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THE REAL INTEREST RATE: AN EMPIRICAL INVESTIGATION

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ABSTRACT

This paper is an empirical exploration of real interest rate movements in the United States over the last fifty years. It focuses on several questions which have repeatedly arisen in the literature. How valid is the hypothesis that the real rate of interest is constant? Does the real rate decline with increases in expected inflation? Are cyclical movements in real variables correlated with real rate movements? How reliable is the Fisher effect where nominal interest rates reflect changes in expected inflation? Does monetary policy affect the real return to saving, with the resulting non-neutral effect on capital formation and productivity? What kind of variation in real interest rates have we experienced in the last fifty years? Have real rates turned negative in the 1970s as is commonly believed, and were they unusually high in the initial stages of the Great Depression?

In pursuing these questions, this paper outlines the methodology and theory used in the empirical analysis. The results indicate that contrary to Fama's finding, there are significant movements in the real rate in both the prewar and postwar period. In particular, there is a significant negative correlation between inflation and the real interest rate, and real rates appear to have been unusually high during the contraction phase of the Great Depression, while unusually low in the high inflation 1970s. The results do not pick up significant correlations between real interest rates and any of the real variables tested. However, the failure to find these correlations is likely to be the result of small cyclical variation of real rates rather than the absence of relationships between real variables and real rates.

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I. INTRODUCTION

Knowledge of how real interest rates move over time is critical to our understanding of macroeconomics. Movements in the real rate are central to the discussion of the transmission mechanisms of monetary policy in the standard ISLM paradigm as well as in modern macroeconometric models.[1] Monetary policy is viewed as affecting the real rate of interest which then affects business and consumers' investment decisions and hence aggregate demand. Real interest rates also play a prominent role in explanations of business cycles and particular business cycle episodes. For example, the apparently unusually high real rates in the early years of the Great Depression is frequently cited as a major factor in that business cycle downturn.[2] The impact of real interest rates on savings-consumption decisions has also been the subject of recent work.[3] If saving responds negatively to the real return on saving, as some have claimed,[4] then declines in the real interest rate can have an adverse effect on capital formation and hence on productivity, a serious concern of policymakers. Clearly, the real interest rate deserves careful study.

This paper is an empirical exploration of real interest rate movements in the United States over the last fifty years. It focuses on several questions which have repeatedly arisen in the literature. How valid is the hypothesis associated with Fama (1975) that the real rate of interest is constant? Does the real rate decline with increased inflation? Are cyclical movements in real variables correlated with real rate movements? How reliable is the Fisher (1930) effect where nominal interest rates reflect changes in expected inflation? What kind of variation in real interest rates have we experienced in the last fifty years?

Have real rates turned negative in the 1970s as is commonly believed, and were they unusually high in the initial stages of the Great Depression?

In pursuing these questions, this paper first outlines in Section II the methodology and theory used in the empirical analysis. The empirical results then follow in Section III, and a final section contains the concluding remarks.

II. THE METHODOLOGY

The real rate of interest for a one-period bond is defined from the Fisher (1930) equation

$$(1) \qquad i_t \equiv rr_t + \pi_t^e$$

where,

i_t = the nominal interest rate earned on a one-period bond maturing at time t--i.e., it is the nominal return from holding the one-period bond from t-1 to t.

π_t^e = the rate of inflation from t-1 to t expected by the bond market at time t-1.

rr_t = the one-period real rate of interest expected by the bond market at time t-1 for the bond maturing at time t.

Hence, the real interest rate, rr_t, is just the difference between the nominal rate of interest and the expected rate of inflation:[5] it is the real return from holding the one-period bond from t-1 to t which is expected at time t-1. Because the real rate is a return expected at the beginning of the period, it is also frequently referred to as the ex ante real rate. This more precise terminology is used to differentiate it from what is termed the ex post real rate: this is the actual real return from holding the one-period bond from t-1 to t. It equals the nominal interest rate minus the actual inflation rate from t-1 to t and can be written as

$$(2) \qquad eprr_t \equiv i_t - \pi_t \equiv rr_t - (\pi_t - \pi_t^e)$$

where,

$eprr_t$ = the one-period ex post real rate for the bond maturing at time t,

π_t = the actual inflation rate from t-1 to t.

Note that the above equations do not allow for taxes. This issue is deferred to a later section of the paper. Note also that for expositional convenience, the ex ante real rate is always referred to as the real rate throughout this paper, while the ex post real rate always refers to the variable defined in (2).

One approach to analyzing the movements in the real rate is to calculate them by using a survey measure of inflation expectations, such as the Livingston data, which is then subtracted from a nominal interest rate. The resulting survey-based measure of the real rate can then be studied directly, for example, by calculating its correlation with relevant variables.[6] The problem with this approach is that it is only as good as the survey measure of inflation expectations, and there are serious doubts as to the quality of these data.[7] As a result, a different methodology is used in the analysis here which involves correlation and ordinary least squares (OLS) regressions with the ex post real rate.

The underlying assumption behind this analysis is the rationality of inflation expectations in the bond market, which implies the following condition:

(3) $E(\pi_t - \pi_t^e | \phi_{t-1}) = 0$

where,

ϕ_{t-1} = information available at time t-1.

This tells us that the forecast error of inflation must be uncorrelated with past available information. There is a large body of evidence that supports this rationality, or equivalently the efficiency of financial markets.[8] Furthermore, tests more specifically directed at the rationality of inflation forecasts in the bond market also support the use of this assumption over long sample periods such as are used here.[9]

If the real rate determined at t-1, rr_t, is correlated with variables, X_{t-1}, which are elements of the available information set ϕ_{t-1}, then we can write,

(4) $rr_t = X_{t-1}\beta + u_t$.

Note that the error term, u_t, is also determined at t-1. Substituting (4) into (2) and writing the inflation forecast error, $\pi_t^e - \pi_t$, as ε_t we get:

(5) $eprr_t = X_{t-1}\beta + u_t - \varepsilon_t$

where

$\varepsilon_t = \pi_t - \pi_t^e$.

Since data on the ex post real rate, $eprr_t$, are observable, in contrast with data on the real rate, equation (5) can be estimated. The question arises: what will be the relationship between OLS estimates from (5), with what would be obtained from (4) if it were estimable? To answer this question, several propositions are demonstrated for the simplest case where the X-variables are non-stochastic. With the appropriate assumptions,[10] corresponding asymptotic results can be generated for the case of stochastic X's, but this additional complexity is avoided here.

The first proposition is that the OLS estimates of β from (4) and (5) are equal in expectation for the non-stochastic X case (and with the appropriate assumption[11] they will be equal in the probability limit in the stochastic X-case).[12] This proposition is derived as follows. Denote the OLS estimate of β in (4) as $\hat{\beta}_{rr}$ and in (5) as $\hat{\beta}_{eprr}$. Then

(6) $$E(\hat{\beta}_{rr}) = E((X'X)^{-1}X'[X\beta + u]) = \beta + E[(X'X)^{-1}X'u]$$

and

(7) $$E(\hat{\beta}_{eprr}) = E[(X'X)^{-1}X'(X\beta + u - \varepsilon)] = \beta + E[(X'X)^{-1}X'u] - E[(X'X)^{-1}X'\varepsilon]$$
$$= E(\hat{\beta}_{rr}) - E[(X'X)^{-1}X'\varepsilon] = E(\hat{\beta}_{rr})$$

because the rationality of inflation expectations in (3) implies that $E(X'\varepsilon) = 0$. Note that this proposition has been demonstrated without an assumption on the properties of u. Thus it is valid even if $\hat{\beta}_{rr}$ is a biased (or inconsistent) estimate of β.

Thus this result tells us that although we cannot observe the real rate, rr_t, we can infer information about its relationship to variables known at time t-1 through OLS regressions with the ex post real rate. Note that any problems of inconsistency that might arise in an OLS estimate of (4) because u_t is correlated with X_t will also exist for the ex post real rate regression. Thus without the knowledge that (4) is exogenous, we cannot interpret OLS results as containing information on causation. They can only provide information on movements in the X-variables.

We will also be concerned with the variance-covariance matrix of the two estimates. Assume that we have a particulary well-behaved problem where the OLS β-estimates are consistent and serial correlation of either the error term in (4) or the composite error term in (5), $u_t - \varepsilon_t$, is ruled out by the following, additional assumptions.[13]

(8) $$E(X'u) = 0 \text{ or equivalently } E(u) = 0$$

$$E(u_t u_{t+s}) = 0 \quad \text{for all } s$$

$$E(u_t^2) = \sigma_u^2 \text{ for all } t$$

$$E(\varepsilon_t^2) = \sigma_\varepsilon^2 \text{ for all } t$$

$$E(u_{t+s}\varepsilon_t) = 0 \quad \text{for all } s > 0$$

Then the variance-covariance matrices of $\hat{\beta}_{rr}$ and $\hat{\beta}_{eprr}$ are:

(9) $$E[(\hat{\beta}_{rr} - \beta)(\hat{\beta}_{rr} - \beta)'] = E[(X'X)^{-1}X'uu'X(X'X)^{-1}] = \sigma_u^2(X'X)^{-1}$$

and

(10) $$E[(\hat{\beta}_{eprr} - \beta)(\hat{\beta}_{eprr} - \beta)'] = E[(X'X)^{-1}X'(uu' + \varepsilon\varepsilon' - 2u'\varepsilon)X(X'X)^{-1}]$$

$$= \sigma_u^2(X'X)^{-1} + \sigma_\varepsilon^2(X'X)^{-1}$$

Note that the derivation of (10) also makes use of the rationality implication in (3) which requires that ε_t is serially uncorrelated and uncorrelated with u's dated at t and before.

A comparison of the two variance-covariance matrices above points out the major difficulty with ex post real rate regressions. Although they yield the same expected value of the coefficient estimates as an actual real rate regression (if it were estimable), the variance-covariance matrix of the estimates will be larger by the term $\sigma_\varepsilon^2(X'X)^{-1}$. Thus statistical tests will have lower power. This problem will be particularly severe if $\sigma_\varepsilon^2 >> \sigma_u^2$. Unfortunately, as has been pointed out by Nelson and Schwert (1977), the forecast errors for inflation are probably extremely large and are much greater in magnitude than the u's. Because the term $\sigma_{\varepsilon}^2(X'X)^{-1}$ will then be much larger than $\sigma_u^2(X'X)^{-1}$, estimates of β from the ex post regressions will be substantially more imprecise than if they were obtainable from an actual real rate regression. In this case it may be very hard to discern significant β-coefficients in the ex post real rate regressions because the statistical tests will have so little power. This is essentially the point made by Nelson and Schwert in their comment on Fama (1975).[14]

We can also use the estimated ex post real rate regressions to provide us with information on how the real rate has moved over time. The fitted values from these regressions are an estimate of the real rate and are denoted as $\hat{rr}_t$: i.e.,

(11) $$\hat{rr}_t = X_{t-1}\hat{\beta}_{eprr}$$

This estimate of the real rate has errors for two reasons. First, some variables or nonlinearities which explain real rate movements are likely to have been left out of the regression model, resulting in a non-zero u_t term. Second, the β-coefficient would be estimated with some error in a finite sample. To see this, we just subtract equation (4) from (11) to yield the error,

(12) $$\hat{rr}_t - rr_t = X_{t-1}(\hat{\beta}_{eprr} - \beta) - u_t$$

The variance of the within-sample error, is easily derived under the above assumptions as follows:[15,16]

$$(13)\quad VAR(\hat{rr}_t - rr_t) = E[(\hat{rr}_t - rr_t)(\hat{rr}_t - rr_t)'] = E[(X_{t-1}(X'X)^{-1}X'(u - \varepsilon) - u_t)$$

$$(u - \varepsilon)'X(X'X)^{-1}X'_{t-1} - u_t)]$$

$$= E[X_{t-1}(X'X)^{-1}X'(u - \varepsilon)(u - \varepsilon)'X(X'X)^{-1}X'_{t-1} + u_t^2 - 2u_t$$

$$(u - \varepsilon)'X(X'X)^{-1}X'_{t-1}]$$

$$= (\sigma_\varepsilon^2 + \sigma_u^2)X_{t-1}(X'X)^{-1}X'_{t-1} + \sigma_u^2 - 2\sigma_u^2 X_{t-1}(X'X)^{-1}X'_{t-1}$$

$$= (\sigma_\varepsilon^2 - \sigma_u^2)X_{t-1}(X'X)^{-1}X'_{t-1} + \sigma_u^2$$

Because we do not know the exact relative size of the σ_ε^2 and σ_u^2 which add up to the overall variance of the composite error term $u - \varepsilon$, the formula in (13) cannot be used directly to yield standard errors of $\hat{rr}_t - rr_t$. However upper and lower bounds for these standard errors can be derived. If the term $X_{t-1}(X'X)^{-1}X'_{t-1}$ is less than one-half as is always the case in the results of this paper, the lower bound for the standard error of $\hat{rr}_t - rr_t$ is reached when all variation of the composite error term is due totally to the forecast error of inflation.[17] With an estimate of the standard error of the ex post real rate regression, $\hat{\sigma}$, this is the case where it is assumed that $\hat{\sigma}_\varepsilon = \hat{\sigma}$ and $\hat{\sigma}_u = 0$. The lower bound estimate of the standard error of $\hat{rr}_t - rr_t$, denoted as $\hat{SE}_\ell(\hat{rr}_t - rr_t)$, is then

$$(14)\quad \hat{SE}_\ell(\hat{rr}_t - rr_t) = \hat{\sigma}\sqrt{X_{t-1}(X'X)^{-1}X'_{t-1}}$$

The upper bound estimate of the standard error, $\hat{SE}_u(\hat{rr}_t - rr_t)$, is reached when all variation of the composite error is due totally to variation in u_t: i.e., $\hat{\sigma}_u = \hat{\sigma}$ and $\hat{\sigma}_\varepsilon = 0$.

$$(15)\quad \hat{SE}_u(\hat{rr}_t - rr_t) = \hat{\sigma}\sqrt{1 - X_{t-1}(X'X)^{-1}X'_{t-1}}$$

Because σ_{ε}^2 is likely to greatly exceed σ_u^2 here, the lower bound estimate is likely to be a far more accurate measure of the true standard error than the upper bound estimate. Thus in the discussion of the empirical results later in the paper, more attention will be devoted to the lower bound estimate. However, the upper bound estimate will also be reported so that someone with a different prior can use this information in deriving their estimates of the standard error.

