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DISCONTINUOUS DISTRIBUTIONS AND MISSING PERSONS: THE MINIMUM WAGE AND UNEMPLOYED YOUTH

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ABSTRACT

The effects of minimum wage legislation on the employment and wage rates of youth are estimated using a new statistical approach. We find that without the minimum, not only would the percent of out-of-school youth who are employed be 4 to 6 percent higher than it is, but also that these youth would earn more. In particular, the expected hourly earnings of youth with market wage rates below the 1978 minimum are 10 percent lower with the minimum than they would be without it. Thus, an effect of the minimum is to increase the concentration of non-employment among low-wage workers and to reduce their earnings relative to higher wage workers as well. The minimum wage accounts for possibly a third of the difference between the employment rates of black and white youth, according to our results.

Our methodology is based on parameterization of the effect of the minimum on the distribution of "market" employment outcomes and market wage rates that would exist in the absence of the minimum. A concomitant of the estimation procedure is joint estimation of market wage and employment functions that would pertain if there were no minimum.

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DISCONTINUOUS DISTRIBUTIONS AND MISSING PERSONS: THE MINIMUM WAGE AND UNEMPLOYED YOUTH

by

Robert H. Meyer and David A. Wise*

Most econometric analysis rests on the assumption that random variables have continuous distributions. But government programs and legislation often impose constraints on individual choice so that empirically observed distributions are discontinuous. The minimum wage has such an effect on the distribution of observed wage rates. Most analysis of the effect of the minimum, however, is based on inferences from aggregate data and disregards these effects. Our analysis not only accounts explicitly for the effects of the minimum on observed distributions of employment and wage rates, but makes these effects an integral part of the process by which the employment and wage impacts of the minimum are estimated.

We have proposed a procedure that estimates the employment and wage effects of the minimum by explicitly parameterizing the relationship between the level of the minimum wage relative to the "market" wage rates that individuals would receive in the absence of the minimum. Indeed our approach parameterizes both observed wage and observed employment outcomes in terms of underlying "market" wage and "market" employment relationships.

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The procedure thus estimates market wage and employment functions for youth, neither of which may be estimated directly from observed data because whether particular data are observed empirically is determined in part by the minimum itself. Parameter estimates are obtained by maximum likelihood.

A concomitant of our procedure is estimation of a market wage function that allows us to compare average market wage rates with the wage rates of persons employed in the presence of a minimum wage, and to compare market wage rates with the expected wage in the presence of the minimum of all youth who would have been employed in its absence. The truncation and concentration effects of the minimum on the observed distribution of wages are explicitly incorporated in our statistical model. In addition, we are able because of the estimation of market wage and employment functions, to compare the distribution of employment versus non-employment by market wage rate when there is no minimum with the distribution when a minimum exists.

To evaluate the results of the minimum wage it is necessary to take account of its effect on employment and its effect on wage rates. Our procedure provides estimates of both of these effects as a direct product of our estimation procedure.

The effect of the minimum is commonly presumed to be greater the higher it is relative to the distribution of wages that would be paid in absence of a minimum. Indeed most time series and cross-section analysis of the effects of the minimum are based on relationships between employment and the ratio of the minimum wage to an average or median wage, often an average adult wage but sometimes the average wage paid to employed youth. Previous studies have been based on aggregate time series data (e.g., Gramlich [1976], Mincer [1976], and Hamermesh [1980]) or aggregate cross-section data (e.g., Welch and Cunningham [1978], Ehrenberg and Marcus [1979], and Cunningham [1980]).

Our analysis differs from these approaches in at least two respects. First, it is based on individual wage and employment data--collected in the May 1978 Current Population Survey. Second, the estimation technique emphasizes explicitly the relationship between the level of the minimum wage relative to the distribution of "market" wage rates that individuals would receive in the absence of a minimum. Some persons who in the absence of the minimum would be paid a wage below it are presumed to receive the legal minimum, while others are presumed to go without work. Still others because of non-coverage or non-compliance may continue to be paid below the minimum. Our goal is to estimate the effect of the minimum wage by explicitly parameterizing and estimating the likelihood that each of these outcomes occurs.

We find that persons who have market wage rates below the minimum, in the presence of the minimum, are paid the minimum with probability approximately .5 and lose their jobs with probability .25. Simulations based on our estimates indicate that employment of out-of-school young men would be 4 to 6 percent higher if the minimum were eliminated. On average, the expected wage of youth is lower with the minimum than without it. In particular, the expected wage of youth whose market wage rates are below the minimum is approximately 10 percent lower with than without the minimum. In addition, the concentration of non-employment among low-wage workers is much more pronounced with the minimum than without it. More succinctly, those who are paid the least without the minimum, are hurt the most by it.

Our model allows estimation of the primary effects of the minimum wage as described by most researchers. In particular, we have presumed that the effect is concentrated on persons who would otherwise be paid below the minimum. The model as set forth in this paper does not allow, for example, for an upward shift in the whole wage distribution because of the minimum. Although we do not believe this to be a first-order effect of the minimum, it is likely to occur to some extent. Under rather plausible assumptions, however, our primary results would not be affected by such shifts. Nonetheless, we will in future research address this possibility directly.

Graphs of wage distributions that serve to motivate our analysis are presented in section I. Our procedure rests on joint estimation of market wage and employment equations. If the disturbance terms in these two equations are uncorrelated, however, the basic parameters of the model may be estimated from a likelihood function based on the conditional distribution of observed wage rates only.[1] For expository purposes, we shall first present in section II this simple model and parameter estimates based on it, without explicitly deriving it as a special case of the two-equation model. This allows the reader to understand the approach in a relatively uncomplicated context that is directly motivated by the graphs

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1. As it turns out, this single-equation conditional wage distribution model is similar to the specification used by Hausman and Wise [1981] to correct for endogenous sampling with a continuous outcome variable.

in Section I.

Then the more complete model based on joint estimation of a market wage function together with a market employment function is presented in section III, together with empirical results based on it. This estimation procedure uses all available information on employment status and wage rates and is not restricted to observations with observed wage rates, unlike the simple model. And in this section, the single equation model is derived as a special case of the more general model. For expository purposes, some of the shortcomings of the simple model are explained for the first time in this section. Indeed for this reason some readers may wish to read the first part of this section before section II. The basic results of the empirical analysis are presented in section IV in the form of simulations. Concluding remarks and discussions are included in section V.

I. Empirical Wage Distributions

To motivate our analysis, we have graphed the empirically observed wage distributions for selected groups of youth.

The graphs are in the form of histograms with breaks at 25 cent intervals. For convenience, one break is at the 1978 minimum wage of $2.65. For most groups, there is a substantial discontinuity in the distribution at this point and it can be easily identified. To facilitate comparison among groups, the histogram includes in the wage interval .90 to 1.15 all persons with wage rates below 1.15 and in the interval 5.90 to 6.15 all persons with wage rates above 5.90. Thus apparent concentration in these intervals must be interpreted accordingly. If the entire distribution were graphed, the graph would approach zero gradually at both tails.

One of the presumptions underlying our analytic approach is that the minimum wage should have a greater impact in low-wage than in high-wage areas. This idea seems intuitively confirmed by comparison of Figures 1 and 2 in the text. The first shows the distribution for non-students 16 to 24 in the states with the lowest quintile of average adult wage rates; the second shows the distribution in states where the average adult wage rate is in the highest quintile. The distributions in both areas show a substantial discontinuity at the minimum, but it is clear that the impact of the minimum on the observed distribution of wage rates is much greater in the low wage areas.

We also presume that the minimum wage should impinge more on youth whose personal attributes are associated with lower earnings than on youth whose personal attributes are associated with higher earnings. For example, older youth with more schooling we assume would be most likely to have wage rates (possibly marginal products) above the minimum wage and thus not be affected substantially by it. This proposition is consistent with the distribution presented in Figure 3. Figure 3 shows the wage distribution for non-student youth 20 to 24 in high-wage areas with 14 years or more schooling. The effect of the minimum is barely apparent in this distribution.

Comparison of the distributions in Figures 4 and 5 for youth 16 to 17 versus 20 to 24 respectively provides further evidence that is also consistent with plausible intuition. Our intuition suggests that persons with attributes associated with low wages should be most affected by the minimum. The graphs strongly support this expectation.

Although these distributions help to motivate our subsequent analysis and in general are consistent with intuition, it is not clear from the graphs what the employment effect of the minimum is. It seems clear that one result is a concentration of wage rates at the minimum, but whether the apparent increase to the minimum of the wage rates of some youth is offset by non-employment of others cannot be inferred from the graphs; thus the motivation for our estimation technique.

There is also another consideration that may be obscured in the graphs, but to which our analysis is directed. For example, the distributions for whites 16 to 24 and for blacks 16 to 24 (neither of which is shown) appear quite similar. But the distributions pertain to employed youth in both groups. Differences between the attributes of employed black and white youth may not be as great as between the attributes of all youth in

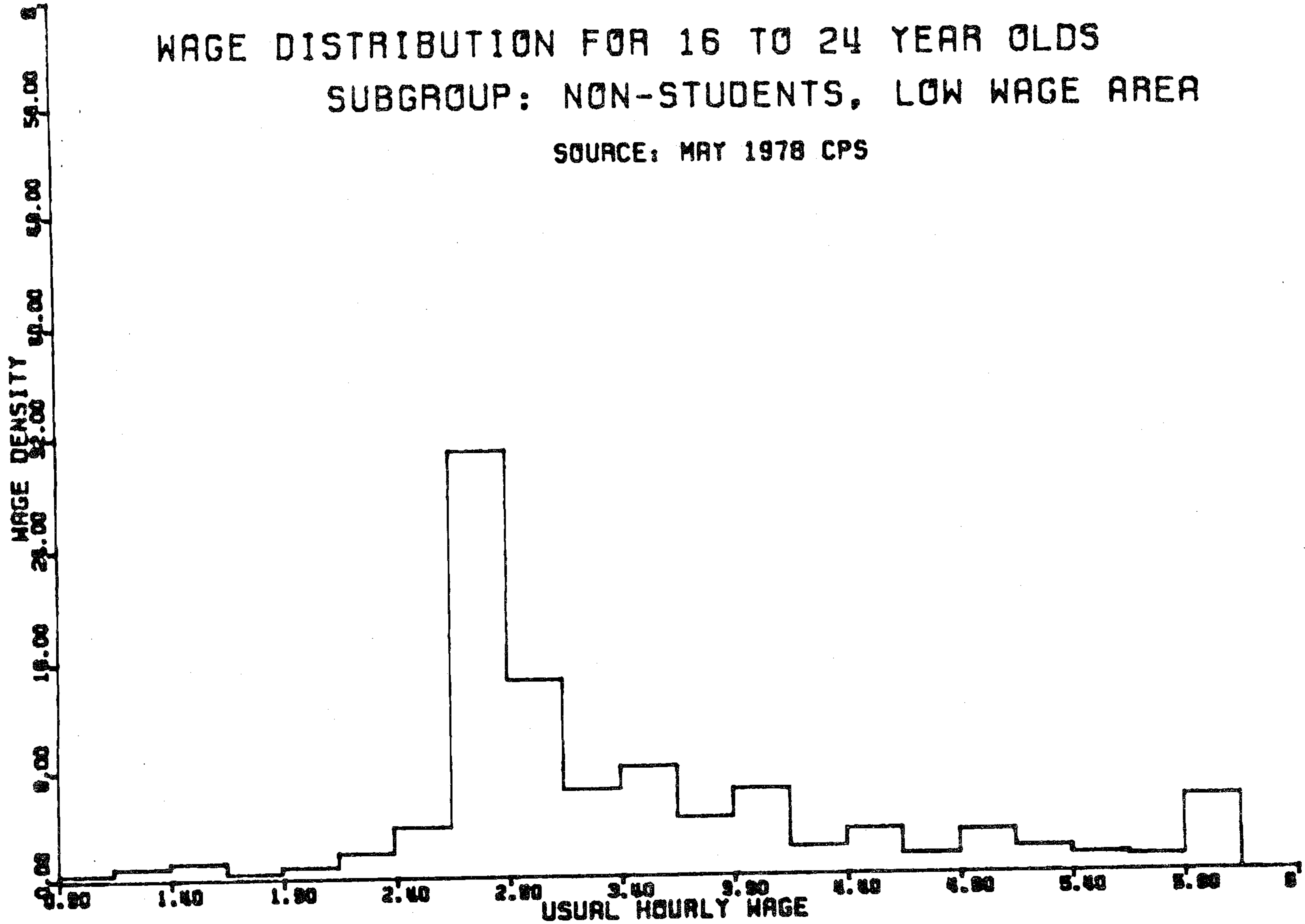
WAGE DISTRIBUTION FOR 16 TO 24 YEAR OLDS
SUBGROUP: NON-STUDENTS, LOW WAGE AREA
SOURCE: MAY 1978 CPS
WAGE DENSITY
USUAL HOURLY WAGE

