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FINANCING AND INVESTMENT IN PLANT AND EQUIPMENT AND RESEARCH AND DEVELOPMENT

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ABSTRACT

In this study a dynamic model of firm behavior is developed which integrates real and financial decisions. The model combines the effects of capital structure and input adjustment costs on the process of capital accumulation. The existence, uniqueness and stability conditions of the long-run equilibrium and the dynamic properties of the factor demand are explored. The equations derived from the theoretical model are estimated using firm cross-section time series data. The results indicate that for both Plant and Equipment (P&E) and Research and Development (R&D), the debt-equity ratio significantly affects the investment demands and the elasticities are highly inelastic. The effect is stronger for P&E than for R&D capital in the long run, while the effects on P&E and R&D investment are quite similar in the short run.

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1. Introduction

The extent to which a firm's capital structure influences capital accumulation has long been a contested theoretical and empirical issue. On the theoretical side work by Stiglitz [21], King [15], Feldstein, Green and Sheshinski [8], Auerbach [2] and Brock and Turnovsky [4] has investigated how alternative financing sources affect the capital stick selected by the firm. However, in these papers it is assumed that the capital stock can be instantaneously adjusted so that capital is treated as a variable factor of production.

There is another view (see Lucas [17], Gould [10], Mussa [19] and Treadway [24]) which postulates that the process of capital accumulation involves the firm incurring adjustment costs. In this framework capital is not instantly variable but rather it is a quasi-fixed factor, which is altered by the investment decisions. Investment functions are generated, which are increasing functions of the demand price of installed capital.

In this paper, by integrating the two approaches, we are able to analyze the influence that the capital structure exerts on investment undertaken by the firm. We develop a model with two quasi-fixed factors--the standard plant and equipment (P&E) capital and research and development (R&D) capital. We are able to characterize the behavior of investment over time and establish the existence, uniqueness and stability of the long run equilibrium.

Financial and real decisions are interrelated in the sense that the firm determines its debt-equity ratio, capital accumulation and labor requirements by maximizing the initial share value. We then establish that this program is equivalent to finding the debt-equity ratio which minimizes the cost of capital and labor requirements which maximize net operating revenues. The firm then uses the maximized net operating revenues and the minimized cost of capital to determine the real investment demands and thereby its capital accumulation plans.

The growth of the capital stocks is governed by the difference between the marginal values of installed to uninstalled capital. The higher the marginal value of installed relative to uninstalled capital the greater the demand for investment. The value of the marginal product denotes the value of installed capital, while the marginal installation costs and the cost of capital characterize the value of uninstalled capital. The latter depends on the level of investment, the debt-equity ratio, the interest and depreciation rates and the price of investment products. The value of the marginal product depends on the stocks of the quasi-fixed factors, the relative price of the variable factors of production and the product price.

Most studies on R&D treat it as a variable input in the production process (see Nadiri [20] and Griliches [11] and the references cited therein). Recently, though, Nadiri and Bitros [21] developed a partial adjustment model with R&D and Schankerman and Nadiri [22] constructed a model with R&D as the sole quasi-fixed factor. In this paper we derive, from intertemporal maximization, and estimate investment demand functions for two quasi-fixed factors, P&E and R&D capital. Moreover, in none of these studies was the purpose to determine the influence of alternative financing sources on capital accumulation.

Empirical evidence for the proposition that financial concerns affect R&D investment is mixed. There are only a few studies which have examined the impact of the capital structure on R&D (see Elliot [6] and Howe and McFetridge [13].) However, in both of these studies an intertemporal maximizing model of firm behavior was not the basis for the estimated equation and, in particular, for the hypothesized relationship between capital structure and R&D.

Research examining the influences of financial behavior on investment in plant and equipment has recently been undertaken by Engle and Foley [7], Von Furstenberg [28], Von Furstenberg, Malkiel and Watson [29] and Summers [24]. These studies, relying to various degrees on a dynamic model of the firm, have shown that industry and sectoral investment demand is significantly affected by changes in the share market values.

Our empirical results, based on a pooled time series, cross-section sample of 49 firms, suggest that the debt-equity ratio exerts a significant but small impact on P&E and R&D investment. In both cases the effect is quite inelastic. In addition, for an increase in the debt-equity ratio, the short-run response for both types of investment are quite similar. As time evolves, however, the effect on P&E becomes relatively stonger, with the long-run result that the percentage decrease in P&E capital is substantially greater than for R&D capital.

Tests were conducted to determine cross-section variations and cross-equation correlations. We found that the disturbance terms for the R&D and P&E investment functions were correlated. Moreover, there were interfirm differences in these equations. Interestingly, the firms which exhibited distinct P&E investment demands were not the same group with differences in R&D investment. Therefore, the majority of firms did not have an identical pair of P&E and R&D investment demand functions.

In section 2 the model is developed, section 3 deals with the short run equilibrium, while section 4 pertains to the dynamics and the long run equilibrium. The empirical work begins in section 5, with the model implementation, and in section 6 we describe the data. Section 7 contains the econometric results and we then conclude.

2. The Model

To begin our analysis of the firm's investment and financing decisions, we assume the technology is governed by

(1) $$y(t) = F[K_p(t), K_r(t), L(t)]$$

where $y(t)$ is output, F is the twice continuously differentiable production function, with positive and diminishing marginal products; $K_p(t)$ is the stock of plant and equipment (P&E); $K_r(t)$ is the stock of research and development (R&D); $L(t)$ is the labor services input. All variables are evaluated in period t.

We assume that the services emanating from the capital stocks are proportional to the stocks themselves.

