organizational changes (Kremer and Maskin 1996, Lindbeck and Snower 1997, Acemoglu 1999). Models favoring causality going from inequality to growth rely, e.g., on credit market imperfections (Loury 1981, Galor and Zeira 1993, Banerjee and Newman 1993, Durlauf 1996, and Galor and Moav 2004); political economy changes (Venieris and Gupta 1986, Alesina and Perotti 1996, Benhabib and Rustichini 1996); and fertility changes by income (Kremer and Chen 2002, and De la Croix and Doepke 2003).

[3] Lucas (1988) also considers spillover effects in goods production, stemming from the average human capital level in a representative-agent model. Tamura (1991) applies a similar spillover effect in human capital production, which results in full income-convergence. De la Croix and Doepke (2003) also use the Lucas-type spillover effect and consider fertility as well as income inequality but reach the same income-convergence result as Tamura. They focus on growth effects stemming from assumed initial inequality in human capital, however, rather than on the joint dynamic evolution of inequalities in education, income, and fertility over the entire development process. Zhong (1998), and Ehrlich and Yuen (2000) develop prototype frameworks similar to ours, but ignore the role of fertility.

[4] Becker (1973) and Burdett & Coles (1997) offer evidence supporting positive assortative mating by intelligence and education. Our assumed fixed distribution of family types still allows for family-group mobility over time, since family group 2 may move closer to, or even overtake, group 1 over the transitional phase (see fn. 14). Alternatively, we can allow for individual mobility as well by allowing for stochastic deviations in ability within initial k subgroups of family type i (=1,2), using a stochastic specification similar to that of Becker and Tomes (1979): $A^{k,i}_{t+1} = \phi A^{k,i}_{t} + (1-\phi)A^{i} + \varepsilon^{k,i}_{t+1}$, where $A^{k,i}_{t+1}$ is the ability level of generation (t+1) in subgroup k, A^{i} is the mean ability level of group i, $(0 < \phi < 1)$ is a weight, and $\varepsilon^{k,i}_{t+1}$ is a stochastic variable, iid, with zero mean and constant variance. It is reasonable to assume that: (a) $\varepsilon^{k,i}_{t+1}$ is revealed a-posteriori; i.e., after parental decisions on quantity and quality of children are made; and (b) social interaction is determined by the average earning capacity and family-group size in group 1 relative to 2 in generation t. Allowing for positive assortative mating within each subgroup we now obtain regression towards the mean ability level A^{i} inside group i in any steady state. Overall income inequality in a steady state will be larger now than in our deterministic model due to the influence of $\varepsilon^{k,i}_{t+1}$. Inequalities in fertility and educational investments, however, will be the same as in the deterministic case, unless we also allow for a stochastic component distinguishing desired and realized fertility. The steady-state comparative-dynamic results of our model hold in this stochastic specification as well.

[5] Knowledge transmission is thus modeled as an exogenous process, assuming that spillover effects cannot be internalized a priori. We also disallow strategic group behavior aimed at benefitting from spillover effects.

[6] The leader-follower specification can be generalized under additional assumptions to a J-group specification (i=1,…J): let the homogeneous members of group 1 be the exclusive source of knowledge transfer, and assume that successful knowledge transfer from a member of group 1 to a member of group i>1 requires **pair-wise interactions**, such as random pairings of agents forming a work team and sharing a school desk or a two-seat assignment on a commuter plane, which allow for **intensive** knowledge transfer. The odds that the member of i>1 is paired with a member of any other group $j\neq i$, is $(N-N^i)/N^i$, where N is the total population. Effective interaction requires, however, that the paired member of group j would be a member of group 1, the conditional probability of which is $N^1/(N-N^i)$. The odds of exclusive interaction between members from groups 1 and i is thus the product of the two $[(N-N^i)/N^i][N^1(N-N^i)] = P^i = N^1/N^i$. This restrictive specification of social interaction yields all the propositions derived in this paper.

[7] This specification of altruism or "companionship", relating parents' utility to children's human capital rather than earning capacity, has the advantage of allowing for interior solutions for all our control and "state" variables, which permits the derivation of all inequality measures as endogenous variables all along the development process, including both the stagnant and growth steady states.

[8] If we allow for differences in survival probabilities from childhood to adulthood, π^i, the necessary stability condition would be equality of the expected numbers of surviving children: $\pi^1 n^1=\pi^2 n^2$.

[9] As propositions 2 and 3 below indicate, in any stable equilibrium steady state, all families spend the same proportion of their potential income on quantity (vn) and quality (θh) of children. Thus all wind up with the same desired fertility level despite their different income levels, because in equilibrium, a lower income ($Y^i_t=\overline{H}^i+H^i_t$) would be offset by a proportionately lower shadow price of fertility ($[v^i+\theta^i h^i_t]Y^i_t$), and the demand for children's quantity n^i is then a function of the ratio of the two.

