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Retirement Flows

Abstract

This paper considers labor market flows of older workers among the states of nonretirement, partial retirement and full retirement. Statistics are presented which describe entry, exit and continuation rates for each state by age, duration dependence, and "reverse flows." One important finding which has implications for the structure of utility functions embedded in life cycle retirement models is the relatively high incidence but short duration of partial retirement. These implications are discussed.

Alan L. Gustman
Department of Economics
Dartmouth College
Hanover, NH 03755
(603) 646-2641

Thomas L. Steinmeier
Department of Economics
Texas Tech University
Lubbock, TX 79409
(806) 742-2203
I. Introduction

The economics literature has generally conceived of the retirement process as a one-way flow from an "in the labor force" status to a "not in the labor force" status. This paper considers a more detailed description of the retirement process—one involving full-time work, partial retirement, and full retirement. The primary interest is in describing the flows of older workers among these three states, both the principal flows from full-time work to full retirement either directly or indirectly through partial retirement, and the much smaller flows in the opposite direction. Information about these flows is useful not only in providing a richer description of the retirement process, but it may also help in establishing values for parameters which are important to the retirement decision and thereby in understanding the nature of that decision.

In the next section of the paper we describe an analytical framework which is sufficient to generate transitions among full-time work, partial retirement, and full retirement. A particularly important point here is that partial retirement is more than simply a reduction in hours below full-time, since it usually is associated with a reduction in the wage rate and frequently is accompanied by a change in employers as well. The following section examines various descriptive statistics related to the retirement process. These include probabilities of older workers being in particular states at given ages, transition rates among the various states, and continuation rates in the states. A final section discusses potential implications of the descriptive statistics for the
II. The Analytical Framework

The analytical framework used in this paper reflects a number of relevant findings presented in our previous work. One of these findings is that partial retirement is indeed a widespread phenomenon (Gustman and Steinmeier, 1981). Between the ages of 65 and 69, partial retirement is as likely as continued full-time work on the individual's main job.\(^1\) More than one-third of the older white males who are not self-employed in the Retirement History Survey indicate that, during at least one of the four sample years up to 1975, they are partially retired. Moreover, the probability of partial retirement remains high even for those who are in good health, do not face mandatory retirement, and are not covered by a pension.

A second important finding is that partially retired workers have significantly lower wage rates than full-time workers (Gustman and Steinmeier, 1982). These lower wage rates may come about for at least two reasons. In a majority of jobs, an individual does not seem to be able to reduce hours below full-time, as indicated both in surveys of individuals and of firms (Gustman and Steinmeier, forthcoming). Under such circumstances, if an older worker wishes to reduce his work effort below full-time, he must quit his main job and take a job that does permit part-time work, and which usually also pays a lower wage rate. In other jobs, the worker can reduce his time below full time without changing jobs.\(^2\) Even here, though, workers who choose to work part-time
report significantly lower wage rates than those who continue working full-time.

These findings may be incorporated into a formal life-cycle model as follows. An individual is presumed to choose a time path for consumption and labor supply so as to maximize lifetime utility:

\[
U = \int_0^T u[C(t), L(t), t; \beta] \, dt
\]

where \(C(t)\) is consumption at time \(t\), \(L(t)\) is leisure at time \(t\), and \(\beta\) is a vector of parameters which determine the nature of the utility function \(u\) at any time \(t\). The maximization of the utility function is subject to the lifetime budget constraint:

\[
\int_0^T d(t) \left\{ W_N[H_N(t), t] + W_P[H_P(t), t] \right\} \, dt + A_o = \int_0^T d(t) C(t) \, dt
\]

where \(d(t)\) is the discount factor to time \(t\), \(W_N[H_N(t), t]\) is the total compensation including changes in pension and Social Security asset values, from working \(H_N(t)\) in the nonretirement job, \(W_P[H_P(t), t]\) is the corresponding compensation for \(H_P(t)\) hours in the partial retirement job, and \(A_o\) is the discounted value of exogenous assets. Further constraints limit the potential quantities of labor supply and relate labor supply to leisure:

\[
0 \leq H_P(t) \leq h_N
\]

\[
H_N(t)H_P(t) = 0
\]

\[
L(t) = 1 - H_N(t) - H_P(t) \geq 0
\]

The first constraint specifies that the individual must work either full
time (where full-time work is a fraction $h_N$ of available time) or not at all in the non-retirement job, while the second specifies that the labor supplied to the second job can range between none and full-time. The third constraint specifies that the individual cannot work on both jobs simultaneously, and the last constraint defines leisure as the time not supplied as labor.

