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RETIREMENT ANNUITY DESIGN IN AN INFLATIONARY CLIMATE

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ABSTRACT

This paper examines the tilt and risk-return characteristics of real retirement incomes provided by variable annuities tied to bills, long-term bonds, stocks and a mixed portfolio which combines all three. The analysis emphasizes the riskiness of the real value of benefits provided by conventional nominal annuities. The Rockefeller Foundation Plan, together with the "ad hoc" cost-of-living adjustments made by many large firms, are interpreted as representative market responses to increased inflation uncertainty. The paper examines the annuity designs implicit in these innovations, and shows them to be variants of the standard variable annuity.

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A pensioner who receives his benefit in the form of a nominal annuity has claim to a stream of payments whose nominal value is certain. The real value of this claim, however, will be highly uncertain if there exists substantial uncertainty regarding the future level of prices. Since this appears to be the case, and since pensioners are presumably concerned with the real value of their retirement incomes, the question of annuity design in an inflationary climate merits increased attention. Our belief is that at least some individuals may find alternatives to the nominal level-payment annuity better suited to their needs or preferences in an environment of substantial inflation uncertainty.[1] If individuals are to make rational choices, they must first understand the risk-return and other characteristics of alternative annuity designs. The primary objective of this paper is to clarify these issues.

The first task is to examine the streams of real benefits that are likely to be provided by variable annuities (VA's). Although equity-based VA's appear to have fallen into some disfavor, perhaps because of the lack of attention accorded the inherent volatility of common stocks, recent work by Bodie (1980, 1981) suggests that VA's tied to bills or short-term bonds may produce income streams that are quite stable in real terms. VA's backed by bills, long-term bonds, common stocks and a mixed portfolio are thus examined, and the results contrasted with those for a graduated-payment, nominal annuity.

The second task is to examine novel annuity designs which have emerged in recent years, in which floors or floors together with ceilings have effectively been added to the standard VA. The Rockefeller Foundation plan, for example, provides cost-of-living adjustments which equal the average prime interest rate for the year less 3% (Heaton (1977)). Once granted, these adjustments are never reduced and thus the annuitant - in effect - has a VA subject to a nominal floor.[2] Annuities provided by the Teachers Insurance and Annuity Association (TIAA) also have a guaranteed nominal floor.[3] In recent years, large firms in both Canada and the United States have frequently made "ad hoc" cost-of-living adjustments to the pensions of retired workers. In Canada, these adjustments have often been financed from pension fund earnings in excess of the plan's valuation rate (Pesando (1981)).[4] Once granted, these adjustments are again permanent. Moreover, there appears to be a ceiling on these adjustments in that the real value of the initial benefit is never increased even if "excess" fund earnings might so permit. The second part of the paper thus examines a variable annuity subject to a nominal floor (VAF) and a variable annuity subject to a nominal floor and a real ceiling (VAFC). The former is suggested by the Rockefeller plan, and both may be viewed as an attempt to formalize the apparent practice of many firms in granting cost-of-living adjustments to retired plan members. Unlike VA's, the cost of VAF's and VAFC's is not known with certainty at their date of purchase.

The paper is organized as follows. The performance of a nominal, level-payment annuity is first contrasted with that of a hypothetical "purchasing power annuity" for the period 1971-1980. The latter is formally equivalent to a VA backed by an index bond yielding a certain real return of zero percent. Theoretical distributions of the real payments from VA's tied to alternative asset bases are then presented, and serve to illustrate the nature of the tradeoff between risk and expected real returns. These payments are also contrasted with those provided by a graduated-payment, nominal annuity. The properties of VAF's are then explored, and simulations conducted to contrast their performance with VA's backed by identical asset portfolios. The same exercise is then repeated for VAFC's. To place the alternative annuity designs in a final perspective, an historical simulation is conducted for the period 1971-1980. A summary and conclusion completes the paper.

1. The Level-Payment, Nominal Annuity

The nominal and real values of the benefits provided by a nominal, level-payment annuity during the period 1971-1980 are illustrated in Table 1. The annuity is purchased at the beginning of 1971 for the sum of $100,000, the annuity is sold at a (nominal) interest rate of 7.5 percent, and the benefits are payable with certainty for 10 years.[5] The real value of the annual, nominal payment declines by more than 50 percent during the decade. Further, it is likely that a substantial portion of this decline was unanticipated. If the anticipated rate of inflation embodied in the nominal rate of interest was 5 percent, then the annuitant would have expected the real value of his benefit to decline at about 5 percent per year.[6] Deviations around this rate of decline would then have been unanticipated.

For illustrative purposes, the performance of a "purchasing power annuity" (PPA) for the period 1971-80 is also shown in Table 1. This annuity is fully linked to the consumer price index and is sold at a certain real return of zero percent. Earlier work by Bodie (1980) indicates that the minimum-variance portfolio (in the absence of short selling) consists of one-month Treasury bills hedged with commodity futures, and that the expected real return on this portfolio would not exceed zero percent. For simplicity, it is assumed that a portfolio could be constructed which would provide a certain real return of zero percent. The PPA is analytically equivalent to a VA tied to an index bond which provides this certain real return. Although the stream of real payments provided by the PPA is certain, there is no requirement that this stream of payments be constant. If RV is the annuity valuation rate used to determine the base value of the annuity payment and if r is the certain real return on the portfolio, then the real value of the annuity payments will change with certainty at an annual rate equal to $(1 + r)/(1 + RV) - 1$. (See Appendix for details.) With $r = 0$ and $RV = 5$, the real benefit declines with certainty at 4.76 percent per year, as shown in Table 1. When r and RV are equal, there is no tilt to the projected stream of real annuity payments. If the real return is uncertain, the previous expression depicts the expected degree of tilting in the real payments stream. If pensioners wish to design a stream of pension payments which is expected to decline in real terms, perhaps due to liquidity constraints or estate motives, this is readily accomplished with vehicles other than the nominal, level-payment annuity. The downward tilt in the real benefit provided by a nominal, level-payment annuity is, of course, equal to the expected rate of inflation.

2. Variable Annuities with Alternative Asset Bases

The limitations of fixed-dollar annuities in an inflationary climate prompted life insurance companies in the 1950's to offer equity-based VA's. As emphasized by Bodie (1980), however, an equity-based VA exposes the annuitant to substantial investment risk even if it is assumed that the real return on equities is unaffected by unanticipated changes in the rate of inflation.[7] The purpose of this section is to explore the real income streams provided by VA's with alternative asset bases.

Theoretical distributions are presented in Table 2 for the real benefits provided by VA's backed by one-month Treasury bills, long-term U.S. government bonds, common stocks, and a mixed portfolio. (The mechanics of a variable annuity are detailed in the Appendix.) The mixed portfolio is the minimum variance portfolio with the same expected return as the long-term bond portfolio.[8] The VA's are purchased for $100,000 and benefits are paid with certainty for 15 years. The real returns on bills, bonds, and stocks are assumed to be lognormally distributed with means of zero, 2.956 and 7.232 percent, respectively, and standard deviations of 1.52, 7.64 and 18.61 percent. The means are the continuous time equivalents of annual returns of zero, 3 and 7.5 percent. These parameters, together with the covariances necessary to construct the mixed portfolio, are based on historical data for the period 1953-80.[9] The valuation rates used to determine the base level of the annuity payments are the annual equivalents of the continuously compounded real rates of return. Examination of the historical data indicates that real bill returns, but not those on stocks and bonds, are serially correlated.[10] For this reason, the theoretical distribution of real benefit payments is also calculated for a bills-based VA on the assumption that real bill returns are serially correlated.

The assumption that real returns are lognormally distributed implies that annuity payments are also lognormally distributed. Since the valuation rates used to calculate the base values of the benefits are the annual equivalents of the expected real returns on the portfolios, median benefit payments show no tendency to rise or to fall over time.[11] Because these payments are lognormally distributed, they exhibit positive skewness and thus the mean payments rise steadily over time. The distribution of real benefits provided by a graduated-payment, nominal annuity is also included in Table 2. For this annuity, all of the uncertainty regarding the real value of the benefit payments stems from price level uncertainty. Thus, an additional set of assumptions is required. The price level is assumed to be lognormally distributed, and the continuously compounded rate of inflation is assumed to follow the first order autoregressive process which characterizes the period 1953-1980.[12] The degree of graduation is set equal to 8 percent, which is the (annual) steady state rate of inflation implied by the autoregression.[13] The purpose of including the graduated, nominal annuity is to emphasize the fact that while its nominal payments are devoid of risk, its real payments are not.