Figure 1

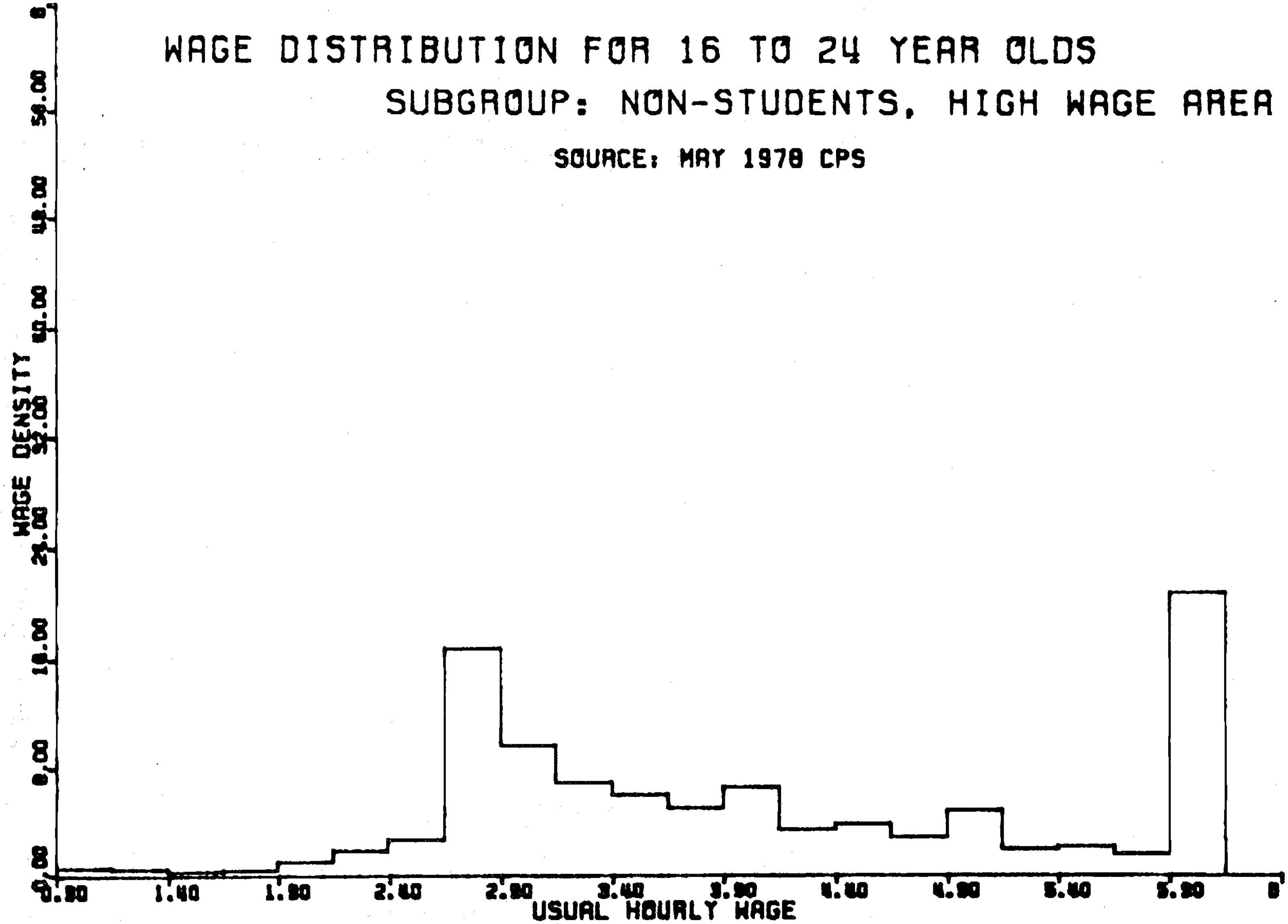
WAGE DISTRIBUTION FOR 16 TO 24 YEAR OLDS
SUBGROUP: NON-STUDENTS, HIGH WAGE AREA
SOURCE: MAY 1978 CPS
WAGE DENSITY
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
USUAL HOURLY WAGE
0.90
1.40
1.90
2.40
2.90
3.40
3.90
4.40
4.90
5.40
5.90

Figure 2

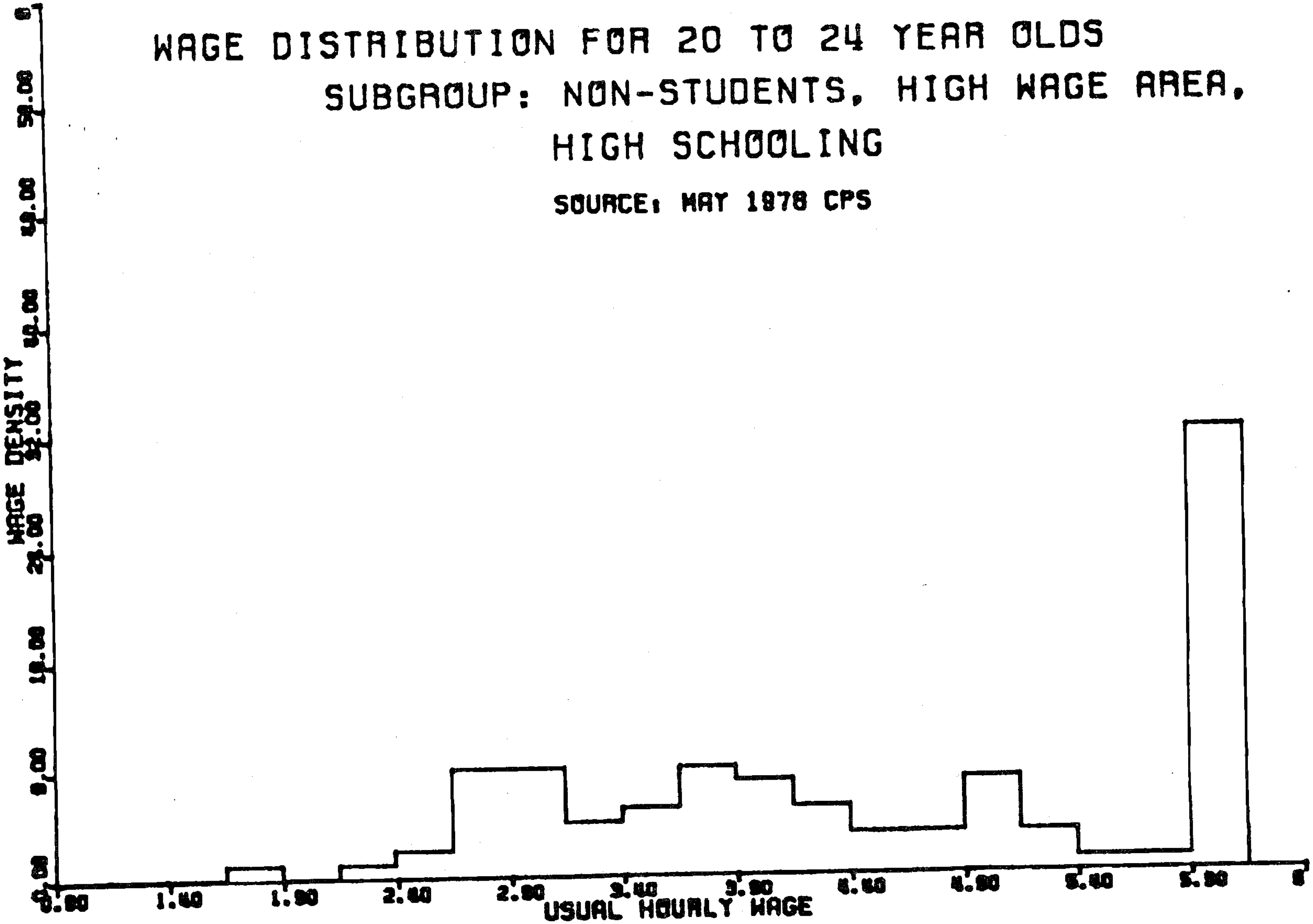
WAGE DISTRIBUTION FOR 20 TO 24 YEAR OLDS
SUBGROUP: NON-STUDENTS, HIGH WAGE AREA,
HIGH SCHOOLING
SOURCE: MAY 1978 CPS
WAGE DENSITY
USUAL HOURLY WAGE

Figure 3

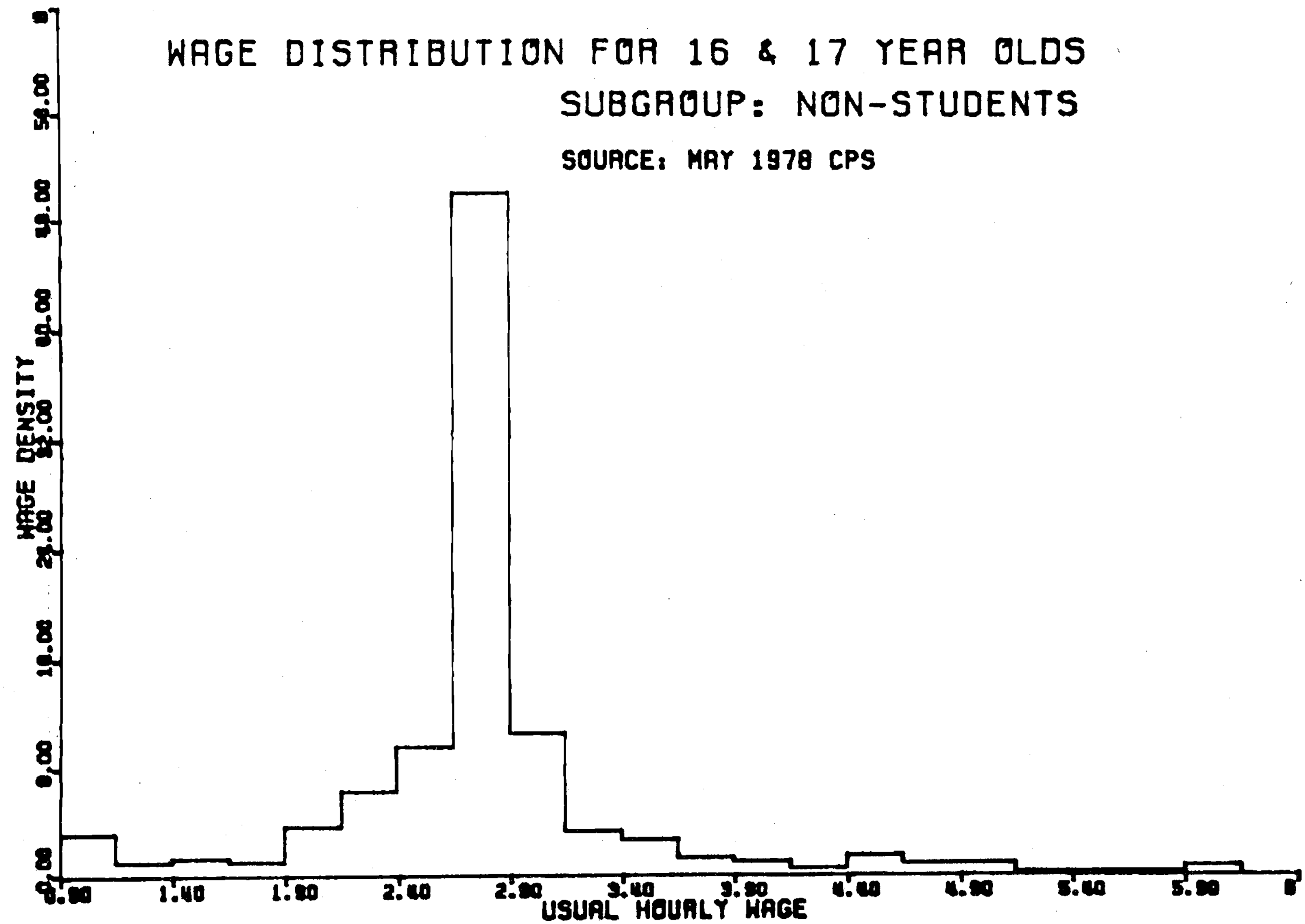
WAGE DISTRIBUTION FOR 16 & 17 YEAR OLDS
SUBGROUP: NON-STUDENTS
SOURCE: MAY 1978 CPS
WAGE DENSITY
USUAL HOURLY WAGE