Within the context of this model, the paths of wages in the two types of jobs (i.e. tenure dependence) will induce bunching of hours. Most people will spend the first part of their working lives in non-retirement jobs, where the wage rate is higher than in the partial retirement job. With increasing age, however, the utility function is likely to change in such a manner that full-time work generates increasing disutility, and at some point individuals will quit working on the non-retirement job. Some of these people will find it advantageous to spend some of the remaining time on a partial retirement job where they can work less than full-time, albeit at a lower wage rate, while others will elect to bypass the stage of partial retirement entirely and move directly to full retirement. Even for those who partially retire, the within-period utility function will continue to shift over time to make work increasingly onerous. Eventually these people too will wish to retire fully. Hence, the sequences that we expect to find most often are non-retirement, possibly followed by partial retirement, and then followed by full retirement.

It is possible that some people may find it desirable to move in the reverse direction from the sequences indicated above. That is, they may work in a partial retirement job after being fully retired,
or they may work in a non-retirement job after being partially or fully retired. Such "reverse" flows may be generated by very substantial jumps in wages in an individual's later years, but this is not very convincing as an explanation for many such flows. A more plausible explanation involves unexpected changes in circumstances which induce an individual to change his mind and return to work in a period when he had anticipated being partially or fully retired. Several possibilities come to mind. An individual may suffer unexpected and large losses in the financial markets and now find he has fewer assets than anticipated. His spouse may suffer from a serious and expensive illness which increases the household's desire for income. Alternatively, he may retire and subsequently find that his enjoyment from retirement is not as great as he anticipated. Any of these circumstances can lead the individual to recalculate the optimal path of labor supply over the remaining lifetime, and the recalculated path may cause the individual to move in the reverse direction from the typical sequences described in the previous paragraph.

III. Descriptive Statistics

In this section we concentrate on some statistical evidence bearing on the magnitudes of the labor force flows associated with the model described in the last section. The data which form the basis of this analysis are from the Retirement History Survey. They pertain only to white males who were not self-employed in a job held while not retired. Four waves of the survey are included ---1969, 1971, 1973, and 1975.
Those included in the sample were 58 to 63 years old in 1969.

Since we are looking at flows, the number of observations in some categories is relatively small. Accordingly, in some cases we have pooled observations for different cohorts and years of the survey. There are two problems with this procedure which the reader should bear in mind. First, the unemployment rate differed widely among the four survey years. It was 3.5%, 5.9%, 4.9%, and 8.5% in 1969, 1971, 1973, and 1975 respectively. Second, there has been a downward trend in male labor force participation rates which, although possibly caused by secular changes in many of the explanatory variables included in a life cycle model, might also include true cohort effects. When we pool these observations, these differences are either hidden or, where we focus on calendar age, may be correlated to some extent with the age variable.

State Probabilities. We first examine the simple percentages of the sample who are not retired, are partially retired, or are fully retired. Table 1 indicates these percentages by survey year and by age. Three aspects of this table would appear to be particularly noteworthy.

First, the departure from the non-retirement state is indeed pervasive between the ages of 58 and 68. The percentage of individuals not retired at all falls from 85% to 8% during this 10-year age span. This is accompanied by a very large increase in the fraction of the sample who are fully retired, and to a somewhat lesser extent, in the fraction of the sample who are partially retired.

Secondly, among those who work at all, partial retirement is more common than non-retirement for individuals past the age of 65. For the
Table 1: Retirement Status by Age and Year

<table>
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<th>Age</th>
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<tr>
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<td>.09</td>
<td>.12</td>
<td>.12</td>
<td>.18</td>
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<td>1971</td>
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<td>.27</td>
<td>.36</td>
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<td>.47</td>
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<td>1973</td>
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five-year span beginning at age 65 and ending at age 69, the fraction of individuals who are partially retired holds fairly steady at between 15% and 20%. During the same period, non-retirement falls from around 20% to just a little more than 5%. These figures reflect a finding already mentioned in the last section, namely, that partial retirement is an important phenomenon, particular in the older age ranges.

