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ABSTRACT

The discount on closed-end funds is widely accepted as proof of investor irrationality. We show,however, that a parsimonious rational model can generate a discount that exhibits many of the characteristics observed in practice. The only required features of the model are that managers have (imperfectly observable) ability to generate excess returns; they sign long-term contracts guaranteeing them a fee each year equal to a fixed fraction of assets under management; and they can leave to earn more money elsewhere if they turn out to be good. With these assumptions, time-varying discounts are not an anomaly in a rational world with competitive investors -- they are required.

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1 Introduction

There is perhaps no empirical regularity cited more often as evidence of investor irrationality than the discount at which closed-end funds trade relative to their net asset value (NAV). In their influential survey of the literature, Lee, Shleifer, and Thaler (1990) identify the four main empirical regularities exhibited by the discount:

1. Closed-end funds are issued at (or above) their NAV, more often than not start trading at a premium to NAV, and then decline.[1]
2. On average, closed end funds trade at a discount relative to their NAV.[2]
3. The discount is subject to wide variation over time and across funds.[3]
4. At termination, price converges to NAV.[4]

They conclude that no rational model could simultaneously explain all of these regularities, that the behavior of closed-end funds is an example of the "limits of arbitrage", and that "The major lesson we take from this analysis is that the demand for securities can influence price, even if that demand is based on irrational beliefs." Other researchers have reached similar conclusions. For example, Pontiff (1996) says that "Pricing theories that are based on fundamentals have had very little, if any, ability to explain discounts."

One problem with this conclusion is that the discount in the UK behaves very like that in the US, while the vast majority of closed-end funds in the UK are held by institutions.[5] This suggests that there might be a rational explanation for at least some of the observed behavior of the closed-end fund discount. In this paper we develop a parsimonious rational model of closed-end fund management in which the discount exhibits all four of the major empirical regularities identified by Lee, Shleifer, and Thaler (1990).

The aim of this paper is not to argue that irrational beliefs or other behavioral factors are not important. Our contribution to the literature is rather to clarify just what features of the discount are also consistent with a rational model. This will help future researchers to identify the regularities that are more likely to be evidence of investor irrationality.

Our model is based on only three main assumptions. First we assume that management talent exists, though it is unobservable. Thus, on average, managers can, by correctly picking stocks, generate returns in excess of their fees. Second, we assume that managers sign long-term contracts with the fund, paying them a fixed percentage of assets under management

[1]See Weiss (1989), Peavy (1990), and Weiss Hanley, Lee, and Seguin (1996).

[2]See Pratt (1966) and Zweig (1973).

[3]See Sharpe and Sosin (1975), Thompson (1978), Richards, Fraser, and Groth (1980), Herzfeld (1980), Anderson (1986), and Brauer (1988).

[4]See Brauer (1984) and Brickley and Schallheim (1985).

[5]For example, Brown (1998) reports that insitutions held 78% of UK closed-end funds in 1986.

each year. Finally, we assume that these contracts are binding on the fund, but not on the manager — the fund cannot prevent the manager from leaving. These three conditions interact with each other to produce exactly the observed empirical regularities. Rather than being anomalies, premia and discounts are *required* if rational investors correctly use the information in past returns to infer management ability.

The behavior of the discount is driven by the tradeoff between managerial ability and fees. Managerial ability adds value to the fund so, in the absence of fees, competitive investors would be willing to pay a premium over net asset value to invest in the fund. Fees subtract value from the fund so, in the absence of managerial ability, investors would only be willing to invest if they could buy shares in the fund at a discount. In the presence of both fees and managerial ability, the fund may trade at either a premium or a discount depending on whether fees or ability dominate. This will change over time as investors see the realized returns on the fund, thereby learning more about the manager's ability.

If performance is bad, investors infer that they have a bad manager, who charges more in fees than the value he creates, and the fund will trade at a discount. Moreover, such a manager is entrenched because of the long term employment contract signed at inception, so the discount is likely to persist. If performance is good, investors infer that they have a good manager, who adds more in value than the fees he charges, and the fund will trade at a premium. However, the manager will eventually either quit the fund for another job that will allow him to capture the benefit he provides, or else he will negotiate a pay increase. Thus premia should be short-lived; most funds will sell at a discount until close to their termination date, when the capitalized value destroyed by the manager becomes small. Investors fully anticipate this behavior, and so expect a closed-end fund to move into discount after its issue date. Investors are still willing to invest at NAV on the IPO because the fee the manager charges is less than the value he is expected to add. In expectation, the additional value added by the manager is exactly cancelled by the expected growth in the discount, and investors get a fair return.

This paper is closely related to the Berk and Green (2002) model of open-end mutual fund management. Both papers use the same economic idea — competition between investors drives the expected returns on an investment fund to the competitive level — but its application to closed-end funds is very different. In Berk and Green (2002), the price of the (open-end) fund is forced to equal NAV at the end of each day. Competition between investors cannot, therefore, change the price of the fund, but instead causes cash in- and out-flows to and from the fund, which drive the returns on the fund to the competitive level. This explains the observed relation between open-end mutual fund flows and performance. With closed-end funds there are no cash in- or out-flows from the fund, but the price is not fixed. Thus, while the assets under management remain fixed, competition between investors causes the fund's *price* to change until the return is again driven to the competitive level. Taken together, the papers show that two different, widely recognized, empirical puzzles — the flow of funds/performance relation in open funds and the discounts of closed-end funds — can both be explained by the same economic fundamentals.

The paper is organized as follows. Section 2 reviews other explanations for the closed-end fund discount. Section 3 develops the formal model, and Section 4 investigates the model's empirical implications. Section 5 concludes the paper.

2 Prior Explanations for the Closed-End Fund Discount[6]

A number of explanations for the closed-end fund discount have been proposed that are based on market frictions such as illiquidity and taxes. If a fund owns a lot of restricted stock, or other illiquid assets, which do not trade freely, its NAV may not accurately reflect its true value, in which case the fact that it does not trade at its NAV is not particularly surprising. Malkiel (1977) and Lee, Shleifer, and Thaler (1991) find that holdings of restricted stock do have some explanatory power for discounts, but these holdings are small or zero for most funds, so cannot fully explain the "anomaly". Seltzer (1989) suggests that funds holding illiquid assets are likely to be overvalued, but this is inconsistent with the fact that funds' price rises when they are open-ended.

Full taxes on a fund's realized capital gains are paid by current shareholders even if most of the gains occurred before they bought their shares. This would imply that funds with large accumulated gains should trade at a discount to NAV. However, Malkiel (1977) finds that even a fund with (a very high) 25% of its assets in unrealized appreciation would see an average discount of only 5%, and moreover the fact that prices rise to NAV on fund liquidation suggests that this factor cannot be the main factor driving discounts.

Brickley, Manaster, and Schallheim (1991) and Kim (1994) suggest an alternative tax-timing explanation based on the idea that holding shares indirectly via a closed-end fund precludes an investor from doing the direct trading in the underlying shares necessary to follow the optimal tax timing strategy.[7] The empirical evidence is mixed. For example, Kim (1994) documents a large increase in the number of closed-end funds after 1986, when changes in the tax-law reduced the tax disadvantage of holding closed-end funds, but DeLong,

[6] The discussion in this section is based on very clear surveys of the literature by Dimson and Minio-Kozerski (1999), Lee, Shleifer, and Thaler (1990), and Anderson and Born (2002).

[7] See, for example, Constantinides (1983, 1984).

Bradford, and Shleifer (1992) document that the discount *increased* between 1985 and 1990. In addition, the tax-timing option cannot explain funds trading at a premium, and should apply to both open and closed-end funds.

Besides frictions, the other main explanations that have been suggested for the discount are fees and managerial ability. The idea that a closed-end fund will trade for a discount if the manager charges fees (but does not add value) was originally proposed by Malkiel (1977). If managers charge fees, and provide nothing of value in return, then the value of the fund to investors should be lower than the fund's NAV. Gemmill and Thomas (2002), Ross (2002a), and Cherkes (2003) show that if the fund pays out a fraction γ to investors each year, and pays fractional management fees of δ each year, the discount is

$$\frac{\delta}{\gamma + \delta}.$$

In particular, if the payout rate to investors is zero, the discount is 100% *regardless* of how small the fractional fee paid to managers each year.[8] Empirically, Malkiel (1977) did not find that fees significantly explained variation in the level of the discount, although Kumar and Noronha (1992) find that differences in fees do explain a small proportion of the cross-sectional variation in discounts. The main drawback of this explanation is that it cannot, alone, explain why closed-end funds trade at a discount, while open-end funds with similar fees trade at NAV. Nor can it explain why closed-end funds are issued at or above their NAV. We will argue in this paper that it is not variation in fees, but rather variation in managerial ability that explains cross-sectional variation in discounts.