[10] In the log utility case, we have an explicit solution for fertility: $n^i_t = [\delta(\beta-1)]/[v^i(1+\delta\beta)]$. Thus, for n^1 to be equal to n^2, $v^1 = v^2$.

[11] More specifically, $\partial G/\partial x=(1-1/E^2)(1-P^2S^2)\partial S^2/\partial x+S^2(P^2+1/E^2)^2\partial E^2/\partial x$ in any equilibrium position, where x is one of our parameters. In a stagnant state, a rise in (e^1/e^2) unambiguously raises $S^2(s)$, while not affecting $E^2(s)$. Thus G(s) will rise or fall depending on whether $P^2(s)\cdot S^2(s)$ is smaller or bigger than 1. An increase in $\overline{H}^1/\overline{H}^2$ will not affect $S^2(s)$ but will raise unambiguously $E^2(s)$ and G(s). In a growth steady state, we cannot make symmetrical predictions because an increase in (e^1/e^2), for example, may also affect $E^2(g)$, as our analysis in section II.B indicates.

[12] Although the fertility level of Hispanic female immigrants is higher than that of the US female population, the rise in the US TFR during the 1980s cannot be entirely attributed to the rise in the share of Hispanic immigrants in the female population (age 15-49) from 2.37 to 4.63% in the 1980s. As US

Census data show, although 93% of Hispanic immigrants report themselves as whites, the rise in the TFR among whites is approximately the same as that of nonwhites, and the TFR of blacks also significantly rose from 2.17 in 1980 to 2.48 in 1990.

[13] Major technological advances raising investment efficiency typically generate structural shifts in the economy favoring employment opportunities for women as well as a rise in housing costs due to urbanization, and are thus accompanied by an increase the unit (opportunity) costs of bearing and raising children (v). Technological advances can thus generate a demographic transition indirectly by our preset model. In EL(1991), where parents are motivated by old-age support from educated children as well as by pure altruism, technological advances can generate a demographic transition directly.

[14] There also is the possibility of mixed cases. For example, a technological shock reaches first family 1 (case b), but government subsidization of education targets family 2 (case c). Alternatively, if a reduction in θ^2 affects family 2 many periods ahead of family 1, or by a sufficiently greater proportion, so that e^1/e^2 actually falls, family 2 can **overtake** family 1, and become the "leading family" in terms of income-generating capacity. Income inequality will then reach a minimum at the point of overtaking, but will rise afterwards until it converges on its GE steady-state level. In this case the time path of income inequality will assume an S shape.

[15] Our family-income inequality measure, E^2, converges on a steady state level, $(H^1/H^2)(g)$, at the growth-equilibrium steady state. De Gregorio and Lee (2002) provide independent support. They estimate a positive relationship between inequality in educational attainments and income inequality.

[16] The Barro-Lee study reports average schooling years for four schooling levels in the population age 15 and up (zero, primary, secondary, and higher) and their population shares. We calculate the mean and standard deviation of this distribution for each country in all sample years.

[17] Also, regression results obtained when using a polynomial of RGDPn of the 4th, 5th, and 6th order showed the same pattern as in all panels of Figure 3.

Table 1: Simulating Comparative Dynamic Effects of Parameter Changes in a Two-agent Economy

Part 1. Stagnant Equilibrium

A^1/θ^1	A^2/θ^2	$\overline{H}^1$	$v^1(v^2)$	γ	$B^1(B^2)$	$n^1(n^2)$	$Y^1=\overline{H}^1+H^1$	$Y^2=\overline{H}^2+H^2$	E	S	$P=N^1/N^2$	Gini
2/1	1/1.01	50	.01	.4	.1	8.053	55.555	1.111	50	5.799	.116	.749
3/1	1/1.01	50	.01	.4	.1	8.243	58.824	1.176	50	15.981	.320	.699
3/1	**1.5/1.01**	50	.01	.4	.1	8.243	58.824	1.176	50	5.799	.116	.749
2/1	**1.5/1.01**	50	.01	.4	.1	8.053	55.555	1.111	50	2.105	.042	.637
2/1	1/1.01	**60**	.01	.4	.1	8.053	66.666	1.111	60	5.799	.097	.765
2/1	1/1.01	50	**.015**	.4	.1	5.344	58.824	1.176	50	5.799	.116	.749
2/1	1/1.01	50	.01	**.45**	.1	8.053	55.555	1.111	50	4.770	.095	.740
2/1	1/1.01	50	.01	.4	**.15**	8.243	55.555	1.111	50	5.799	.116	.749

