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SORTING MODELS OF LABOR MOBILITY, TURNOVER, AND UNEMPLOYMENT

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ABSTRACT

Utilizing a model in which individuals search among lotteries on likely success at different jobs, this paper analyzes both the search decision when unemployed and the implications of the sorting process. The model correctly predicts both the direction and convexity of the age-unemployment relationship and the impact of experience on turnover and wages. Actions taken when unemployed have an important impact on equilibrium turnover rates, unemployment rates, and the work history of the pool of unemployed. The sorting model is used to analyze racial differences in youth unemployment and major empirical features of low income labor markets.

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I. Introduction

At the time of entry into the labor market, few individuals know the type of job for which they are best suited. This uncertainty leads to a process of experimentation in which individuals and employers learn about the appropriateness of the job-individual matchup and terminate this relationship if it is not desirable. The process by which individuals engage in lotteries on appropriate matchups will be referred to in this paper as worker sorting. Different aspects of this process have been considered under various headings, such as job shopping, job matching, and adaptive models of job choice.[1]

The purpose of this essay is to provide a general framework for analyzing the implications of worker sorting for patterns of turnover, unemployment, and wage growth. I will consequently not be primarily concerned with the specific features of the sorting process, but will focus instead on the broader ramifications of sorting for worker mobility. While it will be shown that most of the established empirical patterns of labor mobility are consistent with the sorting model, the framework is not inconsistent with more traditional analyses of mobility, which often contribute to the motivation of the economic processes underlying sorting behavior.[2]

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[1]Although there is some overlap in the issues considered, the following division is representative. See Johnson (1978) and Viscusi (forthcoming-c) for job shopping models, Mortensen (1975, 1978), Becker (1974), Jovanovic (1977), and Mincer and Jovanovic (1979) for matching models, and Viscusi (1979, forthcoming-a,b) for adaptive control models.

[2]More specifically, the analytic foundation of the process leading to sorting is based in part on the insights in the human capital literature, most particularly those regarding specific human capital. See Becker (1964), Lazear (1976), Leighton and Mincer (forthcoming), Mincer (1974), Mincer and Jovanovic (1979), Oi (1962), Parsons (1972), Pencavel (1962), and Rosen (1972).

Section II analyzes the process of job search in which individuals do not search among alternative wage offers but instead confront different lotteries on whether or not a job matchup will be successful. The ramifications of this sorting process for aggregative patterns of unemployment and turnover are the subject of Section III. Special attention is devoted to the implications of the model for the unemployment patterns of black youths and low income workers. Section IV analyzes the impact of experience at the firm on worker separations, quits, layoffs, and wage levels, while Section V considers the age-unemployment relationship. A major strength of the sorting theory is that it correctly predicts both the observed direction of these relationships and the empirically observed convexity.[3]

II. Mobility Lotteries and Optimal Search Procedures

Consider an employment situation in which an individual can reside in one of three possible states S^i, where the superscript i will be used throughout to refer to the individual's present state. State 1 represents a successful job match, S^2 represents an unsuccessful job match, and S^3 is unemployment.[4] Let the periods remaining in the worker's choice problem

[3]See especially Leighton and Mincer (forthcoming) for the most comprehensive analysis of the empirical aspects to be considered. Their analysis also includes a discussion of the importance of sorting and other conceptual underpinnings of labor mobility. Also pertinent are the other papers included in Freeman and Wise (forthcoming).

[4]Following Burdett and Mortensen (1978), one can also develop the model in terms of groups of states S^1 and S^2 for successful and unsuccessful matches across a heterogeneous set of jobs.

be indicated by subscripts. With n periods left, the employment transition probabilities are given by P_n, so that

$$P_n = \begin{bmatrix} b_n & 0 & 1-b_n \\ 0 & c_n & 1-c_n \\ s_n f_n & (1-s_n)f_n & 1-f_n \end{bmatrix},$$

where the ij'th element represents the transition from S^i to S^j with n periods remaining. Each of these transition probabilities can assume values in [0,1], but the rows must sum to 1.

Individuals do not move between successful and unsuccessful job matches without searching for another job while in S^3.[5] The probability of termination of a successful job match is assumed to be less than for an unsuccessful match, so that

$$b_n > c_n .^{6}$$

[5]On-the-job search leading to job changes without intervening unemployment, as in Burdett (1978), is excluded. For short periods, the necessity of an intervening spell of employment does not provide too dissimilar results. With minor amendments, the model also can include labor force dropouts along the lines of Burdett and Mortensen (1979) or Toikka (1976). Doing so would require the addition of a fourth state.

[6]This assumption can be derived rigorously as in Mortensen (1975, 1978), Jovanovic (1977), and Viscusi (forthcoming-a). Cyclical factors and a change in the locale of one's spouse's job are among the influences that lead to termination of successful matches.

The specific economic processes leading to worker turnover from the two employment states need not be considered explicitly for the purposes of this analysis. Cyclical factors, random events that lead the worker to move, and decisions by employers and workers to terminate unsuccessful matchups are among the determinants of these probabilities.