Finally, there appears to be an unmistakable secular trend in the percentages not retired and fully retired. Between 1969 and 1973, the percentage of 62-63 year-olds not retired drops by 8-9 percentage points, and similar drops are observed for 64-65 year olds between 1971 and 1975. The figures for full retirement display an equally large trend in the opposite direction. What is not clear from this table is whether a similarly strong secular trend applies to those under 62 or over 65. For those age ranges the Retirement History Survey includes data from at most two adjacent surveys. The data from these surveys gives some hint of a secular trend for those groups, but it does not appear to be as strong as for the 62-65 year-old group.

Transition Rates Among Retirement States. Table 2 presents the entry and exit rates among the various retirement states. The top half of the table indicates entry rates from specific states two years earlier. For example, of the people who were partially retired, 44.1 percent of them had been not retired two years earlier, 40.0% had been partially retired two years earlier, and so on. The rows of this table sum to one, and the number of observations in each row is indicated along the right side of the table. The bottom half of the table conveys the same
Table 2 Basic Flows Over Two-Year Periods

### Entry Rates

<table>
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<tr>
<th>From Status</th>
<th>Observations</th>
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<td>R</td>
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<td>.711</td>
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### Exit Rates

<table>
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<th>Observations</th>
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</thead>
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<tr>
<td>P</td>
<td>.077</td>
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<tr>
<td>R</td>
<td>.008</td>
</tr>
<tr>
<td>U</td>
<td>.226</td>
</tr>
</tbody>
</table>

1Symbols

- **N**: Not Retired
- **P**: Partially retired
- **R**: Fully retired
- **U**: Unemployed
kind of information on exit rates to specific states two years later. For instance, of the people who were not retired as of a particular survey, 10.5% were partially retired by the next survey two years later, and 26.8% were fully retired two years later. Again, the rows of the table sum to one, and the number of observations in each row is indicated along the right side of the table.

There are three features of particular interest in this table. First, consider the exit rates from non-retirement. For individuals in this age range, about 37.3% of the individuals who are not retired will be either partially or fully retired by the time of the next survey two years later. Of those who leave non-retirement and do not become unemployed, 28.2% (calculated as .105/.373) will partially retire, and the remaining 71.8% will fully retire. These figures underscore the fact that partial retirement is a phenomenon which affects a significant fraction of the labor force during their later years.

A second feature of the table which bears mention is the exit rates for partially retired workers. There is only a 48.5% chance that an individual who is partially retired during one survey will still be partially retired during the next survey two years later. If exit from partial retirement were a random process with a constant hazard rate, this would imply that the average duration in the partial retirement state is a little under three years. The assumption of a constant hazard rate is undoubtedly an oversimplification, but the figure nevertheless suggests that the duration of partial retirement is quite short, particularly when compared to the duration of an individual's
full-time work.

A third interesting feature of the table involves the flows against the normal retirement paths. In the previous section, we noted that for most people, the optimal path of work effort would proceed from non-retirement, possibly through partial retirement, to full retirement. In some cases with unusual wage paths, or in cases where the individual encountered unforeseen events, it is possible that the individual would move in the reverse direction. Table 2 indicates that this does indeed occur. More specifically, of the people who enter partial retirement (and who were not unemployed), about 23.4% \[\text{calculated as } \frac{.135}{.135 + .441}\] had been fully retired in the previous survey, while 76.6% had been not retired. Of the people who left partial retirement and did not become unemployed, 15.3% \[\text{calculated as } \frac{.077}{.077 + .426}\] were not retired in the next survey, while 84.7% were fully retired. The third "reverse" flow, that from full retirement to non-retirement and the corresponding exit rate are both less than one percent.