The flow of funds of the firm is

$$(2) \qquad p(t)y(t) - w(t)L(t) - A[I_p(t)] - E[I_r(t)]$$

$$- r_b(t)B(t) + \overset{\circ}{B}(t) + s(t) - D(t) = 0$$

where $p(t)$ is the output price; $w(t)$ is the wage rate; A is the twice continuously differentiable strictly convex P&E gross investment cost function with $A(0) = A'(0) = 0$, $A' > 0$, $A'' > 0$ for $I_p(t) > 0$; E is the twice continuously differentiable R&D gross investment cost function with $E(0) = E'(0) = 0$, $E' > 0$, $E'' > 0$ for $I_r(t) > 0$;[1] $r_b(t)$ is the interest rate on corporate debt; $\overset{\circ}{B}(t)$ is the change in the value of outstanding debt, $s(t)$ is the value of new shares and $D(t)$ are dividends.[2]

The firm accumulates P&E and R&D according to

$$(3) \qquad \overset{\circ}{K}_p = I_p - \delta K_p, \; K_p(0) > 0$$

$$(4) \qquad \overset{\circ}{K}_r = I_r - \eta K_r, \; K_r(0) > 0$$

where $0 \leqq \delta \leqq 1$, $0 \leqq \eta \leqq 1$, are the fixed depreciation rates for P&E and R&D respectively.[3]

In the determination of share accumulation, we assume as in Auerbach [2] and Feldstein, Green and Sheshinski [8] that the rate of return on equity is dependent on the debt-equity ratio. The larger the debt-equity ratio the higher the rate of return that the shareholders require. We formulate this feature by

$$r_s + H(v) = D/p_sN_s + \mathring{p}_s/p_s, \tag{5}$$

where r_s is the net rate of return, $H(v)$ is the premium required by shareholders when the firm undertakes to issue bonds, where $v = B/p_sN_s$ and $H' > 0$, $H'' > 0$.[4]

The rate of return on shares is comprised of the dividends per share plus (minus) any capital gains (losses). Let $S = p_sN_s$, so $\mathring{S} = \mathring{p}_sN_s + p_s\dot{N}_s$ and by the definition of s we must have $s = p_s\mathring{N}_s$ then equation (5) can be rewritten as

$$\mathring{S} = [r_s + H(v)]\ S - D + s. \tag{6}$$

The corporate share value changes by the return on existing shares plus any new share issues minus any distributions to the shareholders.

We assume that the firm maximizes the initial value of equity, which means that decisions are made in the interest of the shareholders. The initial value is obtained by solving for S(0) from (6);

$$S(0) = \int_0^\infty e^{-\int_0^t r_s du}[D - H(v)S - s]dt. \tag{7}$$

The initial share value equals the present value of the stream of dividends minus both the premium paid to shareholders when there is outstanding debt and any dilution from new share issues.

The program for the firm is obtained by maximizing the right side of equation (7) subject to (1), (2), (3), (4) and (6). The Hamiltonian for this problem is

(8) $$H = (1 - q_4)\,[pF(K_p, K_r, L) - \dot{w}L - A(I_p) - E(I_r) - r_b B + \overset{\circ}{B} - H(v)S] + q_1(I_p - \delta K_p) + q_2(I_r - \eta K_r) + q_3\overset{\circ}{B} + q_4 r_s S,$$

where q_1 to q_4 are the shadow prices associated with the different stocks of real and financial capital.

The optimality conditions are,

(9.1) $$\frac{\partial H}{\partial L} = (1 - q_4)\,[p\frac{\partial F}{\partial L} - w] = 0$$

(9.2) $$\frac{\partial H}{\partial I_p} = -(1 - q_4)A' + q_1 = 0$$

(9.3) $$\frac{\partial H}{\partial I_r} = -(1-q_4)E' + q_2 = 0$$

(9.4) $$\frac{\partial H}{\partial \dot{B}} = 1 - q_4 + q_3 = 0$$

(9.5) $$\dot{q}_1 = (r_s + \delta)q_1 - pF_p(1 - q_4)$$

(9.6) $$\dot{q}_2 = (r_s + \eta)q_2 - pF_r(1-q_4)$$

$$(9.7) \qquad \dot{q}_3 = (r_s + 1)q_3 + (1 - q_4)\,[1 + r_b + H']$$

$$(9.8) \qquad \dot{q}_4 = (1 - q_4)\,[H - H'v].$$

There are, in addition to equations (3) and (4), the transversality and the Legendre-Clebsch (or second order) conditions.

Let us investigate the nature of the firm's intertemporal plan. First, we can see that the determination of the real and financial decisions are recursive. The debt-equity ratio is found from (9.4), (9.7) and (9.8). This debt-equity ratio minimizes the cost of capital. The firm then utilizes this cost of capital to determine the real capital accumulation paths.

To establish the above conclusion note from (9.4) that $1 - q_4 = -q_3$ where q_4 is the cost per dollar of equity, $-q_3$ is the cost per dollar of debt. From (9.2) (or (9.3)) $0 < q_4 < 1$. Thus the cost of financing a dollar of real capital is divided between the debt and equity instruments, since $1 = -q_3 + q_4$.

By combining (9.7) and (9.8) and since $\dot{q}_3 = \dot{q}_4$,

$$(10) \qquad r_s + H(v) = r_b + H'\,(1 + v).$$

The adjusted rate of return on equity equals the interest rate on corporate debt adjusted for the marginal premium needed in light of the higher debt-equity ratio. Equation (10) is a single equation which can be solved for one unknown, the debt-equity ratio. This debt-equity ratio minimizes the cost of capital. Define $r(v) = [r_s + H(v) + vr_b]/(1 + v)$ as the cost of capital. Minimizing r with respect to v yields

$$r^0 = r_b + H'$$

where r^0 is the minimum cost of capital. Substituting r^0 into the definition of r yields equation (10). Notice that if r_b and r_s are constant then the debt-equity ratio is constant for all time.

Second, the labor input decision given by (9.1) is devoid of any intertemporal considerations. Since $1 - q_4 > 0$, at each instant the value of the marginal product is equal to the factor price. The implication is that we can carry out our analysis in terms of operating and capital decisions. First, the firm maximizes net operating revenues, given the capital stocks and prices. This step yields a labor input demand which depends on the stocks of P&E, R&D and w/p. To see this,

$$\max_{(L)} pF(K_p, K_r, L) - w L.$$

The optimality condition to this program is given by equation (9.1). The solution can be denoted as $L = g\,(K_p, K_r, w/p)$.

Substituting the input demand function into the net operating revenue equation, yields the indirect variable profits function

$$R(K_p, K_r, w/p) = p\ F(K_p, K_r, g(K_p, K_r, w/p)) - w\ g(K_p, K_r, w/p).$$

Using the indirect variable profits function and the minimized cost of capital, the real capital accumulation decisions are solved from the following program,

$$\max_{(I_p, I_r)} \int_o^{\infty} e^{-\int_o^t r^0 du} \quad [R(K_p, K_r, w/p) - A(I_p) - E\ (I_r)]dt$$

subject to,

$$\dot{K}_p = I_p - \delta K_p$$

$$\dot{K}_r = I_r - \eta K_r .$$

We can summarize the firm's program in the following manner. First it determines the labor requirements, conditional on the stocks of R&D and P&E, by maximizing net operating revenues. Second, the debt-equity decision is taken which minimizes the cost of capital. Finally, the real investment demands and the capital accumulation plans are determined (by using the maximized net operating revenues, the minimized cost of capital) through maximizing the present value of the flow of funds associated with the two types of real capital.