The distributions of real benefit payments reported in Table 2 mirror the risk-return characteristics of the underlying portfolios. The stream of real benefits provided by the bills-based VA is smaller and more stable than the stream provided by the bonds-based VA, and so on. Acknowledgement of the serial correlation in bill returns produces a more risky stream of benefit payments, especially as the time horizon increases. Even when this serial correlation is acknowledged, however, bills remain the cornerstone of any VA which is intended to limit uncertainty regarding the real value of benefit payments. The importance of diversification is seen in the comparison of the bonds-based VA with the VA tied to the mixed portfolio with the same expected return. Although the median benefits are identical, the standard deviation of the real benefit payment in the 15th year is 22 percent smaller for the VA tied to the mixed portfolio.

Note, finally, the real benefit stream provided by the graduated-payment, nominal annuity. We assume that the implicit expected real return is 3 percent (at an annual rate), and is thus equal to the expected real return on long-term government bonds.[14] This assumption is equivalent to assuming that life companies can hedge graduated-payment, nominal annuities by holding an appropriate sequence of long-term bonds, and that competitive pressures ensure that this is the implicit real yield at which these annuities are sold. Because of the 3 percent return assumption, the median benefits are identical to those for the VA's tied to the government bond and mixed portfolios. The standard deviation of the benefits provided by the nominal annuity is less than those for either of the VA's in year 5, but significantly exceeds them by the 10th year. The significant increase in the riskiness of real benefits provided by the nominal annuity as the annuitant ages merits emphasis. This is a direct reflection of the substantial serial correlation in the inflation rate. These results, especially as the annuitant ages, illustrate how inappropriate it is to argue that VA's are inferior to nominal annuities because they transfer all of the investment risk to the annuitant. The results also highlight the importance of acknowledging the serial correlation in inflation rates in attempting any assessment of the risk of the real benefits provided by nominal annuities.

3. Variable Annuities with Nominal Floors

As noted, the Rockefeller Foundation Plan provides retiring employees with a variable annuity subject to a nominal floor, or VAF. Sun Life Insurance Company of Canada has recently introduced a VAF, tied to Treasury bills, in which "excess" earnings above 3 percent are also used to provide permanent benefit enrichments. The nominal floor in each of these cases is equivalent to the plan sponsor's guaranteeing that the fund will earn at least 3 percent. If the fund earns less than 3 percent in a given period, the plan sponsor fully absorbs the loss. (The mechanics of a VAF, as well as the contrast to a standard VA, are detailed in the Appendix.)

The pension plans provided by most large firms in the United States (and Canada) are defined-benefit plans. These are plans in which the employee receives a benefit equal to a given fraction of his average or of his final earnings for each year of service. Although the promised benefits are nominal, firms - especially in Canada - have typically granted "ad hoc" cost-of-living adjustments to the pensions of retired employees. Once made, these adjustments tend to be permanent. Thus the nominal value of the pension benefit is never reduced even if the fund performs poorly. This is, of course, what happens explicitly under the Rockefeller Foundation Plan, which functions as an ordinary defined-benefit plan during the pre-retirement period.

If the source of these adjustments is pension fund earnings in excess of the plan's valuation rate, and *if* there is no ceiling on the size of the benefit increases, then the plan member effectively owns a VA with a guaranteed nominal floor, or a VAF. Equivalently, he is provided with a traditional VA *plus* a put option on the *nominal* investment earnings of the pension assets with a striking price equal to the plan's valuation rate. *The plan's valuation rate becomes the equivalent of the valuation rate used to set the base payment in a standard VA.* If the nominal return on the pension fund is less than this valuation rate, then the nominal benefit is unchanged and the shortfall is absorbed fully by the plan sponsor.

Let A represent the initial amount in the fund, RV the valuation rate and $\sigma(\tilde{R})$ the measure of the risk of the *nominal* return that is relevant to option pricing. The value (A_{VAF}) of the VAF is:

$$A_{VAF} = A + \mathrm{Put}(A, RV, \sigma(\tilde{R})) \qquad (1)$$

For a given A, the value of the put option is an increasing function of both RV and $\sigma(\tilde{R})$. If the fund is invested exclusively in the risk-free nominal asset and thus earns the certain nominal return R_f, the value of the put option is zero as long as $RV \leqq R_f$. On the other hand, the value of the put option *is* likely to be large if the nominal return on the pension assets is very uncertain, even if RV is well below the expected nominal return on these assets. If the objective of the plan sponsor were to minimize the value of the put option, he would wish to set a low RV and to choose an asset base which would effectively make the VAF into a standard VA. If the sole objective of the employee were to maximize the value of the put option, he would - of course - prefer that the funds be invested in the riskiest asset, or common stocks. Since the employee presumably cannot sell his VAF, he might nonetheless prefer that the fund not be invested in risky assets if he wishes the real retirement income provided by the VAF to be stable. This point is examined below.

Simulation results (1,000 trials) are presented in Table 3 for VAF's tied to (i) bills and (ii) the mixed portfolio described previously. The interest in bills reflects the fact that they represent the cornerstone of any low-risk stream of real annuity payments. Although still active workers may have sufficient flexibility to vary their consumption-saving and work-leisure decisions to permit them to assume considerable investment risk, this is less likely to be the case for retired workers. The interest in the mixed portfolio stems from the desire to monitor - in effect - the value of the put option when the uncertainty in the return on pension assets is increased. Since the value of this option depends upon the nominal return on the pension fund, simulations are performed for both a low inflation (3%) and a high inflation (9%) scenario. The (continuous) nominal return is equal to the sum of the stochastic real return and the continuous time equivalent of these two inflation rates. Both low (3%) and high (8%) valuation rates are included in the simulations for the VAF tied to the mixed portfolio.

When the inflation rate is 3 percent, the expected real return of zero on the bills portfolio implies an expected nominal return of 3 percent. Since the valuation rate is also 3 percent, the floor frequently binds and thus the put option is frequently exercised. The result is that benefits have a higher median and a lower standard deviation than do those provided by the corresponding VA. When the inflation rate rises to 9 percent, the floor never binds and the result is identical to that for the VA. This result occurs because the combination of the high (expected) nominal return relative to the valuation rate and the low standard deviation of bill returns ensures that the realized nominal return always exceeds the valuation rate. Note also that the expected real return of zero together with the valuation rate of 3 percent causes the stream of real benefits to be tilted downward. This is most easily seen for the VA, but occurs as well for the VAF.

When the inflation rate rises from 3 to 9 percent, the put option is occasionally exercised for the mixed portfolio. This is a direct result of its more uncertain return. When the valuation rate (again, the interest rate used to set the base payment) is raised for a standard VA, the sole effect is to tilt the real payments stream downward relative to what it would otherwise have been. When the valuation rate is raised for a VAF, it has the additional effect of raising the value of the put option. When the valuation rate is raised to 8 percent, which is typical of the rates now used to value defined benefit plans in the United States, the striking price of the option rises accordingly. The result is a dramatic rise in the value of the put option in the low inflation scenario. With an expected nominal return of 3 plus 3 equals 6 percent, the nominal return typically falls short of the valuation rate. By the 15th year, the median real benefit is 65 percent greater than that provided by the corresponding VA. In the high inflation scenario, the effective value of the put option falls sharply as realized nominal returns fall short of the valuation rate with much lower frequency.

It is interesting to note that proponents of the Rockefeller Foundation Plan, which functions like a VAF, emphasize the importance of investing the pension fund reserve for retired employees exclusively in short-term commercial paper. If the nominal interest rate on short-term securities remains high relative to the plan's valuation rate of 3 percent, the value of the put option which distinguishes the VAF from a traditional VA will be very small. In effect, the Rockefeller Foundation Plan will have been transformed from a defined-benefit plan in the pre-retirement period to a defined-contribution plan at the date of retirement, with the plan's valuation rate of 3 percent used to capitalize the nominal benefits due at the date of the employee's retirement. If inflation were to recede and thus short-term interest rates to fall, the value of the put option would increase. Thus the annuitants stand to gain and the plan stands to lose from a reduction in the rate of inflation. This fact is of particular interest since the Rockefeller Foundation Plan is widely cited as a means whereby cost-of-living protection can be provided to retired workers, perhaps implying that the plan is particularly attractive to workers if inflation is high.

