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PUBLIC GOODS IN OPEN ECONOMIES WITH HETEROGENEOUS INDIVIDUALS

Joseph E. Stiglitz

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ABSTRACT

This paper formulates a simple model of "perfect community competition." It is shown that (1) the equilibrium is Pareto optimal; (2) communities will, in general, be heterogeneous; not all individuals will have the same tastes; but (3) all individuals of a given skill within the community will have identical perferences; (4) in spite of the heterogeneity of tastes, there is complete unanimity with respect to tax and expenditure policy, and there is no scope for redistribution at the local level; (5) under certain circumstances, everyone's expected utility can be increased by introducing a particular kind of unequal treatment of individuals who are otherwise identical with respect to tastes and production characteristics; (6) when there is not "perfect community competition," the equilibrium will, in general, not be Pareto optimal, and benefit taxation may be desirable.

Joseph E. Stiglitz
Department of Economics
Dickinson Hall
Princeton University
Princeton, New Jersey 08544

(609) 452-4014

PUBLIC GOODS IN OPEN ECONOMIES WITH HETEROGENEOUS INDIVIDUALS*

by

Joseph E. Stiglitz
Princeton University

In the theory of public expenditure there are two critical problems:

(a) There is usually a large number of Pareto optimal allocations (involving different levels and patterns of expenditure on public goods). Each of these has different distributional implications. Economic theory, as such, has little if anything to say about the choice among this set of allocations. This we shall refer to as the indeterminacy problem. (Several different solution concepts have been proposed, e.g., the Lindahl solution, but these obviously are not meant to describe the actual method by which the equilibrium is determined. There are a number of political-economic models providing particular solutions, e.g., a majority voting; the equilibrium then depends on the political system employed.[1] Arrow (1951)

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1 There are two further problems with majority voting: in general it does not yield a determinate solution (see Kramer (1973)) and the majority voting solution depends critically on the tax system employed to raise the revenue for the public expenditure (see, e.g., Atkinson and Stiglitz (1980)).

has established that there does not exist a social choice mechanism for choosing among the Pareto optimal allocations which satisfies the commonly accepted desiderata of (i) non-dictatorship, (ii) transitivity, and (iii) independence of irrelevant alternatives.

(b) It appears difficult to get individuals to reveal their preferences towards public goods; systems which make what individuals have to pay depend on what they say concerning how they value the public good suffer from the free rider problem -- individuals will report an undervaluation of their true benefits since what they enjoy will not depend (significantly) on what they say. On the other hand, any system of benefit taxation which charges individuals for use of the public good is inefficient in the sense that it will restrict consumption even though the marginal cost of using the public good is zero.[1]

Tiebout suggested that, although there are no markets for public goods, individuals could choose communities in which to live, and by their choice of communities they revealed their preferences in exactly the same way that individuals reveal their preferences in their choice of private commodities. Tiebout did not, however, formally model the competition among communities and attempts to do this (see, e.g., Stiglitz (1974)) have shown that the problem is far more complex than Tiebout's intuitive

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[1] A number of recent studies, growing out of the work of Groves and Leyard (1977), has proposed solutions to this revelation problem. There are a number of difficulties with these solutions, which perhaps account for the fact that they have never been employed. For a more extensive discussion, see Mueller (1979) or Atkinson and Stiglitz (1980).

analogy might suggest.[1]

In this paper I wish to prove three general propositions concerning equilibrium when there is competition among communities with heterogeneous individuals:

(a) In a world in which there is competition among communities and communities act competitively to attract inhabitants, in equilibrium the level of public goods and the structure of taxation in each community, given the actions of the other communities, is uniquely determined.[2] The equilibrium is Pareto optimal. We thus resolve the indeterminacy problem.[3]

In other words, the political mechanism has no scope for choice the exact nature of the political mechanism is irrelevant.

In this equilibrium, communities are not homogeneous. There is no reason that the doctors, lawyers, and blue collar workers within each community should have the same tastes. The remarkable result of our analysis is that, under our assumptions, even though individuals' tastes differ, there is complete unanimity with respect to the allocation of resources

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[1] The revelation and indeterminacy problems are not the only critical problems in the theory of public expenditures. In Stiglitz (1981), a third problem is discussed, which I refer to as that of the "management of public good." While for private goods, there are strong incentives for firms to provide the goods which individuals wish to purchase, and to produce them efficiently, the incentives for citizens to obtain information to select good public managers, and the incentive for public managers to provide for the Public Good, are either absent or far from perfect. It is often suggested that competition among local communities serves to improve the quality of the "management of the public good," We shall have little to say about this here.

[2] In our analysis, we assume individuals can belong to only one community, in which they work, and consume public and private goods. In practice, of course, individuals may work in one community, live in another, and join a club for the purpose of enjoying some kinds of public goods. Our analysis can be extended to this more complicated framework.

[3] Similar results are obtained in an extremely insightful paper by E. Berglas (1976).

(both the level and form of taxation and the expenditures on public goods) within each community. The assertion, commonly found in the literature on local public goods, that in equilibrium, all communities will be homogenous, is a consequence of the strong assumptions made in these analysis, and not a general proposition. In our model, the heterogeneity of the community follows from the assumption that individuals of different productive characteristics interact in production, but we could have formulated alternative models in which, for instance, individuals differed with respect to their transportation costs as well as tastes. Communities will then consist of individuals with low transport costs living far from the city center, and indiviudals with high transport costs living near the city center. (See Arnott and Stiglitz (1979, 1981).) Again, individuals with different transportation costs may differ as well in their attitudes towards public goods.[1]

We establish that no community has individuals of the same productivity with different tastes; thus the usual assertion concerning homogenous communities follows as a corollary to our analysis: if there is a single productivity group, communities will be homogeneous.

One further property of the equilibrium is worth noting. All public goods expenditures are paid for by pure rents. (This property of communities which are of optimal size I referred to in my 1974 paper as the Henry George Theorem. In Arnott and Stiglitz (1979) we show that, although the theorem is considerably more general than had previously been established, there were plausible conditions under which it would not obtain.)

[1] Indeed, even if they had the same set of indifference curves, the fact that they live in different locations may well effect their attitudes towards different public goods.

The unanimity theorem we establish here has one further important implication: in a world with competition among communities, there is no scope for redistribution at the local level. If there is to be redistribution, it must be at the national level.

Though our analysis is couched primarily in terms of the vocabulary of the theory of local public goods, it should be clear that our analysis may be viewed equally well as an analysis of equilibrium in an economy with international trade, with free migration of labor, with public goods within each country. Our analysis thus suggests that (in the absence of national loyalties, and restriction on emigration and immigration) in the traditional models involving a large number of small countries, within any country there is effectively no scope either for redistribution or for social choice among alternative public goods/taxation programs.

(b) Under certain circumstances, everyone's expected utility can be increased by introducing a particular kind of unequal treatment of individuals who are otherwise identical with respect to tastes and production characteristics. We form two (or more) communities consisting of individuals of type i and j. Individuals of type i living in Community A are better off than those living in Community B, and conversely for individuals of type j . We then randomly assign individuals of type i ; the possibility of the desirability of inequality whenever there is a kind of convexity in the structure of the economy was noted earlier in Stiglitz (1976).[1] Here, we note that the existence of local public goods introduces the kind of convexity into the structure of the economy which may well make randomization desirable.[2]

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[1] Indeed, it can be viewed as the converse of the well-known arguments for equality in the presence of concavity of Edgeworth, Lerner, and Samuelson.

[2] Even with communities in which all individuals are homogeneous but with transport costs, randomization may be desirable. See Stiglitz (1976).

(c) In a world in which there is competition among communities, but communities ignore migration (so there is no active competition for immigrants), and the level of public goods is determined by majority voting, then the equilibrium will, in general, not be Pareto optimal. When the valuation of public goods differs among individuals the limited competition equilibrium may involve benefit taxation. Benefit taxation may be Pareto optimal in a particular sense to be defined below.

2. The Basic Model

Assume we have an (infinitely) large number of identical islands.[1] We have a number of types of labor. Each type of labor is distinguished by production characteristics and taste characteristics. We let U_k^{ij} be the utility of an individual with taste characteristics j and productive characteristics i who lives in community k. We assume, again for simplicity, that we can write this simply as a function of the wage he receives, w_k^i, the (after-tax) wage of an individual with productivity characteristic i living in community k, prices in community k, p_k, and the supply of public goods, G_k.[2] (p_k and G_k are vectors.)

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[1] The assumption of a set of islands, each with a fixed supply of land, is a convenient way of thinking about community competition, but alternative formulations yield similar results. All that is required is that the output of a particular collection of individuals with, say, n^i individuals with production characteristics i, $F(\underline{n}^i)$, exhibits strongly diminishing returns beyond some point.

[2] We thus assume we cannot differentiate the wages received or prices paid on the basis of taste characteristics. See Section 3.2 below.

(1) $U_k^{ij} = U^{ij}(w_k^i, p_k, G_k)$

There is a certain technological feasibility locus for each island giving the set of wages, w, prices of private goods and production of public goods which are feasible on an island given that it has n^{ij} individuals of type ij:

(2) $T(w_k, p_k, G_k, n_k) = 0$

where $\underline{n}_k$ represents the vector of labor types in the community. (For the moment we assume that trade in commodities is not feasible so that each island is completely isolated.)

A simple example may help explain what is contained in T. Assume that we have only two groups, skilled and unskilled labor. Each island has a production function of the form:

(3) $C + G = F(n^1, n^2)$

where C is aggregate consumption, i.e., there is a single public good and a single private good and the relative production cost is exactly unity. Then equation (2) becomes

(2') $\frac{w^1}{p} n^1 + \frac{w^2}{p} n^2 + G - F(n^1, n^2) = 0.$

We assume that since all individual demand curves are homogeneous of degree 0 in wages and prices, T is homogenous of degree 0 in w and p .[1]

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[1] If the production possibiliites schedule of public and private goods is constant returns to scale, then doubling the vector n doubles output; if we keep w and p constant we double the demand for private goods (per capita demand remains unchanged), and hence if we double G we just exhaust product. Thus, if the underlying production possibilities schedule of public and private goods is constant returns then T is homogeneous of degree 1 in G and n. In that case, there would be only one island with a particular mixture of individuals. We thus postulate diminishing returns to labor.

