NBER WORKING PAPER SERIES

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Working Paper 10080
http://www.nber.org/papers/w10080

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2003

The authors thank Cam Harvey for providing data, Theo Nijman and seminar participants at a joint INQUIRE Europe - INQUIRE UK meeting, the Conference on Dynamic Strategies in Asset Allocation and Risk Management at the EIASM in Brussels, and the editor and the referees for useful comments. This research is funded by a grant from INQUIRE Europe. A more technical version of this article is available on our websites. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

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NBER Working Paper No. 10080
November 2003
JEL No. F30, G11, G12

ABSTRACT

International equity returns are characterized by episodes of high volatility and unusually high correlations coinciding with bear markets. We develop models of asset returns that match these patterns and use them in asset allocation. First, the presence of regimes with different correlations and expected returns is difficult to exploit within a framework focused on global equities. Nevertheless, for all-equity portfolios, a regime-switching strategy dominates static strategies out-of-sample. Second, substantial value is added when an investor chooses between cash, bonds and equity investments. When a persistent bear market hits, the investor switches primarily to cash. There are large market timing benefits because the bear market regimes tend to coincide with periods of relatively high interest rates.

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1 Introduction

International equity returns are more highly correlated with each other in bear markets than in normal times. This asymmetric correlation phenomenon is statistically significant, as shown by Longin and Solnik (2001), while Ang and Bekaert (2002a) show that regime-switching (RS) models perform well at replicating the degree of asymmetric correlations observed in data.[1]

RS models build on the seminal work by Hamilton (1989). In its simplest form, a RS model allows the data to be drawn from two or more possible distributions ("regimes"). At each point of time, there is a certain probability that the process remains in the same regime next period. Alternatively, it might transition to another regime next period. Ang and Bekaert (2002a) find that international equity returns are characterized by two regimes: a normal regime and a bear market regime where returns are, on average, lower and much more volatile than in normal times. Importantly, in the bear market regime, the correlations between various returns are higher than in the normal regime.

RS behavior is not restricted to equity returns: there is also strong evidence of regimes in US and international short-term interest rate data.[2] Short rates are characterized by high persistence and low volatility at low levels, but lower persistence and much higher volatility at higher levels. Again, RS models perfectly capture these features of the data. The regimes in interest rates and equity returns regimes are correlated and are related to the business cycle.

Surprisingly, quantitative asset allocation research usually ignores these salient features of international equity return and interest rate data. The presence of asymmetric correlations in equity returns has so far primarily raised a debate on whether they cast doubt on the benefits of international diversification, in that these benefits are not forthcoming when you need them the most. However, the presence of regimes should be exploitable in an active asset allocation program. The optimal equity portfolio in the high volatility regime is likely to be very different (for example more home-biased) than the optimal portfolio in the normal regime. When bonds and T-bills are considered, optimally exploiting RS may lead to portfolio shifts into bonds or cash when a bear market regime is expected. In this article, we illustrate how the presence of regimes can be incorporated into two asset allocation programs, a global asset allocation setting (with 6 equity markets, and potentially cash) and a market timing setting for US cash, bonds and equity.

In previous work, Clarke and de Silva (1998) show how the existence of two "states" (their terminology) affects mean variance asset allocation, but the article is silent about how the return characteristics in the two states may be extracted from the data. Ramchand and Susmel (1998) estimate a number of RS models on international equity return data, but do not explore how the regimes affect portfolio composition. Das and Uppal (2001) model jumps in correlation using a continuous-time jump model and investigate the implications for asset allocation. However, these jumps are only transitory and cannot fully capture the persistent nature of bear markets. Guidolin and Timmermann (2002) also consider asset allocation implications of a RS model, but they restrict attention to allocating wealth between a risk-free asset and domestic equity. Our work here builds on the framework developed in Ang and Bekaert (2002a), who investigate optimal asset allocation when returns follow various RS processes. Their article restricts attention to returns from the US, UK and Germany.

2 Data

In our first application, we focus on a universe of developed equity markets for a US-based investor. Apart from North America (Canada and the US), we consider the UK and Japan as two large markets, the euro-bloc (which we split into two parts, large and small markets) and the Pacific ex-Japan region. Table 1 details the countries involved. All data are from MSCI and the sample period is from February 1975 until the end of 2000. We measure all returns as simple net returns expressed in US dollars. In our second application, we restrict attention to US returns, allowing the US investor to implement a market-timing strategy between cash (one month T-bills), 10-year bonds and the US stock market, proxied by the S&P 500 index. Here we use a longer sample starting in January 1952 to the end of 2000.

3 A Regime-Switching Model for Equity Portfolios

3.1 Description of the Model

A World CAPM

To build a quantitative model for the 6 international asset classes, we start from the familiar CAPM using the world market return y_t^w (in excess of the US T-bill rate):

$$y_t^w = \mu^w + \sigma^w \varepsilon_t^w. \tag{1}$$

Here μ^w denotes the world market expected excess return and σ^w the conditional volatility. For modeling purposes, we assume that ε_t^w is drawn from a standard normal distribution.

The World CAPM implies a linear Security Market Line: the expected excess return on any security is linear in its beta with respect to the world market. Let individual excess returns for security j be denoted by y^j, then we have:

$$\begin{aligned} y_t^j &= (1-\beta^j)\mu^z + \beta^j \mu^w + \beta^j \sigma^w \varepsilon_t^w + \bar{\sigma}^j \varepsilon_t^j \\ &= \mu^z + \beta^j(\mu^w - \mu^z) + \beta^j \sigma^w \varepsilon_t^w + \bar{\sigma}^j \varepsilon_t^j \end{aligned} \tag{2}$$

The unexpected return on security or portfolio j is now determined by the security's sensitivity to the world market return and by an idiosyncratic term, which has volatility $\bar{\sigma}^j$.

The term $(1-\beta^j)\mu^z$ does not appear in the standard CAPM. The constant μ^z admits a flatter Security Market Line, for which there is strong empirical evidence.[3] The model in equation (2) is a version of Black (1972)'s zero-beta CAPM, which can theoretically be motivated by the presence of differential borrowing and lending rates.

Regime-Switching in the World Market Return

From an asset allocation perspective, nothing could be more boring than a CAPM which prescribes to hold the market portfolio. However, by making one critical change in the set-up of equations (1) and (2), we create a model which not only fits the empirical patterns in international equity returns but also makes quantitative asset allocation potentially fruitful.

Suppose the world expected return and conditional volatility can take on two values, depending on the realization of a regime variable which reflects the world market regime. An economic mechanism behind a world market regime is the world business cycle (expansions or recessions). We denote the world conditional expected return and volatility, which depend on regime i, as $\mu^w(i)$ and $\sigma^w(i)$, respectively. Stock markets are characterized by larger uncertainty and lower returns when a global recession is anticipated, as was the case in 2001.

