NBER WORKING PAPER SERIES

A REEXAMINATION OF PURCHASING POWER PARITY: A MULTI-COUNTRY AND MULTI-PERIOD STUDY

Craig S. Hakkio

Working Paper No. 865

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

March 1982

I would like to thank Robert Chirinko, Robert J. Gordon, Fumio Hayashi and Michael Marrese for helpful comments. This material is based upon work supported by the National Science Foundation under Grant No. DAR-8011371. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #865
March 1982

A Reexamination of Purchasing Power Parity: A Multi-Country and Multi-Period Study

ABSTRACT

This paper presents a systematic analysis of the purchasing power parity hypothesis (PPP). This hypothesis states that the exchange rate is equal to the ratio of the domestic price level to the foreign price level. It has recently been argued that PPP performs poorly in the 1970s. This paper examines several possible explanations for this poor performance. We examine PPP in the 1920s and the 1970s, using monthly and quarterly data, to see if the relationship has changed over time. We also examine PPP in a multi-exchange rate world, allowing a quite general error process so as to allow deviations from PPP to be autocorrelated and correlated across currencies. We are then able to examine the degree to which the world has become more interdependent. We also provide evidence that deviations from PPP may follow a random walk. Finally, the role of the U.S. dollar as base currency is examined. We find, in general, that PPP holds quite well as a long run proposition, but the deviations from PPP tend to persist.

Craig S. Hakkio
Department of Economics
Northwestern University
Evanston, Illinois 60201

(312) 492-5690

I. Introduction

This paper presents a systematic empirical analysis of the purchasing power parity hypothesis (PPP). This hypothesis states that the exchange rate is equal to the ratio of the domestic price level to the foreign price level. A major puzzle is why PPP appears to perform poorly in the 1970s, yet performs well in the 1920s.

Purchasing power parity is a major building block of monetary models of exchange rate determination (see, for example, the essays in Frenkel and Johnson (1978)). The poor performance of PPP has led Dornbusch to conclude that there is "little doubt that the monetary approach to exchange rate determination ...is an unsatisfactory theory of exchange rate determination" (Dornbusch (1980), p. 151). In this paper, it will be argued that this assertion is misleading. It is true that conventional estimation of monetary models may be unsatisfactory, but estimation based on a slightly more general statement of PPP may be satisfactory.

There are several possible explanations for the failure of PPP in the 1970s that should be considered. First, to examine exchange rate behavior in a multi-exchange rate world, we should use multilateral exchange rate models rather than bilateral models. We shall compare the PPP relation in the 1970s to that in the 1920s. To the extent that the world economy is more interdependent in the 1970s than in the 1920s, this should improve our ability to explain exchange rate behavior. In addition, we have a method for determining the degree to which the world has become more interdependent: what has happened to the correlation between exchange rate movements? Second, it can be shown that a greater degree of capital mobility will imply that deviations from PPP approximately follow a random walk. We find that all exchange rates examined in the 1970s follow a random walk, while some exchange rates in the 1920s do not. This observation also supports the hypothesis of greater interdependence.

Several recent studies have found deviations from PPP to be large and persistent. For example, Frenkel (1981) finds that he cannot reject the hypothesis that deviations from PPP are a random walk. Darby (1981, p.9) finds that deviations from PPP follow "a random walk with perhaps a moving-average adjustment process added." Dornbusch (1980, p.46) states that "the short term exchange rate deviate(s) from a PPP path, but there are also cumulative deviations from that path that show substantial peristence." However, all these studies examine exchange rates in a bilateral world, rather than a multilateral world.

In trying to document and explain this puzzle, we shall focus on several aspects of the PPP relationship. In particular, we shall examine PPP along 3 dimensions. First, we shall look at the PPP relationship in two time periods: the 1970s and the 1920s. Second, we shall use two time intervals: monthly and quarterly data in the 1970s (and monthly data in the 1920s). Third, we shall examine PPP country by country and several countries jointly. Such a procedure will enable us to examine exchange rate behavior in a multicountry foreign exchange market.

In section II we look at some simple, single country estimation results. This section provides the evidence for the observation that PPP fails in the 1970s. In section III, we estimate the PPP relationship for several countries simultaneously, assuming a quite general error process. This section allows us to document the degree of interdependence in the 1920s and the 1970s. In Section IV, we consider the extent to which increased capital mobility has led to deviations from PPP following a random walk. Section V inquires into the role played by the U.S. dollar as the base currency. Finally, section VI will discuss and summarize the main results to be drawn from this study.

The PPP relationship to be studied can be written as:

$$\ell n S_{it} = \alpha + \beta \ell n(P_t/P_{it}) + u_{it} \tag{1}$$

S_{it} = domestic (U.S.) price of currency i

P_t = U.S. price level

P_{it} = price level in country i

Most studies of equation (1) focus on bilateral estimation, regressing the U.S. price of the DM on the ratio of U.S. to German price levels. In this paper, we shall estimate equation (1) for several exchange rates (relative to the dollar) simultaneously. That is, we shall examine equation (1) in a time series-cross section framework. The greater cross-sectional variation in the data will allow for more precise estimation of α and β.

As stated earlier, we shall use monthly and quarterly data for the 1970s (June 1973 to December 1979) and the 1920s (January 1921 to May 1925). For the 1970s, we begin with seven countries: the United Kingdom (ENG), France (FRA), Germany (GER), Italy (ITA), Sweden (SWE), Canada (CAN) and Japan (JPN). (See the Data Appendix for more details.) For the 1920s, we focus on five countries: the United Kingdom, France, Italy, Sweden and Japan. We leave Germany out of this sample due to the hyperinflation. To compare the results from the 1920s to the 1970s, we restrict our attention, for monthly data, to the same five countries. For quarterly data, we examine a different subset of countries: the United Kingdom, Germany, Sweden and Canada. For future reference, the three subsets of countries used are:

Q-1970 = United Kingdom, Germany, Sweden, Canada.
1973III to 1979IV

M-1970 = United Kingdom, France, Italy, Sweden, Japan.
June 1973 to December 1979

M-1920 = United Kingdom, France, Italy, Sweden, Japan.
January 1921 to May 1925

II. Single Country Estimation of PPP.

This section examines the PPP relationship in a bilateral world. That is, we look at equation (1), for, say, the U.K. and Germany, but ignore any interaction between these two exchange rates. This is the approach generally used in the literature, and will serve as a summary of the bilateral evidence on PPP. We first estimate, for each of several countries, the following regression equation by OLSQ:

$$\ell n S_{it} = \alpha_i + \beta_i \ln(P_t/P_{it}) + u_{it} \tag{1}$$

Next, we assume u_{it} is AR(1):

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it} \tag{2}$$

We then estimate (1) and (2) by a Cochrane-Orcutt (CORC) procedure. Finally, we recognize that prices and exchange rates may be simultaneously determined, and so we estimate (1) and (2) using instrumental variables (FAIR). For instruments, we use a constant, time, time squared, lagged prices and lagged exchange rates. The results are reported in Table 1. For quarterly data in the 1970s, we see that the OLSQ results indicate that PPP does not hold very well - the estimates of β range from -0.162 (France) to 2.489 (Canada). The use of an AR(1) correction (CORC) leads to some improvement, but the results are still negative. Finally, estimating (1) and (2) using instrumental variables (FAIR) yields slightly better results, β ranges from -1.9 (France) to 2.13 (Germany). In all cases, the estimates of ρ are significantly positive. The estimate of ρ ranges from about .46 (Germany and Sweden) to 1.001 (Canada). We now see the reason for Dornbusch (1980, p.151) to state that the "key link between the exchange rate and PPP fails to hold."

We next examine the PPP relationship in the 1970s using monthly data. The evidence is again weak, and independent of the estimation procedure (OLSQ,CORC,FAIR). If we adopt an asset view of exchange rates, then

> The exchange is never ascertained by estimating the comparative value of money in corn, cloth or any commodity whatever but by estimating the value of the currency of one country, in the currency of another (Ricardo (1821), p. 128 as quoted in Frenkel and Johnson (1978), p. 5).

