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Transition Losses of Partially Mobile Industry-Specific Capital

ABSTRACT

Comparative static models typically assume homogeneous and mobile factors in estimating the economic effects of a tax policy change. Even dynamic models employ a given homogeneous capital stock in two different allocations for the first period of two equilibrium sequences. This malleable capital assumption causes overstatement of early efficiency gains from policies designed to improve factor allocation. On the other hand, immobile factor models would understate such gains by assuming that no capital ever relocates.

The model in this paper attempts to bridge this gap by restricting each industry's capital reduction to its rate of depreciation. The stock of depreciated capital from the previous period represents an industry-specific type of capital which may earn a lower equilibrium return. The usage of mobile capital above this minimum constraint is limited by the total gross saving of the economy, including all industries' depreciation and consumer net saving.

The industry-specific capital model suggests, for example, that previous estimates of the dynamic efficiency gain from full integration of personal and corporate taxes in the U.S. are overstated by about $5 billion. The model could also be used to estimate distributional impacts on individuals with more than proportionate ownership of capital in particular industries.

Don Fullerton
Woodrow Wilson School of Public and International Affairs
Princeton University
Princeton, New Jersey 08544
(609) 452-4811
1. Introduction

General equilibrium tax incidence models from Harberger's (1962) path-breaking two-sector analytical model to larger and more recent computational models have typically assumed perfectly mobile factor supplies in a comparative static framework. McLure (1971) extended the analytical comparative static model to include immobile factors, but still in fixed total supply. He obtained the now-familiar result that an immobile factor is less able to escape taxation whether imposed on that factor, on output, or indeed on the other factor in a sector with a low substitution elasticity. This model compared two short-run static equilibria, so the factor was never in fact given a chance to move between sectors.

The analytical model was extended to a dynamic, growing economy by Feldstein (1974), who showed that the incidence of a factor tax depends on its supply elasticity. His model had one producing sector, however, and did not address the issue of immobile factors.

The size of the tax incidence problem was expanded by computational models in Shoven and Whalley (1972), and again in Fullerton, Shoven, and Whalley (1978). These models have more sectors and other extensions, but are still comparative static with perfect factor mobility. The switch to a growing economy occurs in Fullerton, King, Shoven, and Whalley (1980), where a dynamic version of the model is used to evaluate the integration of personal and corporate income taxes in the U. S. The dynamic model allows the
saving response of one static equilibrium to affect the capital stock of the next static equilibrium in a sequence of calculations. In any given equilibrium, however, there is a fixed total capital stock and no barriers to factor mobility. This homogeneous capital is owned by the twelve consumers in various proportions and can be allocated to any of the nineteen producers. In particular, the capital endowment in the first period of the original (base-case) sequence is used as the stock available in the first period of the simulated (revise-case) sequence. Incidence estimates are obtained by comparing the time path of the economy with the original tax system to the path of the economy resulting from the hypothetical or proposed tax system.

None of these models consider a growing economy with temporarily immobile factors. In fact, this is a great limitation of all comparative equilibrium models, whether static or dynamic. They can describe all attributes of the economy or path associated with an alternative policy regime, but they cannot describe the effects of imposing that policy. They miss the efficiency effects of misallocated factors which are slow to depreciate, transfer, or retrain, and they miss the distributive effects of wide swings in relative prices that are necessary to induce eventual re-location.

The comparative equilibrium models are also logically inconsistent when they use a given capital stock in two different allocations. Consider two possible interpretations of the revised equilibrium calculation. First, if the tax rates were changed today, one could think of moving from the original to the revised equilibrium. The problem with this putty-putty capital assumption is that nonzero net saving over time would imply a dif-
ferent capital stock for the revised calculation.

The second interpretation of the simulation is that it represents a counterfactual world that might have existed today had the tax rates been different from the start. The problem here is that if the different tax rates had caused a welfare gain in each period, and if people saved some of this additional income, then the capital stock would still have to be different in the simulated equilibrium. Similarly, if the revised rates implied a different equilibrium price of capital in each period, and if saving responded to the rate of return, then again, there would have to be a different capital endowment for the counterfactual equilibrium calculation. For positive saving elasticities with respect to income or to the rate of return, neither interpretation should allow the simulation to use the same capital stock as the benchmark.

The model in this paper is an adaptation from Fullerton, King, Shoven, and Whalley. It accepts the first interpretation above, and can observe the movement from the benchmark equilibrium to the revised equilibrium. The tax change is implemented at the beginning of the first period, and saving of the first period still augments the capital stock for the second period. The difference, however, is that capital is not fully malleable, and cannot move out of an industry faster than that industry's rate of depreciation. Essentially, a putty-clay nature for capital is adopted in order to avoid over-estimating the gains that occur when a new tax scheme is implemented and capital moves to a more efficient allocation.

With capital that is specific to a particular industry, it will be possible to measure the transition cost associated with its relocation. The depreciated capital stock of the original equilibrium could be viewed
as a quantity constraint for an industry, a constraint which is not binding if the new tax system would imply an increase or only slight decrease in that industry's use of capital. For an industry with a binding constraint, even though the old capital has lost none of its productive power, its rental price could fall dramatically, reflecting a decreased demand for the industry's output. Those individuals who own capital specific to a shrinking industry will earn a lower return. National income or welfare will not be as great as it would in a situation without these adjustment costs, that is, if capital were immediately perfectly mobile. The model then allows redepreciation in the following periods, constraints disappear, and the economy approaches a new steady state.

The point of this exercise then, is to develop a methodology for measuring the size and duration of some of these transition costs, in an equilibrium setting. Clearly the gains from an efficiency increasing tax change are limited by the short-run losses of individuals who, responding to tax incentives, had invested heavily in undertaxed industries. The general equilibrium setting is important for large tax changes because of the interactive nature of the price mechanism. New relative prices will determine producer decisions on output and factor demands jointly with consumer decisions on purchases and factor supplies. Input costs will affect prices for capital goods as well as consumption goods, while differential income effects across consumer groups can further influence commodity demands and relative prices. These effects hold equally for the low relative price of a temporarily over-abundant factor specific to a contracting industry.

For two reasons, this paper will use the full integration of U. S.
corporate and personal taxes as an example of the new methodology. First, this proposed tax change is expected to have large interindustry reallocations and interpersonal redistributions for which the general equilibrium setting is important. Second, this example will permit direct comparison with published results of the previous model. As shown below, the efficiency gains of this reform were indeed overstated by about $5 billion when transition losses were ignored.

Although this paper continues to use twelve consumers grouped by gross income class, the methodology introduced here would be particularly useful to study tax changes with unbalanced portfolios. An urban/rural distinction could be used for consumer groups in order to capture their disproportionate holdings of agricultural capital and the large immediate redistributive effects of a farm tax change. Similarly, grouping consumers by age or retirement status could capture their disproportionate imputed homeownership income and the redistributive effects of housing tax policy. There may be substantial time required for reallocation of resources as consistent with a new tax treatment of these assets.

