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PROCESS CONSISTENCY AND MONETARY REFORM: FURTHER EVIDENCE AND IMPLICATIONS

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ABSTRACT

In this paper we provide additional evidence that process consistency may have materialized as a restrictive constraint on the money generation process. In addition to recomputing the time series of process consistency probabilities using new data from the German case, we also supply our empirical technique to the data from the other hyperinflations studied by Cagan. We interpret our results as evidence bearing on the type of transversality condition studied by Brock or by Brock and Scheinkman as a sufficient condition to insure a unique equilibrium in optimizing models with perfect foresight and money.

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This century's post-war hyperinflations have displayed rates of price and money supply increase which have isolated them almost as a separate area of monetary theory and experience. Economic researchers have treated hyperinflations primarily as intense monetary experiments, remarkable opportunities to test basic monetary and expectational theories in environments devoid of confounding real movements.[1]

In Flood and Garber (1980a) we devised a model which employed the concept of process consistency, the requirement that a money supply process generate a finite price level, to examine the transition from extreme hyperinflation to monetary reform. Our empirical results, based on the German case, provided evidence in favor of the model, in that the reform occurred at the moment that the probability of process consistency reached its lowest value. Our study can be interpreted, in line with the usual treatment of hyperinflation, as yet another test of abstruse monetary theory provided by the extreme of the hyperinflationary wind-tunnel.[2,3]

In this paper we provide additional evidence that process consistency may have materialized as a restrictive constraint on the money generation process. In addition to recomputing the time series of process consistency probabilities using new data for the German case, we also apply our empirical technique to the data from the other hyperinflations studied by Cagan (1956). In the most extreme cases, Hungary II, Germany and Greece, we find that, as in our earlier study, the probability of process consistency reached its lowest value at the initiation of the reform.

We divide the paper into two sections. Section I is designed simply as a research report on our new results. Section II is a speculation concerning how the results can be interpreted as evidence about some of the more esoteric questions that have arisen recently in monetary theory. In particular, we interpret our results as evidence bearing on a transversality condition studied by Brock (1974) and by Brock and Scheinkman (1980) as a sufficient condition to insure a unique equilibrium in optimizing models with perfect foresight and money.

Since transversality conditions imply that no money supply process can be inconsistent, they also imply that computations of process consistency should be related to monetary reform only by coincidence. That such a coincidence should be present in the most extreme hyperinflations is hard to ignore. Thus, in section IIb we discuss monetary reform in terms of the political climate prevailing at the ends of the big hyperinflations.

I. Empirical Results

In this section we report our constructed time series for process consistency probabilities in a number of hyperinflations. The assumptions and theory behind these computations are those employed in Flood and Garber (1980a). In particular, we assume that the forms of the money demand functions and the money supply processes are those which are analyzed in our previous paper.

$$m_t^d - p_t = \gamma + \alpha[E(p_{t+1} - p_t \mid I_{t-1})] + v_t \quad \text{(money demand)}$$
$$\alpha < 0,\ E(v_t \mid I_{t-1}) = 0,\ E(v_t^2 \mid I_{t-1}) = \sigma_v^2 \tag{1}$$

$$\text{(a)} \quad m_t = m_{t-1} \exp\{\beta_t\} \quad \text{(money supply)}$$
$$\text{(b)} \quad \beta_t = \theta\beta_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma^2),\ (\epsilon_t\epsilon_j) = 0,\ i \neq j \tag{2}$$

the variables m_t^d, m_t, and p_t are the logarithms of money demand, the money stock and the price level, respectively; E is the mathematical expecations operator; and I_{t-1} is an information set which contains all relevant economic data through time t - 1, together with the money demand parameters.[4/] ϵ_t and v_t are random disturbances and γ, α, σ_v^2, σ^2, and θ are parameters. Agents do not know the parameters of money supply; they form beliefs about θ and σ^2 by means of Bayesian posterior probability density functions.

Application of standard methods for the solution of difference equations to (1) yields the following solution for expected price:

$$E(p_t|I_{t-1}) = \frac{\Psi^{-1}}{\alpha}\sum_{i=0}^{\infty}[E(m_{t+i}|I_{t-1}) - \gamma]\Psi^{-i}$$

where $\Psi \equiv \frac{\alpha - 1}{\alpha}$.

If the money supply is expected to grow rapidly enough relative to the money demand parameter embodied in Ψ, the expected (and actual) price solution will be infinite. In such a case, no agent will exchange goods for currency; hence, the supply process will not provide a useful money.

We classify any money supply process which yields a finite price level solution as process consistent. Equivalently, any supply process which yields an infinite expected price solution is called process inconsistent. If an economy is to continue functioning as a monetary economy, agents must believe that any supply process which they perceive as process inconsistent will be overthrown in favor of a process consistent money supply process. Therefore, if there were little likelihood that a supply process is consistent, we would expect a monetary reform.

Since agents are unsure about the values of the parameters of the money supply process is process consistent. However, there is a region of (Θ,σ^2) values for which the supply process is process consistent; therefore, the probability of process consistency at any time can be determined by integrating the posterior probability density function for (Θ,σ^2) over the appropriate region.

Specifically, the construction of the probability of process consistency is an exercise in the integration of the posterior p.d.f.:

$$H(\theta, Z \mid \vec{\beta}) \propto \frac{1}{|1 - \theta|^{T} (\sqrt{2Z})^{T+2}} \exp \{ - \frac{1}{4Z(1-\theta)^2} \sum_{t=1}^{T} (\beta_t - \theta\beta_{t-1})^2 \} \quad (3)$$

over the parameter region

$$-1 < \theta < 1 \quad (4a)$$

$$Z < \log \psi$$

where $Z \equiv \frac{\sigma^2}{2(1-\theta)^2}$. With the money supply data from the various hyperinflations and estimates for α, we can compute the posterior probabilities of process consistency. To insure convergence of the numerical integration we have altered our previous integration technique somewhat; in the appendix we explicitly describe our new method.

a) An Overview of the Hyperinflations

The seven instances which form the basis of most hyperinflation studies encompass a wide range of inflationary intensity. Table I (which is constructed from Cagan's (1956) Table 1, p. 26) arranges the various episodes by the magnitude of price level change experienced during the course of the inflation.

The first three hyperinflations in Table 1 can be categorized as particularly extreme inflationary episodes; the last three were relatively mild; and the Russian case lies somewhere between the other two groups. In studying the extreme hyperinflations, Cagan (1956) was forced to exclude observations from the final months of each episode because he observed far more real balances than predicted by money demand relationships estimated from earlier observations. Following Cagan, others who have studied these inflations have also excluded those months in which prices rose most rapidly. Of the other inflations, only the Polish case required a similar exclusion of observations; Cagan deleted the final two months of Polish data.[5]

For all the hyperinflations except the Greek case, the time of the reform is defined as the time at which the price level or exchange rate stabilized; in these cases, a new money supply process clearly was instituted. In the Greek case, the government announced a monetary reform in November, 1944; the reform proved abortive in that the inflation and money creation continued, but since we pinpoint this time as a switch in a money supply process, it is a possible candidate for the time of reform.

b) The German Case

In our earlier examination of the German case, we employed a time series on money which, partly due to the availability of data, combined monthly observations from the inflation's first years with weekly observations from the last year. In addition, the money supply data from the last weeks of the German inflation included the newly introduced "fixed value" currencies, converted into nominal units by a market exchange rate.