That is, aggregate price levels act as a proxy for monetary condition in the two countries. Since evidence on money demand functions indicates that the relation between prices and money may not be stable for monthly data, the fact that the PPP relation does not hold well for monthly data may not be surprising.

Finally, we estimated equation (1) for the 1920s, using monthly data. The results (using FAIR's method) indicate that PPP held reasonablly well during the 1920s. The exceptions are Italy, Sweden and Japan. The result for Italy is unusual, in that the point estimates of α, β and ρ are quite different (although maybe not significantly so) when the estimates by OLSQ or

CORC are compared to the estimates by FAIR. Einzig (1937) indicates that the financial market in Italy was not very well developed (p. 295) and that "While there was excessive optimism about the prospects of the Italian exchange, there was excessive pessimism about the internal political and economic conditions" (p. 297). Japan may be explained by its relative isolation during the 1920s. Also the 1920s were a period of rapid growth for Japan: between 1919 (a peak year) and 1931 (a tough year), GDP grew 2.3% per year, exports grew 5.8% per year and world trade grew 2.7% per year (Shionoya and Yamozawa (1973), p. 517). Most of Japans trade was with Asia (41% of its 1919 exports), North America (44% of its 1919 exports) and little of its trade was with Europe (10% of its 1919 exports) (Ohkawa and Rosovsky (1973), p. 191). In addition, there was also a large earthquake on September 1, 1923 (Tinbergen (1934), p. 127 and Einzig (1937), p. 208), that may have led to "unusually" large deviations from PPP.

In conclusion, we find that PPP holds reasonably well in the 1920s (even for monthly data) and not so well for the 1970s. In the next three sections we will examine various hypotheses to explain these results.

III Multicountry Estimation of PPP.

A. Description of the General Estimating Equation.

In this section, we consider again equation (1), but consider estimation procedures for a multicountry foreign exchange market. That is, we use a time series-cross section estimation procedure. We assume equations (1) and (2) hold for each country, but we allow the errors to be correlated across countries. The general idea is that if there is a shock to the German-U.S. exchange rate (so as to cause a positive deviation from PPP) then there will likely be a (positive) shock to the French-U.S. exchange rate. This correlation may arise for any of seveal reasons. First, there may be a world shock that effects all exchange rates, such as a world-wide recession. Alternatively, there could be a shock in Germany that is then transmitted to France and so effects both exchange rates. Finally, since all exchange rates are relative to the dollar, any shock in the U.S. will affect all exchange rates. Consider, for example, an unexpected increase in German real income. This will tend to appreciate the Deutsche Mark. If German and Swiss real income innovations are positively correlated, then we should also see an appreciation of the Swiss franc. Alternatively, suppose an increase in German real income leads to an increase in demand for Deutsche Marks and Swiss francs. Then we would expect to observe both the Deutsche Mark and Swiss franc appreciate. That is, an (unexpected) increase in the dollar/Deutsche Mark exchange rate conveys useful information about the dollar/Swiss franc exchange rate that is ignored in single exchange rate estimation procedures.

An exact description of the error term u_{it}, in equation (1) is given below.

$$Eu_{it}^2 = \sigma_{ii} \quad \text{(heteroskedasticity)} \tag{3a}$$

$$Eu_{it}u_{jt} = \sigma_{ij} \quad \text{(mutual correlation)} \tag{3b}$$

$$u_{jt} = \rho_i u_{i,t-1} + \varepsilon_{it} \quad \varepsilon_{it} \sim N(0, \phi_{ii}) \quad \text{(autoregression)} \tag{3c}$$

$$Eu_{i,t-1} \varepsilon_{jt} = 0 \tag{3d}$$

$$E\varepsilon_{it}\varepsilon_{jt} = \phi_{ij}$$

$$E\varepsilon_{it}\varepsilon_{js} = 0 \quad (t \neq s)$$

Equation (3a) allows the variances of the residuals to differ between countries; (3b) allows the residuals to be correlated across countries; (3c) allows the residuals to be autocorrelated for each country. The estimation of equations (1) and (3) is described in Kmenta (1971, pp. 512-514). The procedure employed here is different. Rho-difference equation (1) to obtain

$$\ell nS_{it} = \alpha_i(1-\rho_i) + \beta\ell n(P_t/P_{it}) - \beta\rho_i \ell n(P_{t-1}/P_{i,t-1}) + \rho_i \ell nS_{i,t-1} + \varepsilon_{it} \qquad (4)$$

Equation (4) can then be estimated as a system of N equations (N=4, 5 or 7).

B. Estimation of PPP with a Monetary Model

We first estimate (4) using quarterly data from the 1970s, under the assumption that prices are exogenous. For the sample of all seven currencies, relative to the U.S. dollar, the results are given in Table 2. We see that the estimate of β is close to 1 and significant (the t-statistic is 9.6, and β is 1.080, which is only .7 standard deviations from 1.0). The correlation matrix indicates that the residuals for the European currencies are large and positive, negative for Canada, and small and positive for Japan. Although the assumption of exogenous prices is strong, these results are indicative of those to follow when we relax the assumption of exogenous prices.

The previous estimation procedure assumed that the price ratio, $\ln(P_t/P_{it})$, was exogenous. If this assumption is false, then there is a simultaneous equation bias. To allow for this situation, we can expand the model, to allow prices to be endogenous. From monetary theory, the price level is determined so as to equate real money demand to real money supply. Assuming that the demand for money is a function of income and the interest rate and that income and interest rates are exogenous (exogenous interest rates will be relaxed) we can express the price level as a function of exogenous variables. The expanded model we will consider consists of equations (3) and a set of money demand functions; the model we consider can be written as

$$\ell nS_{it} = \alpha_i(1-\rho_i) + \beta\ell n(P_t/P_{i,t}) - \beta\rho_i \ell n(P_{t-1}/P_{i,t-1}) + \rho_i \ell nS_{i,t-1} + \varepsilon_{it} \qquad i=1,\dots,4 \tag{5a}$$

$$\ell nP_{it} = \ell nM_{it} - \gamma_i \ell nM_{i,t-1} + \gamma_i \ell nP_{i,t-1} - (1-\gamma_i)a_i - b_i \ell ny_{i,t} + b_i\gamma_i \ell ny_{i,t-1} - c_i i_{i,t} + c_i\gamma_i i_{i,t-1} + v_{it} \qquad i=1,\dots,5 \tag{5b}$$

where $M_{i,t}$ = money supply of country i

$y_{i,t}$ = real GNP of country i

$i_{i,t}$ = interest rate in country i

i = 5 denotes the U.S.

In this formulation, there are 29 parameters to estimate $((\beta,\alpha_i,\rho_i(i=1,\dots,4),\ \gamma_j,a_j,b_j,c_j\ (j=1,\dots,5))$. Equations (5a) and (5b) can be estimated using Full Information Maximum Likelihood (FIML). The results are given in Table 3. The estimate of the PPP slope parameter, β, is 1.101, with a standard error of .108 (t-statistic is 10.19). The money demand income elasticity (b_i) is positive (and significant) for the U.K., Germany and Sweden, but is negative, and insignificant, for Canada and the U.S. The money demand interest semi-elasticity (c_i) is negative for Germany, Canada and the U.S., but insignificantly positive for the U.K. and Sweden. One difficulty is that ρ(U.K.) was (insignificantly) greater than 1.0, which indicates non-stationarity. Also, except for the U.K., the money demand autoregressive parameter (γ_i) is small, being insignificantly different from 0.0. One possible explanation is that the serial correlation in money demand functions is being "picked up" by the autregressive error structure of the PPP equation (see Hakkio (1982)).