3. The Short-Run Equilibrium

The short-run equilibrium for the firm is denoted by equations (9.1), (9.2) and (9.3). These equations are independent of each other because labor does not involve any intertemporal considerations and the investment costs only depend on their respective investment flow.

Consider the labor demand. If we assume that increases in the stocks of P&E and R&D increase the marginal product of labor, then with diminishing marginal products, increases in the stocks increase labor demand. In addition, an increase in the real wage decreases labor requirements. Thus

(11) $$L = g(K_p, K_r, w/p); \; g_1 > 0, \; g_2 > 0, \; g_3 < 0.$$

The short run investment demand functions for P&E and R&D are given by (9.2) and (9.3) respectively. We find that,

(12.1) $$I_p = \mathcal{I}(q_1/1 - q_4) \quad \mathcal{I}' > 0$$

(12.2) $$I_r = \mathcal{J}(q_2/1 - q_4) \quad \mathcal{J}' > 0.$$

Gross investment demand is forward looking and each one is an increasing function of its respective demand price and a decreasing function of the per dollar cost of financing the additions to the real capital stocks. An alternative interpretation of equation set (12) is that with the price of uninstalled capital normalized to unity, $1 - q_4$ is the marginal cost of uninstalled capital. Hence investment is an increasing function of the marginal value of installed capital relative to the marginal cost of uninstalled capital. Therefore gross investment is determined by a mechanism similar to Tobin's [23] (see also Abel [1] and Hayashi [12]) "q" theory where, in our context, $q_p = q_1/1-q_4$ and $q_r = q_2/1-q_4$.

4. The Dynamics and the Long-Run Equilibrium

In order to be able to characterize the dynamic behavior of the firm and the long-run equilibrium, we must investigate equation set (9).

First, equation (10) summarizes equations (9.4), (9.7) and (9.8). Second, differentiating (9.2) and (9.3) with respect to time results in

(13.1) $$\dot{q}_4 A' - (1 - q_4)A''\dot{I}_p + \dot{q}_1 = 0$$

(13.2) $$\dot{q}_4 E' - (1 - q_4) E''\dot{I}_r + \dot{q}_2 = 0.$$

Substituting (9.2), (9.8), (10), and (11) and (14.1) into (9.5) provides us with the differential equation for P&E investment

(14) $$A''(I_p) \dot{I}_p = (r_b + H'(v) + \delta)A'(I_p) - pF_p(K_p, K_r, g(K_p, K_r, w/p)).$$

One interpretation of equation (14) is that when the marginal return on new capital, which is $(r_b + H' + \delta)A'$, exceeds the marginal value of existing capital, which is pF_p, then gross investment increases such that $\dot{I}_p > 0$. The converse holds when the marginal return on existing capital exceeds that on new capital. Clearly, when the marginal returns are equal no additional gross investment is undertaken.

Similarly we can determine the differential equation for R&D investment, by using (9.3), (9.8), (10), (11) and (13.2) in (9.6),

(15) $$E''(I_r)\dot{I}_r = (r_b + H'(v) + \eta) E'(I_r) - pF_r(K_p, K_r, g(K_p, K_r, w/p)).$$

Notice that since the debt-equity ratio is determined from (10) then the remaining endogenous variables affecting the path of P&E investment is the P&E investment flow and the two types of real capital.

The dynamic behavior of the firm can now be summarized into four equations, (3), (4), (14) and (15). Let us first determine whether or not there exists a unique long-run equilibrium. Suppose $\dot{K}_p = \dot{K}_r = 0$ then $I_p = \delta K_p$, $I_r = \eta K_r$, and so $\dot{I}_p = \dot{I}_r = 0$. Thus at $\dot{K}_p = \dot{K}_r = 0$, equations (14) and (15) become

(16) $$(r_b + H'(v) + \delta)A' (K_p) = pF_p(K_p, K_r, g(K_p, K_r, w/p))$$

(17) $$(r_b + H'(v) + \eta)E' (K_r) = pF_r(K_p, K_r, g(K_p, K_r, w/p)).$$

The long-run equilibrium levels of the real capital stocks are those which simultaneously solve equations (16) and (17). Let us denote these values as (K_r^e, K_p^e).

The immediate problem is to find the unique solution. To this end we differentiate (16) and (17),[5]

$$(18) \quad \left.\frac{dK_p}{dK_r}\right|_{\dot{K}_p=0} = \frac{(F_{p\ell}F_{r\ell} - F_{pr}F_{\ell\ell})}{[F_{pp}F_{\ell\ell} - F_{p\ell}^2 - (r_b + H' + \delta)(A'' \delta F_{\ell\ell}/p)]} > 0.$$

$$(19) \quad \left.\frac{dK_p}{dK_r}\right|_{\dot{K}_r=0} = \frac{[F_{rr}F_{\ell\ell} - F_{r\ell}^2 - (r_b + H' + \eta)(E'' \eta F_{\ell\ell}/p)]}{(F_{p\ell}F_{r\ell} - F_{pr}F_{\ell\ell})} > 0.$$

Hence in (K_r, K_p) space equations (16) and (17) define direct relationships between the real capital stocks which is illustrated in Figure 1. Moreover, from (16) as $K_p \to 0$ since $A'(0) = 0$ and by assuming $F_p(0, K_r, L) > 0$ then the locus, defined by (16), intersects the K_p - axis. From equation (17), as $K_r \to 0$ since $E'(0) = 0$ and by assuming $F_r(K_p, 0, L) > 0$ then the curve intersects the K_r - axis.

In order for there to exist a unique long-run equilibrium the curves depicted in Figure 1 must intersect only once. A set of sufficient conditions for this to occur are that

$$(20) \qquad F_{ii} F_{\ell\ell} - F_{i\ell}^2 > F_{p\ell} F_{r\ell} - F_{pr} F_{\ell\ell} \qquad i = p, r.$$

If the marginal products of each of the inputs diminish in sufficient magnitude, then the above inequalities are satisfied. We then have

$$\left.\frac{dK_p}{dK_r}\right|_{\dot{K}_p=0} < 1, \quad \left.\frac{dK_p}{dK_r}\right|_{\dot{K}_r=0} > 1$$

and therefore a unique long-run equilibrium.