We postulate that the supply of individuals of each type is fixed at $\bar{n}^{ij}$. Since in this paper we wish to avoid the difficulties raised in our earlier paper (Stiglitz (1977)) concerning what happens when the number of individuals available is not an exact multiple of the optimum number in each community, it is best to think of $\bar{n}^{ij}$ as the portion of the population who are of type ij , when the number of individuals is infinite.[1]

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[1] The assumptions concerning an infinite number of islands and an infinite number of individuals are made not just to simplify the analysis. They play two critical roles. First, the hypothesis concerning competition among communities is plausible only if there are a large number of competing communities. Secondly, as we have noted, there are a large number of problems in the analysis if the number of individuals of any type is not an exact multiple of the optimum number in each community.

The existence of a finite optimum number of individuals (of any particular type) within a community was a question which we addressed in our earlier (1977) study. There we noted that even with diminishing returns to production, the optimal community size could be infinite. We required that there be sufficiently diminishing returns that it offset the natural increasing returns effect associated with the public good. In the absence of this condition, competition among communities is obviously not viable. Here, we simply postulate that on each island, there is sufficiently rapid diminishing returns that the optimum population (to be defined below) for each community is finite.

Although the source of diminishing returns in our analysis arises from the limited supply of land on which individuals work in each island, it should be apparent that similar arguments would hold as a result of either congestion in the use of public goods (the source of diminishing returns in Tiebout's original study) or as a result of increased transport costs in a residential location model with constant returns to scale in production.

2.1 Competitive Equilibrium

There are three basic equilibrium conditions:

(a) Migration Equilibrium. Individuals move freely until they find the community which maximizes their utility, i.e.,

$$\text{(4)} \qquad \text{if } n_k^{ij} > 0, \quad U_k^{ij} = \max_{k \in K} \{U^{ij}\}$$

where K is the set of islands.

(b) Community Competition. Our second equilibrium condition is dependent on the nature of competition among communities. In this section we postulate the community competition equilibrium condition: each community is small; each type of individual in each community recognizes the dependence of in- and out-migration of all other types of labor on the actions of the community. He would like the community to maximize his utility, i.e., for the individual type i^*j^*,

$$\text{(5)} \qquad \max_{\{\underline{n}_k^{ij}, \underline{p}_k, \underline{w}_k, G_k\}} U_k^{i^*j^*}$$

$$\text{(5a)} \qquad \text{s.t.} \quad T = 0$$

and to

$$\text{(5b)} \qquad n_k^{ij} \varepsilon \mathcal{S}_k^{ij}(w_k^i, p_k, G_k)$$

where $\mathcal{S}_k^{ij}$ is the supply correspondence of laborers of type ij to community k.

Thus, we would normally expect that increasing the wage offered to a particular group or increasing the supply of public goods which a particular group finds attractive will induce migration. The exact nature of this supply correspondence under conditions of perfect community competition will be discussed in Section 2.3 below.

The solution to (5) which we shall denote by $(\hat{w}_k^{ij}, \hat{p}_k^{ij}, \hat{G}_k^{ij}, \hat{n}_k^{ij})$, yields the optimal policy (including the optimal size community) from the perspective of i^*j^*. Arnott and Stiglitz (1980) have noted that, in general, the optimal size n of the community will depend on whose perspective is being taken.[1] Similarly, one would normally expect the solution to involve different values of G (public expenditures), w (after-tax wages) and p (consumer prices) depending on whose utility is being maximized. The result we establish here is that, under conditions of competitivity, all individuals will agree on the optimum value of $\underline{n}$, $\underline{p}$, $\underline{w}$, and $\underline{G}$.

The community competition equilibrium condition asserts that the equilibrium action of the community is some feasible action lying within the set of actions preferred by the different individuals. We assume that the actual choices depend, in some way, upon the preferences of the groups within the community. In particular, we postulate that the action taken must lie within the Pareto optimum set for the community. (It is important to emphasize that this assumption need not imply that the economy as a whole is efficient because the population may not be efficiently distributed among different islands, as in Stiglitz (1977).)

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[1] It is important for our analysis that the optimal value of n^{ij} be finite. To assure this requires certain restrictions be placed on the technology. In the case of homogeneous communities, these are discussed in Stiglitz (1977).

(c) The Labor Equilibrium Condition. The final condition is that all laborers must live in some community, i.e.,

(6) $$\sum_{k\varepsilon K} n_k^{ij} = \bar{n}^{ij}$$

2.2 A Dual Problem

Before considering the nature of the market equilibrium let us consider the problem of characterizing the utility possibility frontier of the economy, i.e.,

(7) $$\max \; U^{i'j'}$$

$$\text{s.t.} \quad U^{ij} = \bar{U}^{ij}, \quad ij \neq i'j'$$

and

$$T(w_k, p_k, G_k, n_k) = 0.$$

We can construct the utility possibilities schedule as follows. First, we construct the utility possibiliites schedule for a fixed population. This, in turn, is done by first constructing the utility possibilities schedule for a fixed allocation of public goods. Thus, if there is a single consumption good, and all individuals' utility functions are linear in the private consumption good, the utility possibilities schedule for a fixed population and fixed G is linear, as in Figure 1a. (In the diagrams, we assume there are only two types of individuals. The extension to the general case is straightforward.) If, however, individuals have diminishing utility towards the consumption of private consumption goods, the utility possibilities schedule has the usual concave shape of Figure 1b.

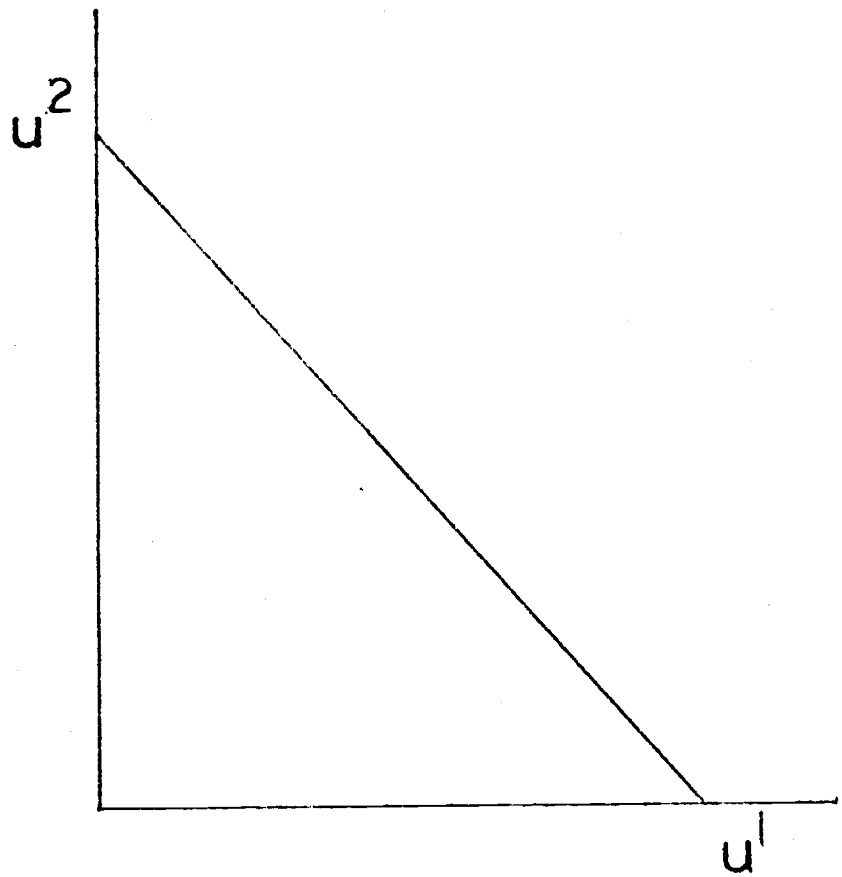

Figure 1a. Utilities possibilities schedule with fixed population and fixed supply of public goods: constant marginal utility of the private consumption good.[1]

(footnote on page 11c)

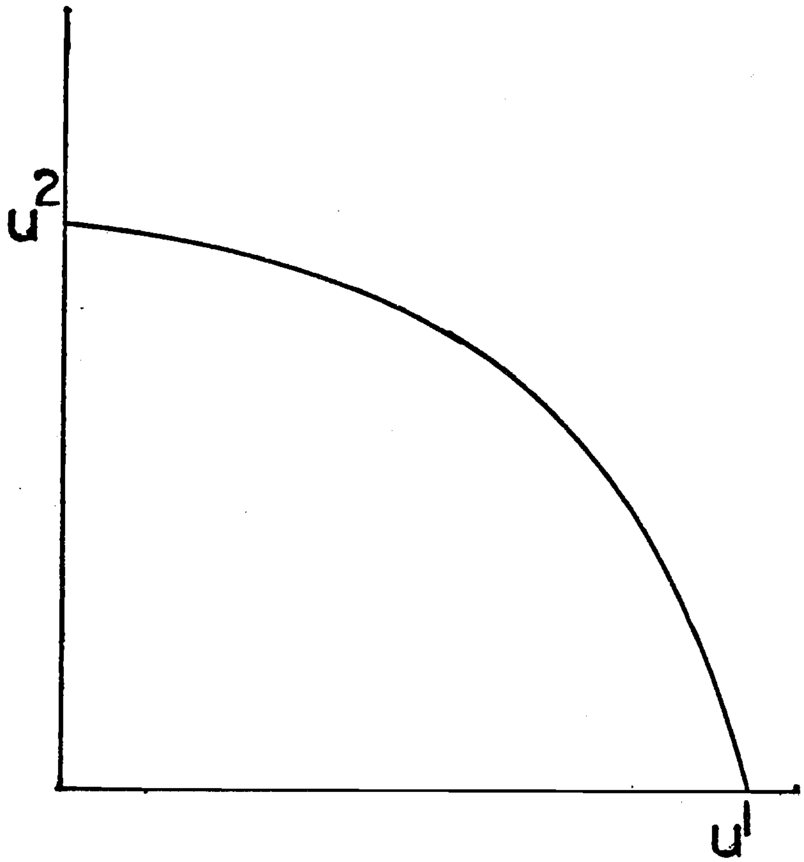

Figure 1b. Utilities possibilities schedule with fixed population and fixed supply of public goods: diminishing marginal utility to private consumption goods.[2]

(footnote on page 11c)

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1 We assume, for instance, a technology of the form represented by equation (3). With n^1 and n^2 fixed,

$$C = c^1 n^1 + c^2 n^2,$$

where c^i is per capita consumtpion of the private good of the individual of type i, and C is, as before, aggregate consumption. Thus, if the utility functions are of the form

$$U^i = c^i + v^i(G) \quad (1')$$

the utility possibilities schedule will be of the form depicted.