If some managers add value, a fund will trade at a discount if investors believe its manager is relatively poor at investing their money (so they do not make back their fees), and at a premium if investors believe the manager is relatively good at investing. Lee, Shleifer, and Thaler (1990) point out that for this to explain the usual overall discount, together with the premium at the IPO, investors must expect superior returns at the IPO, but then (predictably) later expect poor performance. Our model predicts exactly this behavior for the return on the fund's underlying assets. However, when this is combined with the time-series behavior of the discount, investors in the fund's shares always receive the fair rate of return.

Empirical evidence on investors' returns is mixed. Consistent with our model, Malkiel (1977) finds no relation between past performance and discounts. However, Simon (1969) finds that relatively high (low) discounts are associated with relatively worse (better) perfor-

[8]A simple way to see this intuitively is to think of the fund manager as being awarded a fraction δ of the shares remaining in the fund each year. After t years, investors are left with $(1-\delta)^t$ times the number of shares they started with, which goes to zero as $t \to \infty$.

mance, and Roenfeldt and Tuttle (1973) finds a weak relation between the contemporaneous discount and performance.

The explanation most closely related to ours, in that it attempts simultaneously to explain both the issue price and the subsequent discount, is Ross (2002b). Like us, Ross derives a rational model of closed-end funds that explains the post-IPO discounts as a function of the difference between the value added by the manager and the fees charged. Initially, on the IPO, investors are asymmetrically informed about the quality of the manager, and because of the idiosyncrasies of the IPO process, this information is not revealed until after the issue. Hence the fund can be issued at NAV. Although we derive many of same implications, Ross' model differs from ours in a number of important respects. We do not appeal to an information asymmetry — all of our participants are symmetrically informed. Hence, our explanation for why the fund is issued at NAV does not rely on the idiosyncrasies of the IPO process itself. In addition, Ross (2002b) does not examine the implication of a long term labor contract on the fund's performance. Consequently, that paper cannot explain the *predictable* increase in the discount after the IPO, an important contribution of this paper. Ross does not model the intertemporal rational expectations equilibrium and so does not make an inference about the dynamic behavior of the post offering price relative to the pre offering price. However, to be consistent with a rational expectations equilibrium, investors cannot be fooled, so their expectation of the post offering price must be realized on average. In Ross' model this implies that to induce them to participate, on average, the post offering price must equal the pre offering price.

Cherkes (2003) independently develops a related model in which the discount is determined by investors' tradeoff between fees paid and the additional liquidity benefits that accrue through holding the closed-end fund rather than owning the underlying assets directly. Incorporating a liquidity factor allows the model to explain why discounts on related funds tend to move together, something we do not attempt to explain. However, his model does not explain the predictable widening of the discount following the fund's IPO.

3 The Model

Although Section 1 communicated the basic intuition of our model, it says nothing about the size of the effects. To see whether the intuition we describe can produce discounts of the same order of magnitude as observed, this section develops a formal model of closed-end fund management and investment, based on the open-end fund model in Berk and Green (2002). Our intention is to build a parsimonious model of closed-end funds that captures the salient features of the data, rather than to model every idiosyncrasy of the industry. For instance, the typical labor contract in the industry is very much like a CEO's labor contract — the manager serves at the will of board of directors. How this translates into the tenure of a manager is a complicated question and beyond the scope of this paper. Past research has found that proxy contests are rare, and managers are very rarely fired,[9] so we will simply assume that a manager cannot be fired and can, at his or her own discretion, manage the fund until time T when the assets of the fund are distributed to shareholders.[10] We will refer to this date as the *open* date of the fund. Closed-end funds do not in practice have a fixed open date. We could alternatively model the opening of the fund via, for example, some Poisson process with a fixed arrival rate. This would not substantially alter the behavior of the discount near the IPO. The advantage of choosing a fixed open date is that it allows us to use the same model to study the behavior of the discount in both of the periods prior research has identified as important – just after the IPO, and just before the fund is opened.

Assume the (continuously-compounded) return generated by the manager is given by

$$r_t = \hat{r}_t + \alpha - \frac{1}{2\omega} + \epsilon_t, \tag{1}$$

where $\hat{r}_t$ is the return on the (observable) portfolio held at the start of the period,

$$\hat{r}_t = r - \frac{1}{2\zeta} + \xi_t. \tag{2}$$

r_t does not necessarily equal $\hat{r}_t$ as the manager adds value by selecting stocks, so the portfolio composition changes over time. ξ_t is an i.i.d. normal random variable with mean zero and precision ζ, representing the volatility of the assets in the fund at the beginning of the period, and r is the expected return on these assets. ϵ_t is an i.i.d. normal random variable, independent of ξ_t and any risk factor in the economy (it is purely idiosyncratic uncertainty), with mean zero and precision $\omega = 1/\sigma^2$, representing the effect of unobservable changes in portfolio composition during the period. If the manager's skill level, α, is positive (negative) the manager earns an average return greater (less) than the expected return on the portfolio at the beginning of the period. The manager charges a proportional fee, c, per period, so the fund's net asset value at time t, NAV_t, evolves as follows:

$$\text{NAV}_t = \text{NAV}_{t-1}\, e^{r_t - c}. \tag{3}$$

[9] See Brauer (1984, 1988) and Brickley and Schallheim (1985). These papers demonstrate very large gains are associated with managerial turnover, suggesting that large costs must be incurred in order to achieve the turnover, supporting the idea that most managers are entrenched

[10] In reality there are three ways to end a closed-end fund: (1) it can change to an open-end fund structure, (2) it can liquidate, or (3) it can merge with open-end fund. See Brauer (1984) for more details.

Consider a manager who begins managing the fund in period τ. Neither investors nor the manager know the value of α, the manager's skill level. They both have the same prior on α at time τ — normal with mean ϕ_τ and precision $\gamma = 1/\eta^2$. After observing the returns the manager generates, investors and managers alike update their priors. Let ϕ_t denote the mean of the posterior distribution at time t, that is, the expectation of α conditional on seeing all returns up to and including r_t, i.e.

$$\phi_t = E_t[\alpha]. \tag{4}$$

The evolution of the priors is governed by:

Proposition 1 *At time t, the posterior distribution of α is a normal distribution with mean ϕ_t and variance $\frac{1}{\gamma+(t-\tau)\omega}$, where ϕ_t is the solution to*

$$\phi_{t+1} = \phi_t + \frac{\omega}{\gamma + \omega(t-\tau+1)}(r_{t+1} - \hat{r}_{t+1} - \phi_t + \frac{1}{2\omega}). \tag{5}$$

Proof: Follows directly from DeGroot (1970) (Theorem 1, p. 167).

The manager generates an excess return equal to $r_{t+1} - \hat{r}_{t+1}$. By the result above, its mean and variance are given by

$$E_t[r_{t+1} - \hat{r}_{t+1}] = \phi_t - \frac{1}{2\omega}; \tag{6}$$

$$\text{var}_t[r_{t+1} - \hat{r}_{t+1}] = \frac{\gamma + (t-\tau+1)\omega}{\omega(\gamma + (t-\tau)\omega)} \equiv \Delta_t^2. \tag{7}$$

The conditional mean and variance of ϕ_{t+1} then follow directly:

$$E_t[\phi_{t+1}] = \phi_t; \tag{8}$$

$$\text{var}_t[\phi_{t+1}] = \frac{\omega}{[\gamma + \omega(t-\tau+1)]\,[\gamma + \omega(t-\tau)]} \equiv \Phi_t^2. \tag{9}$$

Although the manager cannot be fired, we do allow him or her to leave the fund. In reality, managers will quit (or successfully negotiate a pay raise) when they have a more lucrative outside offer. Rather than explicitly model the arrival of these offers, we will simply assume that if the manager's ability ϕ_t rises above some level $\bar{\phi}$, he is offered more lucrative employment elsewhere and quits. Clearly $\bar{\phi} \geq \phi_\tau$ (otherwise he would never choose to be a closed-end fund manager).

Assume the manager gets a fixed amount of capital to invest at time 0, and that no additional capital enters or leaves the fund until the open date, T, when the proceeds are distributed to the shareholders at net asset value. For convenience, assume that all dividends are reinvested in the fund. Competition between investors implies that, in equilibrium, they cannot earn an expected return greater than r by investing in the fund. Since they are rational, they will not accept an expected return lower than r, so their expected return from investing in the fund must always exactly equal r. Since the expected return the manager generates, after fees, will in general not equal r, the only way this can occur is if the price of the fund adjusts so that it is not equal to the NAV.