Unemployed individuals have a probability f_n of finding an acceptable job, where s_n is the conditional probability of a successful job match given that the offer is acceptable. More specifically, the individual job search will take the form of individuals searching not among alternative wage offers, but rather lotteries on successful and unsuccessful job outcomes with a probability z of S^1 and (1-z) of S^2. Let the density function of possible success probabilities with n periods remaining be $g_n(z)$. If z_n is the lowest probability of S^1 acceptable to the worker, the probability f_n of finding a job in that period is given by

$$(1) \quad f_n = \int_{z_n}^{1} g_n(x)\, dx \; .$$

Similarly, the conditional probability of success s_n given that the job is acceptable is

$$(2) \quad s_n = \frac{\int_{z_n}^{1} x g_n(x)\,dx}{\int_{z_n}^{1} g_n(x)\,dx} \; .$$

The transition probability $s_n f_n$ from S^3 to S^1 can be obtained using equations 1 and 2, and is

$$s_n f_n = \int_{z_n}^{1} x g_n(x) dx \ .$$

Similarly, the transition probability $(1-s_n)f_n$ from S^3 to S^2 is

$$(1-s_n)f_n = \int_{z_n}^{1} (1-x)\ g_n(x) dx \ ,$$

and the value $(1-f_n)$ for remaining in S^3 is

$$(1-f_n) = \int_{0}^{z_n} g_n(x) dx \ .$$

Higher values of z_n increase the probability of remaining unemployed and decrease both probabilities of employment since

$$\frac{\partial(s_n f_n)}{\partial z_n} = -z_n g_n(z_n) < 0 \ ,$$

$$\frac{\partial[(1-s_n)f_n]}{\partial z_n} = -(1-z_n)\ g_n(z_n) < 0 \ ,$$

and

$$\frac{\partial(1-f_n)}{\partial z_n} = g_n(z_n) > 0 \ .$$

Residence in each state S^i with n periods remaining has an associated immediate payoff u_n^i. In addition, there is the discounted present value V_{n-1}^i of starting in that state in the next period. An interest rate r has an associated discount factor $(1+r)^{-1}$, which will be indicated by β.

Entry into a particular S^i has an associated payoff $u_n^i + \beta V_{n-1}^i$. The relative attractiveness of the states is such that

(3) $u_n^1 + \beta V_{n-1}^1 > u_n^3 + \beta V_{n-1}^3 > u_n^2 + \beta V_{n-1}^2$,

for all n.[7]

The recursive solution to the worker's dynamic programming problem in each state is

$$V_n^1 = b_n[u_n^1 + \beta V_{n-1}^1] + (1-b_n)[u_n^3 + \beta V_{n-1}^3] ,$$

$$V_n^2 = c_n[u_n^2 + \beta V_{n-1}^2] + (1-c_n)[u_n^3 + \beta V_{n-1}^3] ,$$

and

(4) $$V_n^3 = s_n f_n[u_n^1 + \beta V_{n-1}^1] + (1-s_n)f_n[u_n^2 + \beta V_{n-1}^2] + (1-f_n)[u_n^3 + \beta V_{n-1}^3] .$$

Any choices in the three states with n periods left are made with respect to the predetermined optimal values V_{n-1}^i.[8]

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[7] A sufficient, but not necessary condition for this to be true is that $u_n^1 > u_n^3 > u_n^2$, for all n.

[8] To avoid complicating the notation further, all values of V in subsequent periods are assumed to be their optimal values determined by backward induction.

The worker in S^3 receives a job offer at random from the distribution characterized by $g_n(z)$. Instead of receiving a wage offer, he learns the probability z that the job match will be successful. The only choice variable is whether or not to accept the job. Thus the worker picks his reservation probability of success z_n in each period. Subsequent lotteries will be generated from a perhaps different distribution so that $g_{n-1}(z)$ need not be identical to $g_n(z)$. However, the values of g in each period are assumed to be independent of the worker's previous experiences, ruling out learning effects and recall of a previous offer.[9]

Two possible variants of sorting can be incorporated in this model. First, the worker who accepts a job may realize immediately whether S^1 prevails so that his job characteristics satisfy $u_n^1 > u_n^2$ and $b_n > c_n$. Alternatively, the rewards in S^1 and S^2 may be indistinguishable ($u_n^1 = u_n^2$) so that the worker and employer are not immediately aware of the outcome.[10] The appropriateness of the match is, however, discovered on a probabilistic basis over time, leading to a higher termination probability for S^2 than S^1. For concreteness, the exposition will be in terms of the first of these situations.

Consider a worker in S^3 with n periods remaining. The optimal decisions in other states at that time are not matters of concern. Moreover, all that must be known about subsequent decisions is that they are

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[9] The parallel of the format with wage search models from a known distribution is quite close. See the survey by Lippman and McCall (1976).

[10] A model developed along these lines must have associated transition probabilities that satisfy equation 3.

made optimally, so that the V^i_{n-1} terms in equation 4 assume their maximum values. The worker's objective is to $\underset{z_n}{\text{Max}}\ V^3_n$, leading to the first-order condition

$$0 = g(z_n)\{-z_n[u^1_n + \beta V^1_{n-1}] - (1-z_n)[u^2_n + \beta V^2_{n-1}]$$
$$+ [u^3_n + \beta V^3_{n-1}]\} ,$$

or

$$(5) \quad z_n\{[u^1_n + \beta V^1_{n-1}] - [u^2_n + \beta V^2_{n-1}]\} = [u^3_n + \beta V^3_{n-1}] - [u^2_n + \beta V^2_{n-1}] .$$

The worker sets the value of z_n so that the product of the reservation success probability and the net gain from being in S^1 instead of S^2 equals the net difference in the value of being unemployed over the value of an unsuccessful job outcome.[11] The optimal value of z_n depends only on the values of being in each state. It is independent of the shape of the $g_n(z)$ distribution, although it does depend on subsequent values of g.