We have recomputed the process consistency probabilities for Germany with two different alterations in our data series. First, we have constructed a purely weekly time series by combining the weekly Reichsbanknote series for 1919-1922 reported in Flood and Garber (1980b) with the weekly series for 1923 used in our (1980a) study.[6] Therefore, any statistical problems which may arise through the combination of weekly and monthly series are eliminated. Second, we have used a weekly money series consisting only of

Reichsbanknotes; the fixed value monies which appeared in the final weeks are ignored.[7]

In Table II, we report the series for the probability of process consistency for each money aggregate. The two money series are identical until September 15, 1923, when nominal railroad money began to appear; upon its appearance at the end of October, 1923, fixed value money was included with nominal currency to form the series for all monies. The date of the first observation on Reichsbanknotes is December 23, 1918. The $\hat{\alpha}$ used to compute log ψ was Cagan's estimate, $\hat{\alpha} = -5.46$ months, converted to weekly time units. The rates of growth of the logarithms of the money supply (the β_t's) were the rate per week.

In the last weeks of the hyperinflation, our original series and the two new series exhibit similar patterns of movement in the process consistency probabilities. The three cycles beginning on August 15, 1923 appear for all three money aggregates. For the original money series and for the all-inclusive weekly series, the probability of process consistency reaches its lowest value on November 15, 1923, the week in which the reform began. For the Reichsbanknote series, the lowest probability is reached on August 31; however, the probability of process consistency for this money series does reach a relatively low value on November 15. The major change observable in our new results is that the probability of process consistency is almost unity prior to August 15, 1923. Our earlier computations indicated that the probability of process consistency moved between .46 and .58 prior to August 15. The results for the weekly series indicate that agents could be fairly confident in their money's process consistency prior to August, 1923.[8]

Table I

Order of Magnitude of Hyperinflations

Country	Pe/Po^{*}	Period
1. Hungary II	3.81×10^{27}	Aug. 1945 - July, 1946
2. Germany	1.02×10^{10}	Aug. 1922 - Nov., 1923
3. Greece	4.7×10^{8}	Nov., 1943 - Nov., 1944
4. Russia	1.24×10^{5}	Dec., 1921 - Jan., 1924
5. Poland	699.0	Jan., 1923 - Jan., 1924
6. Austria	69.9	Oct., 1921 - Aug., 1922
7. Hungary I	44.0	Mar., 1923 - Feb., 1924

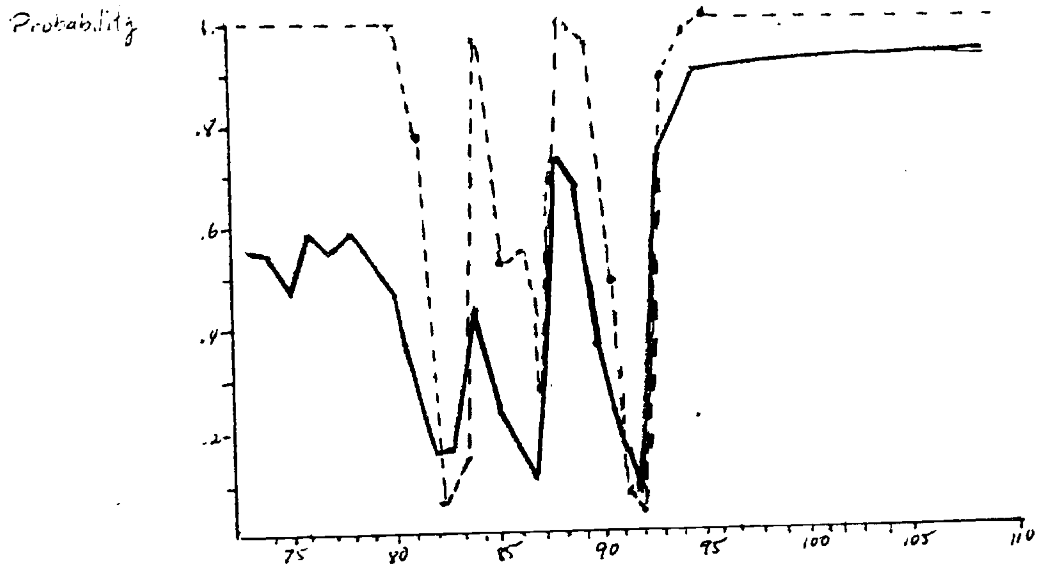
* Pe = price level at end of period
Po = price level at beginning of period

Source: Cagan, Table I

Table II

Probabilities of Process Consistency - Germany (weekly data)

Date	Probability- Includes Fixed Value Money	Probability- Only Reichsbanknotes	Date	Probability- Includes Fixed Value Money	Probability- Only Reichsbanknotes
1923					
May 15	.9999	.9999	Nov 7	.0538	.6480
May 23	.9999	.9999	Nov 15	.0258	.1828
May 31	.9999	.9999	Nov 23	.8922	.8482
June 7	.9999	.9999	Nov 30	.9664	.9888
June 15	.9999	.9999	Dec.7	.9940	.9942
June 23	.9999	.9999	Dec 15	.9953	.9972
June 30	.9999	.9999	Dec 22	.9955	.9972
July 7	.9997	.9997	Dec 31	.9956	.9972
			1924		
July 14	.9999	.9999	Jan 7	.9957	.9974
July 23	.9999	.9999	Jan 15	.9958	-
July 31	.9999	.9999	Jan 23	.9959	-
Aug 7	.9993	.9993	Jan 31	.9960	-
Aug 15	.9998	.9998	Feb 7	.9961	-
Aug 23	.7635	.7635	Feb 15	.9961	-
Aug 31	.0655	.0655	Feb 23	.9962	-
Sept 7	.1323	.1323	Feb 29	.9963	-
Sept 15	.9659	.9783	March 7	.9964	-
Sept 22	.5143	.5662	March 15	.9965	-
Sept 29	.5367	.6454	March 22	.9965	-
Oct 6	.2621	.3346	March 31	.9966	-
Oct 15	.9949	.9969	April 7	.9967	-
Oct 23	.9564	.9696			
Oct 31	.4378	.7112			



Observation Number

(——) - Old Series

(---) - New Series

Figure 1

Probabilities of Process Consistency

Germany

In Figure 1, we reproduce Figure 4 from our previous (1980a) paper; this is simply a diagram of the movement through time of the process consistency probabilities based on the original money series. Superimposed (dashed lines) on Figure 1 is the probability time series computed from the all-inclusive weekly money data. While the timing of the cycles are the same in both cases, the cycles for the all weekly data exhibit a substantially higher amplitude. Observation 92 coincides with November 15, 1923.

c) The Greek Case

We computed the probabilities of process consistency for the Greek inflation from January, 1943 through August, 1946. Data are available at monthly intervals and we used the end of month money series reported in Cleveland and Delivanis (1948). For the first money supply observation we selected July, 1941, the beginning of the Nazi occupation.

An attempt at monetary reform occurred on November 11, 1944; but this reform is generally considered to have been abortive since it was followed by nearly two more years of money stock and price increases. A data problem coincides with this reform attempt because the money stock is reported as of November 11, 1944, rather than as of November 30. Thus, we were forced to interpolate to derive a November 30 observation. We tried two different methods to produce a continuous time series of money observations. In the first method (Series I) a linear interpolation in the double logarithm of the money supply was performed on the November 11 and December 31 observations to yield on observation for November 30. In the second method (Series II) the average monthly β_t's were computed from the observations on October 31 and November 11 and from the observations on

November 11 and December 31; this method is essentially the same as that used in our original German series in that observations taken at varying time intervals are connected in a time series.