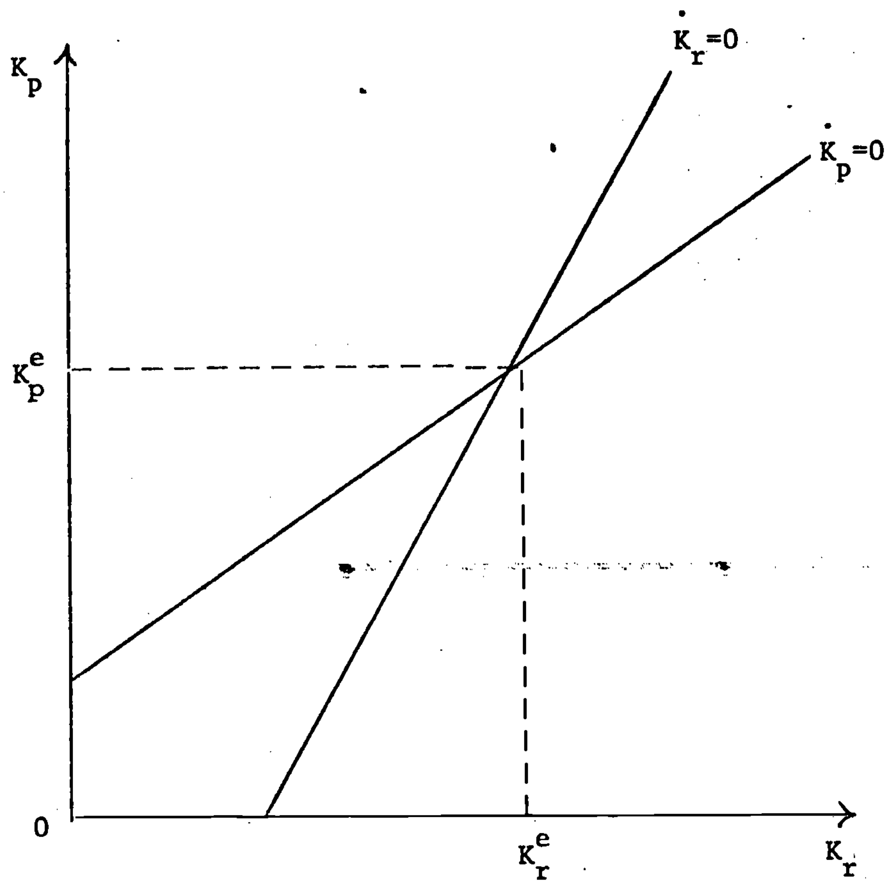

Figure 1. Existence and Uniqueness of the Long-Run Equilibrium

The stability of the long-run equilibrium is illustrated in Figure 2. Figure 2 is a four quadrant diagram which contains Figure 1 in the (K_r, K_p) space. The two remaining spaces to analyze are (I_p, K_p) and (I_r, K_r).

First consider (I_p, K_p). From equation (3), we find $\frac{\partial \dot{K}_p}{\partial I_p} = 1 > 0$, $\frac{\partial \dot{K}_p}{\partial K_p} = -\delta < 0$ and therefore .

(21) $$\left.\frac{dK_p}{dI_p}\right|_{\dot{K}_p=0} = 1/\delta > 0.$$

The $\dot{K}_p = 0$ locus is a straight line through the origin with the slope of $1/\delta$ and above the line $\dot{K}_p < 0$, while below $\dot{K}_p > 0$.

Next, from equation (14) at $\dot{I}_p = 0$, $K_r = K_r^e$ and with the conditions defined by (20),

(22) $$\left.\frac{dK_p}{dI_p}\right|_{\substack{\dot{I}_p=0 \\ K_r=K_r^e}} = \frac{F_{\ell\ell}(r_b + H' + \delta)}{p(F_{pp}F_{\ell\ell} - F_{p\ell}^2)} A'' < 0.$$

Hence the $\dot{I}_p = 0$ locus is negatively sloped. In addition, since $F_p(0, K_r, L) > 0$ and at $\dot{I}_p = 0, A' = pF_p/(r + H' + \delta) > 0$ then the $\dot{I}_p = 0$ locus intersects the I_p - axis. To determine the movement when the firm is off the $\dot{I}_p = 0$ curve, we know from (14) that

$$\left.\frac{\partial \dot{I}_p}{\partial K_p}\right|_{K_r=K_r^e} = \frac{-p}{F_{\ell\ell}A''}[F_{pp}F_{\ell\ell} - F_{p\ell}^2] > 0.$$

Therefore $\dot{I}_p > 0$ for points above the $\dot{I}_p = 0$ locus and $\dot{I}_p < 0$ for points below the curve. In a similar fashion we can derive the nature of the