This section has demonstrated that regressions with ex post real rates have many desirable statistical properties. On the other hand, the potential unreliability of survey measures of inflation expectations makes more direct approaches to inference about real rates suspect. Therefore, even though regressions, or equivalently correlations, with ex post real rates may have low statistical power, they are a dependable way of inferring information about real interest rates. This then is the method of analysis used in this paper. However, the discussion above issues a warning for interpreting the results that follow. The absence of a variable's significant explanatory power in an ex post real rate regression should not be viewed as evidence that the real rate is unrelated to this variable. An alternative, and equally plausible view, is that the statistical tests here just do not have enough power to find this relationship. With this caveat, we can now turn to the actual empirical results in the next section.

III. THE EMPIRICAL RESULTS

THE DATA

The empirical analysis here uses quarterly data on the ex post real rate for three month treasury bills. These ex post rates are at a quarterly rate and are calculated by subtracting the actual, continuously compounded inflation rate (using the seasonally unadjusted Consumer Price Index (CPI)) from the continuously compounded nominal yield on a three month bill maturing at the end of the quarter. The bill rate data were obtained from the bond file of the Center for Research in Security Prices (CRSP) at the University of Chicago.[18] As in Fama (1975), the CPI is used to calculate inflation rates here rather than the Wholesale Price Index (WPI) because only the CPI reflects transaction prices. The dating convention is as follows. The ex post real rate for a quarter is the actual real return on a three month bill held from the beginning to the end of that quarter. All variables that are growth rates, such as money growth, real GNP growth and inflation, are calculated as the change in the log from the last month of the previous quarter to the current quarter and are thus at quarterly rates. The unemployment rate is for the last month in the quarter and the investment to capital ratio is constructed comparably to this ratio in Fama and Gibbons (1980) by dividing the investment over the quarter by the capital stock at the end of the quarter. The variables used in the analysis here are discussed in more detail in the Data Appendix.

Monthly data are not used in the analysis for the following reason. It is not clear what is the appropriate dating for the CPI in a particular month since price quotations have been collected over the entire month. Thus there is no accurate way of matching up the timing of the one month bills with the dating of the CPI. This problem is somewhat less severe when quarterly data are used. The potential timing error of using the CPI in the last month of the quarter as a match for a three month bill maturing at the end of the quarter is smaller as a fraction of the time to maturity than would be the case if the one month bill maturing at the end of the month is matched up, as in Fama (1975), with the CPI in that month. The quarterly ex post real rates should be more accurate than the one month rates as a result. Furthermore, Fama (1975) finds only small differences in coefficient estimates with quarterly versus monthly data, and little additional information is contained in the more noisy monthly series since the standard errors of the coefficients using the monthly data are only slightly smaller than those found with the quarterly data.

The empirical analysis of this paper focuses its main attention on the 1953-1 through 1979-4 sample period as an update to Fama (1975). The 1979-4 quarter was the latest quarter for which the CRSP bill series was available. Fama started his analysis in 1953 as here because the CPI was substantially upgraded in that year. He ended his sample period at the second quarter of 1971 to avoid any possible distortions that the Nixon price controls might have had on the measurement of the CPI. In preliminary results I excluded the price control period from 1971-3 to 1974-4 from my analysis with little appreciable effect on the coefficient estimates or on the major findings.[19] Since it is not obvious that the Nixon price controls should have severely distorted the CPI, data from this period are included in the analysis.

RESULTS: 1953-1 to 1979-4

Tests with ex post real rates in Fama (1975) could not reject the hypothesis that the real rate was constant over the 1953-71 period. However, later work which frequently used more refined statistical tests did find significant rejections of this hypothesis. Included in this work is not only that of Fama's critics, Carlson (1977), Garbade and Wachtel (1978), Hess and Bicksler (1975), Joines (1977) and Nelson and Schwert (1977), but also that by Fama himself, Fama (1976b) and Fama and Gibbons (1980). Extending Fama's original sample period through the end of the 1970s should yield new insights into the question of the constancy of the real rate because the 1971-3 to 1979-4 period has been one of unusual economic turbulence by postwar standards. It contains both the worst recession in the postwar period as well as the highest inflation rates. The greater variation in the data that occurred as a result could make statistical relationships that much more clear-cut.

The null hypothesis of the constancy of the real rate is studied here with two types of tests which are similar to those carried out in Fama (1975). First, we look at the serial correlation structure of the ex post real rate, and we then turn to tests of whether other variables in the available information set, ϕ_{t-1}, are correlated with the ex post real rate. These tests correspond to the weak form and strong form tests discussed in the efficient markets literature.[20]

Table 1 contains the first twelve autocorrelations of the ex post real rate for the 1953-79 period. These autocorrelations are positive and large, and more than half are more than two standard errors away from zero. The constancy of the real rate implies the null hypothesis that all the autocorrelations equal zero. It is formally tested with the adjusted Q-statistic suggested by Ljung and Box (1978). The Q(12) statistic is distributed approximately as $\chi^2(12)$, and its marginal significance level is the probability of getting that value of the test statistic or higher under the null hypothesis: i.e., a marginal significance level less than .01 indicates

TABLE 1

Serial Correlation Structure of the Ex Post Real Rate: 1953-1 to 1979-4

	Lag k												Approximate Standard Error of the Autocorrelations
	1	2	3	4	5	6	7	8	9	10	11	12	
Autocorrelation at Lag k (r_{-k})	.38	.28	.36	.45	.31	.16	.24	.15	.20	.08	.24	.17	.10

Test of $r_1=r_2=,\ldots,=r_{12}=0$

$Q(12) = 102.33$

Marginal Significance level $= 1.94 \times 10^{-16}$

NOTES: Q(12) is the adjusted Q-statistic suggested by Ljung and Box (1978) which is distributed approximately as $\chi^2(12) = n(n+2) \sum_{k=1}^{12} (n-k)^{-1} r_{-k}^2$, where n = number of observations = 108.

Marginal significance level is the probability of setting that value of the Q-statistic or higher under the null hypothesis that $r_1=r_2=,\ldots,=r_{12}=0$.

Approximate standard error $= 1/\sqrt{n}$.

a rejection of the null hypothesis at the one percent level. The rejection of the null hypothesis is very strong. The Q(12) statistic exceeds 100 and its corresponding marginal significance level is extremely small. Thus, despite the possible low power of this test discussed by Nelson and Schwert (1977) and the previous section, the additional data added to Fama's (1975) sample period now result in a clear-cut rejection of the constancy of the real rate.

Table 2 contains tests for whether the ex post real rate, and hence the real rate, has been correlated with other variables whose values were known when the real rate was determined. Because heteroscedasticity can have a major impact on results,[21] Goldfeld-Quandt (1965) as well as Glesjer (1969) tests were performed in order to see if corrections for heteroscedasticity were necessary. They uniformly did not reveal the presence of heteroscedasticity for this sample period and so ordinary least squares was used in estimation.[22]

Considering the recurrent discussion of the Mundell-Tobin effect in the literature, the inflation rate is an obvious candidate as a variable that might be correlated with the real rate.[23] Model 2.1 tests for this correlation and the results do exhibit a strong, significant negative correlation of the ex post real rate with the lagged inflation rate: its coefficient of -.3073 has a t-statistic over six in absolute value with a very low marginal significance level. This result rejecting the constancy of the real rate is consistent with many other studies, such as Lahiri (1976), Carlson (1977), Levi and Makin (1979) and Pearce (1979), which find a significant negative correlation between the real rate and expected inflation.

Since this significant negative relationship between inflation and the real rate is so striking and has such important implications, it deserves

TABLE 2

Tests of the Constancy of the Real Rate: 1953-1 to 1979-4

Dependent Variable: Ex Post Real Rate
Estimation Method: Ordinary Least Squares

	Coefficient of										
Model #	Constant Term	$\pi(-1)$	Time	$Time^2$	$Time^3$	$Time^4$	R^2	SE	DW	F-Statistic	Marginal Significance Level
2.1	.0040	-.3073					.26	.0045	2.16	38.00	1.30×10^{-8}
	(6.26)	(-6.16)									
2.2	.0331		-.2195	.5258	-.4976	.1585	.35	.0043	1.85	13.66	5.73×10^{-9}
	(2.03)		(-2.13)	(2.35)	(-2.47)	(2.48)					
2.3	.0308	-.1554	-.2024	.4872	-.4601	.1467	.37	.0042	2.18	12.02	3.70×10^{-9}
	(1.91)	(-1.99)	(-1.99)	(2.20)	(-2.31)	(2.32)					

NOTES: T-statistics in parentheses.

SE = standard error of the regression.

DW = Durbin-Watson statistic.

Time = time trend = runs from .01 in 1947-1 to 1.32 in 1979-4, superscript of time variable indicates raised to that power.

$\pi(-1)$ = inflation rate at quarterly rate, lagged one quarter

F = F-statistic for null hypothesis that all the coefficients excluding the constant term are zero: distributed as F(1,106) in 2.1, F(4,103) in 2.2 and F(5,102) in 2.3.

Marginal Significance level = probability of getting that value of F or higher under the null hypothesis.

further scrutiny and testing for robustness. One question that arises asks: Is this result dependent on the choice of the CPI as the price index in the empirical analysis? The answer appears to be no. Despite the limitations of the WPI because it is not constructed from transaction prices, it is a plausible alternative to the CPI as a price index for use here. When model 2.1 is reestimated with the variables constructed using the WPI rather than CPI, the significant negative correlation of the ex post real rate and inflation continues to hold up. The coefficient on the lagged inflation rate in this WPI regression equals -.3040 and is almost identical to the corresponding coefficient in Table 2. Its t-statistic is also highly significant, equaling -4.41.

Another question of robustness wonders whether the negative relationship of the real rate and inflation is only consistent with data from the last half of the sample period when inflation rates were high. This is a particularly relevant question because Fama (1975) did not find a significant negative correlation of the real rate and inflation using a sample period that excluded data from the high inflation 1970s. To pursue this question, the sample was split in half, and model 2.1 was estimated over both sample periods. The lagged inflation coefficient in the regression for the first half of the sample, 1953-1 to 1966-2, is not significantly negative: it equals -.0067 with a t-statistic of only -.05. Note, however, that the standard error of this coefficient is quite large, exceeding .13. In the regression for the last half of the sample, 1966-3 to 1979-4, the lagged inflation coefficient is not far from the value of the corresponding coefficient in Table 2: it equals -.3614 and is statistically significant with a t-statistic of -4.23.

The discrepancy between these results could be occurring because, in the early sample period, there just is not enough variation in the real rate to discern an inflation effect. A Chow test for the stability of the coefficients of model 2.1 over these two sample periods confirms this conjecture. The F(2,104) statistic equals 2.47 (its critical value at the 5% level is 3.1) and does not reject the equality of the 2.1 coefficients in the two periods. It appears that Fama's (1975) inability to find the significant correlation of inflation with the ex post real rate was due to the peculiar nature of his particular sample period which, as noted by Shiller (1980), has insufficient variation in the real rate. The results here then show that the postwar period is more accurately characterized as displaying a negative correlation of the real rate and inflation.

Model 2.2 of Table 2 contains a more mechanical test of the constancy of the real rate by postulating that the real rate moves with a fourth-order polynomial in time. A fourth-order polynomial is used in the tests here because additional variables with higher powers of the time trend did not have significant additional explanatory power.[24] The rejection of the constancy of the real rate is also strong in this case, with an extremely small marginal significance level for the null hypothesis that the coefficients on the time trend variables equal zero. When the trend variables are added to model 2.1, as in model 2.3, there is a significant improvement in explanatory power: F(4,102) = 4.33 while the critical F at 5% is approximately 2.5. Here, the marginal significance level that tests the null hypothesis of the constancy of the real rate declines even further.[25]

Table 3 tests whether other variables that have been cited in the literature are correlated with the real rate. The one and four lags of these variables are used in models which either include or exclude the lagged

TABLE 3

Tests for Correlation of Real Rate with Other Variables: 1953-1 to 1979-4

Dependent Variable: Ex Post Real Rate
Estimation Method: Ordinary Least Squares

Model #	Other Explanatory Variable	Coefficient of: Constant Term	$\pi(-1)$	Other Explanatory Variable	R^2	SE	DW	F-test for significant explanatory power of four lags of independent variable when included in this model
3.1	money growth (M1)	.0026 (3.28)		-.1475 (-2.42)	.05	.0051	1.31	3.34
3.2	money growth (M1)	.0043 (5.62)	-.2954 (-5.54)	-.0366 (-.64)	.27	.0045	2.14	.70
3.3	real GNP growth	.0005 (.84)		.0699 (1.46)	.02	.0052	1.29	1.52
3.4	real GNP growth	.0040 (4.96)	-.3064 (-5.90)	.0029 (.07)	.26	.0045	2.16	.98
3.5	GNP gap	.0012 (1.96)		-.0061 (-.33)	.00	.0052	1.21	.79
3.6	GNP gap	.0040 (5.72)	-.3071 (-6.12)	-.0010 (-.06)	.26	.0045	2.16	.61
3.7	unemploy-ment rate	.0042 (2.09)		-.0593 (-1.60)	.02	.0052	1.25	.82
3.8	unemploy-ment rate	.0050 (2.84)	-.3010 (-5.90)	-.0201 (-.61)	.27	.0045	2.16	.16
3.9	investment to capital ratio	.0042 (.59)		-.0321 (-.44)	.00	.0052	1.21	.41
3.10	investment to capital ratio	.0061 (.99)	-.3068 (-6.13)	-.0219 (-.35)	.26	.0045	2.16	.08

NOTES: Data definitions are discussed in the Appendix.

T-statistics in parentheses.