We report process consistency probabilities for both series in Table III; of course, the two series are identical prior to November 11, 1944. The $\hat{\alpha}$ used to derive log ψ is that reported by Cagan, $\hat{\alpha} = -4.09$ months.

The process consistency probability for the drachma fluctuates between .55 and .79 from January, 1943 through April, 1944. Starting in June, 1944 it declines steadily, reaching a value near zero by November 30, the moment the reform started. By January, 1945, the probability of process consistency reaches .84, the highest value to date, and begins a steady rise toward unity. Thus, the Greek case is similar to the German case in that the probability of process consistency reaches its lowest value at the moment of the reform. The reform did not succeed in halting the inflation, but apparently it was successful in restoring the public's confidence in the process consistency of the drachma.

In Figure 2, we plot the probabilities of process consistency for the Series I data. Observation 5 corresponds to May, 1943; observation 22 is November, 1944.

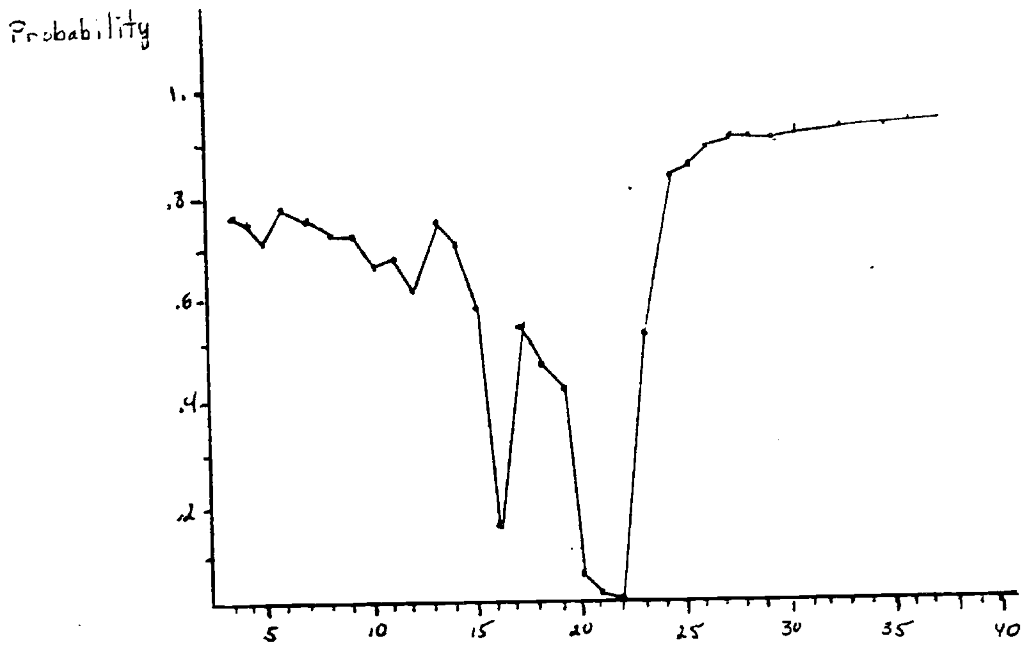
d) The Hungary II Case

The Hungarian II case, like the German case, is complicated by the introduction of a "fixed value" currency during the last months of the inflation. The tax pengo was a demand deposit which was indexed to a cost-of-living price index, although its units were always measured in terms of the nominal pengo. Introduced in January, 1946, it maintained

Table III

Probabilities of Process Consistency - Greece

Date	Probability-Series I	Probability-Series II	Date	Probability-Series I	Probability-Series II
January, 1943	.722	.722	November	$.937 \times 10^{-7}$	$.936 \times 10^{-7}$
February	.771	.771	December	.498	$.921 \times 10^{-8}$
March	.779	.779	January, 1945	.845	.911
April	.748	.748	February	.859	.916
May	.714	.714	March	.887	.929
June	.793	.793	April	.893	.933
July	.780	.780	May	.895	.935
August	.732	.732	June	.901	.939
September	.732	.732	July	.904	.941
October	.730	.730	August	.908	.944
November	.634	.634	September	.910	.946
December	.644	.644	October	.913	.949
January, 1944	.583	.583	November	.916	.951
February	.741	.741	December	.919	.952
March	.710	.710	January, 1946	.921	.954
April	.555	.555	February	.923	.956
May	.149	.149	March	.924	.957
June	.506	.506	April	.928	.959
July	.463	.463	May	.930	.961
August	.383	.383	June	.933	.962
September	.044	.044	July	.935	.964
October	.00022	.00022	August	.937	.965



Observation Number

Figure 2

Probabilities of Process Consistency

Greece

its real value until May. The inflation then became so rapid that the indexation, based on the price index on the day prior to withdrawal, was not sufficient to maintain its real value; and the tax pengo also depreciated rapidly in real terms. Thus, the "fixed value" money became a nominal money in the last months of the inflation. See Nogaro (1948) for a detailed discussion of this inflation.[9]

To account for this difficulty, we again employed two time series for money. The first (Series I) is a series of nominal money only, which, prior to January, 1946, is the sum of paper pengos and demand deposits. From January, 1946 onward, the demand deposits were indexed, so from that date Series I consists only of paper pengos. The second series (Series II) consists of paper pengos plus the nominal value of tax pengos throughout the hyperinflation. Series II seems to be the preferable series, since by the end of July, 1946, the ratio of the nominal value of deposits to the nominal value of notes was on the order of magnitude of 10^{10}, i.e. notes had ceased to be money. With the reform on August 1, 1946, a new currency, the forint was introduced, and the ratio of notes to deposits resumed its pre-tax pengo value, approximately six to one.

In Table IV, we report the money supply figures for the Hungarian II case so that the reader may obtain an idea of the magnitudes involved. The data are the end of month figures. We begin our first money stock observation in June, 1945.[10] Starting in August, 1946, the forint was introduced; it was exchanged with the pengo at a rate of 400 octillion (10^{27}) pengo/forint. We have converted all money totals to pengos.

In Table V we report our computed probabilities of process consistency for both data series. The $\hat{\alpha}$ used to compute log ψ is Cagan's, $\hat{\alpha} = -3.63$ months. The probability series begins with low process consistency

Table IV

Money Supply - Hungary II*

Date	Notes (billion pengo)	Deposits (billion pengo)	Total (billion pengo)	Date	Notes (billion pengo)	Deposits (billion pengo)	Total (billion pengo)
an, 1945	11.1	-		November	$9.37x10^{26}$	$2.45x10^{26}$	$11.82x10^{26}$
ebruary	11.1	-		December	$9.68x10^{26}$	$2.80x10^{26}$	$12.48x10^{26}$
arch	11.1	-		Jan, 1947	$10.17x10^{26}$	$3.05x10^{26}$	$13.22x10^{26}$
pril	11.6	-		February	$10.93x10^{26}$	$3.56x10^{26}$	$14.47x10^{26}$
ay	12.1	-		March	$11.73x10^{26}$	$4.04x10^{26}$	$15.77x10^{26}$
une	14.5	3.2	17.2	April	$12.58x10^{26}$	$4.48x10^{26}$	$17.06x10^{26}$
uly	16.3	3.9	20.2	May	$14.08x10^{26}$	$4.89x10^{26}$	$18.97x10^{26}$
ugust	24.4	4.7	29.1	June	$14.68x10^{26}$	$5.15x10^{26}$	$19.83x10^{26}$
eptember	41.9	6.2	48.1				
ctober	110.	9.6	119.6				
ovember	360.	28.	338.				
ecember	770.	64.	834.				
an, 1946	$1.6x10^{3}$	151.	$1.75x10^{3}$				
ebruary	$5.2x10^{3}$	$1.1x10^{3}$	$6.3x10^{3}$				
arch	$3.4x10^{4}$	$1.2x10^{4}$	$4.6x10^{4}$				
pril	$4.3x10^{5}$	$3.5x10^{5}$	$7.8x10^{5}$				
ay	$6.6x10^{7}$	$1.1x10^{8}$	$1.76x10^{8}$				
une	$6.3x10^{12}$	$1.7x10^{13}$	$2.33x10^{13}$				
uly	$4.7x10^{16}$	$2.4x10^{26}$	$2.4x10^{26}$				
ugust +	$3.56x10^{26}$	$.51x10^{26}$	$4.07x10^{26}$				
eptember	$6.07x10^{26}$	$1.10x10^{26}$	$7.17x10^{26}$				
ctober	$8.43x10^{26}$	$1.172x10^{26}$	$10.15x10^{26}$				