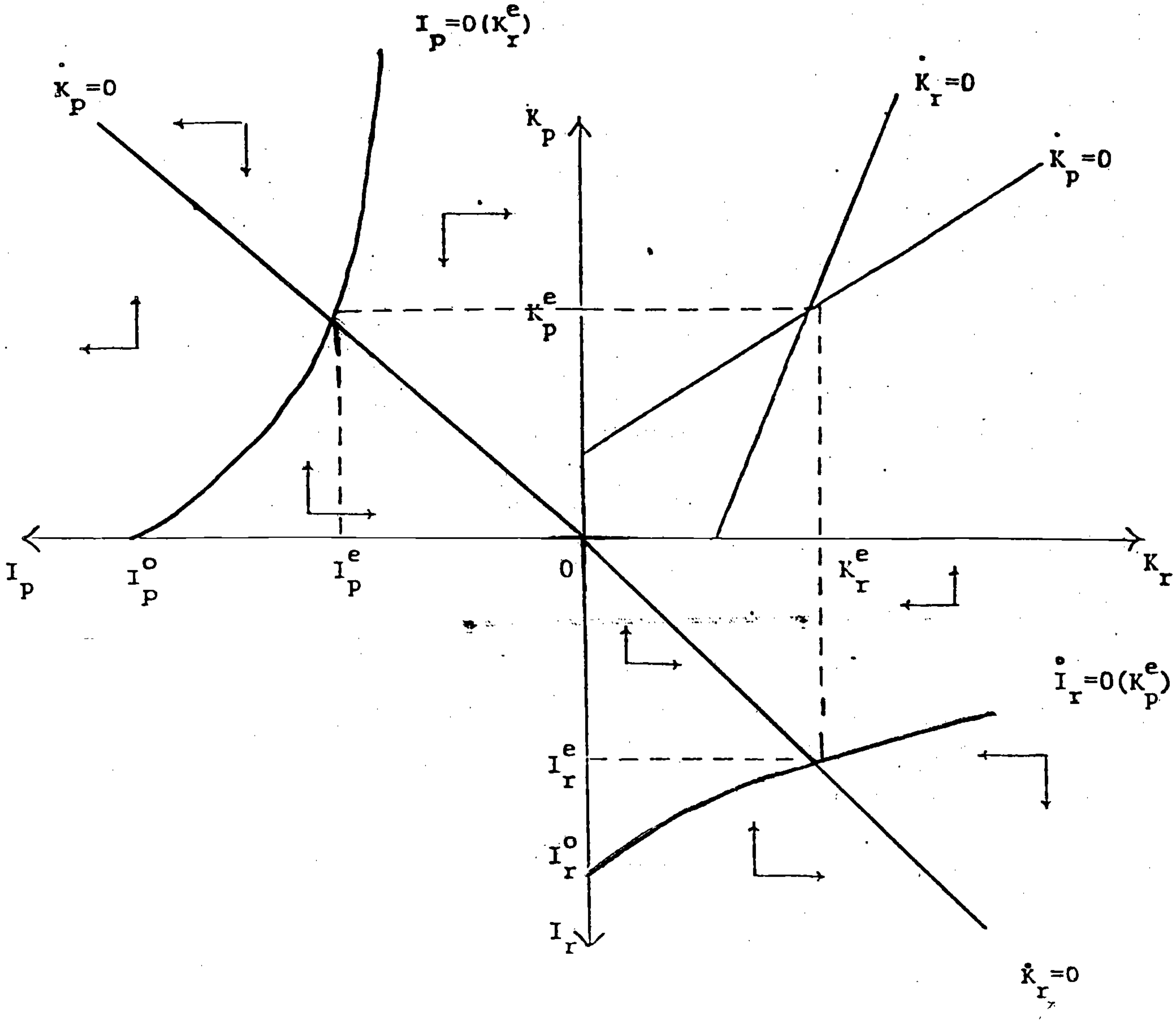

Figure 2. Stability of the Long-Run Equilibrium

curves in (I_r, K_r) space and the behavior of the firm at any point in the space. From these results we can see that the long-run equilibrium is a saddle point.[6]

5. The Empirical Implementation

The equations summarizing the dynamic behavior of the firm are denoted by (3), (4), (14) and (15). We assume that (3) and (4) are non-stochastic and use these equations to define the stocks of P&E and R&D respectively. In order to carry out the estimation of equations (14) and (15), we must specify the investment cost functions $A(I_p)$, $E(I_r)$, the function denoting the premium on the rate of return to shareholders, when debt financing is used, $H(v)$, and the production function $F(K_p, K_r, L)$. We define

$$(23) \qquad A(I_p) = \frac{\alpha}{c} p_p I_p [\log I_p - \log c] \quad , I_p > c \quad , \alpha > 0$$

$$(24) \qquad E(I_r) = \frac{\xi}{d} p_r I_r [\log I_r - \log d] \quad , I_r > d \quad , \xi > 0$$

$$(25) \qquad H(v) = \frac{\gamma}{e} v [\log v - \log e] \quad , v > e \quad , \gamma > 0$$

$$(26) \qquad F(K_p, K_r, L) = A K_p^{\lambda} K_r^{\mu} L^{\beta} \quad , \lambda > 0, \mu > 0, \beta > 0,$$

where p_i $i = p, r$ are the prices of the investment products for P&E and R&D respectively.[7]

Substituting (23) to (26) into (14) and (15) yields

$$(27) \qquad \frac{d(\log I_p)}{dt} = [r_b + \gamma_2 + \gamma_1 \log v + \delta](\log I_p + \frac{\alpha_2}{\alpha_1}) - \frac{\lambda p v}{\alpha_1 p_p K_p}$$

$$(28)\qquad \frac{d(\log I_r)}{dt} = [r_b + \gamma_2 + \gamma_1 \log v + \eta]\,(\log I_r + \frac{\xi_2}{\xi_1}) - \frac{\mu py}{\xi_1 p_r K_r}.$$

where $\alpha_1 = \alpha/c$, $\alpha_2 = \alpha_1(1 - \log c)$, $\xi_1 = \xi/d$, $\xi_2 = \xi_1\,(1 - \log d)$, $\gamma_1 = \gamma/e$, $\gamma_2 = \gamma_1\,(1 - \log e)$. We now have two differential equations in terms of the logarithms of the investment flows. The solution to these equations depends on the time paths of the debt-equity ratio, the prices, the interest rate, output and the capital stocks. For simplicity, as in other dynamic models (see Morrison and Berndt [18]), we assume static expectations.[8] Hence (27) and (28) are first order, nonhomogenous differential equations in terms of log I_p and log I_r with constant coefficients.[9] The solutions are

$$(29)\quad \log I_p(t) = \log I_p^e + [\log I_p(0) - \log I_p^e]e^{-(r_b + \gamma_2 + \gamma_1 \log v + \delta)t}$$

$$(30)\quad \log I_r(t) = \log I_r^e + [\log I_r(0) - \log I_r^e]e^{-(r_b + \gamma_2 + \gamma_1 \log v + \eta)t}$$

where $\log I_p^e = \frac{\lambda py}{{}_1 p_p K_p}\,(r_b + \gamma_2 + \gamma_1 \log v + \delta) - \frac{\alpha_2}{\alpha_1}$ and

$\log I_r^e = \frac{\mu py}{\xi_1 p_r K_r}(r_b + \gamma_2 + \gamma_1 \log v + \eta) - \frac{\xi_2}{\xi_1}$. The superscript e represents the long-run equilibrium values.