2 Under the same conditions noted in footnote 1 above, if the utility function is of the form

$$U^i = u^i(c^i) + v^i(G), \quad \text{with} \quad u^{i\prime} > 0, \quad u^{i\prime\prime} < 0 \quad (1'')$$

the utility possibilities schedule will be of the form depicted.

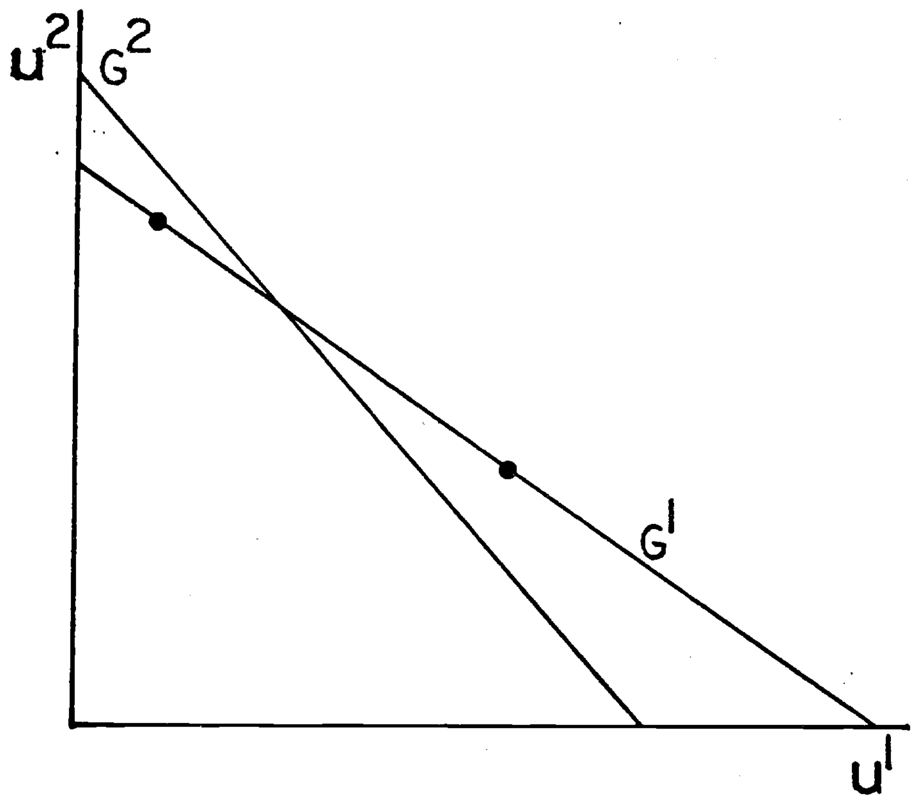


Figure 1c. Effect of an increase in the level of public goods expenditure on Utility Possibilities Schedule.[1] ($G^2 > G^1$)

(footnote on page 11e)

1 Assume, for instance, that

$$U^2 = c^2 + \gamma^2 G$$

while

$$U^1 = c^1 + \gamma^1 G$$

Then, if $c^1 = 0$,

$$U^2 = \frac{F - G}{n^2} + \gamma^2 G$$

where F is aggregate output. Hence

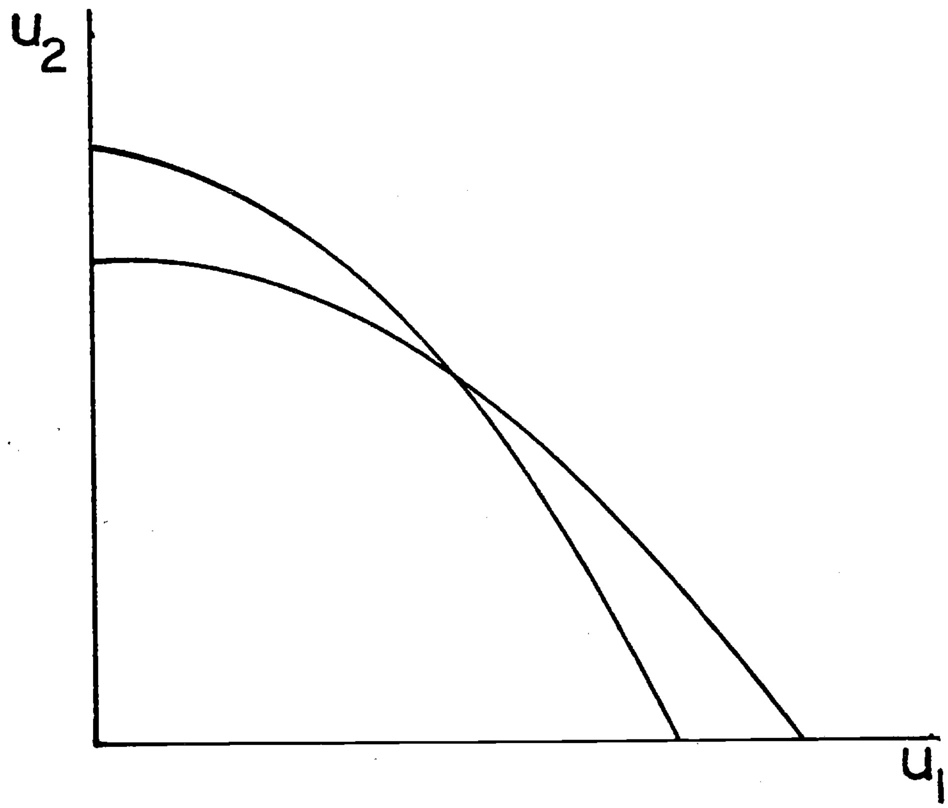
$$\frac{dU^2}{dG} > 0 \quad \text{if} \quad \gamma^2 > \frac{1}{n^2}$$

Similarly, if $c^2 = 0$

$$U^1 = \frac{F - G}{n^1} + \gamma^1 G$$

and

$$\frac{dU^1}{dG} < 0 \quad \text{if} \quad \gamma^1 < \frac{1}{n^1} \quad .$$



<u>Figure 1d</u>. Effects of an increase in the level of public goods expenditure on Utility Possibilities Schedule.

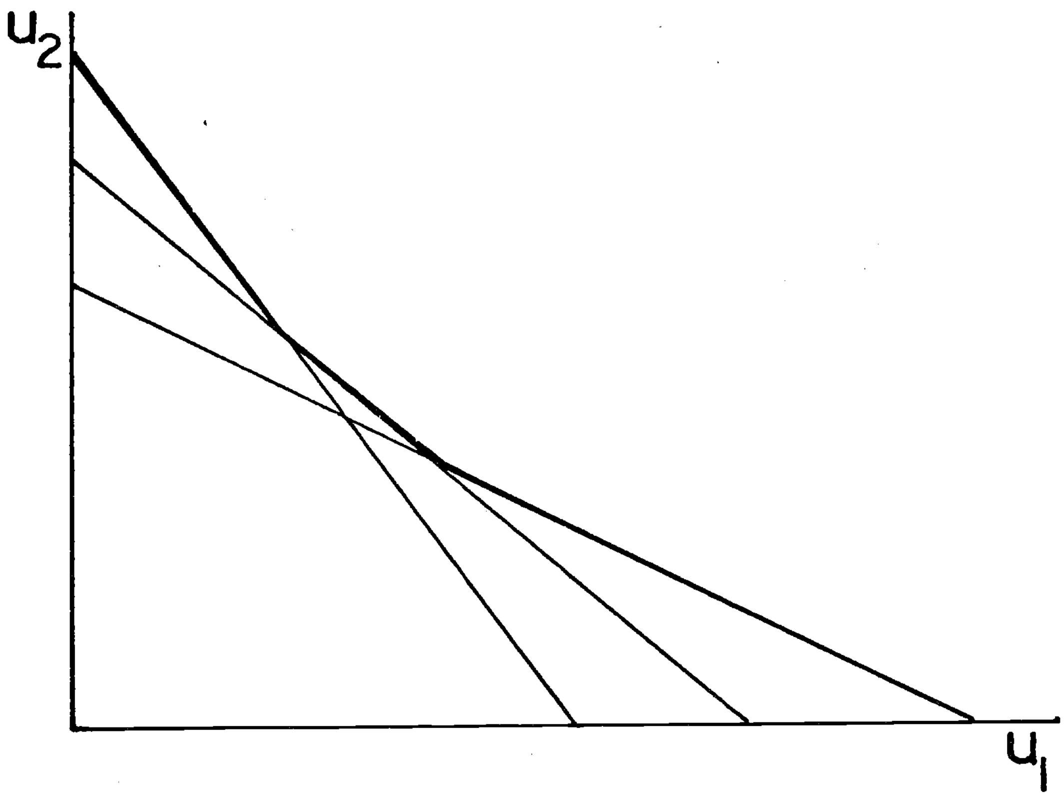
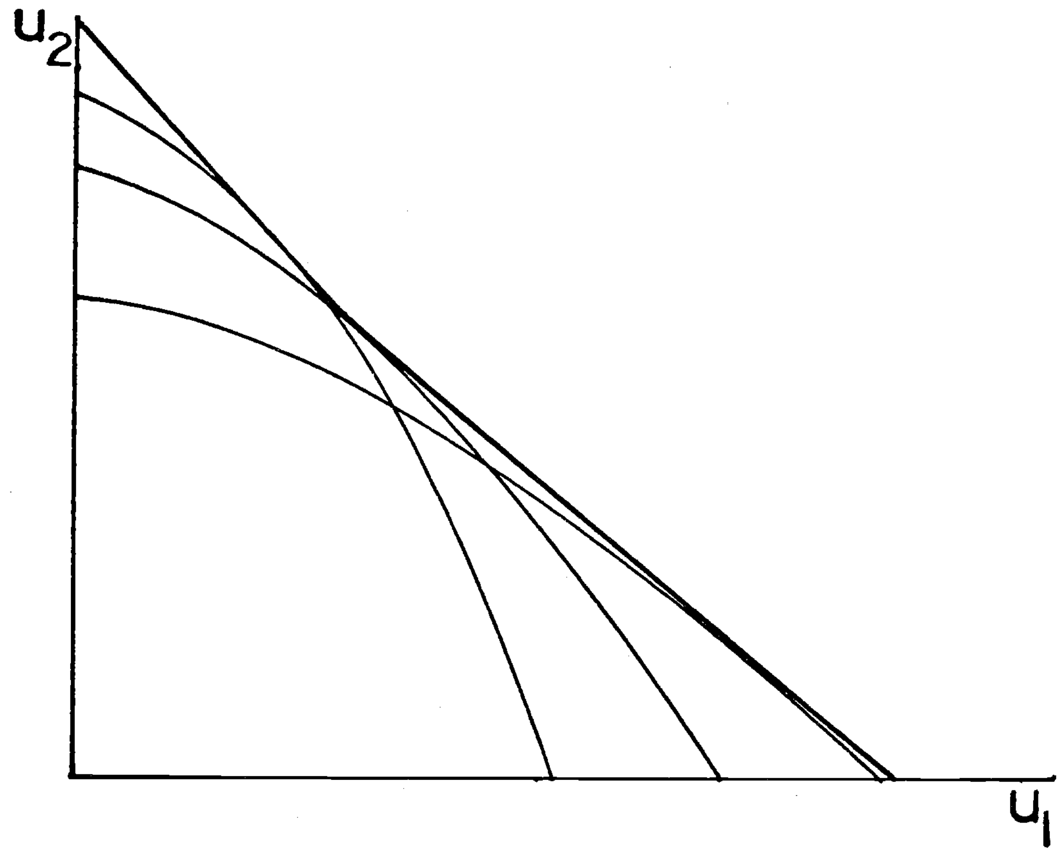