Source: U.N. Monthly Bulletin of Statistics Jan, 1947, No. 1; Dec., 1947, No. 12

+ Data in forints converted to pengo at 400 octillion (10^{27}) pengo/forint

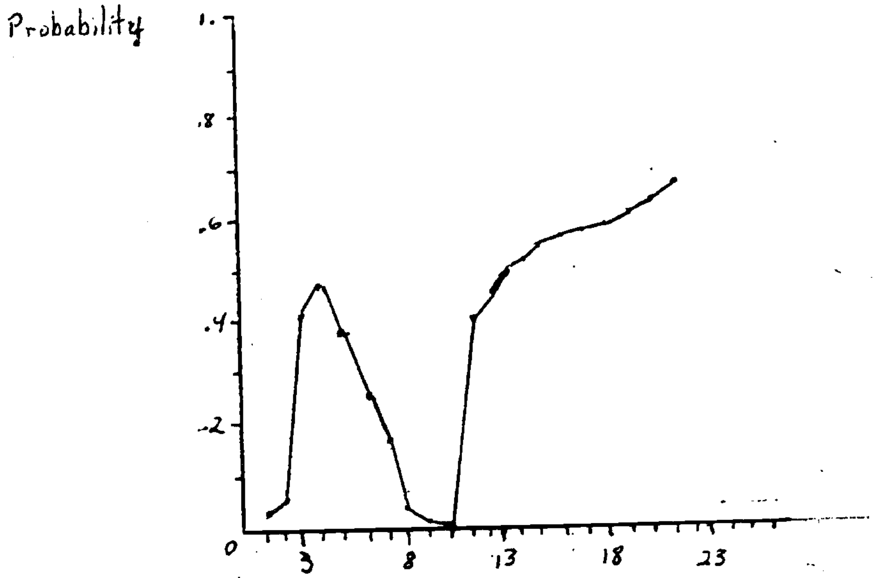
Table V

Probabilities of Process Consistency - Hungary II

Date	Probability Series I	Probability Series II
Nov, 1945	.0322	.0322
Dec.	.0597	.0597
Jan, 1946	.424	.424
Feb.	.485	.461
March	.390	.357
April	.264	.236
May	.203	.157
June	.051	.036
July	.0036	.0028
August	.277	.000058
September	.096	.422
October	.481	.449
November	.502	.474
December	.523	.497
January, 1947	.543	.519
February	.561	.541
March	.579	.561
April	.596	.580
May	.612	.599
June	.628	.616
July	.642	.633

probabilities, which then rise somewhat. This phenomenon results from our starting our money observations in June, 1945; this means that the first few β_t observations, which are increasing, have a strong influence on the posterior p.d.f., causing it to give substantial weight to the non-convergent region. The decline in β_t for November-December, 1945 produces the jump in the process consistency probability. The probabilities then decline until the reform in August, 1946; the Series II process consistency probability reaches its lowest value at the moment of the reform and then rises steadily. We diagram this series in Figure 3; observation three corresponds to January, 1946 while observation 10 corresponds to August, 1946.

The Series I probability reaches its lowest value in July, 1946, rises in August, and declines again in September. This phenomenon can be interpreted from the data in Table IV. Recalling that the process consistency probability for a given observation is computed using money data through the prior observation, we note that the ratio of pengos in June, 1946 to pengos in May, 1946 was an order of magnitude greater than the July-June ratio. The β_t observation for July would then be lower than that for June causing a rise in the process consistency calculation for August. The decline in the probability calculation for September can be explained by the August reform. The July, 1946 money data indicate that the pengo notes had become an insubstantial part of the money stock by that date. The reform on August 1 restored a more normal ratio of notes to deposits (in terms of forints); when the forints are converted to pengos in August, notes appear to have risen by a factor of 10^{10} over July, thereby causing a fall in the process consistency probability measured for September. Obviously, the results for Series I must be treated gingerly.



Observation Number

Figure 3

Probabilities of Process Consistency

Hungary II

e) The Russian Case

As in the German and Hungarian II episodes, the Russian experience is complicated by the introduction of a fixed value currency, the chevronetz, prior to the monetary reform. The reform can be dated around February-March, 1924; the chevrontsi started to circulate in January, 1923. We computed probabilities of process consistency both for the money series consisting of paper rubles alone (Series I) and for the series consisting of paper rubles plus the paper value of the chevronetz circulation (Series II). The data for the quantities of money in circulation and the market exchange rates are available in Katzenellenbaum (1925).

We report the results of these computations in Table VI. The starting date for the money supply series was November, 1917, the beginning of the Bolshevik revolution. The $\hat{\alpha}$ used to compute log ψ was Cagan's $\hat{\alpha} = -3.06$ months. The computed probabilities of process consistency do not approach the low levels reached by the more extreme inflations. The lowest level attained was .526 in February, 1922.[11] There is a fairly substantial decline in the probabilities near the end of the inflation (to .777 for the paper ruble series in December, 1923), but the reform occurred two or three months later.

f) The Polish Case

To study the Polish case, we used a money supply series observed at ten-day intervals. The monetary aggregate (central bank notes in circulation) is the same as that usually used for this case, but the time between observations has been one month. We obtained our data from weekly issues of Economist published contemporaneously with the Polish inflation; starting on August 31, 1922, the money stock was regularly sampled at ten day intervals through the inflation's end in February, 1924.

Table VI

Probabilities of Process Consistency - Russia

Date	Prob. - paper rubles only	Prob. - paper rubles & chevrontsi	Date	Prob.- paper rubles only	Prob. - paper rubles and chevrontsi
April 1, 1921	.996	-	Feb.	.947	.942
May 1	.997	-	March	.952	.946
June 1	.997	-	April	.961	.956
July 1	.997	-	May	.947	.936
Aug. 1	.998	-	June	.951	.933
Sept. 1	.995	-	July	.968	.961
Oct. 1	.990	-	Aug.	.961	.941
Nov. 1	.985	-	Sept.	.954	.898
Dec. 1	.938	-	Oct.	.958	.827
Jan. 1, 1922	.869	-	Nov.	.957	.782
Feb. 1	.526	-	Dec.	.777	.865
March 1	.727	-	Jan. 1, 1924	.920	.873
April 1	.797	-	Feb.	.926	.850
May 1	.771	-	March	.920	.749
June 1	.859	-	April	.804	.793
July 1	.806	-	May	.987	.976
Aug. 1	.902	-	June	.988	.979
Sept. 1	.910	-			
Oct. 1	.915	-			
Nov. 1	.948	-			
Dec. 1	.946	-			
Jan. 1, 1923	.960	-			

Table VII contains the probabilities of process consistency for Poland. Once again Cagan's $\hat{\alpha}$ = - 2.30 months, altered to conform to the ten day time interval, was employed in computing log ψ. The lowest process consistency probability, .927, occurs on December 10, 1923, two months prior to the reform.

g) The Austrian and Hungarian I Cases

For the Austrian and Hungarian I inflations, we used money supply series observed at weekly intervals instead of the monthly series used by other researchers. Weekly data for the Austrian crown are available in Walres des Bordes (1924); we chose January 7, 1920 for our first observation on the Austrian money supply. For Hungary, weekly data exists in Economist's weekly reports on central bank balance sheets in contemporary volumes; we selected December 31, 1921 as our first observation on the Hungarian money stock. For Austria, the reform occurred in September, 1922; for Hungary, the reform can be dated in February, 1924.