Write the price of the closed-end fund at time t as $P_t^\tau(\mathrm{NAV}_t, \phi_t)$, where the current manager started with the fund at time τ, and has current estimated ability ϕ_t. Define

$$D_t^\tau(\phi_t) \equiv \frac{P_t^\tau(\mathrm{NAV}_t, \phi_t)}{\mathrm{NAV}_t}, \tag{10}$$

the price of the fund expressed as a fraction of NAV. The *discount* is then $1 - D_t^\tau(\phi_t)$. At the open date, T, the assets are distributed to investors so

$$D_T^\tau(\phi_T) = 1 \quad \forall\ \phi_T. \tag{11}$$

If the fund were issued at a premium (because the fee charged is less than the value added by the manager), the manager would have an incentive to quit the fund once it was established and start a new fund, since part of his compensation is paid up front in the form of the premium. Consequently, investors would not bid the price up to the full capitalized value. This is costly to the manager, so one would expect him to set the fee at a level where the fund would *not* trade at a premium. Similarly, if the fund were issued at a discount (because the fee charged is more than the value added by the manager), the manager would in effect be choosing to defer compensation. Assuming managers are cash constrained (otherwise there would be no reason for them to market their skill to outside investors), this would not be a preferred outcome from their point of view, so one would expect the fund to be issued at NAV. We therefore assume that the fund is initially sold for its NAV:

$$D_0^0(\phi_0) = 1. \tag{12}$$

When $\phi_t \geq \bar{\phi}$ the old manager quits, a new manager is hired at the same fee, and the fund will again trade at its NAV. This condition pins down the ability level of the new manager hired at time τ, ϕ_τ, and implies that

$$D^\tau_\tau(\phi_\tau) = 1, \tag{13}$$

$$D^\tau_t(\bar{\phi}) = 1 \quad \tau < t \leq T. \tag{14}$$

The following proposition derives the price of the fund as a function of ϕ, the perceived quality of the manager, and the manager's tenure:

Proposition 2 *Let $D^\tau_t(\phi_t)$ denote the price as a fraction of NAV at time t for a fund with open date T, and a manager of perceived ability ϕ_t who started with the fund at date τ. Then $D^\tau_t(\phi_t)$ is not a function of r, and is given recursively by*

$$D^\tau_t(\phi_t) = \begin{cases} 1 & \text{if } \phi_t \geq \bar{\phi} \\ e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}} N\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right] + & \\ \quad \frac{1}{\Phi_t}\int_{-\infty}^{\bar{\phi}} D^\tau_{t+1}(\phi) e^{\frac{\gamma+(t-\tau+1)\omega}{\omega}(\phi-\phi_t)+\phi_t-\frac{1}{2\omega}-c}\, n\left(\frac{\phi-\phi_t}{\Phi_t}\right)\, d\phi & o.w. \end{cases} \tag{15}$$

subject to the boundary condition that $D^\tau_T(\phi) = 1$, where Δ_t and Φ_t are as defined in Equations (7) and (9), and where $N[\cdot]$ and $n(\cdot)$ are the standard Normal cumulative distribution and density functions respectively.

Proof: See Appendix B.1.

When $\bar{\phi} = \infty$ (i.e. the manager cannot leave the fund), this simplifies to

$$D^\tau_t(\phi_t) = e^{\left(\phi_t - c + \frac{T-t}{2(\gamma+(t-\tau)\omega)}\right)(T-t)}. \tag{16}$$

4 Empirical Implications of the Model

This section calibrates the model to the data using results from prior research, and shows that it generates the four empirical regularities listed by Lee, Shleifer, and Thaler (1990), with magnitudes comparable to those observed by previous researchers. Our objective in this section is not to argue that the model can explain every detail of the closed-end fund puzzle, but rather to show that the four most important features of the closed-end fund discount can be produced by a parsimonious rational model.

4.1 Calibration

We begin with ω, the precision of the difference between the return of the fund and the return of the underlying assets held at the beginning of the period. Using US data (1965–85) Pontiff (1997) reports a mean monthly variance across funds of the percentage difference between these two returns equal to 37.33. Adams (2000) finds a value of 11.50 for UK funds (1982–96). As Adams (2000) points out, Pontiff's data cover a particularly volatile period. More recently the US has been much closer to the U.K. experience. Furthermore, the distribution of funds' volatilities is highly skewed — Pontiff reports a median of only 19.62. In light of this we set $\omega = 33$. Expressed on a monthly basis for percentage returns, this translates into a variance of about 25, somewhat less than Pontiff's mean (but still larger than his median).

Lee, Shleifer, and Thaler (1990) report that management fees typically range between 0.5% and 2% per year. We use 1%. Since we do not endogenously model management departure, we must pick an open end date. We choose 10 years.

The remaining three parameters, the prior mean, ϕ_0, and precision, γ, and the level at which the manager quits, $\bar{\phi}$, are not directly observable. Instead we infer their values by experimenting with parameters that would give realistic values for the average discount and maximum premium. Table 1 summarizes the final choice of parameters. Note that these

Variable	Symbol	Value
Percentage fee	c	1%
Years to Open Date	T	10
Mean of prior	ϕ_0	6.2%
Prior precision	γ	42
Return precision	ω	33
Exit mean	$\bar{\phi}$	6.5%

Table 1: **Parameter Values**

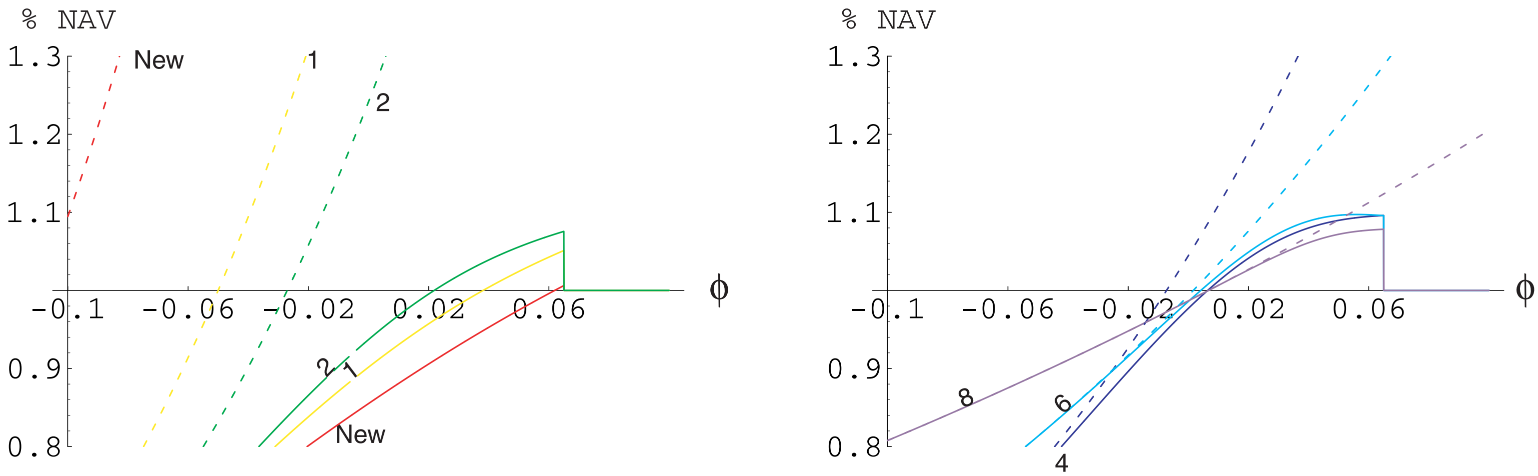
values of ϕ_0 and γ imply, at least initially, that 63% of managers have enough skill to make back what they charge in fees, and since the average manager initially has an α of 6.2% but only charges a 1% fee, his initial excess returns are substantially higher than the fees he charges.

4.2 Discount vs. Ability

Equation (15) can be used recursively to calculate the discount as a function of ability for a fund of any age. Figure 1 plots D_t^τ against ability, ϕ_t, for new managers and for managers with tenures 1, 2, 4, 6, and 8 years. The fund starts with an open date 10 years in the future.

The solid lines are for managers who leave when ϕ is equal to $\bar{\phi} = 0.065$, and the dotted lines show the value of D_t^τ given in Equation (16) for an otherwise identical fund, except that $\bar{\phi} = \infty$ (i.e. the manager cannot leave the fund).