Figure 1_e.



Figures 1e and 1f. Utility possibilities schedule with fixed n but variable G may be convex.

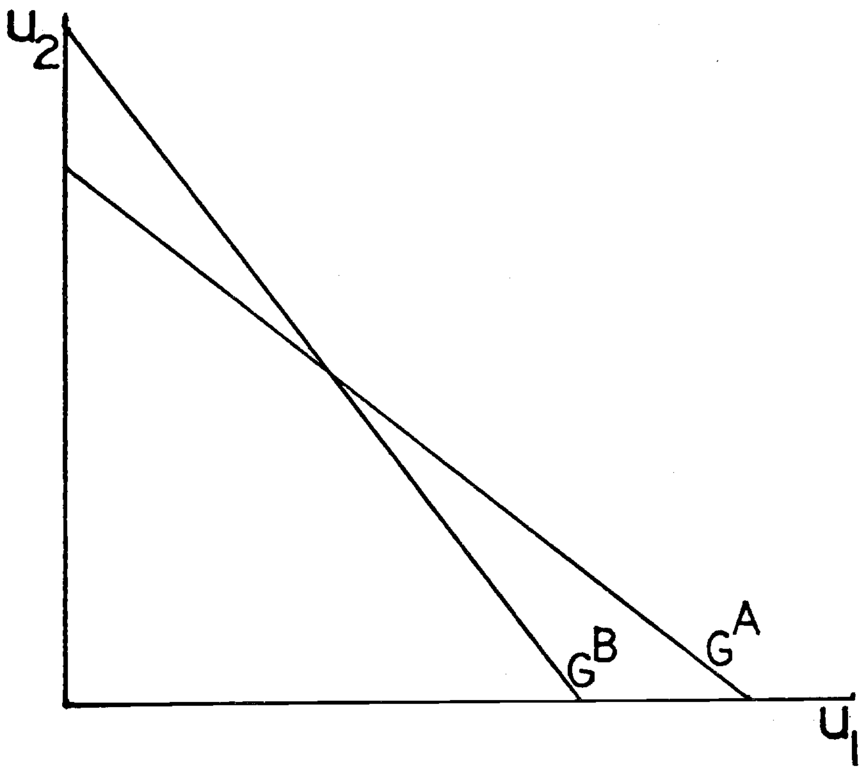


__Figure 1g__. Derivation of utility possibilities schedule with two public goods.[1]

(footnote on page 11j)

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[1] Consider, for example, a situation where there is a single lake, which can be used for boating and swimming. At any moment, however, it can be used only for one or the other (boating makes swimming too dangerous). There are two types of individuals, those who like boating and those who like swimming. The utility of each is a function of the fraction of the time that they can use the lake for their own activity. Thus,

$$U^1 = c^1 + \lambda$$

$$U^2 = c^2 + 1 - \lambda$$

where λ is the fraction of the date that the lake is reserved for the use of swimmers.

Next, we construct the utility possibilities schedule for the same $\underline{n}$, but now, for a different allocation of public goods. Consider the case where the public good is valued positively by only one of the two types of individuals. Then, clearly, even when all the private consumption good is allocated to the individuals of the other type, their utility is lowered. On the other hand, if the level of public goods is sufficiently small, it is clear that an increase in public expenditure will increase the utility of the group which values the public good. Thus, in our first example, with constant marginal utility of private consumption, the new utility possibilities schedule crosses the old one. The utility possibilities schedule for a fixed n is the outer envelope of these utility possibility schedules (Figures 1e and 1f). Thus, it is clear that the utility possibilities schedule, for a fixed population, with variable public goods, may well be convex (rather than concave). This may be the case even when the utility functions are strictly concave. The argument holds equally well if there are two (or more) public goods, one preferred relatively more by one group (Figure 1g).

We now need to consider how changing the population changes the utility possibilities schedule. In Stiglitz (1977), we showed, with a fixed amount of land, how, as we increased the population, the maximum level of utility attainable in a homogeneous population first increased, and then decreased. Here, the argument is analogous. With a fixed number of individuals of the other type(s), as we increase the number of individuals of, say, type ij, the maximum level of utility attainable by group ij may increase, and then decrease. In each of these cases there are two effects. If individuals valued only public goods, so long as the marginal product of an individual is positive, an increase in population would increase output, and hence the supply of public goods, and hence welfare. If individuals valued only private goods, if there is diminishing returns, output per capita will decrease, and hence welfare will decrease. The actual effect on welfare thus represents a mixture of these two effects.

The utility possibilities schedule with variable n is the outer envelope of the utility possibilities schedules with fixed n. Two properties of this utility possibilities schedule should be noted. First, <u>even</u> if the fixed n utilities possibilities schedule is concave, the variable n will not, in general, be. (Clearly, if the fixed n utilities possibilities schedule is convex, the outer envelope will be as well.) This lack of convexity has an important implication which we shall discus at greater length below.

Secondly, as each point on the utility possibilities schedule represents a solution to the problem (5), there is associated with each point a value of <u>n</u>, <u>G</u>, <u>w</u>, and <u>p</u>. In the case of two groups, as we increase the ratio of, say, U^1 to U^2, normally we would expect the associated ratio of n^1 to n^2 to decrease: If individuals of type 1 have to pay individuals of type 2 a lower level of utility, they will wish to have more of them in their community. This is illustrated in Figure 2a.

It is possible, however, that there is a range of values of U^1/U^2 for which the same ratio of n^1/n^2 is optimal, as illustrated in Figure 2b. This can occur when the utility possibilities schedule, for fixed <u>n</u>, is convex, as illustrated in Figure 1b. The utility possibilities schedules corresponding to $(n^1/n^2) = (n^1/n^2)^*$ and $(n^1/n^2) = (n^1/n^2)^{**}$ dominate those corresponding to any intermediate value of n^1/n^2. This simply

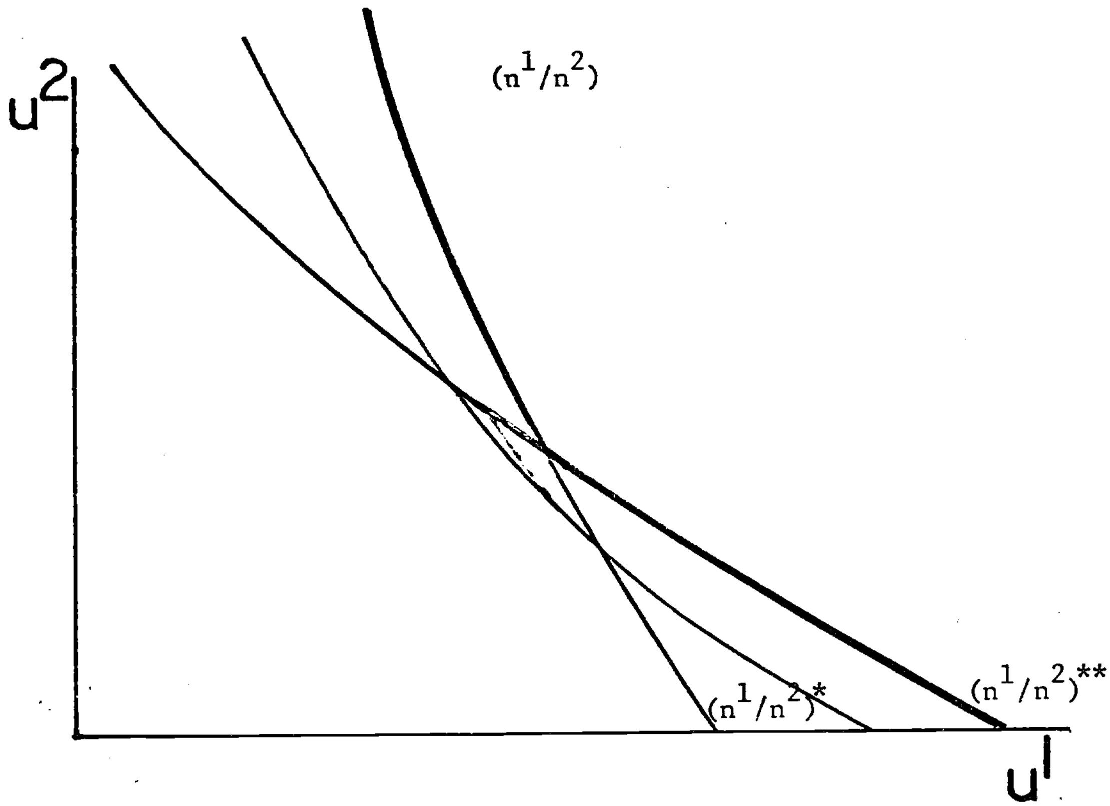

Figure 1h. Derivation of Utility-Possibilities Schedule (variable n and G)