Time differentiating (29) and (30) and taking a discrete approximation for each equation we find that

(31) $$\log I_p(t) = A_o + A_1 \log I_p(t-1) + A_2 \log v(t) + A_3 \log v(t) \log I_p(t-1) + A_4 \frac{p(t)y(t)}{p_p(t)K_p(t-1)}$$

(32) $$\log I_r(t) = B_o + B_1 \log I_r(t-1) + B_2 \log v(t) + B_3 \log v(t) \log I_r(t-1) + B_4 \frac{p(t)y(t)}{p_r(t)K_t(t-1)}$$

where $A_o = -\alpha_2(r_b + \delta + \gamma_2)/\alpha_1$, $A_1 = 1 - r_b - \delta - \gamma_2$
$A_2 = -\alpha_2 \gamma_1/\alpha_1$, $A_3 = -\gamma_1$, $A_4 = \lambda/\alpha_1$, $B_o = -\xi_2 (r_b + \eta + \gamma_2)/\xi_1$,
$B_1 = 1 - r_b - \eta - \gamma_2$, $B_2 = -\xi_2 \gamma_1/\xi_1$, $B_3 = -\gamma_1$ and $B_4 = \mu/\xi_1$.[10]

From equations (31) and (32) we have established that investment (for either P&E or R&D) in any period depends directly on its past value, inversely on the financing costs (represented, in particular, by the debt-equity ratio) and directly on the utilization of the existing stock, as represented by the sales to asset ratio.[11]

6. The Data

Annual data on several variables were collected from a variety of sources indicated below for the period 1959-1966 for forty-nine firms. The selection of firms was dictated by the availability of consistent time series data on R&D expenditures and the stock of R&D. The pooled time-series cross section sample was designed to provide a richer set of information in which to estimate the functions under consideration.

The list of variables and their construction are: Plant and equipment (K_p) is the measure of net stock generated by a perpetual inventory formula

$$K_p(t) = I_p(t) + (1 - \delta) K_p(t - 1)$$

where $I_p(t)$ equals actual expenditures on plant and equipment deflated by the price of investment in P&E. Investment in P&E and its associated price (p_p) were obtained from the McGraw-Hill data series, with the depreciation rate for each firm calculated by summing over time depreciation allowances divided by the gross plant and equipment and then dividing this sum by the number of time periods. The stock of R&D (K_r) was obtained from a similar procedure,

$$K_r(t) = I_r(t) + (1 - \eta)K_r(t - 1).$$

Investment in R&D (I_r) and its associated price (p_r) were obtained from McGraw-Hill data series and we arbitrarily chose $\eta = .1$ to measure the depreciation rate for the stock of knowledge. Debt (B) was obtained from Standard and Poor's data series for long term corporate debt. Equity (S) was also obtained from Standard and Poor's series for the closing common share price multiplied by the number of outstanding common shares. Sales (py) figures were obtained from the Mc-Graw Hill data series.

7. The Empirical Results

In order to render equations (31) and (32) stochastic, we add a random disturbance term to each equation. Moreover, to reflect the fact that the equations can vary among the firms in the sample, because of technological differences, we add to A_o and B_o parameters which are firm-specific. Thus (31) and (32) become,

$$\begin{aligned}(33)\qquad \log I_p(j,t) = {} & A_o(j) + A_1 \log I_p(j,t-1) + A_2 \log v(j,t) \\ & + A_3 \log v(j,t) \log I_p(j,t-1) \\ & + A_4 \frac{p(j,t)\ y(j,t)}{p_p(j,t)\ K_p(j,t-1)} + u_p(j,t)\end{aligned}$$

$$\begin{aligned}(34)\qquad \log I_r(j,t) = {} & B_o(j) + B_1 \log I_r(j,t-1) + B_2 \log v(j,t) \\ & + B_3 \log v(j,t) \log I_r(j,t-1) \\ & + B_4 \frac{p(j,t)\ y(j,t)}{p_p(j,t)\ K_r(j,t-1)} + u_r(j,t)\end{aligned}$$

$$j - 1,\ldots,49 \quad + T = 1960,\ldots,1964.$$

We initially make the following assumptions on the disturbance terms: The joint distribution of $u_i = [u_i(1, 1960),\ldots u_i(49, 1964)]$ is multivariate normal, $E[u_i] = 0$, $E[u_i u_i^t] = \sigma_{ii} I$ for $i = p,r$ and $E[u_p u_r^t] = 0$ where σ_{ii} is the variance of the disturbance terms, I is the identify matrix and 0 is the zero matrix.

The estimation results for P&E investment are presented in Table 1. In this estimation we have assumed that $a_0(j) = a_0(k)$, $b_0(j) = b_0(k)$ for $j,k = 1,\ldots,49$. Initially we impose the restriction that the equations are identical across firms. In Table 1 there are two sets of estimates. The first row refers to the results from estimating equation (33) and the second row refers to a restricted version with $\alpha_1 = \alpha_2$, which implies that $\dot{c} = 1$ in equation (23), and so $A_2 = A_3$ in equation (33). We see that all estimates have the correct sign and the equation fits the data quite well. In addition, from the unrestricted equation, we are unable to reject the hypothesis that $A_2 = A_3$. Thus, imposing the restriction we see from the second row that all estimates are significant. In particular, increases in the debt-equity ratio do indeed decrease P&E investment.

Due to the presence of the interaction between lagged investment and the debt-equity ratio the elasticity varies over the sample. The formula for the short-run elasticity is

$$(35) \qquad e^{s}_{pv} = \frac{\partial \log I_p(j,t)}{\partial \log v(j,t)} = A_2(1 + \log I_p(j,t-1))$$

and for the long-run elasticity

$$(36) \qquad e^{\ell}_{pv} = \frac{\partial \log K_p(j,t)}{\partial \log v(j,t)} = A_2[1 + \log \delta K_p(j,t)]/(1 - A_1 - A_2 \log v(j,t) + A_4 w(j,t))$$

where $w(j,t) = p(j,t)y(j,t)/p_p(j,t)K_p(j,t)$. The long-run elasticity is defined for $\dot{K}_p = 0$, and thereby $I_p = \delta K_p$ and $\dot{I}_p = 0$. The mean

Table 1

Single Equation Basic Model OLS Estimation of P&E Investment
(t Statistics in Parentheses)

	Estimates						
Equation	A_0	A_1	A_2	A_3	A_4	R^2	SEE
Unrestricted	-.055 (-.448)	.934 (26.658)	-.0080 (-.756)	-.0073 (-1.872)	.561 (7.885)	.800	.796
Restricted	-.059 (-.551)	.935 (30.619)	-.008 (-4.229)		.560 (7.936)	.800	.794

value of the short-term elasticity is -.028, while the mean value in the long run is -.193. Thus, in the short run an increase in the debt-equity ratio of 1% leads to a decrease in P&E investment by .028%, while in the long run an increase of 1% leads to a decrease in the P&E capital stock by -.193%. As expected, the long-run elasticity exceeds the short-run. Moreover, both effects are inelastic, with the short-run highly inelastic, but nevertheless significant.