The existing general equilibrium model of Fullerton, Shoven, and Whalley is briefly described in the next section. More thorough explanations can be found in previous papers by those authors, and the familiar reader may want to pass over this section. This paper will therefore emphasize changes to equations and to equilibrium conditions that were necessary to build the new model. These changes are described in section three along with a few of the problems encountered in this modelling effort. Features of the old model which are not specifically mentioned in this section were left intact. Because the particular quantity constraints depend on the tax
replacement, section four begins the full integration example with a
derivation of these minimum capital usages for each industry. Then sec-
tion five reports results of the new model together with estimates of the
effects of integration from the previous model. A final section six pro-
vides conclusion.

2. Carryover Features of the General Equilibrium Model

The production side of the model includes 19 profit-maximizing pro-
ducer goods industries which each use labor and capital in a constant
elasticity of substitution (CES) or Cobb-Douglas production function. Sub-
stitution elasticities are chosen for each industry as a best-guess from
available literature, ranging from 0.6 to one. The *Survey of Current
Business* and unpublished data from the Commerce Department's National Income
Division are used to obtain each industry's payments for labor and capital,\(^1\)/
while quantity usages derive from the convention that a unit of each primary
factor is that which earns one dollar net of taxes in the 1973 benchmark
year. A fixed coefficient input-output matrix is derived from Bureau of
Economic Analysis tables.

An ad valorem tax on each industry's use of capital is comprised of the
corporation income tax, state corporate franchise taxes, and local property
taxes. The Social Security tax and workmen's compensation are modelled
as an ad valorem tax on industry use of labor. Various Federal excise
taxes and indirect business taxes are modelled as an output tax rate for
each of the 19 industries, while state and local sales taxes apply to each
of the 15 consumer goods in the model.

Each producer good can be used directly by government, for export, or
for investment, but indirectly for consumption through a fixed-coefficient
"G" matrix of transition into one of 15 consumer goods with suitable definition for consumer demand. This transition is necessary because the Commerce Department data includes industries such as mining, electrical manufacturing and trade, while the Labor Department's Survey of Consumer Expenditures provides data on purchases of goods like furniture, appliances, and recreation.

Industry and government payments to buy labor and capital services are just matched by total household receipts from the supply of each factor. The Treasury Department's Merged Tax File provides information on labor and capital income for each of the 12 consumer classes as well as tax payments and an estimate of \( \tau_j \), the average marginal income tax rate for each group. These range from a 1% average marginal rate for the first income class to a 40% rate for the highest income class. A progressive income tax system is then modelled as a series of linear schedules, one for each group. Pensions, IRA and Keogh plans are modelled as a 30% saving subsidy to capture the proportion of saving that now has such tax sheltered treatment.

Further tax advantages are modelled in the "personal factor tax," a construct designed to capture industry discriminating features of the personal income tax. Each industry is assigned a fraction \( f_i \) of capital income fully taxable at the personal level, determined by proportions paid through dividends, capital gains, interest and rent. Taxable capital income is subject to \( \tau \), the overall average marginal personal income tax rate. At the consumer level, rebates are given to groups with a \( \tau_j \) less than \( \tau \), while additional tax is collected from others. The personal factor tax acts just like a withholding tax at the industry level, and consumer level
corrections sum to zero. The model thus favors industries with a high proportion of retained earnings, industries with noncorporate investment tax credit, and the housing industry.

Expanded income of each consumer is given by his transfer income plus capital and labor endowments. The latter is defined as 7/4 of labor income to reflect a possible 70 hour week while working 40 hours. Consumer demands are based on budget constrained maximization of the nested CES utility function:

\[ U = \min_{H, \lambda, \xi} U \left( \frac{15}{11} \frac{\lambda_i}{X_i}, \xi, C_f \right) \]  

In the first stage, consumers save some income for future consumption, \( C_f \), and allocate the rest to a subutility function, \( H \), over present consumption goods. The elasticity of substitution between \( C_f \) and \( H \) is based on Boskin's (1978) estimate of 0.4 for the elasticity of saving with respect to the net-of-tax rate of return. Saving in the model is derived from consumer demands for future consumption under the expectation that all present prices, including the price of capital, will prevail in all future periods. Then income for \( H \) is divided between repurchase of leisure, \( \xi \), and a Cobb-Douglas subutility function defined on the 15 consumer goods, \( X_j \). The elasticity of substitution between \( \xi \) and consumer goods is based on a .15 estimate for the elasticity of labor supply with respect to the net-of-tax wage.

Consumer decisions regarding factor supplies are thus made jointly with their consumption decisions. Demands for leisure and for saving will depend on all relative prices whether for factor endowments or for commodity
purchases. The model simultaneously considers the uses and sources sides of income in measuring the gain or loss of any group.

Saving is converted immediately into investment demand for producer goods, with proportions based on national accounting data for fixed private investment and inventories. The foreign trade sector is modelled by the assumption that the net value of exports less imports for each producer good is constant. This simple treatment closes the model, maintains zero trade balance, and allows easy calculation of trade quantities given prices.

The static model is rounded out by specification of the government sector. Revenues from the various taxes described above are used in a balanced budget for transfers, labor, capital, and producer goods. Lump-sum transfers to each consumer group are based on Treasury Department data for social security receipts, welfare, government retirement, food stamps, and similar programs. Government demands for factors and commodities are in a Linear Expenditure System based on Stone-Geary utility.

Because the data set for this model is so comprehensive, the sources are necessarily divergent. The two sides of a single account are often collected by different agencies with different procedures, and thus do not match. In order to use all of this data together, there must be adjustments to insure that each part is consistent with the rest. To do this, some data are accepted as superior and other data are adjusted to match. All industry and government uses of factors are taken to be fixed, so consumers' factor incomes and expenditures must be scaled. Tax receipts, transfers, and government endowments are fixed, so government expenditures must be scaled to balance their budget. Similar adjustments insure that supply equals demand for all goods and factors. 5/

The fully consistent data set then represents a benchmark equilibrium,
where values are separated into prices and quantities by assuming that a physical unit of each good is the amount that sells for one dollar. Elasticity parameters are imposed exogenously as described above, but the model is solved backward to generate the remaining behavioral equation parameters consistent with the data set. Factor employments by industry are used to derive production function weights, just as expenditures are used to derive utility function and demand function weights. The resulting tax rates, function parameters and endowments can be used to solve the model forward, perfectly replicating the benchmark equilibrium. This particular calibration allows for a test of the solution procedure and insures that the various agents' behaviors are mutually consistent in our benchmark data set.