The assumption that interest rates are exogenous will now be relaxed, since both exchange rates and interest rates are determined in a financial market. To allow for this possibility, we estimate the PPP equation (1) using three stage lest squares (3SLS). The instruments are money, money lagged once, income, income lagged once, lagged exchange rates and lagged prices. One can think of this as substituting out interest rates in a rational expectations model. The model that was estimated can be written as

$$\ell n S_{it} = \alpha_i + \beta \ell n\ (P_t/P_{it}) + u_{it} \qquad (6)$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it}$$

$$IV = \{1,\ \ell n M_{it},\ \ell n M_{i,t-1},\ \ell n y_{it},\ \ell n y_{i,t-1},\ \ell n S_{i,t-1},\ \ell n P_{i,t-1} (i=1,\ldots,5)\}$$

A difficulty arose in estimating equation (6), When estimating equation (6) as specified, the system did not converge. All parameters converged, except the constant for Canada (α_4); ρ(Canada) appeared to converge to 1, which indicates nonstationarity. To overcome this lack of convergence,

ρ(Canada) was constrained to be 0.999. With this constraint, the system converged. The estimates of the parameters did not seem to be seriously affected by this constraint, however α_4 has a low t-statistic. The results are:

α_1 = .919 (.176)	α_2 = −.846 (.037)	α_3 = −1.413 (.022)	α_4 = −3.786 (3.964)
ρ_1 = .923 (.070)	ρ_2 = .711 (.083)	ρ_3 = .568 (.112)	$\rho_4 \equiv$ .999

β = .951 (.141)

CORR $(\varepsilon_i, \varepsilon_j)$:

Eng	1.000			
Ger	.662	1.000		
Swe	.510	.846	1.000	
Can	−.354	−.340	−.391	1.000

We see that the estimate of β is again close to one, 0.951 with a standard error of 0.141. The estimates of ρ are all significantly greater than 0. From the correlation matrix we see that the deviations from PPP are highly correlated (positive for Europe and negative for Canada).

C. Comparison of PPP across Time Intervals and Time Periods.

To compare the performance of PPP across time intervals (monthly versus quarterly data) and across time periods (the 1970s and the 1920s), we now reestimate equations (1) and (3). To facilitate comparison, we estimate (1) using 3SLS with instruments being a constant, time, time squared and lagged prices and exchange rates for each model. We also allow α to be different across countries. The results are given in Table 4 (again, ρ(Canada) was constrained to be 0.999).

For quarterly data, all estimates of ρ are less than 1.0, and β is precisely estimated to be .951 with a standard error of .149 (for a t-statistic of 6.4). However, when we look at the monthly data for the 1970s (and a different set of countries), the estimate of β is low (.409 with a standard error of.183). This would seem to indicate that the relationship between prices and exchanges is not very close with monthly data, but does conform to PPP predictions with quarterly data. (This result is similar to the single equation results.)

This evidence is consistent with the hypothesis that PPP is a medium to long run condition. Since the PPP relationship may be viewed as a proxy for the monetary conditions in a country and, in general, monthly money demand functions are not stable, one would not expect PPP to hold well using monthly data. Also, price levels are extremely autocorrelated using monthly data, while exchange rates, being "auction" prices, are much more volatile. It is only when we move to quarterly data that we pick up a relation between exchange rates and prices. One possible method of testing such a possibility would be to use a band spectrum estimation procedure which would "block out" short run movements (this is not done due to the simultaneity problem).

The results for the 1920s appear surprising. The estimate of β is .578, with a standard error of .072, significantly different from both 0 and 1. As stated in section II, this result may be due to the special circumstances of Italy, Sweden and Japan. If we estimate the PPP relation with only the U.K. and France, β equals 1.075 (standard error =.091). If we then include Sweden (exclude Italy and Japan), β equals 1.064 (standard error=.080). Hence, it appears that the poor result (β = .578) is due to the inclusion of Italy and Japan. A possible explanation was given in Section II.

We are now in a position to examine the degree of interdependence. As stated earlier, our measure of interdependence is the correlation of the country error terms (CORR($\varepsilon_i, \varepsilon_j$)). The correlation matrices corresponding to Table 4 are given in Table 5. This definition of interdependence reflects the view that a large correlation implies that deviations from PPP for different exchange rates arise for the same reason or are transmitted very

1970s than in the 1920s. This would be consistent with the hypothesis that shocks in one country are more likely to occur simultaneously in several countries (or be rapidly transmitted to other countries) in the 1970s than in the 1920s. Second, Japan appears to be more isolated from the rest of Europe in the 1920s than in the 1970s, in that the deviations from PPP for Japan were approximately uncorrelated with the PPP deviations for the other European countries, whereas the correlation became positive in the 1970s. That is, since the 1920s, the interdependence of Japan and Western Europe has incresed dramatically.

To summarize this section, by using a time series - cross section estimation procedure, with quarterly data, we found that the PPP relation holds quite well (β is statistically close to 1), although the deviations from PPP tend to be persistent (ρ is close to 1). This is in contrast to the results reported in Section II (using single exchange rate methods) where β ranges from -1.9 to 2.1. The increased precision in estimation arises from exploiting the cross-sectional variation in the data. The correlation of the disturbances is quite large and so improves the estimation of β. However, the results for the monthly data (in the 1970s) indicate that PPP does not hold, even though the correlation matrix of the disturbances is quite large. This seems to indicate that PPP is a long run condition and does not hold for monthly data. If the relation between money and prices is not stable using monthly data (perhaps real income is not exogenous in the short run), and PPP reflects the underlying monetary conditions, then this result should not be surprising.

quickly. In section IV we will consider further evidence concerning interdependence.

In examining the correlation matrix for Q-1970, we find the correlations are quite large. The correlation for Sweden and Germany is 85%; this, however, may only reflect the fact that Sweden and Germany belong to the snake. However, Germany and the U.K. have a correlation of 66%. It is interesting to note that the correlation between Canada and the other countries is negative, and approximately -36%. In addition, the correlations are greatest for Germany, indicating Germany's major role in world affairs. The evidence suggests that a positive PPP deviation for any one of the U.S.-European exchange rates is likely to be coincident with a positive PPP deviation for the other European countries, but with a negative PPP deviation for Canada. It appears, then, that the Canadian exchange rate behavior is different, in this fundamental way, from European exchange rate behavior. One possible explanation is that Canada and the U.S. are close, both economically and geographically, while physically separated from Western Europe. However, no convincing explanation for Canada's anomalous exchange rate behavior has yet been found.

In looking at the monthly data for the 1970s, we find the correlations are still quite large. The correlations with France are the largest (recall that Germany was not included), while the correlations with Japan are the smallest (although positive). If we compare these results with the results from the 1920s, for the same set of countries, we find several interesting results. First, and most important, is the observation that the correlations are all much smaller for the 1920s. I interpret this to mean that the degree of interdependence has increased from the 1920s to the 1970s. That is, unexpected PPP deviations were much more correlated across countries in the

IV. Alternative Representations of PPP.

Roll (1979) argues that under certain conditions, deviations from PPP should follow a random walk. In terms of equations (1) and (2), if $\alpha=0$, $\beta=1$ and $\rho=1$, then the deviations from PPP ($\ell n S_t - \ell n(P_t/P_{it})$) will follow a random walk. The estimates in section III (estimates of ρ close to 1) indicate that the assumption of deviations from PPP being a random walk must be considered.

