NBER WORKING PAPER SERIES

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Working Paper No. 322

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
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February 1979

The research reported here is part of the NBER's research program in economic fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research. The authors wish to thank Kenneth J. Arrow and Jerry Green for stimulating conversations.

* A grant from the Finnish Cultural Foundation is gratefully acknowledged.

** Financial support from NSF Grant SOC78-06162 is gratefully acknowledged.

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Abstract

Stochastic rationing when the market does not clear draws attention because both Drèze (1975) and Benassy (1975) quantity-constrained equilibria have some undesirable features. Gale (1978) gave the existence proof of trade under uncertainty. His stochastic rationing depends on all the individual effective demands. It is too vague to characterize a rationing mechanism. Moreover, his assumption to ensure a non-trivial equilibrium is economically not clear.

In this paper we extend Green (1978) to characterizing the rationing scheme as the individual effective demand times the rationing number which is a function of the aggregate quantity signals. We also construct an economy with money and overlapping generations. We show the existence of the non-trivial equilibrium and provide an example of a non-Walrasian equilibrium at the Walrasian equilibrium prices.

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1. Introduction

One of the key features in the disequilibrium theory is to face the fact that trades take place before all the markets clear through the Walrasian tatonnement process. Moreover, we usually assume the fixed price and the quantity adjustments in the short run. By quantity adjustments we mean the revision of individual optimization plans taking into account the quantity constraints in some markets. However, there are several ways how to formulate this process. Drèze (1975) defined the effective demands as the optimal quantity to maximize utility with quantity limit (the upper and lower bounds). Within the bounds you can trade as much as you want. Therefore, there are no excess effective demand or supply left in the Drèze equilibrium. This is unsatisfactory when we discuss disequilibrium measures such as unemployment and excess capacity of plants. Benassy (1975) proposed the effective demand for the ith good as the amount of the ith good which maximizes the utility given the quantity constraints in all the markets other than the ith. Since economic agents have to solve the different maximization problems as many times as the number of the commodities, the budget constraint does not hold in the effective demand level. This is not satisfactory from the viewpoint of rationality.

The undesirable features in the Drèze or Benassy equilibrium lead researchers to investigate the so-called stochastic rationing schemes. The realization after the announcement of the effective demands are stochastic. Svensson (1977) discussed a stochastic rationing scheme and individual decision making but he did not proceed to establish the market expectational equilibrium which guarantees the rationing scheme is correct in the statistical sense in the market.

Douglas Gale (1977) and Jerry Green (1978) gave the most general treatment of the stochastic rationing equilibria, where individuals in the pure exchange economy have the probability distributions on the realization of his effective demands. With some conditions on the expectation of rationing given the actions taken by other agents Gale and Green showed the existence of the stochastic rationing equilibria.

Although their equilibria show that the budget constraint should be met in the effective demand level, it does not mean the stochastically realized trade is such that everybody balance the budget. Moreover, Gale showed the existence of the non-trivial equilibria, but the sufficient condition is rather vague to be meaningfully interpreted, where the trivial equilibrium is the no-action equilibrium.

In this paper, we specify our economy as an overlapping generations model with production and the government. We impose the ex post budget constraint and the ex post production constraint rather than ones in terms of effective demands. This gives the possibility of discussion of disequilibrium dynamics in the long-run. We also show a meaningful sufficient condition for the non-trivial equilibria.

This framework has also an interesting feature that there may exist a non-Walrasian equilibrium at the Walrasian equilibrium prices, even though expectations and agents are perfectly "rational." This is confirmed by a numerical example in the appendix. Note that in our framework, this phenomenon is caused by the pure pessimism, in contrast to Heller and Starr (1979) whose example of the same effect is not quite correct in that they allow the money endowments to vary (hence money prices are "wrong" though relative prices of goods are "correct.")

Temporary equilibrium models within overlapping generations have been studied by Grandmont and Laroque (1973), Fuchs and Laroque (1976), and Fuchs (1976). They assume that prices move fast enough to clear the spot markets within a period. They define the stationary market equilibrium of the series of prices for the relevant length of time generating the present expected price equal to that price. They showed the conditions for the existence of the stationary market equilibrium; its Pareto efficiency, the local stability of the economy near the equilibrium with respect to the characteristics of its agents and a money stock; and the structural stability with respect to the expectations.

2. Framework and Notations

Our economy is composed of four different types of economic agents: H consumers in each of the young and old generation; firms; and the government. There are I different consumption goods and one type of factor of production named labor. The government issues money which is used as the medium of exchange and as the store of value. A young consumer is endowed with no initial money balance and the positive potential labor force. The young supplies labor, demands consumption goods and saves for the next period. An older consumer with carried-over money balance does not work any more but just consumes before the end of the life. Each of I consumption goods are produced by F identical firms. The production technology of each industry is characterized by a well-behaved neo-classical production function, $f^i(\cdot)$.