Continuation Rates By Age. It is useful to examine in more depth the way these flows, and especially the continuation rates—the diagonal elements of the lower part of Table 2—vary with age. Table 3 reports, by age in the initial year, the percentages of individuals who continue in the same retirement category in the next survey two years later.

How should these continuation rates behave? We know that pension programs and mandatory retirement provisions affect the likelihood of retirement at 62 and 65 either by providing incentives for individuals
### Table 3. Two-Year Continuation Rates by Age

<table>
<thead>
<tr>
<th>Status</th>
<th>Age In Initial Period</th>
<th>58</th>
<th>59</th>
<th>60</th>
<th>61</th>
<th>62</th>
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<th>64</th>
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<th>66</th>
<th>67</th>
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<td>N</td>
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<td>.873</td>
<td>.831</td>
<td>.713</td>
<td>.652</td>
<td>.632</td>
<td>.327</td>
<td>.267</td>
<td>.415</td>
<td>.515</td>
<td>.432</td>
</tr>
<tr>
<td>P</td>
<td>926</td>
<td>.409</td>
<td>.472</td>
<td>.467</td>
<td>.450</td>
<td>.458</td>
<td>.446</td>
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<tr>
<td>R</td>
<td>926</td>
<td>.222\textsuperscript{a}</td>
<td>.267\textsuperscript{a}</td>
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<td>0\textsuperscript{a}</td>
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</table>

\textsuperscript{a} based on sample sizes <25.
to leave their jobs or by forcing them out of their jobs at the age of mandatory retirement. Moreover, while there is controversy about the effects of Social Security at 62, we know that at age 65 the adjustments are no longer actuarially fair, providing a further incentive for retirement. On the supply side, the changing effects of health and family structure and the increasing disutility of work should act to reduce continuation rates in non-retirement below the high levels typical of individuals in their prime working years.

There is indeed evidence of rapidly falling continuation rates for non-retirement up to age 64. These range from 87% at age 58 to 27% at age 64—the age when the strongest economic incentives to leave non-retirement are about to be encountered. The continuation rates for 65 to 67 year-olds lie above those for 64 year-olds, but well below the rates observed for those in their late fifties and early 60's.

For the partially retired, continuation rates tend to stay relatively steady in the 45-50% range up to age 64. Thereafter the continuation rates for full retirement are very high, averaging about 93% and never falling below about 92%. Among the unemployed, there are too few observations to make generalizations about the pattern of the continuation rates.

**Duration Dependence of Continuation Rates.** Table 3 investigates continuation rates by age. A related issue, particularly for partial retirement, is whether the continuation rate depends on how long the individual has been partially retired, i.e., the duration dependence of continuation rates.
To investigate this issue, we selected the individuals who were not retired in 1969 but who were partially retired in 1971. This avoids having to deal with periods of partial retirement already in progress. Moreover, the requirement that individuals be working full-time in 1969 assures that we are looking at individuals who are in the normal sequence and perhaps not quite as likely to be responding to unusual or unexpected circumstances. Of this group, 292 were still in the sample by 1973, and of that number 122, or 41.8%, were still partially retired in 1973. Hence, a person newly partially retired had a 41.8% initial two-year continuation rate.

112 of the 122 individuals who were partially retired both in 1973 and 1971 were in the sample by 1975, and of those individuals 75, or 67.7% were still partially retired at that date. Hence, for individuals with durations in partial retirement of between two and four years, the two-year continuation rate is considerably higher than for individuals with durations of less than two years. It should be kept in mind that these individuals were also growing older with each successive survey, and the evidence from Table 3 indicates that this could be part of the explanation as to why the individuals exhibited higher continuation rates between 1973 and 1975 than between 1971 and 1973. Even so, the magnitude of the increase in the continuation rate from 41.8% to 67.7% is relatively large compared to changes in the continuation rates caused by an additional two years of age, as indicated in Table 3. It would appear that there is some duration dependence in which the continuation rate for partial retirement increases with the length of time
the individual has been partially retired.