To see under what conditions the deviations will be a random walk (or some more complex process (Darby, 1981)), assume that interest rate parity holds:

$$i_t - i_t^* = f_t - s_t \tag{7}$$

where f_t is the log of the forward rate, s_t is the log of the spot rate and i_t^* is the foreign interest rate. If the foreign exchange market is efficient, so that we can write $f_t = s_{t+1}^e$, where s_{t+1}^e is the rational expectation of s_{t+1}, based on information at time t, we can rewrite (7) as

$$i_t - i_t^* = s_{t+1}^e - s_t \tag{7'}$$

If the Fisher relationship holds in both countries, then

$$i_t = r_t + \pi_t^e \tag{8a}$$

$$i_t^* = r_t^* + \pi_t^{*e} \tag{8b}$$

where r_t is the real rate of interest and π_t^e is the expected rate of inflation (between t and t+1). Substituting (8) into (7') and rearranging yields

$$r_t - r_t^* = (s_{t+1}^e - s_t) + (\pi_t^{*e} - \pi_t^e) \qquad (9)$$

Defining $\pi_t^e = p_{t+1}^e - p_t$ (p_t = log of the price level), and rearranging, we obtain

$$s_{t+1}^e - p_{t+1}^e + p_{t+1}^{*e} = s_t - p_t + p_t^* + (r_t - r_t^*) \qquad (10)$$

Equation (10) implies that if capital markets adjust instantaneously, such that real interest rates are equalized across countries (except, perhaps, for a random term), then deviations from PPP will follow a random walk.

Let us define Δ_{it} as the deviation from PPP for country i, $\Delta_{it} = \ln S_{it} - \ln(p_t/p_{it})$. To test if Δ_{it} follows a random walk, consider the following regression:

$$\Delta_{it} = a_i + b_i \Delta_{i,t-1} + \mu_{it} \qquad (i=1, \ldots, N) \qquad (11)$$

If the deviations from PPP follow a random walk, then we should find a = 0 and b = 1. Testing this hypothesis requires care, since for b = 1 we are on the boundary of the permissable (stationary) parameter space. Dickey and Fuller (1981) give test statistics for a = 0 and b = 1 and the empirical distribution function for these test statistics. Table 6 gives the estimates of a and b and the relevant test statistic for the null hypothesis that a = 0 and b = 1. For the 1970s (quarterly and monthly), for all countries, we can not reject the null hypothesis that deviations from PPP follow a random walk. Note that if one examined the estimate of β, and its standard error, one would be tempted to reject the hypotheses of a random walk for Sweden. However, the probability that $\hat{b} < 1$, given $b = 1$, approaches 68% as the sample size gets large (Fuller (1976), p. 370, for the case $a \equiv 0$). It is interesting to note that in the 1920s, the hypothesis that the deviations from PPP are a random walk can be rejected for both France and Italy. This observation, that PPP deviations move from a non-random walk to a random walk from the 1920's to the 1970's, indicates a structural change in French and Italian exchange rate behavior. The explanation for Italy may be a move towards greater capital market integration (recall Einzig's (1937, p. 295) observation). An explanation for France is less clear.

At the beginning of this section, we stated that under certain conditions (interest rate parity, foreign exchange market efficiency, the Fisher relationship and rapid capital mobility) we would expect the deviations from PPP to follow a random walk. The hypothesis of simple market efficiency ($s^e_{t+1} = f_t$) may be suspect; see, for example, Hakkio (1981a) and Hansen and Hodrick (1980). Mishkin (1981) presents evidence that indicates that real interest rates are not equalized across countries, while the evidence presented here is consistent with real rates being equalized. One possible explanation is that the test presented here is not very powerful (in fact, the power is unknown).

If the deviations from PPP follow a random walk, then it may be appropriate to estimate equation (1) in first difference form:

$$\ell nS_{it} - \ell nS_{i,t-1} = \gamma + \beta[\ell n(P_t/P_{it}) - \ell n(P_{t-1}/P_{i,t-1})] + \mu_t \quad (12)$$

There are several interpretations of equation (12) and μ_t. If we assume μ_t is white noise, then (12) is a regression of the rate of depreciation on the differential rate of inflation: the relative version of

PPP. Equations (11) and (12) are the same, only if $\beta = b = 1$. If one first differences equation (1), one obtains equation (12), with $\gamma = 0$ and $\mu_{it} = u_{it} - u_{i,t-1}$. Therefore, if one assumes u_{it} can be written as $\rho u_{i,t-1} + \varepsilon_{it}$ $(\rho \leq 1)$, one can write μ_{it} as $(\rho-1)u_{i,t-1} + \varepsilon_{it}$. If $\rho < 1$, estimating equation (12) by OLSQ is inappropriate since $u_{i,t-1}$ and $\ln(P_{t-1} / P_{i,t-1})$ are correlated (if $\rho=0$ then μ_{it} is MA(1), with a unit root). If $\rho=1$, estimation of (12) by OSLQ is appropriate. Hence, one can view the results in section III as estimating β, given $\rho < 1$, while equation (12) estimates β, given $\rho = 1$.

Table 7 gives estimates of γ and β using OLSQ; we assume the rates of inflation can be treated as exogenous. In all cases, the estimate of γ is insignificantly different from zero. The estimates of β for the quarterly 1970s data are insignificantly different from 1.0, but also insignificantly different from zero, except for the United Kingdom. The same results hold for the other subsets of countries and time periods. The results are similar to the results in Table 1. In Table 8, we assume that inflation is endogenous and we estimate the N-equation system (12) using 3SLS, with instruments being a constant, time, time squared and lagged inflation and rates of depreciation. In this case, we allow the β to be different, but constrain the α_i to be equal. As can be seen from comparing Tables 6 and 7, the results are not very different: $\hat{\beta}$ is imprecisely estimated, often insignificantly different from both 0 and 1.

The right hand side of equation (12) is the inflation rate differential, which we know to be highly autocorrelated, while the left hand side variable, the rate of exchange depreciation, shows little autocorrelation. In other words, equation (12) is trying to "explain" a temporally uncorrelated variable with a temporally correlated variable; a finding of β between 0 and 1 should not be surprising.

In Table 9, the correlation matrix of residuals from equation (12) are given. As in Table 5, the correlations are quite large, and negative for Canada relative to Western Europe, for Q-1970. The correlations are quite large for monthly data, although the PPP relation does not hold well. For the 1920s, the correlations are smaller, and insignificant for Japan.

V. The DM as the Base Currency.

The last topic to be examined is the role of the U.S. dollar as the base currency in the exchange rate equation. It may be that most of the deviations from PPP are a result of movements in the U.S. dollar. If deviations from PPP arise due to transportation costs, then the deviations would be less among the European countries. In addition, the structure of tariff barriers may be more stable among European countries than between the U.S. and European countries. Finally, the behavior of U.S. prices during the 1970s has been influenced by the existence of U.S. price controls (and their removal).

To examine these possibilities, we reestimate the PPP equation using Germany as the base currency. We calculated the DM price of, say, the French franc by using triangular arbitrage: S(DM/fr) = S($/fr)/S($/DM). Denoting the DM price of currency i by $\tilde{S}_i$ and the German price level by $\tilde{P}$, we can consider the following regression equation:

$$\ln\tilde{S}_{it} = \alpha + \beta \ln(\tilde{P}_t/P_{it}) + u_{it} \quad (13a)$$

$$u_{it} = \rho u_{i,t-1} + \varepsilon_{it} \quad (13b)$$

Table 10 reports the results of estimating equation (13), equation by equation, using 2SLS (FAIRs method). Notice that the results for the U.S. need not be the same as for Germany in Table 1, due to the simultaneity and autoregressive correction (the OLSQ results are, of course, the same).

Several results of interest emerge from Table 10. First, the estimates of β are close to one, with "small" standard errors. However, the results for Canada indicate a value of β too large (β = 2.693, standard error = 0.354) and the estimate of β for Japan is -3.340, but with a standard error of 3.392, so that a two standard deviation confidence interval would be (-10.1, 3.4) (in addition, the R^2 for Japan was 0.18). Both these countries are geographically isolated from Western Europe, and so the transport cost argument may be able to explain these two results. In addition, the results when using the U.S. dollar as the base currency were also poor. These results indicate that part of the explanation for the "failure" of PPP (seen in Table 1) may be due to the use of the U.S. as the base currency. The case of Japan and Canada would seem to indicate something peculiar to their country, rather than the foreign exchange market.