F-statistic is distributed as F(4,103) in odd numbered models and F(4,102) in even-numbered models. The critical value of these F-statistics at the 5% level is approximately 2.5.

inflation variable. The most striking finding in this table is that none of the real variables--real GNP growth, the GNP gap which is the percentage difference between potential GNP and real GNP, the unemployment rate and the investment to capital ratio--have significant explanatory power for movements in the ex post real rate.[26] Including lagged inflation in these regressions does not alter this finding. Table 3's lack of success in finding significant correlations should not be viewed as implying that real factors do not affect the real rate. More plausible is the view that the statistical tests here do not have sufficient power to discern co-movements of real variables and the real rate. Since the real variables used here are meant to capture cyclical effects on the real rate, this might occur because there just is not enough cyclical variation in the real rate to be picked up by these tests.[27]

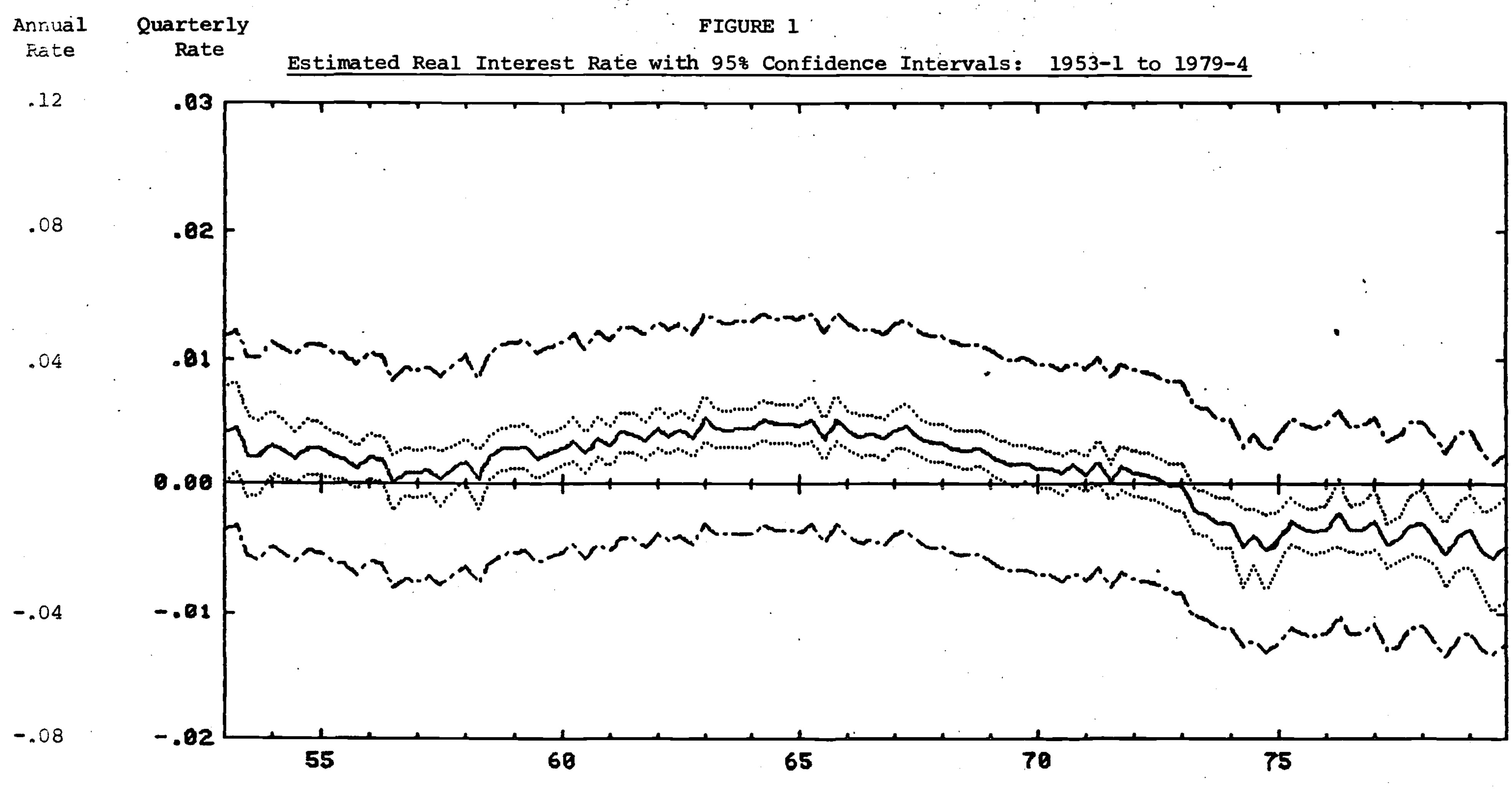
The results with the money growth variable are more interesting. Results with an M2 money growth measure are similar to those found for M1 in Table 3 and are thus not reported.[28] When the lagged inflation rate is excluded from the money growth regressions, the coefficients of the lagged money growth variables are statistically significant and have a negative sum.[29] Thus an increase in money growth has the usual negative relationship with real rates that we would expect from standard monetary theory. However, the 3.2 results indicate that the negative correlation of money growth and the real rate may only arise because of the positive correlation of money growth with inflation: the money growth coefficients are no longer significant when they are added to an ex post real rate regression that includes the lagged inflation rate. Feldstein (1980a) has argued that an increased rate of money growth is non-neutral because it raises inflation and lowers the real return to saving, and this can have an adverse effect on capital formation and productivity. The significant negative correlation of the real rate and money growth found here is in the direction that Feldstein's argument requires, but it can only be taken as evidence supporting his position if we are willing to ascribe causation to the results. Because the money growth variables do not have significant explanatory power in addition to lagged inflation, the evidence for a short-run cyclical effect of monetary policy is weak. Again, this might be attributed to the low power of these statistical tests. But it does point out why it might be so difficult to sort out the issue discussed by Shiller (1980) as to whether the Federal Reserve System can control real interest rates.[30]

To briefly summarize the results in Tables 1-3: The real interest rate has not been constant over the 1953-79 period, and the economic variable most highly correlated with the ex post real rate is the inflation rate. None of the other economic variables add significant explanatory power to the ex post real rate regressions with lagged inflation as an explanatory variable. However, there is something left to explain in real rate movements because time trend variables do add significant explanatory power.

Another approach to understanding these results involves looking at estimates of the real rate derived from fitted values of the most interesting of the regressions. Figure 1 contains the estimates of the real rate derived from the fitted values of the regression with the best fit, #2.3. The inner band surrounding the real rate estimates delimits the region where real rates are within two lower-bound standard errors of the estimated real rate (as calculated from (14)). The outer band is the region where real rates are within two upper-bound standard errors of the estimated real rate (as calculated from (15)). Because, as discussed previously, it is likely that $\sigma_u^2 << \sigma_\varepsilon^2$, the inner band is probably a reasonable characterization of the 95% confidence interval, although it is necessarily an underestimate. Attention will primarily be focused on this inner band when discussing inference in what follows. Yet, the outer band in the figure should warn us to treat such inference with

FIGURE 1

Estimated Real Interest Rate with 95% Confidence Intervals: 1953-1 to 1979-4



Estimated real rate, $\hat{rr}_t$, from Model 2.3

$\hat{rr}_t \pm 2\hat{SE}_\ell(\hat{rr}_t - rr_t)$

$\hat{rr}_t \pm 2\hat{SE}_u(\hat{rr}_t - rr_t)$

suitable caution.

Figure 1 indicates that the real rate tended to be positive in the first twenty years of the sample period, with a declining trend from 1953-56, an upward trend reaching a maximum in the boom years of the mid-1960s and a declining trend thereafter. Using the inner band for inference, the estimated real rates were significantly positive for most of this period and the peak values of over two percent at an annual rate are more than six of the lower-bound standard errors away from zero. The real interest rate appears to turn negative towards the end of 1972 and the downward trend accelerates through the end of 1974, whereupon the real rate stays negative but has no easily discernible trend. Throughout most of the 1973-79 period, the estimated real rates remain significantly negative according to the inner band confidence region, and the most negative estimated real rates of approximately minus two percent at an annual rate are more than five of the lower-bound standard errors away from zero. Thus, as casual inspection of nominal interest rate and inflation data might have led one to suspect, there is strong evidence that real interest rates were positive in the 1950s and 60s, but have turned quite negative in the mid- and late 1970s.

One striking feature of these results is their similarity to those of Garbade and Wachtel (1978) and Fama and Gibbons (1980). They also use ex post real rate data, but find their results with a very different statistical technique involving variable-parameter regression. Both of these studies assume that the real rate is so highly autocorrelated that its movements over time can be approximated as a random walk. There is somewhat more smoothness on their resulting real rate estimates than is the case here, but the overall magnitudes and peaks and troughs of the series are quite close to those in Figure 1.[31] The corroborating evidence from these studies then lends further support to the results here.

Because time trend variables do contain additional explanatory power over the lagged inflation rate in the ex post real rate regressions, there must be some other economic variables left out of the analysis here that help explain real rate movements and are correlated with the trend variables. This leaves us with a puzzle to figure out what these excluded variables might be. One clue comes from a comparison of the estimated real rates from the model which only includes the lagged inflation rate as an explanatory variable with those from the model which also includes the time trends as additional explanatory variables. Figure 2 contains one series of estimated real rates derived from the fitted values of model 2.1 while the other, which has already appeared in Figure 1, is derived from model 2.3. The two series have a roughly similar pattern, but a comparison indicates that the time trend variables make the estimated real rate higher in the 1961-70 period, lower in the 1975-79 period and lower in the 1954-57 period. We might search for relevant economic variables missing from this analysis by noting that their effect on the real rate should be in a similar direction.

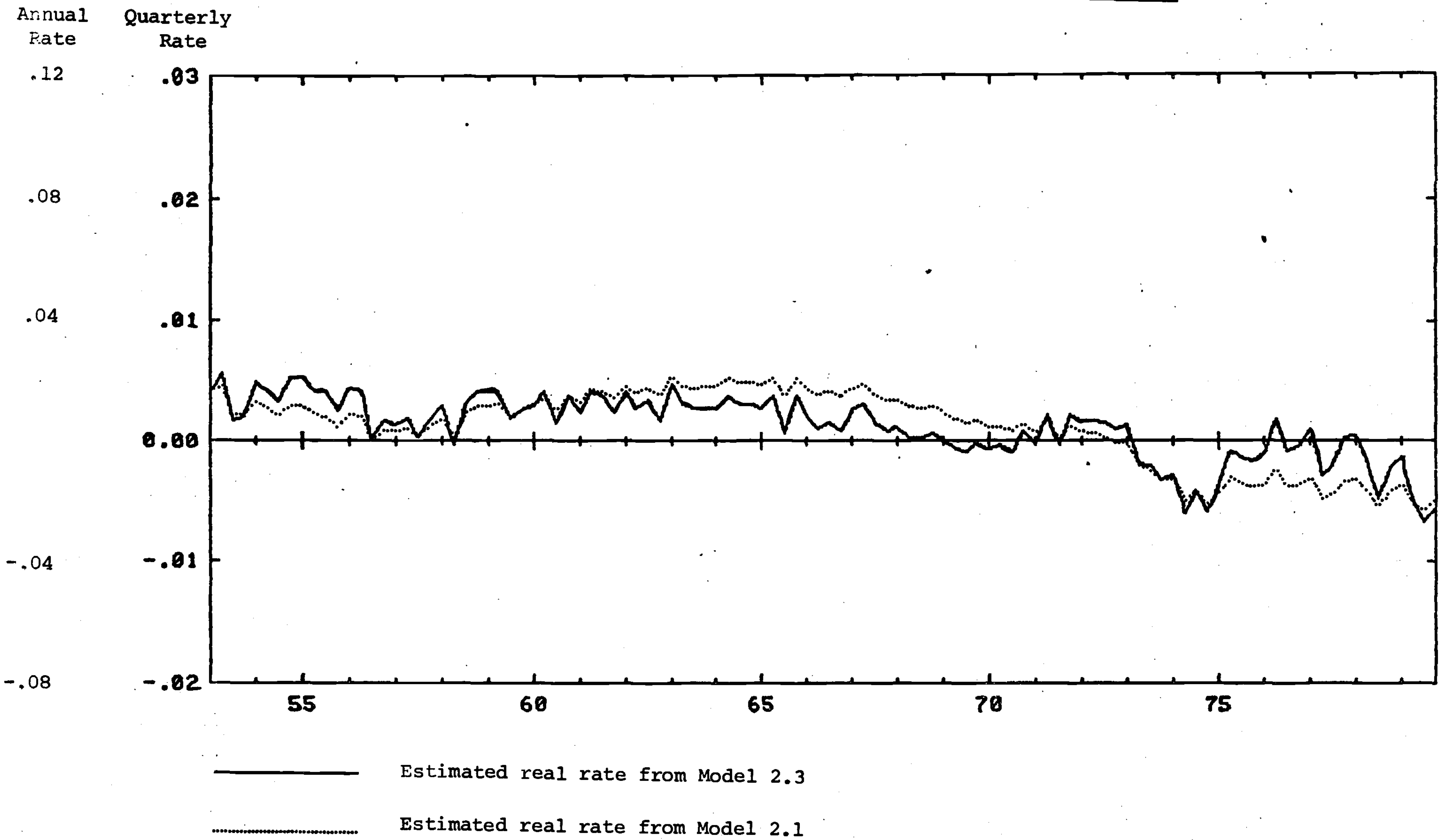
The estimates of the real rate obtained with the approach used here provide another piece of useful information. An estimate of the expected inflation rate, $\hat{\pi}^e_t$, is easily derived by subtracting the estimated real rate, $\hat{rr}_t$, from the nominal interest rate, i_t: i.e.,[32]

$$(16) \qquad \hat{\pi}^e_t = i_t - \hat{rr}_t = i_t - X_{t-1}\hat{\beta}_{eprr}$$

This measure of expected inflation has several advantages over those generated from univariate ARIMA time-series models or from surveys, the alternatives most frequently used in the literature. The principal advantage of this measure is that it is a more accurate predictor of inflation. For the prediction error in this sample, the within-sample standard error of the measure derived from

FIGURE 2

Comparison of Estimated Real Rates from Models 2.1 and 2.3: 1953-1 to 1979-4



an ARIMA (0,1,1) model[33] with a seasonal MA1 term is .0047 versus .0042 for the measure derived from model 2.3.[34] Since Pearce (1979) has shown that a similar ARIMA model forecasts inflation better than the Livingston survey measure, it appears as though the approach outlined here will outperform the Livingston measure as well. Because the true expected inflation rate can probably be characterized as rational, the better, and hence more rational, forecasting performance of the $\hat{\pi}^e_t$ discussed here is an indication that it is a more accurate expected inflation measure than the survey or ARIMA alternatives.