Detailed Flows for Partially Retired Individuals. Table 4 looks at the flows pertaining to partially retired individuals in somewhat greater detail in order to shed some light on the mechanism of partial retirement. In this table, partially retired individuals are separated into three categories according to the relationship between their non-retirement and their partial retirement jobs. The top line of both sections of the table considers individuals who have partially retired in jobs in which they reported themselves not retired in a previous survey, or if the observation is for the first survey, in jobs which they started before the age of 55. The second line refers to individuals who have partially retired in jobs which are different from any jobs in which they reported themselves not retired in prior surveys. The third line indicates individuals who were partially retired, but for whom the job could not be definitely classified with respect to one or the other of these categories.

The information in the table contains a couple of interesting implications. First, it suggests that partial retirement in a job previously reported as a non-retirement job and partial retirement in a different job are relatively distinct paths. Of the individuals leaving partial retirement in a job previously reported as a non-retirement job, only 7.3% (calculated as .039/.531) are found in the next survey to be partially retired in a different job, and of the individuals entering partial retirement in a job not previously reported as a non-retirement job, only 4.4% (calculated as .032/.727) are
Table 4. Detailed Flows For the Partially Retired\(^1\)

<table>
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<th>Entry Rates</th>
<th>From status</th>
<th>Observations</th>
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<td>Pa</td>
<td>.071</td>
<td>0</td>
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</tbody>
</table>

\(^1\) These are two-year rates.

\(^2\) Symbols:  
- Pf: Partially retired in a job previously reported as a non-retirement job or in a job begun before age 55.  
- Po: Partially retired in a different job.  
- Pa: Partially retired, but type of job is ambiguous.  

Other symbols are defined in Table 2.
entering from partial retirement in a job previously reported as a non-retirement job or in a different job, but not both.

Secondly, a comparison between the exit rates of individuals partially retired in jobs previously reported as a non-retirement job and individuals partially retired in different jobs indicates that the behavior of these two groups is generally similar. A person partially retired in a job previously reported as a non-retirement job is a couple of percentage points (9.1% vs. 6.9%) more likely to return to non-retirement, while an individual partially retired in a different job is three percentage points (35.4% vs. 32.4%) more likely to retire completely. Both groups are about equally likely (57.9% vs. 56.8%) to continue partial retirement in some form. Individuals who are partially retired but who cannot be assigned in either one or the other of these categories appear to be somewhat different, with substantially lower probabilities for continuing partial retirement and substantially higher probabilities for complete retirement. There is a suspicion that these unclassified individuals are probably more likely to be in fact partially retired in jobs they have not previously held as non-retirement jobs, but it is not possible to be entirely sure of this.

III. Implications for Retirement Models

One of the aspects of the data described in the last section has particular importance for retirement models of the type presented earlier in this paper. Namely, the data indicate that even though a substantial minority of older individuals pass through a stage of partial retirement, the spells of partial retirement are typically very short.
More than half of these spells appear to last less than two years, and it seems likely that an individual would almost never be partially retired for a significant fraction of his working life. This fact, that partial retirement durations tend to be very short, when considered together with the observed incidence of partial retirement, provides a powerful clue as to the nature of the utility function on which individuals are basing their retirement decisions.

In order to investigate the implications of the descriptive statistics, it is necessary to consider the solution path of the retirement model in somewhat more detail. At any point in time, the solution path maximizes the quantity

\[ Z(t) = u[C(t), L(t), t; \theta] + \lambda_y d(t) S(t) \]

where \( S(t) = Y(t) - C(t) \) is the amount saved in period \( t \) and \( Y(t) = W_N [H_N(t), t] + W_P [H_P(t), t] \) is the net compensation for labor in period \( t \). \( \lambda_y \) may be interpreted as the marginal utility of discounted lifetime income, that is, the marginal utility of relaxing the lifetime budget constraint by one dollar. (See MaCurdy, 1981) It is chosen so that when this optimization is implemented for all time periods, the lifetime budget constraint \( \int_0^T d(t) S(t) dt + A_0 = 0 \) is just satisfied.

The maximization is subject to the constraints of equations (2) through (5), which describe the hours limitations on the two types of employment.