Part 2. Growth Equilibrium

A^1/θ^1	A^2/θ^2	$v^1(v^2)$	γ	$B^1(B^2)$	$n^1(n^2)$	h^1	h^2	$a^1=A^1h^1$	E	S	$P=N^1/N^2$	Gini
40/1	20/1.01	.01	.4	.1	9.448	.05	.0495	2	50	5.799	.116	.749
50/1	20/1.01	.01	.4	.1	9.551	.05	.0495	2.5	85.138	10.131	.119	.804
50/1	**25/1.01**	.01	.4	.1	9.551	.05	.0495	2.5	50	5.799	.116	.749
40/1	**25/1.01**	.01	.4	.1	9.448	.05	.0495	2	29.368	3.320	.113	.667
40/1	20/1.01	**.015**	.4	.1	6.274	.075	.0743	3	50	5.799	.116	.749
40/1	20/1.01	.01	**.45**	.1	9.448	.05	.0495	2	41.502	4.770	.115	.724
40/1	20/1.01	.01	.4	**.15**	9.634	.05	.0495	2	50	5.799	.116	.749

Part 3. Takeoff Triggers

	A^1	A^2	θ^1	θ^2	$v^1(v^2)$	$n^1(n^2)$	h^1	h^2	E	S	$P=N^1/N^2$	Gini
SE	2	1	1	1.01	.01	8.053	.05	.0495	50	5.799	.116	.749
GE	**40**	**20**	1	1.01	.01	9.448	.05	.0495	50	5.799	.116	.749
GE	**40**	1	1	1.01	.01	9.448	.05	.0495	38643	10374	.268	.788
GE	2	1	**1/15**	**1.01/15**	.01	9.316	.75	.743	50	5.799	.116	.749
GE	2	1	**1/15**	**1/15**	.01	9.316	.75	.743	48.857	5.657	.116	.746
GE	2	1	1	1.01	**.11**	.712	.55	.545	50	5.799	.116	.749

Note: Parameters values that deviate from our benchmark values are presented in bold print.

Part 1. Comparative dynamics in the stagnant steady state are simulated by changing A^1/θ^1, A^2/θ^2, $\overline{H}^1$, $v^1(=v^2)$, γ, or $B^1(=B^2)$, holding constant all other parameters: $\overline{H}^2 = 1$, $\sigma = 0.9$, $\delta = 0.9$, and $\beta = 1.2$. Columns for h^1 and h^2 are suppressed, but the corresponding solutions are: .05, .0495 for all rows except row 6; and .075, .0743 for row 6.

Part 2. Comparative dynamics in the growth steady state are simulated by changing A^1/θ^1, A^2/θ^2, $v^1(=v^2)$, γ, or $B^1(=B^2)$, holding constant $\sigma = 0.9$, $\delta = 0.9$, and $\beta = 1.2$.

Part 3. Simulations show the impact of uniform (proportionate) and non-uniform changes in A^i and θ^i, as well as in the common level of $v^1=v^i$ that generate a takeoff from the SE to GE steady state, holding constant: $\overline{H}^1 = 50$, $\overline{H}^2 = 1$, $\sigma = 0.9$, $\delta = 0.9$, $\gamma = 0.4$, $\beta = 1.2$, and $B^1 = B^2 = 0.1$.

Table 2: Fertility Inequality Regressions

Dependent Variable: SD_FERT

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
	OLS	OLS with random effects	OLS with random effects	2SLS	2SLS with random effects	2SLS with random effects
Intercept	1.776852	2.087804	2.181626	1.733183	1.985153	2.157891
	7.45	10.54	10.53	7.34	9.81	10.34
RGDPn	0.000410	0.000249	0.000270	0.000397	0.000310	0.000288
	2.23	1.59	1.83	2.55	2.19	2.21
$RGDPn^2$	-8.49E-08	-5.75E-08	-5.59E-08	-8.56E-08	-7.27E-08	-6.17E-08
	-1.90	-1.60	-1.66	-2.29	-2.33	-2.14
$RGDPn^3$	5.08E-12	3.40E-12	3.25E-12	5.25E-12	4.34E-12	3.64E-12
	1.64	1.44	1.47	2.10	2.24	2.02
AV_FERT	0.056786	0.042325	0.134551	0.071700	0.051122	0.146717
	1.88	2.14	4.18	2.38	2.71	4.90
GOV			-0.004215			-0.005876
			-0.87			-1.28
T			-0.013374			-0.013659
			-3.46			-3.78
Adj. R^2	0.0854	0.1637	0.2824	0.1004	0.2340	0.3852
N	72	72	72	70	70	70

Notes: The dependent variable is the standard deviation of the distribution of surviving children per woman age 40 and over, but we also add the mean fertility levels as a control variable (see text). Data sources are the World Fertility Surveys and the Demographic and Health Surveys (various years). Rows show the estimated coefficients (β) and their z-statistics (β/S_β). This table's regressions employ a random effects specification to account for missing idiosyncratic variables, because the number of observations per country is small. The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in Models 2 and 5. The 2SLS model accounts for endogeneity of RGDPn. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.

[#] The Intercept coefficients represent the mean values of all intercept terms.