In addition quantitative information on how accurate $\hat{\pi}^e_t$ is as a measure of expected inflation can be calculated, while this is not the case for the survey or ARIMA measures. The error in $\hat{\pi}^e_t$ is:

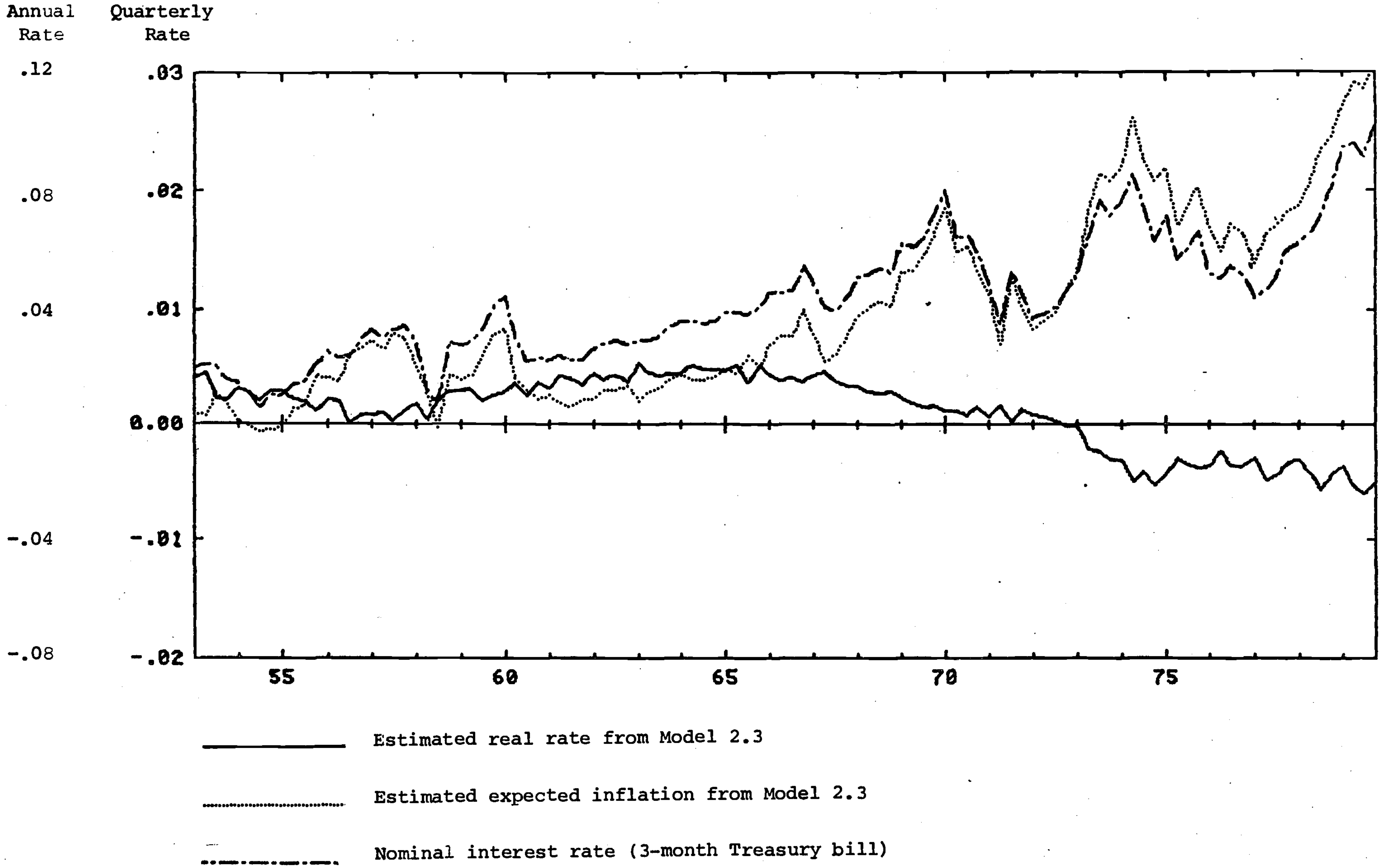
$$(17) \qquad \hat{\pi}^e_t - \pi^e_t = (i_t - \hat{rr}_t) - (i_t - rr_t) = -(\hat{rr}_t - rr_t)$$

Since this error is just the negative of the estimated real rate error, its variance is identical to that of the real rate error in (13) and the lower and upper bound standard errors can be calculated with the formulae in (14) and (15). These standard errors, especially the lower bound which is likely to be more accurate, not only provide confidence intervals for the expected inflation rate, but also can be useful in determining how severe the errors-in-variable bias would be when this expected inflation measure is used in other empirical work.

Figure 3 plots the nominal interest rate along with the estimated real rate from Figure 1 and its corresponding estimated expected inflation rate. The most striking feature of this figure is the presence of the Fisher effect. The correlation of the estimated expected inflation rate and the nominal interest rate is .95. Thus despite the finding here that the real rate is not constant, the basic Fama (1975) position that the Fisher effect is strong in

FIGURE 3

Nominal Interest Rate, Estimated Real Interest Rate and Estimated Expected Inflation: 1953-1 to 1979-4



the postwar period is supported. As would be expected from the significant negative correlation of the ex post real rate and lagged inflation, the negative correlation of the estimated expected inflation rate and the estimated real rate is substantial, equaling -.86.

Probably the most important aspect of Figure 3 from the point of view of policy analysis is the negative correlation (-.67) of movements in the nominal interest rate and the estimated real rate. There is an important moral for policymakers in this negative correlation. Many economists have long warned that increases in nominal interest rates do not necessarily mean that money is tight because movements in the real rate might not be highly correlated with the nominal rate. Figure 3 indicates something even stronger. It says that an increase in nominal interest rates is associated with a lower real rate rather than the reverse. Thus when nominal rates are high, it is more likely that we are in a period of "easy money" with low real rates than the contrary as has frequently been assumed. Targeting on a nominal interest rate can thus cause policy to err in the wrong direction. Therefore, it is a highly dangerous policy approach, and it hopefully has been abandoned by the Fed in the recent revision of its policy procedures in October 1979.

RESULTS: 1931-4 to 1952-4

There are several problems with analysis of ex post real rates in the pre-1953 period. Before the 1953 major revision of the CPI, this price index was not measured that accurately. Particularly troublesome is the fact that prices were often sampled infrequently in constructing the early CPI data, and this could induce spurious correlations with the calculated ex post real rates. Another problem is that for much of this period the nominal interest rate was pegged. From 1937 through 1941 the t-bill rate dropped essentially to zero. Since there is an alternative, readily available asset in the economy which has a zero nominal return--i.e., currency--the bill rate could not fall further. Thus it was effectively pegged during this period. After this, up until the early part of 1951, the bill rate was then actually pegged by the Federal Reserve System. As Fama (1975) has noted, we might not expect Fisher relationships to appear in data where nominal interest rates are pegged.

We must therefore be cautious when we interpret results using these earlier data. However, it is worthwhile studying these results because they might provide further evidence which corroborates the striking results obtained from the postwar data found in Tables 1 and 2. The sample period analyzed here starts in the fourth quarter of 1931 when t-bill data first become available in the CRSP bond file. Goldfeld-Quandt (1965) and Glesjer (1969) tests were again conducted to see if there was heteroscedasticity within the 1931-4 to 1952-4 sample period. No significant heteroscedasticity was found so estimation again proceeded with ordinary least squares.[35]

Tables 4-5 correspond to Tables 1-2 and again test for the constancy of the real rate, but now for the earlier sample period, 1931-4 to 1952-4. As in the previous results, the ex post real rate from 1931-4 to 1952-4 does display large positive autocorrelations for the first four lags; while the autocorrelations at higher lags are not significantly different from zero. The joint test for the significance of the first twelve autocorrelations in Table 4 also corroborates the previous finding that the ex post real rate is serially correlated. The Q(12) statistic is over 40 with a marginal significance level less than .0001, indicating that the real rate was not constant in the 1931-4 to 1952-4 sample period.

Model 5.1 of Table 5 tests for whether the negative relationship between the real rate and lagged inflation also holds up in this earlier

TABLE 4

Serial Correlation Structure of the Ex Post Real Rate: 1931-4 to 1952-4

	Lag k												Approximate Standard Error of the Autocorrelations
	1	2	3	4	5	6	7	8	9	10	11	12	
Autocorrelation at Lag k (r_{-k})	.48	.24	.24	.21	.04	-.12	-.02	-.04	-.15	-.10	-.11	-.03	.11

Test of $r_1=r_2=,\ldots,=r_{12}=0$

$Q(12) = 40.80$

Marginal Significance level = 5.29×10^{-5}

NOTES: See Table 1.

TABLE 5

Tests of the Constancy of the Real Rate: 1931-4 to 1952-3

Dependent Variable: Ex Post Real Rate
Estimation Method: Ordinary Least Squares

Model #	Coefficient of: Constant Term	$\pi(-1)$	Time	$Time^2$	R^2	SE	DW	F-Statistic	Marginal Significance Level
5.1	-.0021 (-1.08)	-.4937 (-5.13)			.24	.0171	2.00	26.28	1.90×10^{-6}
5.2	.0239 (3.55)		-.1381 (-4.12)	.1260 (3.55)	.20	.0176	1.25	10.40	9.43×10^{-5}
5.3	.0161 (2.42)	-.3696 (-3.56)	-.0938 (-2.78)	.0886 (2.55)	.31	.0165	1.94	12.18	1.18×10^{-6}

NOTES: T-statistics in parentheses, R^2, SE and DW, F and marginal significance level are as defined in Table 2 with the F's distributed as F(1,83) for 5.1, F(2,82) for 5.2 and F(3,81) for 5.3.

$\pi(-1)$, Time and $Time^2$ are as defined in Table 2, except that Time runs from .01 in 1931-3 to .88 in 1952-4 sample period. The answer is yes. The coefficient on lagged inflation is again negative and statistically significant: its value of -.4937 is even larger than that found in the postwar data and its t-statistic is greater than five in absolute value. Because the nominal bill rate was effectively pegged after 1937, we might wonder if this negative relationship only occurs as a result of the pegging in the later part of the sample period. To explore this, the sample was split at the 1937-4 quarter, and model 5.1 was estimated for the two periods. The coefficient on lagged inflation in the 1931-4 to 1937-4 sample period equals -.4265 and is statistically significant with a t-statistic of -2.17. The coefficient on lagged inflation in the 1938-1 to 1952-4 sample period equals -.4654 with a t-statistic of -4.04. As would be expected from these results, a Chow test which tests for the stability of the coefficients in 5.1 over the two sample periods cannot reject the null hypothesis that the coefficients are equal: $F(2,81) = 1.16$ with the critical value of F at 5% approximately equal to 3.1.

These results with the earlier sample period provide even more evidence that Fama's (1975) finding that the real rate is constant and is not negatively correlated with inflation is the exception and not the rule. The ex post real rate is found to be significantly negatively correlated with inflation in every sample period except Fama's, and it appears that this is not due to a definite absence of the negative correlation in his sample period, but rather to the lack of variation in the data. Indeed, not too surprisingly considering the nature of the pre-1953 data, it appears as though the strong negative association of real rates and inflation found in the postwar data is on the weak side by historical standards. A Chow test, suitably corrected for heteroscedasticity,[36] comparing the coefficients of 2.1 and 5.1 finds that the coefficients are not stable over the 1931-79 sample period: the null hypothesis of equal coefficients is rejected at the one percent level with F(2,189) = 9.19, while the critical F at 1% is approximately 4.7.

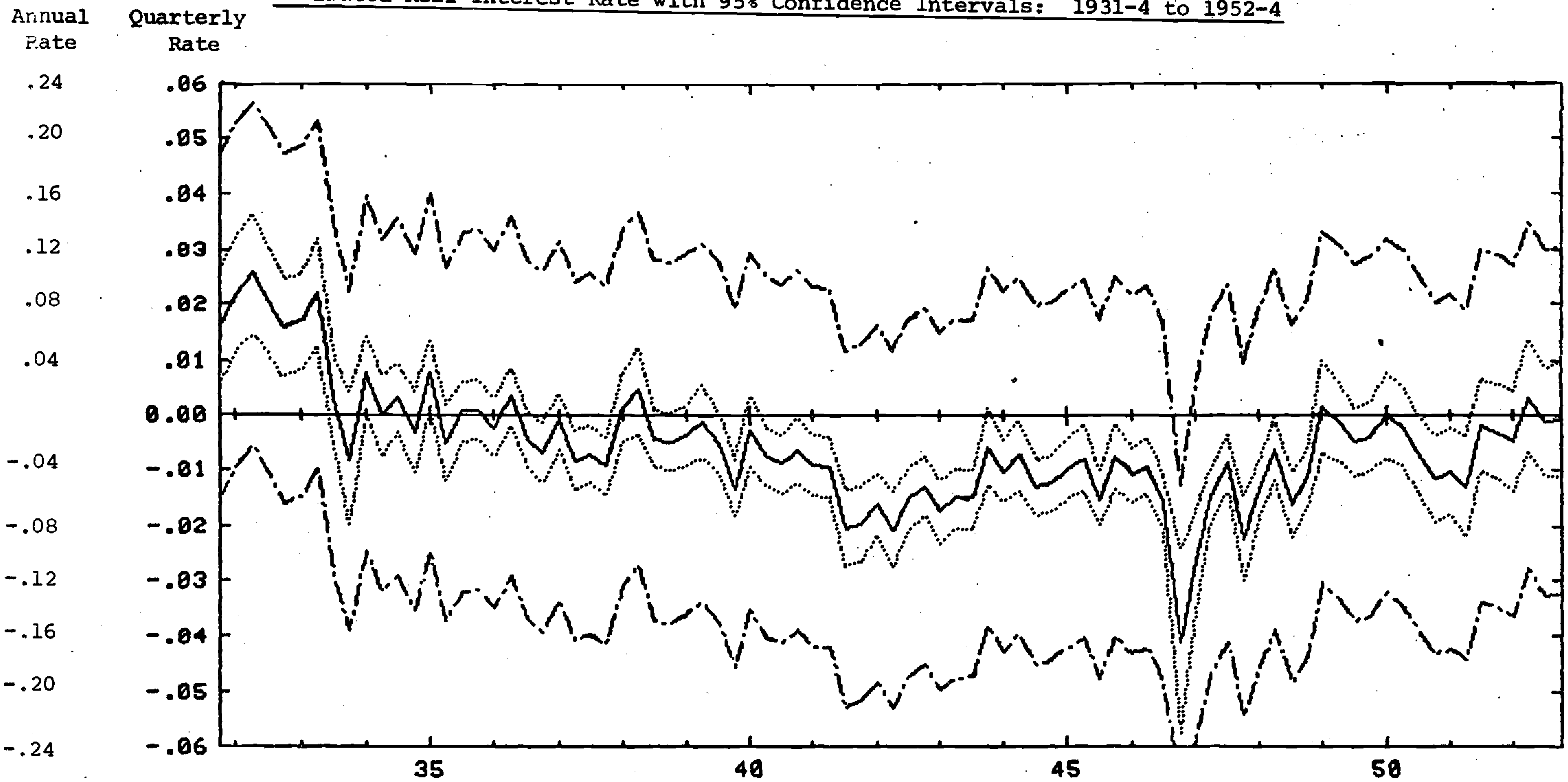
Model 5.2 conducts the more mechanical test of the constancy of the real rate, assuming that the real rate moves with a second-order polynomial in time. A second-order polynomial is used here because additional variables with time trends raised to a higher power did not add significant explanatory power in both the 5.2 and 5.3 regressions.[37] The null hypothesis that the coefficients of the time trend variables equal zero is rejected with a low marginal significance level. When the time trend variables are added to the model with lagged inflation as an explanatory variable, as in 5.3, they do contain significant, additional explanatory power: F(2,81) = 4.13 with the critical F at 5% equal to 3.1. The addition of these variables then leads to an even lower marginal significance level for the null hypothesis implied by the constancy of the real rate. These results again support the postwar findings.