Figure 4

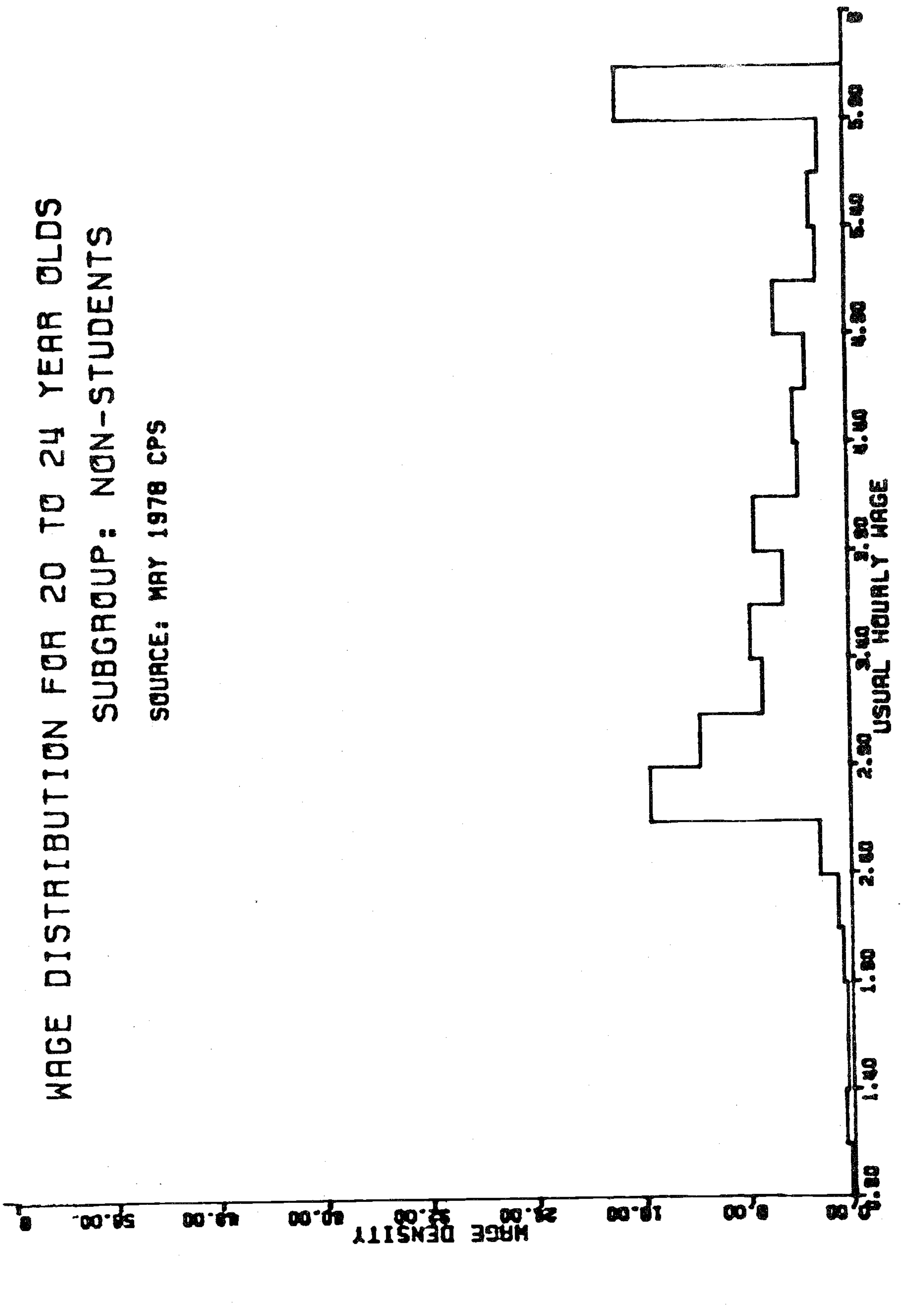
WAGE DISTRIBUTION FOR 20 TO 24 YEAR OLDS
SUBGROUP: NON-STUDENTS
SOURCE: MAY 1978 CPS
USUAL HOURLY WAGE
WAGE DENSITY

Figure 5

the two groups. Many fewer black than white youth are employed. Whether the distributions of wage rates without the minimum would look like those with the minimum cannot be inferred from the graphs, although our methodology allows us to predict such differences.

II. A Simple Approach

Although we shall ultimately obtain estimates based on the joint distribution of observed wage rates and employment status of youth, for expository purposes we shall begin with a model that is based on observed wage rates only. This allows a development that may be intuitively motivated by the empirical wage distributions shown in section I. And it allows us to set forth in a simple context the rationale behind our approach. Then we shall detail a model that treats employment and wage outcomes jointly, a special case of which is the model set forth here.

A. The Model

Consider a group of youth characterized by a vector of measured attributes X. The elements of X include individual measures such as education and age, and also area specific indicators of labor market conditions. Suppose that in the absence of a minimum wage, the distribution in the population of wages paid to employed persons with attributes X would be described by the density function f(X); we shall refer to it as the "underlying" or market distribution of wages. Graphically, think of it as the solid line in Figure 6.

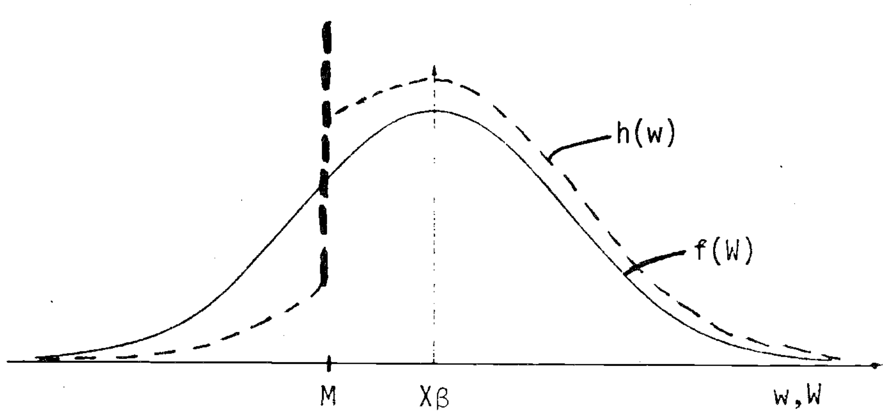

Figure 5

Now suppose that the minimum wage is set at level M. Some persons will continue to be paid a wage below the minimum because they work in non-covered sectors of the economy or on jobs that are not subject to the minimum. And indeed there may be some shifting of employment from covered to non-covered sectors and jobs. Others may be paid below the minimum because of non-compliance. For whatever reason, the net result is that some persons with an underlying wage below the minimum will continue to be hired at a wage below M. To allow for this possibility, we suppose that there is a probability P_1 that persons with an underlying wage below M will receive a wage below this level. (We have not allowed P_1 to depend on the precise value of the underlying wage.)

We also suppose that some persons with an underlying wage below the minimum would after its introduction be paid at the minimum.[1] Although a simple application of marginal productivity theory would imply that persons with an underlying wage below M, would not receive M, there are several possible explanations for such a possibility. One is that employers may pay the minimum to persons they would otherwise pay less than the minimum, but hire fewer or hire them for fewer hours. Whereas without the minimum, a young person may be hired on a permanent basis for eight hours each Saturday, if the youth must be paid the minimum, he may be hired for fewer hours to do only those tasks at which he is most productive. Employers may, for example be less prepared to pay for "slack time."[2]

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1. Welch and Cunningham (1978) impose an extreme form of this assumption, that is, that all persons with market wage rates below the minimum are paid the minimum when it is in effect.

2. Hall (1979) develops this point within a framework based on the theory of employment contracts.

Another possibility is that since the minimum wage applies only to compensation paid directly to an employee, employers can vary the level of non-wage compensation (e.g., on-the-job training or fringe benefits) to offset changes in direct compensation. Individuals with market wages below the minimum may be raised to the minimum in exchange for a comparable reduction in on-the-job training expenditures and fringe benefits. Individuals with market wages above the minimum will be unaffected.[1]

Another explanation is that employers hire at the minimum persons who would otherwise be hired at wage rates below the minimum, but offset this overpayment with slower wage increases--say, with age for example--than would be observed without the minimum.[2]

In addition, employers may find it difficult to identify differences in the quality of young workers, particularly in view of the high turnover in youth employment and the absence of an extensive employment history. If only because of this lack of precision, employers to comply with the legislation may raise to the minimum the wage rates of some employees who would otherwise receive an underlying wage below M. Whatever the reason we suppose that with probability P_2, a person with an underlying wage below the minimum will be employed and paid the minimum.

Those with an underlying market wage below the minimum who are not hired at a wage below M or who are not hired at M are assumed to be without work after the introduction of the minimum. The probability that these

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1. See Mincer and Leighton [1980] for an analysis of the effects of the minimum wage on investment in on-the-job training. Wessels [1980] examines the theoretical aspect of the minimum wage in a model that includes fringe benefits.

2. Lazear [1980] has investigated this possibility, but did not find much empirical support for it.

persons would be without work because of the minimum is $1 - P_1 - P_2$. We assume that the minimum wage does not affect the wages received by youth whose underlying wage is above the minimum. Although it is sometimes argued that the minimum wage tends to shift upward the whole distribution of wage rates, we believe that our model captures the primary postulated effects of the minimum.[1]

These ideas can be described more formally as follows. Suppose that the expected underlying wage of individuals with measured personal and regional attributes X is given by $X\beta$ and that the variance of wage rates among persons with characteristics X is σ^2. This gives rise to a wage distribution f(W) like that shown in Figure 6. That is,

$$W = X\beta + \varepsilon \quad (1)$$

where ε is a disturbance term with variance σ^2.

With a minimum wage M, wage rates may be distributed as represented graphically by the dotted function in Figure 6. The form of this function depends on the values of P_1 and P_2. For example, if P_2 were zero, there would be no pile-up of wages at M, only a jump in the density function at M. If both P_1 and P_2 were zero, the density function would be truncated at M.

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1. In addition, we have not allowed P_1 or P_2 to depend--for persons with market wage rates below M--on the difference between the market wage and the minimum, although in principle we think that they would. We believe, however, that our estimates of P_1 and P_2 are good estimates of the average values that would be obtained if somewhat more realistic assumptions were incorporated in our statistical analysis. Indeed, this conclusion is supported by estimates obtained by dividing the market distribution below the minimum into two intervals and estimating P_1 and P_2 values for each interval.

Let the likelihood of observed wage rates be given by $h(w)$. It may be written as

$$(2) \qquad h(w) = \begin{cases} \dfrac{f(w)\cdot P_1}{D} & \text{if } w < M, \\[2ex] \dfrac{Pr(M \leq W < M + 1) + Pr(W < M)\cdot P_2}{D} & \text{if } M \leq w < M + 1 \\[2ex] \dfrac{f(w)}{D} & \text{if } M + 1 \leq w, \end{cases}$$

where $D = 1 - Pr(W < M)\cdot(1 - P_1 - P_2)$. This formulation--although we shall show below its derivation from a model treating wage rates and employment jointly--may be arrived at by assuming that a random sample is drawn from the underlying distribution of market wage rates. Then, of the values 7 below M, some are set to M (with probability P_2), while others are discarded (with probability $(1 - P_1 - P_2)$). Then $h(w)$ is the distribution of observed wage rates in terms of the underlying distribution f. The denominator D may be thought of as a normalizing factor assuring that the density function integrates to 1. One can also think of $h(w)$ as the conditional distribution of wages, given that a wage is observed.[1] The other elements of the function may be explained in the following way. A value of $w < M$ will be observed with likelihood P_1 times the likelihood of an underlying wage $W = w$. The likelihood of an observed wage at the minimum (1 cent

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1. It is the probability that an individual who would have an observed wage rate in the absence of the minimum will also have one after the introduction of the minimum. Or it is the probability that a person who is employed without the minimum will also be employed with the minimum.

interval) is equal to the likelihood of an underlying wage at the minimum, plus the probability that the underlying wage is below the minimum but is raised to the minimum. Observed wage rates above the minimum follow the distribution of the underlying wage, except that a larger proportion of observed than of underlying wages may be above the minimum, as indicated by the denominator D.