In the previous section (Table 4) we estimated the PPP equation jointly for the U.K., Germany, Sweden and Canada, relative to the U.S. We can estimate a similar equation (for the U.S., the U.K., Sweden and Canada) using the DM as the base currency. The result is:

α(USA) =	.763 (.220)	α(Eng) =	1.638 (.032)	α(Swe) =	-.656 (.396)	α(Can)=	-11.028 (11.978)
ρ(USA) =	.951 (.036)	ρ(Eng)=	.682 (.116)	ρ(Swe) =	.980 (.077)	ρ(Can) $\equiv$	.999
		β =	.886 (.108)				

Instruments were a constant, time, time squared, lagged exchange rates and lagged price ratios. Recall that the estimate of β , when the U.S. was the base currency, was 0.951, with a standard error of 0.149 (see Table 4). The results are close (within one-half standard deviation). This should not be surprising, since by estimating all equations jointly we should be reducing the role played by the dollar. In fact, since the equations with the DM as the base currency are just a linear combination of the equations with the U.S. dollar as the base currency, the joint estimation should produce similar results (the α_i and ρ_i need not be similar due to different base currencies).

We now estimate a PPP equation for Western Europe (the U.K., France, Italy, and Sweden) and the U.S., relative to Germany. The results are given in Table 11, section I. The estimate of β , 1.189 (standard error of 0.060), is significantly greater than 1.0, but precisely estimated. To see the role played by the U.S. dollar, we next estimated the same equation for Western Europe (excluding the dollar). The results are given in Table 11, section II. The results are virtually indistinguishable. Part of the explanation is that the starting values were equal to the final value when the U.S. was excluded. This result tends to indicate that the increased precision is due to the joint estimation, as opposed to the choice of base currency.

VI. Summary and Conclusions.

Using monthly and quarterly data from the 1970s and the 1920s we were able to examine several explanations for the oft-cited failure of PPP to hold empirically. The foreign exchange market is a well-functioning market involved in setting exchange rates for several currencies simultaneously. If one examines PPP in such a setting, one finds that PPP holds quite well as a long run proposition. Part of the "failure" of PPP in the 1970s involved the finding that the relation between exchange rates and prices was very imprecise, cross-sectional estimates of β range from 0.301 (for Canada) to

2.087 (for Germany).

By employing a time series-cross section estimation procedure we were able to precisely estimate β (between 0.9 and 1.1, depending on the exact estimation procedure). The extra precision came from the cross-sectional variation in the data, but also from exploiting the correlation of errors between countries. One can view this as a seemingly unrelated regression procedure in that the regression equations appear unrelated, but are, in fact, closely related due to the correlation of the disturbances (deviations from PPP). As is well-known, taking into account such correlation should improve the efficiency of the estimates — as was the case in this paper. In fact, the correlation between country disturbances was quite large.

A precise estimate of β , close to 1.0, implies that as a long run proposition, PPP is valid. However, the autoregressive parameters, ρ , were large, which indicates that deviations will tend to persist for a long period of time. Hence, the conclusion that deviations from PPP are persistent is correct. However, the conclusion that PPP does not hold well, as a long run proposition, is incorrect. Further evidence that PPP should be viewed as a long run proposition arises when estimating a PPP equation with monthly data: the results indicate that PPP does not hold in the long run ($\beta < 1$). When one allows for deviations from PPP to be correlated and takes this into account in the estimation procedure, the evidence (using quarterly data) supports purchasing power parity.

The implication of this observation is that estimation of a monetary model of exchange rate determination must allow for short run deviations from PPP to persist, but in the long run PPP holds (see Hakkio (1981b)). For efficient estimation, one should estimate several exchange rate equations simultaneously. An efficient estimation procedure would be as follows. Specify money demand functions for all N currencies:

$$(M_i/P_i)^D = L_i(y_{it}, i_{it}, z_{it}) + v_{it} \qquad (14)$$

where z_i is a vector of additional explanatory variables and v_{it} is a serially correlated error term ($v_{it} = \theta_i(L)\mu_{it}$). Next, specify a long run-short run PPP relation:

$$\ln S_{it} = \alpha_i + \beta \ln(P_t/P_{it}) + u_{it} \qquad (15a)$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it} \qquad (15b)$$

where we allow the error terms to be correlated.
Letting $e_t = (\mu_{it}, \ldots, \mu_{Nt}, \varepsilon_{1t}, \ldots, \varepsilon_{N-1,t})$, we can specify that

$$Ee_t e_t' = \Omega = (\sigma_{ij}) \qquad (16)$$

Then, simultaneous estimation of equations (14) and (15) will allow for deviations to persist in the short run, but to disappear in the long run. That is, the conclusion reached by Dornbusch that there is "little doubt that the monetary approach [to exchange rate determination] ... is an unsatisfactory theory of exchange rate determination" (Dornbusch (1980), p. 151) appears to be unwarranted.

In section IV we found that deviations from PPP may follow a random walk. In terms of section III, this would occur if $\alpha = 0$, $\beta = 1$ and $\rho = 1$. We can also derive this observation from the assumptions of an efficient foreign exchange market, real interest rates being equalized across countries and interest rate parity. The implication of this finding is that one should consider estimating exchange rate depreciation models as opposed to exchange rate level models.

Finally, in section V we examined the extent to which using the U.S. dollar as a base currency affect the results. From the single equation results, the dollar appeared to play a significant role. However, from the multi-exchange rate model, the impact of the dollar was not great, as one might have expected. When PPP is examined for only the European countries, with the DM as the base currency, the PPP relation holds well.

To conclude, the major finding is that when examining exchange rate behavior, much more precise statements can be made when models are estimated using all exchange rates and information simultaneously. Much information is lost when one ignores the fact that exchange rates are determined in a world market where exchange rates for several currencies are set simultaneously, so that movements in the dollar/pound rate contain information for the dollar/DM rate. When such additional information is used, the relation between prices and exchange rates is very strong, and conforms to the purchasing power parity prediction. However, deviations from PPP do tend to persist for long periods of time, and, in fact, may follow a random walk (and so never return).

Data Appendix

The data for the period June 1973 to December 1979 was obtained from the December 1979 and April 1981 International Financial Statistics (IFS) tapes. The variables and their IFS codes are given, by country, in the following table.

	Variable				
Country	Exchange Rate	Income	Interest Rate	Money	Prices
Canada	AE	99A.R	60C	34	64
France	AE				64
Germany	AE	99A.R	60B	34	64
Italy	AE				64
Japan	AE				64
Sweden	AE	99B.P	61	34	64
United Kingdom	AE	99B.P	60C	34	64
United States		99A.R	60C	34	64

where the IFS codes stand for:

AE = end of period exchange rate
99A.R = real GNP, 1975 prices
99B.P = real GDP, 1975 prices
60B = call money rate
60C = Treasury bill rate
61 = government bond yield
34 = money
64 = consumer price index

Monthly data was obtained for the spot exchange rate and prices for the period January 1921 to May 1925. The three primary sources were Einzig (EIN, 1937), Tinbergen (TIN, 1934) and various issues of the League of Nations (LON). For each variable, from EIN or TIN, the source, page and column number, respectively, is given.