We assume that the portfolio manager knows which regime is realized at each point of time, but she does not know which regime will be realized next month. Later, we discuss how the identity of the regime can be determined in practice. If we are currently in regime 1, the probability of remaining in that regime is P (and hence the probability of transitioning in the other regime is $1-P$). Similarly, if we are currently in regime 2, Q denotes the probability of staying in the second regime. Technically, the regime variable follows a Markov process with constant transition probabilities P and Q. We present this pictorially in Figure 1. There are three dates presented on the graph. Whereas at each point in time either regime can be realized, we assume, for illustration, that the actual sequence is regime 1, regime 2, and then regime 2 again.

With this change in the model, expected returns and variances now vary through time. Let us investigate time t in Figure 1. The portfolio manager knows that today the world market is in regime 1. The expected return for next period depends on the manager's expectations for the regime realization at time $t+1$ and consequently, she weights the two possible realizations of μ^w with their relevant probabilities. Note that any time when regime 1 is realized, the portfolio manager assesses the expected return to be the same. We denote the expected excess return in regime 1 by $e^w(1)$. However, when the portfolio manager finds the world market in regime 2, as is the case at time $t+1$, she uses $e^w(2)$ as the expected return. To compute this expected return, she now uses the $1-Q$ and Q probabilities to weight $\mu^w(1)$ and $\mu^w(2)$. Note that if $P = 1-Q$, then the regime structure is inconsequential for the expected returns as they are the same across regimes. However, studies like Gray (1996) and Ang and Bekaert (2002b and c) find that both P and Q are well over 50%, indicating that both bull and bear market regimes are persistent.

Analogous to the conditional mean, the conditional variance also depends on the regime. When the portfolio manager is in regime 1, as at time t in Figure 1, she anticipates that there is a probability P that the first regime will continue and that the volatility of world market news will be $\sigma^w(1)$. There is a probability of $1-P$ of transitioning to the, perhaps more volatile, second regime with volatility $\sigma^w(2)$. It is no surprise that the conditional variance is a weighted average of the conditional variances in the two regimes. However, there is an additional jump component in the conditional variance that arises because the conditional mean is also different across the two regimes. We denote the conditional variances in Figure 1 by $\Sigma^w(1)$ and $\Sigma^w(2)$ for regimes 1 and 2, respectively.

Expected Returns and Volatilities for Individual Countries

For the individual assets, we maintain the model of equation (2), except that the world market parameters, $\mu^w(i)$ and $\sigma^w(i)$, now vary across regimes. Since the mean of the world excess return switches between regimes, the expected excess return of country j is given by $(1-\beta^j)\mu^z + \beta^j e^w(i)$ for the current regime i, where $e^w(i)$ are given in Figure 1. Expected returns differ across individual equity markets only through their different betas with respect to the world market.

The conditional variance for the individual assets is quite complex. Intuitively, the conditional variance depends on three components. First, like a standard CAPM, an asset's conditional variance depends on the asset's exposure to systematic risk through the asset's beta. However, the world market return switches regimes, so the market conditional variance now also depends on the regime prevailing at time t. Second, also like a standard CAPM, each asset has an idiosyncratic volatility term unrelated to its systematic (beta) exposure. Finally, the variance of an individual asset depends not only on the realization of the current regime, but also on a jump component, which arises because the conditional means differ across the regimes.[4]

Although the model structure is parsimonious, the model generates rich patterns of stochastic volatility and time-varying correlations. In particular, the model captures the asymmetric correlation structure in international equity returns that motivated our analysis. In any factor model, correlations are higher when factor volatility is higher. Hence, if one regime is more volatile than the other regime, then the correlation between the different asset returns increases in that regime.

Estimating the Model

This model requires only the estimation of P and Q, the world market return process, the μ^z parameter, and a beta and an idiosyncratic volatility term for each country. Because the regime is not observed, the estimation involves inferring from the data which regime prevails at each point in time.[5]

Estimation Results

Table 2 contains the estimation results for the RS Equity Model. The first regime is a normal, quiet regime, where world excess returns are expected to yield 0.90% per month, with volatility of 2.81% per month. However, there is also a volatile regime (standard deviation 5.04% per month) with a lower but imprecisely estimated mean, namely 0.13% per month. The estimate of μ^z is larger than the expected excess equity return in the low volatility regime. The asset betas are estimated very precisely and their magnitudes seem economically appealing. The only surprise is that Japan, which has a rather low average return in the data, is assigned a high beta. However, Japan has the highest volatility of all the equity returns we consider, which the model fits through a high beta and a high idiosyncratic volatility (the highest idiosyncratic volatility across all markets).

In Table 3, Panel A, we report the implied expected excess returns for the six markets. Because the betas are close to 1, expected returns are close to each other in the normal regime. In the bear market regime 2, expected excess returns are dramatically lower and there is more dispersion, with the UK and Japan now having the lowest expected excess returns. In this regime, the zero-beta excess return, μ^z, is higher than the excess return on the world market, causing the high beta countries to have lower expected returns from equation (2). In fact, the expected return for Japan implied by our model is the highest of all markets in the normal regime, but by far the lowest in the bear market regime. North America, followed by Europe small, have the lowest idiosyncratic volatility implied by the model. In the data, these two countries also have the lowest overall volatility.

Panel A of Table 3 also shows the covariance and correlation matrix in the two regimes. Given that the second regime is a high volatility regime, we expect that the model will generate asymmetric correlations, with correlations being higher in the second regime. This is indeed the case, with the correlations in regime 2 being on average some 20% higher than in regime 1.

The estimation procedure also yields inferences on the regimes. Figure 2 shows the cumulative (total) returns on the 6 markets over the sample period and the ex-ante and smoothed regime probabilities. The former is the probability that the regime next month is the low volatility world market regime given current information, the latter is the probability that the regime next month is the low volatility regime given all of the information present in the data sample. Notable high volatile bear markets are the early 1980's, the period right after the October 1987 crash, the early 1990's and a period in 1999.

3.2 Asset Allocation

Mean-Variance Optimization under Regime-Switching

To implement an asset allocation strategy, we use mean-variance optimization with monthly rebalancing, consistent with the data frequency.[6] The standard optimal mean-variance portfolio vector, $w(i)$ in regime i is given by:

$$w(i) = \frac{1}{\gamma}\Sigma(i)^{-1}e(i), \tag{3}$$

where γ is the investor's risk aversion, $\Sigma(i)$ is the covariance matrix associated with regime i and $e(i)$ is the vector of conditional means for regime i.

There are a number of ways we could implement mean-variance optimization. The first issue is that we have to specify the risk-free rate. Each month, we take the 1-month T-bill rate to be the risk-free rate. Hence, the risk-free rate varies over time as we implement the asset allocation program. There are two optimal tangency (all-equity) portfolios the investor would choose in this simple example, one for each regime. An extension of this framework is to add state dependence by using predictor variables for equity returns. Our second application in Section 4 illustrates this possibility.

A second issue is that mean-variance portfolios, based on historical data, may be quite unbalanced, as Green and Hollifield (1992) and Black and Litterman (1992) emphasize. Practical asset allocation programs therefore impose constraints (short-sale constraints for example) or keep asset allocations close to the market capitalization weights. Although it is possible to do this in our application, we choose not to impose constraints at all, but show how mean-variance asset allocations perform in an out-of-sample exercise. This approach highlights the role of regimes in the asset allocation problem, not confounded by the role constraints may play.