Figure 1: **Discount as a Function of Ability:** The plots show the price of the fund, expressed as % NAV, for funds with an open date of 10 years as a function of perceived management ability(ϕ), color coded by management tenure. Red, yellow, green, blue, cyan and violet correspond to a new manager and managers with tenures of 1, 2, 4, 6 and 8 years respectively. (The lines are also marked for readers without access to color.) The parameter values are given in Table 1. The solid lines are for a manager with $\bar{\phi} = 0.065$, and the dotted lines show the discount for an otherwise identical fund with $\bar{\phi} = \infty$.



As the plots show, for low value of ϕ, the relation between ability and the discount is intuitive — a positive change in ability translates into higher expected returns which lead to smaller discounts (D increases). This relation is documented empirically by Chay and Trzcinka (1999). However, at high enough levels of ϕ, the discount and ability are *inversely* related, so an increase in ability translates into an increase in the discount (or, equivalently, a decrease in the premium or a lower value of D). The reason for this behavior is that greater ability implies a higher likelihood that the manager will quit, either now or in the near future (when ϕ hits $\bar{\phi}$), to be replaced with a manager of lower ability. Note that the nonmonotonicity does not only occur at the point $\phi = \bar{\phi}$. For example, for a tenure of 6 years, the graph slopes downwards before the jump. This is because, in this region, as ability increases, even though it is not high enough to cause the manager to leave immediately, it makes it more likely that he will leave in the near future.

The fact that there is a region where $D(\cdot)$ is decreasing is interesting because it potentially could contribute to an empirical phenomenon past researchers have found puzzling.

Lee, Shleifer, and Thaler (1991) and Ross (2002b) find that the empirical relation between past performance and the discount is negative — when funds outperform the market, the discount subsequently increases. As Figure 1 makes clear, good current performance need not translate into future outperformance if the result is that the manager gets an outside offer of employment. Although not part of our model, extremely bad performance might not translate into future underperformance if the resulting update on managerial ability is enough to motivate dissatisfied shareholders to mount a proxy fight to terminate the fund. Potentially, these two effects might explain the empirical puzzle, although evaluating their importance is clearly beyond the scope of the current paper.

Figure 2 shows the initial ability of a newly hired manager as a function of the number of years until the fund's open date. The first manager hired (when the open date is 10 years in the future) has an initial ability of 6.2%. Note that this value is well in excess of the fee (1%). The reason for this is that the long term labor contract offers insurance. The cost of this insurance can be inferred from the dotted line, which shows the ability level of a manager who can never quit the fund.[11] The large difference between the two values (6.9% for the first manager) reflects the value of the insurance. Investors demand much higher qualified managers knowing that the bad ones will become entrenched. This difference shrinks as the number of years to the open date decreases, as Figure 2 shows. When the manager cannot quit, the required initial ability of the manager rises because the benefit derived from the skewness of the log normal distribution is reduced. Concomitantly, the length of the insurance contract decreases, so the required initial ability of a manager who is free to quit decreases. Consequently the two curves in Figure 2 converge to each other.

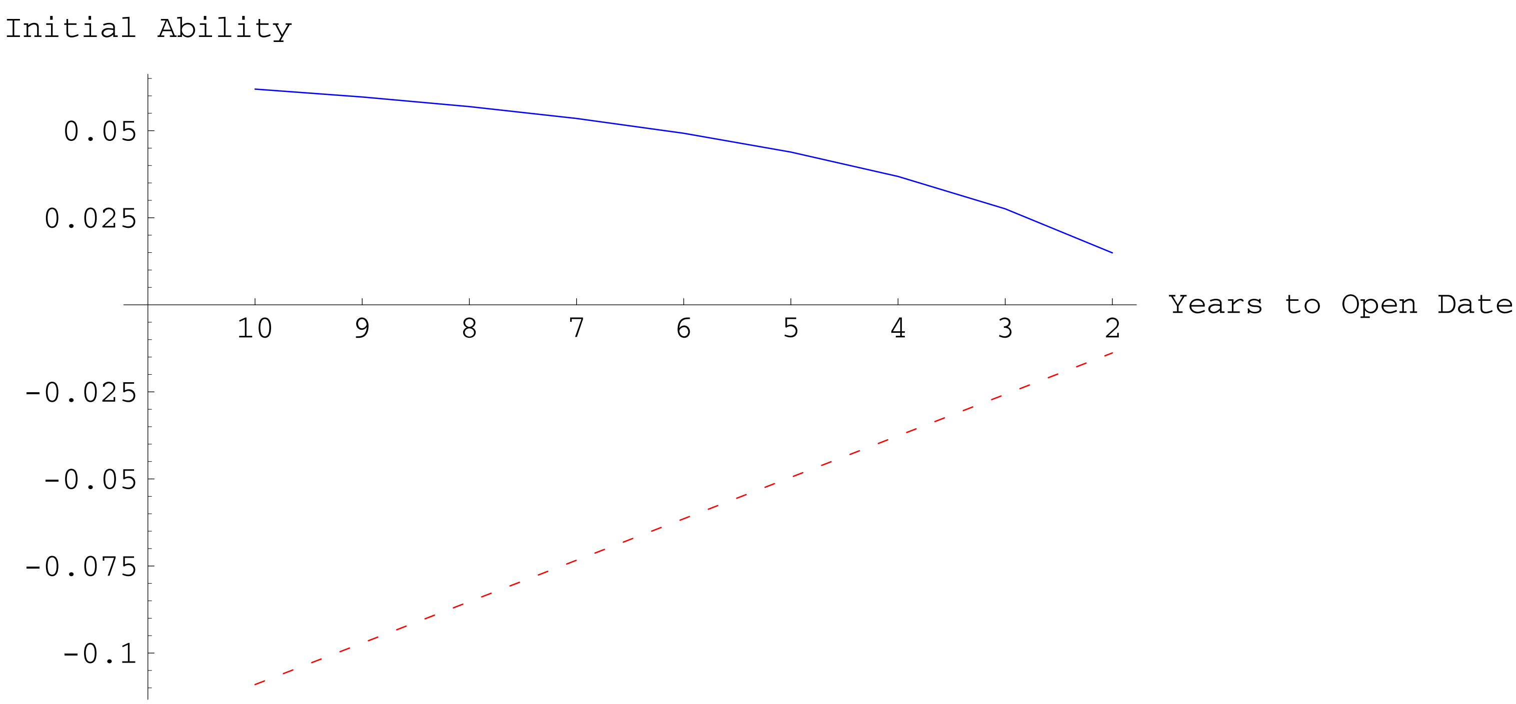
4.3 Evolution of the Average Discount

Three of the four empirical regularities listed by Lee, Shleifer, and Thaler (1990) concern the behavior of the discount of the average fund. In this section we will derive investors' expectations of this behavior within our model, and show that they expect to see exactly the regularities observed in practice.

To compute the expected discount at some point in the future, we will compute the distribution of managerial ability at that time, and then use the relation between the discount and managerial ability derived in Section 3. The distribution of future managerial ability is determined both by investors updating on performance and by managerial turnover. First

[11]Investors are willing to hire such a low quality manager because of the skewness of the lognormal distribution. Since the manager cannot quit, the disproportionate gain if the manager turns out to be good, makes up for the fact that investors expect the average manager to destroy value.

Figure 2: **Initial Ability:** The solid (blue) line shows the initial ability of the manager as a function of the number of years to the open date for a manager who can quit the fund when $\phi > \bar{\phi} = 0.065$. The dashed (red) line is the initial ability of a manager who can commit to remaining with the fund until the open date (i.e., $\bar{\phi} = \infty$). The other parameter values are given in Table 1.



define

$$R_t \equiv \left(\frac{\omega}{\gamma + (t-\tau)\omega}\right)(\alpha - \phi_{t-1} + \epsilon_t),$$

so that

$$\phi_t = \phi_{t-1} + R_t.$$

Note that R_t is normally distributed with mean 0 and variance Φ^2_{t-1}. Assume the current manager was hired at time τ with skill level ϕ_τ given by the solution to (13).