$(n^1/n^2)^{**} < (\hat{n}^1/n^2) < (n^1/n^2)^*$

Some population Ratios Inefficient

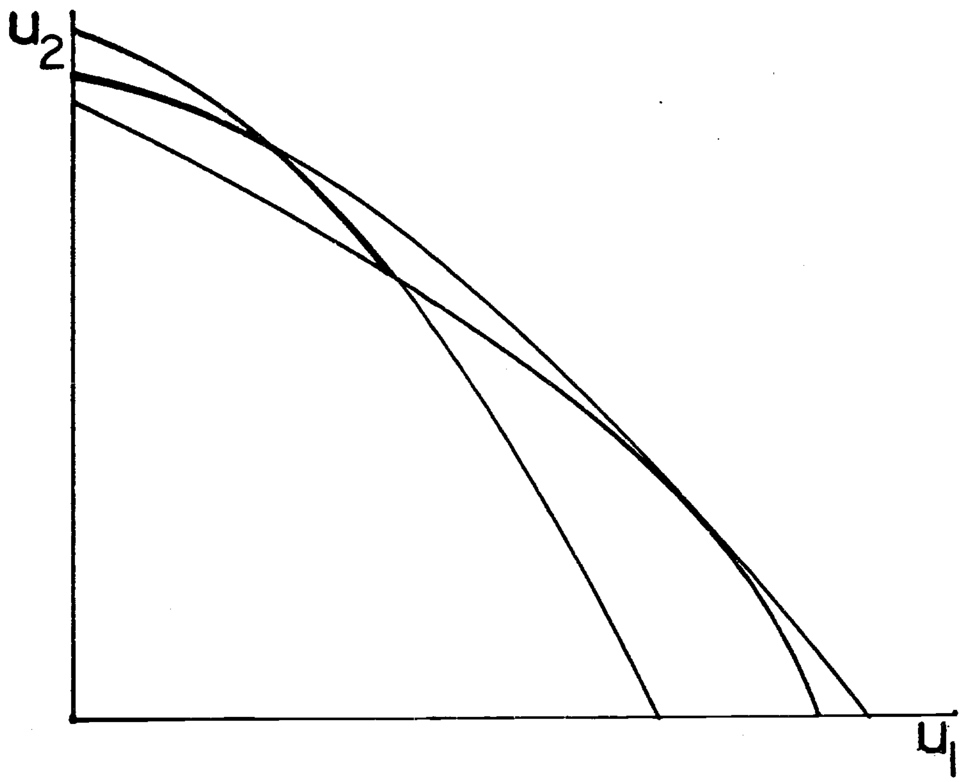

Figure 1g. Derivation of utility possibilities schedule (n and G variable).

says that Pareto optimality entails that communities have either a lot of individuals of type 1 or a lot of individuals of type 2. The extreme case of this is that where all individuals have identical production characteristics and enjoy completely different public goods. We know then that in equilibrium communities will always be homogeneous (under our assumptions).

In both cases, we have assumed that there exists points on the utility possibilities schedule corresponding to arbitrarily large ratios of n^1/n^2 and to arbitrarily small ratios of n^1/n^2. This will be the case so long as the marginal product of any individual of any type is strictly positive (regardless of their numbers).[1] Thus, if we set $w^i = 0$, the wage received by an individual of type i (and hence the consumption of private goods of type i) at zero, an increase in the numbers of type i will unambiguously increase the welfare of individuals of type j, since they allow an increase in either j's private consumption or in the consumption of public goods of the kind that j enjoys.[2]

2.3 The Market Solution

It is now easy to determine the nature of the market equilibrium: we simply find that point on the utility possibilities schedule for which the proportions of the laborers of the different types corresponds to the relative supplies. Thus, in Figure 2a we can immediately see the equilibrium level of (relative) utilities of the two groups; moreover, we

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[1] And assuming, as we have throughout the analysis, that there is no congestion in the public good, or here, no "relative congestion."

[2] Alternative sufficient conditions, involving restrictions on the transformation function T, may easily be derived.

can also see that an increase in the relative supply of individuals of type 1 will lead to a decrease in their level of utility and an increase in that of the other group.

Our analysis can thus be viewed as a straightforward generalization of the traditional theory of demand and supply to incorporate public goods. In the absence of public goods, the relative supply of individuals of types 1 and 2 would determine their relative wages. An increase in the relative supply of individuals of type 1 would reduce their wages, and increase those of type 2 laborers. Now, however, "compensation" may take the form either of wages, with which individuals purchase private goods, or of public goods. In the traditional theory without public goods, if wages of type 1 individuals are too high, there will be an excess supply of them; here, if the utility level of type 1 individuals is too high, there will be an excess supply of them. Each community will wish to have relatively few of the given type. Conversely, if the utility level of type 2 individuals is too low, there will be an excess demand for them. The communities will compete for the individuals of type 2, either by lowering the taxes they have to pay, increasing their wages, or by providing public goods that they particularly like. As a result, the level of utility of type 2 individuals will increase (and, of necessity, the level of utility that can be obtained by individuals of type 1 will decrease.)

Indeed, we can think of Figure 2 simply as a demand curve for laborers of each type (or, more generally, we can think of the solutions to (7) as generating demand correspondences of the form

$$n^{ij} = n^{ij}(U^{11},\ldots.U^{ij},\ldots.).$$

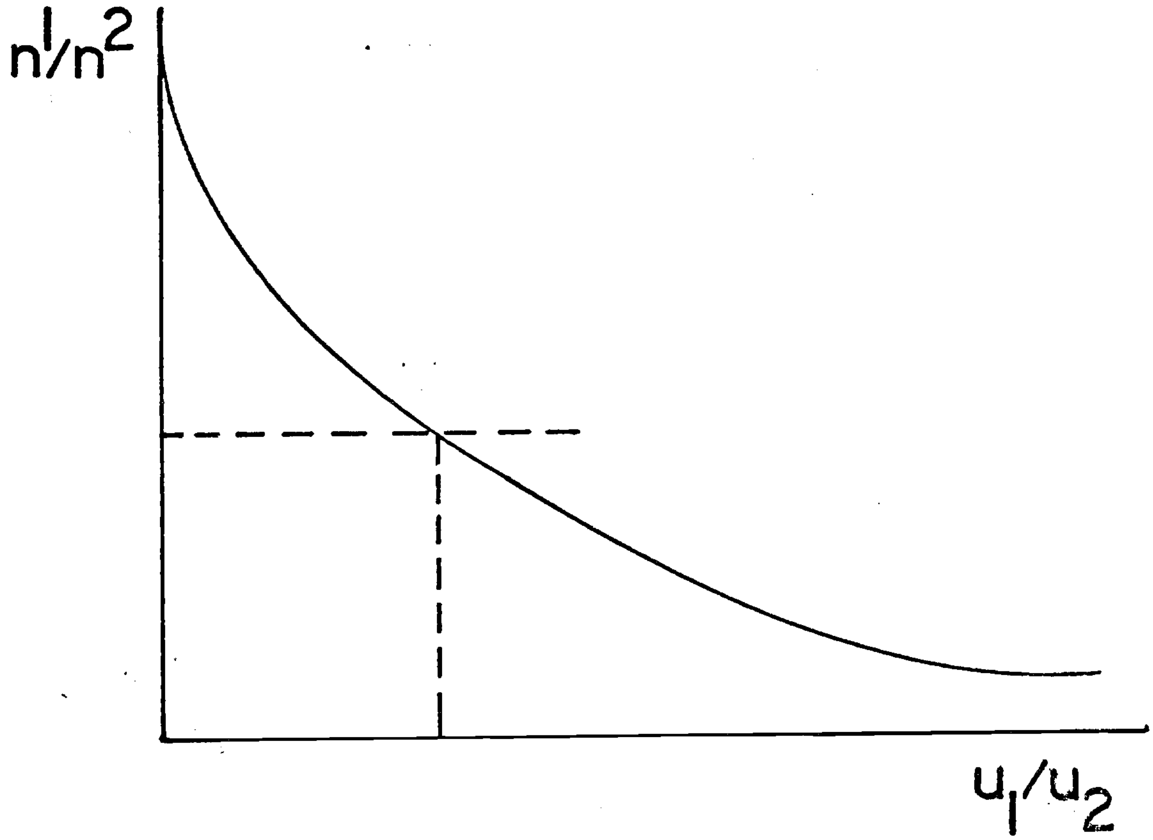

Figure 2a.

