

NBER WORKING PAPER SERIES

REAL WAGES AND THE TERMS OF TRADE:  
ALTERNATIVE INDEXATION RULES FOR AN OPEN ECONOMY

Richard C. Marston

Working Paper No. 1046

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge MA 02138

December 1982

Research support from the National Science Foundation (SES-8006414) is gratefully acknowledged. I would like to thank Joshua Aizenman and Joanna Gray for their helpful comments on an earlier draft as well as participants in the NBER's Summer Institute in International Economic Studies and seminars at the University of Illinois, University of Pennsylvania, and Harvard University. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

REAL WAGES AND THE TERMS OF TRADE:  
ALTERNATIVE INDEXATION RULES FOR AN OPEN ECONOMY

ABSTRACT

This paper examines the desirability of wage indexation in an open economy subject to economic disturbances which change the terms of trade and raise the prices of imported goods. Two indexation rules are considered, the traditional form of indexation to the consumer price index and indexation to the price of domestic goods alone, the latter proposed as a means of limiting the influence of import prices on the economy. The effects of the rules are shown to depend upon how the terms of trade rather than import prices alone respond to disturbances, since changes in the terms of trade determine what adjustments are required in the two real wages faced by firms and labor.

Richard C. Marston  
3404 Steinberg-Dietrich Hall/CC  
Wharton School  
University of Pennsylvania  
Philadelphia, PA 19104

Wage indexation attempts to fix real wages throughout the length of labor contracts regardless of the source of price fluctuations. Many of the economic disturbances experienced recently in industrial countries have called for changes in real wages, especially those disturbances altering the terms of trade between domestic and foreign goods. This paper examines the desirability of indexation when such changes in the terms of trade occur.

There are two real wages in an open economy producing a product differentiated from those abroad: the nominal wage deflated by the price of the domestic good and by a price index composed of domestic and foreign goods. The producer responds to one real wage, while labor responds to the other.<sup>1</sup> When the terms of trade change, these two real wages diverge, so that fixing one real wage through indexation still allows the other real wage to vary. The paper will examine the effects of this interplay between the two real wages, since it is a key feature in the macroeconomic adjustment to economic disturbances.

One suggestion commonly made is to limit wage indexation agreements to domestic prices alone by tying wages to the GNP deflator rather than the consumer price index.<sup>2</sup> The proposal is attractive to many observers because it apparently offers a way of shielding the domestic economy from increases in the prices of imports, whether due to increases in foreign prices or the depreciation of the domestic currency. The paper will examine under what conditions this form of indexation is preferable to the traditional form of indexation to consumer prices. The choice of indexation rule will depend less upon how import prices change than upon how the terms of trade are affected by disturbances.

Most previous studies of optimal wage indexation, including those of Gray (1976) and Fischer (1977b), have confined their analyses to closed

economies.<sup>3</sup> The basic conclusions about indexation in such economies can be summarized as follows: monetary or aggregate demand disturbances call for full wage indexation since fixing real wages prevents output from fluctuating. Only aggregate supply disturbances call for less than full indexation, essentially because the adjustment of the real wage helps to moderate output movements. This paper shows why these propositions must be modified in an open economy.

Indexation is analyzed in the context of a small open economy subject to a variety of disturbances, all of which share a common feature: these disturbances ordinarily result in a rise in the domestic price of imports. Otherwise they are very different in nature, as their descriptions indicate:

- (1) an increase in the foreign money supply which raises the price of foreign goods;
- (2) an increase in the domestic money supply;
- (3) a reduction in demand for the domestic good; and
- (4) a fall in productivity in the domestic industry.

The first disturbance is assumed to occur under fixed exchange rates, since foreign prices have their greatest impact in that regime, while all the remaining disturbances are analyzed under flexible rates. Each will be described more fully later in the paper.

#### I. DESCRIPTION OF THE MODEL

The domestic country trades with the rest of the world but is assumed to be too small to affect foreign output, prices, or interest rates. It produces a good which is an imperfect substitute for foreign goods, but consumes both domestic and foreign goods.  $P_t$  is the price of the domestic good,  $X_t$  is the exchange rate and  $P_t^f$  is the foreign currency price of the foreign good.

(These and other variables are expressed in logs; a list of variables is

provided in an appendix). The consumer price index,  $I_t$ , and the terms of trade,  $T_t$ , are defined as functions of these prices:

$$I_t = aP_t + (1-a)(X_t + P_t^f) ,$$

$$T_t = P_t - (X_t + P_t^f) .$$

The consumer price index is a weighted average of domestic and foreign prices expressed in domestic currency, while the terms of trade measures the price of the domestic good relative to that of the foreign good.

The model of the domestic country consists of three equations, aggregate supply and demand for the domestic good and an equation describing financial behavior. In order to explain the impact of indexation on supply behavior, we begin with a model of aggregate supply in a frictionless economy without contract lags or indexation.

#### A. Supply Behavior in a Frictionless Economy

The terms of trade affect aggregate supply even in an economy without contract lags because producers and labor respond to different real wages. The real wage facing the producer is defined relative to the domestic price, while the real wage that is relevant for labor is defined relative to the consumer price index. The model of supply behavior specified below shows how domestic output is affected by the relative movements of these two real wages. (All variables in this frictionless economy are starred).

$$Y_t^* = c_1 T_t^* + c_2 u_t^s + c_0 . \quad (1)^*$$

$$Y_t^* = (1 - c') L_t^* + u_t^s \quad (1a)$$

$$L_t^{d*} = -(W_t^* - P_t^*)/c' + u_t^s/c' + \ln(1 - c')/c' \quad (1b)$$

$$L_t^{s*} = n(W_t^* - I_t^*) + n_0 \quad (1c)$$

$$\text{where } c_0 = (1 - c')[n \ln(1 - c') + u_0]/(1 + nc')$$

$$c_1 = (1 - c') n (1 - a)/(1 + nc')$$

$$c_2 = (1 + n)/(1 + nc')$$

The model consists of a Cobb-Douglas production function (equation 1a), a labor demand function derived from the production function (equation 1b), and a labor supply function (1c). The production function includes a disturbance term representing changes in productivity,  $u_t^s$ , which is assumed to have a mean of zero and to be serially uncorrelated (as are the other disturbances in the model introduced below). Notice that the producer's real wage appears in the labor demand function, while labor's real wage appears in the supply function.