It is also interesting to note the continued emphasis in policy discussions in Canada on investing pension fund reserves held for retired employees exclusively in short-term securities if "excess" earnings are to be used to provide cost-of-living protection. Since the VAF is virtually identical to a VA when the value of the put option is small, the use of "ad hoc" adjustments may simply reflect the metamorphosis of defined-benefit into defined-contribution plans as the market response to increased inflation uncertainty (Pesando (1982)). Because most large firms had already introduced defined-benefit plans, the use of VAF's - rendered virtually identical to VA's by the combination of low valuation rates and investments concentrated in short-term securities - may be the most convenient way to effect the metamorphosis.[15]

4. Variable Annuities with Nominal Floors and (Cumulative) Real Ceilings

In the preceding section, it was assumed that firms which provide "ad hoc" cost-of-living adjustments could be regarded as providing their employees with VAF's. Although this may well be true for some firms, the reality may also be more complicated. Firms which make *ad hoc* cost-of-living adjustments may impose a ceiling on such increases, and may also bank underwriting losses (when the nominal floor binds) as a first claim on future "excess" earnings. In citing options for pension reform in Canada, the Task Force (1979) considered an "excess" interest scheme which contained a cumulative real ceiling. The real value of any enriched pension could not exceed its initial level and any "excess" earnings above the amount necessary to preserve fully the real value of the pension would be banked against future investment shortfalls. In addition, any underwriting losses incurred by the plan sponsor by virtue of the guaranteed nominal floor would be banked, would accumulate at a market rate of interest, and would represent a prior claim on future "excess" earnings. Only after any accumulated losses borne by the plan sponsor were repaid would "excess" earnings be used to enrich pensions in pay. Significantly, this illustrative scheme was chosen for study after the Federal government solicited input from both firms and members of the employee benefits industry.

The most important feature of a VAFC relative to a VAF is its banking provisions. (This is perhaps most easily seen by considering the case in which there is a real floor equal to the initial benefit. In this case, the annuity would be constant in real terms and the banking provisions would mirror the underwriting experience of a plan sponsor who provided a fully indexed pension and held assets other than index bonds in the pension fund.) Nonetheless, it is useful to consider the options inherent in a VAFC without reference to the banking provisions. By virtue of the ceiling on the real value of the pension benefit, the worker has - in effect - sold a call option on "excess" investment earnings above those sufficient to provide full cost-of-living protection. Since the nominal return on the plan's assets is the real return plus the inflation rate, this is equivalent to the worker's having sold a call option on real investment earnings in excess of the valuation rate. Let A_{VAFC} represent the value of the variable annuity subject to both a floor and a ceiling; let $\sigma(\tilde{r})$ be the measure of risk of the real return that is relevant to option pricing, and let A_{VAF} and A be as defined in (1). Then:

$$A_{VAFC} = A + \text{Put}(A, RV, \sigma(\tilde{R})) - \text{Call}(A, RV, \sigma(\tilde{r}))$$

$$= A_{VAF} - \text{Call}\ (A, RV, \sigma(\tilde{r})) \qquad (2)$$

Unlike a VAF, whose value to the beneficiary is at least as great as that of a standard VA, the value of a VAFC may be greater or less than that of the corresponding VA, depending on the relative values of the put and the call. For a given A, the value of the call option is a decreasing function of the plan's valuation rate and an increasing function of the risk of the real return on the plan's assets. The value of the call option will be zero if the pension fund is invested exclusively in a risk-free real asset and if the risk-free real rate of return $r_f \leqq RV$. As noted by Bodie (1980), there is no risk-free real asset. A pension fund invested exclusively in bills will, however, earn a real return which is quite stable and which has an expected value of (approximately) zero. The value of this call option will thus be close to zero if (1) the fund holds only bills or their equivalent, and (2) the valuation rate is above (say) 3 percent.[16] In this case, the value of the VAFC will equal that of the VAF. If, in addition, the anticipated rate of inflation is sufficiently high that the nominal bill yield significantly exceeds RV, then the value of the put option contained in both the VAF and VAFC will equal zero and thus both will be equal in value to the corresponding VA.

Consider first (Table 4) the distribution of real benefits by a VAFC tied to a bills portfolio when the inflation rate is low. Because the projected stream of real annuity payments is tilted downward (since the expected real return of zero is less than the valuation rate), the ceiling binds rarely and only in the initial years of the annuity pay-out. Median benefits fall short of those provided by a VAF, primarily due to the banking provisions, but exceed those of a VA. In the high inflation scenario, the VAFC provides benefits which simply reproduce those of a VA. This result, which was anticipated in the discussion of (2), may be empirically relevant. If so, this might explain the apparent lack of attention that is sometimes accorded this issue. The Rockefeller Foundation Plan, for example, makes no reference as to whether or not a ceiling exists on the cost-of-living increases. Because the valuation rate of 3 percent exceeds the expected real return on a portfolio of short-term commercial paper (or its surrogate, the prime rate), the question of whether or not there is a ceiling may simply not be empirically relevant.

For the mixed portfolio with a valuation rate of 3 percent, there is no tilt to the projected stream of real benefits provided by the corresponding VA. For the VAFC, unlike the VAF, the benefit payments are similar in both the low and the high inflation scenarios. This is, of course, due to the banking provisions. For both scenarios, the ceiling binds frequently (i.e., the call option is exercised) as evidenced by the fact that median benefits remain at the ceiling in all years. Although we do not attempt to explicitly evaluate them, it would appear that the value of the sponsor's call option exceeds the value of the annuitant's put in these two cases. Note that the median and mean benefits are lower than those of the corresponding VA in all years. Furthermore, in contrast to both the VA and the VAF, the mean benefit for the VAFC is well below its own median, reflecting the reverse skewness induced by the truncation of the upper tail of the distribution. The dramatic decline in the standard deviation relative to both the VA and VAF is also a result of this truncation and therefore reflects not a reduction in risk from the annuitant's perspective, but rather the loss of upside potential. Further evidence that in these two cases the value of the VAFC is considerably less than that of the corresponding VA is provided by Table 4' which shows the distribution of the real accumulation in the "bank" at the end of year 15. When this number turns out to be positive at the end of a simulation run it means that the years of "excess" earnings from the portfolio were more than enough to compensate for the years of shortfall.

Raising the valuation rate, as noted in the discussion of (2), reduces the value of the call option. When the valuation rate is set at 8 percent, median benefits do exceed those provided by the VA for all years in the low inflation scenario, although they remain less than those provided by the VAF. When inflation is high, and thus the permitted real erosion in the value of benefits is also high, the stream of payments provided by the VAFC and VA are quite similar. The ceiling frequently binds, but the "excess" funds so banked are then used to enrich nominal benefits in subsequent years.

To sum up, three empirical results merit emphasis. First, if the pension fund is invested exclusively in bills, the VAFC will provide benefits similar to those provided by a standard VA if (1) the inflation rate is high relative to the plan's valuation rate and (2) the valuation rate is (say) 3 percent or more and thus significantly exceeds the expected real return on bills. In this case, the value of each of the put and the call options is (approximately) equal to zero. Second, when the expected real return on the plan's assets is equal to the valuation rate, the real benefits provided by a VAFC are likely to be far more stable than those provided by either a VAF or the traditional VA. This result, in essence, reflects the procedure for banking the gains or losses experienced by the plan sponsor. Third, by choosing appropriate combinations of RV and asset allocations, it would appear possible to "cancel" the values of the put and the call (without setting each equal to zero) and to create a number of VAFC's all having a value equal to A. But there would have to be mutual agreement between sponsor and annuitant about the portfolio's composition, and some mechanism for monitoring adherence to it.

As noted, our interest in the VAFC is motivated by the possibility that it may formalize the behavior of at least some firms which make "ad hoc" cost-of-living adjustments. If so, and if the stream of real payments is smoothed relative to those obtainable from (say) a bills-based VA, then firms must be compensated for their underwriting the attendant investment risk. In principle, this should be reflected in compensating wage differentials. Since the VAFC does not alter the efficient frontier, it will be the basic risk-return tradeoffs available in the capital market which dictate the size of these compensating wage differentials.

5. Alternative Annuity Designs: Historical Simulations for the Period: 1971-1980

Historical simulations of the nominal and real benefits provided by VA's, VAF's, and VAFC's for the period 1971-1980 are presented in Tables 5A-5D. As in Table 1, the initial capital in 1971 is $100,000 and the payments are made with certainty for 10 years. VAF's and VAFC's must, of course, be underwritten by the plan sponsor (or life insurance company) and their cost may exceed or fall short of the initial capital.[17] Two valuation rates, zero and five percent, are used in the simulations. The former is the expected real return on the minumum variance portfolio (i.e., bills) while the latter is typical of the rates actually used in the early 1970's to value defined benefit plans.

Consider first the bill results. When the valuation rate is zero, the real benefit provided by the VA declines from $10,100 in 1971 to $8,794 in 1980 or by 13 percent. This erosion is only modest in view of the substantial unanticipated inflation that appears to have occurred in the 1970's. Because the floor never binds, the VAF produces benefits identical to the VA. Because the ceiling binds in 1971 and 1972, thus causing "excess" earnings to be banked for future use, the real benefits provided by the VAFC diverge from those provided by the VA and VAF. With a valuation rate of 5 percent, the floor binds twice (1971 and 1972) so that the final benefit provided by the VAF exceeds that provided by the VA. Because of the banking feature, which requires that the plan sponsor be compensated for prior underwriting losses, the stream of benefits provided by the VAFC differs from that provided by the VA.