Panel B of Table 3 shows the tangency portfolios in regime 1 and regime 2. In the normal regime 1, the investor places 42% of her wealth in the North American portfolio, which is not too far from the average relative market capitalization over the sample period. The European and Pacific indices are over-weighted relative to their market capitalizations, but the UK and Japanese markets are under-weighted, due to the implied high volatility of the UK and Japanese markets. (There is even a small short position for the Japanese market.) In regime 2, the investor resolutely switches towards the less volatile markets, which includes North America. This does not mean the portfolio is now home-biased because the investor also invests more heavily in the European markets, allocating more than 50% of her wealth to Europe small. The short position in Japan is now quite substantial, exceeding 50%.

Figure 3 shows the essence of the implications of RS for asset allocation. The solid line represents the frontier using the unconditional moments, ignoring regime switches. The other frontiers are the ones applicable in the two regimes. The frontier near the top represents the normal regime. The risk-return trade-off is generally better here, because the investor takes into account that, given that the regime is persistent, the likelihood of a bear market regime with high volatility next period is small. The Sharpe ratio available along the capital allocation line (the line emanating from the risk-free rate on the vertical axis tangent to the frontier) is 0.871. In the bear market regime, the risk-return trade-off worsens and the investor selects a very different portfolio, only realizing a Sharpe ratio of 0.268 with the tangency portfolio. When we average the moments in the two regimes, we obtain an unconditional frontier implied from the RS model. The best possible Sharpe ratio for this frontier is 0.505. Note that the world market portfolio (using average market capitalization weights) is inefficient; it is inside the unconditional frontier.

Theoretically, the presence of two regimes and two frontiers means that the RS investment opportunity set dominates the investment opportunity set offered by one frontier. In particular, in regime 1, the unconditional tangency portfolio yields a Sharpe ratio of 0.619. The investor could improve this trade-off to 0.871 holding the risk-free asset and the optimal tangency portfolio for this low variance regime. In regime 2, the unconditional tangency portfolio yields a

Sharpe ratio of only 0.129, which could be improved to 0.268 holding the optimal tangency portfolio for the high variance regime.

Practical Implementation

We show the results of an asset allocation strategy starting with $1 in 1985. The analysis is out-of-sample. The RS model is estimated up to time t, and the RS and non-regime dependent weights are computed using information available only up to time t. The model is re-estimated every month. The non-regime dependent strategy uses means and covariances estimated from data up to time t. Our performance criterion is the ex-post Sharpe ratio realized by the various strategies.

The RS strategy requires the risk-free rate and the realization of the regime. For the first, we simply take the available one-month Treasury bill. To infer the regime, the investor computes the regime probability from current information, which is a by-product of the estimation of the RS model. If the regime probability is larger than a half for regime 1, the investor classifies the regime as 1, otherwise she classifies it as 2. This calculation does not require any further data input.

Table 4 reports that over the out-sample, the RS strategy's Sharpe Ratio is 1.07, more than double the out-sample world market portfolio Sharpe Ratio (0.52). This is also higher than the non-regime dependent Sharpe Ratio (0.90). The RS strategy does so well because over this sample period the US market records very large returns, Japan performs very poorly, and the world market portfolio features a relative large Japanese equity allocation. In fact, the US Sharpe ratio over the period is 0.65! In the normal regime, the all-equity portfolio for the RS model has a very large weight on North America (see Panel B of Table 3). In the bear market regime, the RS strategy has a very large short position in Japanese equities.

Figure 4 shows how wealth cumulates over time in these strategies. The large North American and the short Japanese positions imply that both the RS and the non-regime dependent strategies out-perform the world market and the North American market consistently. Nevertheless, the out-performance is particularly striking for the last 5 years. It is also over the last 5 years that the RS strategy outperforms the non-regime dependent strategy particularly successfully.

Given that this example is highly stylized and our results may be intimately linked to a perhaps special historical period, we do not want to claim that the success of the RS strategy shown here is a good indicator for future success. For example, not all investors will feel comfortable implementing the relatively large short positions implied by the model. The important conclusion to draw is that RS strategies have the potential to out-perform because they set up a defensive portfolio in the bear market regime that hedges against high correlations and low returns. This conclusion would remain valid in the presence of short-sale constraints because this portfolio essentially tilts the allocations towards the lowest volatility assets. This portfolio need not be completely home-biased, and in our example still involves substantial net international positions. It is likely that in any practical implementation of a RS model, which relies less on historical moments, or is based on a different sample period, the optimal portfolios should be even more internationally diversified. In Ang and Bekaert (2002a), this is actually the case.

4 A Regime-Switching Market-Timing Model

4.1 Description of the Model

The Statistical Model

When short rates are low, subsequent equity returns tend to be high. Hence, when a bear market regime is expected, the optimal asset allocation response may be to switch to a safe asset or a bond. The model we explore in this section considers asset allocation among three assets, cash, a 10 year (constant maturity) bond and an equity index (all for the US). We formulate the model in excess returns. We use r_t to denote the risk-free rate (the nominal T-bill rate), r_t^b as the excess bond return and r_t^e as the excess return on US equity.

The Market-Timing Model is given by:

$$\begin{aligned} r_t &= \mu^r(i) + \rho(i) r_{t-1} + \varepsilon_t^1 \\ r_t^b &= \mu^b + \varepsilon_t^2 \\ r_t^e &= \mu^e + \varepsilon_t^3, \end{aligned} \quad (4)$$

which allows the short rate to exhibit different behavior over each regime i. The error terms $\varepsilon_t = (\varepsilon_t^1 \, \varepsilon_t^2 \, \varepsilon_t^3)'$ are drawn from a normal distribution with zero mean but with a covariance that switches across the regimes, so the conditional volatility of all assets is regime-dependent.

The short rate follows an autoregressive process, but the constant term $\mu^r(i)$ and the autoregressive parameter $\rho(i)$ depend on the regime. Many articles, like Ang and Bekaert (2002b), demonstrate that the data support such a model, where one regime captures normal times in which interest rates are highly persistent and not too variable ($\rho(1)$ is close to 1), and another regime captures times of volatile, higher interest rates which revert quickly to lower rates ($\rho(2) < \rho(1)$).

In the RS Equity Model of Section 3, the transition probabilities between the regimes, P and Q were constant. In our Market-Timing Model, we allow the interest rate to influence the transition probabilities. Hence, P_t and Q_t are now time-varying.[7] For example, if interest rates are high, this might affect the probability of staying in the "normal" or "bad" regimes differently than if interest rates are low. Consequently, the short rate predicts transitions in the regime and hence implies time-variation in expected returns. The predictive power of nominal interest rates for equity premiums has a long tradition in finance going back to at least Fama and Schwert (1977), but most studies allow only linear predictability, entering through the conditional mean. If we allow the conditional means of excess bond and equity returns to become regime-dependent and also allow the lagged short rate to enter the conditional mean, these coefficients are estimated with little precision. We cannot reject our model relative to this more intricate specification.