Solving this model results in the aggregate supply equation (1)\*. Output is a function of the terms of trade as well as the productivity disturbance. How the terms of trade affect output can be illustrated by Figure 1 which shows how the nominal wage and employment are determined.

Consider first the effects of an equal rise in domestic and foreign prices which keeps the terms of trade constant.

$$0 < P_t^* - \bar{P} = I_t^* - \bar{I} = (X_t^* - \bar{X}) + (P_t^{f*} - \bar{P}^f) .$$

The price changes are defined relative to prices in a stationary equilibrium  $(\bar{P}, \bar{I}, \bar{X}, \bar{P}^f)$  where all disturbances are equal to zero. As will be seen below, this change in prices could be due to (1) an unanticipated increase in the foreign money supply under fixed exchange rates which induces a rise in the price of the foreign good, or (2) an unanticipated increase in the domestic money supply under flexible exchange rates which induces a depreciation of the domestic currency. Labor demand and supply both respond

proportionally to these price changes with the demand and supply curves shifting up to point b. Labor employed and therefore output remain constant since the nominal wage rises as much as prices, thus keeping real wages constant.<sup>4</sup>

Now consider a case where the foreign exchange rate, and therefore the domestic currency price of the foreign good, rises more than the domestic price so that the terms of trade fall. (For simplicity, the price of the foreign good in foreign currency is assumed to remain constant.) This price behavior could be due to a shift in demand away from the domestic good under flexible rates:

$$0 < P_t^* - \bar{P} < I_t^* - \bar{I} < X_t^* - \bar{X}.$$

These price changes induce a fall in the amount of labor employed and output. The labor supply curve shifts up by the change in the consumer price index (to  $L_t^{s'}$ ), while the labor demand curve shifts up by the change in the domestic price (to  $L_t^{d''}$ ). Labor employed must fall (to c). Both real wages change: the real wage faced by the producer must rise while the real wage earned by labor must fall.

In equations (1d) and (1e) below we express the two real wages as functions of the terms of trade and the supply disturbance (the latter of which we assume equal to zero for purposes of the present discussion).

$$(W_t^* - P_t^*) - (\bar{W} - \bar{P}) = \frac{-nc'(1-a)(T_t^* - \bar{T}) + u_t^s}{1 + nc'} \quad (1d)$$

$$(W_t^* - I_t^*) - (\bar{W} - \bar{I}) = \frac{(1-a)(T_t^* - \bar{T}) + u_t^s}{1 + nc'} \quad (1e)$$

These two equations confirm the diagrammatic analysis: as long as the terms of trade are constant, real wages remain constant whether defined in terms

of  $P_t^*$  or  $I_t^*$ . But if the terms of trade fall, the producer's real wage rises, so producers reduce output; at the same time, however, the fall in the terms of trade lowers labor's real wage.

Only in special cases is either real wage constant. If labor supply is completely inelastic ( $n = 0$ ), then the change in the terms of trade has no effect upon the producer's real wage and therefore upon output in this frictionless economy. Alternatively, if labor supply is infinitely elastic ( $n = \infty$ ), then labor's real wage remains constant. But in the general case, changes in the terms of trade call for changes in both real wages. That will pose problems in an economy with contract lags and indexation if indexation holds one or the other real wages constant.

#### B. Supply Behavior in an Economy with Contract Lags

When there are lags in labor contracts, current wages do not generally reflect current prices unless there is some type of indexation. The model specified below assumes that there is a one period contract lag so that wages contracted for period  $t$ ,  $W'_t$ , reflect labor market conditions expected as of period  $t-1$ .<sup>5</sup> (See equation 1f).

$$Y_t = c(1-b-b')(P_t - {}_{t-1}EP_t) + cb(1-a)[T_t - {}_{t-1}ET_t] \quad (1)$$

$$+ u_t^s/c' + c_1 {}_{t-1}ET_t + c_0$$

$$W'_t = ({}_{t-1}EP_t + nc' {}_{t-1}EI_t + \ln(1-c') - c' n_0)/(1 + nc') \quad (1f)$$

$$W_t = W'_t + b(I_t - {}_{t-1}EI_t) + b'(P_t - {}_{t-1}EP_t) \quad (1g)$$

where  $c = (1 - c')/c'$ .

The model allows for two alternative types of indexation: consumer price indexation, where wages are tied to the general price level, and domestic price indexation, where wages are tied to the price of domestic output alone. If there is consumer price indexation, current wages are adjusted by a fraction  $b$  of the difference between current and expected consumer prices  $(I_t - {}_{t-1}EI_t)$ .<sup>6</sup> If there is domestic price indexation, the adjustment is by a fraction  $b'$  of the difference between current and expected producer prices  $(P_t - {}_{t-1}EP_t)$ . (See equation 1g). The indexing parameters,  $b$  and  $b'$ , are assumed to lie in the range between zero indexation ( $b = b' = 0$ ) and full indexation (with either  $b = 1$  or  $b' = 1$ ).

The amount of labor demanded and output produced depends upon the actual wage,  $W_t$ , and the domestic price. Equation (1), the supply equation for domestic output, is obtained by substituting the actual wage from equation (1g) into the non-starred equivalent of the labor demand function (1b), and substituting the latter into the non-starred equivalent of the production function (1a).<sup>7</sup> The result is a formidable-looking supply equation that expresses output as a function of two price prediction errors as well as the supply disturbance. As in a closed economy (where  $a = 1$ ), output is positively related to unanticipated changes in the price of domestic output. But output is also positively related to unanticipated changes in the terms of trade if wages are indexed at all to the consumer price index (i.e.,  $b > 0$ ), since then wages and therefore output respond to unanticipated changes in both domestic and foreign prices.