For convenience, we shall consider the minimum wage to be an interval (it may be arbitrarily small, say 1 cent), going from M_1 to M_2. We shall also assume that W, or a transformation of W(e.g., ln W) is distributed normally. Then if Φ is taken to be a standardized normal distribution function, h(w) is given by

$$(3) \qquad h(w) = \begin{cases} \dfrac{f(w)\cdot P_1}{D} & \text{of } w < M_1 , \\[2ex] \dfrac{\Phi[(M_2-X\beta)/\sigma] - \Phi[(M_1-X\beta)/\sigma] + P_2\cdot\Phi[M_1-X\beta)/\sigma]}{D} & \text{if } M_1 < w < M_2 , \\[2ex] \dfrac{f(w)}{D} & \text{if } M_2 < W , \end{cases}$$

where $D = 1 - \Phi[(M_1 - X\beta)/\sigma]\cdot(1 - P_1 - P_2)$. We have used this specification because it allows us conveniently to test the sensitivity of our results to inclusion of wage rates somewhat above the minimum with those at the minimum.[1]

Suppose that among N persons with observed wage rates, N_1 are below M, N_2 are "at" M and, N_3 are above M. For these N persons indexed by i, the log-likelihood of the realized observations would be

$$(4) \qquad L = \sum_{i=1}^{N_1} \ln h(w_i) + \sum_{i=1}^{N_2} \ln h(w_i) + \sum_{i=1}^{N_3} \ln h(w_i),$$

with the specification of $h(w_i)$ for each group taken from equation (3). This function is maximized with respect to β, σ, P_1 and P_2.

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1. Following standard practice, the log of wages is used as the dependent variable in our wage model. Since our results are likely to be sensitive to this distributional assumption, we have also experimented with other transformations of wages, in particular the Box-Cox transformation:

$$w^{(\lambda)} = \begin{cases} \dfrac{w^{\lambda} - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \log w & \text{if } \lambda = 0 \end{cases}$$

We find that the predicted nonemployment resulting from the minimum wage is least when wages are assumed to be log normal (i.e., $\lambda = 0$) and greatest when nominal wages are assumed to be normally distributed (i.e., $\lambda = 1$). Estimates of the empirical distribution of wage rates compared with the predicted distribution based on the log normal density are presented below.

B. Parameter Estimates Based on the Simple Model

The variables used in the estimation are defined as follows:

Age: Age in years.

School: Number of years of school completed.

Race: Equal to 1 for blacks and zero otherwise.

Sex: Equal to 1 for women and zero for men.

Union: Equal to 1 for union members and zero otherwise.

Part-time: Equal to 1 for persons working part-time and zero otherwise.

City: Equal to 1 if the person lives in an urban area and zero otherwise.

Never Married: Equal to 1 if the person has never married and zero if married, widowed, or divorced.

Area Wage: The average wage of adult manufacturing workers in the SMSA or state in which the person lives.

Area Unemployment: The adult unemployment rate in the SMSA or state in which the person lives.

Northeast: Equal to 1 if the person lives in the Northeast and zero otherwise.

South: Equal to 1 if the person lives in the South and zero otherwise.

West: Equal to 1 if the person lives in the West and zero otherwise.

Wage: The dependent variable. The logarithm of the hourly wage rate, except where noted.

1. Comparison with Least Squares Results

Estimates of the parameters in equation (3) for a sample of all out-of-school young men and women aged 16 to 24 are shown in Table 1. To serve as an informal check for general consistency of our results with the assumptions motivating our model, we have also compared our wage function parameter estimates with least squares estimates. We shall not emphasize the empirical significance of the estimates in this section; they are treated as illustrative. Following the discussion of these results, we will compare estimates based on our model for selected subgroups of youth. Simulated effects of the minimum on the employment of these subgroups are presented in section IV.

Recall that a concomitant of our procedure is to estimate the "market" wage of an individual given his attributes. We suppose that the youth whose wage rates we measure are only a portion of those who would have measured wage rates in the absence of the minimum. In particular, some persons who would otherwise be employed and thus have an observed market wage below the minimum do not have an observed wage rate. We proceeded as though our sample were drawn from a group that, in the absence of the minimum, would have measured wages, but if an individual had a market wage less than M, the observation was retained, assigned the value M, or thrown out with probabilities P_1, P_2, and $1 - P_1 - P_2$ respectively.[1]

Consider first the wage function parameter estimates, the β's. To motivate the relationship between our estimates and the least squares results, we have graphed in Figure 7 the hypothesized market relationship

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1. We did not constrain by functional form $P_1 + P_2$ to be less than 1.

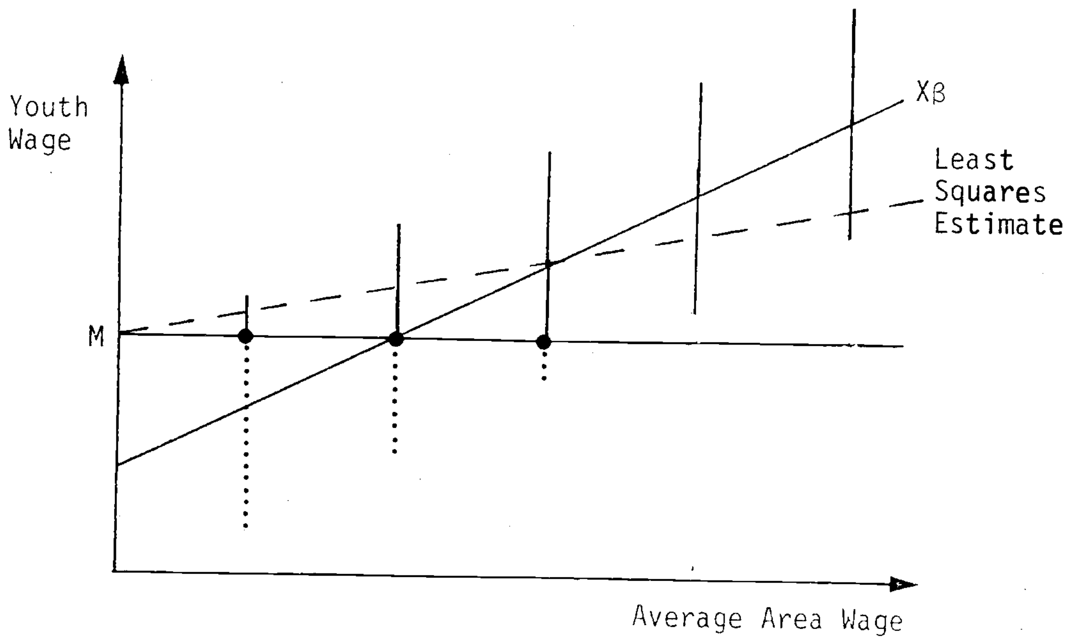

Figure 7

between average area wage rates and youth wages.

With the establishment of the minimum wage at M, some persons who would be employed and have observed market wage rates below M are not employed and thus not in the sample, while others have wage rates equal to M; some remain employed with wage rates below M. Thus as shown by the dashed line in Figure 7, if the minimum had the hypothesized effect on employment, least squares estimates would underestimate the relationship between average area wage rates and the underlying market wage.[1]

We see from Table 1, that this is indeed the case. Our estimates of the coefficient on area wage is 22 percent higher than the least squares estimate. Similar expectations and estimated results apply to other variables.

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1. This truncated distribution result is similar to the case discussed by Hausman and Wise [1977], in which the truncation was complete and at the upper end of the distribution.

For example, our schooling coefficient is 28 percent higher than the least squares estimate; our coefficient on age is 28 percent higher. Apparently persons from low wage areas and with personal attributes that are associated with lower market wages tend either to be excluded from the sample or to have wage distributions with concentrations at the minimum.[1]

And our estimated values of P_1 and P_2 are consistent with the relationship between our estimated slope parameters and least squares estimates. Our estimated value of P_1 (.231) indicates the 23 percent of persons with market wages below the minimum continue to be employed at wages below M, while the estimate of P_2 (.338) indicates that 34 percent of this group receive wages equal to the minimum. Thus 43 percent $(1 - P_1 - P_2)$ of those who have market wages below the minimum and would otherwise be employed are not employed, according to these estimates.

We would also expect--based on Figure 7 for example-- that given characteristics X, the variance of observed wage rates would be lower than the variance of market wage rates, and our estimates are consistent with this intuition. Our estimate of the standard error of wage rates (σ) is .335 while the least squares estimate is .296.

Because our methodology emphasizes the interaction between an individual's market wage and the impact of the minimum--in particular that the effect will be greater on workers with lower market wages--we also emphasize the substantial estimated effects of race and sex on market wage rates. Holding the other variables constant, women earn 22 percent less than men and

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1. Estimates for subgroups (not shown) are also consistent with this expectation. For youth 16 to 17, who have relatively low market wages, our estimated area wage coefficient of .065 is 1.6 times as large as the least squares estimate of .040. For women 16 to 24 our estimate is 2.7 times as large as the least squares estimate (.049 versus .018 respectively).

Table 1. Parameter Estimates for Out of School Youth 16 to 24 With Least Squares Comparison.

Variable	Parameter Estimate and (Asymptotic t-Statistic)	Least Squares Comparison
Age	0.041 (13.311)	0.032 (12.192)
School	0.041 (12.314)	0.032 (10.992)
Black	-0.069 (3.543)	-0.059 (3.698)
Women	-0.224 (18.279)	-0.186 (18.762)
Union	0.373 (24.503)	0.325 (25.699)
Part-time	-0.189 (13.458)	-0.138 (11.326)
Never Married	-0.095 (7.264)	-0.075 (6.823)
City	-0.018 (1.400)	-0.020 (1.827)
Area Wage	0.066 (8.824)	0.054 (8.305)
Area Unemployment	0.005 (1.073)	0.007 (1.635)
Northeast	-0.004 (0.203)	-0.006 (0.330)
South	0.028 (1.550)	0.033 (2.115)
West	0.122 (6.575)	0.099 (6.419)
Constant	-0.459 (5.387)	-0.037 (0.529)
P_1	0.231	--
P_2	0.338 (13.084)	--
σ	0.335 (98.044)	0.296 --
R^2	--	0.422
N	4000	4000

blacks 7 percent less than whites. These estimates suggest that we should predict a greater impact of the minimum on women and blacks than on white men.

Union members in this age group according to our estimates have a "market" wage 37 percent higher than non-union youth, holding other attributes constant. The wage rates of most union members are well above the minimum. Youth working part-time earn 19 percent less than those working full-time.

In the subsequent analysis we have eliminated the union and part-time variables because they are essentially endogenous. Both union and membership and part-time work are likely to depend in part on education and age, for example, and part-time at least may indeed be affected by the minimum wage. These variables also are available only for persons who are employed and would thus have to be inferred in the two-equation model described below. To limit the number of variables in the model, we also have eliminated the three regional dummies. In general, we found that regional effects--other than for the West--were not significantly different from zero after inclusion of the area wage and unemployment variables.