Country	Wholesale Price Index	Spot Exchange Rate
France	TIN, 72-73, 34	EIN, 450-458, 2
Italy	LON, Table 10	EIN, 450-458, 4
Japan	TIN, 131-132, 13	TIN, 131-132, 12
Sweden	TIN, 195-196, 27	TIN, 195-196, 25
United Kingdom	TIN, 105-106, 21	EIN, 450-458, 1
United States	TIN, 210-211, 28	

Table 1

Purchasing Power Parity

$$\ell n\ S_t = \alpha + \beta\ \ell n\ (P_t/P_t{*}) + u_t$$

$$u_t - \rho u_{t-1} + \varepsilon_t$$

Country	Procedure	α	β	ρ	R^2	D.W.	ser.
		Quarterly Data	1973 III - 1979 IV				
ENG	OLSQ	.755	.599		.46	.19	.093
		(.020)	(.132)				
	CORC	1.307	.972	.979	.90	1.64	.040
		(.502)	(.470)	(.041)			
	FAIR	1.611	1.454	.977	.89	1.78	.041
		(.691)	(.850)	(.043			
FRA	OLSQ	-1.516	-.162		.01	.41	.068
		(.016)	(.308)				
	CORC	-1.569	-1.312	.795	.64	1.22	.041
		(.064)	(1.019)	(.121)			
	FAIR	-1.601	-1.900	.819	.63	1.21	.042
		(.080)	(1.287)	(.115)			
GER	OLSQ	- .887	1.934		.85	.98	.055
		(.012)	(.163)				
	CORC	- .901	2.069	.396	.91	1.75	.044
		(.018)	(.215)	(.184)			
	FAIR	- .905	2.130	.400	.91	1.77	.044
		(.018)	(.219)	(.183)			

Table 1 (continued)

Country	Procedure	α	β	ρ	R^2	D.W.	ser.
ITA	OLSQ	-6.543	.912		.75	.43	.075
		(.018)	(.107)				
	CORC	-6.644	.345	.818	.89	2.06	.046
		(.082)	(.363)	(.115)			
	FAIR	-6.628	.423	.806	.89	2.05	.046
		(.080)	(.367)	(.118)			
SWE	OLSQ	-1.448	.449		.17	1.12	.046
		(.013)	(.201)				
	CORC	-1.448	.417	.437	.31	1.56	.043
		(.022)	.325)	(.180)			
	FAIR	-1.454	.297	.447	.31	1.55	.043
		(.024)	(.358)	(.179)			
CAN	OLSQ	- .019	2.489	-	.55	.20	.049
		(.011)	(.462)				
	CORC	2.249	1.059	1.002	.92	1.48	.021
		(1.853)	(.734)				
	FAIR	2.140	.483	1.003	.92	1.51	.021
		(1.652)	(.914)				

Table 1 (continued)

JPN	OLSQ	-5.563	.375		.02	.10	.156
		(.031)	(.620)				
	CORC	-5.471	-.017	.957	.91	.83	.049
		(.221)	(.535)	(.058)			
	FAIR	-5.475	.878	.933	.90	1.00	.051
		(.164)	(.715)	(.072)			

Monthly Data June 1973 - December 1979

ENG	OLSQ	.759	.639		.50	.10	.088
		(.011)	(.073)				
	CORC	.652	-.088	.961	.95	1.78	.028
		(.109)	(.324)	(.031)			
	FAIR	.663	-.030	.958	.95	1.77	.028
		(.139)	(.605)	(.033)			
FRA	OLSQ	-1.516	-.026		.00	.20	.067
		(.090)	(.175)				
	CORC	-1.544	-.733	.899	.81	2.29	.029
		(.045)	(.687)	(.050)			
	FAIR	-1.571	-1.367	.916	.81	2.33	.029
		(.057)	(.996)	(.045)			
ITA	OLSQ	-6.541	.934		.78	.15	.069
		(.096)	(.057)				
	CORC	-6.558	.764	.932	.97	2.02	.027
		(.068)	(.289)	(.041)			
	FAIR	-6.590	.582	.939	.97	2.04	.026
		(.086)	(.374)	(.039)			

Table 1 (continued)

SWE	OLSQ	-1.448 (.007)	.492 (.109)		.21	.44	.044
	CORC	-1.458 (.020)	.289 (.279)	.786 (.070)	.69	1.88	.027
	FAIR	-1.456 (.022)	.311 (.330)	.790 (.069)	.69	1.91	.027
JPN	OLSQ	-5.570 (.018)	.360 (.349)		.01	.04	.151
	CORC	-5.503 (.171)	.359 (.326)	.979 (.023)	.96	1.89	.031
	FAIR	-5.506 (.140)	1.303 (.695)	.972 (.027)	.95	1.90	.032
		Monthly Data	January 1921 – May 1925				
ENG	OLSQ	1.833 (.025)	.671 (.046)		.81	.30	.034
	CORC	1.832 (.074)	.666 (.144)	.858 (.071)	.94	1.96	.018
	FAIR	1.946 (.129)	.895 (.255)	.850 (.073)	.94	2.06	.018
FRA	OLSQ	-1.159 (.081)	1.111 (.056)		.88	.84	.068
	CORC	-1.119 (.146)	1.138 (.101)	.578 (.113)	.92	1.76	.056
	FAIR	-1.242 (.161)	1.051 (.112)	.584 (.113)	.92	1.76	.056

Table 1 (continued)

ITA	OLSQ	-1.535 (.576)	.895 (.325)		.13	.92	.110
	CORC	-1.082 (.775)	1.145 (.435)	.505 (.120)	.39	1.58	.091
	FAIR	-2.655 (1.740)	.261 (.640)	.526 (.118)	.34	1.52	.095
SWE	OLSQ	-1.081 (.018)	.487 (.029)		.85	.50	.029
	CORC	-1.298 (.080)	.054 (.149)	.895 (.062)	.93	1.00	.018
	FAIR	-1.105 (.046)	.439 (.080)	.727 (.095)	.93	1.02	.019
JPN	OLSQ	3.802 (.141)	-.034 (.192)		.00	.06	.084
	CORC	3.798 (.158)	.128 (.113)	.981 (.027)	.94	1.24	.020
	FAIR	3.563 (.237)	-.207 (.267)	.972 (.033)	.93	1.25	.022

Notes: Standard errors are in parentheses; ser is the standard error of the regression. CORC is the Cochrane - Orcutt correction procedure for AR(1) errors. FAIR is Fair's two-stage least squares procedure when the error is AR(1) The instruments used were a constant, time, time squared, the lagged exchange rate and the lagged price ratio.

Table 2

$$\ln S_{it} = \alpha_i + \beta \ln(P_t/P_{it}) + u_{it}$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it}$$

Quarterly Data 1973III - 1979IV

Country	α	ρ	β(constrained)
United Kingdom	.953 (.174)	.920 (.061)	1.080 (.112)
France	-1.439 (.063)	.852 (.063)	
Germany	-.845 (.055)	.812 (.069)	
Italy	-6.520 (.034)	.672 (.094)	
Sweden	-1.412 (.025)	.635 (.094)	
Canada	-.096 (.071)	.916 (.065)	
Japan	-5.522 (.080)	.868 (.058)	

Correlation Matrix of Residuals:

ENG	1.000						
FRA	.614	1.000					
GER	.613	.765	1.000				
ITA	.598	.735	.418	1.000			
SWE	.515	.742	.875	.475	1.000		
CAN	-.456	-.505	-.341	-.594	-.405	1.000	
JPN	.224	.283	.465	.033	.351	-.236	1.000

Notes: Standard errors are in paranthesess. β was constrained to be equal for all countries.

Table 3

Full Information Maximum Likelihood Estimation
Purchasing Power Parity

Quarterly Data 1973 III - 1979 IV

$$\ell nS_{it} = \alpha_i + \beta \ell n(P_t / P_{it}) + u_{it}$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it}$$

$$\ell nM_{it} - \ell nP_{it} = a_i + b\ell ny_{it} + c_i i_{i,t} + \omega_{it}$$

$$\omega_{it} = \gamma_i \omega_{i,t-1} + v_{it}$$

Country	α_i	a_i	b_i	c_i	ρ_i	γ_i
United Kingdom	32.442 (1668.4)	3.785 (.232)	.393 (.071)	.000 (.002)	1.003 (.171)	.745 (.089)
Germany	-.858 (.032)	-9.859 (.896)	1.490 (.128)	-.014 (.003)	.682 (.079)	.170 (.143)
Sweden	-1.410 (.018)	-3.915 (.310)	.638 (.072)	.017 (.010)	.472 (.081)	.212 (.147)
Canada	1.349 (6.966)	-1.356 (.672)	-.005 (.134)	-.011 (.003)	.979 (.103)	.088 (.117)
United States		2.512 (1.068)	-.195 (.146)	-.001 (.004)		.074 (.153)

β = 1.101
(.108)

Note: Standard errors are in parentheses.