Before closing the model with aggregate demand and financial equations, we can use equation (1) to review some of the results of the domestic indexation literature (where  $a = 1$ ).

(1) If there are monetary disturbances or aggregate demand disturbances in a closed economy, then output changes in proportion to the resulting unanticipated change in prices:

$P_t - {}_{t-1}E_t P_t$ . However, if there is full indexation, which can be represented by either  $b=1$  or  $b'=1$  since the price of domestic output and the consumer price index are identical in this simple closed economy, then output is shielded from this disturbance.

(2) If there are supply disturbances, such as a fall in productivity,  $u_t^s < 0$ , then output is affected both directly (proportionally to  $u_t^s$ ) and indirectly (through the resulting unanticipated rise in the domestic price.). With full indexation, the change in output is even larger, since indexation prevents the rise in the domestic price from dampening the direct effect of the disturbance on output. Thus indexation increases the effect of the disturbance on output.

Both of these results are modified by the introduction of foreign goods and the terms of trade into the economy.

### C. Closing the model

Aggregate demand is assumed to be a function of the terms of trade, the real interest rate, and a stochastic disturbance,  $u_t^d$ .

$$Y_t = -g_p T_t - g_r [r_t - ({}_t E_{t+1} I_t - I_t)] + g_0 + u_t^d. \quad (2)$$

A rise in the terms of trade (rise in the domestic price relative to the foreign price) is assumed to reduce aggregate demand, as is a rise in the real interest rate. In the case of perfect substitution between foreign and domestic goods, where  $g_p$  becomes infinite in size, this aggregate demand relationship reduces to the familiar purchasing power parity relationship.

In equation (3), the demand for money is expressed as a function of the interest rate and real income, with money balances and income being deflated by the general price level.

$$M_t - I_t = -k_1 r_t + k_2(P_t + Y_t - I_t) + k_0, \quad (3)$$

$$r_t = r_t^f + {}_tEX_{t+1} - X_t, \quad (3a)$$

$$M_t = \bar{M} + u_t^m. \quad (3b)$$

The domestic bond is assumed to be a perfect substitute for the foreign bond, so the domestic interest rate is equal to the expected return on the foreign bond. (Equation 3a). Under flexible exchange rates, the supply of money is assumed to be equal to a base level plus a random disturbance term,  $u_t^m$ . (Equation 3b). Under fixed exchange rates, the supply of money is determined endogenously by equation (3).

## II. MONETARY DISTURBANCES

We begin by analyzing two types of disturbances for which indexation is ideally suited, foreign and domestic monetary disturbances. In the appendix we determine the impact of the foreign monetary disturbance on the foreign price and interest rate under the assumption that the foreign country is optimally (i.e., fully) indexed. If  $u_t^{mf}$  denotes the foreign monetary disturbance, which like the domestic disturbances has a mean of zero and is serially uncorrelated, then the changes in the foreign price and interest rate are as given below:

$$P_t^f - \bar{P}^f = u_t^{mf} / (1 + k_1) = -(r_t^f - \bar{r}^f).$$

The increase in the foreign money supply raises the price of foreign output, but lowers the foreign interest rate.

The effects of both monetary disturbances on the domestic economy can be shown after solving the three equations of the domestic model (1 - 3) for  $Y_t$ ,  $P_t$  and  $X_t$ . We solve the model in two stages. First, using (1) and (2) we express  $Y_t$  and  $P_t$  as functions of  $X_t$  and the foreign monetary disturbance. The resulting expressions, equations (4) and (5) below, describe demand and supply behavior in both exchange rate regimes:

$$Y_t - \bar{Y} = \frac{(g_p + g_r a)c(1-b'-b)}{[g_p + g_r a + c(1-b'-ba)]} [(X_t - \bar{X}) + u_t^{mf}/(1+k_1)] , \quad (4)$$

$$P_t - \bar{P} = \frac{[g_p + g_r a + cb(1-a)]}{[g_p + g_r a + c(1-b'-ba)]} [(X_t - \bar{X}) + u_t^{mf}/(1+k_1)] , \quad (5)$$

$$X_t - \bar{X} = -\frac{u_t^{mf}}{1+k_1} + \frac{[g_p + g_r a + c(1-b'-ba)]}{D} u_t^m , \text{ where} \quad (6)$$

$$D = (1+k_1)[g_p + g_r a + c(1-b'-ba)] + c(1-b'-b)[k_2(g_p + g_r a) + (1-k_2)(1-a) - 1] > 0.$$

Under fixed exchange rates,  $X_t$  is kept equal to  $\bar{X}$ . Under flexible rates,  $X_t$  can be expressed as in equation (6) as a function of both domestic and foreign disturbances by solving all three equations, (1) - (3), for the full reduced form.

#### A. Foreign Monetary Disturbance under Fixed Exchange Rates

Indexation to the price of domestic output might be thought to insulate the economy completely from changes in foreign prices. According to this reasoning the domestic wage and price, and thus domestic output as well, can be shielded from the foreign disturbance if only foreign goods are excluded from the price index used for indexation. Yet such reasoning is wrong. Domestic price indexation does have the important effect of insulating domestic output from this disturbance, but so does indexation to the consumer price index. Neither type of indexation, on the other hand, shields the

domestic wage or price from the disturbance.

Consider indexation to the consumer price level first. Full indexation ( $b=1, b'=0$ ) ensures that the rise in the price of the foreign good is fully transmitted to the domestic wage; the price of the domestic good, in turn, rises in proportion to the wage. As a result, the domestic price rises as much as the foreign price:

$$P_t - \bar{P} = P_t^f - \bar{P}^f = u_t^{mf} / (1 + k_1) .$$

Thus with full indexation, the terms of trade remain constant, and so also does domestic output. It is because the domestic wage and price adjust fully to the disturbance that the terms of trade and domestic output can be shielded from the disturbance.

Indexation to the domestic price alone ( $b'=1, b=0$ ) has the same effects on the economy as indexation to the consumer price level since with the terms of trade constant it cannot matter which type of indexation is employed. To see why this is true, consider the following relationship between the two real wages:

$$W_t - P_t = W_t - I_t - (1-a)T_t .$$

As long as the terms of trade are constant, fixing one real wage through indexation also fixes the other real wage. So the two types of indexation are equivalent, with both types insulating domestic output completely from the disturbance.