The sharpest contrast among the alternative annuity designs occurs with the riskiest asset base, which is common stocks. Consider only the results when the valuation rate equals 5 percent. Although this rate is less than the expected real rate of return on common stocks, the real benefits provided by the VA, in fact, decline sharply. This result simply reflects the poor performance of the stock market during the decade. The value of the nominal floor (i.e., the put option) is high as evidenced by the fact that the real benefit provided by the VAF in 1980 is almost twice that provided by the VA. The tendency for the VAFC to stablize the real stream of benefit payments is readily apparent. In 1973 and 1974, for example, annuitants are partially insulated from the precipitous declines in the stock market. When the stock market recovers in 1975, however, real benefits continue to decline as "excess" fund earnings are first used to repay the plan sponsors for the net underwriting losses they incurred in the previous years.

The final comparison is between the bond portfolio and the mixed portfolio with the identical expected return. Because of the very adverse performance of the bond market, the real benefit by 1980 is much higher for the VA when it is tied to the mixed portfolio. This *ex post* result is consistent with the greater *ex ante* risk of the bond portfolio. The comparisons of the results for the VA's, VAF's and VAFC's are quite straightforward, and only the continuing tendency for real benefits to be stabilized under the VAFC merits note.

6 Summary and Conclusion

Nominal annuities, whether level-payment or graduated, expose the annuitant to substantial uncertainty regarding the real value of his retirement income. The exclusive source of this uncertainty is uncertainty regarding the future level of prices, and hence of the future rate of inflation. Standard VA's backed by Treasury bills or their equivalent provide much more stable real retirement incomes, even when consideration is given to the serial correlation in real bill returns. VA's backed by common stocks, long-term government bonds, and a mixed portfolio illustrate the risk-return tradeoffs inherent in the alternative portfolios. These should be of interest to plan sponsors who may wish, without increasing their own costs, to provide increased annuity choices to plan members.[18]

The cost of a VAF, which is a VA with a nominal floor, is not known with certainty on the date the annuity is purchased and must be underwritten by a plan sponsor or life company. The plan provided by the Rockefeller Foundation functions, in effect, like a VAF. If the objective is to provide a stable stream of real benefits, then a VAF must also be linked to a bills portfolio. When nominal interest rates are high and the valuation rate is low, this VAF will produce results virtually identical to those of a bills-based VA. This is the case for the Rockefeller Foundation Plan. In effect, the Rockefeller Foundation Plan functions as a defined-benefit plan in the pre-retirement period, and becomes a defined-contribution plan at the date of the employee's retirement. This metamorphosis of defined-benefit plans into defined-contribution plans appears to have occurred extensively in Canada, and may represent a market response to increased inflation uncertainty.

Because the "ad hoc" adjustments made by firms are never (to our knowledge) more than those necessary to fully offset the impact of inflation, it is likely that the behavior of many firms is more complicated than that suggested by the VAF. We therefore analyze a VA subject to a nominal floor and a real ceiling, in which underwriting losses and gains by the plan sponsor are banked from one period to the next. Under stylized conditions which might well be met in practice, a VAFC tied to bills would closely replicate the benefits provided by a bills-based VA. More generally, due to the interaction of the floor and ceiling with the banking provisions, it is possible for a VAFC to provide substantially more stable real benefits than a VA tied to the same asset base. Because the risk-return tradeoffs available in the capital market have not changed, however, sponsoring firms in these cases would presumably extract compensating wage differentials from their employees _if_ mean benefits were unaffected. As noted in the text, however, mean benefits will be reduced if the implicit call option (pertaining to the real ceiling) proves to be more valuable than the implicit put option (pertaining to the nominal floor).

FOOTNOTES

1 For simplicity, the discussion proceeds as if the sole source of wealth of the retiring plan member is his claim to a private pension. If he has other sources of wealth, then the risk-return characteristics of his pension benefit must be analyzed in the context of his total portfolio. For a discussion which explicitly focuses on social security as an additional source of wealth, see Feldstein (1981). See L. Summers' paper in this volume for a discussion of the extent to which households may be able to diversify away the inflation risk implicit in nominal pension benefits.

2 A separate provision in the Rockefeller Foundation plan provides that the cost-of-living adjustment equal at least 4 percent if the inflation rate as measured by the Consumer Price Index exceeds 4 percent. Otherwise, however, the floor is that cited in the text (Heaton (1981)). There is no reference to a ceiling on the size of the cost-of-living adjustments. Subsequent discussion of the Rockefeller plan ignores the separate floor provision.

3 TIAA, which manages one of the largest pension plans in the United States, offers its members two annuity designs which resemble VA's which contain a nominal floor. The older of these, the traditional TIAA annuity, has a guaranteed minimum nominal floor. This floor is embodied in the guaranteed return of 3%. Unlike the Rockefeller plan (which is characteristic of the hybrid annuities examined at length in the text), this nominal floor does not ratchet upwards over time. The asset base in the TIAA annuity consists of a portfolio

dominated by long-term bonds, mortgages and other fixed-interest loans. TIAA pays to its beneficiaries a variable benefit which has been "smoothed" relative to what it would be under a standard VA design. by ignoring unrealized capital gains and losses on these dollar-fixed investments. One consequence of this smoothing is that the guaranteed rate cited previously is far less likely to bind. Another consequence is that there can be cross-subsidization of different generations of annuitants. Currently, for example, TIAA is paying a total nominal rate of return of 11% to new retirees, while the risk-free nominal rate of return in the capital markets is well in excess of that. The interest rate used to determine the initial benefit (called the Assumed Interest Rate or A.I.R.) is also equal to 11%, so the expected nominal benefit stream is level. Recently, TIAA has offered its members an alternative design (called the Graded Benefit Payment Method), which differs in two respects from the older one. First, the expected nominal benefit stream has been given an upward tilt by using an A.I.R. of 4% to determine the initial benefit level. Secondly, the guaranteed nominal floor ratchets upward whenever the interest rate declared in each period actually exceeds 4%. Earnings above 4% are credited at the end of the year and - in effect - are used to purchase an additional TIAA annuity with its own guarantees and dividends. (The interest rate used in calculating the increase in the nominal floor is the guaranteed rate of 3%.)

It is worth noting that TIAA has been shortening the average maturity of its portfolio in recent years. If this process were to continue, the TIAA graded payment annuity would come to look more like the Rockefeller Plan annuity.

4 Ontario's Select Committee on Pensions (1981) has recommended that the use of "excess" investment earnings to provide inflation protection be mandated by law. No reference is made to floors and/or ceilings in the proposed scheme, which the analysis in this paper shows to be of crucial importance.

5 For the purposes at hand, there is no advantage in explicitly incorporating mortality factors into the analysis. Mortality is thus ignored in all of the illustrations presented in the paper.

6 More precisely, each year's real benefit would be equal to the previous year's benefit divided by 1.05.

7 In fact, real equity returns appear to be negatively correlated with unanticipated inflation, as noted by Bodie (1976) and Pesando and Rea (1977). Feldstein (1980) attributes this result to the fact that inflation raises the effective tax rate on corporate-source income.

8 The mixed portfolio consists of 52% bonds, 29% bills and 19% stocks. We do not refer to this as an efficient portfolio for two reasons. First, our portfolio proportions are derived from a single-period variance-minimization procedure which ignores the serial correlation in bill returns. Second, the efficiency of an annuity for a particular household can only be determined if we know all of the household's other assets and liabilities.

9 As noted by Bodie (1981), the mean realized real return on bonds is in fact negative during this period. The mean real return on bonds

was set equal to an annual rate of 3%, whose continuous time equivalent is 2.956%, while the other parameters were based on the observed means, variances and covariances.

10 First order autoregressions were performed for the logarithms of the real annual wealth relatives of bills, bonds and stocks. The results are as follows:

Bills: $r_t = \underset{(.205)}{-.044} + \underset{(.136)}{.768}\, r_{t-1}$ $R^2 = .559$ SEE = 1.04 (% per year)

Bonds: $r_t = \underset{(1.493)}{-1.619} + \underset{(.207)}{.261}\, r_{t-1}$ $R^2 = .056$ SEE = 7.64

Stocks: $r_t = \underset{(3.849)}{5.847} - \underset{(.201)}{.021}\, r_{t-1}$ $R^2 = .0004$ SEE = 19.28

Bracketed figures are standard errors.