The worker's choice of z_n responds in the expected fashion to changes in u^i_n, since

$$\frac{\partial z_n}{\partial u^1_n} < 0, \ \frac{\partial z_n}{\partial u^2_n} < 0, \text{ and } \frac{\partial z_n}{\partial u^3_n} > 0 .$$

Increases in the value of either employed state lower the reservation acceptance probability, while increases in the unemployment state payoff make the worker more selective as he raises z_n.

[11] See Howard (1971) for a detailed motivation of optimization procedures within the context of Markov decision models.

The role of the mobility lottery distribution can be seen by examining the role of $g_{n-1}(z, \psi)$, where ψ is some parameter. Only the V^3_{n-1} term in equation 5 depends on $g_{n-1}(z, \psi)$, so that

$$\text{sign}\left(\frac{\partial z_n}{\partial \psi}\right) = \text{sign}\left(\frac{\partial V^3_{n-1}}{\partial \psi}\right) .$$

From the counterpart of equation 5 for n-1 periods left, we know that z_{n-1} is independent of $g_{n-1}(z, \psi)$ so that z_{n-1} need not be written as a function of the parameter ψ that will be used to alter $g_{n-1}(z, \psi)$. Using equation 4, V^3_{n-1} can be formulated as

$$\begin{aligned} V^3_{n-1} = {} & [\int_{z_{n-1}}^{1} x\, g_{n-1}(x, \psi)\, dx][u^1_{n-1} + \beta V^1_{n-2}] + \\ & [\int_{z_{n-1}}^{1} (1-x) g_{n-1}(x, \psi)\, dx][u^2_{n-1} + \beta V^2_{n-2}] + \\ & [\int_{0}^{z_{n-1}} g_{n-1}(x, \psi)\, dx][u^3_{n-1} + \beta V^3_{n-2}] . \end{aligned}$$

Differentiating with respect to ψ, one obtains

$$(6)\quad \frac{\partial V^3_{n-1}}{\partial \psi} = \{[u^1_{n-1} + \beta V^1_{n-2}] - [u^2_{n-1} + \beta V^2_{n-2}]\} \frac{\partial}{\partial \psi} \int_{z_{n-1}}^{1} x\, g_{n-1}(x, \psi)\, dx$$

$$+ \{[u^2_{n-1} + \beta V^2_{n-2}] - [u^3_{n-1} + \beta V^3_{n-2}]\} \frac{\partial}{\partial \psi} \int_{z_{n-1}}^{1} g_{n-1}(x, \psi)\, dx ,$$

where the first term in braces is positive and the second is negative.

Consider a shift in g_{n-1} distribution toward the upper tail so that both of the $\frac{\partial}{\partial \psi}$ terms are positive. This would occur if, for example, $g_{n-1}(x, \psi)$ shifted to the right by some constant amount. The first derivative term is the increased probability of a successful job outcome, $s_{n-1}f_{n-1}$, from a marginal increase in ψ. This value is weighted by the net advantage of residence in S^1 over S^2. The second derivative term is the increase in the probability of finding a job, f_{n-1}, weighted by the (negative) difference between the values of states S^2 and S^3. If both of the $\frac{\partial}{\partial \psi}$ terms are positive, z_n will be an increasing function of ψ if the value of the increased probability of a successful job match offsets the drop in utility associated with the increased chance of finding a job that is not a successful match.

Whether one considers changes in the distribution of job offers, payoffs, or other parameters of the job choice problem, any effect that alters z_n will influence both the probability of finding a job f_n and the conditional probability of a successful job outcome in opposite fashion. Upon differentiation of equations 1 and 2, one obtains

$$(7)\quad \frac{\partial f_n}{\partial z_n} = - g_n(z_n) < 0 ,$$

and

$$(8)\quad \frac{\partial s_n}{\partial z_n} = \frac{g_n(z_n)[-z_n f_n + \int_{z_n}^{1} x g_n(x)dx]}{(f_n)^2} > 0 .$$

These equations will be utilized in Section III to assess whether different groups' unemployment experiences are attributable to differences in job offer distribution or differences in other parameters of the decision problem.

III. Determinants of Equilibrium Outcomes

A. Steady State Frequencies

Suppose that the average transition matrix P for a labor market group is given by

$$P = \begin{bmatrix} b & 0 & 1-b \\ 0 & c & 1-c \\ sf & (1-s)f & 1-f \end{bmatrix} ,$$

which is the same as P_n except that the period subscripts have been dropped assuming that the group's average behavior is time-invariant. The equilibrium proportions a_i of the group in each of the states S^i are given by

$$(9) \quad a_1 = \frac{sf(1-c)}{sf(1-c) + (1-s)f(1-b) + (1-b)(1-c)} ,$$

$$(10) \quad a_2 = \frac{(1-s)f(1-b)}{sf(1-c) + (1-s)f(1-b) + (1-b)(1-c)} ,$$

and

$$a_3 = \frac{(1-b)(1-c)}{sf(1-c) + (1-s)f(1-b) + (1-b)(1-c)} .$$

The nature in which the group proportions depend on the different parameters y (where y includes s,f,b, and c) is summarized in Table I. Groups with higher conditional probabilities of success s will exhibit greater concentrations in S^1 and lower concentrations in S^2 and S^3. Increasing the probability f of finding an acceptable job reduces the unemployment rate and increases both employment state frequencies. Finally,

Table I

Summary of $\partial a_i/\partial y$ Effects

y	a_1	a_2	a_3
s	+	-	-
f	+	+	-
b	+	-	-
c	-	+	-

increases in b and c raise the values of a_1 and a_2, respectively, while decreasing all other a_i's.