Cagan's $\hat{\alpha}$'s, altered to account for the weekly observation period, were employed; for Austria, $\hat{\alpha}$ = - 8.55 months while for Hungary, $\hat{\alpha}$ = - 8.70 months. The Austrian probability of process consistency was computed for each week in the period March 15, 1922 to December 15, 1922; for all observations the probability of process consistency is at least .9999. For Hungary, the probabilities were computed for each week in the period March 15, 1923 to March 31, 1924; for all but two observations, the probability of process consistency was at least .99. For two weeks August 23 and August 31, 1923, the probability fell to .983.

Table VII

Probabilities of Process Consistency - Poland

Date	Prob.	Date	Prob.
July 20, 1923	.989	Feb. 29	.992
July 31	.989	March 10	.990
Aug. 10	.986	March 20	.994
Aug. 20	.989	March 31	.994
Aug. 31	.989	April 10	.995
Sept. 10	.975	April 20	.996
Sept. 20	.987	April 27	.996
Sept. 30	.988		
Oct. 10	.974		
Oct. 20	.986		
Oct. 31	.981		
Nov. 10	.932		
Nov. 20	.980		
Nov. 30	.965		
Dec. 10	.927		
Dec. 20	.983		
Dec. 31	.969		
Jan. 10, 1924	.942		
Jan. 20	.989		
Jan. 31	.983		
Feb. 10	.940		
Feb. 20	.989		

II. Implications for Behavior at Infinity

The time series of process consistency probabilities presented in section I are a set of data which we constructed. The computation of these numbers is based on a simple theory of money demand and supply in severe hyperinflation; and the theory seems to derive some support from the probabilities' attaining their lowest values at the moment of reform. This support depends on only three observations, so to push the results into other theoretical domains may seem brash. However, these episodes are as close to infinity as any observable monetary economy has ever approached. Therefore, we propose to employ them as the only available evidence of limiting behavior in a monetary economy.

Our results concern the movement of prices to infinity in finite time as the rate of money creation expands; somehow process inconsistency forces a monetary reform. However, in the context of monetary models with explicit optimizing behavior, transversality conditions or conditions on utility functions are imposed which prevent real balances from reaching zero in finite time (or, in some cases, from approaching zero in the limit) regardless of the rate of money growth. The purpose of these transversality conditions is to force unique solutions for equilibrium price sequences, but they are strong enough to preclude not only embarrassing multiple price solutions but also process inconsistency.

Propositions in economics about behavior at infinity usually do not suffer from the indignity of being confronted by evidence; in fact, our data have a very limited scope for distinguishing the appropriateness of the transversality conditions. However small the scope, in this section we explore the use of our results as such evidence since other means of confronting these propositions are not readily available. First, we discuss the nature of the transversality conditions and briefly review the literature in which they appear. Next, we discuss how our results may be used as evidence for or against these hypotheses. We warn the reader at this point that only the results of the German case seem capable of distinguishing whether or not process inconsistency can occur; even in that case, the distinction is not manifest.

a) Transversality Conditions

In simple models of money demand, like that in equation 1, the possibility of multiple, explosive price solutions arises. In developing price solutions in such models, a transversality condition is often imposed to yield a unique solution. For instance, Sargent and Wallace (1973, p. 331) impose the condition

$$\lim_{n \to \infty} \psi^{-(n - 1)} \underset{t}{E} x_{t+n} = 0$$

in solving for the current inflation rate. Here $\psi = \frac{\alpha - 1}{\alpha}$ as in (4b) and $\underset{t}{E}x_{t+n}$ is the current expectation of the inflation rate n periods from now. In Flood and Garber (1980b, p. 762-3) we explicitly show that this restriction on the growth rate of anticipated inflation rates is strong enough to exclude multiple solutions and to restrict attention to money supply processes which are not too explosive; money demand behavior does not preclude process inconsistency, however.

Dissatisfied with these mechanical rules to determine money demand, theorists in the last decade have proposed somewhat less mechanical models to determine money demand through optimizing behavior in a dynamic framework.

Brock (1974) accounted for money demand by placing real balances in the utility function. Each period, the typical agent in Brock's model seeks to maximize (in Brock's notation)

$$\sum_{t=1}^{\infty} \beta^t[u(c_t) + v(m_t) + b(l_t)]$$

subject to a budget constraint. The current period utility function is additively separable in consumption, c_t, real balances, m_t, and labor supply, l_t. The restriction (p. 753) that $v'(0) = +\infty$ is strong enough to rule out price solutions in which real balances approach zero; in essence, the possession of real balances becomes so urgent when m_t is small that agents will hold an amount of m_t, bounded away from zero, even when the rate of money creation approaches infinity.[12/] Aimed at preventing multiple price solutions, Brock's condition implies money demand behavior for which all money supply processes are process consistent since the price level cannot be infinite in finite time.

Rather than by placing money directly in the utility function, other theorists have sought to explain money demand by placing explicit payment or saving technologies in models with explicitly optimizing agents. The overlapping generations models, e.g. Samuelson (1958), Lucas (1972), Wallace (1980), produce a demand for money because money serves to transfer consumption from one period of life to another. These models do not rule out multiple price solutions in which prices may rise to infinity.

Wallace does not consider this "tenuousness" of money to be a problem; as in Lucas (1972) he is satisfied (p. 55) to assume away multiple solutions by assuming that current prices are functions only of the current state of the economy, where the current state does not include time. This turns out to be a weaker requirement than is imposed by assuming some transversality condition, since the assumption that the current prices is a function of the state of the economy does not preclude process inconsistency.[13/]

However, other researchers, desiring to close off the possibility of multiple solutions by explicit behavioral assumptions, have suggested the use of transversality-type conditions in the overlapping generations model. Scheinkman (1980) proposes to attach a transversality condition of a type first used by Brock (1978) to overlapping generations models. If the young person's utility function is $u(c_1, c_2) = u(c_1) + \delta u(c_2)$, where c_i is consumption in the ith period of life (Scheinkman's notation), then

$$\lim_{x \to 0} xu'(x) > 0$$

is sufficient to produce a unique price solution (see Scheinkman, pp. 95 - 96). The reasoning here is that consumption is urgent enough at low levels that the marginal utility of consumption approaches infinity at least at the same rate that x (or real balances) approaches zero. Scheinkman demonstrates that this condition is equivalent to $\lim_{\mu \to \infty} \mu x_\mu > 0$, where μ is the rate of growth of money and real balances, x_μ, are a function of the rate of money growth; this prevents the price level from being infinite in finite time.