Figure 4 plots the estimate of the real interest rate derived from the fitted values of 5.3. The upper and lower bound 95% confidence bands are derived as for Figure 1 from the formulae in (14) and (15). One interesting feature of this figure is the much greater variability of the estimated real rate in this early sample period than in the later, postwar period. (This result is brought out more clearly in Figure A1 in Appendix I, which displays the estimated real rate for both sample periods in one graph.) This might be an indication of the greater economic stability in the later period.

Some economists have challenged the Keynesian position that money was "easy" during the contractionary phase of the Great Depression by stating that real interest rates were probably quite high during this period although nominal interest rates were low.[38] This view is given strong support in Figure 4. The estimated real rates from 1931-4 to 1933-1 range from 6.2% to 10.2% at an annual rate. Using the lower bound standard errors, not only are the

FIGURE 4

Estimated Real Interest Rate with 95% Confidence Intervals: 1931-4 to 1952-4



Estimated real rate, $\hat{rr}_t$, from Model 5.3

$\hat{rr}_t \pm 2\hat{SE}_\ell(\hat{rr}_t - rr_t)$

$\hat{rr}_t \pm 2\hat{SE}_u(\hat{rr}_t - rr_t)$

estimated real rates significantly higher than zero--sometimes by over four standard errors--but they also appear to be significantly higher than any of the estimated real rates for the postwar period. By the criterion of real rates, the evidence indicates that since 1931 money has never been tighter than in the contraction phase of the Great Depression.

The estimated real rate begins to decline as the economy recovers after 1933-2 and bounces around zero until 1938-2. From then on until 1952-4, the estimated real rate is almost always negative and is significantly so (by the criterion of the lower-bound standard errors) for most of this period. Indeed the estimated real rate is frequently less than -5% and reaches its low point of -16.4% in 1946-4 when it is five of the lower bound standard errors away from zero. Thus the period where nominal interest rates were pegged, also appears to have been a period of negative real rates.

Figure 5 contains the estimated real rates derived from the model with only lagged inflation as an explanatory variable versus those from the model which also includes the time trends as explanatory variables. Because the trend variables do significantly help to explain the ex post real rate, as discussed before, they can provide information on what effect left-out economic variables might be having on the real rate. The comparison of the two estimated real rate series in Figure 5 indicates that the left-out variables make the real rate higher in the 1931-37 period and lower thereafter.

Figure 6, which plots the estimated real interest rate from 5.3, its corresponding estimated expected inflation and the nominal interest rate for the 1931-52 period, is comparable to Figure 3. Here we see some striking differences between the earlier and later sample periods. We again see the negative correlation of expected inflation and real rates as we might expect, although it is even stronger than before, equaling -.99. However, we no

FIGURE 5

Comparison of Estimated Real Rates from Models 5.1 and 5.3: 1931-4 to 1952-4

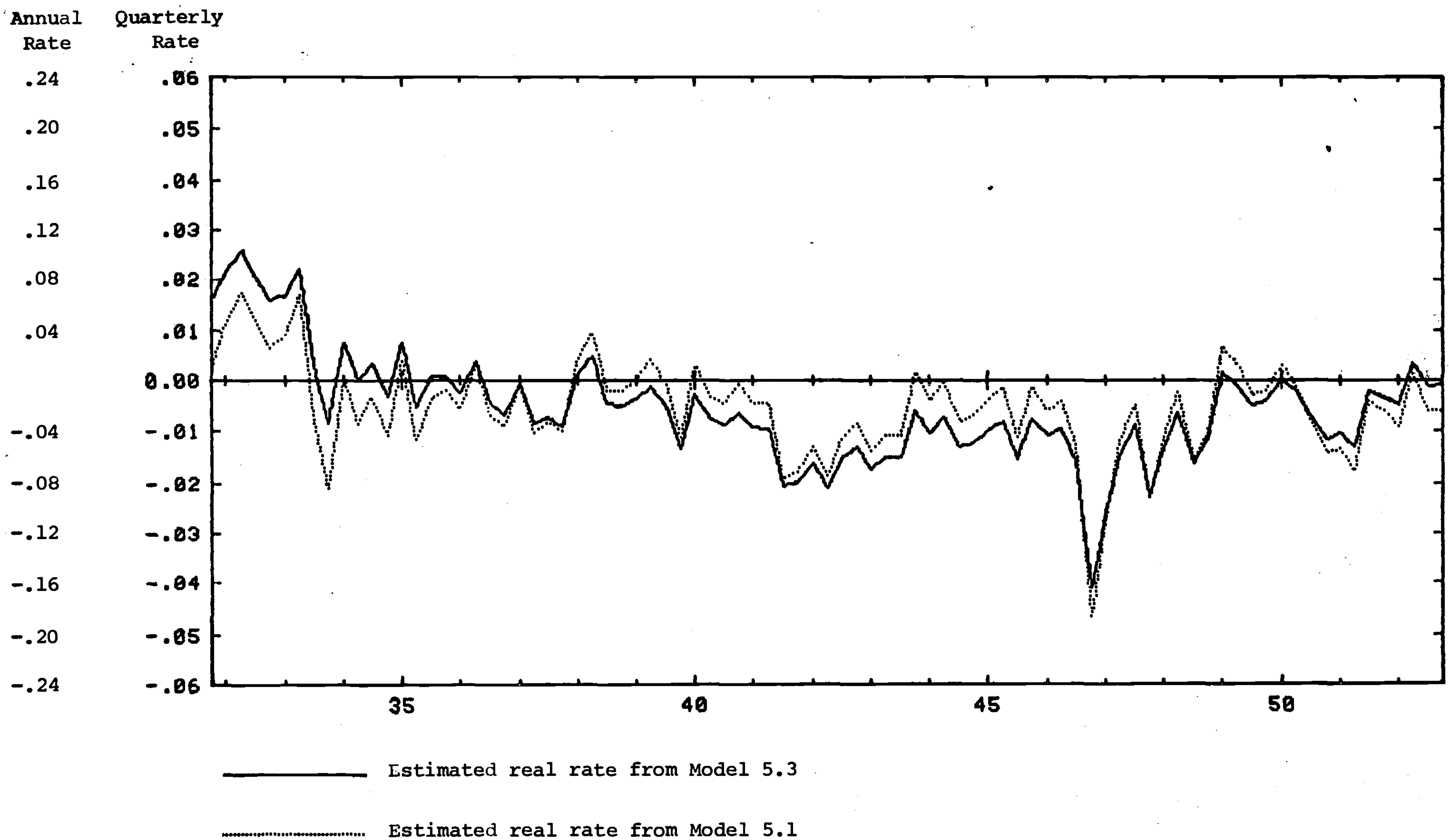
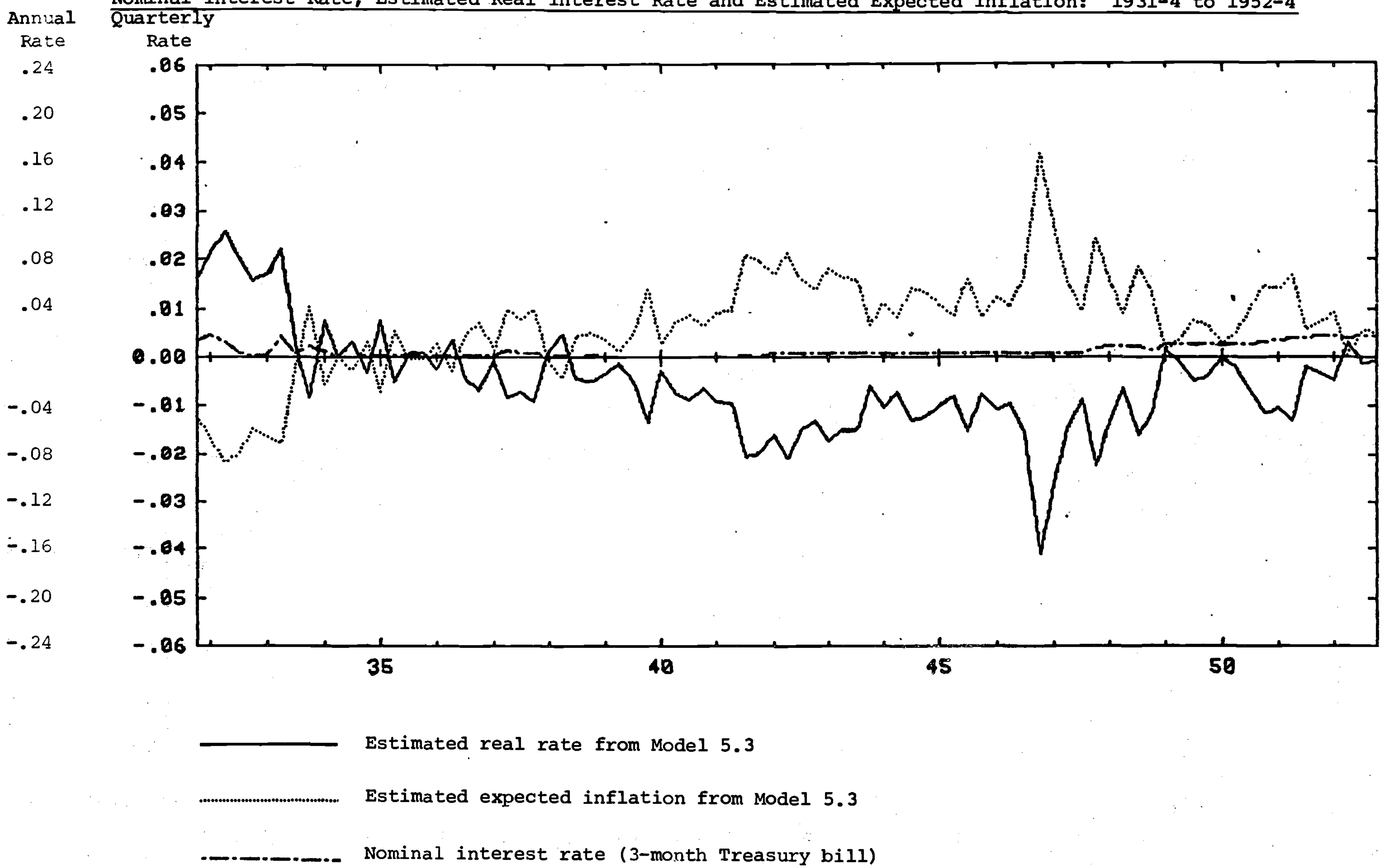


FIGURE 6

Nominal Interest Rate, Estimated Real Interest Rate and Estimated Expected Inflation: 1931-4 to 1952-4



longer find any evidence of the Fisher effect that was so visible in the post-war period. The correlation of expected inflation and the nominal interest rates is not even positive in this early sample period: it is quite small and is slightly negative, equaling -.13. None of these results is particularly surprising because as Figure 6 illustrates, there was very little movement in nominal interest rates in this period. With substantial variation in the expected inflation rate, this must necessarily result in a very strong negative correlation of the real rate and expected inflation and the absence of a Fisher effect.

Figure 6 does support the important policy-oriented conclusion of Figure 3. Movements in nominal interest rates contain little information about the movements in real interest rates. In this early sample period, the correlation of the nominal with the real interest rate is only .25. It was just as easy for the monetary authorities in the pre-1953 period to confuse tight versus easy monetary policy as a result of focusing on nominal rather than real interest rates, as it was during the more recent period. The policy mistakes of the Fed during the contraction phase of the Great Depression clearly attest to this fact.

RESULTS: TAX EFFECTS ON THE REAL RATE

So far, the consequences of taxes on nominal interest payments have been ignored. As has been emphasized by Darby (1975) and especially Feldstein (1976, 1980a,b) this can be misleading for the postwar period. Clearly, what is relevant to the firm's decision to invest or the consumer's decision to save versus consume is the real after-tax interest rate. However, as has been illustrated by Feldstein and Summers (1978) and Shiller (1980), the effective tax rate on interest payments can vary tremendously for different individuals and firms, particularly when they are making different types of decisions. It is easy to find cases where the effective tax rate on interest payments ranges from zero on up to the top marginal tax rate. As a result, it is extremely difficult to know what is the appropriate tax rate on interest payments for the overall economy. This is the primary reason why the analysis in this paper focuses first on real interest rates ignoring the effect of taxes.

To get a flavor of how taxes might affect the previous results, the empirical analysis in Tables 1-3 has been redone with an after-tax ex post real rate which has been calculated under the assumption that the effective marginal tax rate is a third. This tax rate was chosen because it is approximately in the middle of the range of several authors' estimates of the marginal tax rate for interest payments in the postwar period.[39] There is no claim here that this tax rate is an accurate estimate of the effective tax rate on interest, but the results using it will provide us with a reasonable idea of how important tax effects might be to some of the conclusions reached earlier.