2. Estimates for Selected Groups of Men

Parameter estimates for selected groups of out-of-school male youth are shown in Table 2. The estimates for each group are based on all the observations in the survey that are in that group. Of most interest are the values for P_1 and P_2. For all male youth between 16 and 24, the estimates indicate that approximately 32 percent of persons who would have jobs with market wages below the minimum are excluded from employment by the minimum. Black youth are somewhat more likely than whites to be

Table 2. Parameter Estimates for Selected Groups of Out of School Male Youth[a]

Variable	Blacks and Whites 16-24	Blacks 16-24	Whites 16-24	Blacks and Whites 20-24	Blacks and Whites 16-17
Age	0.065 (15.943)	0.078 (5.268)	0.063 (15.414)	0.046 (7.552)	0.068 (1.332)
School	0.034 (8.636)	0.049 (3.203)	0.032 (8.068)	0.030 (6.842)	0.010 (0.508)
Black	-0.107 (4.078)	--	--	-0.103 (3.534)	-0.003 (0.030)
Never Married	-0.196 (11.482)	-0.147 (2.526)	-0.192 (11.114)	-0.196 (10.772)	-0.218 (2.094)
City	-0.032 (2.005)	-0.030 (0.555)	-0.031 (1.887)	-0.036 (1.903)	-0.024 (0.468)
Area Wage	0.084 (10.773)	0.082 (3.484)	0.083 (10.248)	0.081 (8.876)	0.065 (2.591)
Area Unemployment	0.008 (1.517)	0.024 (1.352)	0.007 (1.233)	0.012 (1.879)	-0.031 (1.857)
Constant	-0.822 (7.476)	-1.522 (3.718)	-0.754 (6.787)	-0.361 (2.283)	-0.323 (0.383)
P_1	0.229 (9.202)	0.212 (3.219)	0.245 (8.619)	0.232 (6.020)	0.341 (4.477)
P_2	0.451 (9.719)	0.410 (2.989)	0.467 (9.313)	0.454 (7.179)	0.512 (4.255)
σ	0.373 (64.293)	0.363 (20.174)	0.368 (62.883)	0.373 (54.150)	0.278 (25.974)
N	3005	268	2737	2131	231

a. T-statistics are in parenthesis.

excluded--38 percent versus 29 percent.[1] According to these estimates, differential effects of the minimum on these groups arise both from differences among the groups in market wage rates, dependent on the personal characteristics X, and differences in the probability of being hired given a market wage rate below the minimum.

The estimates for 16 to 24 year old black and white youth together imply that on average blacks earn 11 percent less than whites with the same measured characteristics (the coefficient on the black variable is -.107). For the younger group, however, the estimated black versus white effect is zero.

The estimates of P_1 and P_2 also differ by age group. Teenagers with market wages below the minimum are more likely than youth 20 to 24 to be hired at these wage rates. Differences in the jobs held by the two groups relative to the minimum wage legislation coverage we believe to be a likely explanation for this finding. We cannot rule out differences in compliance rates, however.

On the other hand, teenagers who would otherwise be hired at market rates less than the minimum are slightly more likely than 20 to 24 year olds to be hired at the minimum. One explanation is that according to our estimates the variance of underlying market rates is smaller for teenagers than for older young persons. Thus among out-of-school teenagers, market rates below the minimum are bunched closer to the minimum

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1. It is possible that the black youth are less likely than whites to be hired at the minimum--if their market wage rates are below the minimum--because the expected value of market wage rates below the minimum is lower for blacks than for whites, and for this reason their wage rates less likely to be raised to the minimum.

than are the sub-minimum market rates of older youth. It also seems probable to us that given measured characteristics X, sub-minimum market wage rates are more likely among older workers than among teenagers to be associated with poor employee attributes. If this were true, presumably employers would be less willing to "take a chance" with older workers and hire them at the minimum.

In sum, these estimates indicate that approximately 32 percent of men 16 to 24 with market wage rates below the minimum are without work because of it. Contrary to our expectation at least, our estimates imply that less than 20 percent of 16 to 17 year olds with market wage rates below the minimum are displaced by it.

3. An Empirical versus a Predicted Distribution

Unlike most more traditional methods of analysis, the distributional assumptions play a key role in our work. It has become standard practice to assume that wage functions are log-normal, and the results reported above are based on a log-normal distribution as well. However, to check the sensitivity of our results to this assumption and to determine a "best" fit, we also experimented with other distributions, using a Box-Cox transformation of wage rates.[1]

A comparison of the empirical distribution of wage rates by interval for all male youth 16 to 24 versus the predicted distribution based on the log-normal wage distribution is shown in Figure 8. It appears from the graph that the fit is quite close, especially at the tails where alternative

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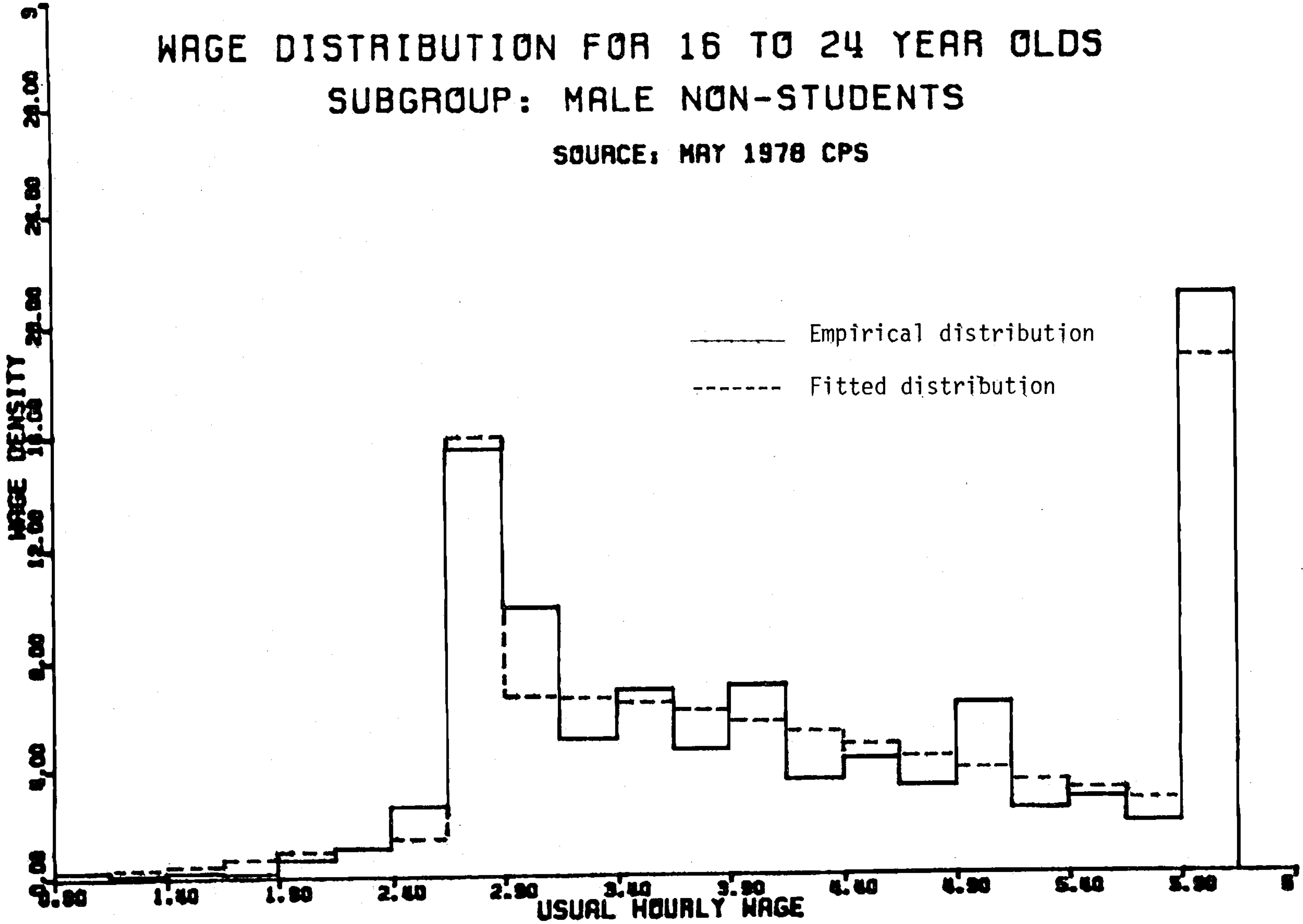
1. See footnote 1, page 15.

distributions are likely to give different results. Thus if we can fit the tails in particular, we have added confidence in our results. The actual percentages below the minimum, at the minimum (interval), and above \$5.90 are 4.9, 15.6, and 21.1 respectively; the predicted percentages are 5.0, 16.1, and 18.8. No continuous distribution, of course, can capture precisely the pile-up of wage rates at "magnet" values like \$3.00, \$4.00, or \$5.00.

A somewhat some formal way to measure the fit is to calculate a chi-square statistic based on the differences between the empirical and predicted frequencies within the intervals. The statistic:

$$\chi^2 = \sum_{j=1}^{J} \frac{(n_j - \hat{n}_j)^2}{\hat{n}_j},$$

(where n_j is the number of observations in the j^{th} interval, and J is the number of intervals) has a chi-square distribution with N-(J - 1 + K) degrees of freedom, where K is the number of parameters estimated in our model. Among a wide range of distributions that we tried, the log-normal gives the smallest chi-square value. It is very much smaller than the chi-square value based on the assumption of normality for example (286.1 versus 548.7).



WAGE DISTRIBUTION FOR 16 TO 24 YEAR OLDS
SUBGROUP: MALE NON-STUDENTS
SOURCE: MAY 1978 CPS
Empirical distribution
Fitted distribution
WAGE DENSITY
USUAL HOURLY WAGE
0.90
1.40
1.90
2.40
2.90
3.40
3.90
4.40
4.90
5.40
5.90

Figure 8

III. A More Complete Model: Employment and Wages

The results of the simple model are based only on the distribution of wage rates among youth who reported an hourly wage rate (plus a small number of youth for whom we could calculate a wage from reported weekly earnings and hours). Among the 24,305 youth 16 to 24 included in the May 1978 Current Population Survey, the distribution by employment and hourly versus salaried workers is as follows:

Category	Percent	
Total	100.0	
Not Employed	41.3	
Employed-Salaried	19.6	100.0
Salary Reported		57.6
Salary Not Reported		42.4
Employed-Hourly	39.1	100.0
Wage Reported		90.7
Wage Not Reported		9.3

The estimates in Section II were based on the distribution of wages among sub-groups of out-of-school youth with reported wage rates.[1]

The data, however, contain much more information than wage rates. In particular, they contain information on employment status. Although it is plausible under our assumptions to base estimates only on the wage data and from them to infer employment effects, it is clear that more robust estimates could be obtained by combining the wage data with the information on employment status. Intuitively, it appears that the addition of employment data

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1. To obtain adequate sample sizes we used all youth in some categories.

should make the estimates less dependent on distributional assumtions. In using both employment and wage data, we must also account for those youth who are employed, but for whom we do not have a reported wage.

We will show that the single equation model is a special case of this more general model and is a correct model if the disturbance terms in the wage and employment equations are uncorrelated. But in this model, a zero correlation does not mean that employment and wage equations can be estimated separately with no loss of information. Indeed a market employment equation cannot be estimated without considering a wage function as well. And estimating the two equations jointly provides additional information on wage rates, even with a zero correlation. As usual, the use of more information constrains the parameter estimates to reflect more empirical fact and to this extent provides better estimates, but in this case the information does not "separate" as might be expected on the basis of experience with more standard models.

A. A Two-Equation Model

In addition to an underlying wage distribution that would exist in the absence of the minimum wage, we shall incorporate explicitly an underlying employment relationship. Again, it is useful to think of a group of individuals with measured attributes X. Suppose that in the absence of a minimum wage, the employment and wage relationships would be of the form

$$E = X\alpha + \varepsilon_1 \, ,$$
$$W = X\beta + \varepsilon_2 \, , \qquad (5)$$
$$R \equiv \text{probability of a reported hourly wage.}$$

with E an unobserved index variable with the property that an individual is employed if $E > 0$, and where ε_1 and ε_2 are disturbance terms with covariance matrix

(6) $$\Sigma = \begin{bmatrix} 1 & \rho\sigma \\ & \sigma^2 \end{bmatrix}.$$

Given X, R is assumed to be uncorrelated with E and W, although R could in principle depend on X and need not be the same for each person with observed attributes X.