Under flexible exchange rates, the foreign monetary disturbance leads to an appreciation of the domestic currency. The appreciation completely insulates all domestic variables from the disturbance, a familiar result for this type of disturbance, so the degree of indexation is irrelevant.<sup>8</sup>

B. Domestic Monetary Disturbance under Flexible Rates

This disturbance affects the domestic economy much like the previous disturbance since an increase in the domestic money supply under the flexible rates leads to a depreciation of the domestic currency and a rise in the domestic currency price of the foreign good. In the absence of indexation, domestic output and the price of that output both increase in response to the depreciation. But if wages are fully indexed, whether to the consumer price index or the domestic price alone, then the rise in the domestic price is equal to the change in the exchange rate:

$$(P_t - \bar{P}) = (X_t - \bar{X}) = u_t^m / (1+k_1) .$$

Since the terms of trade are constant, indexation insulates domestic output just as in the case of the foreign monetary disturbance.

We note that under fixed exchange rates, the assumption of perfect capital mobility ensures that domestic output and price are completely insulated from the disturbance (regardless of the degree of indexation), another common result in the literature on exchange rate regimes.

III. DISTURBANCES THAT CHANGE THE TERMS OF TRADE

The two monetary disturbances are ideal candidates for indexation because neither disturbance has real effects in the absence of contract lags. (Recall the geometric analysis of the frictionless economy above.) Indexation allows nominal wages to adjust fully to the disturbance just as they would in the frictionless economy. Indexation is less well suited for the other two disturbances to be considered, however, since both disturbances have real effects even in the frictionless economy. The choice of indexation rule makes a difference, moreover, since both disturbances induce changes in the terms of trade thereby opening a wedge between the two real wages.

Before considering these disturbances, we must discuss the proper criterion for judging indexation since an ambiguity arises in the case of these disturbances. We might judge indexation according to whether or not it minimized the effects of disturbances on domestic output, a traditional objective of economic policy. Gray (1976), however, proposed an alternative criterion: the stabilization of output relative to that prevailing in a frictionless economy without contract lags. In our notation, the criterion involves minimizing the value of the loss function,  $Z$ , which is expressed in terms of deviations of actual  $Y_t$  from output in an economy without contract lags,  $Y_t^*$ :

$$Z = E[(Y_t - Y_t^*)^2].$$

The rationale for this criterion lies in the inefficiencies associated with being off labor demand and supply curves because of contract lags. This criterion is the proper one if the fundamental objective of indexation is to undo the distortions caused by contract lags. In a closed economy, the two criterion differ whenever there are supply disturbances, since output in a frictionless economy is not then constant. In an open economy, the two criteria also differ for any disturbance that changes the terms of trade since  $Y_t^*$  is a function of the terms of trade.<sup>9</sup> The two criteria were identical for the two monetary disturbances considered above since neither disturbance affects  $Y_t^*$ . But the two criteria will differ for both of the disturbances considered below.

The two remaining disturbances to be considered are as follows: a reduction in the demand for the domestic good and a fall in productivity in the domestic industry, both under flexible exchange rates. Since the terms of trade are central to the discussion, we solve the domestic model (equations

1-3) to express output and the terms of trade as a function of these two disturbances:

$$Y_t - \bar{Y} = \frac{cb(1-a)(1+k_1) + [k_1 + (1-k_2)(1-a)]c(1-b'-b)}{D} u_t^d \quad (7)$$

$$+ \frac{(g_p + g_r a)(1+k_1)}{c'D} u_t^s .$$

$$T_t - \bar{T} = \frac{[1 + k_1 + k_2 c(1-b'-b)]}{D} u_t^d - \frac{(1+k_1)}{c'D} u_t^s , \quad (8)$$

where  $\bar{Y}$  and  $\bar{T}$  are the values of the variables in a stationary equilibrium in which all disturbances are equal to zero. For purposes of comparison, output and the terms of trade in the frictionless economy are also given below:

$$Y_t^* - \bar{Y} = \frac{c_1}{(g_p + g_r a + c_1)} u_t^d + \frac{c_2(g_p + g_r a)}{(g_p + g_r a + c_1)} u_t^s \quad (7)^*$$

$$T_t^* - \bar{T} = \frac{u_t^d}{(g_p + g_r a + c_1)} - \frac{c_2 u_t^s}{(g_p + g_r a + c_1)} \quad (8)^*$$

These expressions are obtained by solving equation (1)\* and the starred equivalents of equations (2) and (3).

#### A. Aggregate Demand Disturbance

If there is an unanticipated fall in the demand for the home good ( $u_t^d < 0$ ), then in the frictionless economy without contract lags domestic output and the terms of trade fall. (See equations (7)\* and (8)\*). Because the terms of trade change, the economy behaves differently under the two types of indexation:

##### Consumer Price Indexation (b=1)

Output and the terms of trade still fall if wages are fully indexed to consumer prices:

$$(Y_t - \bar{Y})_{b=1} = \frac{c(1-a)}{(g_p + g_r a + c(1-a))} u_t^d < 0 \quad (9)$$

$$(T_t - \bar{T})_{b=1} = \frac{u_t^d}{(g_p + g_r a + c(1-a))} < 0. \quad (10)$$

Indexing to the consumer price index ensures that labor's real wage remains constant. But because the terms of trade change, the real wage facing producers cannot also remain constant. And it is the producer's real wage which determines the behavior of output. Recall the relationship between the two real wages:  $W_t - P_t = (W_t - I_t) - (1-a)T_t$ . With the terms of trade falling, the producer's real wage must rise. Thus output must fall even with complete indexation.