Let $f_t^{\phi_\tau,\tau}(\phi)$ denote the p.d.f. of ϕ_t for a manager who starts with a fund at time $\tau < t$, conditional on not having left the fund prior to t, where the fund has open date T, and the manager has initial prior expected ability ability ϕ_τ. Let $F_t^{\phi_\tau,\tau}(\phi)$ be the corresponding c.d.f., i.e.

$$\begin{aligned} F_t^{\phi_\tau,\tau}(\phi) &\equiv P[(\phi_t < \phi) \mid (\phi_s < \bar{\phi} \text{ s.t. } s = \tau+1, \ldots, t-1)], \\ f_t^{\phi_\tau,\tau}(\phi) &\equiv \frac{dF_t^{\phi_\tau,\tau}(\phi)}{d\phi}. \end{aligned}$$

It will often be simpler to work with the improper p.d.f. and c.d.f.,

$$\begin{aligned} \tilde{F}_t^{\phi_\tau,\tau}(\phi) &\equiv P[(\phi_t < \phi) \cap (\phi_s < \bar{\phi} \text{ s.t. } s = \tau+1, \ldots, t-1)], \\ \tilde{f}_t^{\phi_\tau,\tau}(\phi) &\equiv \frac{d\tilde{F}_t^{\phi_\tau,\tau}(\phi)}{d\phi}. \end{aligned}$$

Lemma 1 in Appendix A provides expressions relating the proper and improper forms, and Lemma 2 derives explicit representations for $f_t^{\phi_\tau,\tau}(\phi)$ and $\tilde{f}_t^{\phi_\tau,\tau}(\phi)$. The next proposition derives the expected discount.

Proposition 3 *The expected discount, at time t, of a fund that started at time 0 with open date T is*

$$\sum_{\tau=0}^{t-1} H_\tau \left[\int_{-\infty}^{\bar{\phi}} \tilde{f}_t^{\phi_\tau,\tau}(\phi)\, D_t^\tau(\phi)\, d\phi + G_t^{\phi_\tau,\tau}\right], \tag{17}$$

where $G_t^{\phi_\tau,\tau} \equiv \int_{\bar{\phi}}^{\infty} \tilde{f}_t^{\phi_\tau,\tau}(\phi)\, d\phi$ is the probability that a manager who starts with a fund at time τ leaves the fund at time t, and where H_t is the probability that a fund that starts at time zero with open date T will have a manager leave (equivalently, have a manager start) at time t. H_t is defined recursively by

$$H_t = \sum_{\tau=0}^{t-1} H_\tau G_{\tau,t}^{\phi_\tau},$$

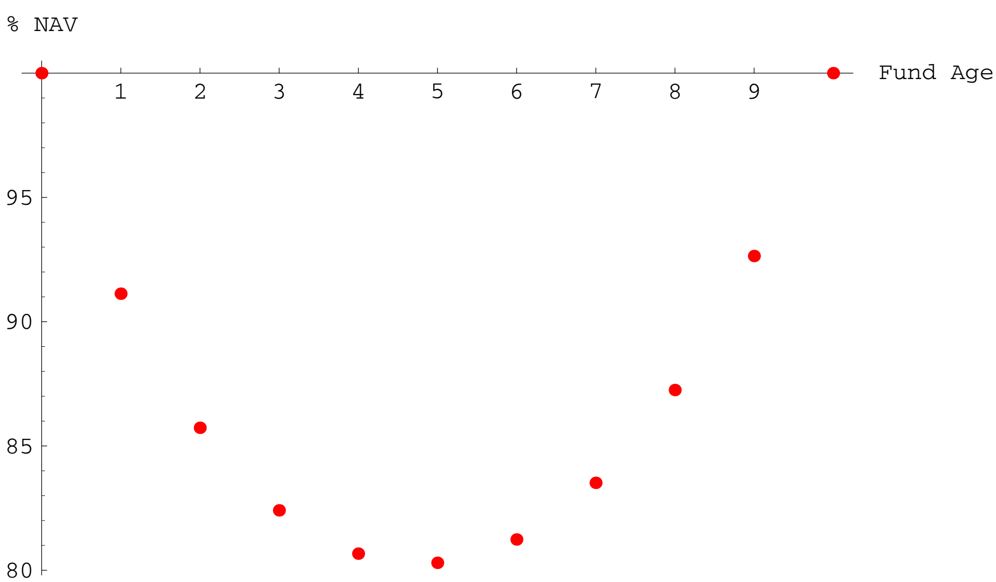
with the boundary condition $H_0 = 1$.

Proof: See Appendix B.2.

Figure 3 plots the expected value of the price of the fund, expressed as % NAV, as a function of the age of the fund. The fund is issued at par with an open-end date 10 years in the future. Initially, the expected discount increases over time, since the expected quality of the manager declines — good managers leave, while bad managers become entrenched. The total capitalized value dissipated by a poor manager depends how long the manager will be with the fund. As we approach the open date, even a very poor manager does not have time to hurt investors much, and the price must converge back to NAV.

The plot reproduces three of the four empirical regularities enumerated by Lee, Shleifer, and Thaler (1990): The fund is issued at par and is expected to fall into discount; it is expected always to trade at a discount; and this discount is expected to disappear as the fund approaches its open-date.

Figure 3: **Expected Discount as a Function of Fund Age** The plot shows the price of the fund, expressed as % NAV, as a function of fund age. The parameter values are given in Table 1.



The overall level of the discount is similar to that observed in the data. Lee, Shleifer, and Thaler (1991) analyze US funds between 1965 and 1985, and report that discounts towards the end of this period tend to be between 10% and 20%. From 1980–98, the average discount on US stock funds varied from about 0–20%,[12] and discounts at the end of the 1990s were around 10% in the UK and 5% in the US. Anderson and Born (2002) report that in February

[12] See Dimson and Minio-Kozerski (1999).

2001 the average discount for all equity funds (worldwide) was 10.9%.

Note also that the expected discount grows quite rapidly — the price drops to 91% of NAV after only 1 year. This fast initial decline is also observed in practice. For example, Weiss (1989) finds that after just twelve weeks the average discount for US stock funds in her sample is just over 2%, and after 24 weeks it is 10%.

4.4 Distribution of the Discount

Only one of the four regularities listed by Lee, Shleifer, and Thaler (1990) remains to be explained — the wide dispersion in discounts across funds. In this section we will investigate the model's implications for the cross-sectional distribution of discounts.

To compute the distribution of the discount at some point in the future, we will combine the distribution of managerial ability, $f_t^{\phi_\tau,\tau}(\phi)$, calculated in Section 4.3 with the relation between the discount and ability, $D_t^\tau(\phi)$, calculated in Section 3. This is relatively straightforward to do, with one caveat. $D_t^\tau(\phi)$ is not necessarily monotonic, so it does not have a unique inverse. However, as Figure 1 illustrates, we can split the range of ϕ into two subsets, on each of which $D_t^\tau(\phi)$ is monotonic. Let $\hat{\phi}$ be the value of ϕ at which $D_t^\tau(\phi)$ reaches a maximum, let $\phi_t^\tau(D)$ denote the inverse of $D_t^\tau(\phi_t)$ over the region $(-\infty, \hat{\phi}]$, and let $\hat{\phi}_t^\tau(D)$ denote the inverse of $D_t^\tau(\phi_t)$ over the region $(\hat{\phi}, \bar{\phi})$. The following proposition then derives the distribution of the discount as a function of management tenure. The proof is straightforward and left to the reader.

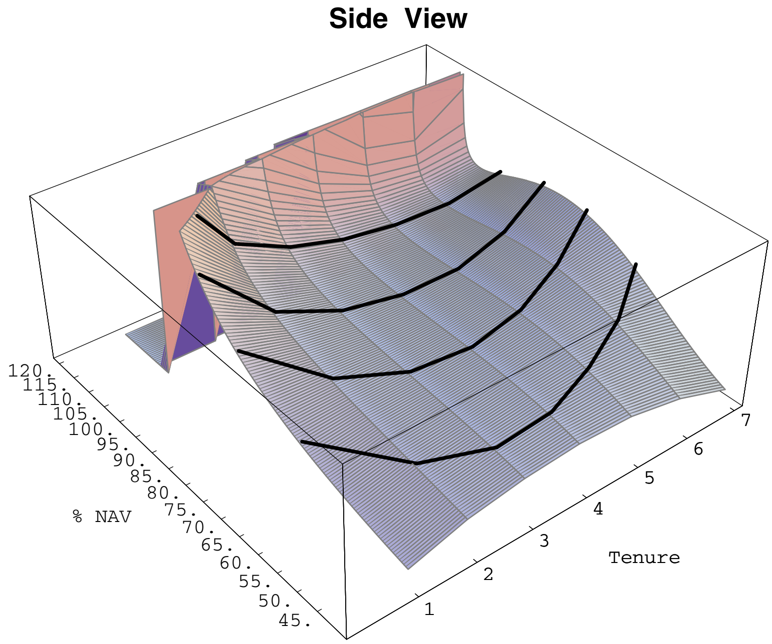
Proposition 4 *Let $g_t^\tau(D)$ be the p.d.f. of D, the price of the fund expressed as a fraction of net asset value, for a fund at time t with a manager that started at time τ. Then*