What is particularly striking about the results is that a difference in a parameter for one of the states has widespread ramifications for the group's labor mobility patterns. Consider two groups of workers that differ only in that one group has a lower conditional probability of success s.[12] As these individuals leave the ranks of the unemployed, they will tend to be concentrated in unsuccessful job matches in S^2. These jobs are associated with higher turnover rates, which in turn leads to an increase in the level of unemployment a_3 even though both groups were assumed to have the same probability f of finding an acceptable job once employed.

This interdependence is reflected in the characterizations in the literature on the secondary labor market and individuals in dead-end jobs.[13] There is a strong correlation among high turnover, low wage jobs, and unemployment rates. As the sorting models indicate, this correlation need not be attributable to institutional factors or labor market discrimination once employed, but may be due to the dynamics of the mobility process, and, more specifically, to differences in groups' job search activities. This matter will be explored in greater detail in Section III.C.

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[12]Other things equal, a lower z_n is required to lower s and an increase in f will occur as well. The analysis here assumes that s is the only difference in behavior. The joint variation of s and f is analyzed below.

[13]For detailed discussion along these lines, see Doeringer and Piore (1971) and Hall (1972).

B. Turnover Histories and the Quality of the Workforce

The nature of the interdependence in the labor mobility process is forcefully illustrated by the work histories associated with different labor force groups. Since there is greater turnover from S^2, one's intuition might suggest that S^3 would eventually consist largely of individuals with unsuccessful job histories. To assess whether this is the case, suppose that the transition matrix P is time-invariant and that the average quality Q^* of any group of workers is the ratio of the proportion in that group who are successfully matched (i.e., those in S^1) to the proportion who are unsuccessfully matched (i.e., those in S^2). The average quality of the inflow of workers from S^3 to new jobs is always $s/(1-s)$ since the fraction s will always be in S^1 and (1-s) will enter S^2. Now consider the equilibrium Q^* of the workers leaving their jobs in any period. If the total population of workers is N, then the number of successfully matched workers entering S^3 in any period is $Na_1(1-b)$, while the corresponding number of unsuccessful entrants to S^3 is $Na_2(1-c)$. Consequently, the value of Q^* for the flow into the unemployed state is

$$Q^* = \frac{Na_1(1-b)}{Na_2(1-c)} = \frac{s}{1-s} \quad ,$$

as one can verify by substituting for a_1 and a_2 from equations 9 and 10.

Any value of s will yield an equilibrium quality of workers that is the same for new hires, those entering the ranks of the unemployed, and the stock of unemployed.[14] Moreover, the value of Q^* depends only on s, the conditional probability of success for acceptable jobs. Labor market groups with lower values of s generated during the job search process will consequently be associated with more turnover from unsuccessful job matches and a stock of unemployed who were unsuccessfully matched on their previous jobs. For the equilibrium values of the labor mobility process, there is not simply some positive correlation among these values. Rather, all of the Q^* values are identical and dependent only on s.

C. Application to Black-White Youth Unemployment Differences

The sorting model can be profitably applied to the analysis of a particularly salient problem, the wide discrepancy between rates of unemployment for black and white youths. Despite slight differences in emphasis in various studies, the general consensus is that, while blacks have a somewhat lower probability of finding a job once unemployed, the dominant cause of the unemployment discrepancy is that young blacks exhibit much higher rates of turnover.

Although this problem is manifested through departures from employment (S^1 and S^2), the source of the discrepancy may not be due to any aspect of the functioning of the employment process. Rather differences while unemployed in the nature of the search strategies can generate observed differences in employment even if the transition probabilities while in S^1 and S^2 are identical for both groups.

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[14] If exits from S^3 are random and the Q^* for the outflow from S^3 and for the inflow are identical, the value of Q^* for the preceding job held by the stock of workers in S^3 will also have the same equilibrium Q^*.

More specifically, suppose that some economic factors lead blacks to be less selective in their search so that they adopt a lower z_n. One such influence may be a lower u_n^3 value because remaining unemployed may be less desirable for poorer black youths. The reduced z_n will lead blacks to be matched less successfully to jobs in S^2 from which they will exhibit higher turnover and become unemployed.