Estimation Results

The first regime is a "normal" regime, where the short rate is nearly a random walk ($\rho(1) = 0.99$), shocks to the interest rate are not very variable (standard deviation 0.02% per month) and shocks to excess bond and equity returns are less volatile (standard deviations 1.75% and 3.41% per month, respectively). In the second regime, there are large, rapidly mean-reverting, volatile interest rates. Here, the short rate is much less persistent ($\rho(2) = 0.94$) and interest rates have a conditional volatility of 0.09% per month. Bond and equity return shocks are also much more volatile, with standard deviations of 3.98% and 5.55% per month, respectively. The mean for the excess bond return is 0.07% per month, and the mean excess equity return is 0.68% per month.

Figure 5 graphs the transition probability functions. P is the probability of staying in regime 1, given that we are currently in regime 1. As interest rates rise, the probability of transitioning into the high volatility and bear market regime becomes higher. Q is the probability of remaining in regime 2, given that we are currently in regime 2. In the second regime, as interest rates move higher, the probability of staying in a bear market increases. A constrained model where P and Q are constant is strongly statistically rejected. Hence, non-linear predictability is an important feature of the data. The long-run probability of the normal regime implied by the model is 0.7014.

4.2 Asset Allocation

Mean-Variance Asset Allocation in the Market-Timing Model

We follow the same mean-variance strategy as Section 3.2, except now the optimal asset allocation vector is a function of the expected excess returns on the two risky assets, the bond and equity, and their covariance matrix.[8]

To obtain intuition on the asset allocation weights for this model, Figure 6 graphs the optimal asset allocations to bonds and stocks (which add to 1 minus the weight assigned to the risk-free asset) as a function of the short rate at the estimated parameters. We set the risk aversion level to $\gamma = 5$. In regime 1, if interest rates are low enough, the investor borrows at the risk-free rate and invests a small fraction of her portfolio in bonds and more than 100% in equities. As interest rates rise, equities become less attractive as the probability of switching to the high variance regime increases. Bonds also become less attractive and because the bond premium is very small, it quickly becomes optimal to short bonds. In the second regime, the investor always shorts bonds, but the investment in equities is never higher than 80%. The main hedge for volatility clearly is the risk-free asset, not a bond investment.

Because the interest rate is so important in this model, the optimal asset allocation varies over time with different realizations of the interest rate. Figure 7 shows optimal asset allocation weights for all three assets across time for the full-sample, assuming that the investor uses the moments implied by the full sample estimation. Note that during the 1987 crash, the investor is heavily invested in equity. After the crash the investor shifts this equity portion into risk-free holdings. Importantly, the asset allocations show only infrequent large changes in asset allocation, which coincide with regime changes. Because interest rates are relatively smooth and persistent, the month-to-month changes in asset allocation are often modest.

Out-of-Sample Performance of the Market-Timing Model

We consider an out-of-sample exercise, similar to the exercise in Section 3.2, starting with $1 in 1985. We show the mean return, volatility and Sharpe ratio to following the optimal RS strategies for the Market-Timing Model and compare it to a strategy that simply uses unconditional moments. The results are reported in Table 5. The Market-Timing Model's strategy is more volatile, but delivers higher average returns, than a non-regime dependent strategy. The Market-Timing Model is the best performing model in terms of Sharpe ratios, but Sharpe ratios become quite low for highly risk averse people.

Figure 8 shows that the superior performance is not due to a few isolated months in the sample, but that the last 5 years do play an important role in giving the RS strategies an edge. During these years, the Market-Timing Model allocates more money to equity and benefits handsomely from the US bull market. However, the RS strategy's positions are more leveraged and although they have higher returns, they also have higher volatility.

5 Conclusion

There is much evidence in the academic literature that both expected returns and volatility vary through time. Moreover, in high volatility environments across the world, equity returns become more highly correlated and do not perform very well. If this is true, active portfolio management should be able to exploit these regime changes to add value. In this article, we show how this can be formally accomplished. Our results are meant to be illustrative. On the one hand, we exaggerate the performance of the models, because we do not take transaction costs into account. Of course, the RS strategies are relatively robust to transactions costs because they are designed to exploit low frequency changes in expected returns and volatilities. Because the probability of staying within the same regime is relatively high, portfolio turnover is low. On the other hand, we greatly undersell the potential of regime-switching (RS) models, because we did not try to estimate the best possible model, do an extensive model search, or incorporate performance-enhancing constraints.

There is a long list of extensions that can be accommodated in the framework and are likely to improve performance. First, equity portfolio allocation programs typically are compensated based on tracking error relative to an index. Therefore, active management often starts from expected returns reverse-engineered from a benchmark, as in Black and Litterman (1992), and deviates from the benchmark towards the predictions of a proprietary model. Instead, we have used only historical data.

Second, in international asset allocation, it is often the case that the equity benchmarks are hedged against currency risk. Ang and Bekaert (2002a) show that the RS Equity Model can be extended to allow both currency hedged and non-hedged returns. In this case, the asset allocation model yields the optimal currency hedge ratio.

Third, we have assumed that there is only one regime variable. However, it would be interesting to test whether there are country-specific regimes, and whether the regimes in short rates and equity returns are less than perfectly correlated.

Finally, in the optimization we have only focused on first and second moments, but many investors prefer positive skewness and dislike kurtosis. RS models have non-trivial higher order moments, because they can be interpreted as a time-varying mixture of normals model. For investors with preferences involving higher order moments of returns, RS models are a viable alternative to consider.

Our current results point to two robust conclusions. First, whereas it is possible to add value in all equity portfolios, the presence of a bear market, high correlation regime does not negate the benefits of international diversification. Although portfolios in that regime are more home-biased, they still involve significant international exposures. Second, it is most valuable to consider RS models in tactical asset allocation programs that allow switching to a risk-free asset.

Notes

[1]See also Erb, Harvey and Viskanta (1994) and Campbell, Koedijk and Kofman (2002).

[2]See Gray (1996), Bekaert, Hodrick and Marshall (2001) and Ang and Bekaert (2002b and c).

[3]See Black, Jensen and Scholes (1972) for an early example.

[4] The expected return for asset j with beta β^j in regime i is $e^j(i) = (1 - \beta^j)\mu^z + \beta^j e^w(i)$. There are two possible variance matrices for unexpected returns next period, given by $\Omega^j(i) = (\beta^j)^2[\sigma^w(i)]^2 + [\bar{\sigma}^j]^2$, for $i = 1, 2$. The conditional variance of asset j in regime 1 is then $[\sigma^j(1)]^2 = P\Omega^j(1)^2 + (1-P)[\Omega^j(2)^2] + P(1-P)[e^j(2) - e^j(1)]^2$ and the conditional variance of asset j in regime 2 is: $[\sigma^j(2)]^2 = (1 - Q)\Omega^j(1)^2 + Q[\Omega^j(2)^2] + Q(1 - Q)[e^j(2) - e^j(1)]^2$.

[5]See Hamilton (1994) and Gray (1996) for estimation methods of RS models using maximum likelihood techniques.

[6]Because the first and second moments of our model vary through time, investors with different horizons may hold different portfolios. However, Brandt (1999) and Ang and Bekaert (2002a) show that the differences across these portfolios are not large and we ignore them here.