If we substitute for \( S(t) \) in equation (6), the maximand in this
problem becomes

(7) \[ Z(t) = u[C(t), L(t), t; \beta] + \lambda_Y d(t) [Y(t) - C(t)]. \]

Since \( C(t) \) appears neither in the definition of \( Y(t) \) nor in any of the constraints in equations (2) through (5), the value of \( C(t) \) which maximizes equation (7) may be found simply by differentiating the equation and setting the result equal to zero:

(8) \[
\frac{dZ(t)}{dC(t)} = u_C[C(t), L(t), t; \beta] - \lambda_Y d(t) = 0
\]

where \( u_C \) indicates the partial derivative with respect to the first argument. This equation may then be solved for the optimal \( C^*(t) \) as a function of \( L(t) \) and \( \lambda_Y d(t) \):

\[ C^*(t) = C^* [L(t), t; \beta, \lambda_Y d(t)] \]

This may in turn be substituted into equation (7) to yield:

(9) \[
Z(t) = u[C^* [L(t), t; \beta, \lambda_Y d(t)], L(t), t; \beta] + \lambda_Y d(t) \{Y(t) - C^* [L(t), t; \beta, \lambda_Y d(t)]\} = Z[Y(t), L(t), t; \beta, \lambda_Y d(t)]
\]

At a particular point in time, this means that the individual may be
Figure 1. The Earnings-Leisure Choice at Time $t$
viewed as maximizing a utility function involving only income and leisure, instead of consumption and leisure as in equation (6). The maximization is done subject to the definition of Y(t) and the constraints of equations (2) through (5).

Figure 1 illustrates the maximization problem facing the individual at time t. The indifference curve II is one of a set of such curves implied by equation (9). All of these indifference curves are vertical displacements of one another, or equivalently, they all have the same slope along any vertical line. The budget constraint for the individual at time t consists of point A plus the line segments between B and C. The point A corresponds to the earnings and leisure available if the individual chooses to work on the non-retirement job. The series of line segments between B and C represent potential income opportunities if the individual works on the partial retirement job, allowing for effects such as the reduction in Social Security benefits after a disregard amount. The individual chooses the point along this constraint which enables him to reach the highest indifference curve. This may occur at point A, in which case the individual is non-retired, or at some point between B and C, which corresponds to partial retirement, or at C, which represents full retirement. Notice that a value of zero income at time t, which is associated with point C, does not mean that consumption, or income from Social Security, pensions or other programs would be zero should the outcome associated with point C be chosen.

Over time, this diagram changes in some important respects. First,
the indifference curves will rotate clockwise, -i.e., other things the same they will be steeper as an individual becomes older, reflecting the fact that a given amount of work is likely to generate an increasing amount of disutility with increasing age. Point A may shift downward as well, since past a certain age both Social Security and private pensions may reduce effective compensation. The budget line between B and C, may also be affected, but here we would not expect the effects to be too great, particularly for the part of the constraint which lies below the Social Security disregard amount. In this range, Social Security will not change the effective compensation, and partial retirement jobs are unlikely to involve pension plans which alter the effective compensation.

Now consider the implications of the two facts noted above: first that a substantial minority of older workers go through a phase of partial retirement, and second that for most of them the period in partial retirement is fairly short. In Figure 1, there are two ways in which an individual might find it optimal to retire partially for a relatively short period of time. One possibility is that the individual has a set of indifference curves with just the right degree of curvature so that when he leaves point A, the tangency with the budget segment BC will already be very close to C. In this case, only a slight rotation of the curve or a slight decline in the partial retirement wage rate will be sufficient to induce him to retire fully after only a short period of time in partial retirement. This might be a satisfactory explanation for an occasional individual who partially
retired for a short period of time, but it seems more unlikely that almost everyone who partially retires should have just the right curvature of the indifference curves so that almost all of them partially retire for only a short period in spite of the fact that they face a wide range of wages in the nonretirement and partial retirement jobs.

This leaves a second explanation for the fact that most people who partially retire do so only for a short period of time, specifically, that the indifference curves are rotating fairly rapidly. Here, when the individual leaves the non-retirement job, the indifference curve for an individual who partially retires may have a tangency at any point along BC. If the indifference curves are rotating rapidly, the point of tangency will travel along BC towards C fairly rapidly, and the individual will again retire after a fairly brief spell in partial retirement.