An important issue is whether the observed racial differences can be possibly attributed solely to exogenous factors that affect the choice z_n. First, it should be noted that the values of b and c may differ by race, so that lower levels of z_n and the associated lower values of s_n need not be the cause of the observed patterns, though they are consistent with the data. Second, if blacks do have lower z_n values, one can determine whether their mobility lottery offer distributions are identical to those of whites. Since $\partial f_n/\partial z_n$ is negative, a lower s (due to a lower z_n) for blacks must be accompanied by a higher job finding rate f if the distribution g(z) is identical across races. Since unemployed blacks have slightly lower probabilities of finding jobs, if lower s values do indeed contribute to the observed mobility patterns, then blacks must also have different g(z) distributions with smaller upper right tails.[15]

Differences in groups' choices of z_n has an additional ramification as well. Suppose that economic factors associated with the disadvantaged

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[15] For a discussion of the empirical differences in the f values for black and white youths and the other patterns considered here, see Leighton and Mincer (forthcoming), the other papers in Freeman and Wise (forthcoming), and Flanagan (1978).

status of a labor force group have led them to lower their z_n values for the mobility lottery search process. Then that group's job finding rate will be raised so that examination of exits from unemployment for these individuals will yield an overly optimistic portrayal of the functioning of the labor market for the unemployed, while at the same time leading to greater turnover (since $\partial s_n/\partial z_n > 0$) that suggests that the problem is an intrinsic instability of the nature of jobs. Thus, the endogeneity of mobility decisions may mask important differences in the behavior of the unemployed. Moreover, group differences in search strategies will lead to different turnover propensities that one might attribute incorrectly to the functioning of the employment process rather than individual search decisions. Both the locus and cause of labor mobility differences may be misunderstood.

IV. On-the-Job Experience and Labor Mobility

A central, and perhaps most important, determinant of empirically observed patterns of labor mobility is the years of experience the worker has had with his present employer. Firm-specific experience has a negative influence on overall job separations, quits, and layoffs, and a positive effect on worker earnings.[16] The economic motivations for these effects include the role of specific human capital formation, which enhances the employee's attractiveness to the firm. Such concerns are implicit in the models developed here since they contribute to the higher termination rate of unsuccessful matches.

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[16] See, for example, Becker (1964), Jovanovic (1977), Lazear (1976), Leighton and Mincer (forthcoming), Mincer (1974), Mincer and Jovanovic (1979), Mortensen (1978), Oi (1962), Pencavel (1972), Rosen (1972), and Viscusi (1979, forthcoming-a).

The focus of this section is not on the economic underpinnings of the probabilitistic structure of the mobility patterns analyzed in Sections II and III. Rather I will attempt to assess whether the rather unrestrictive stochastic structures described previously are consistent with empirical observations. Both the first and second derivatives of the relationship between firm-specific experience and the aforementioned variables will be matters of concern.

Let t indicate the years of the worker's experience at the firm. Then the probability $P(S^1|t)$ that the worker is matched successfully in S^1 given t years of experience is given by

$$P(S^1|t) = \frac{P(S^1)P(t|S^1)}{P(S^1)P(t|S^1) + P(S^2)P(t|S^2)} = \frac{1}{1 + \frac{P(S^2)P(t|S^2)}{P(S^1)P(t|S^1)}},$$

which simplifies to

$$(11)\quad P(S^1|t) = \frac{1}{1 + \frac{P(S^2 \cap t)}{P(S^1 \cap t)}} = \frac{1}{1 + (\frac{c}{b})^t}.$$

The strong correlation between specific experience and successful job matches is reflected in the limiting result that

$$\lim_{t\to\infty} P(S^1|t) = 1,$$

since b > c. In contrast, from the earlier asymptotic results, we know that as the worker's age n increases,

$$\lim_{n\to\infty} P(S^1|n) = a_1 < 1.$$

The superior empirical performance of specific experience (or tenure) variables over that of age variables is not unexpected since large values of experience are quite accurate predictors of successful job matches, which, in the context of the sorting models, are the key determinants of mobility.

The impact of t on mobility patterns depends on how it alters the value of the conditional probability $P(S^1|t)$.[17] Logarithmic differentiation of equation 11 yields

$$(12)\quad \frac{\partial P(S^1|t)}{\partial t} = - [1 + (\tfrac{c}{b})^t]^{-2}(\tfrac{c}{b})^t \ln(\tfrac{c}{b}) > 0 ,$$

and

$$(13)\quad \frac{\partial^2 P(S^1|t)}{\partial t^2} = \frac{[\ln(\tfrac{c}{b})]^2[(\tfrac{c}{b})^t - 1](\tfrac{c}{b})^t}{[1 + (\tfrac{c}{b})^t]^3} < 0 .$$

The relation between the conditional probability of a successful job match is positive, but $P(S^1|t)$ increases at a diminishing rate.

The relations in equations 12 and 13 provide all of the information required to assess the impact of specific experience within the context of the sorting models. Let $P(Sep|S^i)$ indicate the separation probability for a worker in state i. By our earlier assumption,

$$P(Sep|S^1) = (1-b) < P(Sep|S^2) = (1-c) .$$

The probability of a separation for a worker with experience t is a conditional weighted average of the probabilities for each state, so

[17] Since $P(S^2|t) = 1 - P(S^1|t)$, this information is all that is required to assess the conditional probabilities of success for employed workers.

that

$$P(Sep|t) = P(Sep|S^1)P(S^1|t) + P(Sep|S^2)P(S^2|t)$$

or

$$P(Sep|t) = P(Sep|S^2) + P(S^1|t)[P(Sep|S^1) - P(Sep|S^2)] .$$

Upon differentiation by t, one obtains the result using equations 12 and 13 that

$$\frac{\partial P(Sep|t)}{\partial t} = \frac{\partial P(S^1|t)}{\partial t}[P(Sep|S^1) - P(Sep|S^2)] < 0 ,$$

and

$$\frac{\partial^2 P(Sep|t)}{\partial t^2} = \frac{\partial^2 P(S^1|t)}{\partial t^2}[P(Sep|S^1) - P(Sep|S^2)] > 0 .$$

As illustrated in Figure I, the sorting model predicts a convex negative relationship between the probability of separation and firm-specific experience t. The limiting value of the separation probability is the turnover rate (1-b) for S^1. The initial separation probability for new hires is an average of the separation rates for each state, with the probability of a successful job match being the weight. More specifically, the initial separation value is s(1-b) + (1-s)(1-c).