[7] Specifically, we set $P_t = \frac{\exp(a_1 + b_1 r_t)}{1 + \exp(a_1 + b_1 r_t)}$ and $Q_t = \frac{\exp(a_2 + b_2 r_t)}{1 + \exp(a_2 + b_2 r_t)}$.

[8]The determination of conditional expected returns and variances is similar to the procedure in Section 3.1, except that the transition probabilities vary over time.

References

Ang, A., and G. Bekaert. 2002a. “International Asset Allocation with Regime Shifts.” *Review of Financial Studies*, vol. 15, no. 4 (Fall):1137-1187.

Ang, A., and G. Bekaert. 2002b. “Regime Switches in Interest Rates.” *Journal of Business and Economic Statistics*, vol. 20, no. 2 (April):163-182.

Ang, A., and G. Bekaert. 2002c. “Short Rate Nonlinearities and Regime Switches.” *Journal of Economic Dynamics and Control*, vol. 26, no. 7-8 (July):1243-1274.

Bekaert, G., R.J. Hodrick, and D. Marshall. 2001. “Peso Problem Explanations for Term Structure Anomalies.” *Journal of Monetary Economics*, vol. 48, no. 2 (October):241-270.

Black, F. 1972. “Capital Market Equilibrium with Restricted Borrowing.” *Journal of Business*, vol. 45, no. 3 (July):444-454.

Black, F., M. Jensen, and M. Scholes. 1972. “The Capital Asset Pricing Model: Some Empirical Tests.” *Studies in the Theory of Capital Markets*, Jensen. M., ed., New York: Praeger.

Black, F., and R. Litterman. 1992. “Global Portfolio Optimization.” *Financial Analysts Journal*, September:28-43.

Brandt, M.W. 1999. “Estimating Portfolio and Consumption Choice: A Conditional Euler Equations Approach.” *Journal of Finance*, vol. 54, no. 5 (October):1609-1646.

Campbell, R., K. Koedijk, and P. Kofman. 2002. “Increased Correlation in Bear Markets.” *Financial Analyst Journal*, January-February:87-94.

Clarke, R.G., and H. de Silva. 1998. “State-Dependent Asset Allocation.” *Journal of Portfolio Management*, vol. 24, no. 2, (Winter):57-64.

Das, S.R., and R. Uppal. 2001. “Systemic Risk and Portfolio Choice.” working paper, London Business School.

Erb, C.B., C.R. Harvey, and T.E. Viskanta. 1994. “Forecasting International Equity Correlations.” *Financial Analysts Journal*, November-December:32-45.

Fama, E., and G.W. Schwert. 1977, “Asset Returns and Inflation.” *Journal of Financial Economics*, vol. 5, no.2:115-146.

Gray, S.F. 1996. “Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process.” *Journal of Financial Economics*, vol. 42, no. 1 (September):27-62.

Green, R., and B. Hollifield. 1992. “When Will Mean-Variance Efficient Portfolios be Well Diversified?” *Journal of Finance*, vol. 47, no. 5 (December):1785-1809.

Guidolin, M., and A. Timmermann. 2002. “Optimal Portfolio Choice under Regime Switching, Skew and Kurtosis Preferences.” working paper, UC San Diego.

Hamilton, J.D. 1989. “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle.” *Econometrica*, vol. 57, no. 2 (March):357-384.

Longin, F., and B. Solnik. 2001. “Correlation Structure of International Equity Markets During Extremely Volatile Periods.” *Journal of Finance*, vol. 56, no. 2 (April):649-676.

Ramchand, L., and R. Susmel. 1998. “Cross Correlations Across Major International Markets.” *Journal of Empirical Finance*, vol. 5, no. 4 (October):397-416.

Table 1: Composition of International Returns

North America	UK	Japan	Europe large	Europe small	Pacific ex-Japan
Canada			France	Austria	Australia
US			Germany	Belgium	New Zealand
			Italy	Denmark	Singapore
				Finland	
				Ireland	
				Netherlands	
				Norway	
				Spain	
				Sweden	
				Switzerland	

The table lists the country composition of the geographic returns. Within each geographic region, we construct monthly simple returns, value-weighted in US dollars.

Table 2: Regime-Switching Equity Model Parameter Estimates

Transition Probabilities and μ_z

	P	Q	μ^z
Estimate	0.8917	0.8692	0.74
Std error	0.0741	0.1330	0.68

World Market

	$\mu(1)$	$\mu(2)$	$\sigma(1)$	$\sigma(2)$
Estimate	0.90	0.13	2.81	5.04
Std error	0.32	0.62	0.44	0.55

Country Betas β

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	0.88	1.03	1.21	0.90	0.89	0.92
Std error	0.03	0.06	0.07	0.05	0.04	0.07

Idiosyncratic Volatilities $\bar{\sigma}$

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	2.40	4.50	4.62	3.87	2.72	4.99
Std error	0.09	0.18	0.19	0.16	0.11	0.20

All parameters are monthly and are expressed in percentages, except for the transition probabilities P and Q.

Table 3: Regime-Switching Equity Model Asset Allocation

Panel A: Regime-Dependent Means and Covariances

Regime-Dependent Excess Returns

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Regime 1	9.64	9.76	9.90	9.65	9.65	9.67
Regime 2	3.47	2.54	1.42	3.36	3.39	3.22

Regime-Dependent Covariances/Correlations

Regime 1						
N Amer	1.35	[0.44]	[0.48]	[0.45]	[0.54]	[0.38]
UK	0.90	3.08	[0.37]	[0.35]	[0.42]	[0.29]
Japan	1.06	1.25	3.60	[0.38]	[0.46]	[0.32]
Eur lg	0.79	0.92	1.08	2.30	[0.43]	[0.30]
Eur sm	0.78	0.91	1.07	0.80	1.53	[0.36]
Pac	0.81	0.94	1.11	0.82	0.82	3.33
Regime 2						
N Amer	2.37	[0.64]	[0.68]	[0.65]	[0.73]	[0.58]
UK	2.10	4.49	[0.58]	[0.55]	[0.63]	[0.49]
Japan	2.47	2.89	5.53	[0.58]	[0.66]	[0.52]
Eur lg	1.83	2.14	2.52	3.36	[0.63]	[0.49]
Eur sm	1.82	2.13	2.50	1.85	2.58	[0.56]
Pac	1.88	2.20	2.58	1.91	1.90	4.45

Panel B: Tangency Portfolio Weights

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Regime 1	0.42	0.06	-0.01	0.15	0.31	0.08
Regime 2	0.79	-0.14	-0.55	0.25	0.54	0.10
Unconditional	0.52	0.04	-0.16	0.18	0.37	0.09
Ave Mkt Cap	0.50	0.09	0.22	0.08	0.08	0.02

We report the regime-dependent means and covariances of excess returns implied by the estimates of the RS Equity Model in Table 2. Panel A reports the regime-dependent excess return means and covariances, where we list correlations in the upper-right triangular matrix in square brackets. All numbers are listed in percentages, and are annualized. Panel B reports the mean variance efficient (MVE) (tangency) portfolios, computed using an interest rate of 7.67%, which is the average 1-month T-bill rate over the sample. The Ave Mkt Cap denotes the average market capitalization, averaged across the sample.