11 The real benefit in year t is given by $b_t = B_o \prod_{i=1}^{t} \frac{e^{\tilde{r}_i}}{(1+RV)}$ where B_o is the initial projected annuity payment, $\tilde{r}_i$ is the realization of the stochastic logarithic real return in year i, and RV is the annuity valuation rate. Since b_t is the product of lognormal variates, it is also lognormally distributed: $\log(b_t) = \log(B_o) + \sum_{i=1}^{t} \tilde{r}_i - t \log(1+RV)$. Since we have chosen RV such that $E(r_i) = \log(1+RV)$, the median value of b_t equals B_o for all t. By contrast, $E(b_t) = B_o e^{\frac{1}{2}\sigma_t^2}$ where σ_t^2 equals the variance of $\sum_{i=1}^{t} \tilde{r}_i$. If there is no serial correlation in the $\tilde{r}_i$ series, $\sigma_t^2 = t\sigma^2$ where σ^2 is the variance of r_i in a single year.

Rea (1981) also discusses the design of a variable annuity which produces a payments stream which is expected to remain constant in real terms.

12 The first order annual autoregression, based on the consumer price index, is:

$$\Pi_t = \underset{(0.597)}{0.794} + \underset{(0.117)}{0.902}\,\Pi_{t-1} \qquad R^2 = .750 \qquad \text{SEE} = 2.003 \text{ (\% per year)}$$

where $\Pi_t = \log(P_t/P_{t-1})$ and P_t is the price level at time t.

13

Assume Π_t follows the first order autoregressive process, $\Pi_t = \alpha + \rho\Pi_{t-1} + \varepsilon_t$, where ε_t is distributed $N(0,\sigma)$. Then $\Pi^* = \alpha/(1-\rho)$ is the steady-state rate of inflation. Note that $\log(P_t) = \log(P_o) + \sum_{i=1}^{t} \tilde{\Pi}_t$ where $\tilde{\Pi}_t$ is the realization of the inflation process. Let $P_o = 1$ and let $\Pi_o = \Pi^*$. Then median of $\log(P_t) = t\Pi^*$; median of $P_t = e^{t\Pi^*}$; and variance of $\log(P_t) = \sigma^2_t = \frac{\sigma^2}{(1-\rho)^2}\left[t-\rho\frac{(1-\rho^t)}{(1-\rho)}\left(2-\rho\frac{(1-\rho^t)}{(1+\rho)}\right)\right]$. If B_t is the known nominal benefit in period t, then the real benefit $b_t = B_t/P_t$. Thus $\log(b_t) = \log(B_t) - \log(P_t)$. Let $B_t = B_o e^{gt}$ where g is the rate of graduation, and let $\mu_t = (g-\Pi^*)t$. Median $b_t = B_t \div$ median $P_t = B_o e^{\mu_t}$. Since B_t is graduated so as to increase at the anticipated inflation rate, $\mu_t = 0$ and median $b_t = B_o$. Since B_t is non-stochastic, variance of $\log(b_t)$ = variance of $\log(P_t) = \sigma^2_t$. Mean $b_t = B_o e^{\frac{1}{2}\sigma^2 t}$ and the variance of $b_t = B_o^2 e^{\sigma^2 t}(e^{\sigma^2 t} - 1)$.

14

The continuously compounded nominal interest rate (R) is thus equivalent to an annual rate of 11%, since the (annual) steady state rate of inflation built into the illustration is 8%.

15

_If_ there were no nominal floor on the pension benefits, any decision to channel pension fund reserves exclusively into bills or their equivalent would have an unambiguous interpretation. Workers, who presumably cannot diversify away the inflation risk inherent in nominal pension benefits, are sufficiently risk averse that they will pay the price (i.e. a low expected real return on their pension wealth) of stabilizing their real retirement incomes.

16 Remember that the standard deviation of the continuously compounded real bill return is only 1.52 percent per annum, so that the expected real return of zero is about two standard deviations less than 3 percent.

17 The plan sponsor could underwrite VAF's or VAFC's on either a pay-go or a fully-funded basis. This issue is not explored in this paper.

18 If the sponsor provides a defined-benefit plan, the lump sum necessary to purchase the requisite annuity could be made available to the employee, who could then choose his preferred VA. If the promised pension is purely nominal (and the firm has no tradition of providing ad hoc adjustments), then discounting the promised payments by the risk-free nominal rate R (as well as by mortality) would identify the lump sum to be offered to the employee.

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Table 1

ILLUSTRATION OF TRADITIONAL, LEVEL-PAYMENT ANNUITY AND

PURCHASING POWER ANNUITY FOR PERIOD 1971-1980

		Traditional Annuity[a]		Purchasing Power Annuity[b]			
				No Tilting (RV=0)		Tilting (RV=5)	
Year	Inflation Rate (CPI)	Nominal Value	Real Value	Nominal Value	Real Value	Nominal Value	Real Value
1971	3.3	14,568	14,095	10,336	10,000	12,748	12,334
1972	7.4	14,568	13,630	10,688	10,000	12,555	11,746
1973	8.8	14,568	12,527	11,629	10,000	13,009	11,187
1974	12.2	14,568	11,165	13,047	10,000	13,901	10,654
1975	7.0	14,568	10,434	13,962	10,000	14,167	10,147
1976	4.8	14,568	9,955	14,634	10,000	14,142	9,664
1977	6.8	14,568	9,324	15,624	10,000	14,380	9,204
1978	9.0	14,568	8,551	17,035	10,000	14,932	8,765
1979	13.3	14,568	7,547	19,303	10,000	16,114	8,343
1980	12.4	14,568	6,714	21,698	10,000	17,251	7,951

Notes: [a] Assumes that the nominal interest rate is 7.5%, initial capital is $100,000, annuity is payable with certainty for 10 years and annuity payments are made at the end of the year.

[b] Assumes that life insurance company can purchase an index bond which offers a real return of zero. If π is the inflation rate in period t, then the nominal annuity payment $B_t = B_{t-1} * (1 + \pi_t)/(1 + RV)$ where RV is the interest rate used to determine the base annuity payment (B_o). B_o equals $RV * A/(1 - (1+RV)^{-T})$ where A is the initial capital and T is the number of years the annuity is payable. For RV = 0, B_o equals A/T.

Table 2

VARIABLE ANNUITIES WITH ALTERNATIVE ASSET BASES: THEORETICAL DISTRIBUTIONS

Portfolio	Expected Real[a] Return (%)	Standard Deviation (%)	Annuity Valuation Rate (%)	Base Annuity[b] Payment ($)	Annuity Payment in Year 5 median	Annuity Payment in Year 5 mean	Annuity Payment in Year 5 std.dev.	Annuity Payment in Year 10 median	Annuity Payment in Year 10 mean	Annuity Payment in Year 10 std.dev.	Annuity Payment in Year 15 median	Annuity Payment in Year 15 mean	Annuity Payment in Year 15 std.dev.
Bills	0	1.52	0	6,667	6,667	6,670	227	6,667	6,674	321	6,667	6,678	393
Bills (Serial[c] Correlation)	0	1.04	0	6,667	6,667	6,677	361	6,667	6,701	679	6,667	6,731	933
Bonds	2.956	7.64	3	8,377	8,377	8,500	1,463	8,377	8,625	2,115	8,377	8,752	2,647
Stocks	7.232	18.61	7.5	11,329	11,329	12,354	5,372	11,329	13,471	8,666	11,329	14,689	12,974
Mixed[d] Portfolio	2.956	6.08	3	8,377	8,377	8,455	1,155	8,377	8,533	1,656	8,377	8,613	2,056

Graduated Payment[e], Nominal Annuity (Graduation Rate=8%)	2.956	-	3	8,377	8,377	8,445	1,080	8,377	8,723	2,531	8,377	9,219	4,236

[a] mean of the logarithm of the real annual wealth relative. Valuation rate is the equivalent annual rate.

[b] Initial capital is $100,000, annual payments are made with certainty for 15 years, payments are in constant dollars.

[c] Based on the following autoregression for the annual real return on bills: $r_t = .76r_{t-1}+e_t$ with $\sigma_e = 1.04\%$ per year and $r_o = 0$.

[d] Mixed portfolio, consisting of bonds (52%), bills (29%) and stocks (19%), minimizes the variance of the annual real return for the given mean.

[e] Uncertainty regarding the real annuity payments stems solely from uncertainty regarding the price level, which is assumed to be log normally distributed. The continuously compounded rate of inflation (π) follows the following autoregressive process: $\pi_t = .77 + .9\pi_{t-1} + U_t$ with $\sigma_U = 2.00\%$ per year and $\pi_o = 7.7\%$. The steady state inflation rate of 7.7% is equivalent to the annual graduation rate of 8%. The graduated-payment nominal annuity assumes an expected real return equal to that of bonds (i.e., 2.956%).