A variant of Scarf's (1973) algorithm is used to solve for prices in a short-run competitive equilibrium such that abnormal profits are zero and supply equals demand for each good or factor. Simplex dimensions are required only for labor, capital, and government revenue since a knowledge of these three "prices" is sufficient to evaluate all agent behavior. Producer good prices are based on factor prices and zero profits, while consumer good prices are based on producer good prices through the G transition matrix. A complete set of prices, quantities, incomes, and allocations are calculated for every equilibrium. Since it is not based on differential calculus, the computational model could handle any number of market distortions such as quotas or externalities by adding a simplex dimension to account for each one. It can measure discrete changes in any tax or distortion without linearity assumptions and without ignoring income effects. There could be any number of sectors and agents, and any
specification of demand so long as it satisfies Walras' Law. For now, however, the model assumes no involuntary unemployment of factors, no externalities, and no other distortions.

The dynamic model is derived by first assuming that the 1973 consistent data set or benchmark equilibrium is one that lies on a steady state growth path. Observed saving behavior and the capital endowment are translated into an annual growth rate for capital, \( n \), equal to .0275661, and this growth rate is also attributed to effective labor units. This exogenous growth rate for labor is split evenly between population growth and Harrod-neutral technical progress. The benchmark sequence of equilibria is then calculated by maintaining all tax rates and preferences fixed, increasing labor exogenously, and allowing saving to augment capital endowments over time.\(^6\) By construction this sequence will have constant factor ratios and constant prices all equal to one.

Simulations are performed by altering tax rates appropriately while retaining preference parameters and the exogenous labor growth rate. Saving and other behaviors conform to the specified elasticities, growth of capital diverges from the steady state rate, and the economy begins to approach a new path with a new capital/labor ratio. Sequences are compared by discounting the \( H \) composites of instantaneous consumption through time with appropriate terminal conditions. Only leisure and present consumption are included in this welfare measure because saving is reflected in later consumption of the sequence. The sequence is discounted at a 4% rate and includes only the initial population. Otherwise, the importance of future periods would be sensitive to population growth, including total rather than average consumption.
Finally, the welfare gain or loss of a tax change is the aggregate compensating variation, defined as the number of dollars at new prices that would be required for each consumer to attain the old sequence of consumption values. The model thus incorporates both interindustry and intertemporal tax distortions and efficiency changes.

3. Constrained Factor Movements

Let $K^o_i$ represent the $i^{th}$ sector's use of capital in the benchmark equilibrium with the old tax regime and no mobility problems. These quantities were measured using the units convention on capital income net of the corporate income tax, property tax, and the portion of the personal income tax attributable to capital. The total availability of capital is given by

$$K^o_T = \sum_{i=1}^{20} K^o_i$$

(2)

where the twenty capital users include nineteen industries plus general government. If a tax change were to occur, suppose that the constrained minimum use of "old" capital for the $i^{th}$ sector is indicated by $K^c_i$. The derivation of these numbers is based on annual depreciation rates $d_i$ and depends on the particular application. Later sections will specify which sectors are to be constrained in the full integration example, and which are to have no minimum usage requirement (a $K^c_i$ of zero). There exists a pool of new or mobile capital which might be used by any sector, given simply by

$$K^m_T = K^o_T - \sum_{i=1}^{N} K^c_i$$

(3)
where \( N \) is the number of constrained sectors. Thus \( N \) heterogeneous old capital stocks are each specific to the industry in which it was originally used. One type of new capital can be allocated to any industry, and one type of labor is still mobile and homogeneous.

Although there are now \( N + 2 \) factors of production, at most three of these appear in the production function of any one industry. One can imagine a world with no technological change, where old and new capital types are perfect substitutes in an industry, and the two types of capital together are imperfectly substitutable for labor. Rental of an old loom is equivalent to rental of a new loom in the textile industry, for example, and new looms are implicitly purchased as investment goods by net savers. If the textile industry is not expanding, then none of the new saving is used to purchase looms and none of the new capital is used in that industry. If the new tax rates would cause the industry to contract faster than its rate of depreciation, however, there will be "too many" old looms available, in fixed supply, with an equilibrium rental price that is lower than the price of new capital.

Abstracting from the possibility of a corner solution, there is a positive equilibrium price at which all of the old capital of an industry gets used by that industry. Where \( K_i \) denotes the demand for an industry specific capital type, and \( K^C_i \) denotes its supply, the market is assumed to clear at a price \( P_{Ki} > J/ \) This price may be lower than the price of new capital, \( P_K \), but it may not be greater than \( P_K \). If \( P_{Ki} < P_K \) then none of the new capital would be used, since the two are perfect substitutes. If \( P_{Ki} \) were > \( P_K \) then the producer would try to use none of the old capital, driving down its equilibrium price, until \( P_{Ki} \leq P_K \).
There are a number of problems with the model as it has been presented so far. First, it ignores certain price rigidities and their resulting disequilibrium effects. It does not consider under-utilization of capacity nor involuntary unemployment of labor. These phenomena can be better handled in a different type of model, however. The equilibrium model can be appropriately adapted to consider these quantity constraints, and the method described here can still measure significant economic effects.

Second, we have introduced heterogeneous capital types but assumed homogeneity within each type. Industries actually use various types of plant and equipment, and some of these are more readily used in other industries. Even a dramatically contracting industry could easily move out of office space, delivery trucks and warehouses.

Third, the possibility of industry-specific labor could be considered in similar context. Because of special trade skills, not all of an industry's labor force can readily relocate to other uses. Rather than a criticism, however, this comment reflects the very usefulness of the method described here. Special skills could indeed attract lower wages after shifts in government policies. The capital tax examples here should only be viewed as illustrative.

Fourth, the suggestion of time-dated investment is ignored. We can imagine a model where capital/labor ratios are flexible ex ante, but fixed ex post. Then constrained capital, given by the depreciated stock of the previous period, would have to be used in a fixed proportion to labor. If the adjustment took more than one period, then each period's investment could be fixed with a different ratio to labor, and is thus a different type. The
problem with such a full-scale model is only its computational expense: each of these different capital types would require another dimension on the price simplex, while costs increase with roughly the cube of the number of dimensions.9/ 

Fifth, other modelling possibilities exist. A limited form of time-dating could be undertaken, or various types of technological change could be considered. Many of the model's input numbers such as depreciation rates, growth rates, and tax rates should be viewed as illustrative.

Sixth, and finally, there is a small computational problem with modelling old and new capital as perfect substitutes. As mentioned above, $P_{K1}$ cannot be greater than $P_K$ or else there would be no demand for $K_1$ and its price falls. If $P_{K1}$ is less than $P_K$ then only old capital is used in that industry. If prices are equal, they both may be used. Such factor demand functions cannot be used with this algorithm for an equilibrium solution, because of discontinuity.