Brock and Scheinkman (1980, p.229) argue that any device which prevents real balances from falling below some arbitrarily small $\varepsilon > 0$ excludes the possibility of multiple solutions with the price level rising to infinity. As alternatives to the transversality conditions, such devices may enter through the introduction of some tax payable in money or through some payments technology which cannot speed up sufficiently for available real balances to implement trade in all available real goods. The first argument is a theme developed by Starr (1980, 1974), but it does not seem to hold in a continuous-time model since money is a stock and taxes are flows.[14/] Lucas' (1980) model, an example of a transactions technology which sets a floor on real balances, also depends on a fixed payments period. Since, as Barro (1970) demonstrated, payments accelerate in a hyperinflation, Lucas' model with an endogenous payments interval added to it would not rule out multiple price paths in the absence of a transversality condition. Also, Scheinkman (1980) shows that adding some auxiliary barter capacity to Lucas' model reestablishes the possibility of multiple soultions. It seems that these $\varepsilon > 0$ conditions are merely step-children of the transversality assumptions. In any case, to assume $\varepsilon > 0$ for any rate of money growth is to exclude the possibility of process inconsistency.

b) Process Inconsistency vs. Transversality Conditions

Since the transversality conditions or $\varepsilon > 0$ conditions imply that no money supply process can be inconsistent, they also imply that computations of process consistency probabilities should be related to the advent of monetary reform only by coincidence. Since in all three cases of extreme hyperinflation the probabilities of process consistency reach their lowest values at the moment of the reform, one is tempted to conclude that more than a chance association is involved and to allocate these results as the first entries in the "empirical evidence against" column of the scoresheets for optimizing monetary theories.

That such a conclusion is premature lies in our neglect of the political mechanism that forces a reform. In our (1980a) model of monetary reform we postulated an "amorphous political mechanism" which transforms a high probability of process inconsistency into a rationally high subjective probability of reform. We interpreted the simultaneity of thé German reform with the lowest process consistency probability as evidence in favor or our theory. However, we cannot count the result as evidence against monetary theories which preclude process inconsistency unless we can show that they predict a different result. To do this we must attach to them some political mechanism which somehow forces a reform and determine if the mechanism should force a reform at a different time from our "amorphous mechanism".

The urgency of consumption and the inability to transact or pay taxes when real balances are low all suggest a political mechanism of reform which is triggered into action when real balances fall to some critical level because of the central bank's money creation process. It seems reasonable to associate the timing of reform with the date at which real balances reach their lowest levels. Indeed, in the Hungarian and Greek cases, reform occurs just after the lowest level of real balances is attained,[15/] as reference to Table VIII will indicate. Since, prior to the reform dates in these two inflations, money grew at ever more rapid rates, our series of process consistency probabilities should decline steadily and also reach its lowest value on the reform date simply as a result of our computation method. Therefore, the coincidence of the reform dates with the low points in our process consistency probability series should not be a surprise, even though process consistency has no operational meaning. We conclude that the data from the Greek and Hungary II cases do not distinguish between these two theories of monetary reform and certainly cannot serve as evidence against the transversality condition.

Table VIII

Real Balances - Extreme Hyperinflations

Germany

Date	Reichsbanknotes only (million goldmark)	All Money (million goldmark)
July 14, 1923	572.3	-
July 23	381.7	-
July 31	166.4	-
August 7	79.3	-
August 15	181.0	-
August 23	226.4	-
August 31	270.3	-
Sept. 7	93.6	-
Sept. 15	147.8	-
Sept. 22	329.3	338.7
Sept. 29	740.7	751.8
Oct. 6	328.4	337.4
Oct. 15	137.7	141.1
Oct. 23	39.3	47.6
Oct. 31	144.6	300.3
Nov. 7	127.7	373.8
Nov. 15	154.7	458.7
Nov. 23	223.9	941.3
Nov. 30	400.2	1487.8
Dec. 7	390.0	1810.7
Dec. 15	414.2	1958.5
Dec. 22	474.6	2131.1
Dec. 31	496.5	2273.6

Source: Statistiches Reichsamt, 1925 pp. 47-49.

Greece

Date	Real Balances June 1941 = 120
Jan, 1944	15.9
Feb.	13.5
March	10.6
April	11.5
May	8.27
June	10.1
July	5.36
August	5.03
Sept.	3.31
Oct.	3.50
Nov. 11	.33
Dec.	2.21
Jan. 1946	8.9
Feb.	15.0
March	20.0
April	32.5
May	28.3
June	54.1
July	56.1
August	40.5

Source: Cleveland and Delivanis, Statistical Appendices

Table VIII (continued)

Hungary II*

Date	$\log_{10}$ [Real Balances](December, 1939 = 1
July, 1945	-2.08
August	-2.14
September	-2.26
October	-2.69
November	-2.9
December	-3.07
January, 1946	-2.97
February	-3.2
March	-2.98
April	-3.03
May	-3.17
June	-2.98
July	-4.59

*Source: Cagan (1956), p. 110, Table B9.

The German case is somewhat different; for the money series with Reichsbanknotes alone, real balances declined until October 23, 1923 and rose substantially in November, 1923, the month of the reform. For the data which includes all monies, the low point is again October with substantial increases in real balances in November. Process consistency probabilities reach the lowest point at the moment of the reform (for the data with all monies). The simple reform mechanism based on levels of real balances should have forced the reform a month prior to its actual occurrence. The reform mechanism based on process consistency probabilities places the reform at the correct moment (for one of the German money series). This result seems to distinguish between the two hypotheses and to provide evidence against the transversality - ($\varepsilon > 0$) conditions. However, the distinction is weak since one can readily postulate a lag in the implementation of the reform, which, though caused by the dearth of real balances in October, did not take effect until November.[16]

III. Conclusion

In this study we have applied our technique for measuring process consistency probabilities to all the hyperinflationary episodes examined by Cagan. As in our previous work, we find that for the severe hyperinflations the process consistency probabilities attain their lowest values at the times of the monetary reforms; however, the phenomena observed at the ends of these inflations easily can be interpreted as evidence which supports monetary theories whose assumptions preclude any operational meaning for process inconsistency. Only in the German case do the data distinguish between these two types of theories. For obvious reasons, we lean toward interpreting the German result as evidence in favor of the possibility that process inconsistency may materialize and against those monetary theories in which transversality conditions or other similar devices exclude the possibility. Even the German evidence seems weak, so we are wary of making too strong a case for its powers of distinction. However, since it appears to us that there exist no other means of empirically confronting this array of constraints attached to optimizing models of money, we present our results, because it is possible that they may alter some priors, though we are not sure of the direction of change.

Appendix

In this appendix we describe the method which we used to integrate numerically the p.d.f. in (3) over the region in (4). The function $H(\theta, Z)$ is bi-modal; one mode occurs in a region where $\theta > 1$ and one is in a region where $\theta < 1$. The function has properties such that for given $\bar{Z}$, $\lim_{\theta \to +1} H(\theta, \bar{Z}) = 0$ and $\lim_{\theta \to -1} H(\theta, \bar{Z}) = 0$. For given $\bar{\theta}$, $\lim_{Z \to +0} H(\bar{\theta}, Z) = 0$. Along the Z-axis, for given $\bar{\theta}$, there is a single inflection point. In the θ-direction for $\theta < 1$ and given $\bar{Z}$, there is a single inflection point; similarly, for $\theta > 1$ and given $\bar{Z}$, there is a single inflection point. The function is diagrammed in Figure 4. Point A is an inflection point of the function for given $\bar{\theta}$; B is an inflection point for given $\bar{Z}$; and C and D are the two modal values.