The prices of the consumption goods, $(P_1, P_2, \ldots, P_I)$ and the nominal wage, w, are fixed within a period. The aggregate demand and supply "signals" for each market, $\{\alpha\}$, are announced and distributed among the economic agents without costs. Y_i^d and Y_i^s denote the aggregate demand and supply signal in the ith market, respectively, and L^d and L^s denote those of the labor market, respectively. According to the aggregate signals and the individual effective demand (or supply), the economic agent faces the stochastic rationing of trade. An individual agent is assumed to take the aggregate signals as given and to think that his individual effective demand is so small compared to the market signals that change in his own individual effective demand is negligible. This assumption is similar to the price-taker assumption in the Walrasian economy. The realization of trades occur after all the economic agents submit their effective demands and supplies. The stochastic rationing mechanism will be defined in the next section.

Let us sketch the economic agents decision-making. Consumers, whether they are young or old, maximize their expected utility. Older consumers, (k = 1,..., H) plan which goods they should order out of the predetermined money balance which is the consequence of the preceding period. Young consumers (h = 1, ..., H) have to take into account the possibility of future rationing as well as the present one. A consumer, h, submits his effective demand for the ith, consumption goods, y_i^h, and his effective supply of labor, ℓ^h, after maximizing the expected utility function with the budget constraint in the "worst" case of rationing possibility. Therefore, the effective demand (or supply) is the amount up to which the consumer is "ready" to buy (or sell). Since an older consumer is not allowed to work, a younger consumer plans the non-negative money balance for all possible cases of rationing.

A firm, f, in the ith industry which is assumed to be risk-neutral maximizes its expected profit. It submits the effective demand for labor, ℓ_i^f, and the effective supply of the consumption goods, y_i^f . The effective demand for labor and the effective supply of the goods have to be technologically possible in the "worst" rationing case, i.e., the maximum possible assignment of sales and the minimum assignment of hiring labor. Therefore, the effective demand and supply are again the amounts that the firm is "ready" to accept.

Since all the markets meet simultaneously, the effective demand and supply is final once they are submitted. The origin is assumed to be included in the production possibility set, hence the firms never plan the negative expected profit. However, the realization of events may be such that a firm incurs losses from hoarding the excess labor relative to restrictive rationing on sales. Considering that some firms do make losses in the real world, we do not think this is a bad assumption. We assume that all the realized profits and losses are taxed or subsidized by the government. Therefore, the only store of value available for the young generation is money. The government demands the constant amount of the consumption goods $(y_1^g, y_2^g, \ldots, y_I^g)$ and labor services ℓ^g.

A <u>temporary equilibrium with stochastic rationing</u>, given the fixed prices, is defined as the signals of aggregate demands and supplies which induce economic agents to submit individual effective demands and supplies which exactly summed up to the signals.

<u>Definition 2.1</u>

A temporary equilibrium with stochastic rationing is the set of the quantity signals with the following conditions:

$$Y_i^d = \sum_{h=1}^{2H} y_i^h(\alpha) + y^g \qquad \text{where } y_i^h(\alpha) \in \zeta_i^h(\alpha)$$

$$Y_i^s = \sum_{f=1}^{F} y_i^f(\alpha) \qquad \text{where } y_i^f(\alpha) \in \zeta_i^f(\alpha)$$

$$L^d = \sum_{i=1}^{I} \sum_{f=1}^{F} \ell_i^f(\alpha) + \ell^g \qquad \text{where } \ell_i^f(\alpha) \in \zeta_i^f(\alpha)$$

$$L^s = \sum_{h=1}^{H} \ell_i^h(\alpha) \qquad \text{where } \ell_i^h(\alpha) \in \zeta_i^h(\alpha)$$

$$\alpha \equiv (Y_{1t}^d, \quad Y_{2t}^d, \ldots, Y_{It}^d, \quad Y_{1t}^s, \quad Y_{2t}^s, \ldots,; L_t^d, L_t^s) \; .$$

where a subscript t denotes period t, and ζ denote individual demand and supply correspondences.

In this kind of expectational equilibrium, we usually have the trivial equilibrium of no-action (see Gale (1977), the last section). That is, if the signals show zero aggregate demand and supply, then individuals feel they cannot sell anything. This gives zero purchasing power and zero effective demands. Therefore, the signals with zero demand and supply are confirmed as an equilibrium. We are going to show that the existence of government and the old generation with sure purchasing power, the rationing probability distribution function bounded away from zero, and the well-behaved production functions are enough to prove that the trivial equilibrium vanishes.

3. Stochastic Rationing

Let us describe the stochastic rationing scheme we are going to adopt. The firm announces its "effective demand" for labor, ℓ_i^f, and its "effective supply" of the ith consumption goods, y_i^f. The consumer announces his "effective demand" for the consumption goods, $(y_i^h, y_2^h, \ldots, y_I^h)$ and his "effective supply" of labor, ℓ^h. Since the prices are fixed during the period there is stochastic rationing in the market to make trades feasible. Rationing is an assignment of trade, which is called "realization," by random drawing according to a known distribution. We are going to adopt the following axioms established by Green (1978):

I) The probability distribution of rationing to the jth agent in the ith (or labor) market depends only on the jth effective demand (or supply) and the quantity signals of the ith market. That is, the distribution is independent across the markets. Moreover, the distributions are the same if the agents in the same market offer the same effective demand (or supply).