When the factor prices are taken from the simplex for another iteration, it should be possible to calculate each producer's factor demands. Under the described scheme, if $P_{K1}$ is slightly less than $P_K$ for some industry, then all demand for capital falls on old capital. If the supply of this old capital is not sufficient, the vector labelling will cause an increase in $P_{K1}$. If the next iteration provides a $P_{K1}$ that is greater than $P_K$, there will be no demand for old capital, a large excess supply, and vector labelling which causes $P_{K1}$ to fall again. Because the algorithm uses small but finite steps in its search on the price simplex, it might iterate forever around the proper solution of $P_{K1} = P_K$ which would allow use of both old and new capital. The algorithm would not solve because large
shifts in demand (from only old capital to only new capital) occur around a single relative price.

This problem occurs because old and new capital are perfect substitutes, and it can be solved by making the two imperfect substitutes. With a high elasticity of substitution between old and new capital, we can model a production function that furnishes large but continuous shifts between capital types as their relative prices change. The adjustment is similar to that described above, but it is smoother. In particular, we use a nested CES production function for each industry

\[ Q = A \left[ \frac{\sigma_{KL}}{\sigma} + (1 - \alpha) \bar{K} \right] \tag{4} \]

where the \( \alpha \) and \( A \) parameters are derived from the data by the backwards solution procedure and \( \sigma \) is the elasticity of substitution between capital and labor, \( L \). This function is the same as before, except that homogeneous capital is replaced by a composite of two capital types, \( \bar{K} \).

For only the \( N \) constrained industries

\[ \bar{K} = \left[ \frac{\sigma_{KL}}{\sigma} + \frac{\sigma_{K_i}}{\sigma_{K_i}} \right] \frac{\sigma_{K}}{\sigma_{K_i}} \tag{5} \]

where \( K \) is new (malleable) capital, and \( K_i \) is old (industry-specific) capital. The parameter \( \sigma_{K} \) is the arbitrarily high elasticity of substitution between capital types, and values between 10 and 50 are attempted as described below.

Producers minimize the unit cost of output in a two-stage procedure. First, differentiate the Lagrangean
\[ L_1 = P_L L + \bar{P}_K \bar{K} + \lambda_1 \left[ 1 - A \left( \frac{\sigma - 1}{\sigma} L^\sigma + (1 - \alpha) \bar{K}^\sigma \right) \right] \] (6)

with respect to \( L, \bar{K} \) and \( \lambda_1 \), where \( \bar{P}_K \) is a price index, a composite of \( P_K \) and \( P_{Ki} \). Solve for the factor demand functions

\[ R_L = A^{-1} \left[ (1 - \alpha) \left( \frac{\alpha P_K}{(1 - \alpha) P_L} \right)^{1-\sigma} + \alpha \right] \] (7)

\[ R_{\bar{K}} = A^{-1} \left[ \alpha \left( \frac{(1 - \alpha) P_L}{\alpha P_K} \right)^{1-\sigma} + (1 - \alpha) \right] \frac{\sigma}{1-\sigma} \] (8)

giving the labor and composite capital requirements per unit of output. These are identical to the previous model except for the bar above the \( K \). Producers then go on to minimize the cost of a unit of composite capital. Differentiate the Lagrangean

\[ L_2 = P_K K + P_{Ki} K_i + \lambda_2 \left[ 1 - \left( \frac{\sigma_{K-1}}{\sigma_K} K + \frac{\sigma_{K-1}}{\sigma_K} K_i \right) \right] \] (9)

with respect to \( K, K_i \) and \( \lambda_2 \) to obtain the demand functions

\[ P_K = \left[ \left( \frac{P_{Ki}}{P_K} \right)^{1-\sigma_K} + 1 \right] ^{\frac{\sigma_K}{1-\sigma_K}} \] (10)
giving the old and new capital requirements per unit of the \( \tilde{K} \) composite. It is then simple to obtain the value for \( K \) per unit of output as \( R_K \cdot R_K \) and the value for \( K_i \) per unit of output as \( R_{Ki} \cdot R_K \). Labor per unit output is just \( R_L \).

If we solve for \( \lambda_2 \) in the process of minimizing the cost of composite capital, we get the appropriate composite price index for capital,

\[
\bar{p}_K = \lambda_2 = \left[ \frac{1}{\sigma_K} \frac{p_K^{1-\sigma_K} + p_{Ki}^{1-\sigma_K}}{1-\sigma_K} \right]^{1-\sigma_K} \tag{12}
\]

The algorithm first uses gross-of-tax factor prices to calculate \( \bar{p}_K \) for each industry. It then derives the factor requirements per unit output as shown above. Factor prices also determine producer good prices and endowment incomes as before, so that commodity demands and derived total factor demands can be obtained. These functional forms result in a factor demand ratio given by

\[
\frac{K}{K_i} = \left[ \frac{p_{Ki}}{p_K} \right]^{\sigma_K} \tag{13}
\]

For a high enough \( \sigma_K \), this behavioral rule has the desired property that \( p_{Ki} \) slightly lower than \( p_K \) causes demand for old capital that is much greater than that of new capital. If the prices happen to be equal, then
the two demands are equal. With $\sigma_K$ less than infinite, however, producers will always use some $K$ in addition to their $K_i$. This behavior seems reasonable if investors buy some new capital (perhaps to test) even though excess old capital exists. The price of new capital will always exceed the price of old capital unless significant amounts of new capital are used. Table 1 shows some examples of the sensitivity of factor demands to relative price where the general relationship is given by equation (13). In examples below, $\sigma_K$ of 40 was used because the computer was unable to solve with higher values.

### Table 1
RELATIVE USE OF NEW TO OLD CAPITAL FOR DIFFERENT PRICE RATIOS

<table>
<thead>
<tr>
<th>$K/K_i$</th>
<th>$\sigma_K = 10$</th>
<th>$\sigma_K = 20$</th>
<th>$\sigma_K = 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{Ki}/P_K = .95$</td>
<td>.5987</td>
<td>.3585</td>
<td>.1285</td>
</tr>
<tr>
<td>$P_{Ki}/P_K = .90$</td>
<td>.3487</td>
<td>.1216</td>
<td>.0148</td>
</tr>
</tbody>
</table>

General government does not have an overall production function like that of equation (4), but it does have a final demand for capital based on its own Stone-Geary utility function. This structure is easily adapted for capital constraints by using the previous capital demand functions for composite capital, using equation (5) for the breakdown of old and new capital demands, and giving government its own composite price index based on equation (12).

Some adaptations are also required for dynamic sequencing. The first period of the revised sequence uses specific capital types $K^c_i$. 
which are based on the depreciated capital stock of the previous period. Mobile capital $K$ is given by equation (3), and the equilibrium is calculated with a new set of tax schedule specifications. The net saving response of the first period is still used to augment the total capital stock for the next period, but it augments the malleable capital stock in particular. The constraints $K_i^c$ are re-depreciated, and the depreciation is also used to augment mobile capital.