Turning to the results for R&D investment, we see from Table 2 that the estimation of the restricted version of equation (34) elicits significant coefficients with the correct sign. Once again the fit is quite strong. An interesting feature is that changes in the debt-equity ratio do indeed exert a significant impact on R&D investment. The short-run elasticity is

$$e^{s}_{rv} = \frac{\partial \log I_r(j,t)}{\partial \log v(j,t)} = B_2(1 + \log I_r(j,t-1))$$

and in the long run

$$e^{\ell}_{rv} = \frac{\partial \log K_p(j,t)}{\partial \log v(j,t)} = B_2[1 + \log \eta K_r(j,t)]/1 - B_1$$

$$- B_2 \log v(j,t) + B_4 z(j,t))$$

where $z(j,t) = p(j,t)y(j,t)/p_r(j,t)K_r(j,t)$. The mean value for the short run is -.017 and for the long run the mean value is -.0985. Once again the short-run effect is less than the long-run. In addition, the influence of changes in the debt-equity ratio is substantially smaller than for P&E, both in the short and long runs. Indeed, the initial

Table 2

Single Equation Basic Model OLS Estimation of R&D Investment
(t Statistics in Parentheses)

	Estimates						
Equation	B_0	B_1	B_2	B_3	B_4	R^2	SEE
Unrestricted	.127 (1.228)	.924 (27.515)	-.018 (-1.941)	-.028 (-.837)	.022 (4.475)	.834	.852
Restricted	.050 (.635)	.948 (34.154)	-.006 (-3.000)		.021 (4.294)	.833	.854

impact of a percentage increase in the debt-equity ratio causes the percentage decrease in P&E investment to be approximately 65% greater than the percentage decrease in R&D investment. As time passes, the spread in percentage changes enlarges, such that in the long run the mean value of P&E capital elasticity with respect to the debt-equity ratio is approximately double the R&D capital elasticity.

Differences in investment demand functions among firms can arise through the different production technologies. In order to account for any cross section variations, we drop the assumption that the intercepts in equations (33) and (34) are the same for all firms. In our sample the 49 firms are classified into 10 different 2-digit SIC groupings. Hence we introduce 9 binary variables into the regression equations. The 10 different firm groupings are represented as 10 different equations, for each investment category, with the equation differences reflected through the binary variable coefficients.

The single equation estimates of this covariance model for the restricted version of the P&E investment equation are presented in Table 3. We find that the nonbinary variable coefficients have the correct sign, they are significant and the fit is good. The coefficient on the debt-equity ratio is -.005 as opposed to -.008 as found in Table 1. Thus, the mean value of the short-run elasticity is now -.017 and the mean value for the long-run is -.103. The introduction of cross section variations in P&E investment has caused the effects emanating from changes in the debt-equity ratio to become weaker. This occurs in the short and long runs with the short-run elasticity falling by 40% and the long-run decreasing by roughly 47%.

Table 3

Single Equation Covariance Model OLS Estimation
of P&E and R&D Investment
(t Statistics in Parentheses)

Estimates	Equation P&E	Equation R&D	Estimates
A_0	.077 (.380)	-.049 (-.278)	B_0
A_1	.858 (22.221)	.904 (26.915)	B_1
A_2	-.005 (-2.635)	-.006 (-2.572)	B_2
A_4	.555 (7.778)	.022 (4.501)	B_4
d_{p1}	-.007 (.025)	-.043 (-.155)	d_{r1}
d_{p2}	.384 (1.863)	.304 (1.369)	d_{r2}
d_{p3}	.490 (1.825)	.017 (.063)	d_{r3}
d_{p4}	.468 (1.210)	.680 (1.663)	d_{r4}
d_{p5}	.041 (.188)	-.141 (-.616)	d_{r5}
d_{p6}	-.153 (-.806)	.357 (1.822)	d_{r6}
d_{p7}	-.389 (-1.579)	-.082 (-.324)	d_{r7}
d_{p8}	.104 (.403)	.750 (2.684)	d_{r8}
d_{p9}	.071 (.184)	-.336 (-.814)	d_{r9}
R^2	.813	.847	R^2
SEE	.784	.832	SEE

With respect to the interfirm differences we see that there is a suggestion of cross section variation arising from groups 3, 4 and 8 represented respectively by the binary variables d_{p2}, d_{p3} and d_{p4}.

Table 3 also shows the restricted R&D investment demand estimation. The fit, signs, significance and values of the nonbinary variable coefficients is similar to those found in Table 2. In addition, unlike the case for P&E the mean value of the short-run elasticity has not changed, it is still -.017. The mean value of the long-run elasticity is now -.06. Therefore, we find that the long-run effect of a change in the debt-equity ratio, as for P&E, has significantly decreased with the introduction of cross section differences. Interestingly, a 1% increase in the debt-equity ratio causes the same decrease, in the short run, for P&E and R&D investment. However, as time evolves the effect on P&E capital increases relative to the effect on R&D capital, such that in the long run the debt-equity elasticity of P&E is 72% larger than for R&D.

The binary variables also illustrate that groups 5, 7 and 9 (as represented by d_{r4}, d_{r6} and d_{r8}) exhibit technological differences such that their R&D investment demand equations are distinct from the cross section average. In fact, with group 9, d_{r8} = .750, and therefore the distinct technology for this group causes the intercept in its R&D equation to be -.049 + .750 = .701, which is significantly above the average value of -.049.

We have proceeded in the estimation as if the P&E and R&D investment equations are independent of each other. However, as they are both derived from the same dynamic program characterizing firm behavior, it seems reasonable to expect some stochastic relationship to be present between the equations. Thus, we drop the assumption that $E[u_p u_r^t] = 0$ and now assume that $E[u_p u_r^t] = \sigma_{pr} I$, where σ_{pr} is the covariance between the disturbance terms of the two investment equations.

Equations (33) and (34) are now seemingly unrelated regression equations and are jointly estimated by maximum likelihood to account for the cross equation correlation among the disturbance terms. We also include the binary variables to account for the cross section differences within each equation. The estimates are presented in Table 4.

The effect of changes in the debt-equity ratio on P&E investment in the short run is the same as when the equations were estimated separately, that is, -.017. However, the mean value of the long-run elasticities is now -.088. This shows a significant decline in the long-run magnitude from the single equation estimate of -.103. The effects on R&D do not change when joint estimation is undertaken. The short- and long-run elasticities are, respectively, -.017 and -.06. Therefore, we can summarize our findings on the debt-equity effects in the following way. An increase in the ratio initially causes the same response on P&E and R&D investment. As time passes, though, the P&E effect becomes relatively larger, and indeed in the long run the effect is about 47% greater for P&E compared to R&D capital. Nevertheless, for both types of capital the effects, both short- and long-run, are highly inelastic.