If the composition of separations in the two states is not too similar, then one will also have a relationship between worker quits (Quit) and permanent layoffs (Fire) that satisfy

$$P(Quit|S^1) < P(Quit|S^2) ,$$

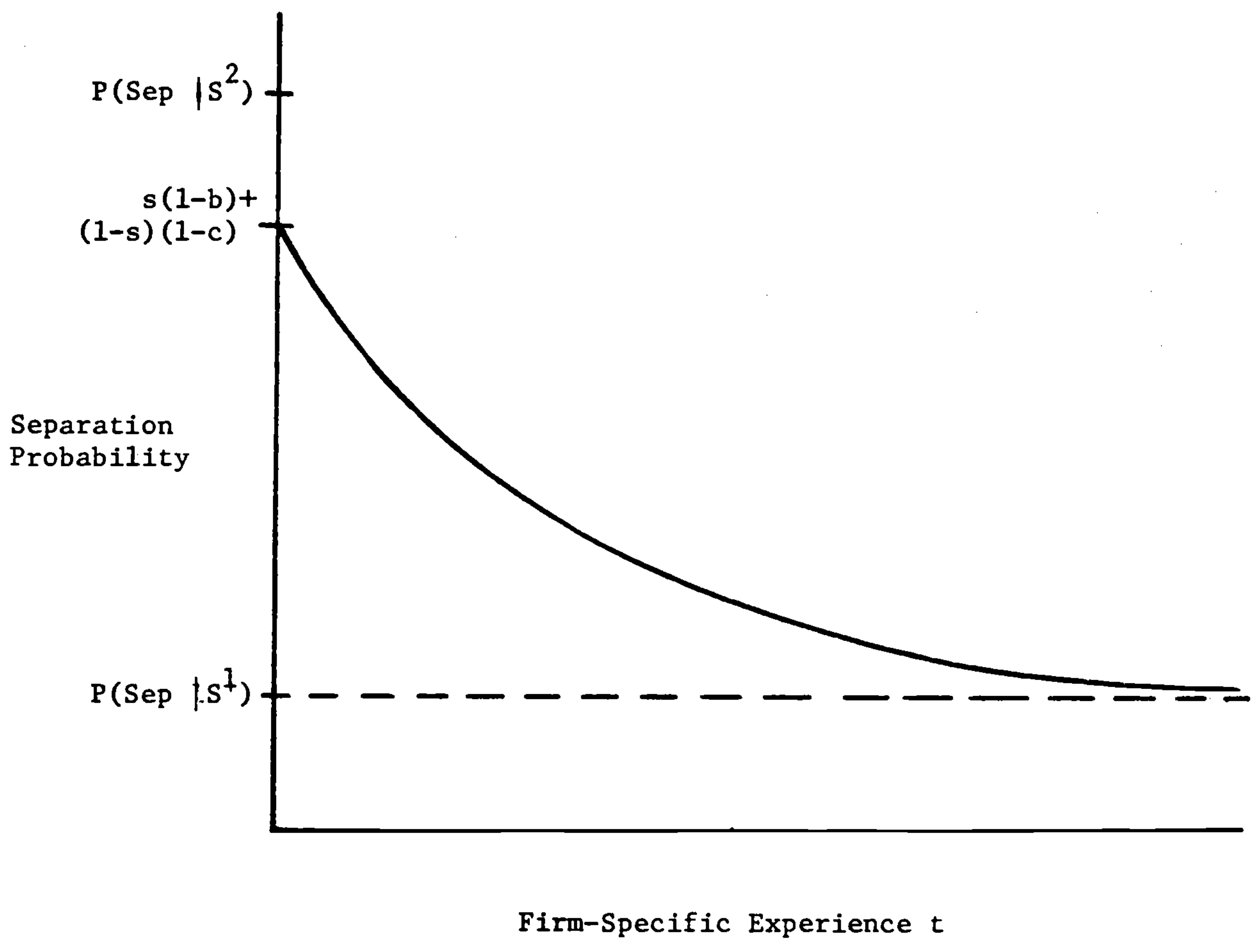
and

$$P(Fire|S^1) < P(Fire|S^2) .$$[18]

[18]In most analyses of the matching process as in empirical work, the distinction between quits and involuntary terminations is somewhat blurred.

Figure I

Firm-Specific Experience and Separation Probabilities



With these stipulations one can derive a similar negative, convex relation between t and both quits and layoffs.

In addition to generating predictions regarding worker turnover propensities, the sorting model can also be used to analyze the earnings-experience relationship. Let the state rewards u^i be wage levels for S^1 and S^2 and let $\bar{u}(t)$ be the expected wage rate with t years of firm-specific experience.