Table 3

VARIABLE ANNUITIES WITH GUARANTEED NOMINAL FLOORS

Portfolio	Expected[a] Real Return (%)	Standard Deviation (%)	Inflation Rate (%)	Annuity Valuation Rate (%)	Base Annuity[b] Payment ($)	Annuity Payment in Year 5			Annuity Payment in Year 10			Annuity Payment in Year 15		
						median	mean	std.dev.	median	mean	std.dev.	median	mean	std.dev.
Bills	0	1.52	3	3	8,377	7,424 (7,219)	7,447 (7,223)	150 (245)	6,604 (6,203)	6,619 (6,222)	188 (302)	5,867 (5,357)	5,888 (5,375)	204 (313)
Bills	0	1.52	9	3	8,377	7,221 (7,221)	7,334 (7,224)	248 (248)	6,219 (6,219)	6,229 (6,229)	312 (312)	5,365 (5,365)	5,373 (5,373)	322 (322)
Mixed[c]	2.956	6.08	3	3	8,377	8,851 (8,366)	8,943 (8,448)	926 (1,180)	9,369 (8,459)	9,616 (8,595)	1,434 (1,687)	10,020 (8,512)	10,326 (8,724)	1,976 (2,139)
Mixed[c]	2.956	6.08	9	3	8,377	8,473 (8,407)	8,564 (8,489)	1,120 (1,176)	8,633 (8,461)	8,738 (8,577)	1,589 (1,655)	8,684 (8,406)	8,918 (8,671)	1,974 (2,047)
Mixed[c]	2.956	6.08	3	8	11,683	10,787 (9,192)	10,948 (9,289)	747 (1,225)	10,103 (7,326)	10,276 (7,446)	997 (1,407)	9,483 (5,773)	9,629 (5,922)	1,129 (1,339)
Mixed[c]	2.956	6.08	9	8	11,683	9,586 (9,226)	9,701 (9,276)	1,035 (1,263)	7,960 (7,219)	8,070 (7,380)	1,190 (1,390)	6,549 (5,708)	6,706 (5,879)	1,229 (1,370)

[a] Mean of the logarithm of the real annual wealth relative. The nominal return is the sum of the simulated real return plus the continuous time equivalent of the annual inflation rate noted in the table.

[b] Initial capital is $100,000, annual payments are made with certainty for 15 years, payments are in constant dollars.

[c] Same as in Table 2.

Bracketed results are those for a variable annuity without the nominal floor.

Table 4

VARIABLE ANNUITIES WITH GUARANTEED NOMINAL FLOORS AND CUMULATIVE REAL CEILINGS

Portfolio	Expected Real Return (%)[a]	Standard Deviation (%)	Inflation Rate (%)	Annuity Valuation Rate (%)	Base Annuity Payment ($)[b]	Annuity Payment in Year 5			Annuity Payment in Year 10			Annuity Payment in Year 15		
						median	mean	std.dev.	median	mean	std.dev.	median	mean	std.dev.
Bills	0	1.52	3	3	8,377	7,314 (7,219)	7,360 (7,223)	151 (245)	6,348 (6,203)	6,403 (6,222)	185 (302)	5,482 (5,357)	5,543 (5,375)	187 (313)
Bills	0	1.52	9	3	8,377	7,221 (7,221)	7,224 (7,224)	248 (248)	6,219 (6,219)	6,229 (6,229)	312 (312)	5,365 (5,365)	5,373 (5,373)	322 (322)
Mixed[c]	2.956	6.08	3	3	8,377	8,377 (8,366)	8,080 (8,448)	393 (1,180)	8,377 (8,459)	7,900 (8,595)	690 (1,687)	8,377 (8,512)	7,725 (8,724)	973 (2,139)
Mixed[c]	2.956	6.08	9	3	8,377	8,377 (8,400)	7,995 (8,489)	585 (1,176)	8,377 (8,461)	7,865 (8,577)	796 (1,655)	8,377 (8,406)	7,806 (8,671)	971 (2,049)
Mixed[c]	2.956	6.08	3	8	11,683	10,106 (9,192)	10,399 (9,289)	462 (1,226)	8,769 (9,348)	9,063 (7,446)	585 (1,407)	7,564 (5,773)	7,834 (5,922)	561 (1,339)
Mixed[c]	2.956	6.08	9	8	11,683	9,257 (9,226)	9,372 (9,276)	1,129 (1,263)	7,263 (7,219)	7,428 (7,380)	1,420 (1,390)	5,574 (5,708)	5,746 (5,879)	1,470 (1,370)

[a] Mean of the logarithm of the real annual wealth relative. The nominal return is the sum of the simulated real return plus the continuous time equivalent of the annual inflation rate noted in the table.

[b] Initial capital is $100,000, annual payments are made with certainty for 15 years, payments are in constant dollars.

[c] Same as in Table 2.

Bracketed figures are those for a variable annuity without the floor and ceiling.

Table 4'

Amount in "Bank" at End of Year 15 for VAFC's in Table 4

					Amount in Bank ($)				
Portfolio	Expected Real Return	Standard Deviation	Inflation Rate	RV	Median	Mean	Minimum	Maximum	Standard Deviation
Bills	0	1.52	3	3	-1,800	-2,209	-9,421	0	1,787
Bills	0	1.52	9	3	0	0	0	0	0
Mixed	2.956	6.08	3	3	2,591	10,024	-23,458	122,557	16,933
Mixed	2.956	6.08	9	3	3,381	10,898	-1,408	127,940	15,459
Mixed	2.956	6.08	3	8	-23,836	-24,112	-61,225	17,841	12,835
Mixed	2.956	6.08	9	8	-63	-770	-13,194	2,988	1,649

Table 5A

ALTERNATIVE ANNUITY DESIGNS: SIMULATIONS FOR A BILLS ONLY PORTFOLIO, 1971-1980

Year	Inflation Rate (%)[a]	Return On United States T-bills (%)	Variable Annuity[b]				Variable Annuity with Nominal Floor				Variable Annuity with Nominal Floor, Real Ceiling			
			RV=0		RV=5		RV=0		RV=5		RV=0		RV=5	
			Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real
1971	3.36	4.39	10,439	10,100	12,876	12,457	10,439	10,100	12,950	12,529	10,336	10,000	12,950	12,529
1972	3.41	3.84	10,840	10,142	12,733	11,913	10,840	10,142	12,950	12,116	10,688	10,000	12,950	12,116
1973	8.80	6.93	11,591	9,967	12,968	11,151	11,591	9,967	13,189	11,341	11,626	9,997	12,950	11,136
1974	12.20	8.00	12,518	9,594	13,337	10,222	12,518	9,594	13,565	10,396	12,556	9,623	13,284	10,181
1975	7.01	5.80	13,245	9,486	13,439	9,625	13,244	9,486	13,669	9,790	13,284	9,514	13,385	9,586
1976	4.81	5.08	13,917	9,510	13,449	9,190	13,917	9,510	13,679	9,347	13,959	9,539	13,396	9,154
1977	6.77	5.12	14,629	9,363	13,465	8,618	14,629	9,363	13,695	8,765	14,673	9,391	13,411	8,583
1978	9.03	7.03	15,657	9,191	13,726	8,057	15,657	9,191	13,959	8,194	15,705	9,219	13,670	8,024
1979	13.31	10.38	17,284	8,954	14,429	7,475	17,284	8,954	14,675	7,602	17,335	8,980	14,371	7,445
1980	12.41	10.40	19,082	8,794	15,170	6,991	19,082	8,794	15,429	7,111	19,138	8,820	15,110	6,964

Notes: [a] Based on the consumer price index. Inflation and security return data are from Ibbotson and Sinquefeld, Stocks, Bonds, Bills and Inflation, Financial Analysts Research Foundation, 1977, updated by the authors.

[b] The initial capital is $100,000; the annuity is payable with certainty for 10 years; annuity payments are made at the end of the year; RV is the interest rate used to determine the initial level of the projected stream of annuity payments.

Table 5B

ALTERNATIVE ANNUITY DESIGNS: SIMULATIONS FOR A STOCKS ONLY PORTFOLIO, 1971-1980

Year	Inflation Rate (%)[a]	Nominal Return on Stocks (%)	Variable Annuity[b]				Variable Annuity with Nominal Floor				Variable Annuity with Nominal Floor, Real Ceiling			
			RV=0		RV=5		RV=0		RV=5		RV=0		RV=5	
			Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real
1971	3.36	14.31	11,431	11,059	14,098	13,640	11,431	11,059	14,099	13,641	10,336	10,000	13,386	12,950
1972	3.41	18.98	13,601	12,725	15,976	14,947	13,601	12,725	15,976	14,947	10,688	10,000	13,842	12,950
1973	8.80	-14.66	11,607	9,981	12,985	11,166	13,601	11,696	15,976	13,738	11,629	10,000	13,842	11,903
1974	12.20	-26.48	8,533	6,540	9,092	6,968	13,601	10,424	15,976	12,244	11,629	8,913	13,842	10,609
1975	7.01	37.20	11,708	8,385	11,879	8,508	18,660	13,364	20,875	14,951	11,629	8,329	13,842	9,914
1976	4.81	23.84	14,499	9,908	14,012	9,575	23,109	15,791	24,621	16,824	14,359	9,812	13,842	9,459
1977	6.77	-7.18	13,458	8,613	12,386	7,927	23,109	14,790	24,621	15,758	14,359	9,188	13,842	8,859
1978	9.03	6.39	14,318	8,405	12,550	7,367	24,585	14,431	24,947	14,644	14,359	8,429	13,842	8,125
1979	13.31	18.44	16,958	8,785	14,157	7,334	29,119	15,085	28,140	14,578	16,039	8,309	13,842	7,171
1980	12.41	32.01	22,386	10,317	17,797	8,202	38,440	17,715	35,378	16,304	21,173	9,578	13,842	6,379

Notes: a Based on the consumer price index. Inflation and security return data are from Ibbotson and Sinquefield, Stocks, Bonds, Bills and Inflation, Financial Analysts Research Foundation, 1977, updated by the authors.

b The initial capital is $100,000; the annuity is payable with certainty for 10 years; annuity payments are made at the end of the year; RV is the interest rate used to determine the initial level of the projected stream of annuity payments.