Table 3: Educational Attainments Inequality Regressions

Dependent Variable: SD_SCHYR

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
	OLS	OLS with fixed effects	OLS with fixed effects	2SLS	2SLS with fixed effects	2SLS with fixed effects
Intercept	2.634408	2.074857[#]	2.512515[#]	2.720585	2.227975[#]	2.560042[#]
	37.43			34.86		
RGDPn	9.29E-05	-3.79E-05	-4.36E-05	1.27E-04	-7.17E-06	-1.10E-06
	2.71	-1.02	-1.32	3.60	-0.14	-0.02
$RGDPn^2$	-1.20E-08	4.29E-09	2.75E-09	-1.51E-08	2.68E-09	2.35E-11
	-3.79	1.48	1.07	-4.74	0.79	0.01
$RGDPn^3$	3.03E-13	-1.59E-13	-1.19E-13	3.81E-13	-1.21E-13	-5.98E-14
	3.48	-2.21	-1.87	4.42	-1.52	-0.85
AV_SCHYR	0.205215	0.351542	0.111584	0.189881	0.302893	0.051998
	11.97	18.40	4.50	10.26	11.64	1.79
GOV			0.000617			0.002328
			0.25			0.60
T			0.029080			0.032278
			12.92			12.38
Adj. R^2	0.3320	0.4891	0.6022	0.3170	0.4262	0.5746
N	721	721	721	575	575	571

Notes: The dependent variable is the standard deviation in the distribution of schooling years attained in the population age 15 and over, but we also add the mean fertility levels as a control variable (see text). The data source is Barro and Lee (2000). Rows show the estimated coefficients (β) and their z-statistics (β/S_β). Data on the dependent variable are available every five years. The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in Models 2 and 5. The 2SLS model accounts for endogeneity of RGDPn. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.

[#] The Intercept coefficients represent the mean values of all intercept terms.

Table 4: Income Inequality Regressions: GINI Coefficient (G)

Dependent Variable: GINI

	Model 1 OLS	Model 2 OLS with fixed effects	Model 3 OLS with fixed effects	Model 4 2SLS	Model 5 2SLS with fixed effects	Model 6 2SLS with fixed effects
Intercept	41.948920 26.34	35.729530[#]	39.392580[#]	41.489030 24.10	34.189310[#]	41.346450[#]
RGDPn	0.001583 2.94	0.001095 2.62	0.001081 2.03	0.001925 3.45	0.001367 2.37	0.000853 1.17
$RGDPn^2$	-2.45E-07 -5.22	-1.08E-07 -3.54	-1.04E-07 -3.00	-2.71E-07 -5.75	-1.20E-07 -3.03	-8.36E-08 -1.88
$RGDPn^3$	7.04E-12 5.96	2.81E-12 3.99	2.70E-12 3.43	7.55E-12 6.51	2.93E-12 3.38	2.10E-12 2.18
GOV			-0.170141 -2.00			-0.297255 -2.48
T			-0.027090 -0.66			0.003112 0.05
Adj. R^2	0.4108	0.0691	0.0932	0.4891	0.0578	0.0882
N	318	318	310	263	263	263

Notes: The dependent variable is the GINI coefficient, based on household income data. The data source is Dollar and Kraay (2001). Rows show the estimated coefficients (β) and their z-statistics (β/S_β). The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in Models 2 and 5. We derive the 2SLS regressions in models 4-6, which account for endogeneity of RGDPn, from the subset of countries for which both income and educational attainments data are available, in order to see if income and educational inequality paths exhibit a similar shape. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.

[#] The Intercept coefficients represent the mean values of all intercept terms.

Table 5: Income Inequality Regressions: Inter-Quintile Ratio (E)

Dependent Variable: QUINT

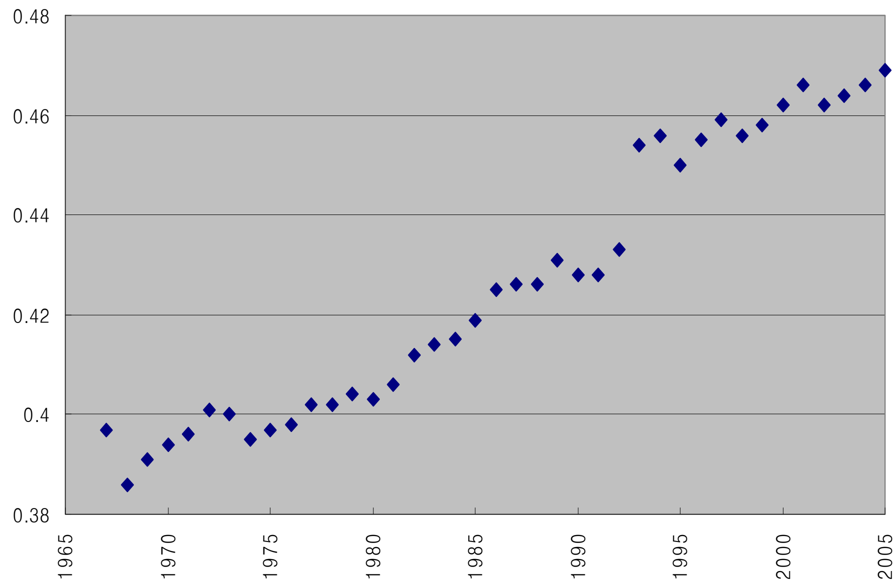
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
	OLS	OLS with fixed effects	OLS with fixed effects	2SLS	2SLS with fixed effects	2SLS with fixed effects
Intercept	9.110258 7.52	4.513832#	2.290842#	9.194144 6.35	7.860770#	9.954935#
RGDPn	0.001440 3.47	0.001294 3.82	0.001478 3.37	0.001622 3.40	0.000626 1.47	0.000485 0.87
$RGDPn^2$	-1.80E-07 -4.92	-1.00E-07 -3.98	-1.15E-07 -3.95	-1.98E-07 -4.87	-6.36E-08 -2.14	-5.25E-08 -1.53
$RGDPn^3$	4.97E-12 5.34	2.32E-12 3.96	2.69E-12 4.02	5.37E-12 5.34	1.66E-12 2.53	1.41E-12 1.87
GOV			0.103520 1.50			-0.086037 -0.95
T			-0.005237 -0.16			-0.005231 -0.12
Adj. R^2	0.2498	0.0663	0.0788	0.2908	0.0447	0.0505
N	289	289	281	240	240	240