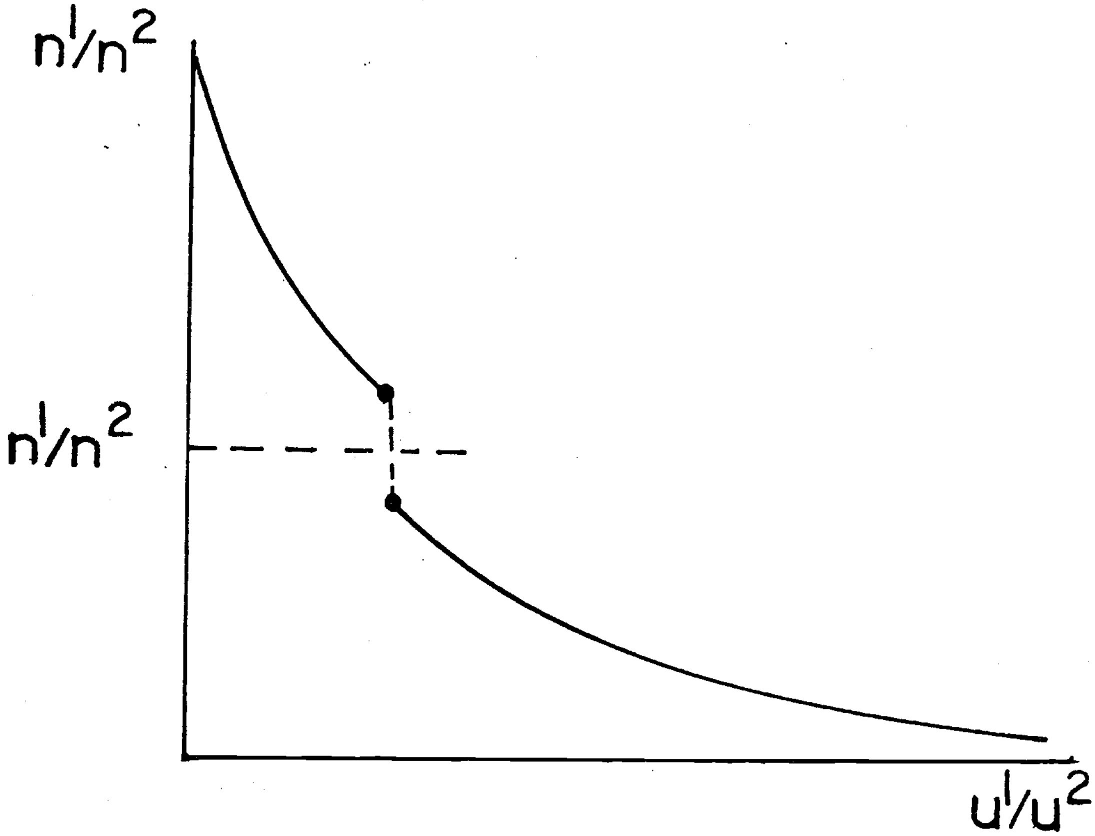

Figure 2b.

Equilibrium is thus a (feasible) allocation for which the demand for each type of laborer equals the supply.

Our assumptions assure us that no matter how large, or how small, the relative supplies of a particular type of individual, there will be a solution, i.e. corresponding point on the utility possibilities schedule.

Points of discontinuity along the demand curve for labor (as illustrated in Figure 2b) raise some interesting questions, but pose no serious problem for the analysis. We postpone these questions until Section 3. More formally, we now establish

Proposition 1. Any set of values of $\{w_k, p_k, G_k, n_k\}$ such that

$$\Sigma n_k = \bar{n}_k$$

and which is the solution to (7) for some value of $\bar{U}^{ij}$ is an equilibrium.

Proof. In the market equilibrium all communities are utility takers, and in equilibrium there must be a single level of utility enjoyed by all (other) communities, $\underline{U^{ij*}}$. Thus, the community in question must, to attract anyone, guarantee at least a utility level of U^{ij*}. At U^{ij*} it can obtain an infinitely large supply of labor. Thus, each group's optimal policy for the community is given by the solution to (even assuming that the number of individuals of its own type is also a control variable):

(8) $\max \; U^{i'j'}$

s.t.

(8a) $U^{ij} \geq \bar{U}^{ij*}$

and

(8b) $$T(w_k,p_k,G_k,n_k) = 0.$$

But this is identical to problem (7). Hence the values of the variables in the solution to this are precisely the same as the values of the corresponding variables in that problem. In particular, in equilibrium, the constraints (8a) will be binding: the level of utility for any group is exactly what it obtains everywhere else; and the level of utility generated for group $i'j'$ is the same as that group could have attained elsewhere.

Proposition 2. The equilibria characterized in proposition 1 are the only possible equilibria.

Assume that there is some set of $\{w_k,p_k,G_k,n_k\}$ satisfying (6) (demand for labor equals supply) and $U_k^{ij} = U_{k'}^{ij}$, all k, $k'\varepsilon K$ (migration equilibrium)), but which is not the solution to (7). There are two alternative (equivalent) ways of seeing why this could not be an equilibrium.

(a) Land Developer. Since there is assumed to be an infinite supply of islands the price of an island is zero. Thus, any land developer could occupy an island, increase his own welfare and attract (an infinite) supply of labor at the given utility levels. Thus the original equilibrium could not have been an equilibrium.

(b) For some group (on each island) there is some policy which increases its welfare without reducing that of anyone else. The excess or shortage of labor of each type is sent to or taken from a large number of other communities and hence has a negligible effect on each.

Corollary 1. There is no disagreement among the members of the community about the optimal policy of the community regardless of the differences in tastes and productivities.

Corollary 2. Every competitive equilibrium is Pareto optimal.[1]

3. Characterizing Equilibrium

In this section, we establish two results characterizing the equilibrium (if it exists). We simplify the problem by assuming the technology is of the form

$$C + G = F(n)$$

(For simplicity we assume a single public good.)

3.1 The Henry George Theorem

If there is an equilibrium, it cannot pay any land developer to organize a new island; hence if there is to be an equilibrium

$$\max_{\{n,G\}} F(n) - \Sigma w^{ij}(G;U^{ij})n^{ij} - G = 0 \tag{9}$$

where $w^{ij}(G;U^{ij})$ is the after-tax wage required to induce an individual of type ij to come to the community when the equilibrium level of utility yielded in the equilibrium for him is U^{ij}. Thus the equilibrium is characterized by

$$F_i = \min_j w^{ij} \equiv w^i \tag{10}$$

All laborers of a given productivity hired within a community receive the same after-tax wage; there is no benefit taxation. _Each factor gets paid its marginal product._

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[1] This result depends critically on our assumption of identical islands. See Stiglitz (1977)

If profits are to be zero, using (9) and (10),

(11) $$G = F - \sum_i F_i n_i$$

where

(12) $$n_i = \sum_j n^{ij}$$

Expenditure on public goods is equal to rents.

3.2 Taste Variability

We show that, within any community, all individuals of a given productivity must be identical (in the sense that their marginal rate of substitution between the public and private good must be the same).

Assume the vector n_k has been chosen optimally for the community. Then the land developer's profits are maximized at the point where

(13) $$-\sum_{ij} \frac{\partial w^{ij}}{\partial G} n^{ij} = 1 \ .$$

This is just the familiar condition that the sum of the marginal rates of substitution equal the marginal rate of transformation. But note that as the developer increases G it would pay him to "switch" to having only the individuals of productivity i who value the public good a great deal; and as it reduces G it would pay him to switch to having only the individuals of productivity i who do not value the public good (at the margin). In Figure 3, there is, in effect, a kink in the supply function of laborers of productivity i. But this implies that profits could be increased by either increasing or decreasing G; and hence

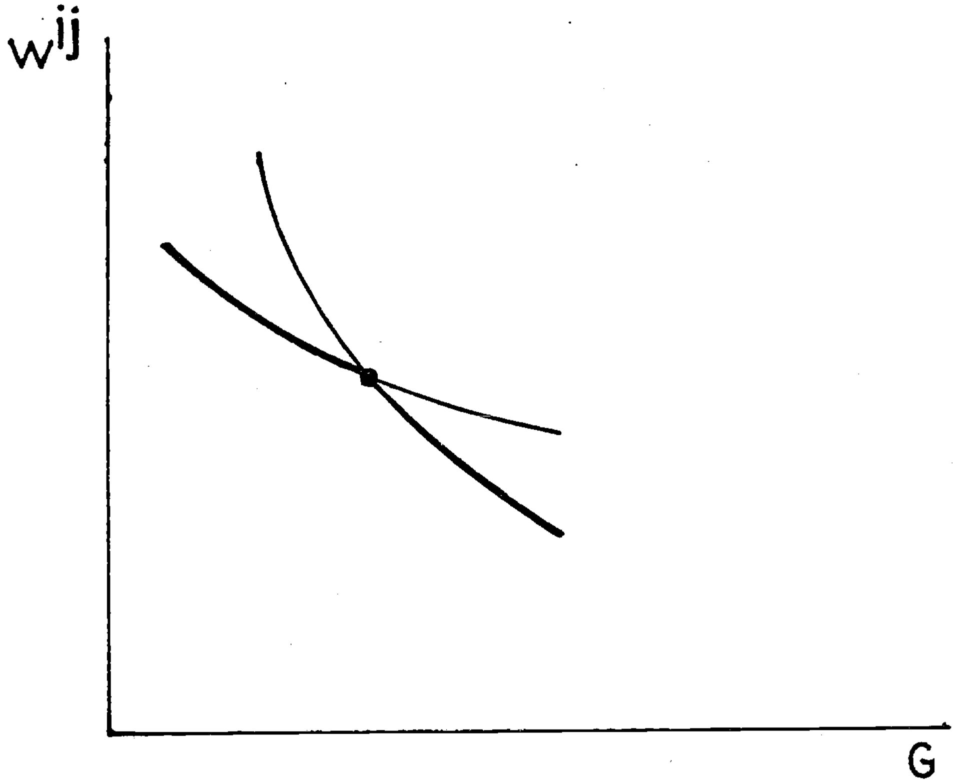

Figure 3.

the original allocation could not have been an equilibrium.[1] For instance, if there are two productivity groups in the population and each productivity group has two taste sub-groups, and if the relative proportions of productivity groups differ in the different taste sub-groups, then equilibrium will, in general, consist of there being three kinds of islands: those in which all workers (of both productivities) are low demanders; those in which there is a low demander of type 1 and a high demander of type 2; and those in which there are high demanders of both types. In Figure 4 we illustrate an equilibrium with three communities in which relative wages are not equalized in the different communities.