Table 5C

ALTERNATIVE ANNUITY DESIGNS: SIMULATIONS FOR A BONDS ONLY PORTFOLIO, 1971-1980

Year	Inflation Rate (%)[a]	Nominal Return On United States Bonds (%)	Variable Annuity[b]				Variable Annuity with Nominal Floor				Variable Annuity with Nominal Floor, Real Ceiling			
			RV=0		RV=5		RV=0		RV=5		RV=0		RV=5	
			Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real
1971	3.36	13.23	11,322	10,954	13,966	13,512	11,323	10,955	13,966	13,512	10,336	10,000	13,386	12,950
1972	3.41	5.68	11,966	11,195	14,056	13,151	11,966	11,195	14,056	13,151	10,688	10,000	13,842	12,950
1973	8.80	-1.11	11,834	10,176	13,239	11,384	11,966	10,290	14,056	12,087	11,629	10,000	13,842	11,903
1974	12.20	4.35	12,348	9,464	13,156	10,083	12,487	9,570	14,056	10,773	12,721	9,750	13,842	10,609
1975	7.01	9.19	13,484	9,657	13,680	9,798	13,634	9,765	14,617	10,469	13,890	9,948	13,842	9,914
1976	4.81	16.75	15,742	10,757	15,212	10,395	15,918	10,877	16,253	11,106	14,634	10,000	15,041	10,278
1977	6.77	-0.67	15,636	10,007	14,391	9,210	15,918	10,188	16,253	10,402	15,625	10,000	15,041	9,626
1978	9.03	1.03	15,797	9,273	13,847	8,128	16,082	9,440	16,253	9,541	16,965	9,959	15,041	8,829
1979	13.31	-1.22	15,605	8,084	13,026	6,748	16,082	8,331	16,253	8,420	16,965	8,789	15,041	7,792
1980	12.41	-4.45	14,911	6,872	11,854	5,463	16,082	7,412	16,253	7,490	16,965	7,818	15,041	6,932

Notes: [a] Based on the consumer price index. Inflation and security return data are from Ibbotson and Sinquefeld, Stocks, Bonds, Bills and Inflation, Financial Analysts Research Foundation, 1977, updated by the authors.

[b] The initial capital is $100,000; the annuity is payable with certainty for 10 years; annuity payments are made at the end of the year; RV is the interest rate used to determine the initial level of the projected stream of annuity payments.

Table 5D

ALTERNATIVE ANNUITY DESIGNS: SIMULATIONS FOR A U.S. MIXED PORTFOLIO, 1971 - 1980

Year	Inflation Rate (%)[a]	Nominal Return on Mixed[c] Portfolio (%)	Variable Annuity[b]				Variable Annuity with Nominal Floor				Variable Annuity with Nominal Floor, Real Ceiling			
			RV=0		RV=5		RV=0		RV=5		RV=0		RV=5	
			Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real
1971	3.36	10.89	11,089	10,729	13,678	13,233	11,089	10,729	13,677	13,232	10,336	10,000	13,386	12,950
1972	3.41	7.70	11,943	11,174	14,030	13,126	11,943	11,174	14,029	13,125	10,688	10,000	13,842	12,950
1973	8.80	-1.40	11,777	10,127	13,175	11,329	11,943	10,270	14,029	12,064	11,629	10,000	13,842	11,903
1974	12.20	-0.52	11,716	8,979	12,483	9,567	11,943	9,153	14,029	10,752	12,027	9,218	13,842	10,609
1975	7.01	13.59	13,308	9,531	13,503	9,671	13,567	9,717	15,177	10,870	13,661	9,784	13,842	9,914
1976	4.81	14.75	15,271	10,435	14,758	10,085	15,568	10,638	16,587	11,334	14,634	10,000	14,234	9,727
1977	6.77	-0.25	15,233	9,749	14,020	8,973	15,568	9,964	16,587	10,616	15,625	10,000	14,234	9,110
1978	9.03	3.79	15,809	9,280	13,857	8,134	16,157	9,484	16,587	9,737	16,592	9,740	14,234	8,355
1979	13.31	5.89	16,742	8,673	13,975	7,240	17,109	8,863	16,727	8,665	17,570	9,102	14,234	7,374
1980	12.41	6.82	17,882	8,241	14,217	6,552	18,276	8,423	17,017	7,842	18,768	8,649	14,234	6,560

Notes: [a] Based on the consumer price index. Inflation and security return data are from Ibbotson and Sinquefeld, Stocks, Bonds, Bills and Inflation, Financial Analysts Research Foundation, 1977, updated by the authors.

[b] The initial capital is $100,000; the annuity is payable with certainty for 10 years; annuity payments are made at the end of the year; RV is the interest rate used to determine the initial level of the projected stream of annuity payments.

[c] Mixed portfolio consists of bonds (52%), bills (29%) and stocks (19%).

Appendix

Description of Alternative Annuity Designs

I. Notation

R_t = Nominal rate of return earned on the fund in year t.

RV = Interest rate used to determine the base value of the annuity payment; also called the annuity valuation rate or valuation rate.

r_t = Real rate of return earned on the fund in year t.

B_t = Nominal benefit payment received at the end of year t.

B_0 = Base value of the benefit; i.e., the value of B_1 if $R_1 = RV$.

b_t = Real benefit received at the end of year t.

A_t = Nominal value of the amount left in the fund at the end of year t after B_t is paid out.

P_t = Consumer price level at the end of year t with P_0 set equal to one.

T = Number of years the annuity lasts.

II. Terms of the Annuities

For all annuities the base value of the annuity payment is determined by:

$$B_0 = \begin{cases} A_0/T & \text{if} \quad RV = 0 \\ A_0 RV[1 - (1+RV)^{-T}]^{-1} & \text{if} \quad RV > 0 \end{cases}$$

We assume that benefit payments start at the end of the first year so B_0 is not actually paid out, but rather serves as the base value for computing the first year's benefit, B_1.

For the standard variable annuity the nominal benefit is:

$$B_t = B_{t-1} \frac{(1+R_t)}{(1+RV)}$$

and the real benefit:

$$b_t = b_{t-1} \frac{(1+r_t)}{(1+RV)}$$

or

$$b_t = B_t/P_t$$

For a nominal annuity R_t is nonstochastic so

$$B_t = B_{t-1} \frac{(1+R)}{(1+RV)}$$

and the rate of graduation in the nominal benefit payments is:

$$\frac{(1+R)}{(1+RV)} - 1$$

Note that if RV = R, we have the conventional level-payment nominal annuity.

For a purchasing power annuity r_t is nonstochastic so

$$b_t = b_{t-1} \frac{(1+r)}{(1+RV)}$$

and the rate of graduation in the real benefit stream is:

$$\frac{(1+r)}{(1+RV)} - 1$$

For the VAF, the variable annuity with a nominal floor, the nominal benefit is given by:

$$B_t = \begin{cases} B_{t-1} \frac{(1+R_t)}{(1+RV)} & \text{if } R_t > RV \\ B_{t-1} & \text{if } R_t \leq RV \end{cases}$$

and the real benefit by:

$$b_t = B_t/P_t$$

The VAFC, the variable annuity with a nominal floor and real ceiling, is complicated. The benefit calculation follows an iterative procedure that can be seen by a simple flow chart (Flow Chart 1).

To create an algebraic flow chart, we need some additional notation:

K_t = Amount of money in the "bank"; $K_0 = 0$.

X_t = Amount of money available to increase the benefit stream.