Table 4: All-Equity Portfolio Allocation with the Regime-Switching Equity Model

	World Market	N America	Regime Dependent	Non-Regime Dependent
Mean ret	13.73	15.84	21.46	20.04
Stdev ret	14.86	15.21	14.51	15.67
Sharpe Ratio	0.52	0.65	1.07	0.90

We consider all-equity portfolio holdings on an out-sample of the last 180 months (Jan 1985 to Dec 2000). The model is estimated up to time t, and the regime-dependent and non regime-dependent weights are computed using information available only up to time t. We use the actual 1 month T-bill yield at time t as the risk-free asset. The model is re-estimated every month. The non-regime dependent strategy estimates means and covariances from data up to time t. The Non-Regime Dependent Allocations are computed with static one-period mean-variance utility, using the returns up to time t. The columns labelled ‘World Market’ and ‘N America’ refer to returns on holding a 100% world market and 100% North American portfolio, respectively. All returns are annualized and are reported in percentages.

Table 5: Out-of-Sample Portfolio Allocation Back-Testing with the Market-Timing Model

Regime-Dependent Allocations

Risk Aversion γ	2	3	4	5	10
Mean ret	25.29	17.69	13.89	11.61	7.05
Stdev ret	34.53	23.02	17.27	13.82	6.91
Sharpe Ratio	0.58	0.54	0.50	0.47	0.27

Non Regime-Dependent Allocations

Risk Aversion γ	2	3	4	5	10
Mean ret	17.65	12.60	10.07	8.55	5.52
Stdev ret	26.25	17.50	13.13	10.50	5.26
Sharpe Ratio	0.48	0.42	0.37	0.32	0.07

We present the mean, standard deviation and Sharpe ratios of out-of-sample returns following the Market-Timing Model and a naïve non-regime dependent strategy over an out-sample of the last 15 years (Jan 1985 to Dec 2000) are used. Over the out-sample, the model is estimated up to time t, and the regime-dependent and non regime-dependent weights are computed using information available only up to time t. The model is re-estimated every month. The non-regime dependent strategy estimates means and covariances from data up to time t. All returns are annualized and are reported in percentages.

Figure 1: A Regime-Switching Model for the World Market

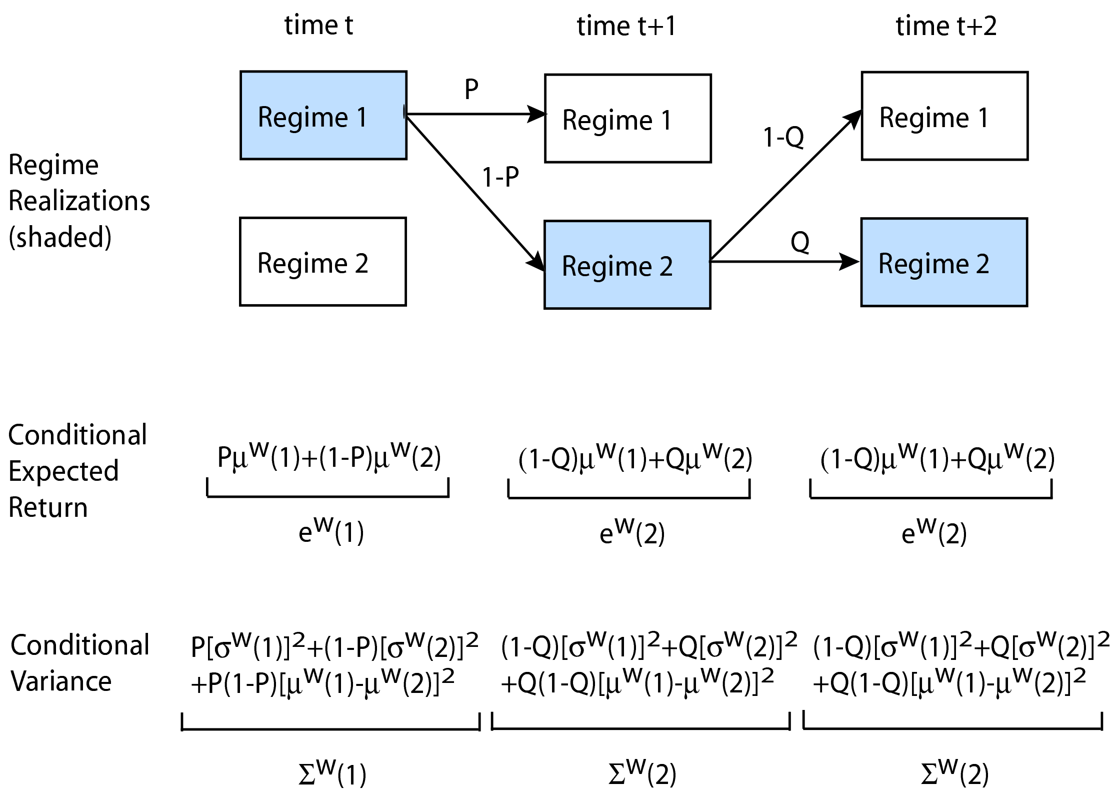
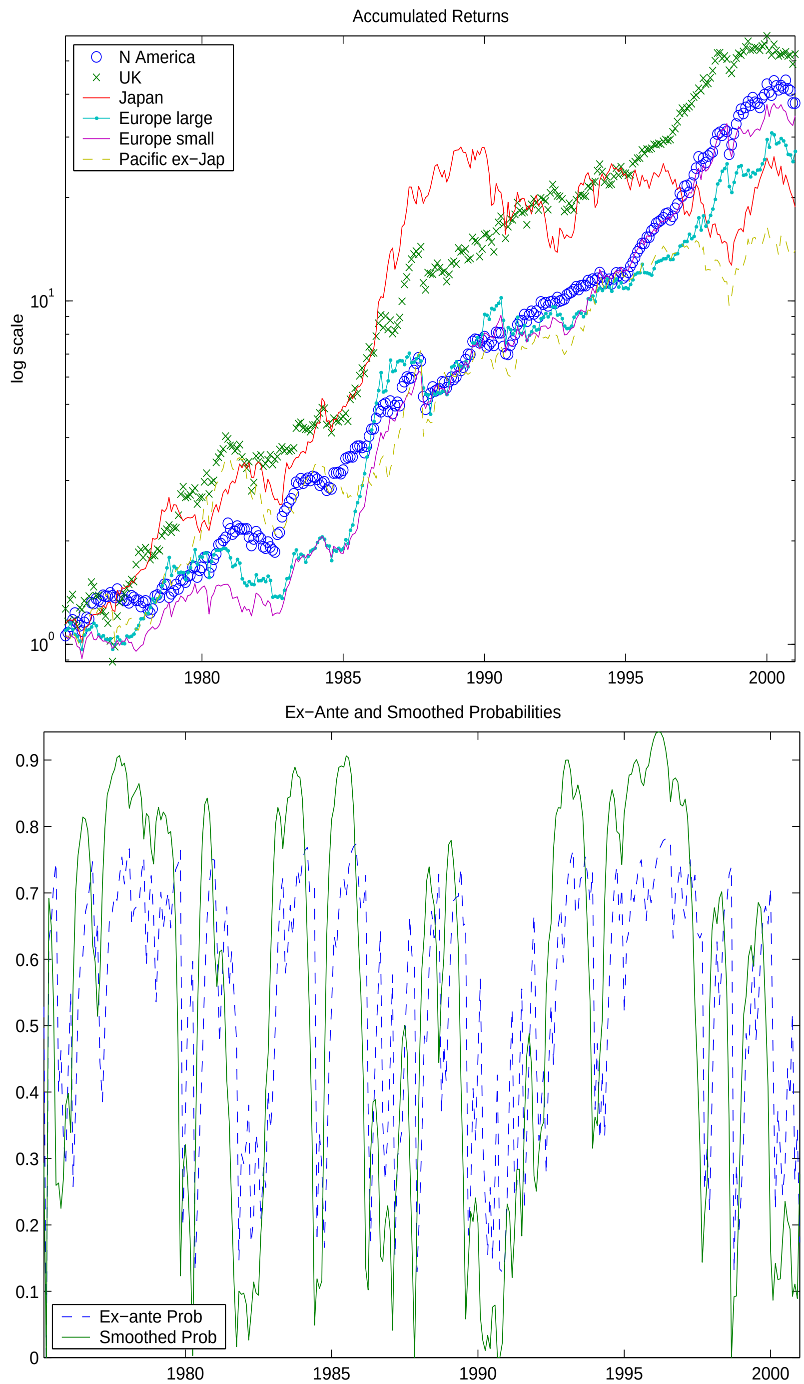
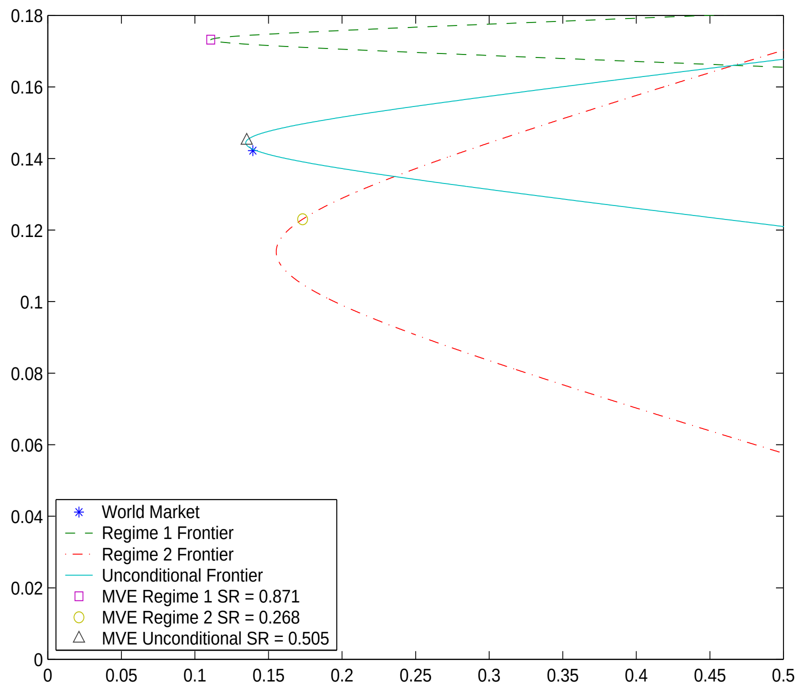