Following a procedure similar to that employed for separations, one obtains the result that

$$\bar{u}(t) = u^2 + P(S^1|t)[u^1 - u^2] .$$

Differentiation with respect to t yields

$$\frac{\partial \bar{u}(t)}{\partial t} = \frac{\partial P(S^1|t)}{\partial t} [u^1 - u^2] > 0 ,$$

and

$$\frac{\partial^2 \bar{u}(t)}{\partial t} = \frac{\partial^2 P(S^1|t)}{\partial t^2} [u^1 - u^2] < 0 .$$

Figure II sketches the positive, concave functional relationship between $\bar{u}$ and t. The value of $\bar{u}$ starts at $su^1 + (1-s)u^2$ and approaches a limiting value of u^1.

What is particularly striking about all of these results is that the sorting model involves a rather simple probabilistic structure and that no restrictive assumptions about the magnitude of different probabilities were made. Nevertheless, the sorting models correctly predicted both the direction and convexity of the relationship between firm-specific experiences and both wages and turnover.[19]

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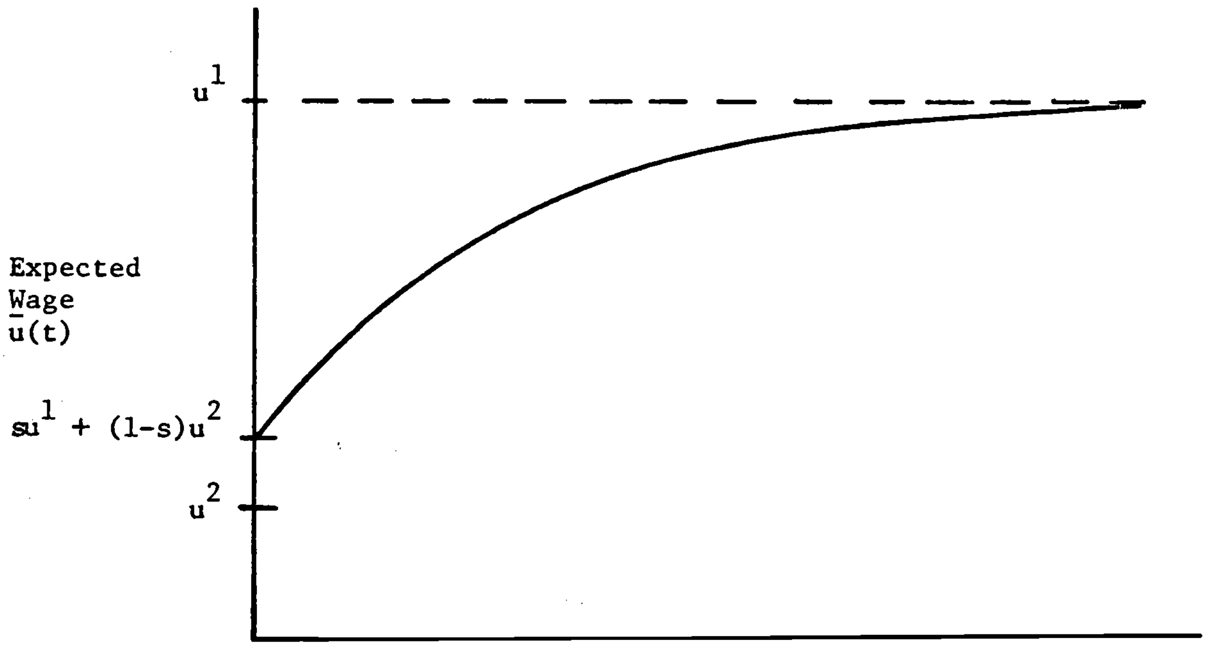
[19] The direction and convexity of these relations is examined most thoroughly by Leighton and Mincer (forthcoming).

Figure II

Firm-Specific Experience and

Expected Wage Levels



Firm-Specific Experience t

V. Worker Age and Unemployment

Perhaps the most salient characteristic of unemployment patterns is that youth unemployment rates are considerably higher than those of their older counterparts. A major source of this difference is that older workers have more on-the-job experience, are more likely to be matched appropriately to jobs, and will have lower turnover rates, thus entering the ranks of the unemployed less frequently.[20] The analysis here will abstract from such age-related effects and will focus on the impact of age <u>per se</u> on unemployment.

Consider the following merged mobility transition matrix,

$$\begin{bmatrix} q & 1-q \\ f & 1-f \end{bmatrix},$$

where S^1 is employment, S^2 is unemployment, f is still the probability of finding a job, q is the probability that one will not leave the job, and $q > f$.[21] New labor force entrants are assumed to enter the labor force in S^2. Their age n reflects the number of periods they have been in the labor force.[22]

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[20] On an empirical basis, these influences are probably best reflected in the firm-specific experience (or tenure) variables, as the analysis in Section IV would suggest.

[21] If b equals c in the larger version of the model, the merged Markov process involves no loss in generality. Otherwise, the restriction to two-states can be viewed as a simplification. Analysis of 3-state models is too complex to yield general closed-form solutions, such as that in equation 14, once one leaves the realm of numerical examples.

[22] This format also accords with usual empirical procedures, which typically set the overall experience measure equal to Age-Years of Schooling - 6.

The matter of interest is the nature of the conditional probability of unemployment, given the worker's age, or $P(S^2|n)$. Using the z-transform analysis described in Howard (1971), one can calculate the closed form expression for this value as

$$(14) \quad P(S^2|n) = [\frac{1-q}{1+f-q}] + (q-f)^n [\frac{f}{1+f-q}] \ .$$

The first bracketed term is the steady state probability that the worker will be in S^2. The final terms in this expression are associated with the impact of n on $P(S^2|n)$ and have a limiting value of 0.