II) Rationing does not change the side of the market i.e., a demander remains buying the goods, and a supplier selling the goods.

III) Agents are not forced to buy (or sell) more than the amount they announce as the effective demand (or supply).

IV) The mean of the probability distribution of rationing balances demand and supply.

V) The probability distribution of rationing is continuous when endowed with the topology of weakly convergence.

The fourth condition requires that the realized trade balances only its "mean." It is debatable whether we can actually devise a rationing scheme which balances "realized" aggregate demand with supply "with certainty" keeping individual rationing stochastic. We will come back to this point later.

The fifth condition is the standard technical assumption.

Green showed that the rationing scheme satisfying the above axioms is representable in the following form:

$$
\begin{aligned}
\tilde{y}_i^f &= y_i^f \; \tilde{s}_i^f \, (y_i^f,\ Y_i^d,\ Y_i^s) \\
\tilde{y}_i^h &= y_i^h \; \tilde{s}_i^h \, (y_i^h,\ Y_i^d,\ Y_i^s) \\
\tilde{\ell}_i^f &= \ell_i^f \; \tilde{s}_\ell^f \, (\ell_i^f,\ L^d,\ L^s) \\
\tilde{\ell}^h &= \ell^h \; \tilde{s}_i^h \, (\ell_i^h,\ L^d,\ L^s)
\end{aligned}
\tag{3. 1}
$$

where y_i^f and y_i^h are the effective supply from firm f, and the effective demand from household h in the ith consumption goods market; ℓ_i^f is the effective labor demand from the firm f in the ith industry; and ℓ_h is the effective labor from household h; $\tilde{s}$ are stochastic functions depending on the individual effective demand and the market quantity signals. The stochastic rationing functions have the means which are independent of the induvidual offers.

Notice that all the firms in the ith industry are identical in the production function and behavior. Therefore, we have the following simplified stochastic rationing function:

$$\tilde{y}_i^f = y_i^f \ \tilde{s}_i^f \ (Y_i^d, \ Y_i^s)$$

We also assume the similar rationing function on the other side of the market and also in the labor market. There we have

$$\tilde{y}_i^f = y_i^f \ \tilde{s}_i^f \ (Y_i^d, \ Y_i^s)$$

$$\tilde{y}_i^h = y_i^h \ \tilde{s}_i^h \ (Y_i^d, \ Y_i^s)$$

(3, 1')

$$\tilde{\ell}_i^f = \ell_i^f \ \tilde{s}_\ell^f \ (L^d, \ L^s)$$

$$\tilde{\ell}^h = \ell^h \ \tilde{s}_\ell^h \ (L^d, \ L^s)$$

This is an ad hoc assumption, which implies that an economic agent expects the proportional rationing independent of his action.

In order to ensure the axioms above, we have certain requirements on the supports of the probability distributions.

Assumption 3.1

$0 \leq \tilde{s}_i^f \leq 1$ $\quad \forall_i, \forall_f$, with probability one

$0 \leq \tilde{s}_i^h \leq 1$ $\quad \forall_i, \forall_h$, with probability one

$0 \leq \tilde{s}_\ell^f \leq 1$ $\quad \forall_\ell, \forall_f$, with probability one

$0 \leq \tilde{s}_\ell^h \leq 1$ $\quad \forall_\ell, \forall_h$, with probability one

This assumption ensures axioms II) and III). The next assumption implies axiom IV).

Assumption 3.2

$$\sum_{f=1}^{F} E(\tilde{y}_i^f) = \sum_{h=1}^{2H} E(\tilde{y}_i^h) + y_i^g$$

$$\sum_{i=1}^{I} \sum_{f=1}^{F} E(\tilde{\ell}_i^f) + \ell^g = \sum_{h=1}^{H} E(\tilde{\ell}^h)$$

Although Assumption 3.1 is necessary to ensure the axioms II) and III), it seems too weak an assumption to derive a meaningful equilibrium. The combination of this assumption with the budget constraint and the production feasibility in the "worst" cases will end up with the trivial equilibrium. We are going to assume stronger assumptions about the response of distributions to the aggregate signals.

A usual assumption in the disequilibrium macroeconomics is that the trade is realized at the short side of the market. This assumption implies that the probability distribution is degenerate at unity for agents in the short side of the market. For example, suppose that the excess demand prevails in the ith market. And as the disequilibrium deepens, the long side anticipates the "worse" distribution. We are going to characterize this idea by putting restrictions on the upper bound and the lower bound of the support of the probability distribution. The following are the assumptions on rationing that the firm faces.