Figure 2: Ex-Ante and Smoothed Proabilities of the Beta Model



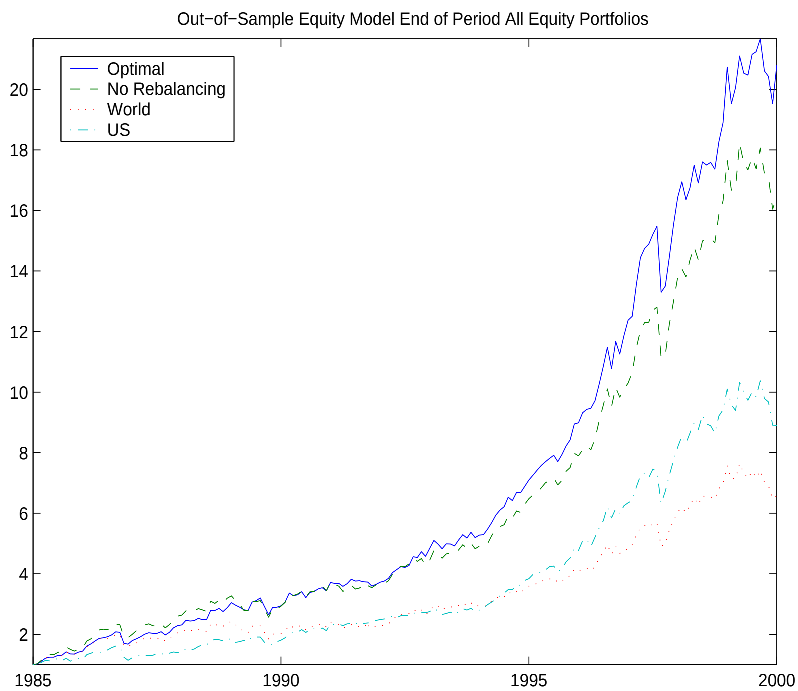
The top plot shows the accumulated total returns of $1 at Jan 1975, through the same until Dec 2000 of each of the geographic regions. The bottom plot shows the ex-ante probabilities (using information up until time $t-1$) and the smoothed probabilities (using all sample information) of being in the first regime, where the first regime is the world low variance regime.

Figure 3: Mean-Standard Deviation Frontiers of the Regime-Switching Equity Model



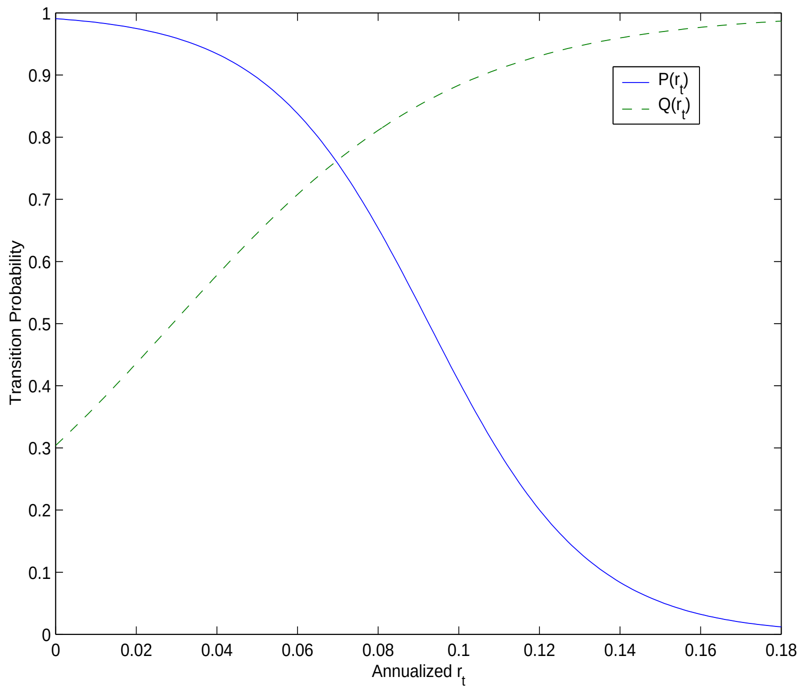
We plot the mean-variance frontier of regime 1 (the world low variance regime), regime 2 (high variance regime), and the unconditional mean-variance frontier, which averages across the two regimes. The mean variance efficient (tangengy) MVE portfolios for each frontier are also marked. The mean and variance have been annualized by multiplying by 12. We also mark the position of the World Market as an asterix.