Tables 6-8 contain the estimated results using the after-tax ex post real rate. Because only a crude adjustment has been made here for taxes, these results do not deserve a detailed discussion. A few brief comments should be sufficient to give us a flavor of how tax effects would affect our views. The tests for the constancy of the after-tax real rate in Tables 6 and 7 reject the null hypothesis at even lower marginal significance levels than in Tables 1 and 2. Adjusting the ex post real rate for taxes increases the variation in this variable and this is what leads to the even stronger positive serial correlation found in Table 6 and the stronger rejections of the constancy of the after-tax real rate. Because the tax adjustment forces the after-tax real rate to decline as the nominal interest rate rises along with the inflation

TABLE 6

Serial Correlation Structure of the After-Tax Ex Post Real Rate: 1953-1 to 1979-4

	Lag k 1	2	3	4	5	6	7	8	9	10	11	12	Approximate Standard Error of the Autocorrelations
Autocorrelation at Lag k (r_{-k})	.56	.48	.50	.52	.41	.29	.31	.22	.26	.18	.29	.22	.10

Test of $r_1=r_2=,\ldots,=r_{12}=0$

$Q(12) = 194.38$

Marginal Significance level = 4.70×10^{-35}

NOTES: See Table 1.

After-tax ex post real rate calculated with tax rate of .33.

TABLE 7

Tests of the Constancy of the After-Tax Real Rate: 1953-1 to 1979-4

Dependent Variable: Ex Post Real Rate, Adjusted for .33 Marginal Tax Rate
Estimation Method: Ordinary Least Squares

	Coefficient of										
Model #	Constant Term	$\pi(-1)$	Time	$Time^2$	$Time^3$	$Time^4$	R^2	SE	DW	F-Statistic	Marginal Significance Level
7.1	.0020 (3.13)	-.4766 (-9.42)					.46	.0046	2.40	88.79	1.80×10^{-11}
7.2	.0254 (1.49)		-.1728 (-1.60)	.4106 (1.75)	-.3917 (-1.85)	.1243 (1.86)	.49	.0045	1.60	24.29	3.58×10^{-14}
7.3	.0216 (1.32)	-.2564 (-3.23)	-.1444 (-1.39)	.3469 (1.54)	-.3297 (-1.63)	.1049 (1.63)	.53	.0043	2.22	23.30	8.00×10^{-12}

NOTES: See Table 2.

TABLE 8

Tests for Correlation of After-Tax Real Rate with Other Variables: 1953-1 to 1979-4

Dependent Variable: Ex Post Real Rate, Adjusted for .33 Marginal Tax Rate
Estimation Method: Ordinary Least Squares

Model #	Independent Variable	Coefficient of: Constant Term	Coefficient of: π(-1)	Coefficient of: Independent Variable Lagged Once	R^2	SE	DW	F-test for significant explanatory power of four lags of independent variable when included in this model
8.1	money growth (M1)	-.0001 (-.15)		-.2321 (-3.30)	.09	.0058	.99	7.29
8.2	money growth (M1)	.0025 (3.21)	-.4569 (-8.47)	-.0605 (-1.04)	.46	.0046	2.37	1.20
8.3	real GNP growth	-.0032 (-4.30)		.0875 (1.55)	.02	.0061	.90	1.44
8.4	real GNP growth	.0022 (2.75)	-.4822 (-9.16)	-.0178 (-.41)	.46	.0046	2.40	1.08
8.5	GNP gap	-.0024 (-3.41)		-.0001 (-.00)	.00	.0062	.82	.70
8.6	GNP gap	.0019 (2.66)	-.4778 (-9.40)	.0078 (.48)	.46	.0046	2.40	.77
8.7	unemploy-ment rate	.0013 (.55)		-.0719 (-1.65)	.02	.0061	.86	1.10
8.8	unemploy-ment rate	.0025 (1.42)	-.4733 (-9.13)	-.0103 (-.31)	.46	.0046	2.39	.26
8.9	investment to capital ratio	.0046 (.54)		-.0722 (-.84)	.01	.0062	.82	.82
8.10	investment to capital ratio	.0075 (1.20)	-.4754 (-9.39)	-.0564 (-.88)	.46	.0046	2.41	.27

NOTES: See Table 3.

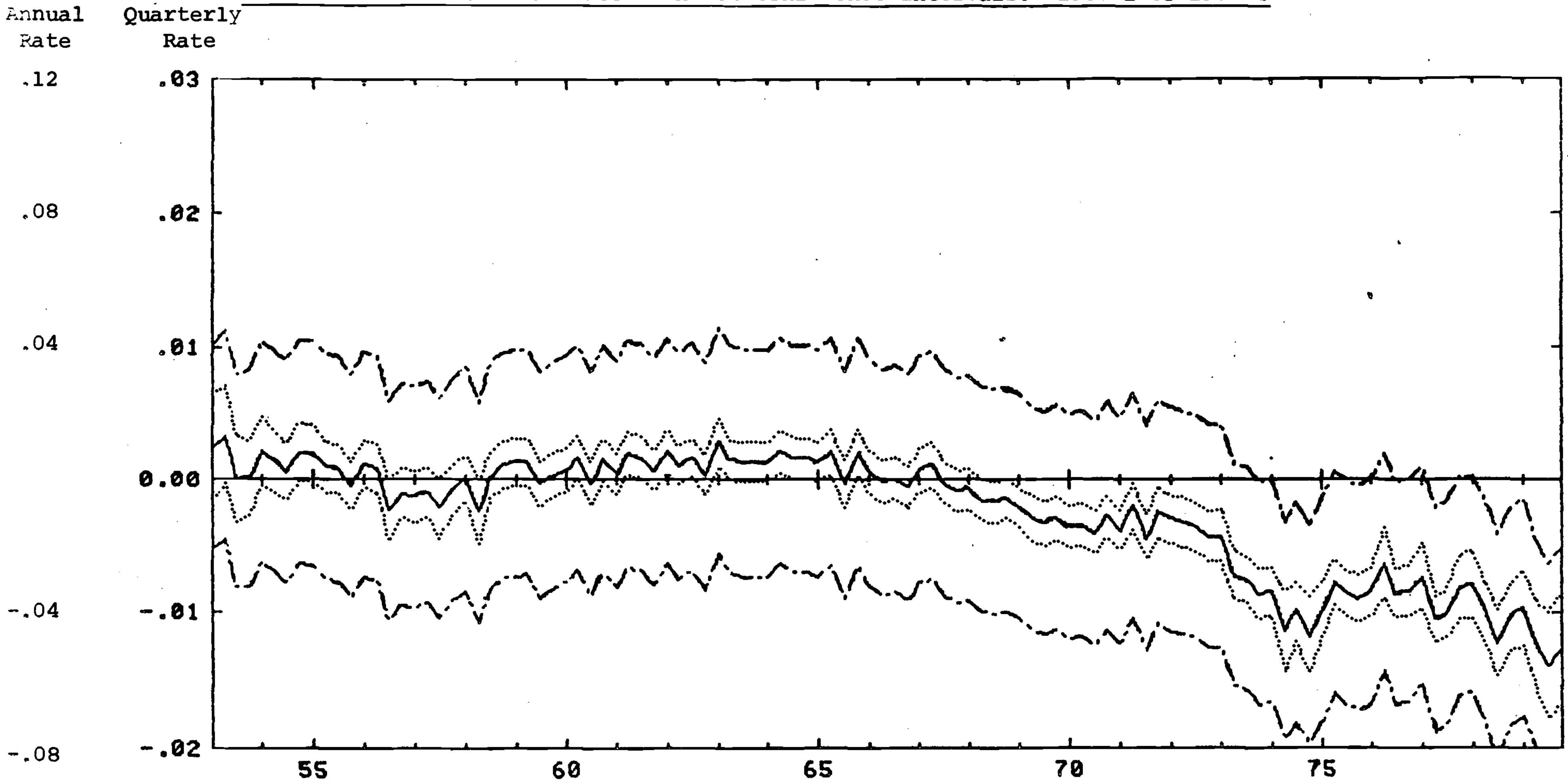
rate, the coefficient on lagged inflation becomes even more negative and statistically significant. Tax adjustment of the ex post real rate thus generates conclusions similar to and even stronger than those found when it was not adjusted. Clearly, use of a different tax rate would not alter the direction of these results.

Table 8 shows that the conclusions on the correlation of real rates with other variables do not change when a tax adjustment is made. Feldstein (1980a) makes the case that monetary policy will be non-neutral, because even if it leaves the unadjusted real rate unchanged when it causes higher inflation, the after-tax real rate will fall. The results in Table 3 indicate that, even without a tax adjustment, the real rate has fallen with faster money growth. However, as the 8.1 and 8.2 results show, when the real rate is adjusted for taxes, this negative correlation is even stronger.

The estimated after-tax real rate derived from the fitted values of 7.3 along with its upper and lower bound 95% confidence intervals is plotted in Figure 7. In the first half of the 1953-79 period, the after-tax real rates appear to be mostly positive, yet even the lower bound confidence interval rarely rejects the hypothesis that the after-tax real rate is zero. In the late 1960s the estimated after-tax real rate turns negative and from 1969 through the end of the sample period it is significantly negative using the lower-bound standard errors. Note that towards the end of the period, even the upper-bound standard errors indicate that it is significantly negative. In the 1970s, the estimated after-tax real rate is frequently below -4% at an annual rate and is more than six of the lower-bound standard errors away from zero.

FIGURE 7

Estimated After-Tax Real Rate with 95% Confidence Intervals: 1953-1 to 1979-4



Estimated after-tax real rate, $\hat{rr}_t^{at}$, from Model 7.3

$\hat{rr}_t^{at} \pm 2\hat{SE}_\ell(\hat{rr}_t^{at} - rr_t)$

$\hat{rr}_t^{at} \pm 2\hat{SE}_u(\hat{rr}_t^{at} - rr_t)$

FIGURE 8

Comparison of Estimated After-Tax Real Rates from Models 7.1 and 7.3: 1953-1 to 1979-4

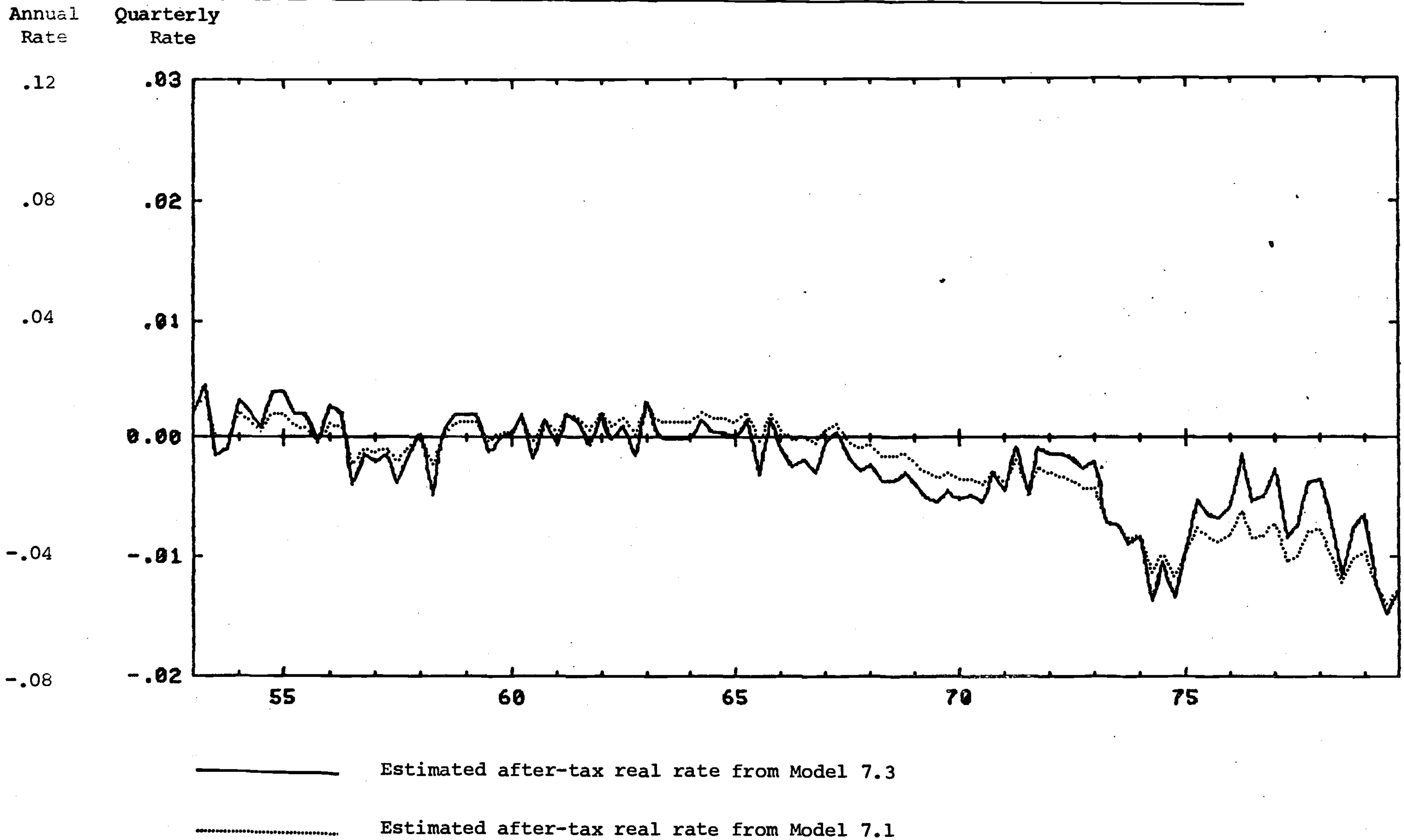
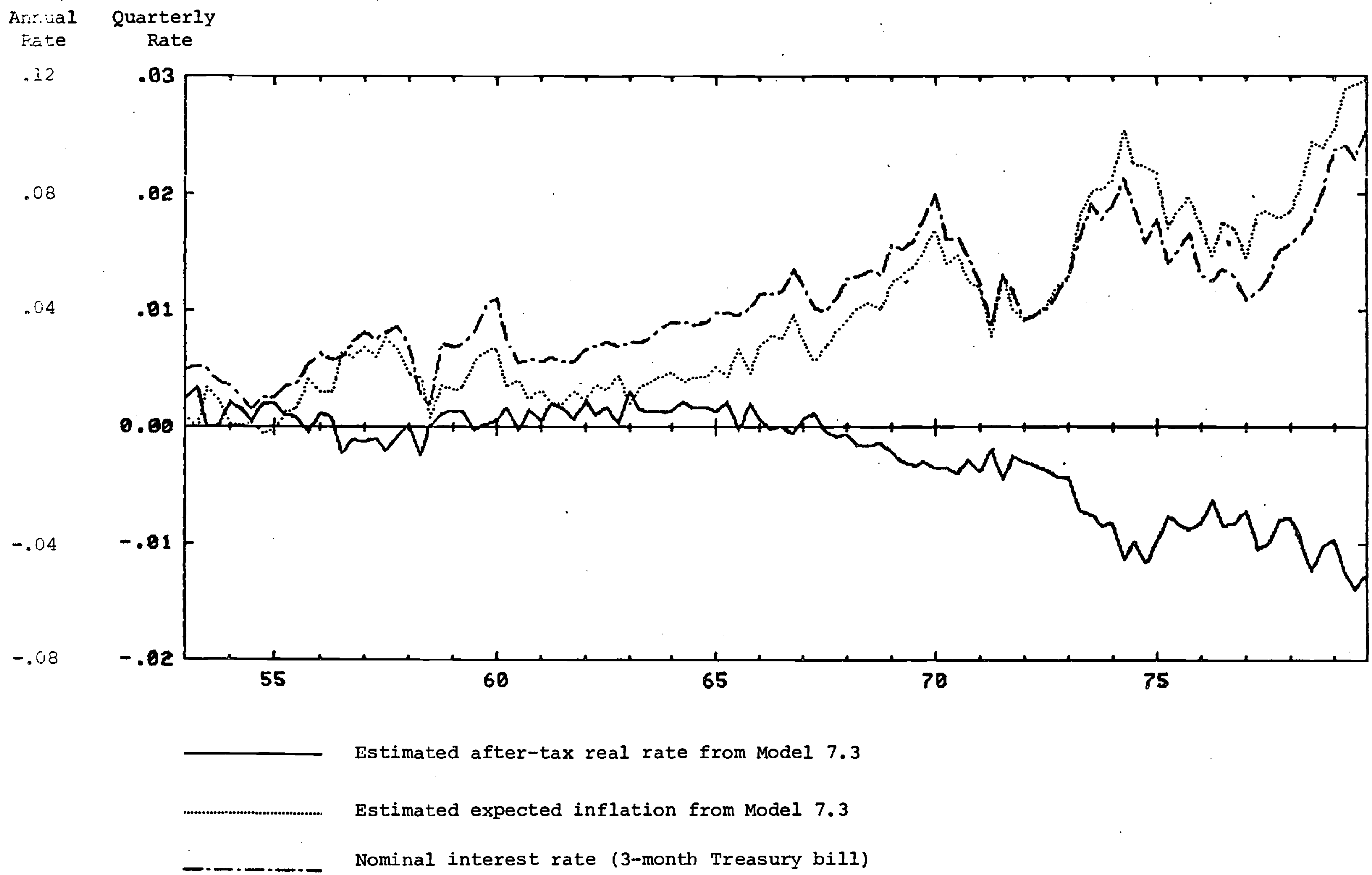