If the re-depreciated constraints are low enough to become non-binding, then the model should eliminate them as constraints for two reasons. First, the factor demand functions described above are not correct for a significant use of mobile capital. (If new capital exceeds old capital use, then its price would be lower.) Second, there are computational savings from reducing the simplex dimensions. The form of the production function provides a neat procedure for checking constraints, since all industries must use some of the mobile capital. If a substantial amount is used in one period, then it is logical to presume that the constraint will not be binding in the next period. Thus if an industry uses more new capital than its total depreciation in some period, the constraint is eliminated, the old capital is added to the malleable capital, and the dimension size is reduced by one.

When all industries are off their constraints, the short-run adjustment period has ended. A single period's fixed capital stock is now properly allocated among sectors. But the long-run adjustments will continue while the saving response pushes the economy's overall capital/labor ratio toward the steady state ratio.
Next, the model needs to designate the ownership of all factors, including industry-specific capital. An important application of this methodology would investigate the losses of individuals who have disproportionate investments in industries whose relative tax position worsens. At this point, to concentrate on the methodology itself, distributional issues will be ignored. For simplicity, all types of capital are divided among consumers according to their previous total endowment of capital. This procedure is equivalent to adding up net-of-tax capital income, distributing it among consumers according to fixed shares, and ignoring portfolio effects. The risk avoidance behavior of spreading investments among all industries is not an unreasonable assumption.11/

In previous dynamic sequencing, the government was assumed to save at the steady state rate \( n \). For comparability, this procedure is retained, and this saving is added to the stock of mobile capital. Consumer saving is added to the mobile capital stock as well. The total value of new and old capital in the next period are still distributed among consumers and government, but in their new proportions of total capital ownership. All other functional forms remain unchanged.

4. Derivation of Constraints

A result of the specification just described is that if an expanding industry wanted to use more capital than twice the depreciated old capital, the old price could be driven above the new capital price. To avoid this difficulty, and to save execution time as discussed earlier, the number of simplex dimensions can be limited by considering old capital prices for only those industries that are expected to contract. This procedure is implemented by first making a static comparison from the previous model, with
malleable capital and no constraints. The vector of capital use by industry in the simulation is compared to the vector of depreciated original capital use to see which industries would disinvest faster than their depreciation rate. Old capital stocks need only be considered in these industries.

Because of capital depreciation, overall growth, and built-in flexibility on the part of industrial decision makers, a major policy tax change may be required before minimum capital constraints have a perceptible influence on capital allocations. A good example of such a major tax change is the "full integration" of personal and corporate income tax systems. When the "double taxation" of capital income in the corporate industries is eliminated, they will draw mobile capital from other industries in an effort to expand faster than the overall rate of growth. Efficiency gains of the new-tax regime cannot be fully realized if capital cannot fully re-allocate. The full integration plan is documented in Fullerton, King, Shoven and Whalley (1980), where the earlier model with unconstrained capital movement was used to evaluate its effects.

First, the twenty sectors are defined in column 1 of Table 2. Government enterprises is an industry comparable to the other private industries (except that it has a large output subsidy). General government collects taxes and has final demand for goods and factors. An element $K_i^0$ of the vector shown in column 2 of Table 2 indicates the $i^{th}$ industry's use of capital in the original (benchmark) equilibrium, net of all taxes.

Annual depreciation rates $d_i$ are obtainable from data within the model's production sector, and from sources consistent with it.12/
### Table 2

**The Derivation of Capital Constraints**

<table>
<thead>
<tr>
<th>Industry</th>
<th>( K_1^0 )</th>
<th>Depreciation</th>
<th>( d_1 )</th>
<th>( K_1^u )</th>
<th>( K_1^0 \left( 1 - \frac{d_1}{1+n} \right) )</th>
<th>( K_1^0 \left( 1 - \frac{d_1}{1+n} \right)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, Forestry, Fisheries</td>
<td>22,767.8</td>
<td>8,867.</td>
<td>.01558</td>
<td>22,202.5</td>
<td>21,811.87</td>
<td>20,896.07</td>
</tr>
<tr>
<td>Mining</td>
<td>874.8</td>
<td>1,222.</td>
<td>.05588</td>
<td>896.3</td>
<td>803.74</td>
<td>738.47</td>
</tr>
<tr>
<td>Crude Petroleum and Gas</td>
<td>2,466.2</td>
<td>613.</td>
<td>.00959</td>
<td>2,415.5</td>
<td>2,374.22</td>
<td>2,287.53</td>
</tr>
<tr>
<td>Construction</td>
<td>835.9</td>
<td>3,529.</td>
<td>.16886</td>
<td>1,303.9</td>
<td>676.14</td>
<td>546.89</td>
</tr>
<tr>
<td>Food and Tobacco</td>
<td>1,308.3</td>
<td>2,300.</td>
<td>.07032</td>
<td>1,859.8</td>
<td>1,183.65</td>
<td>1,070.90</td>
</tr>
<tr>
<td>Textile, Apparel, Leather</td>
<td>802.8</td>
<td>1,149.</td>
<td>.05725</td>
<td>1,224.1</td>
<td>736.50</td>
<td>675.71</td>
</tr>
<tr>
<td>Paper and Printing</td>
<td>2,097.2</td>
<td>2,344.</td>
<td>.04711</td>
<td>2,719.6</td>
<td>1,949.65</td>
<td>1,812.52</td>
</tr>
<tr>
<td>Petroleum Refining</td>
<td>6,555.7</td>
<td>1,231.</td>
<td>.00751</td>
<td>6,494.0</td>
<td>6,331.93</td>
<td>6,115.79</td>
</tr>
<tr>
<td>Chemicals and Rubber</td>
<td>2,964.7</td>
<td>3,851.</td>
<td>.05196</td>
<td>4,099.1</td>
<td>2,735.26</td>
<td>2,523.58</td>
</tr>
<tr>
<td>Lumber, Furniture, Stone</td>
<td>3,311.1</td>
<td>1,907.</td>
<td>.02304</td>
<td>3,703.2</td>
<td>3,148.04</td>
<td>2,993.01</td>
</tr>
<tr>
<td>Metals, Machinery</td>
<td>7,900.3</td>
<td>8,217.</td>
<td>.04160</td>
<td>10,068.1</td>
<td>7,368.48</td>
<td>6,872.49</td>
</tr>
<tr>
<td>Transportation Equipment</td>
<td>100.9</td>
<td>888.</td>
<td>.35200</td>
<td>172.8</td>
<td>63.63</td>
<td>40.13</td>
</tr>
<tr>
<td>Motor Vehicles</td>
<td>3,684.4</td>
<td>1,338.</td>
<td>.01453</td>
<td>4,547.0</td>
<td>3,533.52</td>
<td>3,388.77</td>
</tr>
<tr>
<td>Transportation, Communication, Utilities</td>
<td>9,881.2</td>
<td>20,086.</td>
<td>.08131</td>
<td>10,742.0</td>
<td>8,834.20</td>
<td>7,898.18</td>
</tr>
<tr>
<td>Trade</td>
<td>7,515.6</td>
<td>9,561.</td>
<td>.05089</td>
<td>9,897.3</td>
<td>6,941.80</td>
<td>6,411.82</td>
</tr>
<tr>
<td>Finance and Insurance</td>
<td>6,126.4</td>
<td>3,655.</td>
<td>.02386</td>
<td>6,799.7</td>
<td>5,819.74</td>
<td>5,528.46</td>
</tr>
<tr>
<td>Real Estate</td>
<td>52,616.1</td>
<td>32,192.</td>
<td>.02447</td>
<td>50,069.6</td>
<td>49,951.50</td>
<td>47,421.83</td>
</tr>
<tr>
<td>Services</td>
<td>9,456.7</td>
<td>8,889.</td>
<td>.03760</td>
<td>9,206.3</td>
<td>8,856.97</td>
<td>8,295.30</td>
</tr>
<tr>
<td>Government Enterprises</td>
<td>6,180.7</td>
<td>...</td>
<td>.03167</td>
<td>5,768.8</td>
<td>5,824.44</td>
<td>5,488.70</td>
</tr>
<tr>
<td>General Government</td>
<td>97,961.2</td>
<td>...</td>
<td>.03167</td>
<td>91,215.4</td>
<td>92,314.01</td>
<td>86,992.47</td>
</tr>
<tr>
<td>TOTAL</td>
<td>245,406.0</td>
<td>111,849.</td>
<td>.03167</td>
<td>245,405.0</td>
<td>231,529.29</td>
<td>217,998.62</td>
</tr>
</tbody>
</table>