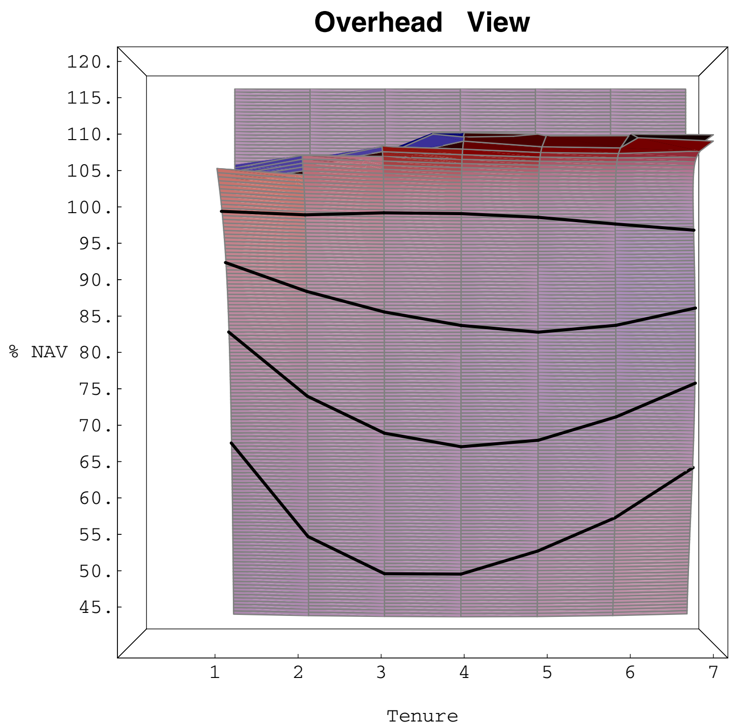
$$g_t^\tau(D) = \begin{cases} f_t^{\phi_\tau,\tau}(\phi_t^\tau(D)) \, \frac{d}{dD}\phi_t^\tau(D) & -\infty < D \leq D_t^\tau(\bar{\phi}) \\ f_t^{\phi_\tau,\tau}(\phi_t^\tau(D)) \, \frac{d}{dD}\phi_t^\tau(D) - f_t^{\phi_\tau,\tau}(\hat{\phi}_t^\tau(D)) \, \frac{d}{dD}\hat{\phi}_t^\tau(D) & D_t^\tau(\bar{\phi}) < D < D_t^\tau(\hat{\phi}) \end{cases} .$$

Figure 4 plots g using the parameters in Table 1. The four contours that are shown mark the 0.2, 0.4, 0.6 and 0.8 levels on the corresponding c.d.f. That is, taking the lower plot in Figure 4, the lower black line denotes the 0.2 contour — the probability that a discount greater than the point on the line will be observed is 20%. The model generates a wide dispersion in discounts. For example for a manager who has been at the fund for 4 years, the probability that the discount will be over 55% (the price, expressed at a % of NAV being less than 45%) is over 20%. Yet there is still a 20% probability of the fund trading at a premium.

Significant dispersion in funds' discounts occurs very early in practice. For example, Weiss (1989) finds that after 24 weeks, while the average discount of the 22 US stock funds

Figure 4: **Distribution of the Discount** The plots show, as a function of how long the manager has been with the fund (tenure), the p.d.f. of price of the fund expressed at a percentage of NAV (% NAV). The upper plot is a view of the surface from the side. The lower plot is a view of the same surface from above. The black lines mark the 0.2, 0.4, 0.6 and 0.8 contours of the c.d.f. That is, in the upper plot the right most line is the 0.2 contour, so 20% of the probability mass lies to the right of this line.





she studies is 10%, two of the funds still trade at a premium. Figure 4 shows that 20% of funds trade at a premium at the end of the first year. In addition, Thompson (1978) notes that discounts in excess of "20% are quite frequent, with discounts exceeding 30 and 40% not uncommon," consistent with our model.

4.5 NAV return

Lee, Shleifer, and Thaler (1990) point out that in order for a model with differential management ability to generate the observed time series behavior of average discounts, investors must systematically expect management performance to decline over time. Our model predicts exactly this behavior.

Investors' expectations of managerial ability can be measured by the excess expected return on the fund's NAV. Let

$$R_{t+1}^{NAV} \equiv \frac{\text{NAV}_{t+1}}{\text{NAV}_t} = e^{r_{t+1}-c} \tag{18}$$

be the (gross) realized NAV return. Conditional on information available at time t, and using standard properties of the lognormal distribution, Equations (6) and (7) tell us that the expected time-t NAV return for the first manager of the fund is

$$E_t[R_{t+1}^{NAV}] = e^{r+\phi_t-c+\frac{1}{2(\gamma+t\omega)}}. \tag{19}$$

This is higher than the fair return if

$$e^{r+\phi_t-c+\frac{1}{2(\gamma+t\omega)}} \quad > \quad e^r, \tag{20}$$

that is, if

$$\phi_t - c + \frac{1}{2(\gamma+t\omega)} \quad > \quad 0.$$

Therefore define the continuously compounded excess NAV return as

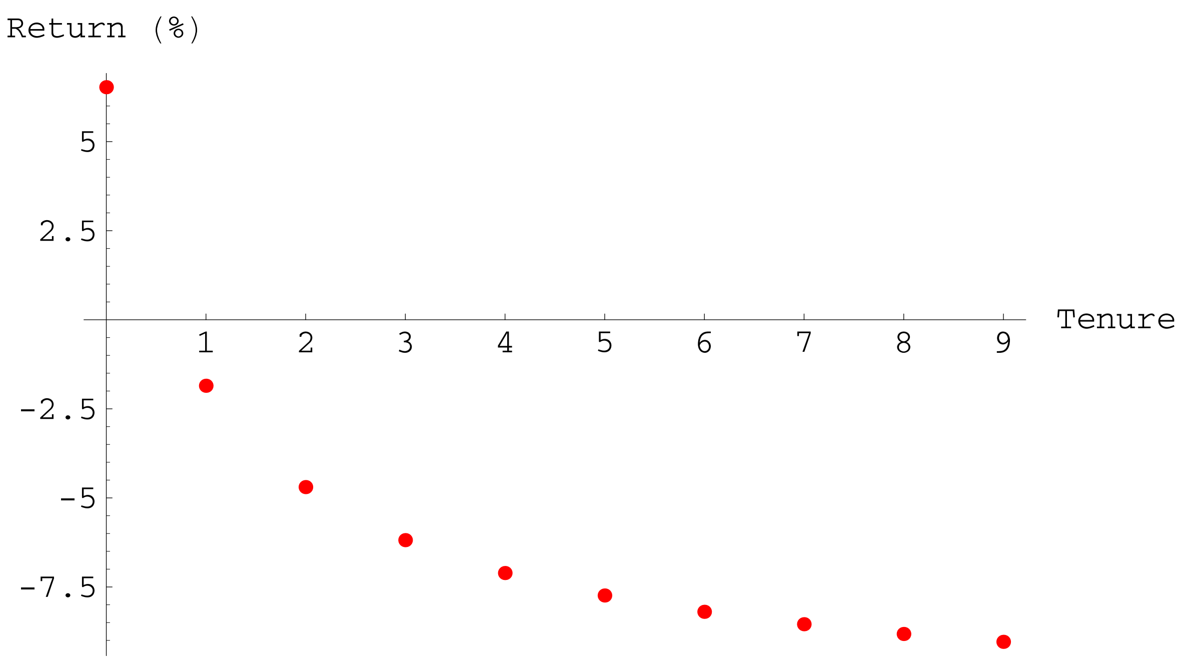
$$\mu_t \equiv \phi_t - c + \frac{1}{2(\gamma+t\omega)}. \tag{21}$$

Investors' expectations of the future performance of the fund's initial manager are given by

$$E_0[\mu_t \mid \phi_1, \ldots, \phi_t < \bar{\phi}] = \int_{-\infty}^{\bar{\phi}} \left(\phi - c + \frac{1}{2(\gamma+t\omega)}\right) f_t^{\phi_\tau,\tau}(\phi)\, d\phi. \tag{22}$$

Figure 5 plots this expectation as a function of managerial tenure, t. The parameters used are given in Table 1. The NAV return is initially expected to outperform the benchmark (the return on the assets held at the beginning of the period), but as the tenure of the manager increases, it is expected to underperform. With our calibrated parameters, this deterioration in performance is very rapid – underperformance occurs by the second year of the manager's tenure. This is because, at these parameter values, investors learn about managerial ability very quickly. Hence, most good managers leave early, accounting for the rapid decline in expected NAV returns.