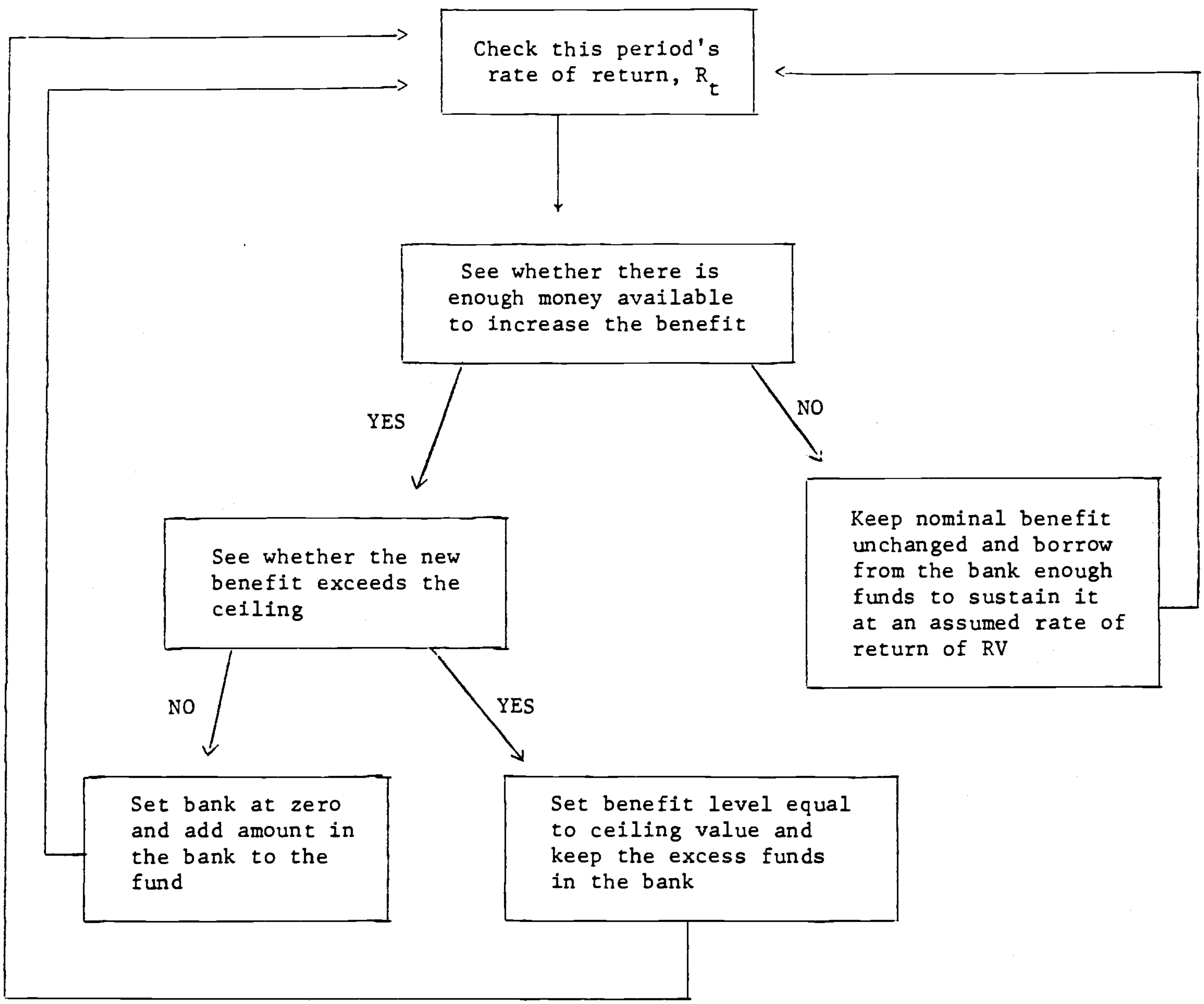
F_t = Present value of a $1 annuity due for T-t+1 years at an interest rate of RV.

$\hat{B}_t$ = Benefit which would be payable in the absence of the real ceiling.

The benefit calculation follows the following iterative procedure (Flow Chart 2).

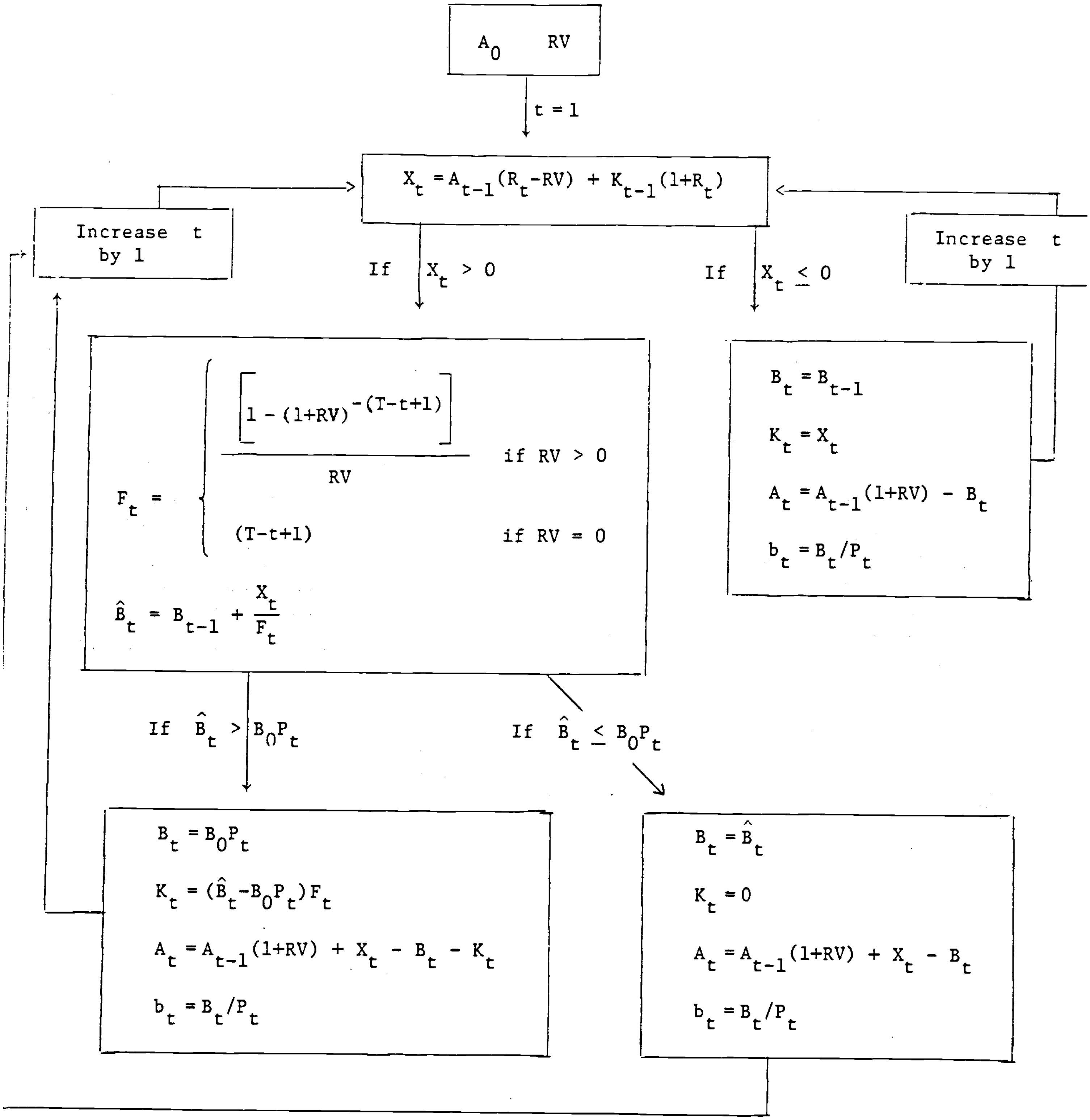
Flow Chart 1

Flow Chart Showing the Iterative Procedure for Calculating Benefit Stream for a VAFC



Flow Chart 2

Algebraic Flow Chart Showing the Iterative Procedure
for Calculating Benefit Stream for a VAFC



In the following table we demonstrate how the procedure works for the VAFC based on the stocks only portfolio reported in Table 5. In this example, A_0 = \$100,000 and RV = .05. We present the calculation for the first three years only.

	t	R_t	P_t	X_t	F_t	$\hat{B}_t$
	0	.05	1.0000	--	--	--
1971	1	.1431	1.0336	9,310	8.1078	14,099
1972	2	.1898	1.0688	20,180	7.4632	16,090
1973	3	-.1466	1.1629	-3,274	6.7864	--

	B_0P_t	B_t	K_t	A_t	b_t
	--	12,950	0	100,000	12,950
1971	13,386	13,386	5,781	95,142	12,950
1972	13,842	13,842	16,774	89,464	12,950
1973	--	13,842	-3,274	80,095	11,903