**Column**

1) Gives our aggregation to twenty sectors.
2) \( K_1^0 \) is capital income net of all taxes in the original equilibrium; with unitary prices these give allocation of capital units among industries.
3) Depreciation is in millions of 1973 dollars; derivation and data are described in the text.
4) \( d_1 \) is the depreciation rate, equal to (3) times .04 over (2).
5) \( K_1^u \) is the allocation of capital in the unconstrained "full integration" case; the difference between total \( K_1^0 \) and \( K_1^0 \) indicates the specified accuracy for equilibrium calculations.
6) Minimum capital constraints after one year.
7) Minimum capital constraints after two years.
The estimates of economic depreciation are shown in column 3 of Table 2 in millions of 1973 dollars. These figures are multiplied by $\gamma$, the average net-of-tax rate of return to get depreciation of capital services units: the dollar depreciation numbers are part of gross saving and must be converted by $\gamma = 0.04$ like consumers' net saving to get new units of capital. Dividing by column 2, we have rates of depreciation, shown in column 4. The average depreciation rate for private industry is used for the government enterprises industry and for general government.

For purposes of comparison, the unconstrained uses of capital under the full integration tax replacement, $K_u^i$, are shown in column 5 of Table 2. This is the new allocation of capital if the old version of the model is used. The minimum required use of old capital for each industry will be $K^c_i$, but the size of this constraint depends on the length of each period and the timing of investment. If, for example, the industries are notified of the tax change just after they had made investment decisions, then constraints would be given by $K^c_i = K^o_i$, the quantity of capital ready to begin the period under the expectations of the old-tax regime. Comparison of columns 2 and 5 indicates that under such a modelling, six industries (numbers 1, 3, 8, 17, 18, and 19) plus general government (20) would hit their respective constraints, since these are the groups that use less capital in the unconstrained calculations.

If, on the other hand, industries are notified of the tax change just prior to investment decisions, then constraints would be given by the depreciated capital stocks of the previous period \(^{13}\):

$$K^c_i = K^o_i \left[ \frac{1 - \frac{1}{1 + n}}{1 + \frac{1}{1 + n}} \right]_{NYRS}$$

(14)

where NYRS is the number of years in each period. If periods are very long, then these constraints could easily be nonbinding.
They are less than $K^0_1$ because of gross investment over time to cover both depreciation at rate $d_1$ and net investment at rate $n$ in the steady state.

In continuous time there is no difference between the two methods. In the discrete time model, however, with periods of one year, the latter method results in the constraints of column 6 in Table 2. Looking at the unconstrained simulation of the full integration tax replacement, it appears that only two constraints are transgressed (numbers 19 and 20). This comparison of columns 5 and 6 is not necessarily an accurate guide to the number of binding constraints, however, since $K^u_1$ were completely mobile.

If one sector were compelled to use more capital, then less is available to other industries. These other sectors may hit constraints, as is indeed the case, shown below.

Column 7 gives the constraints after two years. It is rather surprising that none of the sectors are restrained after such a short period, but there are two partial explanations. First, the tax change may not be dramatic enough to require greater capital movements. Second, overall growth of the economy quickly allows producers to adjust factor ratios without disinvesting. In particular, for industries whose capital tax rates have been raised in relative terms, growth of the labor force at rate $n$ provides an easy means to a lower capital/labor ratio.

For three major reasons, it was decided to model the timing of investment as after the tax-change announcement. First, this may be a more realistic treatment since policy decision makers do not typically operate in secret. Producers get signals of pending tax change proposals and may hold back investment plans until tax rate uncertainty has been resolved. Second, the column 6 constraints are smaller than those of column 2, fewer will be binding, and computation will be less expensive. Third, this choice will result in lower estimates for the effects of constraints. In this re-
spect, it gives an underestimate for the transition cost of partially mobile capital in adjusting to full integration. This cost is still shown below to be significant.

5. Simulation Results

Previous simulations of full integration used dynamic sequences with five periods of ten years each. The present value of all efficiency gains from this model was $179.977 billion if the equal tax yield was acquired through multiplicative scaling of personal income tax rates. This result is directly comparable to the new constrained model, except that a ten year period is sufficient to depreciate away all constraints. In fact the surprising result of the previous section was that periods as short as one year were required to meet binding constraints. For comparison purposes, full integration was resimulated on the old model for ten periods of one year each, where all of the standard parameter values were applied. The present value of efficiency gains (EG) were then $174.402 billion.

This figure is taken as the standard of comparison for the constrained model's results. If new present value of efficiency gains are lower, then the difference (ΔEG) is taken as the loss due to partially mobile capital. Most of these losses will occur in the constrained transition year(s), but effects might be felt later as well.

An array of results are displayed in Table 3. The unconstrained results appearing in the left-hand column include the efficiency gain just mentioned and the three relative "prices" of the equilibrium solution. All solution vectors are normalized by the price of labor which is always taken as one. The other two dimensions give the relative price of capital and the tax scalar used for equal yield calculations. The scalar of 1.123 indicates
that personal tax rates must be 12.3 percent higher to recoup the revenue
lost with full integration. The price of capital rises from 1.0 to 1.105
when integration is imposed, but falls over ten years to 1.092 as capital
deepening takes place in the economy.