3.3 Variability Across Communities

In our earlier discussion, we noted that it was possible that the "demand" functions for labor might not be continuous: there might be no Pareto optimal allocation entailing a ratio of n^1 to n^2 between $(n_1/n_2)^*$ and $(n_1/n_2)^{**}$, as illustrated in Figure 2b. The nature of the equilibrium in this case is straightforward: there will be some communities with $(n^1/n^2) = (n_1/n_2)^*$ and some with $(n_1/n_2) = (n_1/n_2)^{**}$. The relative proportions of the two types of communities will depend on the relative supply of laborers of each type.

These equilibria have several interesting properties:

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1 That is, if at G^*, $-\frac{dw^{ij}}{dG} > -\frac{dw^{ij'}}{dG}$, then $-n^i\frac{\partial w^{ij}}{\partial G} > 1$ while $-n^i\frac{\partial w^{ij'}}{\partial G} < 1$. Hence, by setting $n^{ij'} = 0$ (and increasing n^{ij} a corresponding amount) and increasing G, or by setting $n^{ij} = 0$ (and increasing $n^{ij'}$ by a corresponding amount) and decreasing G, profits can be increased.

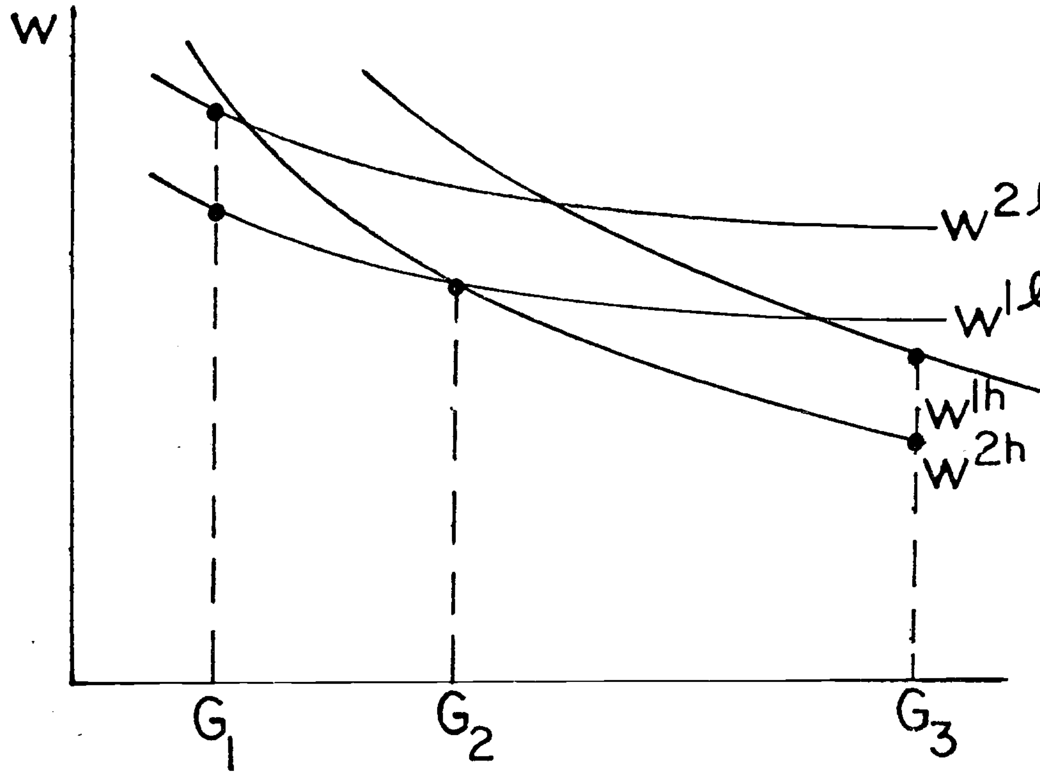

Figure 4.

First, note that changes in the relative supplies of laborers of different types may have no effect on the level of utilities attained; changes in the relative supply only change the relative proportions of communities of each type.

Secondly, because the relative proportions of individuals within the different communities differ, the equilibrium will not be characterized by production efficiency; a rearrangement of the population among the islands would increase group output, but it would not result in a Pareto optimal improvement.

Indeed, the economy may not be productively efficient even when the "demand" functions for laborers are continuous.

3.4 Randomization and Convex Utility Possibilities Schedules

We noted in our analysis that the utility possibilities schedule may well be convex, even though all individuals have concave utility functions (so individuals are risk averse) and all production functions are strictly concave. Assume that the labor supply is such that the market equilibrium occurs at a point E on the utility possibilities schedule where the schedule is convex. Assume, for simplicity, that the "demand" curve for laborers of each type is continuous near the equilibrium. We can then find points on the utility possibilities schedules, A, and B, with corresponding levels of relative demand for the two types of labor, as illustrated in Figure 5, such that we can form communities of type A and type B (Pareto efficient communities) with relative supplies of the two types of labor corresponding to the points A and B on the utility possibilities schedule), in such proportions as to fully absorb the available labor, and such that the expected utility of both type 1 and type 2 individuals is higher than in the original equilibrium, E. In community A, individuals

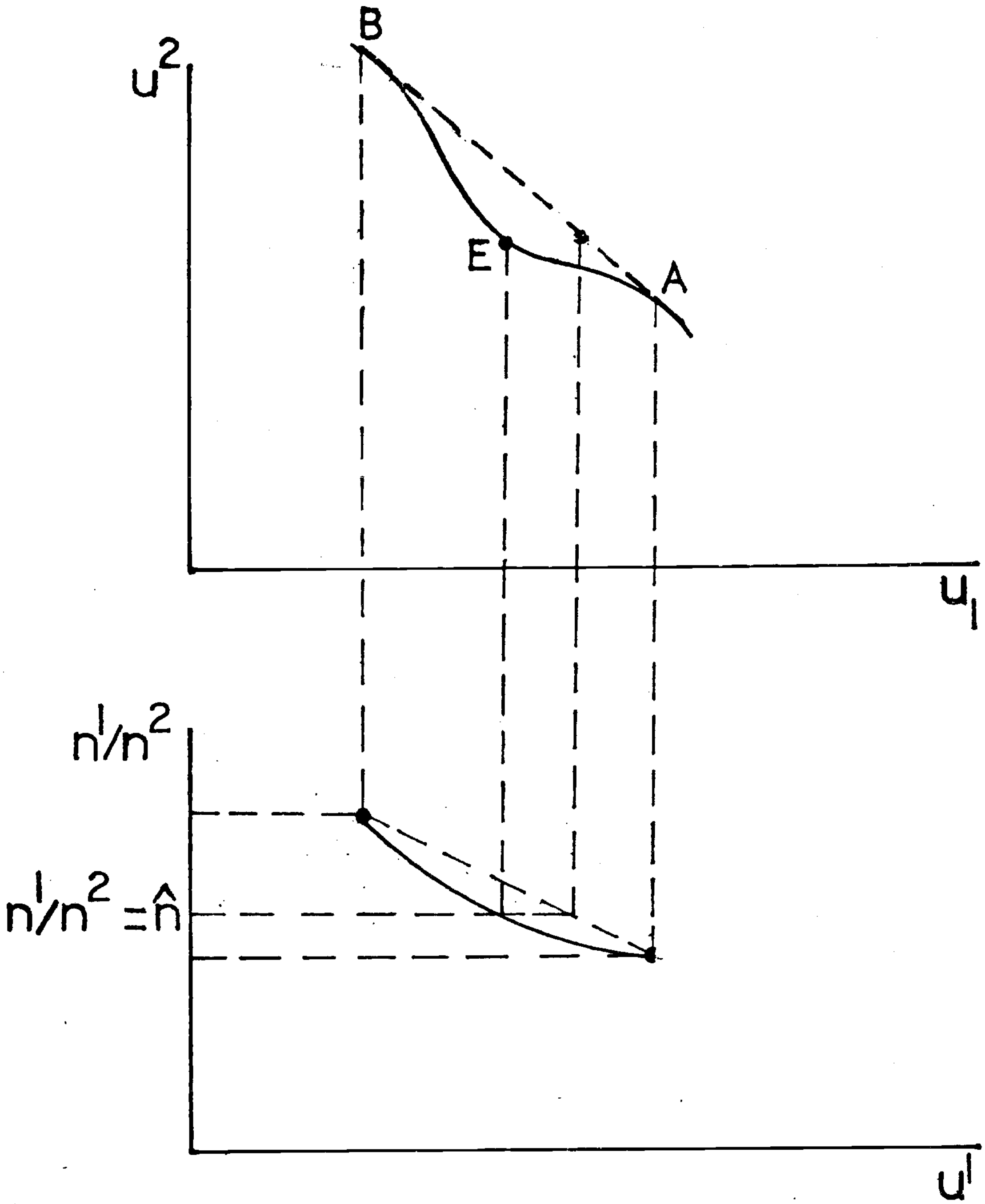

Figure 5.

of type 1 are better off than at E, but in community B they are worse off, and conversely for indiviudals of type 2. It is clear that Pareto efficiency (in terms of maximizing ex ante expected utility) requires randomization.

4. Limited Competition

Critical in the above analysis was the assumption that each community realized that its policies affected the migration of individuals into and out of it; that is, perceptions about the "supply functions" were correct and that there were so many islands and individuals that each community effectively faced a horizontal supply schedule of laborers of each type (at a particular utility level).

Assume, at the other extreme, that each community simply acts myopically, ignoring the consequences (in its decision about public goods allocation) for migration. We replace the Condition 2 with the following condition:

The set of feasible allocations are viewed by each individual to be those satisfying

$$T(w_k, p_k, G_k, n_k) = 0$$

for fixed n_k.

Each individual ranks the feasible allocations simply by the effect on his own utility. There is some social decision rule which aggregates the different preferences. It satisfies the minimal condition of being Pareto optimal within the set of policies which are viewed to be feasible. We call any such allocation a "limited competition" equilibrium, in contrast to the "full competitive equilibrium" analyzed in Section 2.

We can now prove:

Proposition 3. Every full competitive equilibrium is a limited competitive equilibrium, but there are limited competition equilibria which are not full competitive equilibria and which are not Pareto optimal.