Figure 5: **Expected NAV Excess Return as a Function of Managerial Tenure:** The plot shows the one period expected NAV risk adjusted return, $E_0[\mu_t \mid \phi_1, \ldots, \phi_t < \bar{\phi}]$ for funds with an open date of 10 years as a function of management tenure (t). The parameter values are given in Table 1.



5 Conclusions

Lee, Shleifer, and Thaler (1990) summarize the finance profession's understanding of the closed-end fund puzzle by listing four main empirical regularities exhibited by the discount on these funds. In this paper we show that all four of these empirical regularities can be generated by a parsimonious rational model.

Appendix

A Lemmas

Lemma 1

$$\tilde{F}_t^{\phi_\tau,\tau}(\phi) = F_t^{\phi_\tau,\tau}(\phi) \prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}), \tag{23}$$

$$\tilde{f}_t^{\phi_\tau,\tau}(\phi) = f_t^{\phi_\tau,\tau}(\phi) \prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}). \tag{24}$$

Proof: Equation (23) follows immediately from writing the joint probability as the product of conditional probabilities. Equation (24) then follows by differentiating Equation (23).

Lemma 2 $f_t^{\phi_\tau,\tau}(\phi)$ *and* $\tilde{f}_t^{\phi_\tau,\tau}(\phi)$ *are defined recursively as follows:*

$$f_t^{\phi_\tau,\tau}(\phi) = \frac{\int_{-\infty}^{\bar{\phi}} n\left(\frac{\phi-x}{\Phi_{t-1}}\right) f_{t-1}^{\phi_\tau,\tau}(x)\,dx}{\Phi_{t-1} F_{t-1}^{\phi_\tau,\tau}(\bar{\phi})}, \tag{25}$$

$$\tilde{f}_t^{\phi_\tau,\tau}(\phi) = \frac{\int_{-\infty}^{\bar{\phi}} n\left(\frac{\phi-x}{\Phi_{t-1}}\right) \tilde{f}_{t-1}^{\phi_\tau,\tau}(x)\,dx}{\Phi_{t-1}}, \tag{26}$$

subject to the boundary condition

$$f_{\tau+1}^{\phi_\tau,\tau}(\phi) = \tilde{f}_{\tau+1}^{\phi_\tau,\tau}(\phi) = \frac{1}{\Phi_\tau} n\left(\frac{\phi-\phi_\tau}{\Phi_\tau}\right),$$

where $n(\cdot)$ *is a standard normal density function,*

$$n(u) = \frac{1}{\sqrt{2\,\pi}} e^{-\frac{1}{2}u^2}.$$

Proof:

$$\begin{aligned} \tilde{F}_t^{\phi_\tau,\tau}(\phi) = P[(\phi_t < \phi) \cap (\phi_{t-1} < \bar{\phi}) \cap (\text{all prior } \phi_s < \bar{\phi})] &= \\ \int_{-\infty}^{\bar{\phi}} P[\phi_t < \phi \mid \phi_{t-1} = x]\, \tilde{f}_{t-1}^{\phi_\tau,\tau}(x)\,dx &= \\ \int_{-\infty}^{\bar{\phi}} N\left(\frac{\phi - x}{\Phi_{t-1}}\right) \tilde{f}_{t-1}^{\phi_\tau,\tau}(x)\,dx. \end{aligned}$$

Differentiating this with respect to ϕ yields Equation (26), and this in conjunction with Equations (24) yields Equation (25). Finally, Equations (8) and (9) tell us immediately that

$$f_{\tau+1}^{\phi_\tau,\tau}(\phi) = \tilde{f}_{\tau+1}^{\phi_\tau,\tau}(\phi) = \frac{1}{\Phi_\tau} n\left(\frac{\phi - \phi_\tau}{\Phi_\tau}\right).$$

B Proofs

B.1 Proof of Proposition 2

First, from Equation 5 it is simple to show that $\phi_{t+1} = \bar{\phi}$ when $r_{t+1} = r_{t+1}^*(\phi_t)$, defined by

$$r_{t+1}^*(\phi_t) \equiv (\bar{\phi} - \phi_t)\frac{\gamma + (t - \tau + 1)\omega}{\omega} + \hat{r}_{t+1} + \phi_t - \frac{1}{2\omega}. \tag{27}$$

Thus, whenever $r_{t+1} \geq r_{t+1}^*(\phi_t)$, the fund will trade for NAV_{t+1} in period $t+1$.

Competitive investors must all earn the fair rate of return, r, in equilibrium, so the price of the fund today is the expected value of the price tomorrow, discounted at r. This can be split into two pieces, as

$$\begin{aligned} P_t^\tau(\mathrm{NAV}_t, \phi_t) \quad = \quad & E_t\left[\mathrm{NAV}_{t+1}e^{-r}|r_{t+1} > r_{t+1}^*(\phi_t)\right]\ P\left[r_{t+1} > r_{t+1}^*(\phi_t)\right] + \\ & E_t\left[P_{t+1}^\tau(\mathrm{NAV}_{t+1}, \phi_{t+1})e^{-r}|r_{t+1} \leq r_{t+1}^*(\phi_t)\right]\ P\left[r_{t+1} \leq r_{t+1}^*(\phi_t)\right]. \end{aligned} \tag{28}$$

Now,

$$\begin{aligned} P\left[r_{t+1} \leq r_{t+1}^*(\phi_t)\right] &= P\left[\phi_{t+1} \leq \bar{\phi}\right] \\ &= \mathrm{N}\left[\frac{(\bar{\phi} - \phi_t)}{\Phi_t}\right], \end{aligned} \tag{29}$$

from Equations (8) and (9). Similarly,

$$P\left[r_{t+1} > r_{t+1}^*(\phi_t)\right] = 1 - \mathrm{N}\left[\frac{(\bar{\phi} - \phi_t)}{\Phi_t}\right] = \mathrm{N}\left[-\frac{(\bar{\phi} - \phi_t)}{\Phi_t}\right]. \tag{30}$$

From Equation (3) we have

$$
\begin{aligned}
& E_t\left[\mathrm{NAV}_{t+1}e^{-r}|r_{t+1} > r^*_{t+1}(\phi_t)\right]\ P\left[r_{t+1} > r^*_{t+1}(\phi_t)\right] \\
&= E_t\left[\mathrm{NAV}_t e^{r_{t+1}-c-r}|r_{t+1} > r^*_{t+1}(\phi_t)\right]\ P\left[r_{t+1} > r^*_{t+1}(\phi_t)\right], \\
&= \mathrm{NAV}_t e^{-c-r} E_t\left[e^{r_{t+1}-\hat{r}_{t+1}} e^{\hat{r}_{t+1}}|r_{t+1} > r^*_{t+1}(\phi_t)\right]\ P\left[r_{t+1} > r^*_{t+1}(\phi_t)\right], \\
&= \mathrm{NAV}_t e^{-c-r} E_t\left[e^{\hat{r}_{t+1}}|r_{t+1} > r^*_{t+1}(\phi_t)\right] E_t\left[e^{r_{t+1}-\hat{r}_{t+1}}|r_{t+1} > r^*_{t+1}(\phi_t)\right] P\left[r_{t+1} > r^*_{t+1}(\phi_t)\right], \\
&= \mathrm{NAV}_t e^{-c} E_t\left[e^{r_{t+1}-\hat{r}_{t+1}}|r_{t+1} > r^*_{t+1}(\phi_t)\right] P\left[r_{t+1} > r^*_{t+1}(\phi_t)\right], \\
&= \mathrm{NAV}_t e^{-c} E_t\left[e^{r_{t+1}-\hat{r}_{t+1}}|(r_{t+1}-\hat{r}_{t+1}) > (\bar{\phi}-\phi_t)\frac{\gamma+(t-\tau+1)\omega}{\omega} + \phi_t - \frac{1}{2\omega}\right] P\left[r_{t+1} > r^*_{t+1}(\phi_t)\right], \\
&= \mathrm{NAV}_t e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}}\ \mathrm{N}\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right], \qquad (31)
\end{aligned}
$$

where the fourth line follows from the independence of ξ_t and ϵ_t, and the last line follows from a well-known property of the lognormal distribution [see (Ingersoll, 1987, p.14)] and the last line uses (27). Substituting (29), (30) and (31) into (28) yields

$$
\begin{aligned}
P_t(\mathrm{NAV}_t, \phi_t) &= \mathrm{NAV}_t\, e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}}\ \mathrm{N}\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right]\ + \qquad (32) \\
& \quad E_t\left[P_{t+1}(\phi_{t+1})e^{-r}|r_{t+1} \leq r^*_{t+1}(\phi_t)\right]\ \mathrm{N}\left[\frac{(\bar{\phi}-\phi_t)}{\Phi_t}\right].
\end{aligned}
$$