Table 4

3SLS Estimation of Purchasing Power Parity

$$\ell n\ S_{it} = \alpha_i + \beta \ell n(P_t/P_{it}) + u_{it}$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it}$$

$$IV = \{c,\ TIME, TIME^2,\ (\ell nS_{i,t-1}, i=1,\ldots N,),\ (\ell n(P_{t-1}/P_{i,t-1}),\ i=1,\ldots,N)\}$$

Country	α	ρ	β(constrained)
	Quarterly Data 1973 III - 1979 IV		
United Kingdom	.919 (.177)	.923 (.070)	.951 (.149)
Germany	-.847 (.037)	.711 (.084)	
Sweden	-1.413 (.022)	.568 (.112)	
Canada	-3.786 (3.964)	.999	
	Monthly Data July 1973 - December 1979		
United Kingdom	.744 (.080)	.957 (.028)	.409 (.183)
France	-1.486 (.102)	.970 (.030)	
Italy	-6.602 (.069)	.948 (.025)	
Sweden	-1.446 (.017)	.794 (.051)	
Japan	-5.466 (.191)	.979 (.020)	

Table 4 (continued)

	Monthly Data	January 1921 - May 1925	
United Kingdom	1.847 (.093)	.941 (.063)	.578 (.072)
France	-1.937 (.119)	.818 (.074)	
Italy	-2.091 (.131)	.494 (.108)	
Sweden	-1.038 (.041)	.773 (.080)	
Japan	4.035 (.927)	.991 (.035)	

Notes: The system of equations was estimated using the 3SLS procedure on version 3.3 of TSP. The standard errors are reported in parentheses. The starting values were obtained from single equation 2SLS using FAIRs method, Table 1. The value of ρ for Canada (Q-1970) was constrained to be 0.999.

Table 5

Correlation Matrices

$$\ell n\, S_{it} = \alpha_i + \beta \ell n(P_t/P)_{it} + u_{it}$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it}$$

$$IV = \{c,\ TIME,\ TIME^2,\ (\ell n S_{i,t-1}),\ \ell n(p_{i,t-1}),\ i = 1,\ldots,\ 4 \text{ or } 5\}$$

Quarterly Data 1973 III - 1979 IV

ENG	1.000			
GER	.662	1.000		
SWE	.510	.846	1.000	
CAN	-.354	-.340	-.391	1.000

Monthly Data July 1973 - December 1979

ENG	1.000				
FRA	.583	1.000			
ITA	.489	.627	1.000		
SWE	.492	.738	.503	1.000	
JAP	.403	.585	.362	.428	1.000

Monthly Data January 1921 - May 1925

ENG	1.000				
FRA	.359	1.000			
ITA	.203	.306	1.000		
SWE	.470	.311	-.022	1.000	
JAP	-.160	-.004	-.167	-.001	1.000

Notes: The correlation matrix was calculated from the estimated variance-covariance matrix of the residuals. The corresponding parameter estimates are given in Table 4.

Table 6

Test of Deviations from PPP being a Random Walk

$$\Delta_t = a + b\ \Delta_{t-1} + u_t$$

$$\Delta_t = \ell n S_t - \ell n\ (P_t/P_t^*)$$

Data Set	Country	a	b	$\hat{\sigma}_0^2$	$S^2 e\mu$	Φ_1
Q-1970	United Kingdom	-.015 (.065)	1.036 (.084)	$.163 * 10^{-2}$	$.158 * 10^{-2}$	1.360
	Germany	-.143 (.114)	.829 (.132)	$.271 * 10^{-2}$	$.274 * 10^{-2}$	.892
	Sweden	-.611 (.255)	.569 (.179)	$.234 * 10^{-2}$	$.203 * 10^{-2}$	2.956
	Canada	-.005 (.005)	1.008 (.076)	$.417 * 10^{-3}$	$.424 * 10^{-3}$	.797
M-1970	United Kingdom	.009 (.027)	.993 (.035)	$.859 * 10^{-3}$	$.871 * 10^{-3}$	.454
	France	-.069 (.065)	.953 (.044)	$.905 * 10^{-3}$	$.911 * 10^{-3}$	.718
	Italy	-.338 (.293)	.948 (.045)	$.698 * 10^{-3}$	$.703 * 10^{-3}$	.772
	Sweden	-.244 (.094)	.828 (.066)	$.849 * 10^{-3}$	$.800 * 10^{-3}$	3.369
	Japan	-.143 (.130)	.974 (.023)	$.991 * 10^{-3}$	$.997 * 10^{-3}$	.746

Table 6 (continued)

M-1920	United Kingdom	.273 (.111)	.863 (.055)	$.387 * 10^{-3}$	$.356 * 10^{-3}$	3.270
	France	-.523 (.148)	.603 (.113)	$.391 * 10^{-2}$	$.326 * 10^{-2}$	6.114
	Italy	-.668 (.157)	.502 (.116)	$.109 * 10^{-1}$	$.828 * 10^{-2}$	9.340
	Sweden	-.050 (.035)	.940 (.044)	$.612 * 10^{-3}$	$.603 * 10^{-3}$	1.388
	Japan	.095 (.190)	.979 (.042)	$.886 * 10^{-3}$	$.913 * 10^{-3}$	.226

Notes: Each equation was estimated by OLS. The coefficients and standard errors are reported. $\hat{\sigma}_0^2$ is the maximum likelihood estimate of var(u), under the null hypothesis that $(\alpha, \beta) = (0,1)$. $S_{e\mu}^2$ is the variance of u under the alternative hypothesis that $(a, b) \neq (0,1)$. Φ_1 is the usual regression "F-test." Notation is from Dickey and Fuller (1981). The 95% critical value for n = 25 is 5.18, n = 50 is 4.86 and n = 75 is 4.71, where n = number of observations (n = 25 for Q-1970, n = 79 for M-1970 and n = 53 for M-1920.)

Table 7

Purchasing Power Parity - First Differences

$$(\ell n\ S_t - \ell n\ S_{t-1}) = \alpha + \beta\ [\ell n\ (P_t/P_t{*}) - \ell n\ (P_{t-1}/P_{t-1}{*})] + u_t$$

Data Set	Country	α	β	D.W.	R^2	ser
Q-1970	United Kingdom	.013 (.011)	1.033 (.443)	1.72	.191	.040
	Germany	.008 (.017)	.504 (1.314)	2.06	.006	.054
	Sweden	.002 (.010)	.445 (.816)	2.06	.013	.050
	Canada	-.005 (.004)	1.068 (.734)	1.47	.084	.021
M-1970	United Kingdom	-.002 (.004)	.044 (.339)	1.78	.000	.028
	France	-.000 (.004)	-.226 (.964)	2.41	.001	.030
	Italy	.003 (.004)	1.235 (.485)	2.14	.079	.027
	Sweden	-.000 (.003)	-.061 (.506)	2.08	.000	.029
	Japan	.001 (.004)	.331 (.321)	1.91	.014	.031

Table 7 (continued)

M-1920	United Kingdom	.001 (.003)	.514 (.161)	2.03	.169	.018
	France	.002 (.009)	1.261 (.247)	2.08	.343	.063
	Italy	.005 (.015)	1.303 (.643)	2.05	.076	.106
	Sweden	.004 (.003)	.045 (.175)	.98	.001	.020
	Japan	-.003 (.003)	.136 (.111)	1.26	.029	.020

Notes: Each equation was estimated by OLSQ. Standard errors are reported in parentheses.