The first constraint considered was for general government because
this user of capital seemed to be furthest below its depreciated old
capital in the unconstrained simulation. (Table 2 shows two sectors with
column 5 unconstrained equilibrium capital demand less than column 6 con-
straints, but sector 20 has the larger differential.) Once this single
constraint takes effect, however, several more industries' capital demands
fall below their constraints. The columns of Table 3 show the results of
different simulations where additional constraints are successively im-
posed. The order of consideration for these additional constraints is
essentially arbitrary, but they generally appear in declining order of im-
portance.

With one constraint for sector 20, the return to old capital is in-
deed lower than that of new capital, as seen from comparison of row 5a
and 6a. Government does not want that much capital, so its price falls
to 1.049 in relative terms. With government capital restricted, there is
less mobile capital for other users, and its price rises to 1.149 in the
new equilibrium. Government's demand for old capital at these prices is
92314 units (equal to the constraint as shown in Table 2), and its demand
for new capital is 2465 units. These factors have a .0267 ratio in accor-
dance with equation (13).

The tax scalar falls to 1.096 in this equilibrium because government's
cost of capital is lower. Its composite price of capital is 1.067, based
on equation (12). Its endowment income has not fallen proportionately
TABLE 3
THE RESULTS OF FULL INTEGRATION FOR SIX SIMULATIONS WITH INCREASING DIMENSIONALITY

<table>
<thead>
<tr>
<th>1) Number of constraints</th>
<th>None</th>
<th>One</th>
<th>Two</th>
<th>Three</th>
<th>Four</th>
<th>Five</th>
</tr>
</thead>
<tbody>
<tr>
<td>2) Number of dimensions</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>3) Tax scalar</td>
<td>1.123</td>
<td>1.096</td>
<td>1.101</td>
<td>1.113</td>
<td>1.114</td>
<td>1.115</td>
</tr>
<tr>
<td>4) Price of labor</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>5) Price of mobile capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. period one</td>
<td>1.105</td>
<td>1.149</td>
<td>1.155</td>
<td>1.176</td>
<td>1.181</td>
<td>1.189</td>
</tr>
<tr>
<td>b. period ten</td>
<td>1.092</td>
<td>1.092</td>
<td>1.092</td>
<td>1.092</td>
<td>1.092</td>
<td>1.092</td>
</tr>
<tr>
<td>6) Price of old capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. sector 20</td>
<td>...</td>
<td>1.049</td>
<td>1.053</td>
<td>1.065</td>
<td>1.067</td>
<td>1.071</td>
</tr>
<tr>
<td>b. sector 1</td>
<td>...</td>
<td>...</td>
<td>1.064</td>
<td>1.078</td>
<td>1.081</td>
<td>1.086</td>
</tr>
<tr>
<td>c. sector 17</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>1.068</td>
<td>1.071</td>
<td>1.075</td>
</tr>
<tr>
<td>d. sector 19</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>1.062</td>
<td>1.065</td>
</tr>
<tr>
<td>e. sector 18</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>1.096</td>
</tr>
<tr>
<td>8) ΔEG: Transition losses of partially mobile capital, in billions of 1973$</td>
<td>...</td>
<td>1.959</td>
<td>2.698</td>
<td>4.628</td>
<td>4.673</td>
<td>5.196</td>
</tr>
</tbody>
</table>
because of the diversification assumption, and the result is that less tax revenue is required to make the same real purchases as in the benchmark. Finally, the efficiency gains of this computation are $172.443 billion, lower by $1.959 billion than the unconstrained computation.

Once general government (sector 20) is constrained, the capital demands of agriculture (1), real estate (17), government enterprises (19), and services (18) also fall below their depreciated old capital stocks. As these constraints are imposed, row 5a of Table 3 shows that the price of mobile capital rises. Less is available to other industries. The demand for and price of government-specific capital also rises, since government's small relative demand for mobile capital gets even smaller as its price rises. The somewhat more costly nature of government's capital causes a higher tax revenue scalar, as shown in row 3. Looking across sub-rows of 6 indicates that any industry-specific capital type earns a somewhat higher return when other industries are also constrained. Every additional constraint reduces the quantity of mobile capital in the model, drives up its price, and makes the old capital relatively more desirable. In all cases, when the model redeprecimates constraints after the first period, it finds that new capital demand exceeds depreciation and thus removes constraints for the second period.

Row 7 shows that, indeed, the efficiency gains of full integration are reduced as more types of capital are partially immobile. These reductions are reported in Row 8 and vary between two and five billion dollars. The question that immediately arises is whether these are large or small numbers. Five billion is small compared to the $174 billion present value of EG, but other considerations are more relevant for three reasons.
First, this five billion dollars is a large absolute amount regardless of its ratio to any other number. Second, since this $5 billion is really a loss to the first period, it could be compared to the $6.2 billion annual efficiency gain of the static comparisons reported in Fullerton, King, Shoven, and Whalley (1980). This comparison makes it a large relative number as well as a large absolute number.

Third, the comparison with the $174 billion present value of efficiency gains depends heavily on the discount rate, set at four percent for these calculations. A higher discount rate makes the first period $5 billion a larger relative number since it occurs earlier. 16/

Before leaving the new constrained model, there are a few other insights to be gleaned from it. No particular capital owners are injured by constraints because of diversification, but capital owners generally are injured relative to labor. Though the price of mobile capital increases with constraints, the prices of old capital types fall. The net results are a lower overall return to capital and a U shaped burden distribution since the capital/labor ratio of income falls and then rises again over the twelve income classes in the model.

Other simulations varying \( \sigma_K \) also reveal an interesting economic result. As this substitution elasticity is raised, old and new capital prices come closer together. Since the constrained industry provides only a small part of the demand for \( K \), however, its price does not move as much as \( P_{K_1} \) moves. When the two factors become closer substitutes, the industry decreases demand for \( K \) and increases demand for \( K_1 \) which has the lower price. The greater demand for \( K_1 \) relative to its fixed supply results in a higher equilibrium price. Since \( P_{K_1} \) levels off as \( \sigma_K \) is
increased in all cases, it is fair to presume that perfectly substitutable capital stocks would still show $P_{ki}$ less than $P_K$. Reported results used the highest possible $c_K$ of 40.

Finally, these simulations provide information about the computational expense of the algorithm as applied to this model. As expected, additional simplex dimensions increase computer cost more than proportionately. However, there seems to be an acceleration in the rate of increase. At first, costs rise by less than the square of the number of dimensions, and later by more than the square. The addition of the eighth price raises cost by more than the cube of the number of dimensions. These geometrically increasing costs serve to reinforce the adoption of ad hoc assumptions made earlier to limit the number of dimensions.