Assumption 3.3

(i)

$$\underline{s}_i^f \leq \tilde{s}_i^f \leq \bar{s}_i^f \quad , \qquad \text{with probability one}$$

$$\underline{s}_\ell^f \leq \tilde{s}_\ell^f \leq \bar{s}_\ell^f \quad , \qquad \text{with probability one}$$

where

(ii)

$$\underline{s}_i^f = g^i(Y^d, Y_i^s) \geq 0 \quad , \qquad \text{with strict inequality if } Y_i^d > 0$$

$$\underline{s}_\ell^f = g^\ell(L^d, L^s) \geq 0 \quad , \qquad \text{with strict inequality if } L^s > 0$$

(iii)

$$\bar{s}_i^f = h^i(Y_i^d, Y_i^s) \leq 1$$

$$\bar{s}_\ell^f = h^\ell(L^d, L^s) \leq 1 \quad ;$$

(iv) Functions g and h are continuous with respect to their arguments.

Assumption 3.4

$$(s_i^f)_m \equiv E(\tilde{s}_i^f)$$

$$(s_i^\ell)_m \equiv E(\tilde{s}_i^\ell)$$

where

$$(s_i^f)_m = m^i(Y_i^d,\ Y_i^s) \quad , \quad \text{continuous with respect to its arguments}$$

$$(s_i^\ell)_m = m^\ell(L^d,\ L^s) \quad , \quad \text{continuous with respect to its arguments}$$

Example

We will give an example of a rationing scheme in an economy where all firms in an industry and all consumers are identical. The following rationing scheme preserves the equal treatment of individuals (i.e., the name does not matter), rational expectations on the quantity rationing scheme, the randomness for individuals; and balances of the aggregate demand and supply not only in the mean but also in realization with probability one. The rationing scheme also preserves the above-mentioned assumptions on the support of the distribution.

Take the representative firm in the ith consumption good market: his stochastic realization of supply, $\tilde{y}_i^f$ is now the effective demand multiplied by a stochastic "rationing number" $\tilde{s}$ which depends on the market signals. $\tilde{s}_i^f$ is distributed uniformly between $\underline{s}_i^f$ and $\bar{s}_i^f$ in the following manner: (we omit the obvious subscripts and superscripts here)

$$\begin{aligned}
\bar{s} &= 1 && \text{if } Y^d/Y^s \geqq 1 \\
&= 1/2 + (Y^d/2Y^s) && \text{if } (1/2) \leqq Y^d/Y^s \leqq 1 \\
&= 3Y^d/2Y^s && \text{if } \quad 0 \leqq Y^d/Y^s \leqq 1/2 \\
s_m &= \min(Y^d/Y^s,\ 1) && Y^d/Y^s \geqq 0
\end{aligned}$$

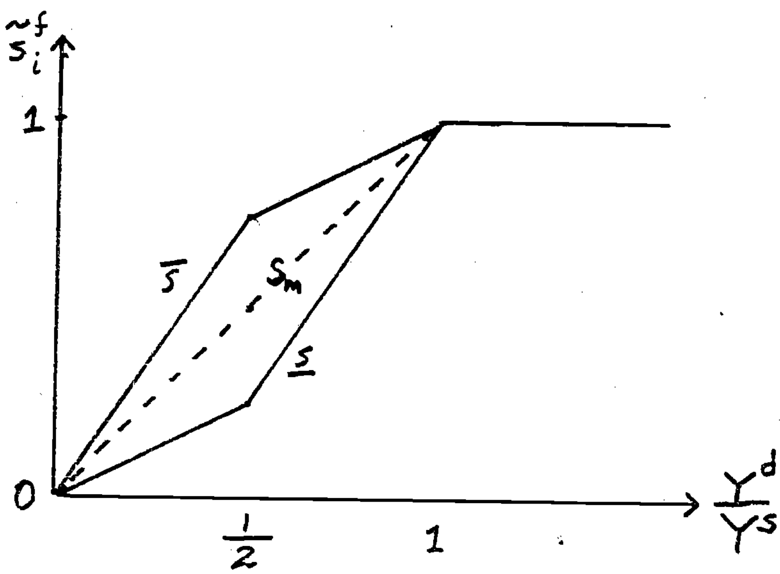

Fig 1

$$\underline{s} = 1 \qquad \text{if} \qquad Y^d/Y^s \geqq 1$$

$$= -1/2 + 3Y^d/2Y^s \qquad \text{if} \qquad 1/2 \leqq Y^d/Y^s \leqq 1$$

$$= Y^d/2Y^s \qquad \text{if} \qquad 0 \leqq Y^d/Y^s \leqq 1/2 \quad .$$

We also need one technical assumption. There is an even number of firms. Now we explain the procedure of rationing. Suppose the market signal, Y^d/Y^s, is less than one. (I) First stage is to make the lottery tickets stochastically: we are going to create F (an even number of firms) tickets. For the first ticket, draw a random number between $\underline{s}$ and $\bar{s}$. The second ticket is $s_m + (s_m - n_1)$ where n_1 is the number of the first ticket. Repeat this process F/2 times; (II) The second stage is the random drawing to decide the ordering of firms to draw the prepared tickets; (III) The third stage is that the first firm decided by the second stage draw randomly one ticket from the urn prepared by the first stage, and the drawn ticket is removed from the urn. Repeat the process F times to exhaust firms and tickets.