#### Domestic Price Indexation (b'=1)

This type of indexation keeps the producer's real wage constant, so output itself remains constant despite a fall in the terms of trade:

$$(Y_t - \bar{Y})_{b'=1} = 0 \quad (9)'$$

$$(T_t - \bar{T})_{b'=1} = \frac{u_t^d}{(g_p + g_r a)} < 0. \quad (10)'$$

Because the producer's real wage cannot adjust, the terms of trade must absorb the full impact of the disturbance, falling enough to keep aggregate demand constant. Thus output and employment are shielded from the disturbance. If the objective of indexation is to keep output constant, then domestic price indexation is optimal.

If instead, the objective of indexation is to minimize deviations from output in an economy without contract lags, then neither type of indexation is optimal. Consumer price indexation induces output to fall more than it would

in a frictionless economy, while domestic price indexation prevents output from falling at all.

$$(Y_t - \bar{Y})_{b=1} \leq (Y_t^* - \bar{Y}) \leq (Y_t - \bar{Y})_{b'=1} = 0$$

The reason should be clear from the discussion of behavior in the frictionless economy in Section I. This type of disturbance calls for a rise in the producer's real wage and a fall in labor's real wage. (See equations (1d) and (1e)). If labor resists the fall in its real wage by adopting consumer price indexation, output falls even more than in an economy without contract lags. If the wage is indexed to the domestic price, in contrast, the producer's real wage cannot rise and so no real adjustments take place. Each type of indexation leads to deviations from the frictionless norm, one causing an excessive reduction in output, the other preventing any real adjustment at all.

Only in polar cases does either type of indexation lead to output variation equal to that in an economy without contract lags. If labor supply is inelastic ( $n = 0$ ), then as discussed in Section I, output remains constant in the economy without contract lags, so domestic price indexation is optimal. (Notice that the two criteria merge in this case.) Alternatively, if labor supply is infinitely elastic ( $n = \infty$ ), then labor's real wage remains constant in the frictionless economy; so indexing to the consumer price index must result in the same output behavior as in the frictionless economy.<sup>10</sup> But otherwise, both types of indexation are suboptimal (measured relative to behavior in the frictionless economy).

In the general case where neither form of indexation is optimal, we might choose between the two indexation schemes according to which more closely approximates behavior in the economy without contract lags. It should be

clear from the above discussion that the choice would depend upon the sensitivity of labor supply to real wages. As long as labor supply is relatively insensitive to real wages domestic price indexation would be preferred: how insensitive labor supply must be depends upon other parameters in the model.<sup>11</sup>

Alternatively, we might retain the traditional form of indexation to consumer prices, but choose an indexation parameter less than one. The indexation parameter that keeps actual output equal to that in a frictionless economy is

$$\hat{b} = 1 - \frac{c(1-a)(1+k_1)}{c(1+c'n)[k_2(1-a) + k_2c_1 - k_1a]}$$

This parameter is equal to one if the labor supply curve is infinitely elastic ( $n = \infty$ ), but less than one otherwise.<sup>12</sup> In general, partial indexation is called for, unlike the case of a closed economy.

We can summarize the discussion of the decline in demand for the domestic good as follows:

- (1) Domestic price indexation can stabilize output completely in the presence of this disturbance, while unlike in a closed economy consumer price indexation allows output to fall.
- (2) If the policy objective is to minimize deviations from output in an economy without contract lags, neither form of indexation is generally optimal, although domestic price indexation is still preferred as long as the labor supply elasticity is relatively low.
- (3) If consumer price indexation is retained, the optimal indexation parameter is generally less than one, again in contrast with the case of a closed economy.

B. Aggregate Supply Disturbance

The supply disturbance to be considered is a decline in productivity ( $u_t^s < 0$ ). This disturbance lowers domestic output in the frictionless economy, but raises the terms of trade as well as the price of domestic output. (See equations (7)\* and (8)\*). The decline in productivity has an indeterminate effect on the exchange rate; with domestic output falling but the price of that output rising, the transactions demand for money can either fall or rise. Even if the domestic currency depreciates, however, the exchange rate still rises less than the price of domestic output, so the terms of trade must rise. The general price level, moreover, also rises less than the domestic price (and may actually fall) because  $I_t$  is just a weighted average of the prices of the two goods.

With the terms of trade rising, the two real wages respond differently to the disturbance.<sup>13</sup>

$$(W_t^* - P_t^*) - (\bar{W} - \bar{P}) = \frac{[g_p + g_r a + n(1-a)]}{(1+c'n)(g_p + g_r a + c_1)} u_t^s < 0 \quad (1d)'$$

$$(W_t^* - I_t^*) - (\bar{W} - \bar{I}) = \frac{[g_p + g_r a - (1-a)]}{(1+c'n)(g_p + g_r a + c_1)} u_t^s \begin{matrix} < \\ > \end{matrix} 0. \quad (1e)'$$

The real wage facing the producer falls as a result of the productivity decline just as in a closed economy. Labor's real wage never falls as much as the producer's real wage because the rise in the terms of trade moderates that fall by opening a wedge between  $P_t$  and  $I_t$ . In fact, as equation (1e)' suggests, labor's real wage can actually rise in response to this disturbance if, for example, the domestic currency appreciates enough to lower the general price level. But a rise in that real wage can be ruled out as long as the elasticity of aggregate demand with respect to the terms of trade exceeds a

critical value less than one:  $g_p > 1 - a - g_r a$ . For the remainder of this section, we assume that this elasticity condition is fulfilled,<sup>14</sup> so that labor's real wage also falls in response to this disturbance.

The effect of indexation again depends crucially upon which type of indexation is employed. We begin by describing domestic price indexation:

Domestic Price Indexation (b'=1)

In the case of a productivity disturbance, keeping the producer's real wage fixed does not keep output fixed since the disturbance directly affects output. With full indexation, output falls and the terms of trade rise by:

$$(Y_t - \bar{Y})_{b'=1} = u_t^s / c' < 0 \quad (11)'$$

$$(T_t - \bar{T})_{b'=1} = - \frac{u_t^s}{(g_p + g_r a)c'} > 0. \quad (12)'$$

Because the producer's real wage is prevented from falling, in fact, output variation is greater with domestic price indexation than in the economy without contract lags. Adjustment of the real wage in the frictionless economy helps to minimize the effects of the disturbance on output, while indexation prevents that adjustment.