6. Conclusion

Comparative static models typically assume homogeneous and mobile factors in estimating the economic effects of a tax policy change. Even dynamic models employ a given homogeneous capital stock in two different allocations for the first period of two equilibrium sequences. This malleable capital assumption causes overstatement of early efficiency gains from policies designed to improve factor allocation. On the other hand, immobile factor models would understate such gains by assuming that no capital ever relocates.

The model in this paper attempts to bridge this gap by restricting each industry's capital reduction to its rate of depreciation. The stock of depreciated capital from the previous period represents an industry-specific type of capital which may earn a lower equilibrium return. The usage of mobile capital above this minimum constraint is limited by the total gross
saving of the economy, including all industries' depreciation and consumer net saving.

The industry-specific capital model suggests, for example, that previous estimates of the dynamic efficiency gain from full integration of personal and corporate taxes in the U.S. are overstated by about $5 billion. The model could also be used to estimate distributional impacts on individuals with more than proportionate ownership of capital in particular industries.
FOOTNOTES

1. Labor compensation includes all wages, salaries, commissions, and tips, while capital earnings include net interest paid, net rent paid, and corporate profits with capital consumption adjustments and inventory valuation adjustments. Non-corporate profits were divided between labor and capital on the basis of full-time-equivalent hours and average wage for each industry. Some industries were averaged over several years to avoid recording transitory effects.


3. All dividends are 96% taxable, reflecting the 4% that fall under the $100 exclusion in 1973. All retained earnings are 73% taxable, reflecting the value of tax deferral and rate advantages for capital gains, but including the taxation of purely nominal gains. Interest and rents are fully taxable except for the imputed net rent of owner occupied homes, while the non-corporate investment tax credit also appears as a personal tax reduction varying by industry.

4. Portfolio effects are ignored because dividends, capital gains, interest, rent, and other capital income types are summed to obtain capital endowments. Since capital is homogeneous, consumers will gain or lose according to their total endowment in the model.

5. In particular, the input-output matrix does not conform to the requirement that gross output of each good can be measured by the column sum plus value added or the row sum plus final demand. Iterative row and
column scaling converges to a consistent matrix, and similar scaling satisfies similar conditions for the expenditure matrix.

6. A dollar of saving is converted into capital service rental units through multiplication by $\gamma$, the real after-tax rate of return. The model assumes that 25 dollars of saving can purchase a capital asset that will earn one dollar per period net of depreciation and taxes. That is, .04 is used for $\gamma$.

7. In equilibrium, all of the constrained capital stock appears in the industry's production function, but there is no constant added to the functional form: the use of old capital is a variable from the producer's viewpoint. Also, there is no monopsony in the purchase of specific capital, since competition among firms of a single industry would insure that zero profits exist at the equilibrium.

8. Harvey Galper and the Office of Tax Analysis are responsible for this useful and enlightening suggestion.

9. The existing methodology requires three dimensions, with labor, capital, and government revenue on the price simplex. The methodology adopted here would require $N + 3$ dimensions, with extra prices for the specific capital types of only the few contracting industries which are given constraints. The methodology just suggested would require prices for each time-dated type of capital, and for all expanding industries as well. This requirement can be seen by considering the determination of factor use ratios from factor cost ratios. The new tax regime changes the capital tax rate of each industry in a different way, so every industry desires a new capital/labor ratio. Any time-dated capital stock with a specific ratio different from the desired ratio would earn a
lower equilibrium return than new capital with a flexible ratio. Since there are twenty sectors and twenty capital types with specific labor ratios, there would be twenty-three simplex dimensions in only the first period of adjustment.

10. These production functions have suppressed i subscripts for most of the parameters. Only $\sigma_k$ will be the same for all industries.

11. Because government is also endowed with some capital ownership, it is also given a balanced portfolio. The implication is that the proportion of government's capital endowment which is used in the $i^{th}$ sector is the same as the proportion of other consumers' endowments used in that sector. This procedure is adopted once again to short-cut distributional problems, but an equally valid solution might assign all of government's endowment for use in government. The choice will not affect measurements of transition loses reported below, but the latter suggestion has complications. If the price of government-specific capital falls due to immobility, it can severely affect tax rates through the equal yield feature. Personal tax rates are adjusted to obtain exactly enough revenue to make the same real government purchases as in the benchmark equilibrium. With the portfolio model adopted here, the government's income is lowered whenever a capital constraint lowers consumers' incomes. Government's cost of capital falls whenever its old capital is constrained for use there.

12. For manufacturing industries, Coen (1976) provides estimates of 1973 depreciation, and for other industries, national accounts provide the capital consumption allowance. When possible, the capital consumption adjustment is subtracted, to better approximate economic depreciation.
The July 1976 Survey of Current Business shows 1973 capital consumption allowances, and also shows the capital consumption adjustment for agriculture and real estate.

13. The benchmark equilibrium is assumed to be part of a steady state growth path where $n$ is the annual growth rate (derived earlier as .0275661). The previous year's use of capital must have been $K_{i-1}^o/(1+n)$ to grow at this rate and yield $K_i^o$. When the previous year's capital is depreciated, we have (14).


15. Because the consideration of each additional constraint raises the transition loss estimate, row 8 of Table 3 seems to imply the following marginal costs of each constraint:

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>SECTOR</th>
<th>$ BILLIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>General government</td>
<td>1.959</td>
</tr>
<tr>
<td>1</td>
<td>Agriculture</td>
<td>.739</td>
</tr>
<tr>
<td>17</td>
<td>Real estate</td>
<td>1.930</td>
</tr>
<tr>
<td>19</td>
<td>Government enterprises</td>
<td>.045</td>
</tr>
<tr>
<td>18</td>
<td>Services</td>
<td>.523</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.196</td>
</tr>
</tbody>
</table>

The order of consideration is arbitrary, however, for once sector 20 is constrained, all four other sector demands for capital fall below $K_i^c$. When sector 20 is paired with each other sector, the following marginal costs result:
This comparison emphasizes interlinking effects for which a general equilibrium model is required.

16. Furthermore, the transition losses of the first period are actually somewhat larger than five billion. Because the first period's mobile capital price is higher than the unconstrained calculations, and because of the positive saving elasticity with respect to this net rate of return, the first period saving is increased even though incomes are lower. Because the present tax system distorts the choice between present and future consumption, and because full integration does not fully offset this distortion, this extra saving inducement is a net efficiency gaining change. Since this small effect serves to increase the efficiency gains of row 7, the ΔEG numbers of row 8 are smaller than the effect of first period transition losses alone. This effect is not a large one, however, since the price of capital still falls to 1.092 over time. A careful inspection of the complete results indicate that all later prices and quantities of the constrained and unconstrained sequences are practically identical.
REFERENCES


8. Scarf, Herbert E., with the collaboration of Terje Hansen (1973), The Computation of Economic Equilibria, New Haven, Conn.: Yale University Press.