By the first stage, the sum of the rationing tickets add up to $s_m F \equiv FY^d/Y^s$ with certainty. Each firm is assumed to submit the same effective demand y_i^f. That means that the actual rationing is $Fy_i^f\, Y^d/Y^s$. From the definition of temporary equilibrium with stochastic rationing, $F\, y_i^f = Y^s$. Therefore, the aggregate <u>realization</u> of rationing is always Y^d. That is, the rationing scheme balances the aggregate demand and supply in realization with probability one. However, from the individual point of view, the rationing is stochastic and the distribution of rationing number is exactly explained above. Although the third stage of drawing means that a distribution of rationing tickets is dependent on other drawings, the second stage erases this problem. Hence, the identical distribution of rationing with anonymity as a whole system is preserved.

Note also that $\bar{s}$, s_m, and $\underline{s}$ are continuous with respect to Y^d and Y^s for $Y^d \geqq 0$ and $Y^s \geqq 0$, and satisfy assumptions made.

4. The Firms' Behavior

There are F firms in each of I industries of consumption goods. All the firms in the ith industry are identical. The representative firm f in the ith industry produces output $\hat{y}_i^f$ by the actual labor input $\hat{\ell}_i^f$.

The production technology is represented by the neoclassical production function with the following properties:

Assumption 4.1

By the actual input, $\hat{\ell}_i^f$, the firm can produce the actual output, $\hat{y}_i^f$, which is

$$f^i(\hat{\ell}_i^f) \geq \hat{y}_i^f \geq 0 \qquad \text{for} \quad \hat{\ell}_i^f \geq 0 \qquad \forall_i \ \forall_f \ .$$

Assumption 4.2

$$f^i(0) = 0 \qquad \forall_i \ .$$

The first assumption means there is no technological externality. The second assumption means that we cannot produce anything without inputs, or that no action is possible.

Assumption 4.3

$f^i(\hat{\ell}_i^f)$ is a concave and increasing function.

or alternatively,

Assumption 4.3'

$f^i(\hat{\ell}_i^f)$ is a strictly concave and increasing function.

Assumption 4.4 [Well behavedness]

(i) There exists a continuous first derivatives for f^i, V_i .

(ii) $f^{i\,\prime}(0) = \infty$

(iii) $f^{i\,\prime}(\infty) = 0$.

These are the standard neo-classical assumptions on a production function.

Now we can set up the maximization problem. The representative firm's decision is to maximize the expected profit:

(4.1) $$\max_{y_i^f, \ell_i^f} E(p_i \tilde{y}_i^f - w \tilde{\ell}_i^f)$$

subject to

(4.2) $$\tilde{y}_i^f = y_i^f \tilde{s}_i (Y_i^d, Y_i^s)$$

(4.3) $$\tilde{\ell}_i^f = \ell_i^f \tilde{s}_\ell (L^d, L^s)$$

(4.4) $$y_i^f \bar{s}_i \leq f^i(\ell \underline{s}_\ell)$$

Here the rationing proportions and their bounds are without superscript f by axiom I of Green (see p. 3-1 above). The same will be done later for consumers.

(4.4) means that in the "realization" terms, the firm has to be able to fulfill its promise of effective supply even if the most rationing in hiring labor and the least rationing in sales happen to be the case. Considering that we are maximizing the profit,

$$y_i^f = (1/\bar{s}_i) f^i(\ell \underline{s}_\ell) \ . \tag{4.5}$$

substituting (4.5) into (4.1)

$$\underset{\ell_i^f}{\text{Max}} \quad E \quad P_i f^i(\ell_i^f \underline{s}_\ell) \tilde{s}_i / \bar{s}_i - w \ell_i^f \tilde{s}_\ell$$

The first order condition is

$$P_i \underline{s}_\ell (s_i)_m \; f^{i\,\prime}(\ell_i^f \underline{s}_\ell)/\bar{s}_i - w(s_\ell)_m = 0 \ . \tag{4.6}$$

Since the market signals are exogenous to the firm, Assumption 4.3 guarantees that $\{\ell_i^f\}$ satisfying (4.6), say $\{(\ell_i^f)^*\}$ maximizes the expected profit. Assumptions 4.1 and 4.4 imply $(\ell_i^f)^*$ is strictly positive but finite, if $L^s > 0$ and $Y_i^d > 0$.

By assumptions 3.3 and 3.4 and 4.4, we find the upper hemi-continuous correspondence ζ^i from the space of the quantity signals to the space of the effective demand for labor and the effective supply of the ith consumption goods.

$$\ell_i^f = \zeta_{\ell i}^f \; (Y_i^d;\ Y_i^s;\ L^d;\ L^s) \qquad \forall (P^i,\ w) >> 0$$

$$Y_i^f = \zeta_{yi}^f \; (\cdot) \qquad \forall (P^i,\ w) >> 0 \ .$$

If so, let $[y(n),\ell(n)]$ ε line segment between $[\bar{y}(n),\bar{\ell}(n)]$ and $[y^o,\ell^o]$ such that

$$w_t(n)\ell(n)\underline{s}_t(\alpha_t(n) - \sum_i p_{it}(n)y_i(n)\bar{s}_{it}(\alpha_t(n)) = 0 .$$

Such a point clearly exists and satisfies $[y(n),\ell(n)]\varepsilon\beta_t^h(\bar{\alpha}_t(n))$.