Table 8

3SLS Estimation of Purchasing Power Parity - First Differences

$$(\ell nS_{it} - \ell nS_{i,t-1}) = \alpha + \beta_i[\ell n (P_t/P_{it}) - \ell n (P_{t-1}/P_{i,t-1})]$$

$$IV = \{c, TIME, TIME^2, (\Delta\ell nS_{i,t}, i = 1, . . ., N), (\Delta \ell n (P_t/P_{it}), i = 1, . . ., N)\}$$

Country	α (constrained)	β
Quarterly Data 1973 III - 1979 IV		
United Kingdom	$.746 * 10^{-3}$ $(3.150 * 10^{-3})$	.986 (.358)
Germany		.672 (.445)
Sweden		.619 (.546)
Canada		.910 (.750)
Monthly Data July 1973 - December 1979		
United Kingdom	$.850 * 10^{-3}$ $(2.740 * 10^{-3})$	.603 (.414)
France		.478 (.958)

Table 8 (continued)

Italy		.929 (.385)
Sweden		-.610 (.660)
Japan		-.250 (.491)
Monthly Data	January 1921 - May 1925	
United Kingdom	$.152 * 10^{-2}$ $(.182 * 10^{-2})$	.337 (.154)
France		.771 (.325)
Italy		1.189 (.780)
Sweden		.122 (.189)
Japan		.303 (.195)

Notes: The system of equations was estimated using the 3SLS procedure on Version 3.3 of TSP. The standard errors are reported in parentheses. The starting values are obtained from single equation OLS results.

Table 9

Correlation Matrices

$$\ell nS_{it} - \ell nS_{i,t-1} = \alpha + \beta[\ell n(P_t/P_{it}) - \ell n(P_{t-1}/P_{i,t-1})] + u_{it}$$

$$IV = \{c, TIME, TIME^2, (\ell nS_{i,t-1} - \ell nS_{i,t-2}), (\ell n(P_{t-1}/P_{i,t-1}) - \ell n(P_{t-2}/P_{i,t-2}), i=1,\ldots 4 \text{ or } 5\}$$

Quarterly Data 1973 III - 1979 IV

ENG	1.000			
GER	.545	1.000		
SWE	.426	.856	1.000	
CAN	-.415	-.257	-.230	1.000

Monthly Data July 1973 - December 1979

ENG	1.000				
FRA	.580	1.000			
ITA	.474	.661	1.000		
SWE	.513	.736	.565	1.000	
JPN	.426	.581	.446	.482	1.000

Monthly Data January 1921 - May 1925

ENG	1.000				
FRA	.412	1.000			
ITA	.297	.253	1.000		
SWE	.611	.226	.187	1.000	
JPN	-.144	.004	-.007	.002	1.000

Notes: The correlation matrix was calculated from the estimted variance-covariance matrix of the residuals. The corresponding parameter estimates are given in Table 8.

Table 10

Purchasing Power Parity - Germany as Base Country

$$\ln \tilde{S}_{it} = \alpha + \beta \ln(\tilde{P}_t / P_{it}) + u_{it}$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it}$$

$$IV = \{c, \text{TIME}, \text{TIME}^2, \ln \tilde{S}_{i,t-1}, \ln(\tilde{P}_{t-1}/P_{i,t-1})\}$$

1973 III to 1979 IV

Country	α	β	ρ	R^2	DW	ser
United States	.905 (.018)	2.130 (.219)	.400 (.183)	.91	1.77	.044
United Kingdom	1.652 (.034)	.971 (.136)	.654 (.151)	.95	1.60	.043
France	-.619 (.035)	1.015 (.234)	.723 (.138)	.91	1.77	.034
Italy	-5.650 (.022)	1.198 (.086)	.400 (.183)	.96	1.95	.049
Sweden	-.498 (.037)	1.518 (.214)	.803 (.119)	.98	1.53	.023
Canada	.922 (.039)	2.693 (.354)	.630 (.155)	.94	1.90	.055
Japan	-5.138 (.359)	-3.340 (3.392)	.851 (.105)	.18	1.90	.074

Notes: Each equation was estimated using FAIRs method. Standard errors in parentheses.

Table 11

Purchasing Power Parity - Germany as Base Country

3SLS

$$\ell n\ \tilde{S}_{it} = \alpha_i + \beta\ \ell n\ (\tilde{P}_t / P_{it}) + \mu_{it}$$

$$\mu_{it} = \rho_i \mu_{i,t-1} + \varepsilon_{it}$$

$$IV = c,\ TIME,\ TIME^2,\ (\ \ell n\ \tilde{S}_{i,t-1},\ \ell n\ (\tilde{P}_{t-1} / P_{i,t-1}),\ i=1,\ldots,N)\}$$

1973 III - 1979 IV

Country	I			II		
	α	ρ	β	α	ρ	β
United States	.717 (5.201)	.997 (.108)	1.189 (.060)			
United Kingdom	1.694 (.035)	.711 (.084)		1.694 (.035)	.711 (.093)	1.189 (.065)
France	-.589 (.039)	.816 (.083)		-.589 (.039)	.816 (.087)	
Italy	-5.649 (.019)	.430 (.110)		-5.649 (.020)	.430 (.115)	
Sweden	-.534 (.077)	.933 (.083)		-.534 (.077)	.933 (.085)	

Notes: Standard errors are in parentheses. Starting values for the second set of results were the final results from the first set of results.

References

(1) Darby, Michael R. "Does Purchasing Power Parity Work?" N.B.E.R. Working Paper No. 607. December 1980.

(2) Dickey, David A. and Wayne A. Fuller. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," Econometrica 49 (July 1981): 1057-1072.

(3) Dornbusch, Rudiger, "Exchange Rate Economics: Where do We Stand?" BPEA, 1:1980, pp. 143-185.

(4) Enzig, Paul, The Theory of Foreign Exchange, MacMillan and Co., London 1937.

(5) Frenkel, Jacob A. and Harry G. Johnson, The Economics of of Exchange Rates: Selected Studies, Addison-Wesly, 1978.

(6) Frenkel, Jacob A., "Collapse of Purchasing Power Parity during the 1970s," N.B.E.R. Conference Paper No. 73, October 1980.

(7) Frenkel, Jacob A., "Purchasing Power Parity, Doctrinal Perspective and Evidence from the 1920s," Journal of Internation Economics, 8 (May 1978); 169-191.

(8) Fuller, Wayne A., Introduction to Time Series, Wiley, 1976.

(9) Hakkio, Craig S., "Expectations and the Forward Exchange Rate," International Economic Review, forthcoming, October 1981a.

(10) Hakkio, Craig S., "Exchange Rate Determination and the Demand for Money," N.B.E.R. Working Paper No. 766, September 1981b.

(11) Hansen, Lars Peter and Robert J. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," Journal of Political Economy, 88 (October 1980): 829-53.

(12) Kmenta, Jan. Elements of Econometrics, New York: The MacMillan Co., 1971.

(13) Mishkin, Frederick M., "The Real Interest Rate: A Multi-Country Empirical Study," Unpublished manuscript, The University of Chicago, September 1981.

(14) Officer, Lawrence H., "The Purchasing-Power-Parity Theory of Exchange Rates: A Review Article," IMF Staff Papers, XXIII (March 1976): 1-60.

(15) Ohkawa, Kazushi and Henry Rosovsky. Japanese Economic

Growth, Stanford, California: Stanford University Press, 1973.

(16) Roll, Richard, "Violations of Purchasing Power Parity and their Implications for Efficient Commodity Markets," in Sauret and Szego, eds. International Finance and Trade, vol. 1, Ballinger, 1979.

(17) Shionoya, Yuichi and Ippei Yamazawa, "Industrial Growth and Foreign Trade in Pre-War Japan," (pp. 515-548) in Economic Growth, the Japanese Experience since the Meiji Era, vol. 2, edited by K. Ohkawa and Yujiro Hayami. Tokyo: The Japan Economic Research Center, 1973.

(18) Tinbergen, Jan, ed. International Abstract of Economic Statistics, 1919 - 1930, International Conference of Economic Services, London, 1934.