Notes: The dependent variable is the share of total income received by the top, relative to the bottom, quintile of families in the population. The data source is Dollar and Kraay (2001), and only household income data are used. Rows show the estimated coefficients (β) and their z-statistics (β/S_β). The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in Models 2 and 5. We derive the 2SLS regressions in models 4-6, which account for endogeneity of RGDPn, from the subset of countries for which both income and educational attainments data are available, in order to see if income and educational inequality paths exhibit a similar shape. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.

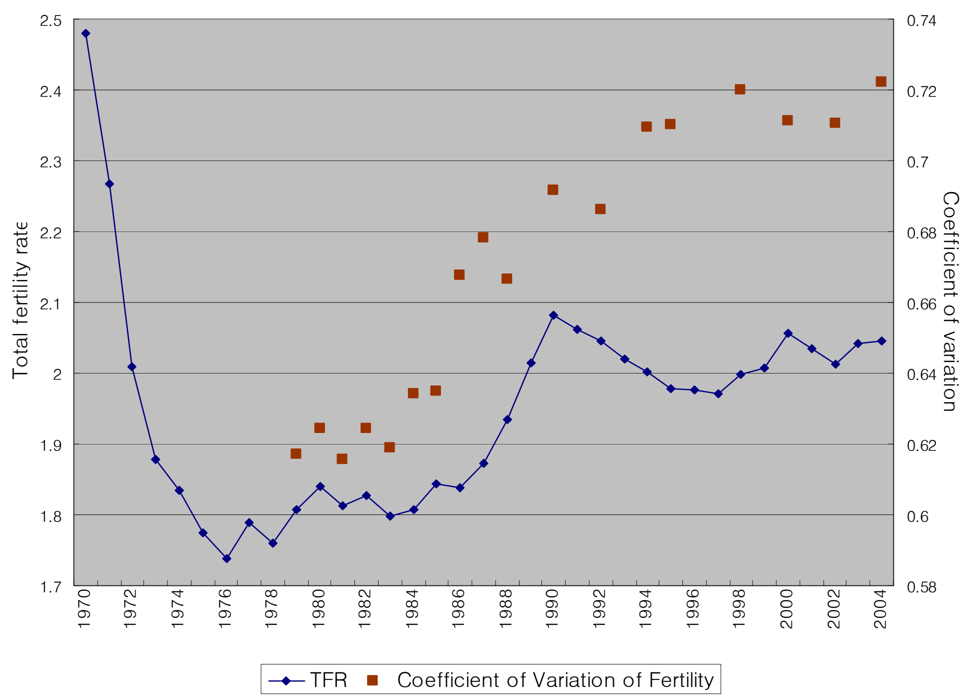
[#] The Intercept coefficients represent the mean values of all intercept terms.

Figure A: Changes in the Dynamic Patterns of the Gini Coefficient, Total Fertility Rate, and Coefficient of Variation of the Fertility Distribution in the U.S. in Recent Decades

a. Gini Coefficient



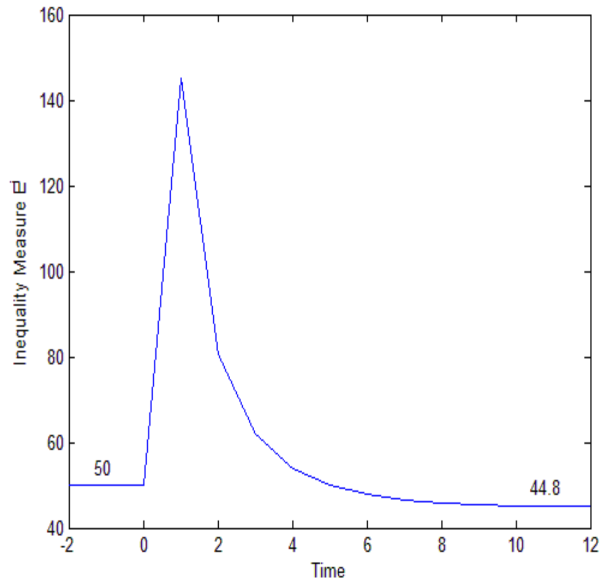
b. Total Fertility Rate and Coefficient of Variation of Fertility Distribution