Finally, it is routine to check that in all cases

$$[y(n),\ell(n)] \to [y^o,\ell^o], \text{ as } n \to \infty .$$

QED

Lemma 5.2

$\beta_{t+1}^h(\cdot,\bar{\alpha}_t,y_t^h,\ell_t^h)$ is a compact-valued, convex-valued and continuous correspondence in the set $\{p_{t+1}\varepsilon\mathbb{R}_+^I \mid p_{i,t+1} > 0\ \forall_i\}$, when $(\bar{\alpha}_t,y_t^h,\ell_t^h)$ is given and satisfies $w_t\ell_t^h\underline{s}_t(\alpha_t) - \sum_{i=1}^I p_{i,t}\ y_{it}^h\bar{s}_{it}(\alpha_t) > 0$. Lower hemi-continuity may fail if the last assumption is violated.

Proof: Entirely routine. QED

There are, thus, some problems with continuity, which will be dealt with later.

The next step is to relate expected future signals ${}_t\alpha_{t+1}$ to current ones $\bar{\alpha}_t$. This is done in the usual way (Grandmont) (1977 or 1978): ${}_t\alpha_{t+1}$ has a probability distribution which depends continuously on α_t in the topology of weak convergence of probability measures. Let $\rho(\alpha_t)$ be the distribution of ${}_t\alpha_{t+1}$, given α_t. The problems of non-continuity, especially in lemma 5.2, necessitate the following assumptions.

Proposition 2.2 [Continuity or Single-valued Mapping]

Suppose Assumptions 3.1-3.4 and 4.1-4.4 with assumption 4.3' instead of 4.3, then Ψ is single-valued.

Proof is trivial

Proposition 2.3 [Positive Response]

If $Y_i^d > 0$ and $L^s > 0$, then $\zeta_{yi}^f > 0$, $\zeta_{\ell i}^f > 0$.

Proof is obvious from Assumption 3.3(ii).

5. The Consumer's Behavior

Let us now consider a representative consumer h. To begin with we introduce the notations with subscript t:

$0 \leq y^h_{it}$ = effective demand for good i by consumer h at period t,

$0 \leq \ell^h_t$ = effective supply of labor by h at period t,

Y^d_{it} = aggregate effective demand for good i at period t,

L^s_t = aggregate effective supply of labor at period t.

Similarly, we denote by Y^s_{it}, L^d_t the aggregate demand for labor. They will appear as signals in the rationing mechanisms. Also, we let

$0 \leq \bar{L}^h_t$ = the endowment of labor of consumer h,

p_{it} = price of good i at period t,

w_t = nominal wage at period t,

$\tilde{y}^h_{it}$ = realized purchases of good i by consumer h at period t,

$\tilde{\ell}^h_t$ = realized sales of labor by consumer h at period t.

Finally, if subscript i is dropped the notation signifies the corresponding I-dimensional vector, where I is the number of goods in each period.

At each period a generation of consumers h = 1, ..., H is born. They live for two periods. In the 1st period a consumer sells labor and buys goods. In the 2nd period he is retired and only buys goods with money balances he has carried forward from the 1st period of his lifetime. No planned bequests exist, so a household has no initial money balances. Any involuntary bequests caused by rationing are taxed away by the government.

Since realizations are random it is natural to assume that the household has a preference relation $\succsim_h$ over probability distributions of realizations. We shall adopt the simpler hypothesis of expected utility maximization i.e., $\succsim_h$ can be represented by adopting a utility function $u_h : \mathbb{R}^{2I+1} \to \mathbb{R}$ such that

$$\int u_h(\tilde{y}_t^h, \bar{L}_t^h - \tilde{\ell}_t^h; \tilde{y}_{t+1}^h) d\mu$$

represents $\succsim_{ht}$, where μ is the probability distribution of consequences. $u_h(\ )$ will be taken to be concave. (For a further discussion of this issue see Grandmont (1972, 1977 and 1978).)

To relate realized transactions to effective demands and supplies we follow the approach of Green (1978) with one further simplification: the (stochastic) fractions of effective demand/supply that are realized are independent of the effective demand/supply of each individual consumer. The fractions do depend on aggregate effective demand and supply in the market. The assumptions of sign-preservation, continuity, etc. made by Green (1978) hold. Let ω_t, ω_{t+1}, denote the random elements in the rationings in each period. Then, we have

$$\tilde{y}_{it}^h = y_{it}^h \ \tilde{s}_{it}(Y_{it}^d, Y_{it}^s, \omega_t) \ , \quad i = 1, \ldots, I$$
$$h = 1, \ldots, H$$

$$\tilde{\ell}_t^h = \ell_t^h \ \tilde{s}_{\ell t} \ (L_t^d, L_t^s, \omega_t)$$

$$\tilde{y}^h_{i,t+1} = y^h_{i,t+1} \quad \tilde{s}_{i,t+1} \; (Y^d_{i,t+1}, \, Y^s_{i,t+1}, \, \omega_{t+1}) \; , \; i = 1, \, \ldots, \, I,$$

where the aggregate signals in period (t + 1) are now perceptions by the agent. Note that the proportions of rationing do not depend on the agent and the feasibility and budget constraints that each consumer has to obey are the followings:

Almost surely with respect to ω_t, ω_{t+1}

$$\text{I} \qquad \begin{aligned} &\bar{L}^h_t - \tilde{\ell}^h_t \geq 0 \\ &\tilde{y}^h_t \geq \theta \\ &\tilde{y}^h_t \geq 0 \\ &\tilde{y}^h_{t+1} \geq \theta, \end{aligned}$$

where θ is the zero vector and the notation for vector inequalities is the familiar one $\geq$, $>$, $>>$;

Almost surely with respect to ω_t, ω_{t+1}

$$\text{II} \qquad \begin{aligned} &\tilde{m}_t = w_t \; \tilde{\ell}^h_t - \sum_{i=i}^{I} p_{it} \; \tilde{y}^h_{it} \geq 0 \\ &\text{and} \\ &\sum_{i=1}^{I} {}_t p_{i,t+1} \quad {}_t y^h_{i,t+1} \leq \tilde{m}_t \; . \end{aligned}$$

where $({}_t p_{i,t+1})$ is the ith price of period (t + 1) anticipated at period t, and $({}_t y^h_{i,m+1})$ is the effective demand of the ith consumption goods from household h in generation t placed in period (t + 1). Condition (II) rules out the possibility of bankruptcy or default. This is so because they are assigned undefined expected disutility (compare Green (1978)). Note also that the second inequality in (II) involves effective demands $y^h_{i,t+1}$ rather than realizations. This is natural since a consumer does not plan any bequests, and furthermore, realization will never exceed the demand.

Let us now introduce notations for the signals that influence the behavior of the consumer. Let

$$\bar{\alpha}_t = (p_t, w_t\ (Y^d_{it}, Y^s_i\ )^I_{i=1}\ (L^d_t, L^s_t)) \equiv (p_t, w_t; \alpha_t)$$

$$_t\alpha_{t+1} = (p_{t+1}\ (Y^d_{i,t+1}, Y^s_{i,t+1})^I_{i=1}\ )\ .$$

be the signals of period t and the signals of (t+1) anticipated at period t.

$$\beta^h(\bar{\alpha}_t, {}_t\alpha_{t+1}) = \left\{ (y^h_t, \ell^h_t, y^h_{t+1}) \in \mathbb{R}^{2I+1}_+ \;\middle|\; \begin{array}{l} \text{a.s.}(\omega_t)\quad w_t \ell^h_t \tilde{s}_{\ell t}(\alpha_t,\omega_t) s_{it}(\alpha_t, \omega_t) \\ \qquad - \sum_{i=1}^{I} p_{it} y^h_{it} \tilde{s}_{it} \geq 0 , \\ \bar{L}^h_t - \ell^h_t \tilde{s}_{\ell t}(\alpha_t,\omega_t) \geq 0 , \\ \sum_{i=1}^{I} p_{i,t+1}\, y^h_{i,t+1} \leq w_t \ell^h_t \tilde{s}_{\ell t}(\alpha_t,\omega_t) \\ \qquad - \sum_{i=1}^{I} p_{it} y^h_{it} \tilde{s}(\alpha_t, w_t) \end{array} \right\}$$

be the set of possible actions by the consumer.

We make the following assumptions

Assumption 5.1

The distributions of $s_{\ell t}$, s_{it} are non-degenerate distributions satisfying

$$\forall \omega_t \qquad \tilde{s}_{it}(\ ) \in [\underline{s}_{it}, \bar{s}_{it}] \subset [0,1], \ i = 1, \ldots I$$

$$\tilde{s}_{\ell t}(\ ) \in [\underline{s}_{\ell t}, \bar{s}_{\ell t}] \subset [0,1]$$

Here the bounds may depend on α_t and do so continuously.

Then we have the following situation:

We can break the correspondence $\beta(\ )$ into two parts

$$\beta^h(\bar{\alpha}_t, {}_t\alpha_{t+1}) = \beta^h_t(\bar{\alpha}_t) \times \beta^h_{t+1}({}_t\alpha_{t+1}, \bar{\alpha}_t, y^h_t, \ell^h_t) \quad , \qquad \text{where}$$

$$\beta^h_t(\bar{\alpha}_t) = \left\{ (y^h_t, \ell^h_t) \in \mathbb{R}^{I+1}_+ \;\middle|\; \text{a.s.}(\omega_t) \ w_t \ell^h_t \tilde{s}_{\ell t}(\alpha_t, \omega_t) - \sum_{i=1}^{I} p_{it} y^h_{it} \tilde{s}_{it}(\alpha_t, \omega_t) \geq 0 \right.$$
$$\left. \bar{L}^h_t - \ell^h_t \tilde{s}_{\ell t}(\alpha_t, \omega_t) \geq 0 \right\}$$

$$\beta^h_{t+1}(\cdot) = \left\{ y^h_{t+1} \in \mathbb{R}^I_+ \;\middle|\; \text{a.s.}(\omega_t) \sum_{i=1}^{I} p_{i,t+1} y^h_{i,t+1} \leq w_t \ell^h_t \tilde{s}_{\ell t}(\alpha_t, \omega_t) \right.$$
$$\left. - \sum_{i=1}^{I} p_{it} y^h_{it} \tilde{s}_{it}(\alpha_t, \omega_t) \right\} .$$