Logarithmic differentiation of equation 14 yields

$$\frac{\partial P(S^2|n)}{\partial n} = [\frac{f}{1+f-q}] (q-f)^n \ \ell n(q-f) < 0 \ ,$$

and

$$\frac{\partial^2 P(S^2|n)}{\partial n^2} = [\frac{f}{1+f-q}] (q-f)^n [\ell n(q-f)]^2 > 0 \ .$$

As illustrated in Figure III, the probability of unemployment diminishes at a decreasing rate with respect to age, as is found empirically.[23]

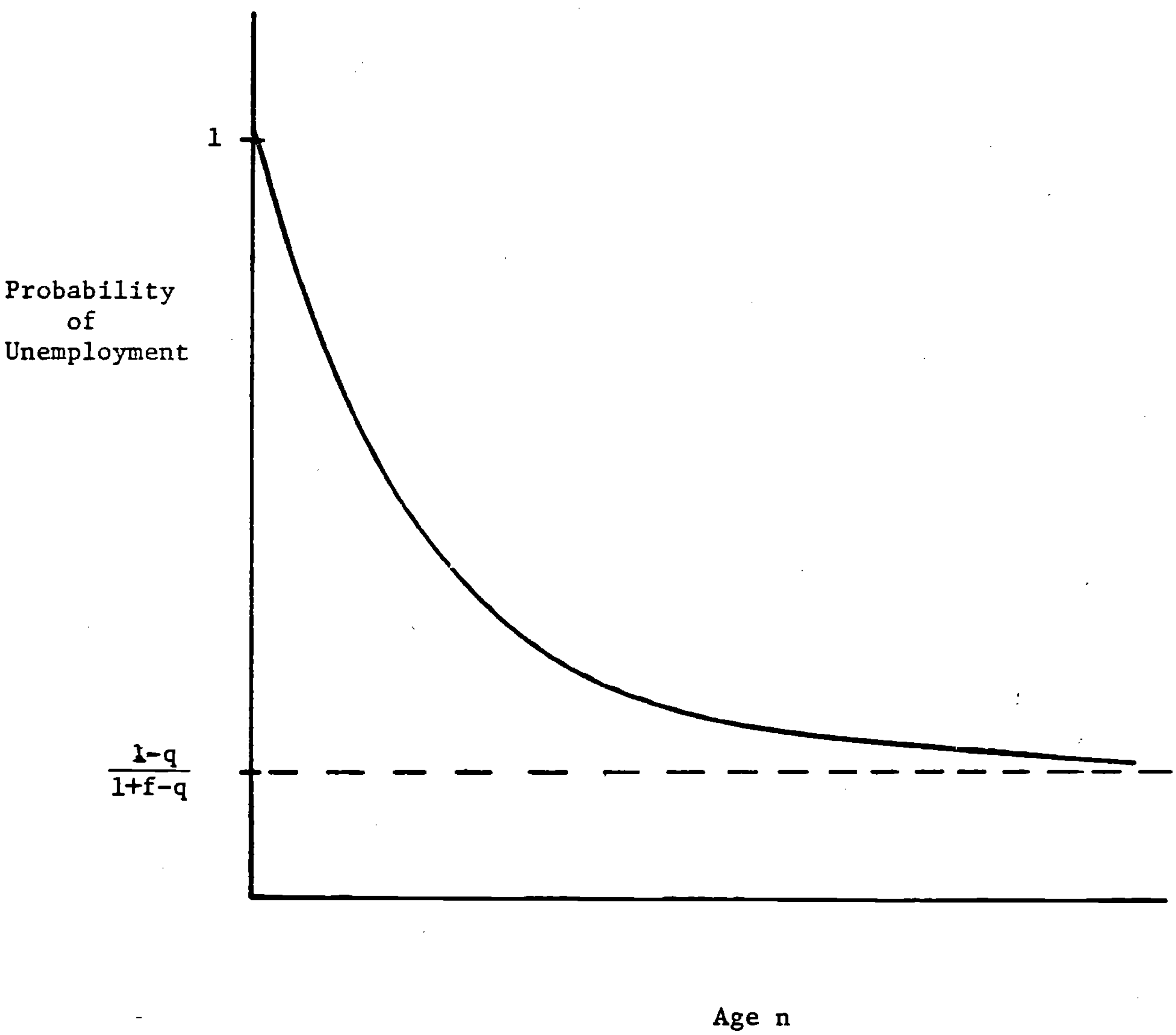
VI. Conclusion

The sorting models of labor mobility introduced the process of mobility lottery search into the analysis of unemployment and assessed its implications for subsequent worker actions. With relatively minimal assumptions concerning relative empirical magnitudes, these simple models correctly predict both the direction and convexity of the age-unemployment

[23]A recent analysis of this relationship is provided by Leighton and Mincer (forthcoming).

Figure III

Age-Unemployment Relationship



relation as well as the impact of enterprise-specific experience on wages and various turnover measures.

Application of the sorting framework to observed mobility patterns suggests that the endogeneity of the mobility lottery decision may lead to turnover differences that are incorrectly attributed to employment policies and unemployment patterns that disguise important group differences in the nature of unemployment. The theory also implies that differences in black-white unemployment and turnover cannot be simply due to less selective search policies by blacks. If blacks do indeed have lower reservation probabilities for acceptable job matches, then their opportunities must also differ from those of whites.

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