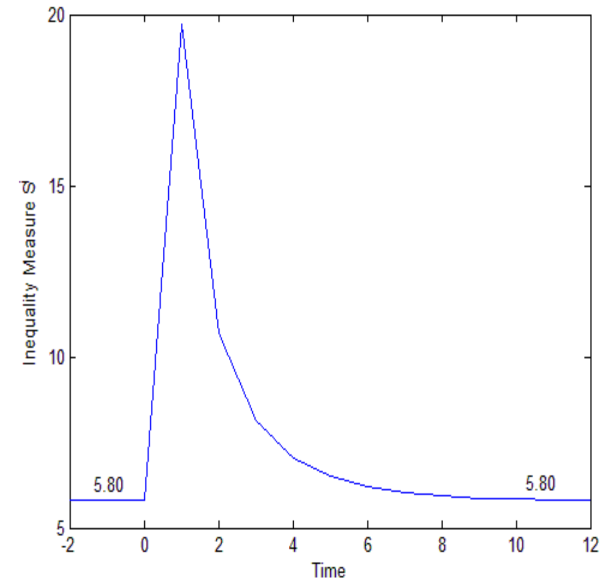
Note: Figure b shows the coefficient of variation of the distribution of surviving children per woman age 40-44.

Figure 1: Simulated Time Paths of the Evolution of Key Endogenous Variables over the Process of Development: A Uniform Productivity Shock Affecting Family 1 Ahead of Family 2

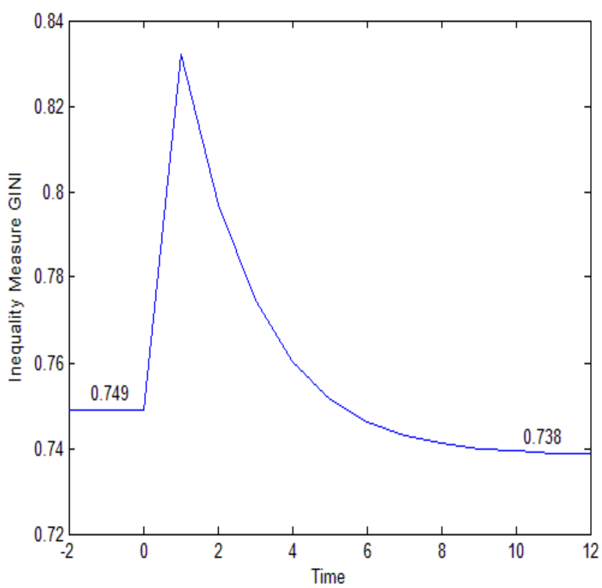
a. Family-Income Inequality (E)



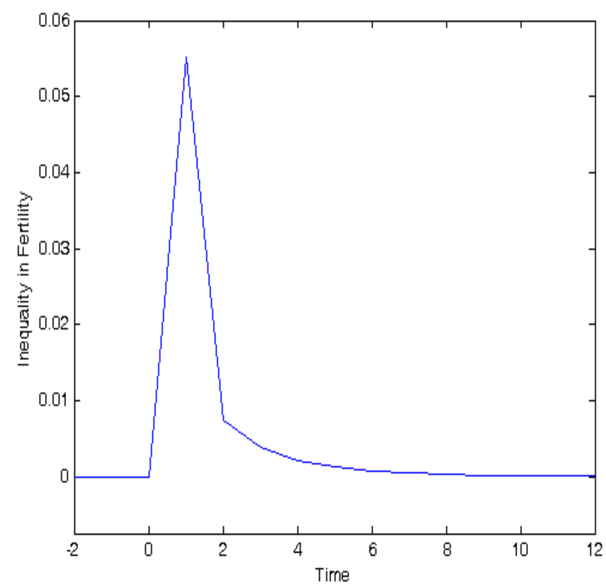
b. Income-Group Inequality (S)



c. Gini Coefficient (G)