For expository purposes we shall pause for a moment and consider a diagram that relates the values of E, W, and R to the possible outcomes in the presence of a minimum wage, as shown in Figure 9. The entries within the diagram pertain to outcomes with a minimum wage. The notation on the top and bottom outside margins of the diagram pertain to underlying values of the employment and wage variables. On the right outside margin is indicated whether, among persons who would be employed in the absence of a minimum, a wage would be reported. The lined area indicates the proportion of the group who would not be employed with a minimum wage. Those with $E < 0$ would not be employed without the minimum and added to this group are those with $W < M$ who are not employed with a minimum--the two areas indicated by $1 - P_1 - P_2$. Some of the latter group would have a reported wage and others would not. We observe hourly wage rates for persons schematically included in the shaded area. (This was the group used in the procedure described in Section II. From this group we estimated P_1 and P_2.) The remaining group we observe to be employed but we don't observe their wage rates. Our goal then is to describe the probability of the possible outcomes.

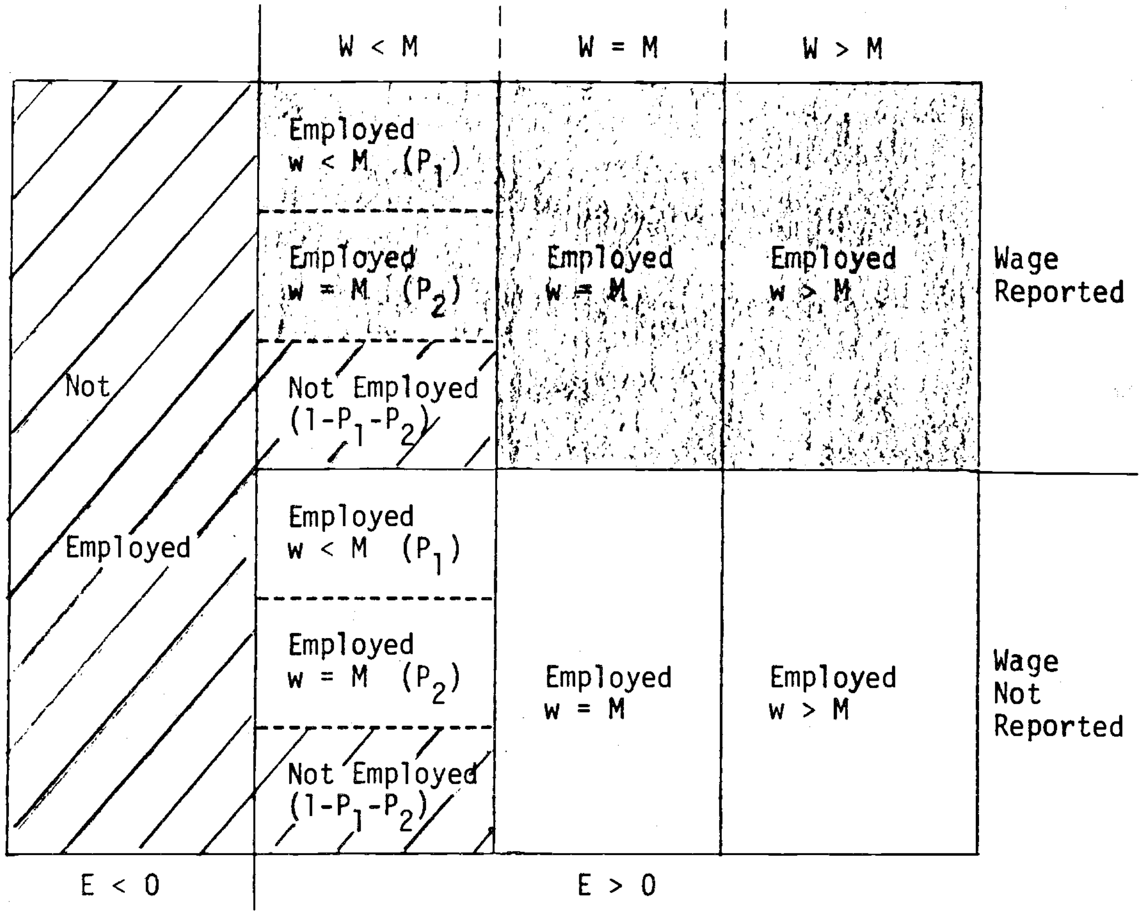

Figure 9

To do this we assume that E and W (a transformation of the wage rate) are distributed bivariate normal. To facilitate computation--and we believe without appreciably altering the results--we suppose, as noted above, that the unmeasured determinants of the underlying employment and wage equations on the one hand and the unmeasured determinants of whether a wage is reported on the other, are not correlated. This allows us to proceed with a bivariate instead of a trivariate distribution.[1] For the ease of exposition we have only specified two relationships in equation (5). We

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1. We shall not explain this in detail but without this assumption, the development would proceed much as we have laid it out except that we would have to evaluate trivariate integrals in some instances.

might more formally have added a third, say $S = X\delta + \varepsilon_3$ where an employed worker has an observed wage if $S > 0$. If ε_3 is uncorrelated with ε_1 and ε_2, however, expressions like $\Pr(E > 0, W = w, S > 0)$ can be written as $\Pr(E > 0, W = w)\cdot\Pr(S > 0)$. Our assumptions lead to expressions like these and rather than carry the third equation throughout the analysis, we have suppressed it, simply letting R indicate the probability of a reported hourly wage. (Extension of this reasoning demonstrates also that if ε_1 and ε_2 are uncorrelated, then consistent estimates of P_1 and P_2 and the parameters of the market wage function are obtained by the procedure used in Section II. We shall return to this.)

The possible outcomes--corresponding to the diagram--are as follows:[1]

(i) Pr[Not employed]

$$= \Pr[E < 0] + \Pr[E > 0 \text{ and } W < M_1]\cdot(1 - P_1 - P_2)$$

$$= 1 - \Phi[X\alpha] + \Phi_2\left[X\alpha, \frac{M_1 - X\beta}{\sigma}; -\rho\right]\cdot(1 - P_1 - P_2) \qquad = \Pr(1)$$

(ii) Pr[Employed with a wage w less than M]

$$= \Pr[E > 0, W = w]\cdot P_1\cdot R$$

$$= \Pr[E > 0 | W = w]\cdot f(W)\cdot P_1\cdot R$$

$$= \Phi\left[\frac{X\alpha + (\rho/\sigma)(w - X\beta)}{(1 - \rho^2)^{\frac{1}{2}}}\right]\cdot\frac{1}{\sigma}\phi\left(\frac{w - X\beta}{\sigma}\right)\cdot P_1\cdot R = \Pr(2)\cdot R$$

1. As in the single equation model we consider the minimum to be an interval, going from M_1 to M_2.

(7) (iii) Pr[Employed with a wage w equal to M]

$$= \Pr[E > 0,\ M_1 < W < M_2]\cdot R$$

$$+ \Pr[E > 0,\ W < M]\cdot P_2\cdot R$$

$$= \left(\Phi\left[X\alpha,\ \frac{M_2 - X\beta}{\sigma}\ ;\ -\rho\right] - \Phi\left[X\alpha,\ \frac{M_1 - X\beta}{\sigma}\ ;\ -\rho\right]\right)\cdot R$$

$$+ \Phi_2\left[X\alpha,\ \frac{M_2 - X\beta}{\sigma}\ ;\ -\rho\right]\cdot P_2\cdot R \qquad = \Pr(3)\cdot R$$

(iv) Pr[Employed with a wage w greater than M]

$$= \Pr[E > 0,\ W = w]\cdot R$$

$$= \Pr[E > 0 | W = w]\cdot f(W)\cdot R$$

$$= \Phi\left[\frac{X\alpha + (\rho/\sigma)(w - X\beta)}{(1 - \rho^2)^{\frac{1}{2}}}\right]\frac{1}{\sigma}\phi\left(\frac{w - X\beta}{\sigma}\right)\cdot R \qquad = \Pr(4)\cdot R$$

(v) Pr[Employed without a wage]

$$= \Pr[E > 0,\ W < M]\cdot(P_1 + P_2)\cdot(1 - R)$$

$$+ \Pr[E > 0,\ W \geq M]\cdot(1 - R)$$

$$= \Phi_2\left[X\alpha,\ \frac{M - X\beta}{\sigma}\ ;\ -\rho\right]\cdot(P_1 + P_2)\cdot(1 - R)$$

$$+ \Phi_2\left[X\alpha,\ -\frac{M - X\beta}{\sigma}\ ;\ \rho\right]\cdot(1 - R) \qquad = \Pr(5)\cdot(1 - R)$$

We see from (i) that the probability that an individual is not employed is given by the probability of not being employed without the minimum, $\Pr[E < 0]$; plus the probability that without the minimum he would be employed at a wage below M, times the probability that he is <u>not</u> employed below the minimum or at the minimum $(1 - P_1 - P_2)$. Similar explanations pertain to the remaining expressions.

The log-likelihood function for N observations is then given by

$$(8) \qquad L = \sum_{i=1}^{N_1} \ln Pr(1)_i + \sum_{i=1}^{N_2} \ln Pr(2)_i + \ldots + \sum_{i=1}^{N_5} \ln Pr(5)_i$$

$$+ (N_2 + N_3 + N_4) \ln R + N_5 \ln (1-R) \, ,$$

where i indexes individuals and $N_1 + N_2 + \ldots + N_5 = N$. Thus as long as R does not depend on parameters that enter elsewhere in the likelihood function, it may be disregarded in estimation. Equation (8) is maximized with respect to α, β, σ, P_1, P_2, and ρ.

Now suppose that, given X, E and W are uncorrelated so that $\rho = 0$. Equation (7) may then be rewritten as follows:

(9)

(i) Pr[Not employed]
$$= 1 - \Phi[X\alpha] + \Phi[X\alpha]\cdot\Phi[(M - X\beta)/\sigma]\cdot(1 - P_1 - P_2)$$

(ii) Pr[Employed with a wage w less than M]
$$= \Phi[X\alpha]\cdot f(w)\cdot P_1\cdot R$$

(iii) Pr[Employed with a wage w equal to M]
$$= \left(\Phi\left[X\alpha, \frac{M_2 - X\beta}{\sigma}; -\rho\right] - \Phi\left[X\alpha, \frac{M_1 - X\beta}{\sigma}; -\rho\right]\right)\cdot R + \Phi[X\alpha]\cdot\Phi[(M_1 - X\beta)/\sigma]\cdot P_2\cdot R$$

(iv) Pr[Employed with a wage w greater than M]
$$= \Phi[X\alpha]\cdot f(w)\cdot R$$

(v) Pr[Employed without a wage]
$$= \Phi[X\alpha]\cdot\Phi[(M - X\beta)/\sigma](P_1 + P_2)(1 - R)$$
$$\Phi[X\alpha]\{1 - \Phi[(M - X\beta)/\sigma]\}(1 - R)$$

The probability of having an observed wage is equal to 1 - (i) - (v), which is given by

(10)
$$\begin{aligned} &\Pr[\text{Employed with an Observed Wage}] \\ &\quad = R\cdot\Phi[X\alpha]\{1 - \Phi[(M - X\beta)/\sigma](1 - P_1 - P_2)\} \\ &\quad = R\cdot\Phi[X\alpha]\cdot D \ , \end{aligned}$$

where D is as defined in equation (2). The distribution of observed wage rates, conditional on observing a wage, can be derived by dividing equations (9,ii), (9,iii), and (9,iv), by (10). This gives the same result as equation (2) in Section II describing h(w), since the expression $R\cdot\Phi[X\alpha]$ multiplies each term in the numerator and denominator of each of the three parts of the conditional density function (and cancels out). Thus, given our assumptions, consistent estimates can be obtained from the single equation model if $\rho = 0$.

There is no analagous employment equation that does not depend on the wage function, however. Thus even though $\rho = 0$, estimation of the two equations jointly provides information that cannot be duplicated by estimating each separately.