In the retirement model, then, the descriptive statistics thus suggest that the indifference curves in Figure 1 have enough curvature that at least some individuals partially retire, and that the curves are rotating fairly rapidly, becoming significantly steeper as the individual ages. This is an interesting and important implication of the data, but it leaves unanswered one final question: What do these results imply about the utility function in the original structural model, namely \( u[C(t), L(t), t; \theta] \)? To examine this issue, consider the specific function

\[
(10) \quad u[C(t), L(t), t; \theta] = [\theta(t)/\rho] \cdot [C(t)^\rho + b(\theta, t)L(t)^\rho], \quad \rho < 1
\]

where \( \theta(t) \) is the time preference discount factor and \( \sigma = 1/(1-\rho) \).
is within-period elasticity of substitution between consumption and leisure. The indifference curves implied by this utility function have slope $S_u = -b(\bar{e}, t) \left[ \frac{L(t)}{C(t)} \right]^{\rho-1}$. The corresponding indifference curves in Figure 1 have slope $S_Z = -b(\bar{e}, t) \frac{\theta(t)}{\beta(t)} L(t)^{\rho-1} / \{ \gamma d(t) \}$. For a given point in Figure 1, $S_Z$ changes over time with the quantity $b(\bar{e}, t) \theta(t)/d(t)$, while for a given point in the consumption-leisure space of $u$, $S_u$ changes according $b(\bar{e},t)$. Unless the rate of time preference exceeds the discount rate by a considerable amount, both sets of indifference curves will be rotating rapidly if either is. Thus the fact that few individuals who partially retire do so for long, which implies that the indifference curves in Figure 1 are rotating rapidly, implies that the indifference curves corresponding to the utility function in the structural model are also rotating rapidly as the individual ages.

IV. Conclusion

The descriptive statistics presented in the paper impose some important requirements for a good structural retirement model. First, a good model should be able to explain the behavior of continuation rates, especially the sharp dip in continuation rates in the years immediately prior to 65. It seems likely that the explanation for this dip lies in the effect of pension and Social Security benefit formulae, mandatory retirement and other factors affecting the budget line. Certainly, models which explain these continuation rates in terms of ad hoc, discrete, age-related changes in slopes of the indifference curves should be regarded with a certain amount of suspicion, particularly if the models are intended to be used to predict the effects of hypo-
theoretical changes in Social Security or pension rules. Secondly, a good structural model must deal with the minority of observations for which the flows appear to be in a reverse direction compared with the normal sequence. In particular, it is necessary to resolve the question whether these reverse flows are the result of expected but unusual paths of the wages in the full-time or partial retirement jobs, or as seems more likely, whether these reverse flows signify responses to unforeseen events or to miscalculations. In the latter case, the proper model may be a stochastic model in which the individual recalculates the optimal labor supply path in each period conditional on his past decisions, taking into account current or expected future changes which were not foreseen when he made his previous calculations.

The statistics also suggest an important characteristic of the lifetime utility function that individuals are attempting to maximize. Specifically, the fact that a significant number partially retire but that few of them remain in the state for very long implies that the indifference curves of the individuals, measured either in the earnings-leisure space or the consumption-leisure space, may be relatively convex but rotating fairly rapidly with age. If confirmed by further studies, this would be an important finding, for the rapidity with which these indifference curves rotate is an important factor in determining the effects of potential changes in such important programs as Social Security and private pensions on the amount of labor individuals wish to supply to the market.
FOOTNOTES

1 In that paper, the main job is defined as the job held by the individual at age 55.

2 Counting each observation for a given employer only one time, we found that for a sample of older white males who are not self-employed, 53% of the partially retired are on jobs on which they had previously worked full-time, and the remainder are in jobs on which they have not previously reported working full-time (Gustman and Steinmeier, 1982).

3 Including a bequest motive in the budget constraint would leave the discussion unchanged.

4 In this formal model, the "partial retirement" job may refer to a job distinct from the main job, or it may refer to the opportunity to remain in the main job and work less than full-time at a reduced wage.

5 A closely related model could be developed with the assumption that labor supplied to the partial retirement job must fall in a more restricted range.