Figure 4: Out-of-Sample Wealth for the Regime-Switching Equity Model



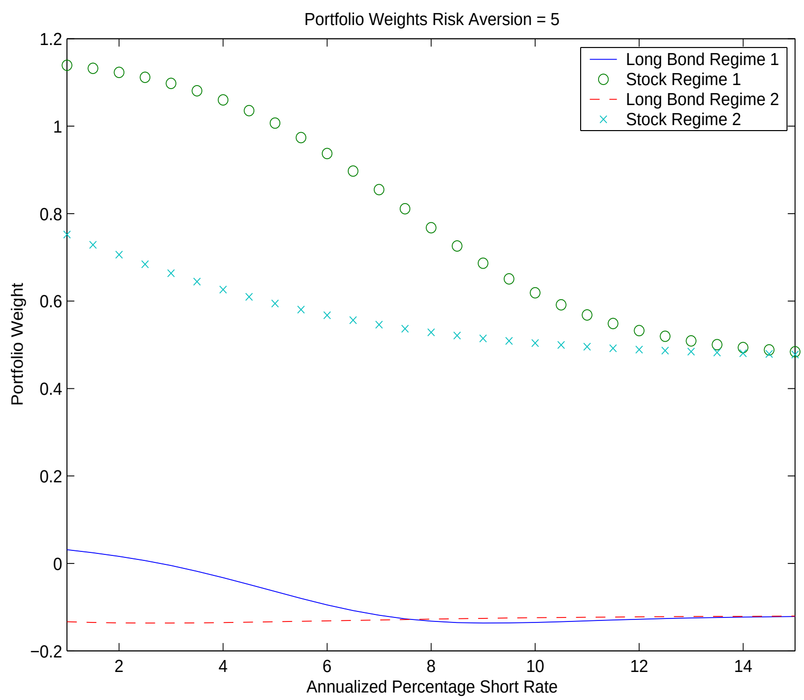
We show the out-of-sample wealth for the value of $1 at Jan 1985 for the Regime-Switching Equity Model, contrasted with a static mean-variance strategy, and the returns for the world and US portfolios.

Figure 5: Transition Probabilities of the Market-Timing Model



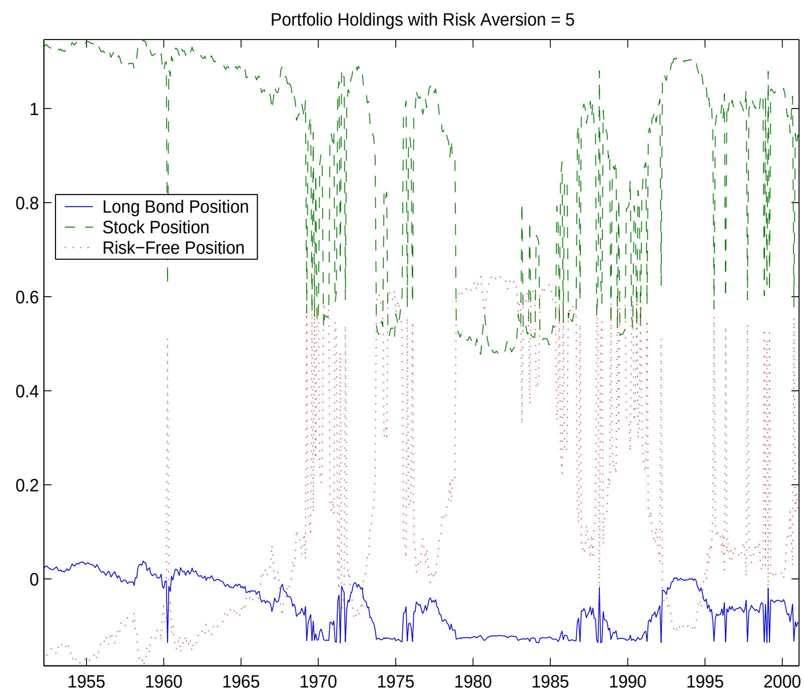
We graph the probability of staying in the normal regime next period, P_t, given that we are currently in the normal regime at t, as a function of r_t. We also graph the probability Q_t of staying in the bear market regime next period, given that we are currently in the bear market regime at t.

Figure 6: Asset Allocation of the Market-Timing Model as a Function of the Short Rate



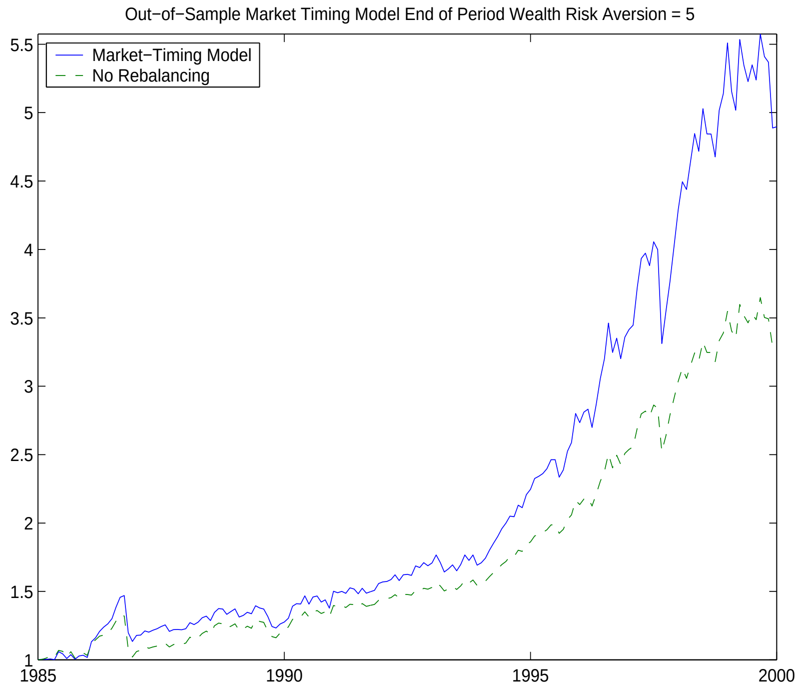
The figure plots the position in bonds and stocks as a function of the short rate for the Market-Timing Model.

Figure 7: Asset Allocation of the Market-Timing Model Across Time



We show the position in bonds, stocks and the risk-free asset across time for the Market-Timing Model.

Figure 8: Out-of-Sample Wealth for the Market-Timing Model



We show the out-of-sample wealth for the value of $1 at Jan 1985 for the Market Timing Model and the static mean-variance strategy.