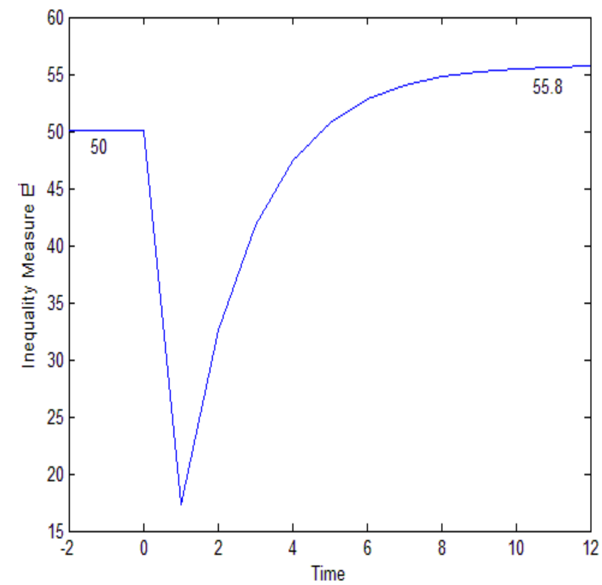
d. Inequality in Fertility



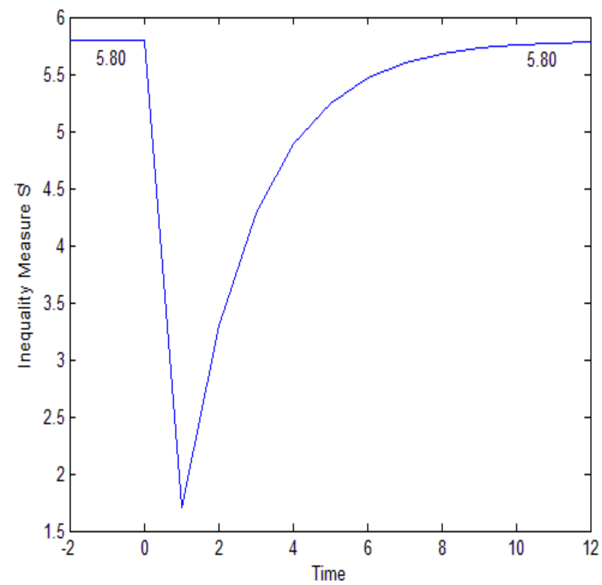
Note: Parameter values used in these simulations are: $\theta^1=1$, $\theta^2=1.01$, $H^1_0=50$, $H^2_0 = 1$, $B^1=B^2=0.1$, $v^1=v^2=0.01$, $\gamma = 0.4$, $\sigma = 0.9$, $\delta = 0.9$, and $\beta = 1.2$. Prior to period 1, the economy is in a stable SE steady state, with $A^1=2$ and $A^2=1$. In period 1, family 1 alone experiences a once-and-for-all increase in A^1 to 40. In period 2, also family 2 experiences the same proportional increase in A^2 to 20.

Figure 2: Simulated Time Paths of the Evolution of Key Endogenous Variables over the Process of Development: A Uniform Productivity Shock Affecting Family 2 Ahead of Family 1

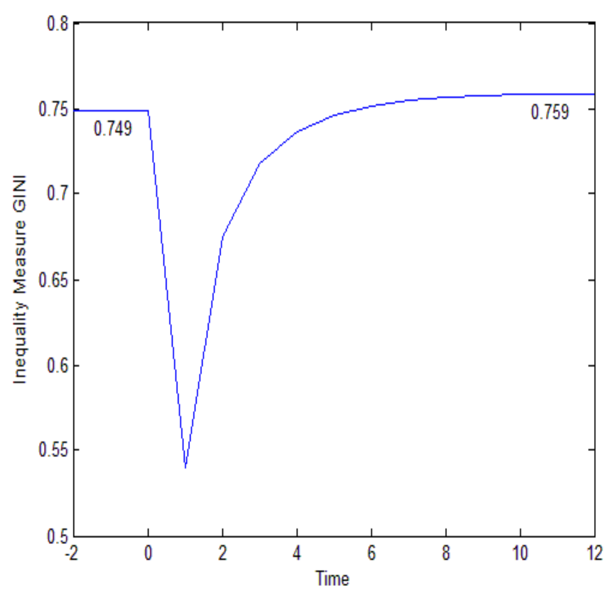
a. Family-Income Inequality (E)



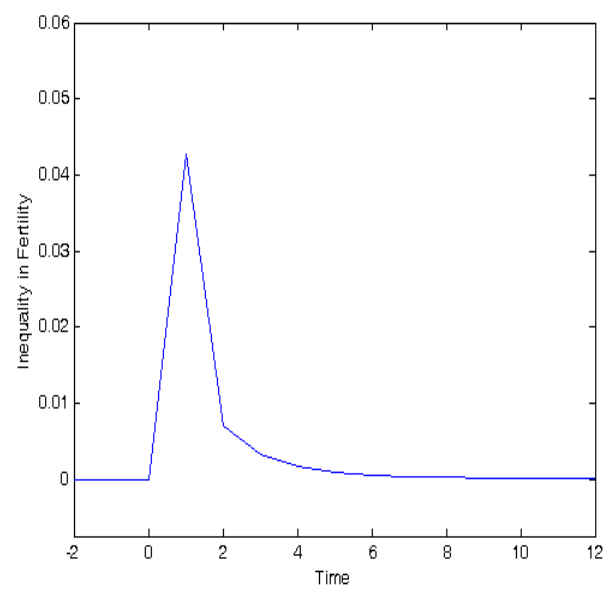
b. Income-Group Inequality (S)



c. Gini Coefficient (G)