6 A fourth category, not included in Table 1, consists of anyone who reported that their major activity during the survey week was looking for work. With the exception of one cell (61-year-olds in 1971), the percentage in this category never exceeded 2 percent. People were classified as not retired, partially retired, or fully retired on the
basis of their answers to the question "Do you consider yourself to be completely retired, partially retired, or not retired at all?"

7 The figures in Table 2 exclude cases where the individual dropped out of the sample in the subsequent survey (for exit rates) or was not in the sample in the previous survey (for entry rates). The principal reasons for being out of the sample were death and non-response. The percentages of individual who dropped out of the sample by the next survey were 10.1% for not retired workers, 11.4% for partially retired workers, 15.0% for retired workers, and 10.2% for unemployed workers. Only 2.2% of the individuals who dropped out of the sample subsequently re-entered, and most of those who re-entered did so into the full retirement state.

8 With a constant hazard rate, durations are distributed with the exponential density function \( f(t) = \frac{\gamma}{\exp(\gamma t)} \). If 51.5% of this distribution lies between zero and two, \( \gamma \) may be calculated as .362. The mean of the distribution is then calculated as 2.76 years.

9 There is little reliable work on the incentive effects for partial retirement. For some discussion, see Gustman and Steinmeier (1981 and forthcoming). Reduced form retirement equations which include partial retirement as an outcome are reported in the former paper.

10 Given the sizes of the two samples, the difference between the 41.8% figure and the 67.7% figure is statistically significant at better than a 1% confidence level.
11 Note especially that this group includes anyone who was partially retired during the initial survey but whose current job began after age 55.

12 The fact that $\lambda_Y$ appears in $Z[\cdot]$ means that the function cannot be viewed as constant from individual to individual, since $\lambda_Y$ depends on earnings opportunities in other years.

13 This may be shown by examining the slope of an indifference curve at any point in the diagram. This slope is given by $S_Z = Z_L/Z_Y$. From equation (9) $Z_L$ does not depend on $Y(t)$, so that it may be written $Z_L[L(t), t; \beta, \lambda Yd(t)]$, and $Z_Y = \lambda Yd(t)$. Thus

$$S_Z = \frac{Z_L[L(t), t; \beta, \lambda Yd(t)]}{[\lambda Yd(t)]}.$$

Since $Y(t)$ does not appear either directly or indirectly in this expression, the slope of the indifference curve at time $t$ depends only on $L(t)$ and hence all the curves must have the same slope.

14 For a related discussion, see Blinder (1982). Note that since this budget constraint refers to compensation, the slopes of the segments reflect not only any current Social Security benefit reductions, but also the effects of any deferred benefits. Also, this budget constraint does not include a "guarantee" amount (i.e., a vertical segment at $L(t) = 1$), since any such guarantee should really be attributed to compensation in previous years.

15 This is similar to the utility function used by Gordon and Blinder in their study (1980). Note that it would not make sense to choose $u$ to
be linear homogeneous, for the resulting indifference curves in Figure 1 would necessarily be straight lines. This may be shown as follows. In a linear homogeneous function, both $u_C$ and $u_L$ are strict functions of the ratio $C(t)/L(t)$, and hence of each other. By equation (8), $u_C$ is equal to $\lambda \gamma d(t)$, which is independent of earnings and leisure at time $t$, giving the result. The Gordon-Blinder function does satisfy the criterion that the degree of homogeneity should be less than 1, yielding convex indifference curves in Figure 1.

16 If $\rho$ is close to 1, the indifference curves associated with both $u$ and $Z$ have little curvature. Hence the existence of a substantial amount of partial retirement at reduced compensation rates would suggest that $\rho$ cannot be close to 1 for all individuals. This reasoning is contrary to Gordon and Blinder's empirical finding that $\rho = 0.9$, relatively close to 1.

17 It may be argued that a more elaborate model than ours is appropriate because discontinuities at particular ages may result from the influence of some socially acceptable retirement age, which in turn is influenced by program parameters. But to analyze the effects of changes in retirement policy, the role of a socially acceptable retirement age should be modeled explicitly since the effects of these age terms may be altered by the policy change.
References