We integrated this p.d.f. over 36 contiguous regions in the (θ, Z) plane; the probability weights outside of those regions were never significant. The regions were determined as follows. The θ-axis between .6 and 1.3 was divided into twelve segments, six on either side of θ-axis between .6 and 1.3 was on the θ-axis which define the division were .6, .7, .8, .9, .95, 975, 1.00, 1.01, 1.03, 1.05, 1.075, 1.15, and 1.3. For a given segment of the θ-axis, the Z-axis was divided into three segments. The position of these segments depended on the average over the θ-segment of the p.d.f.'s Z-direction inflection points. For instance, the first θ segment, between .6 and .7, was subdivided into 25 subsegments for the numerical integration; for the endpoint of each subsegment there is a value of Z which locates the inflection point of the p.d.f. Define Z* as the average of these Z's across the given θ segment. Then for the θ segment between .6 and .7 the Z-axis was segmented from zero to Z*, from Z* to log ψ, and from log ψ .5. If, by chance,

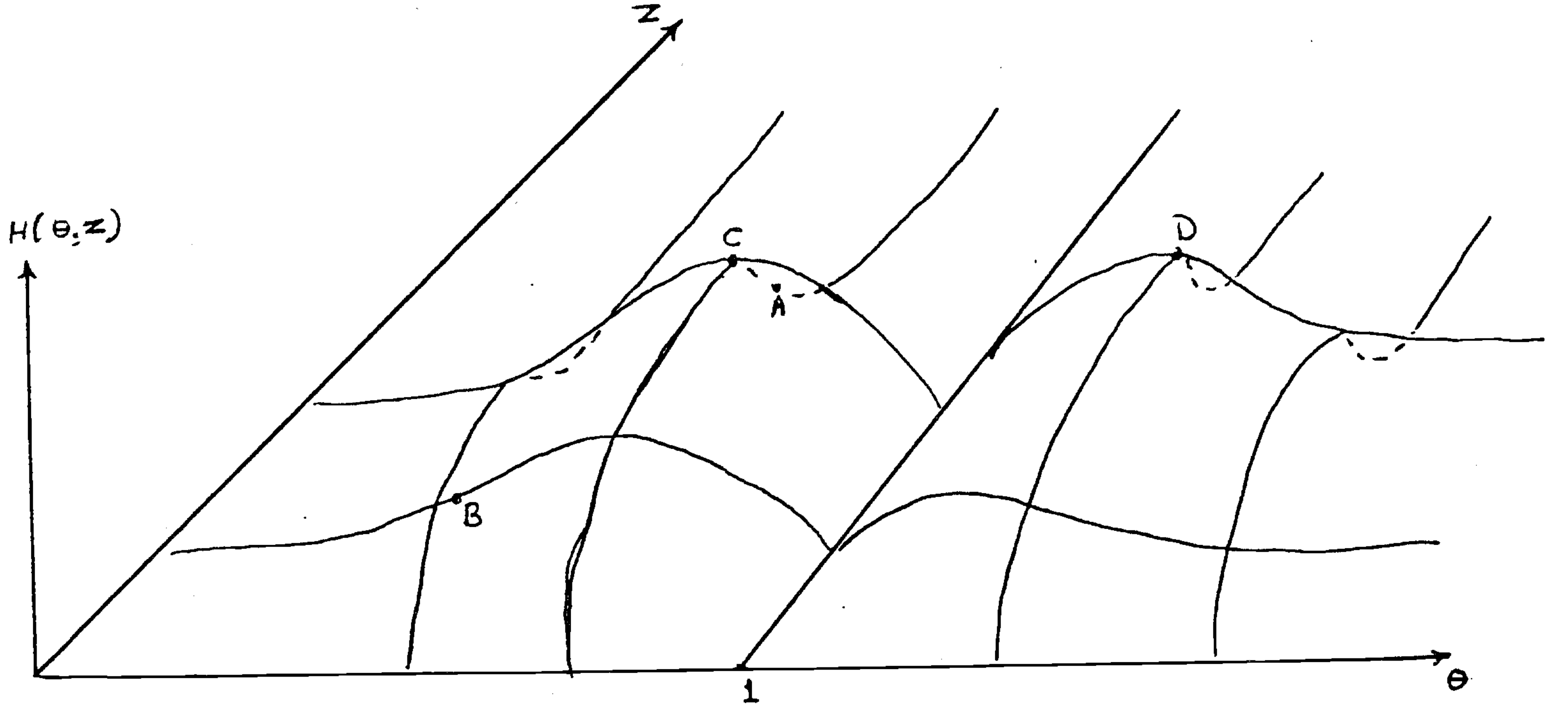


Figure 4

Diagram of $H(\theta, Z)$

$Z^* > \log \psi$, then the segments were zero to $\log \psi$, $\log \psi$ to Z^*, and Z^* to .5.

The regions of integration for a given observation in a given hyperinflation might then appear as in Figure 5. Region 1 is the rectangle defined by $[.6 \leq \theta \leq .7,\ 0 \leq Z \leq Z_1^*]$, where Z_1^* is the average of the inflection points for the $.6 \leq \theta \leq .7$ segment. Region 2 is the rectangle $[.6 \leq \theta \leq .7,\ Z_1^* \leq Z \leq \log \psi]$; Region 3 is the rectangle $[.6 \leq \theta \leq .7,\ \log \psi \leq Z \leq .5]$; region 4 is the rectangle $[.7 \leq \theta \leq .8,\ 0 \leq Z \leq Z_2^*]$, etc. For purposes of the numerical integration, each rectangle, except the two rectangles containing the modal values of (θ, Z), was subdivided by splitting its sides into 25 subsegments; thus, the p.d.f. was evaluated at 625 points in each region. The sides of the modal rectangles were divided into 50 subsegments, so in these two regions the p.d.f. was evaluated at 2500 points.

We took this care in setting our regions and grid sizes because of the nature of the p.d.f. Typically, $H(\theta, Z)$ produced a very narrow ridge running in the θ direction; the probability weight was so concentrated, that it was easy to miss significant portions of it by making the mesh of our grids too large. Using the inflection points of the p.d.f. assured that we would catch most of the probability weight. The same reasoning lay behind our selecting a finer mesh in the modal regions.

To carry out the integration, we used Simpson's rule twice. See Zellner (1971, Appendix C) for a description of this method.