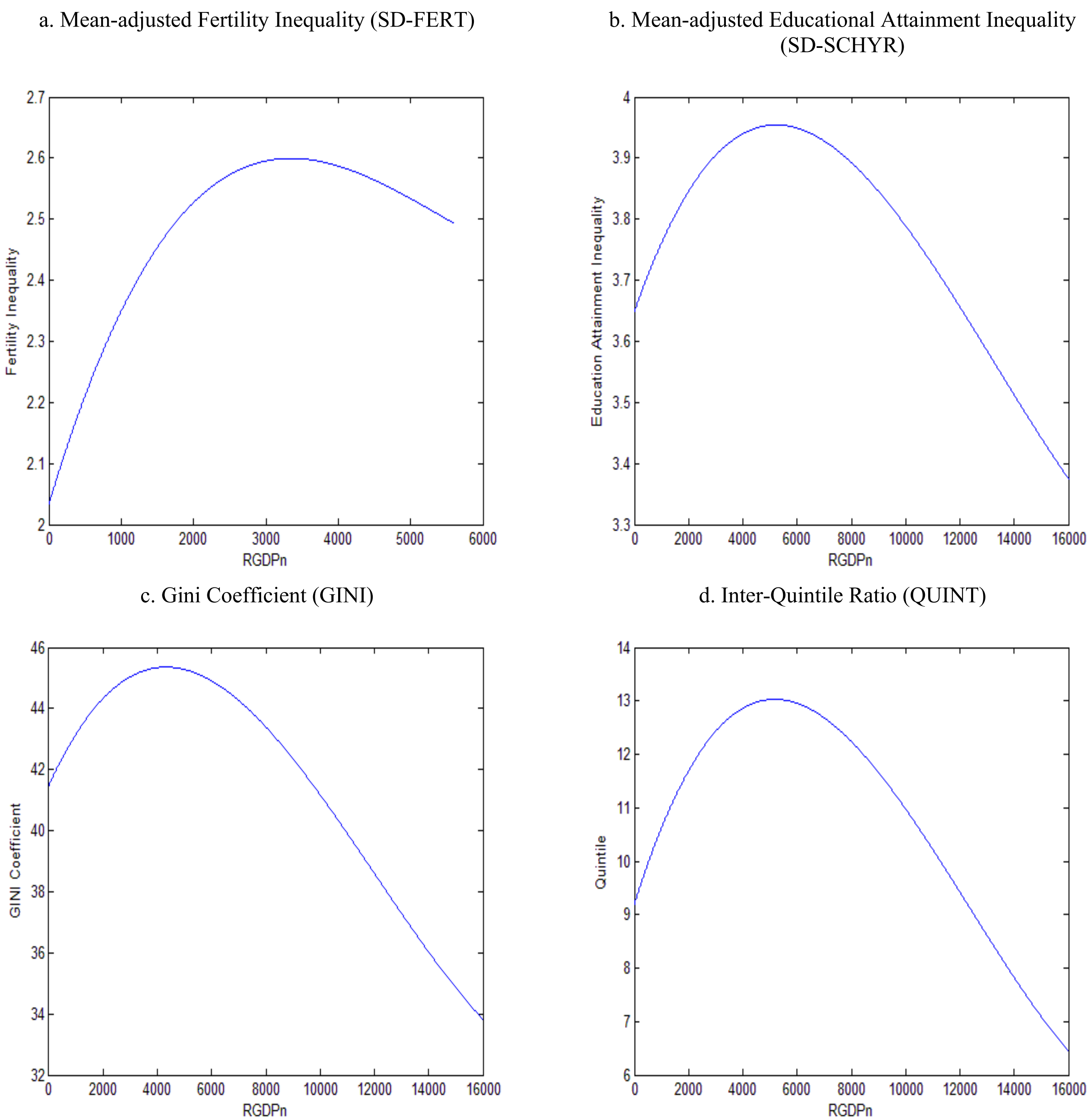


d. Inequality in Fertility



Note: Parameter values used in the simulations for Figure 2: $A^1=2$, $A^2=1$, $H^1_0=50$, $H^2_0 = 1$, $B^1=B^2=0.1$, $v^1=v^2=0.01$, $\gamma = 0.4$, $\sigma = 0.9$, $\delta = 0.9$, and $\beta = 1.2$. Prior to period 1, the economy is in a stable SE steady state with $\theta^1=1$ and $\theta^2=1.01$. In period 1, family 2 alone experiences a once-and-for-all reduction in θ^2 to 1.01/20. In period 2, also family 1 experiences the same proportional decrease in θ^1 to 1/20.

Figure 3: Fitted Regression Lines Linking the Evolution of Key Inequality Measures and Per-Capital Income Levels within and across Countries, based on 2SLS Panel Regression Results



Note: Panels a, b, c, and d are based on the regression results of Model 4 in Tables 2 - 5, respectively. The RGDPn values on the x-axes of all panels cover 90% of the observations on RGDPn used in our regressions.