Consumer Price Indexation (b=1)

In the case of consumer price indexation, output still falls, and the terms of trade rise:

$$(Y_t - \bar{Y})_{b=1} = \frac{(g_p + g_r a) u_t^s}{(g_p + g_r a + c(1-a))c'} < 0 \quad (11)$$

$$(T_t - \bar{T})_{b=1} = \frac{-u_t^s}{(g_p + g_r a + c(1-a))c'} > 0. \quad (12)$$

Choosing this form of indexation, however, always results in less output variation than when domestic price indexation is adopted. (Compare (11) and

(11)'). The reason is clear from examining the relationship between the two real wages and the terms of trade:  $W_t - P_t = (W_t - I_t) - (1-a)T_t$ . By fixing labor's real wage, consumer price indexation still allows the producer's real wage to fall since the terms of trade rise. With some adjustment in the real wage facing them, producers reduce output less than when wages are tied to domestic prices. So the decline in output is smaller in absolute terms under consumer price indexation than under domestic price indexation.

Yet how does output variation under consumer price indexation compare with that in the frictionless economy? Once again, we can determine the answer most easily by examining the real wage in the frictionless economy (1e)'. In the absence of contract lags, labor's real wage declines when productivity falls as long as the elasticity condition is fulfilled. When there are contract lags and indexation of wages to the consumer price index, this real wage is prevented from falling. Thus output falls more with consumer price indexation than in the frictionless economy, the same result found in a closed economy.<sup>15</sup>

It is evident from the preceding that of the two forms of indexation domestic price indexation leads to greater variation in output both in absolute terms and relative to output in the frictionless economy since

$$(Y_t - \bar{Y})_{b'=1} < (Y_t - \bar{Y})_{b=1} \leq (Y_t^* - \bar{Y}) < 0 .$$

Thus consumer price indexation is preferable to domestic price indexation whichever criteria is used.

Since full indexation even to the consumer price index is generally not optimal, we might wish to determine the optimal indexation parameter as we did in the case of the demand disturbance. We would again find that partial indexation is optimal.<sup>16</sup>

$$\hat{b} = 1 - \frac{(1+k_1)[g_p + g_r a - (1-a)]}{(1+n)[k_1 a + k_2(g_p + g_r a) - k_2(1-a)]}$$

That is, the indexation parameter that keeps output equal to that in a frictionless economy is less than one.<sup>17</sup>

We can summarize this discussion of the decline in productivity as follows:

- (1) Neither type of indexation can keep output constant because the productivity disturbance directly affects output.
- (2) Domestic price indexation results in greater output variation than consumer price indexation both in absolute terms and relative to output variation in an economy without contract lags.
- (3) If some form of consumer price indexation is adopted, the optimal indexation parameter is generally less than one, just as in a closed economy.

#### IV. CONCLUSIONS

The relationship between the terms of trade and real wages provides the key to understanding the effects of wage indexation in an open economy. All of the disturbances considered above raise the domestic price of imported goods, but the desirability of indexation differs according to how the terms of trade rather than the import price itself responds to the disturbance.

Of the two forms of indexation studied, domestic price indexation shows advantages over consumer price indexation only in the case of aggregate demand disturbances. A decline in demand for the domestic good leads to a fall in the terms of trade; stabilizing labor's real wage through consumer price indexation still allows the producer's real wage, and therefore output, to vary when the terms of trade fall, whereas domestic price indexation stabilizes output completely. For aggregate supply disturbances, on the other

hand, domestic price indexation leads to greater variation of domestic output both in absolute terms and relative to output in a frictionless economy. For disturbances which leave the terms of trade unchanged, moreover, it does not matter which indexation rule is chosen since fixing one real wage also fixes the other. This is the case for monetary disturbances, including foreign monetary disturbances which raise the price of foreign goods. Thus only limited support is found for deliberately excluding foreign prices from wage indexation schemes.

For the more traditional form of wage indexation--indexation to the consumer price index--the analysis confirms some of the results found in a closed economy, but modifies others. The chief difference arises in the case of aggregate demand disturbances. Full indexation prevents labor's real wage from adjusting to the terms of trade. Only with partial indexation does output vary as it would in the economy without contract lags.

FOOTNOTES

<sup>1</sup>For previous analyses of the labor market emphasizing this difference between the two real wages, see Salop (1974), Purvis (1979), Sachs (1980), Flood and Marion (1982), and Marston (1982).

<sup>2</sup>Countries where such proposals have been debated include Denmark, Israel, and the United Kingdom. See, for example, the study of wage indexation by Braun (1976). Israel opted instead for a partial (70%) adjustment to the CPI which was meant to exclude on an average basis only those increases in the CPI due to rises in import prices or indirect taxes. For a discussion, see the Bank of Israel (1975). Recently the Congressional Budget Office (1981) has compared a variety of indexation rules for the United States including tying wages and federal benefits to the GNP deflator specifically to exclude import prices.

<sup>3</sup>An important exception is the recent study by Flood and Marion (1982) of optimal indexation in an open economy. For most of their study they assume that there is a single world good so that the law of one price holds, whereas here changes in the terms of trade are central to the analysis. Where the results are similar is in the case of a productivity disturbance, since they show how their results are modified for this disturbance when the domestic and foreign goods are distinct. See the discussion of the productivity disturbance below.

<sup>4</sup>Output changes are directly proportional to employment changes as long as there are no supply disturbances. (See equation 1a).

<sup>5</sup>This contracting approach to the aggregate supply function has been criticized by Barro (1977) who offers an alternative description of labor market behavior which gives Pareto optimal outcomes. Barro develops a model where wages are set in contracts but where employment is kept at its Pareto

optimal level at all times. Such contracts dominate the simple ones considered here, but raise problems of moral hazard because of the contingent contracts required, as Barro himself recognizes, and seem to be very different from actual labor contracts, as Fischer (1977a) observed in his comment on Barro's paper.