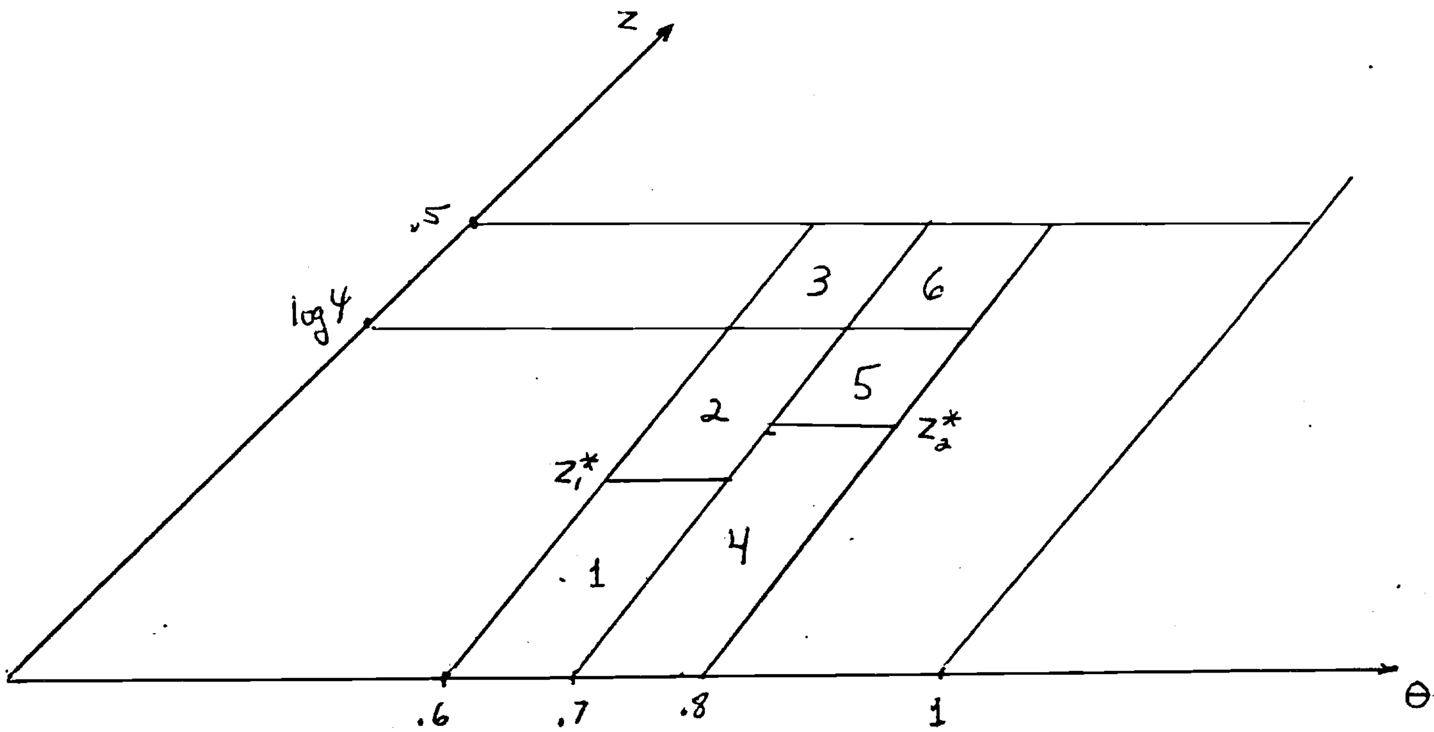


Figure 5

Regions of Integration

Notes

1/ Less emphasis has been given to the role of hyperinflation as part of the continuous economic process, producing effects which may still manifest themselves long after the monetary reform.

2/ In addition, our model is capable of removing the discontinuity between an extreme hyperinflation and its post-reform era.

3/ Some of our previous work involving the German case has been criticized on the grounds that hyperinflation data contains massive measurement errors. We find this critique hard to take seriously. A relevant measure of the importance of measurement error in data might be the ratio of variance induced by measurement error to total variance. A measure of unity would indicate worthless data. Because total variance in the data is so astoundingly huge, we feel, as did Cagan (1956, pp. 47-51), that measurement error is not a serious problem.

4/ We assumed that no current information is used by agents forming inflation expectations because we wanted a simple method to capture agents' inability to observe current money currently. A more complex alternative would involve agents observing prices currently and forming an estimate of current money from that price information. The imposition of this additional complication is not pursued. If we did pursue this complication our results would be a weighted average of our present results and the results assuming agents to have full current information about money and prices. The full current information results may be obtained from those presented below by pushing all of our probabilities back in time one observation interval.

5/ Cagan suggested that this phenomenon may have been due to agents' anticipation of monetary reform. If this suggestion is correct, then agents in the extreme inflations must have had a reason to believe in impending reform while agents in the mild inflations had no such reason.

The need to exclude the last two months of the Polish data as outliers may have arisen in Cagan's empirical technique; Barro (1970) managed to incorporate the entire Polish data set in estimating his model.

6/ The Reichsbanknotes were the currency which drove the hyperinflation. While other currencies constituted a third of the money supply in 1918, they remained relatively stable during the following years, so that by the end of 1922 they represented only 1.5 percent of the money stock.

7/ Cagan felt that the "fixed value" currencies should be left out of the money totals since they were not depreciating in terms of goods. However, these currencies had no backing; they were promises to pay in some asset denominated in gold which would be available at some point in the future.

8/ One reason that this change may have occurred is that use of a weekly series quadruples the number of pre-1923 observations from 50 to 200; any given 1923 observation would then produce less of an alteration in the posterior p.d.f. However, beginning in August the observations on β_t deviated substantially enough from the predictable values that the posterior p.d.f.'s (at least in terms of the probabilities associated with the convergent regions) were essentially the same.

9/ Bomberger and Makinen (1980) have recently reiterated Nogaro's observations in a more modern idiom.

10/ The Russian invasion of Hungary occurred in the early months of 1945, so this is a reasonable time to start with diffuse priors. Before 1945, there was a substantial money creation, but with the Russian invasion Hungarian Nazis ran off with the currency reserves, so there was little money creation before June, 1945.

11/ There was an attempt to reform the currency in early 1922 by replacing the old ruble with a new ruble. Table 1 indicates the degree of success of this reform.

12/ For details of this result the discussion of Figures 2.1 and 3.1 in Brock (1974), p. 756 and p. 759, is helpful. Brock requires still stronger restrictions on $v(m_t)$ to preclude price solutions in which the nominal price of goods converges to zero, but this need not concern us here.

13/ This is true provided that the equilibrium price function is selected from the proper domain. Lucas (1972) restricts the domain so that the price solution cannot be infinite in finite time in order to fit the assumptions of the Banach fix-point theorem required in his existence proof. He shows that a finite solution will always exist regardless of the anticipated growth rate of money. In the context of Lucas' model, in which money is transferred to agents according to their money holdings, the transfer offsets any anticipated depreciation in real value due to inflationary money creation. Process inconsistency cannot arise as an issue in such a case. In a model in which money is transferred randomly to agents, process inconsistency may be possible. In that case, restricting the price solution to a domain which bounds real balances away from zero may lead to a unique solution which makes no economic sense.

14/ Assuming that real balances never fall below ε implies that the government can extract arbitrarily large amounts of real resources by increasing the rate of money creation; at some point, the government will be unable to extract the direct tax ε regardless of the enforcement measures it may take because it will already be extracting most income and wealth indirectly through this inflation tax.

The tax argument also does not seem to have much empirical relevance. For instance in the German case, real taxes collapsed as inflation rates accelerated; thus, the rise in prices seems to force the ε of real taxes payable to the government in cash to collapse toward zero. We reproduce here a table from Bresciani-Turroni indicating the sequence of tax revenue during the German hyperinflation.

15/ Of course, we must ignore the large quantities of foreign currencies which circulated as money in each of the severe hyperinflations. We have also neglected the effects of this replacement of domestic by foreign currency on the α parameter in computing our process consistency probability series; if α were actually larger than we assume, than the probability of process inconsistency would also be larger.

16/ However, the revolutions and social unrest of November, 1923 then become difficult to explain (see Flood and Garber (1980a), footnote). The legal framework establishing the Rentenbank, the vehicle of reform, was passed in the middle of October, 1923 (See Bresciani-Turroni, pp. 334-348).

German Tax Revenue

1921	Government Tax Revenue (Goldmarks)
April	352.3
May	411.8
June	350.3
July	304.8
August	256.3
September	196.5
October	173.0
November	112.5
December	175.3
January	192.6
February	194.2
March	207.8
	2,927.4
1922	**Income**
April	190.3
May	254.9
June	235.1
July	183.4
August	116.4
September	90.8
October	66.2
November	60.6
December	73.4
January	65.9
February	50.8
March	100.3
	1,488.1
1923*	**Income**
April	150.6
May	123.3
June	48.2
July	48.3
August	78.1
September	55.6
October	14.5
	518.6

Source: Bresciani-Turroni, Appendix, Table I.

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