Dividing by NAV_t gives

$$
\begin{aligned}
D_t^{\tau}(\phi_t) &= e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}}\ \mathrm{N}\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right]\ + \\
&\qquad E_t\left[\frac{P_{t+1}(\phi_{t+1})}{\text{NAV}_t} e^{-r} | r_{t+1} \leq r^*_{t+1}(\phi_t)\right]\ \mathrm{N}\left[\frac{(\bar{\phi}-\phi_t)}{\Phi_t}\right], \\
&= e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}}\ \mathrm{N}\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right]\ + \\
&\qquad E_t\left[D^T_{t+1}(\phi_{t+1}) e^{r_{t+1}-r-c} | \phi_{t+1} \leq \bar{\phi}\right]\ \mathrm{N}\left[\frac{(\bar{\phi}-\phi_t)}{\Phi_t}\right], \\
&= e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}}\ \mathrm{N}\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right]\ + \\
&\qquad E_t\left[D^T_{t+1}(\phi_{t+1}) e^{r_{t+1}-\hat{r}_{t+1}-c} e^{\hat{r}_{t+1}-r} | \phi_{t+1} \leq \bar{\phi}\right]\ \mathrm{N}\left[\frac{(\bar{\phi}-\phi_t)}{\Phi_t}\right], \\
&= e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}}\ \mathrm{N}\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right]\ + \\
&\qquad E_t\left[e^{\hat{r}_{t+1}-r} | \phi_{t+1} \leq \bar{\phi}\right] E_t\left[D^T_{t+1}(\phi_{t+1}) e^{r_{t+1}-\hat{r}_{t+1}-c} | \phi_{t+1} \leq \bar{\phi}\right]\ \mathrm{N}\left[\frac{(\bar{\phi}-\phi_t)}{\Phi_t}\right], \\
&= e^{\phi_t - c + \frac{1}{2(\gamma+(t-\tau)\omega)}}\ \mathrm{N}\left[-\frac{(\bar{\phi}-\phi_t)}{\Phi_t} + \Delta_t\right]\ + \\
&\qquad \frac{1}{\Phi_t}\int_{-\infty}^{\bar{\phi}} D^{\tau}_{t+1}(\phi)\, e^{\frac{\gamma+(t-\tau+1)\omega}{\omega}(\phi-\phi_t)+\phi_t-\frac{1}{2\omega}-c}\, n\!\left(\frac{\phi-\phi_t}{\Phi_t}\right)\, d\phi,
\end{aligned}
\tag{33}
$$

the last line following from Equation (27). Independence of r follows by inspection.

B.2 Proof of Proposition 3

Let $G_t^{\phi_\tau,\tau}$ be the probability that a manager who starts with a fund at time τ leaves the fund at time t. Then, from the definition of $\tilde{f}_t^{\phi_\tau,\tau}(\phi)$,

$$G_t^{\phi_\tau,\tau} = \int_{\bar{\phi}}^{\infty} \tilde{f}_t^{\phi_\tau,\tau}(\phi)\, d\phi.$$

From Lemma 1 in Appendix A,

$$
\begin{aligned}
\int_{\bar{\phi}}^{\infty} \tilde{f}_t^{\phi_\tau,\tau}(\phi)\, d\phi, &= \int_{\bar{\phi}}^{\infty} f_t^{\phi_\tau,\tau}(\phi)\, d\phi \prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}) \\
&= \left(1 - F_t^{\phi_\tau,\tau}(\bar{\phi})\right) \prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}).
\end{aligned}
$$

Hence

$$G_t^{\phi_\tau,\tau} = \left(1 - F_t^{\phi_\tau,\tau}(\bar{\phi})\right) \prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}).$$

Now let H_t denote the probability that a fund that starts at time zero with open date T will have a manager leave (equivalently, have a manager start) at time t. Then

$$\begin{aligned}
H_t &= \mathrm{P}[\text{Manager leaves at } t] \\
&= \mathrm{P}[\text{Manager leaves at } t|\text{started at } 0]\mathrm{P}[\text{started at } 0] \\
&\quad +\mathrm{P}[\text{Manager leaves at } t|\text{started at } 1]\mathrm{P}[\text{started at } 1] + \dots \\
&\quad +\mathrm{P}[\text{Manager leaves at } t|\text{started at } t-1]\mathrm{P}[\text{started at } t-1] \\
&= G_{0,t}^{\phi_0} 1 + G_{1,t}^{\phi_1} H_1 + \cdots + G_{t-1,t}^{\phi_t} H_{t-1}
\end{aligned}$$

I.e.

$$H_t = \sum_{\tau=0}^{t-1} H_\tau G_{\tau,t}^{\phi_\tau} \tag{34}$$

with the boundary condition $H_0 = 1$. Now to calculate the expected discount:

$$\begin{aligned}
E_0(D_t) &= E_0(D_t \mid \text{Last new manager started @ } 0) P[\text{Last new manager started @ } 0]) \\
&\quad + E_0(D_t \mid \text{Last new manager started @ } 1) P[\text{Last new manager started @ } 1]) + \dots \\
&\quad + E_0(D_t \mid \text{Last new manager started @ } t-1) P[\text{Last new manager started @ } t-1]).
\end{aligned}$$

Taking one of these terms, we have

$$E_0(D_t \mid \text{Last new manager started @ } \tau) = \int_{-\infty}^{\bar{\phi}} f_t^{\phi_\tau,\tau}(\phi)\, D_t^\tau(\phi)\, d\phi + \left(1 - F_t^{\phi_\tau,\tau}(\bar{\phi})\right), \tag{35}$$

where the integral is the contribution to the expectation if the manager does not leave in period t (i.e., if $\phi_t < \bar{\phi}$), and the second term accounts for the possibility that the manager will leave in period t, in which case the discount will automatically reset to 1. Also,

$$\begin{aligned}
P[\text{Last new manager started @ } \tau] &= P[(\text{No manager left between } \tau+1 \text{ and } t-1) \cap \\
&\qquad (\text{Manager started @ } \tau)], \\
&= H_\tau \prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}). \qquad (36)
\end{aligned}$$

Combining these results and summing across τ,

$$\begin{aligned} E_0(D_t) &= \sum_{\tau=0}^{t-1} H_\tau \left[\left(\prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}) \int_{-\infty}^{\bar{\phi}} f_t^{\phi_\tau,\tau}(\phi)\, D_t^\tau(\phi)\, d\phi \right) + \left(1 - F_t^{\phi_\tau,\tau}(\bar{\phi})\right) \prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}) \right] \\ &= \sum_{\tau=0}^{t-1} H_\tau \left[\left(\prod_{s=\tau+1}^{t-1} F_s^{\phi_\tau,\tau}(\bar{\phi}) \int_{-\infty}^{\bar{\phi}} f_t^{\phi_\tau,\tau}(\phi)\, D_t^\tau(\phi)\, d\phi \right) + G_t^{\phi_\tau,\tau} \right] \end{aligned}$$

where the last line follows directly from above. The result in the proposition then follows immediately from Lemma 1.

C Corollaries

Corollary 1 *Conditional on information available at time t, ϕ_{t+s} is normally distributed, with mean ϕ_t and variance*

$$\frac{\omega s}{[\gamma + \omega(t-\tau)]\,[\gamma + \omega(t+s-\tau)]}.$$

Proof: Iterating Equation (5), ϕ_{t+s} is the sum of normal random variables, so is itself normal. From Equation (5), its mean is ϕ_t. Finally, by independence of the ϵ_t we have

$$\begin{aligned} var_t[\phi_{t+s}] &= \sum_{i=0}^{s-1} \Phi_{t+i}^2, \\ &= \sum_{i=0}^{s-1} \frac{\omega}{[\gamma + \omega(t+i-\tau+1)]\,[\gamma + \omega(t+i-\tau)]}, \\ &= \sum_{i=0}^{s-1} \left\{ \frac{1}{[\gamma + \omega(t+i-\tau)]} - \frac{1}{[\gamma + \omega(t+i-\tau+1)]} \right\}, \\ &= \frac{1}{[\gamma + \omega(t-\tau)]} - \frac{1}{[\gamma + \omega(t+s-\tau)]}, \\ &= \frac{\omega s}{[\gamma + \omega(t-\tau)]\,[\gamma + \omega(t+s-\tau)]}. \end{aligned}$$

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