FIGURE 9

Nominal Interest Rate, Estimated After-Tax Real Rate and Estimated Expected Inflation: 1953-1 to 1979-4



Figures 8 and 9 which correspond to 2 and 3 are included in the paper for the sake of completeness. They reveal no surprises. As is evident from Figure 8, the time trend variables play a similar role in altering the estimate of the after-tax real rate that they do for the estimate of the real rate unadjusted for taxes. Furthermore, the estimate of expected inflation and its relationship to the nominal interest rate in Figure 9 is similar to that found in Figure 3.[40] Figure 9 does illustrate the expected finding that the correlations of the estimated after-tax real rate with the estimates of expected inflation and the nominal interest rate are even more negative--the respective correlations equal -.96 and -.80--than is the case when the real rate is unadjusted for taxes. The conclusion that the nominal interest rate is a misleading guide to the tightness of monetary policy is then given even greater support when we view the after-tax real rate as the vehicle through which monetary policy is transmitted.

IV. CONCLUSIONS

This paper has analyzed the movements of real interest rates over the 1931-79 period with a statistical methodology that has the advantage of reliability. Despite the low statistical power of the approach used here, many interesting findings have been gleaned from the data. These findings are listed below.

1. The hypothesis that the real rate is constant is strongly rejected both for the 1953-79 period as well as the 1931-52 period. Fama's (1975) finding that the constancy of the real rate could not be rejected is the exception and not the

rule. His result seems to stem from the lack of variation in the real rate for his sample period. The adjustment of the real rate for taxes lends even stronger support to the position that the after-tax real rate, which is probably more relevant to economic decisions, is not constant.

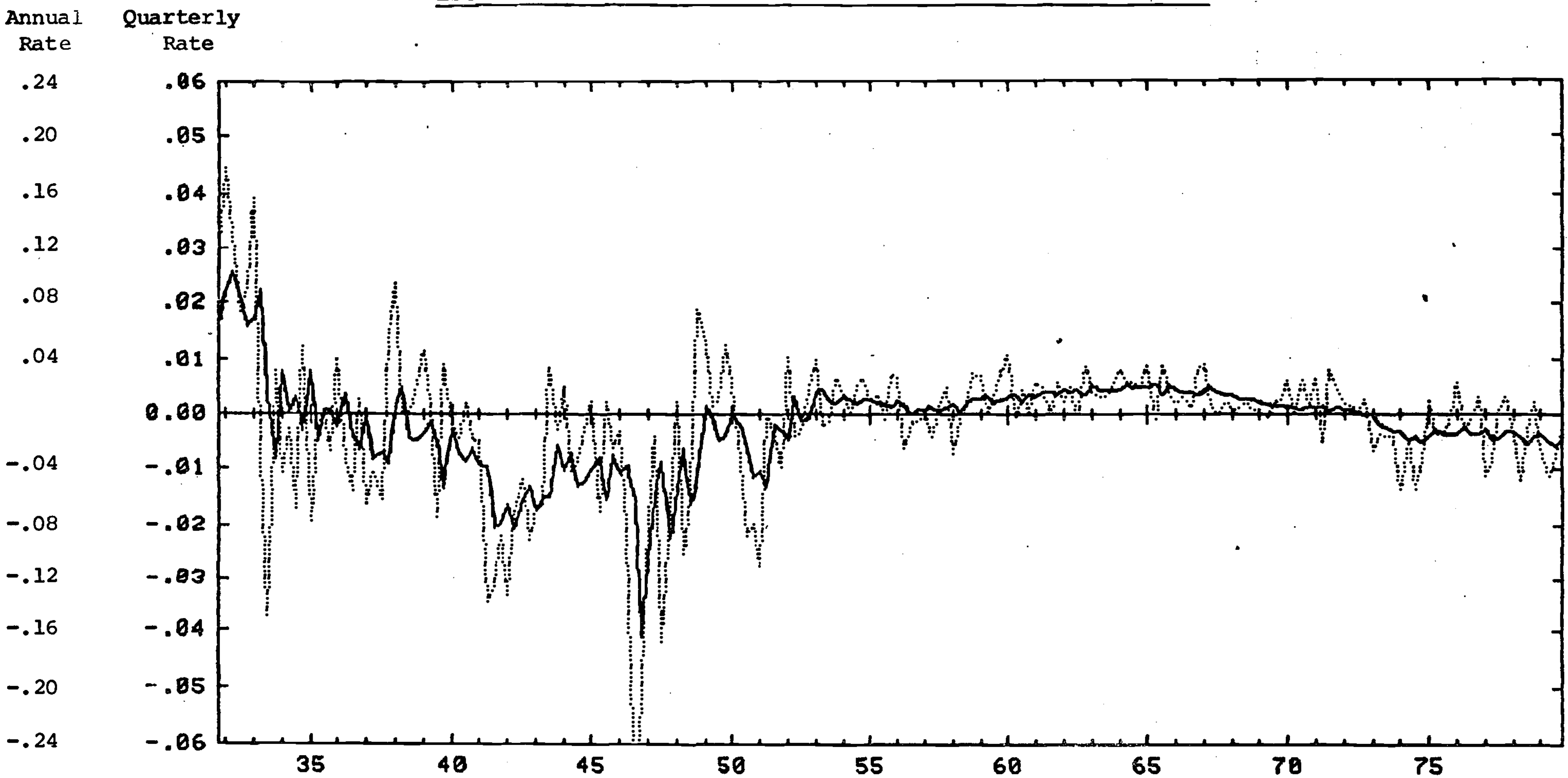
2. The real rate, whether adjusted or unadjusted for taxes, is negatively correlated with inflation. This result is found for both the 1953-79 and 1931-52 sample period, and again Fama's (1975) failure to find this result appears to be due to a lack of variation in the real rate data.
3. Movements in real variables are not significantly correlated with ex post real rates, whether adjusted or unadjusted for taxes. The failure to find these correlations is more likely the result of small cyclical variation of real rates rather than the absence of relationships between real variables and real rates.
4. Increased money growth is associated with a decline in real rates. This non-neutrality result is even stronger when the real rate is adjusted for taxes. Little evidence was found that money growth affects real rates other than through its effect on inflation, but again this could be due to the low power of the statistical tests used here.
5. The real interest rate appears to have been positive in the 1950s and 1960s but has since turned negative in the mid- and late 1970s. When the real rate is adjusted for taxes, there is less evidence that it has been positive in the postwar period and there is strong evidence that it has been negative since 1969.

6. Real rates were extremely high during the contraction phase of the Great Depression and have never been as high since. From the perspective of real interest rates, money has never been tighter than during this period.
7. The period where nominal interest rates were effectively pegged from 1938-1951 has been a period where real interest rates have been negative.
8. Estimates of expected inflation rates have been derived from models of the real rate, and these estimates have several advantages over those obtained from surveys or Box-Jenkins' ARIMA models. The estimates of expected inflation confirms Fama's (1975) view that in the postwar period there is a strong Fisher effect where expected inflation and nominal interest rates move together. However, no Fisher effect is evident in the pre-1953 sample period.
9. Movements in nominal interest rates are not a reliable indicator of movements in real rates, whether adjusted for taxes or not. This is true of both the 1931-52 and 1953-79 sample period. The correlation of estimated real rates and nominal interest rates is low in the 1931-52 sample period and is even negative in the postwar period.

The most important policy implication from these results is the recommendation that policymakers, politicians and the public should not focus on nominal interest rates as an indicator of the tightness of monetary policy. Not only do the results here indicate that nominal interest rates contain little information on real interest rates and hence on the tightness of monetary policy, but they also indicate that nominal interest rates have been a highly misleading indicator of monetary tightness during some crucial business cycle episodes. For example, real interest rates appear to have been extremely high during the Great Depression downturn, and yet, by the criterion of low nominal interest rates, this period appears to have been one of "easy money" rather than the reverse. Knowledge of the true state of monetary tightness might have led the Fed to take more appropriate policy actions. Similarly, the period of the 1970s appeared to be one of "tight money" by the nominal interest rate criterion, yet real rates appear to have been quite low. Knowledge of how "easy" money truly was might have encouraged a less expansionary monetary policy, resulting in less inflation.

FIGURE A1

Estimated Ex Post and Ex Ante Real Rates: 1931-4 to 1979-4



Estimated ex ante real rate from Figures 1 and 4.

Ex post real rate.

DATA APPENDIX

VARIABLES AND SOURCES OF DATA

π_t = inflation rate (at quarterly rate) = $\log(CPI_t/CPI_{t-1})$.

CPI_t = seasonally unadjusted consumer price index for the last month of quarter t: postwar data obtained from the Survey of Current Business and Business Statistics; prewar data obtained from the BLS.

i_t = the nominal return (at a quarterly rate) over the quarter of a 3-month treasury bill maturing at the end of the quarter = $\log(100/PB_t)$.

PB_t = the average of the bid and asked price at the beginning of the quarter for the 3-month bill maturing at the end of quarter t: obtained from the Center for Research in Security Prices (CRSP) bond file.

$eprr_t$ = the ex post real rate for the 3-month bill maturing at the end of quarter t = $i_t - \pi_t$.

money growth $(M1)_t$ = the log of M1 for the last month of quarter t minus the log of M1 for the last month of quarter t-1: data obtained from Business Statistics and the Survey of Current Business.

real GNP growth$_t$ = the log of real GNP for quarter t (GNP_t) minus the log of real GNP for quarter t-1: data obtained from Business Statistics and the Survey of Current Business.

GNP gap$_t$ = $\frac{POTGNP_t - GNP_t}{POTGNP_t}$: $POTGNP_t$ is potential GNP for quarter t, estimated by the Council of Economic Advisors and obtained from them.

unemployment rate$_t$ = the unemployment rate for the last month of the quarter t: obtained from Business Statistics and the Survey of Current Business.

investment to capital ratio_t = expenditures on housing and business fixed investment in plant and durable equipment in quarter t, divided by the stock of residential, plant and durable equipment capital at the end of the quarter t: data obtained from the (MPS) Quarterly Econometric Model data bank at the Board of Governors of the Federal Reserve.

FOOTNOTES

[1]For example, see Modigliani (1974).

[2]See Mayer (1978) and Meltzer (1976).

[3]Howrey and Hymans (1978), Boskin (1978), and Summers (1978).

[4]Recent empirical results on this issue are quite mixed. For example, Boskin's (1978) study has been strongly challenged by Howrey and Hymans (1978).

[5]All returns, inflation and interest rates discussed in this paper are continuously compounded, so that additional second-order terms are not necessary in the Fisher equation (1).

[6]For example, see Gibson (1972), Cargill (1976), Lahiri (1976), Carlson (1977), Levi and Makin (1979) and Tanzi (1980).

[7]One obvious danger with survey data is that there may have been very little incentive for the respondents to answer accurately. A more subtle point that is often unrecognized in the literature is that the behavior of market expectations need not reflect the average expectations of participants in that market. Market expectations are frequently believed to be rational, but not because all, most or even the average market participant is also believed to be rational. Rather, rational expectations are plausible because market expectations can be driven to the rational expectations equilibrium by the elimination of unexploited profit opportunities. This arbitrage view of expectations formation clearly allows the average expectations of market participants to differ from the market's expectations. (Mishkin (1978) and (1981b) discuss this point more extensively.) On theoretical grounds alone then, we should be skeptical of using survey data to measure inflation

expectations in a market.