B. Estimates Based on the Two-Equation Model

Parameter estimates for the two-equation model based on all out-of-school young men 16 to 24 and for those 20 to 24 are presented in Table 3. They may be compared with the single equation estimates in the first and fourth columns of Table 2. The sample used in the single equation estimation can be thought of as contained in but comprising only part of the data used in the two-equation estimation. The sample of all men in the 16 to 24 age group, within which the 3005 with an hourly wage rate were included, is distributed by employment category as follows:

Category	Percent		Number
Total	100.0		5997
Not Employed	15.4		926
Employed	84.6	100.0	5071
Wage Rate Known		59.3	3005
Wage Rate Unknown		40.7	2066

For those 20 to 24 only, the distribution is:

Category	Percent		Number
Total	100.0		4278
Not Employed	12.7		542
Employed	87.3	100.0	3736
Wage Rate Known		57.0	2131
Wage Rate Unknown		43.0	1605

The estimates of ρ in Table 3 for both the 16 to 24 and the 20 to 24 age groups are essentially zero. This in itself would suggest, under our assumptions, that the single-equation estimates should be approximately

Table 3. Two-Equation Estimates for Men 16 to 24 and 20 to 24

Variable	Age Group 16 to 24		Age Group 20 to 24	
	Wage Equation	Employment Equation	Wage Equation	Employment Equation
Age	0.063 (15.905)	0.012 (0.718)	0.045 (7.261)	0.006 (0.233)
School	0.033 (4.911)	0.078 (6.958)	0.029 (4.272)	0.068 (4.748)
Black	-0.104 (1.691)	-0.605 (9.113)	-0.104 (1.673)	-0.526 (5.979)
Never Married	-0.185 (5.509)	-0.468 (7.093)	-0.197 (5.038)	-0.476 (5.990)
City	-0.026 (1.245)	-0.206 (3.701)	-0.036 (1.461)	-0.209 (2.774)
Area Wage	0.082 (8.799)	-0.065 (2.173)	0.081 (6.639)	-0.088 (2.195)
Area Unemployment	0.009 (0.875)	-0.107 (5.902)	0.012 (1.042)	-0.121 (4.942)
Constant	-0.747 (7.159)	1.598 (2.972)	-0.322 (2.046)	2.150 (2.788)
P_1	0.272 (10.354)		0.266 (6.595)	
P_2	0.533 (9.571)		0.455 (6.761)	
ρ	-0.047 (0.073)		0.007 (0.009)	
σ	0.369 (57.500)		0.379 (56.136)	
N	5997		4278	

of the correct order of magnitude, although the simulated results using both salary and hourly workers may differ from those based on hourly wage workers only. The two-equation estimates for P_1 and P_2 don't differ greatly from the single-equation counterparts--.27 versus .23 and .53 versus .45 respectively for the 16 to 24 age groups and .27 versus .23 and .46 versus .45 for those 20 to 24. Recall that the single equation estimates are based only on hourly wage workers. The two-equation model incorporates both hourly and salaried workers--for whom we do not have a wage rate--and we assume that the same wage function and P_1 and P_2 values apply to both groups. The impact of the minimum on salaried workers may well differ from the effect on wage workers; the information we have on the salaried group is apparently too weak to verify this.[1]

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1. Because there is a large potential error in hourly wage rates estimated by using reported salaries and normal hours worked, much of the precision of actual wage rates is lost. The wage distributions for salaried workers that we generated using the ratio of salaries to hours, however, suggest that salaried workers with market wages below the minimum may be more likely than comparable hourly workers to be employed below the minimum, and less likely to be employed at the minimum.

IV. Simulations

We shall present first simulations based on the single-equation model and then additional ones based on the two-equation specification. The single-equation version is somewhat easier to work with and, because the relevant parameter estimates do not differ greatly from one to the other, we have presented some details that could be obtained based on the two-equation estimates but were not. Although the estimated employment impact of the minimum varies somewhat depending on the model, the general implications do not. Some results, however, are impossible to infer from the single-equation estimates only. Such results are presented in the second part of this section. In particular, we are able to infer the distribution of non-employment by market wage rate, with and without the minimum.

A. Simulations Based on the Single-Equation Model

From the estimates in Table 1 we may by simulation obtain estimates of the effects of the minimum wage on the employment and wage rates of these youth. These estimates are summarized in the tabulation below. The simulations use all observations used in estimation and allow for weighting of these observations depending on the likelihood that an individual with attributes X who would have been employed with the minimum is not observed with a wage because of the minimum. The numbers thus pertain to <u>hourly wage workers only</u>. All of the estimates pertain to 1978 as well. Thus the implication of lowering the minimum, for example, from \$2.65 to \$2.30 should be thought of in terms of 1978 dollars. The simulation methodology is explained in detail in Appendix A.

	Blacks & Whites 16-24	Blacks 16-24	Whites 16-24	Blacks & Whites 20-24	Blacks & Whites 16-17
1. Percent increase in employment if no minimum	6.8	11.9	5.5	3.6	9.5
2. Employment elasticity re reduction of minimum from 2.65 to 2.30	.195	.300	.166	.127	.233
3. Employment elasticity re increase of minimum from 2.65 to 3.10	-.222	-.309	-.193	-.171	-.178
4. Expected wage, given the minimum, of those employed	4.14	3.80	4.18	4.54	2.75
5. Expected wage, given the minimum of all those who would have been employed without the minimum	3.78	3.29	3.88	4.32	2.53
6. Expected market wage of all persons who would have been employed without minimum	3.87	3.43	3.95	4.39	2.49

If there were no minimum wage, according to row 1, the number of male youth between 16 and 24 with jobs would be 6.8 percent higher than it is (1978) now. It would be only 3.6 percent higher for those 20 to 24 and 9.5 percent higher among those 16 to 17.

Around the level of the minimum wage, the estimates of employment elasticities with respect to changes in the minimum are approximately 20 percent, but are considerably lower for the older group. Our methodology allows the elasticity to vary depending upon the level of the minimum relative to the underlying distribution of wage rates. Thus while standard estimates require that a single estimated elasticity be used to extrapolate employment effects for all levels of the minimum, our procedure allows the employment effect of an incremental change in the minimum to depend on its level. The closer the minimum is to the central tendency of the wage distribution, the greater the elasticity. A comparison of the elasticities for 20 to 24 and 16 to 17 year olds reveals this property. The minimum wage is much lower than the central tendency of the wage distribution of the older youth but above the central tendency of the wage rates of the younger group.

Our estimates suggest that reductions in the minimum would have relatively large effects down to about $2.00; but are close to zero below $1.50. For the group as a whole, the estimated marginal effects on employment of successive reductions in the minimum are as follows:

Reduction	Percent Increase in Employment
$2.65 to 2.30	2.6
2.30 to 2.00	1.8
2.00 to 1.70	1.2
1.70 to 1.50	0.5

At a minimum of $1.50, our simulations indicate that the expected wage of youth ($4.07) is approximately equal to the estimated expected market wage of $4.05. As shown above, at about $1.50 further reduction in the minimum would have virtually no effect on employment.

The fourth row of the tabulation on page 44 shows the expected wage of those employed, given the existing minimum. By comparison of the fifth and sixth rows, it can be seen that because some youth who would be employed in the absence of the minimum are not employed, the expected wage of the total group that would have been employed without the minimum is lower with it than without it. The expected market wage of the total group is shown in row 6. The increase of wages of some youth from below the minimum up to the minimum is more than offset by non-employment (zero wages) of others. The average difference is 9 cents per hour.[1]

The expected market wage of $2.49 for the 16 to 17 age group is well below the minimum wage of $2.65. To the extent that this figure is accepted, it is not surprising that the estimated employment effect is relatively large for this group.

According to our model, the wage effects are concentrated on persons who would otherwise be paid below the minimum. Thus for these sub-minimum workers, the loss in expected wages is greater than indicated by the numbers above. For all youth with sub-minimum market wages, the expected wage is 10 percent lower with than without the minimum ($2.06 versus $1.83). The loss is 9.1 percent for whites ($2.08 versus $1.89) and 13.1 percent for blacks ($1.98 versus $1.72). It is 14.3 percent for teenagers 16 and 17 ($2.17 versus $1.86). Only for older youth 20

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1. The estimates in rows 4, 5, and 6 were obtained by estimating logarithm values first and then converting these to absolute values. Thus there may be some error because of the non-linearities involved, but we believe that the relative magnitudes are not affected substantially.

to 24 is there essentially no effect of the minimum wage on the expected wage of sub-minimum wage workers (an estimated gain of 2.4 percent).

Finally, we have applied the parameter estimates based on hourly wage workers to all those employed, both hourly and salaried. This allows us to estimate the total number of out-of-school young men that would be employed if the minimum were eliminated. For the total group, we can also compare observed employment ratios with simulated ratios without the minimum. These values are given, by selected subgroups, in the tabulation below.

It is often argued that the minimum wage has a greater effect on black than on white youth employment, presumably because of the lower levels of education and other wage related attributes among black youth. Our results are consistent with this claim. That is, according to these estimates, if the minimum were eliminated, employment among black youth would be increased by 12 percent, while employment of white youth would be increased by only 5 percent. Nonetheless, only 30 percent of the difference between the employment ratios of black and white youth is due to the minimum, according to our estimates.

	Blacks & Whites 16-24	Blacks 16-24	Whites 16-24	Blacks & Whites 20-24	Blacks & Whites 16-17
1. Observed employment ratio with the minimum.	84.6	66.6	87.1	87.3	70.2
2. Percent increase in employment if no minimum.	6.6	12.2	5.3	3.5	9.5
3. Employment ration without the minimum.	90.1	77.4	91.7	90.4	76.8

B. Simulations based on the Two-Equation Model

Simulations based on the two-equation model for young men 16 to 24 are shown in the tabulation below. The tabulation shows the simulated distributions

Simulated Employment and Wages, Men 16 to 24

	Wage Rate Below Minimum	Wage Rate Above Minimum	Total
	Without a Minimum Wage		
Employed	924	4322	5246
Not Employed	170	579	749
Total	1094	4901	5995
	With the Minimum Wage		
Employed	744	4322	5066
Not Employed	350	579	929
Total	1094	4901	5995

of the 5997 persons in our sample, with and without the minimum. Of persons with market wage rates below the minimum, who without the minimum are employed, 20 percent (180) are without work with the minimum. According to these simulations, elimination of the minimum would increase total employment among young men by 3.6 percent. (The percent employed would increase from 84.6 percent to 87.5 percent.) The single-equation model based on 3005 hourly wage employees only implied that their number would be increased by 6.8 percent, if the minimum were eliminated. The single-equation wage model when applied to <u>all</u> 5066 <u>employed</u> persons (i.e., hourly and salaried workers) predicts an increase of 6.6 percent, as compared with 3.6 percent based on the two-equation model. It is to be expected that because a smaller proportion of salaried than hourly workers have market wages below the minimum, the percentage effect on both groups would be smaller than on hourly workers only. The difference between the two-equation and single-equation results, however, is apparently due only in small part to this fact.

There are other characteristics of the simulations that we find striking. Without a minimum, among youth with market wages below the minimum, 16 percent would not be employed, while of those with market wage rates above the minimum, 12 percent would not be employed. With the minimum, 32 percent of the sub-minimum wage group are not employed. Thus the results suggest that low wage workers would be disproportionately without work in either case, but the minimum wage magnifies substantially the difference between the employment rates of the two groups. Without a minimum, only 23 percent of non-employment is accounted for by those with sub-minimum market wages, while with the minimum this group accounts for 38 percent of non-employment.

It is of course impossible to infer these results without jointly estimating the wage and employment equations together. We need both market employment and market wage estimates, neither of which can be estimated without taking account of the effect of the minimum itself on each of them.

Analagous simulations for the age group 20 to 24 are presented below.

Simulated Employment and Wages, Men 20 to 24

	Market Wage Rate Below Minimum	Market Wage Rate Above Minimum	Total
	Without a Minimum Wage		
Employed	437	3437	3874
Not Employed	62	341	403
Total	499	3778	4277
	With the Minimum Wage		
Employed	298	1057	3735
Not Employed	201	100	542
Total	499	3778	4277

Employment among this age group, if the minimum were eliminated, would be 3.7 percent higher that it is--91.0 percent instead of 87.3 percent--according to these estimates. (For this age group, virtually the same estimate is obtained by applying the single-equation parameter estimates to all employed young men, both hourly and salaried.) And again, we observe that the minimum tends to increase the concentration of non-employment among low-wage youth. Without the minimum, the estimated 12.4 percent of those with below-minimum market wages who are not employed account for only 15.4 percent of non-employment, while with the minimum the 40.3 percent of the sub-minimum group who are not employed account for 37.1 percent of those without work.