In the context of joint estimation, cross section variations have also become significant for both types of investment. For P&E, groupings

Table 4

Seemingly Unrelated Regression Equations Covariance Model
Maximum Likelihood Estimation of P&E and R&D Investment
(Asymptotic t Statistics in Parentheses)

Estimates	Equation P&E	Equation R&D	Estimates	Equation System
A_0	.296 (1.515)	.008 (.043)	B_0	
A_1	.809 (22.354)	.872 (27.646)	B_1	
A_2	-.005 (-2.605)	-.006 (-2.968)	B_2	
A_4	.427 (6.371)	.020 (4.550)	B_4	
d_{p1}	-.065 (-.257)	-.078 (-.290)	d_{r1}	
d_{p2}	.381 (1.904)	.353 (1.636)	d_{r2}	
d_{p3}	.524 (2.013)	.032 (.122)	d_{r3}	
d_{p4}	.487 (1.293)	.706 (1.774)	d_{r4}	
d_{p5}	-.028 (-.130)	-.188 (-.847)	d_{r5}	
d_{p6}	-.223 (-1.209)	.361 (1.893)	d_{r6}	
d_{p7}	-.485 (-2.027)	-.082 (-.355)	d_{r7}	
d_{p8}	.147 (.169)	.817 (3.010)	d_{r8}	
d_{p9}	.063 (.168)	-.270 (-.673)	d_{r9}	

Table 4 (continued)

Estimates	Equation		Estimates	Equation System
	P&E	R&D		
Individual Equation R^2	.810	.846		
Individual Equation SEE	.769	.812		
R^2				.948
Chi-Square (24 DF)				724.88

3, 4 and 8 exhibit differences from the cross section average. Groupings 3 and 4 invest in P&E more than the average. Indeed, the intercept term for group 3 is more than twice as large as the average. The investment for group 8 is smaller, with an intercept of -.189 compared to the average of .296.

R&D investment shows interfirm differences. As observed in single equation estimates, group 9 invests in R&D substantially more than the average. Now we also find a tendency for groups 5 and 7 to significantly vary from the average, with their R&D investment levels larger. A final interesting result is that, not only are there cross section differences, but the firms that have different P&E investment demand functions are not those which have difference R&D investment demand functions. Taken together, the package of P&E and R&D equations for each grouping appears to be quite distinct.

8. Conclusion

In this paper we developed a dynamic model of firm behavior which integrates real and financial decisions. The firm, at each instant, determines its labor requirements, the debt-equity ratio and the real investment demands for plant and equipment and research and development. Although the firm determines these elements simultaneously by maximizing the initial value of equity, in effect the decision process is sequential. The debt-equity ratio is found by minimizing the cost of capital and the labor input emanates from the maximization of net operating revenues. Gathering these two parts permits us to characterize the accumulation of the real capital stocks.

The estimated equations derived from the theoretical model have the property that investment demand depends on the lagged investment flow, the cost of additional capital (as reflected by the debt-equity ratio), and the utilization of the existing capital stock as measured by the sales to asset ratio for each particular type of real capital. The equations were estimated to account for the statistically significant cross equation and cross section differences. For both P&E and R&D the debt-equity ratio significantly affects the investment demands and the elasticities are highly inelastic. In addition, the effect is stronger for P&E than for R&D capital in the long run, while the effects on P&E and R&D investment are quite similar in the short run.The impact of a percentage increase in the debt-equity ratio causes the percentage decline in P&E capital to be approximately one and one half times the percentage decrease in R&D.

Footnotes

1. We can view P&E investment costs as reflecting a constant price, p_p, and installation costs. The latter are represented by $C(I_p)$, which is increasing and strictly convex. Thus, $A(I_p) = p_p I + C(I_p)$. Following the discussion of Mussa [19] the installation costs are internal, although separable from the capital stocks and labor.

2. We assume that the lending and borrowing interest rates on corporate debt are identical. In addition, we assume that there is only one type of bond and share.

3. Henceforth we drop the symbol (t) for notational convenience.

4. The function H(v) summarizes, in a simple way, the bankruptcy costs resulting from the firm's choice of a debt-equity ratio. See Gordon [9] and Lintner [16].

5. We assume that $F_{pr} > 0$, so that now all cross partial derivatives are positive.

6. We cannot illustrate the dynamic path of the firm because the $\dot{I}_p = 0$ locus depends on I_p, K_p and K_r, while the $\dot{I}_r = 0$ locus depends on I_r, K_p and K_r.

7. The functions satisfy the properties needed in the specification of the model. The production function is a first order logarithmic approximation to any arbitrary production function and we do not restrict the degree of returns to scale.

8. An alternative would be to use equation set (12) which relates P&E and R&D investment respectively to $q_p = q_1/1 - q_4$ and $q_r = q_2/1 - q_4$. These are the shadow prices defining Tobin's q's for

each type of capital. However, in order to obtain observable variables for the shadow prices, the share value of the firm (S) must be homogeneous of degree 1 in the capital stocks, labor services and the investment flows (see Hayashi [12] and Summers [24]). This implies that the technology must exhibit constant returns to scale, unit adjustment costs are homogeneous of degree 0 in the respective investment flow and capital stock, and the debt to asset ratio is fixed. Clearly, at the firm level, the assumption of constant returns to scale is quite restrictive. In addition, the fixed debt-asset ratio abstracts from the financing decision and its influence on capital accumulation, which we are attempting to test.

9. The static expectations assumption yields investment equations which are of the accelerator variety. In our context the accelerator is in terms of the logarithms of investment. For the theoretical development of the accelerator model see Treadway [27] and for empirical surveys see Eisner [5] and Jorgenson [14].

10. We are treating $(r_b + \delta + \gamma)$ and $(r_b + \eta + \gamma)$ as parameters. This seems reasonable given that the terms do not vary over the firms, and δ, η and γ do not vary over time. In addition, r_b is relatively constant over this period.

11. In the "q" approach to estimating investment functions it has been found that investment is related to the lagged value of q rather than to the contemporaneous value (see Summers [24]). This result is not predicted by the theory and indeed may be quite troublesome. The reason is that by relating investment to the lagged value of the ratio of the shadow prices of installed to uninstalled

capital involves an assumption concerning the firm's expectations, which is not explicitly accounted for in the theory.

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