In addition, a recent empirical study by Pearce (1979) raises further doubts about the accuracy as a measure of bond market expectations of the survey data most frequently used in this literature, Livingston's. Pearce finds that over the 1959-76 period the Livingston survey data predict inflation substantially worse than a simple (0,1,1) ARIMA model estimated only on data available at the time of the forecast. As Pearce notes, this result implies that the Livingston expectations data do not fully exploit information on past inflation rates and are thus inconsistent with rationality. If the bond market is believed to be efficient and hence rational--and there is evidence to support this view--then Pearce's finding suggests that the Livingston data do not accurately measure bond market expectations of inflation. For example, using the sample period closest to Pearce's 1959-2 to 1976-4, I conducted the same test found in Mishkin (1981b) for the rationality of inflation expectations in the bond market and could not reject rationality. The $\chi^2(6)$ statistic was 3.4, while its critical level at 5% is 12.6. Pearce also regresses nominal interest rates on both the Livingston measure of inflation expectations as well as the ARIMA model measure. Not only is the Livingston measure outperformed by the ARIMA measure in terms of goodness of fit, but also the Livingston measure adds no significant additional explanatory power when added to a Fisher equation using the ARIMA measure, while the reverse is not true. Given that a strong Fisher effect (positive correlation) between expected inflation and nominal interest rates is expected in the postwar period, this evidence is also not supportive of using the Livingston data in the Fisher equation (1).

[8] See Fama (1970).

[9] See Mishkin (1981b).

[10] For example, an assumption like plim $\frac{X'u}{n} = 0$, where n is the number of observations, performs a similar role in the stochastic X case as the assumption that $E(u) = 0$ (or equivalently $E(X'u) = 0$) performs in the non-stochastic X case.

[11] I.e., plim $\frac{X'X}{n}$ exists.

[12] An even stronger statement can be made if a particular form of Muthian expectations discussed by Sargent (1979, Chapter 10) is used to form the ex ante real rate. If the ex ante real rate is formed by least-squares projection on information that includes X_{t-1}, then $X'\varepsilon = 0$ and the OLS estimate of β from (4) will exactly equal that in (5), even in finite samples. This is a manifestation of the law of iterated projections discussed in Sargent (1979, Chapter 10).

[13] Cumby, Huizinga and Obstfeld (1980) point out that in a setup of the type discussed here that even if the u's are serially uncorrelated as are the ε's because of rationality, this does not rule out serial correlation in the composite error term $u - \varepsilon$. This occurs because rationality does not rule out correlation of the forecast errors with future values of variables such as u. This is why the fifth assumption in (8) is needed. If serial correlation does exist in the composite error term, correcting it with standard techniques leads to inconsistent estimates. For example, see Flood and Garber (1980). An estimator that avoids this problem is discussed in Cumby, Huizinga and Obstfeld (1980).

[14] A similar argument to that here is another way to explain the position taken by Fama (1976a) and Mishkin (1978) that when actual returns have large variability relative to expected returns, accurate specification of the model of market equilibrium is not critical to tests of market efficiency.

[15]Note that this formula is not invalidated if $E(X'u) \neq 0$ as long as u is a linear function of X. I.e., if $u = X\delta + \eta$ where η is also a linear function of X and so on, then $E(\hat{rr}_t - rr_t) = 0$. Thus consistency of the $\hat{\beta}$ estimate is not a necessary condition for the validity of the 95% confidence intervals used in the figures. However, exclusion of relevant variables can invalidate the formula.

[16]The variance of a post sample error would be $(\sigma_\varepsilon^2 + \sigma_u^2)X_{t-1}(X'X)^{-1}X_{t-1}' + \sigma_u^2$ which is larger than the within-sample variance in (13). This occurs because the term $E(u_t(u - \varepsilon)')$ equals zero in this case. Because the real rate estimates later in the text are all within sample, the within-sample variance, rather than the post sample variance, is discussed in the text.

[17]If we denote $X_{t-1}(X'X)^{-1}X_{t-1}' = Q$ and $\hat{\sigma}_u^2 = \alpha\hat{\sigma}^2$, then the estimated $\widehat{VAR}(\hat{rr}_t) = [(1 - \alpha)\hat{\sigma}^2 - \alpha\hat{\sigma}^2]Q + \alpha\hat{\sigma}^2$. $\frac{d\widehat{VAR}(\hat{rr}_t)}{d\alpha} = \hat{\sigma}^2[1 - 2Q] > 0$ if $Q < 1/2$. Thus the lower bound estimate is reached when $\alpha=0$ and the upper bound estimate when $\alpha=1$.

[18]The data used here are very similar to that used by Fama (1975). Using his sample period I was able to reproduce very closely his results with quarterly data.

[19]For example, excluding the 1971-3 to 1974-4 quarters from the 1953-79 sample period, the coefficient on $\pi(-1)$ in model 1.2 dropped to -.2469 but remained significantly negative with a t-statistic of -4.41. A Chow test of the 2.1 model splitting the 1953-79 sample period into its price control and non-price control periods did not reveal significant changes in the parameter estimates. $F(2,104) = 2.36$ while the critical F at 5% is approximately 3.1.

[20]See Fama (1970).

[21]For example, see the discussion of the Shiller (1979) results in Mishkin (1978) and (1981a).

[22]For example, in model 2.1 a Goldfeld-Quandt (1965) test which excluded 16 observations yielded the $F(44,44)$ statistic of 1.51, while the critical F at the 5% level is approximately 1.7. A Glesjer (1969) test where the absolute value of the residuals in 2.1 was regressed against a time trend yielded a t-statistic on the trend variable of only .94. I also regressed the absolute value of the residuals from the Table 2 model against lagged inflation because some recent literature has suggested that the variability of inflation forecasting errors, ε's, might rise with inflation. The evidence for this was weak. For example, with the 2.1 residuals the t-statistic on lagged inflation was only .72. Furthermore, a heteroscedasticity correction of the type suggested by Glesjer (1969)--i.e., weighting by the fitted values from this regression--has only a slight effect on the results.

[23]A proxy for the expected inflation rate derived from an ARIMA (0,1,1) inflation model with a seasonal MA1 term was also used in estimation and the results were very similar. For example, in a model of the form 2.1 when the expected inflation proxy replaced the lagged inflation rate, its coefficient was -.3658 with a t-statistic of -7.03. I preferred not using this proxy variable in the text because the results there clearly indicate that other information besides past inflation rates are used in the bond market to forecast inflation.

[24]For example, the null hypothesis that when added to model 2.2, the coefficients on four additional powers of time, up to the eighth power, equal zero could not be rejected. $F(4,99) = .95$ while the critical F at 5% is approximately 2.5.

[25]One issue not discussed in the text is the seasonality of the real rate. For a detailed discussion of this issue, see Fama and Schwert (1979) and Shiller (1980). I corroborated both of these studies' findings that seasonality is present in the ex post real rate. For example, a test of whether seasonal dummies entered model 2.1 did reject the null hypothesis that coefficients and seasonal dummies equal zero--($F(3,103) = 5.93$ with a critical value at 5% of 2.7)--yet the results on the coefficient of lagged inflation changed hardly at all. The coefficient on lagged inflation was estimated to be -.3170 with a t-statistic of -6.67. However, more time is not spent on this issue here for two reasons. One is that an extensive treatment of this issue is contained in the two studies mentioned above. Second, as the discussion in Fama and Schwert (1979) indicates, the fact that seasonality exists in the real rate is not evidence against the constancy of the real rate net of storage costs, and this is the more interesting hypothesis anyway. In addition, the fixed-weight nature of the CPI may induce some seasonality in the measured ex post real rate that is spurious. When the relative price of a good in the CPI consumption bundle is seasonally high--fruit during the winter is an example--we expect that there will be substitution away from this good. Thus the seasonality in the true price index should be less than in the measured fixed-weight index. In the data here, the seasonality in ex post real rates comes predominantly from the price index. Hence, the seasonality of the measured ex post real rate constructed using the fixed-weight index will be overstated.

[26]An industrial production growth variable as well as other variations of the investment-to-capital-ratio variable suggested by Fama and Gibbons (1980), including one using a new order series for durable goods, were also tested as in Table 3. Again, no significant correlations were found with the ex post real rate.

[27] Fama (1976b) has suggested that the variability of inflation forecast errors (as measured by the variability of the ex post real rate), as well as the variability of expected inflation (as measured by the variability of the change in nominal t-bill rates), might affect risk premiums and hence real rates. Experiments with variables similar to Fama's (1976b) did indicate that the variability of these variables was significantly negatively correlated with ex post real rates. However when lagged inflation was also included in these regressions, the coefficients of these variables no longer remained significant.

Fama and Gibbons (1980) suggest that expectations of higher returns on investments and hence the investment-capital ratio will be positively correlated with real rates. As Table 3 indicates, experiments with the investment-capital ratio did not yield significant correlations. The failure of this variable could have resulted because it is too loosely correlated with expectations of investment opportunities. One crude variable that might reflect these expectations would be the real value of the Standard and Poor's index of stock prices which measures one component of the valuation of firms. Results with this variable were somewhat, but not strongly, encouraging. One lag of this variable was almost, but not quite, significantly correlated with ex post real rates (t = 1.95) and had the expected positive sign. Four lags were significant in a regression that excluded lagged inflation: F(4,103) = 3.23, while the critical value at 5% is approximately 2.5. In regressions with lagged inflation included, one lag of this variable was again almost significant (t = 1.95) and four lags were not significant (F(4,102) = 1.97). The results are encouraging enough so that further work on this issue might be worthwhile.

[28] When M2 money growth replaces the M1 measure in 3.1, the coefficient on the M2 growth variable lagged only once is -.1475 with a t-statistic of -2.65, and the F-statistic on the four lags is 2.89 with the sum of the

coefficients equal to -.2155. In the 3.2 result, the coefficient on M2 growth lagged once is -.0571 with a t-statistic of -1.11 and the F-statistic on the four lags is .66.

[29]In the regression with four lags of the M1 money growth variable, the sum of the M1 money growth coefficients is -.2781.

[30]Seasonally unadjusted M1 and M2 data were also used in the 3.1 and 3.2 regressions. In 3.1, the coefficient on the money growth variable lagged only once is insignificantly positive, but the four lagged money growth variables do have significant explanatory power: for M1 $F(4,103) = 5.40$ and for M2, $F(4,103) = 3.35$. Furthermore, the sum of their coefficients is again negative: -.2626 for M1 and -.1276 for M2. In 3.2, the coefficient on M1 lagged only once is significantly positive for M1--it is .0367 with a t-statistic of 2.26--while the coefficient on M2 lagged only once is positive and insignificant. The four lagged M2 variables again do not have significant additional explanatory power--$F(4,102) = 1.87$ with the critical F at 5% equal to 2.5--yet the four lagged M1 variables do--$F(4,102) = 3.85$. However, there is no easily discernible pattern to these coefficients which are both positive and negative and sum to -.0240. This result is intriguing and should be pursued further. Overall then, the evidence with seasonally unadjusted M1 and M2 data is more mixed, although it does continue to support the suggestion that increased money growth is non-neutral because it lowers the real rate by raising inflation.

[31]Garbade and Wachtel (1978) report the standard error of their estimate α_t as the standard error of the estimated real rate. Hence this standard error is also a lower-bound estimate because implicitly they are assuming that $\sigma_u = 0$. Their standard errors are also similar in magnitude to the lower-bound standard errors of Figure 1.

[32]The nominal interest rate adjusted with a constant is one such estimate, if, as in Fama (1975), the real rate is assumed to be constant.

[33]This ARIMA model was identified with the Box-Jenkins (1970) procedure and was subjected to the usual diagnostic checks and found to be adequate. Pearce (1979) also specifies inflation as an ARIMA (0,1,1) but does not need to specify any seasonal terms since he worked with seasonally adjusted data rather than unadjusted data as here.

[34]If seasonal dummies are added to model 2.3, the standard error of the $\hat{\pi}_t^e$ estimate drops even further to .0039. Thus this comparison becomes even more favorable to the $\hat{\pi}_t^e$ estimate.

[35]For example, in model 5.1, a Goldfeld-Quandt test which excluded seventeen observations yielded the F(32,32) statistic of 1.21 while the critical F at 5% is approximately 1.8. A Glesjer test where the absolute value of the residuals in 5.1 was regressed against a time trend variable yielded a t-statistic on the trend variable of -1.66. A similar test when the absolute value of the residuals was regressed on lagged inflation yielded a t-statistic on the lagged inflation coefficient of -.58.

[36]A comparison of the standard error of 2.1 and 5.1 indicates that the variance of the error term is over fourteen times higher in the pre-1953 than in the 1953-79 sample period. This is a very significant rejection of the null hypothesis that there is homoscedasticity: F(83,106) = 14.43 with the critical F at 1% approximately equal to 1.7. The Chow test mentioned above used weighted least squares in estimation where the data for the 1931-52 observations were weighted by the standard error of 5.1 and the 1953-79 observations by the standard error of 2.1.

[37]In the 5.2 model three additional trend variables raised to the third, fourth and fifth power did add significant explanatory power: $F(3,79) = 3.03$ with the critical F at 5% equal to 2.7. However, in the 5.3 model these variables did not add significant explanatory power: $F(3,78) = 1.74$ with the critical F at 5% equal to 2.7. In any case, the same conclusions were reached using either the second-order or the fifth-order polynomial in the above analysis.

[38]For example, Meltzer (1976) and Mayer (1978).

[39]For example, Wright (1969), Darby (1975) and Tanzi (1980). Note that Levi and Makin (1978) also use this tax rate on interest payments for illustrative purposes.

[40]Once lagged inflation, which is highly correlated with the lagged nominal interest rate, is included as an explanatory variable in a real rate regression, the results for additional explanatory variables are expected to be very similar, regardless of whether the real rate is adjusted or unadjusted for taxes. To see this, just realize that if the lagged nominal interest rate were an explanatory variable, then a regression with the after-tax real rate as the dependent variable is just a linear transformation of the regression with the unadjusted real rate as the dependent variable. In this case the results on additional explanatory variables would be identical in either regression. They should therefore be quite similar if the lagged nominal interest rate is replaced by the lagged inflation rate. This same point explains why the estimates of expected inflation from the after-tax versus unadjusted real rate models 2.3 and 7.3 should also be so similar.

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