V. Discussion and Conclusions

Our results imply that if there were no minimum wage, the number of out-of-school young men who are employed would be 4 to 6 percent higher than it is now. Among hourly workers, the effect is apparently the largest. Possibly one-half of the potential increase in employment could be gained by a 15 percent decrease in the minimum. Although the potential percentage increase in employment is greater for younger than for older youth, more older youth are employed. In 1978, for example, there were 601,000 employed males 16 and 17 who were not in school, and 6,735,000 male students 20 to 24. Thus a 9.5 percent increase for those 16 to 17 (from our single-equation results) would be 57,000, whereas a 3.6 percent increase for 20 to 24 year olds would be 242,000. These data apply to out-of-school youth, however, and most youth 16 to 17 are in school.

Our estimates imply also that the likelihood that a male non-student youth 16 or 17 with a market wage below the minimum is employed at or below the minimum is greater than the likelihood for older workers--.85 versus .69. Thus for example, whether a youth minimum is desirable, as opposed say to a reduction in the minimum, depends on the goals of the reduction. The effect on individuals of different ages may not be the same as the aggregate effects by age group.

The average wage paid to youth according to our estimates is lower with the minimum than it would be without it. Although those youth who are employed earn more on average than they would without the minimum, the increase for these youth is more than offset by the non-employment of others. Thus those least well-off without the minimum bear a disproportionate share of the cost of the minimum wage legislation. Because increases in wage rates come to a large extent with work experience, reduced work experience when young results in substantially reduced wage rates when older. Thus the total effect of the minimum on these low-market-wage workers is likely to be greater than the effect implied by the point-in-time estimates reported in the paper.

There are, of course, several possible effects of the minimum wage that our analysis does not address. We have set forth a model that we believe captures the primary postulated effects of the minimum wage as they are described by most researchers. In particular, we have assumed that the effect would be concentrated on persons who would otherwise receive wages below the minimum. Although economic theory suggests that substitution of higher quality for lower quality workers, for example, may raise the wage rates of workers with market wages above the minimum, the first order effect is thought to be on low wage persons. It may also be that increases in the minimum wage have an inflationary effect on the wage rates paid to all workers and thus shift upward the underlying distribution of wage rates. Such effects could be estimated if both time series and cross-section data were used and we will do that in future research.

It can be demonstrated that a purely inflationary shift in the underlying distribution would affect our estimated elasticities with respect to a change in the minimum, but not our estimated total employment effects, were the minimum to be eliminated. (This is explained in more detail in Appendix A.)

The minimum wage may also affect school attendance rates. Thus far we have restricted our formal analysis to out-of-school youth. A possible extension of our model would incorporate a school attendance equation or would use the model as is to obtain separate estimated effects for youth in school.

It may also be that the minimum wage affects hours worked, even among employed youth. For example, youth may be more likely to work part-time with a minimum than without it.[1] Explicit allowance for this possibility, as well as effects on school attendance, we believe would tend if anything to increase the employment effect of the minimum if employment were "adjusted" to account for these possible effects.[2]

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1. Most part-time workers are students and are therefore excluded in large part from our analysis.

2. Sherwin Rosen in his discussion has also pointed out that our data excludes military personnel and the minimum wage may interact with enlistments, possibly for young men just out of high school in particular.

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APPENDIX A: SIMULATIONS

Single Equation Model: Recall that D_i is the probability that a person with attributes X_i who would be employed without the minimum will be employed when it is in effect. Thus for each person in our sample with an observed wage rate and attributes X_i, the expected number of persons with attributes X_i who would be employed without the minimum is simply $1/D_i$. Given a sample of size N of employed persons, the number T that we predict would have been employed without the minimum is

$$T = \sum_{i=1}^{N} \frac{1}{D_i} . \tag{A-1}$$

For any particular minimum j indicated by M_j, the predicted number L_j of jobs lost is given by

$$L_j = \sum_{i=1}^{N} \frac{Pr(W_i < M_j)(1 - P_1 - P_2)}{D_i} . \tag{A-2}$$

The change in employment resulting from a shift in the minimum from the current M_0 to some M_j is then $L_0 - L_j$. Then:

(1) The percent increase in employment that would result if the minimum were eliminated is $(T - N)/N$.

(2) The employment elasticity with respect to a reduction in the minimum from 2.65 to 2.30 is $[(L_{2.65} - L_{2.30})/N]/[(2.65 - 2.30)/2.65]$.

(3) The employment elasticity with respect to an increase in the minimum from 2.65 to 3.10 is $[(L_{2.65} - L_{3.10})/N]/[(2.65 - 3.10)/2.65]$..

(4) Given the minimum, the expected wage of those employed is given by

$$\text{(A-3)} \qquad \frac{1}{T-L} \sum_{i=1}^{N} \left\{ \frac{Pr(W_i \geq M)}{D_i} \cdot E(W_i \mid W_i \geq M) + \frac{Pr(W_i < M)}{D_i} [P_2 \cdot M + P_1 \cdot E(W_i \mid W_i < M)] \right\}$$

This is simply the expected value of the density in equation (3), averaged over persons in the sample.

(5) Given the minimum, the expected wage of those who would have been employed without the minimum is given by

$$\text{(A-4)} \qquad \frac{1}{T} \sum_{i=1}^{N} \{ \cdot \},$$

where the term in brackets is the same as under (4) above.

(6) The expected market wage of all persons who would have been employed without the minimum is given by

$$\text{(A-5)} \qquad \frac{1}{T} \sum_{i=1}^{N} e^{X_i \beta} / D_i$$

Two-Equation Model: Let N now represent the total number of persons in the sample--employed and not employed, with and without an observed wage rate. Without a minimum, the total number T employed is given by

$$T = \sum_{i=1}^{N} Pr(E_i > 0) = \sum_{i=1}^{N} \Phi\,[X_i\hat{\alpha}]. \tag{A-6}$$

The number not employed is given by

$$N - T .$$

The number with wage rates below M is

$$\underline{N} = \sum_{i=1}^{N} Pr(W_i < M) = \sum_{i=1}^{N} \Phi\left[\frac{M - X_i\beta}{\sigma}\right]. \tag{A-7}$$

The number with wage rates above M is given by

$$\overline{N} = N - \underline{N} .$$

The other entries in the top part of the tabulations on pages 47 and 49 may be calculated using the estimated parameters in the bivariate normal function of our model. For example, the number not employed and with wage rates W less than M is given by

$$\begin{aligned} &\sum_{i=1}^{N} Pr(E_i < 0 \text{ and } W_i < M) \\ &= \sum_{i=1}^{N} \Phi_2\left[-X_i\alpha, \frac{M - X_i\beta}{\sigma} ; \rho\right] . \end{aligned} \tag{A-8}$$

The other three entries are obtained analagously.

With the minimum, the number L of jobs lost is given by

$$L = \sum_{i=1}^{N} \Pr(E_i > 0 \text{ and } W_i < M)\cdot(1 - P_1 - P_2)$$

(A-9)

$$= \sum_{i=1}^{N} \Phi_2[X_i\alpha, \frac{M-X_i\beta}{\sigma}; -\rho]\cdot(1 - P_1 - P_2).$$

The number employed is

$$T - L .$$

The number not employed is

$$N - (T - L)$$

The number not employed and with wage rates less than M is given by

(A-10)

$$\sum_{i=1}^{N} [\Pr(E_i < 0 \text{ and } W_i < M)$$

$$+ \Pr(E_i > 0 \text{ and } W_i < M)\cdot(1 - P_1 - P_2)]$$

The other elements in the bottom part of the tabulations on pages 47 and 49 are the same as in the top part.

An alternative method of simulating the job loss resulting from the minimum is based on a method analagous to the one used in the single equation simulations. It is based only on employed persons. (In practice this alternative method and the one described above give almost identical results.) Analagous to D_i, we define a $D_i(2)$, which in the conditional probability that a person who would be employed without the minimum will still be employed when it is in effect. It is the conditional probability of employment, given that $E_i > 0$, and may be written as

(A-11)

$$D_i(2) = 1 - \frac{Pr(E_i > 0,\ W_i < M)}{Pr(E_i > 0)}\cdot(1 - P_1 - P_2)$$

$$= 1 - \frac{\Phi_2[X_i\alpha,\ \frac{M - X_i\beta}{\sigma}\ ;\ -\rho]}{\Phi[X\alpha]}\cdot(1 - P_1 - P_2)$$

If $\tilde{N}$ is the total number employed with the minimum in effect, then the total number T that would have been employed without the minimum is given by

(A-12)

$$T = \sum_{i=1}^{\tilde{N}} \frac{1}{D_i(2)} .$$

For any minimum j denoted by M_j, the number L_j of lost jobs is given by

(A-13)

$$L_j = \sum_{i=1}^{\tilde{N}} \frac{Pr(W_i < M_j \mid E_i > 0)\cdot(1 - P_1 - P_2)}{D_i(2)}$$

$$= \sum_{i=1}^{\tilde{N}} \frac{1}{D_i(2)} \cdot \frac{\Phi[X_i\alpha,\ \frac{M_j - X_i\beta}{\sigma};\ -\rho}{\Phi[X\alpha]}\cdot(1 - P_1 - P_2)$$

In particular, the number of jobs lost at the existing minimum M_0 is L_0 and employment $\tilde{N}$ at the existing minimum is given by

$$\tilde{N} = T - L_0.$$

<u>A Shift in the Wage Distribution</u>: As mentioned in the text, it is sometimes argued that the minimum wage has a purely inflationary effect on all wages; that is, it shifts upward the wage rates paid to all workers.

Within the context of our model, we could in this case think of the effect of the minimum in two parts: first, it shifts all wages upward, and second, persons remain employed or lose their jobs according to the mechanism we have described, but with respect to this "shifted" distribution. Using only cross-section data, we are unable to estimate the magnitude of such shifts if they do occur, but we could do so using both cross section and time series data. We will do this in future research. In the meantime, we note that such shifts would not affect our estimates of lost employment resulting from the minimum, although they would affect our employment elasticity estimates with respect to a change in the minimum.

That is, with a purely inflationary increase in the underlying distribution, the total employment loss is the increase in employment that would result, given the shifted distribution, if the minimum were eliminated.

To test the sensitivity of our elasticity estimates to such shifts--with changes in the minimum--we have calculated them assuming selected shifts in the underlying wage distribution, using the single equation model. As in the text we begin by assuming that without a minimum the logarithm of the wage rate is given by

(A-14) $$\ln W = X\beta + \varepsilon .$$

But in this case, we assume in addition that with a minimum M_j, all wages are shifted upward by an amount $S(M_j)$, so that without the discontinuities caused by the minimum, wage rates would be given by

(A-15) $$W(M_j) = W \cdot S(M_j),$$

and $\ln W(M_j)$ by

(A-16) $$\ln W(M_j) = X\beta + \varepsilon + \ln S(M_j) .$$

In this case, the shift associated with the minimum M is simply embodied in our estimated constant term.

Relative to the underlying wage rates with the current minimum M, wage rates with another minimum M_j would be given by

(A-17) $$W(M_j) = W(M) \cdot \frac{S(M_j)}{S(M)} ,$$

with

(A-18) $$\ln W(M_j) = X\beta + \varepsilon + \ln \frac{S(M_j)}{S(M)} .$$

Our model of course does not provide estimates of the last term, but the sensitivity of our elasticity results can be checked by substituting for $X\hat{\beta}$, in the lost employment calculations, $X\hat{\beta} + K$, where K is a selected value for $\ln [S(M_j)/S(M)]$. For example, we could assume that a 15 percent reduction in the minimum would shift the underlying distribution down by 5 percent so that $S(M_j)/S(M)$ would be .95.

We can demonstrate now that a shift as described above would not affect the estimate of jobs lost as a result of the minimum. To see this, we have rewritten equation (A-2), to allow for the shift parameter, as

(A-19) $$L_j = \sum_{i=1}^{\tilde{N}} \frac{\Phi\left[\frac{M_j - X_i\beta - \ln(S(M_j)/S(M))}{\sigma}\right] \cdot (1-P_1-P_2)}{D_i}$$

The denominator depends only on the current minimum. At the current minimum, $M_j = M$ and the shift term is equal to zero. Thus our estimate of the non-employment effect of the present minimum is not affected by possible shifting of the underlying wage distribution. However, simulated employment elasticities based on a comparison of L_j with some L_k will depend on the shift term. They will be somewhat lower, depending on the magnitude of the shift.