The first part of the proposition is obvious.

There are two kinds of situations giving rise to non-optimality. First, the size of communities may not be optimal. Assume all individuals are identical. Then, under certain conditions, it can be shown that the level of utility attainable on an island is a function of the number of individuals on the island, as depicted in Figure 6. There exists an "optimal" number of individuals. Any distribution of the population among a set of islands in which all islands have the same population, in which for the given population the supply of public goods is optimal, is a limited competition equilibrium; but clearly it is not Pareto optimal unless the number is precisely equal to the optimal number. If the number exceeds the optimum, the equilibrium is stable in the sense that any individual who migrates to another island lowers the utility on that island and raises the utility in his original island, and thus has an incentive to return to his original island.

The second kind of inefficiency arises from the heterogeneity of individuals. With majority voting there will be a tendency of islands to concentrate in one type of individual. This is limited by the fact that if different types of individuals are complementary in production, as the relative proportion of one group increases, its productivity declines. But the resulting equilibria are likely to be inefficient. Consider the following example. Assume there are two groups which enter symmetrically in production (but are complementary) and have different tastes; there are two public goods, G_1 and G_2, preferred relatively strongly by groups 1 and 2.

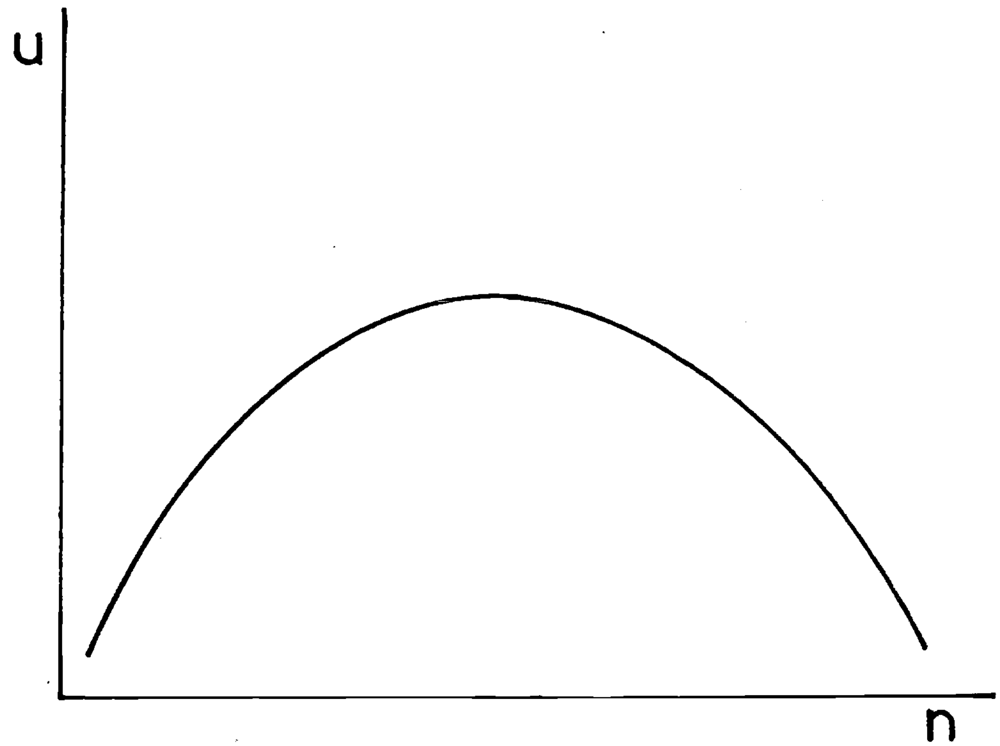

Figure 6.

Assume the relative cost of production for the two goods is unity and for both groups the sub-utility function of public goods can be written $\delta_1^i G_1 + \delta_2^i G_2$ with $\delta_1^1 = 1$ for group 1 and $\delta_2^1 < 1$ for group 1, and symmetrically for group 2. Finally, assume that we use as our social decision rule majority voting. Because we have only two groups and two alternatives we do not get into any of the usual problems with majority voting; if we assume that there are slightly more individuals of type 1 than type 2, then the majority voting equilibrium for a closed economy would be straightforward: it is only the preferences of type 1 that count, and provided that lump sum taxes can be imposed the equilibrium is Pareto optimal. Now, however, the equilibrium will in general not be Pareto optimal.

Let

$$C + G_1 + G_2 = F(n_1, n_2)$$

as before. Then, equilibrium consists of two types of communities, those in which n_1 are in the majority, and those in which n_2 are in the majority; the differences in wages just compensate for the differences in public goods supply: if we assume precisely the same number of individuals in the two groups the (asymmetric) equilibrium is described by the equations below.

(i) Within each community the supply is determined by majority vote:

(14a) $u'(c_1^1) = n_1 + n_2$ for community 1

(14b) $u'(c_2^2) = n_1 + n_2$ for community 2

(15a) $G_1 = 0$ for community 1

(15b) $G_2 = 0$ for community 2

(16a) $\quad c_1^1 = F_1 - t, \quad c_2^1 = F_2 - t \qquad$ for community 1

(16b) $\quad c_2^2 = F_2 - t, \quad c_1^2 = F_1 - t \qquad$ for community 2

where we have assumed that the government can only impose uniform lump sum taxes.[1]

(ii) Individuals are indifferent about migrating (letting $\delta \equiv \delta_j^i$, $i \neq j$)

(17a) $\quad u(c_1^1) + G_1 = u(c_2^1) + G_2$

(17b) $\quad u(c_1^2) + \delta G_1 = u(c_2^2) + G_2$

or, in the symmetric equilibrium

(18) $\quad u(F_2 - t) - u(F_1 - t) = (1 - \delta)(F - F_1 n_1 - F_2 n_2 + t(n_1 + n_2))$

Hence, we obtain the result that any set of (n_1, n_2, t) satisfying (18) and

(19) $\quad u'(F_1 - t) = n_1 + n_2$

is a limited competitive equilibrium. We solve (19) for t as a function of n_1 and n_2:

$$F_1 - t = u'^{-1}(n_1 + n_2)$$

or

$$t = F_1 - u'^{-1}(n_1 + n_2)$$

$$\frac{dt}{dn_1} = F_{11} - \frac{1}{u''}$$

$$\frac{dt}{dn_2} = F_{12} - \frac{1}{u''}$$

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[1] Unless some such restriction is imposed on the let of admissible taxes, the majority will attempt to confiscate the wealth of the minority; the only equilibria then entail separate communities.

There is a whole range of values of (n_1,n_2) yielding an equilibrium. But all entail production inefficiency (i.e., gross output is less than it would be if all individuals lived in communities with $n_1 = n_2$. Mixing the communities more would, on the other hand, entail a "loss" from having imperfectly matched public goods. To see that for some values of δ all limited competition equilibria are inefficient, consider the effect of combining two communities, redistributing the labor force equally among them, and mixing the two public goods equally. The increment in gross output can be approximated by

$$F(\frac{n_1 + n_2}{2}, \frac{n_1 + n_2}{2}) - F(n_1,n_2) \approx (F_{11} - F_{12})(\frac{n_1 - n_2}{2})^2$$

But we know

$$(1 -\delta)G = u(F_2 - t) - u(F_1 - t)$$

$$\approx u'(F_{21} - F_{11} - F_{22} + F_{12}) \frac{\Delta n}{2} = u'(F_{21} - F_{11})\Delta n$$

$$\approx 2\bar{n}(F_{21} - F_{11})\Delta n$$

(when $\bar{n} = \frac{n_1 + n_2}{2}$)

Substituting, we obtain the gain in output per worker as

$$-\frac{(1 - \ )^2 G^2}{4} \Big/ (u')^2 \ (F_{11} - F_{12})$$

A lower bound on the gain in utility is thus given by

$$u'\Delta F - \ G \approx G \left[\frac{(1 - \delta)^2 G}{\delta\bar{n}(F_{12} - F_{11})} - \delta \right] > 0$$

provided δ , $\bar{n}$, and $|F_{12} - F_{11}|$ are small enough.

That the majority voting equilibrium, even when "internally" Pareto optimal, is not "internationally" Pareto optimal, has some other important implications. Note that in the previous section there was no scope for benefit taxation: within any community the after-tax wage was a function only of the individual's productivity -- not of his tastes (and indeed there was no mixing of tastes for a given productivity). Now, however, there may be some scope for benefit taxation; first, there may exist within any community individuals of different tastes and the same productivity; secondly, there may be no way of identifying who is a "low" demander from who is a high demander without resorting to benefit taxation. Thirdly, observe that although there is a consumption inefficiency thereby induced, there may be no other way of identifying a particular group (low demanders), and hence there may be no non-discriminatory equilibrium in which a particular group is as well off as in the one involving benefit taxation -- it is in this sense that benefit taxation may be Pareto optimal. Finally, note that the total social loss from benefit taxation may be less than the induced reduction in consumption of the public good, the usual focus of the discussion of inefficiency; for benefit taxation may entail greater production efficiency -- in the absence of benefit taxation migration equilibrium may entail wide discrepancies in wage ratios in different communities.[1]

5. Concluding Comments

In this paper we have formulated a model of "perfect community competition" analogous to the "perfect competitive model" for conventional communities. To attain Pareto optimality communities must be aware of the competitive environment in which they operate. The model has one particularly interesting

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[1] In Stiglitz (1981), I provide a simple example in which without benefit taxation, the "high demanders" and "low demanders" have separate communities; with benefit taxation they live together. The latter is Pareto superior to the former.

feature: there is unanimity within each community about the level and pattern of allocation of public expenditure, even though individual tastes differ. There is no "social choice" problem nor is there any scope for redistribution.

The model has other implications which are undoubtedly unrealistic; within each community all individuals of a given productivity are identical.

Clearly there is an important grain of truth in the Tiebout hypothesis. Whether, however, the "perfect community competitive" model is a good model for the determination of the supply the public goods and the allocation of individuals among communities -- as good, say, as the corresponding competitive model for private goods -- remains a moot question.

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