<sup>6</sup>  ${}_{t-1}E I_t$  is the expected value of the general price level at period  $t$  based on information available at period  $t-1$ . The expectation, like others in the model, is assumed to be formed rationally on the basis of behavior in the model.

<sup>7</sup> Cukierman (1980) shows that the effects of indexation are sometimes changed if employment is determined by labor supply rather than labor demand. Although the employment rule deserves further study, we have chosen not to further complicate our analysis with alternative rules.

<sup>8</sup> If the foreign country were less than fully indexed, however, flexible rates would not insulate the economy from a foreign monetary disturbance. See Marston (1982) for a discussion of this case. The disturbance would raise foreign output which would affect domestic aggregate demand much like the aggregate demand disturbance to be discussed below.

<sup>9</sup> When the terms of trade can change, real income is not equivalent to real output. We might alternatively specify the objective function in terms of real income,  $Y_t + P_t - I_t$ . The criterion would be expressed in terms of the deviation of actual real income from real income in an economy without contract lags as follows:

$$\begin{aligned} Z' &= E\left[\left((Y_t + P_t - I_t) - (Y_t^* + P_t^* - I_t^*)\right)^2\right] \\ &= E\left[\left((Y_t - Y_t^*) + (1-a)(T_t - T_t^*)\right)^2\right] \end{aligned}$$

This expression, however, can be simplified so that only output appears in the

loss function. (Flood and Marion (1982) make a similar point). If we take the aggregate demand function (2) and subtract its (starred) equivalent in the frictionless economy for the case where all foreign variables are constant, we obtain:

$$Y_t - Y_t^* = -(g_p + g_r a)(T_t - T_t^*) .$$

The objective then simplifies to

$$Z' = E[K^2(Y_t - Y_t^*)^2],$$

where  $K = (g_p + g_r a - (1-a))/(g_p + g_r a)$ . All of the results in the text for real output carry through for real income except in the special case where  $g_p + g_r a = 1 - a$ . In that case, real income is always identical in the economies with and without contract lags, with the deviation in output exactly offsetting the deviation in the terms of trade.

<sup>10</sup>When labor supply is infinitely elastic, output in the frictionless economy changes by

$$Y_t^* - \bar{Y} = c(1-a) u_t^d / (g_p + g_r a + c(1-a)) = (Y_t - \bar{Y})_{b=1},$$

since  $c_1 = c(1-a)$  in that case.

<sup>11</sup>To choose between the two forms of indexation, we compare the change in output in each indexation scheme relative to  $Y_t^* - \bar{Y}$ . Output variation under domestic price indexation is closer to  $Y_t^* - \bar{Y}$  if and only if

$$\frac{[(g_p + g_r a) - c'n(g_p + g_r a + c(1-a))]}{(1 + c'n)} > 0.$$

Note that this expression is positive when labor supply is inelastic ( $n = 0$ ), but negative when labor supply is infinitely elastic ( $n = \infty$ ). An example of an intermediate case is as follows: Suppose that the elasticity of aggregate demand with respect to the terms of trade exceeds a critical value less than

one,  $g_p > 1 - a - g_r a$ , a condition discussed below in connection with productivity disturbances; the expression must be positive (i.e., domestic price indexation is preferred) if the labor supply elasticity is less than or equal to one,  $n \leq 1$ .

<sup>12</sup>Note that  $\hat{b} = 1$  if  $a = 1$  as in a closed economy. We assume that the denominator in the expression above is positive which is the case even for  $n = 0$  (i.e. so that  $c_1 = 0$ ) as long as  $k_2(1-a) > k_1 a$ .

<sup>13</sup>These expressions are derived by substituting (8)\* in (1d) and (1e).

<sup>14</sup>This is the same condition encountered earlier in the discussion of real income. Unless  $g_r a > 1 - a$ , the condition is somewhat more stringent than the Marshall-Lerner condition which implies only that  $g_p > 0$ . The possibility of an appreciation of the domestic currency occurs with a low price elasticity because the domestic price may then increase enough to raise the transactions demand for money.

<sup>15</sup>One exception arises when the labor supply curve is infinitely elastic ( $n = \infty$ ). The real wage of labor in the frictionless economy then remains constant (see 1e'), so consumer price indexation leads to the same variation of output as in the frictionless economy.

<sup>16</sup>Flood and Marion (1982) similarly find that partial indexation is optimal for an open economy under fixed exchange rates, at least if the domestic and foreign goods are distinct, although not when there is only one good in the world (since then the domestic price is tied to the foreign price and hence uncorrelated with the productivity disturbance). Note that with the exchange rate flexible, partial indexation is optimal even when there is only one good (a case which can be represented in our model by making  $g_p$  infinite).

<sup>17</sup>Note that as mentioned above, full indexation ( $\hat{b} = 1$ ) is optimal if labor supply is infinitely elastic. By assuming earlier that

$g_p > 1 - a - g_r a$ , we ruled out another case where  $\hat{b} = 1$  (i.e., where this inequality is replaced by an equality), as well as cases where  $\hat{b} > 1$  (i.e., for very low elasticities).

APPENDIX: MODEL OF THE FOREIGN COUNTRY

$$Y_t^f = c_0^f + c^f(1-b^f)(P_t^f - {}_{t-1}E P_t^f) \quad (A1)$$

$$Y_t^f = g_0^f - g_r^f(r_t^f - ({}_tE P_{t+1}^f - P_t^f)) \quad (A2)$$

$$(\bar{M}^f + u_t^{mf}) - P_t^f = -k_1 r_t^f + k_2 Y_t^f + k_0 . \quad (A3)$$

The model of the foreign country parallels that of the domestic country except that there is only one good involved. Aggregate supply, therefore, is a function of the price of that good alone; output responds to unanticipated changes in that price if there is incomplete indexation (or  $b^f$  less than one). Note that for simplicity the money demand function is assumed to have the same income and interest rate elasticities as the domestic country.

If there is a monetary disturbance ( $u_t^{mf} > 0$ ), full indexation ( $b^f = 1$ ) is optimal since it can shield output fully from this disturbance. With full indexation, the foreign price rises and the foreign interest rate falls in proportion to the disturbance as follows:

$$P_t^f - \bar{P}^f = u_t^{mf} / (1 + k_1) = -(r_t^f - \bar{r}^f) .$$

These are the expressions which appear in the text above.

LIST OF VARIABLES

All variables are in logarithms (except interest rates). In the frictionless economy, the corresponding variables are starred.

- $Y_t$  : domestic output,
- $P_t$  : price of domestic output,
- $P_t^f$  : price of foreign output,
- $X_t$  : exchange rate (domestic currency price of foreign currency),
- $T_t$  : terms of trade ( $T_t = P_t - X_t - P_t^f$ ),
- $r_t, r_t^f$  : domestic, foreign interest rates,
- $I_t$  : general price level of the domestic country,
- ${}^E J_{t+1}$  : expected value of a variable at  $t+1$ ,  $J_{t+1}$ , based on information available at  $t$ ,
- $L_t$  : units of labor,
- $W_t$  : domestic nominal wage,
- $W_t'$  : contract wage,
- $M_t$  : domestic money supply,
- $u_t^d, u_t^s$  : domestic aggregate demand, supply disturbances,
- $u_t^m, u_t^{mf}$  : domestic, foreign monetary disturbances.

REFERENCES

- Bank of Israel, 1975, Annual Report.
- Barro, R. J., 1977, Long-term contracting, sticky prices, and monetary policy, *Journal of Monetary Economic* 3, 305-16.
- Braun, A. R., 1976, Indexation of wages and salaries in developed economies, *International Monetary Fund Staff Papers* 23, 226-71.
- Congressional Budget Office, 1981, Indexing with the Consumer Price Index: Problems and Alternatives (Congressional Budget Office, Washington, D.C.)
- Cukierman, A., 1980, The effects of wage indexation on macroeconomic fluctuations: a generalization, *Journal of Monetary Economics* 6, 147-70.
- Fischer, S., 1977, "Long-term contracting, sticky prices, and monetary policy": a comment, *Journal of Monetary Economics* 3, 317-23.
- Fischer, S., 1977, Wage indexation and macroeconomic stability, in: K. Brunner and A. H. Meltzer, eds., *Stabilization of the Domestic and International Economy* (North-Holland, Amsterdam) 107-47.
- Flood, R. P. and N. P. Marion, 1982, The transmission of disturbances under alternative exchange-rate regimes with optimal indexing, *Quarterly Journal of Economics* 97, 43-66.
- Gray, J. A., 1976, Wage indexation: a macroeconomic approach, *Journal of Monetary Economics* 2, 221-35.
- Marston, R. C., 1982, Wages, relative prices and the choice between fixed and flexible exchange rates, *Canadian Journal of Economics* 15, 87-103.
- Purvis, D. D., 1979, Wage responsiveness and the insulation properties of a flexible exchange rate, in: A. Linbeck, ed., *Inflation and Employment in Open Economies* (North-Holland, Amsterdam) 225-45.
- Sachs, J., 1980, Wages, flexible exchange rates, and macroeconomic policy, *Quarterly Journal of Economics* 94, 731-47.

Salop, J., 1974, Devaluation and the balance of trade under flexible wages,  
in: G. Horwich and P. Samuelson, eds., Trade Stability and Macroeconomics  
(Academic Press, New York).

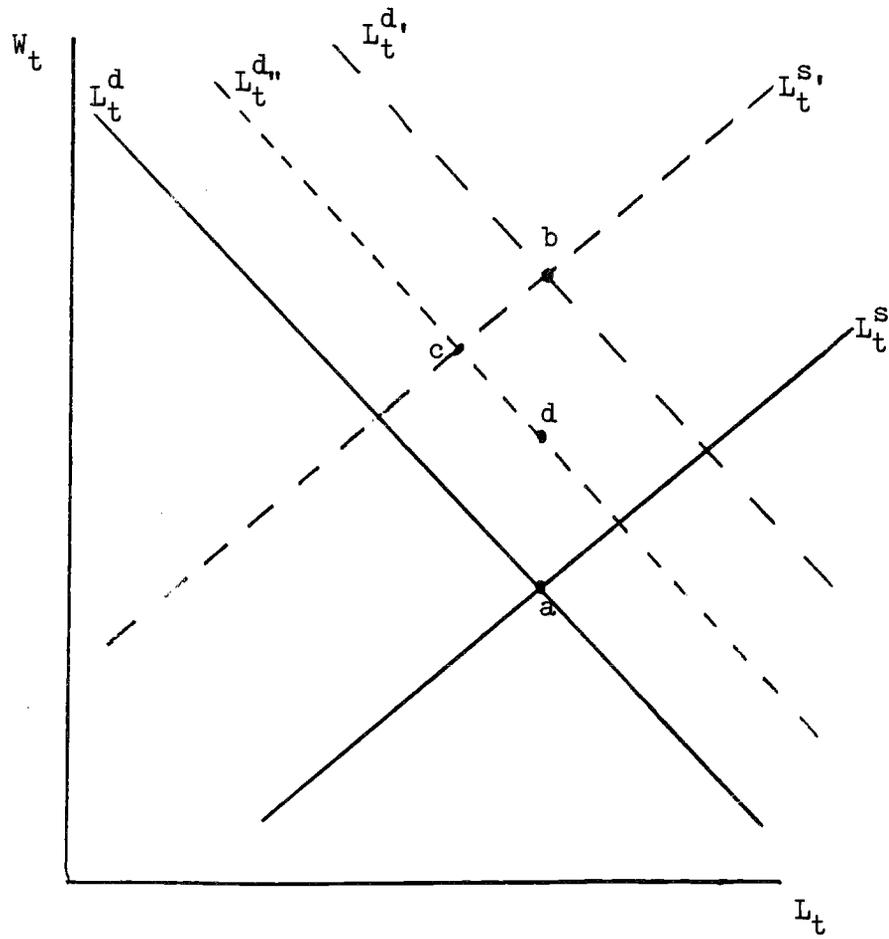


Figure 1: Labor Market in an Economy Without Contract lags