Since the rationing mechanism is sign-preserving, realizations are always non-negative whenever the effective demands are non-negative, i.e., $\tilde{y}^h_t \geq \theta$, $\tilde{y}^h_{t+1} \geq \theta$, $\tilde{\ell}^h_t \geq 0$ are guaranteed. Therefore,

Lemma 5.1

$\beta^h_t(\bar{\alpha}_t)$ is a compact-valued, convex-valued and upper hemi-continuous correspondence. It is also lower hemi-continuous $\forall \alpha_t$ for which $\underline{s}_{\ell t} > 0$.

Proof: Upper hemi-continuity is shown in Green. Each image set is convex-valued since we have the rationing proportion, s, independent of the individual effective demands, unlike Green. For lower hemi-continuity, let

$\alpha_t(n) \to \bar{\alpha}_t(\infty)$ as $n \to \infty$ and choose $(y^o, \ell^o) \varepsilon \beta_t^h(\bar{\alpha}_t(\infty))$. We have two possibilities:

1) $w_t(\infty)\ell^o \underline{s}_{\ell t}(\alpha_t(\infty)) - \sum_{i=1}^{I} p_{it} y_i^o \bar{s}_{it}(\alpha_t(\infty)) > 0$.

Then for all $n \geq n_o$

$$w_t(n)\ell^o \underline{s}_{\ell t}(\alpha_t(n)) - \sum_{i=1}^{I} p_{it} y_i^o \bar{s}_{it}(\alpha_t(n)) > 0$$

and we can choose

$$[y(n),\ \ell(n)] = \begin{matrix} (y^o,\ \ell^o),\ n \geq n_o \\ (\theta,\ 0),\ n < n_o \end{matrix} \quad .$$

2) $w_t(\infty)\ell^o \underline{s}_{\ t}(\alpha_t(\infty)) - \sum_i p_{it}(\infty) y^o \bar{s}_{it}(\alpha_t(\infty)) = 0$. Since $\underline{s}_{\ell t}(\ ) > 0$ there exist $[\bar{y}(n),\ \bar{\ell}(n)]$ such that the corresponding physical realization requirements hold and $w_t(n)\bar{\ell}(n)\underline{s}_{\ell t}(\bar{\alpha}_t(n) - \sum_i p_{it}(n)\bar{y}(n)\bar{s}_{it}(\bar{\alpha}_t(n)) > 0$. There are now two possibilities for a given n.

a) $w_t \ell^o \underline{s}_{\ell t}(\alpha_t(n)) - \sum_i p_{it}(n) y_i^o \bar{s}_{it}(\alpha_t(n)) \geq 0$.

If so, choose

$$[y(n), \ell(n)] = [y^o,\ \ell^o]$$

b) $w_t \ell^o \underline{s}_{\ell t}(\alpha_t(n)) - \sum_i p_{it}(n) y_i^o \bar{s}_{it}(\alpha_t(n)) < 0$

If so, let $[y(n),\ell(n)]$ ε line segment between $[\bar{y}(n),\bar{\ell}(n)]$ and $[y^o,\ell^o]$ such that

$$w_t(n)\ell(n)\underline{s}_t(\alpha_t(n) - \sum_i p_{it}(n)y_i(n)\bar{s}_{it}(\alpha_t(n)) = 0 \ .$$

Such a point clearly exists and satisfies $[y(n),\ell(n)]\varepsilon\beta^h_t(\bar{\alpha}_t(n))$.

Finally, it is routine to check that in all cases

$$[y(n),\ell(n)] \to [y^o,\ell^o], \text{ as } n \to \infty \ .$$

QED

Lemma 5.2

$\beta^h_{t+1}(\cdot,\bar{\alpha}_t,y^h_t,\ell^h_t)$ is a compact-valued, convex-valued and continuous correspondence in the set $\{p_{t+1}\varepsilon\mathbb{R}^I_+ \mid p_{i,t+1} > 0\ \forall_i\}$, when $(\bar{\alpha}_t,y^h_t,\ell^h_t)$ is given and satisfies $w_t\ell^h_t\underline{s}_t(\alpha_t) - \sum^I_{i=1} p_{i,t}\ y^h_{it}\bar{s}_{it}(\alpha_t) > 0$. Lower hemi-continuity may fail if the last assumption is violated.

Proof: Entirely routine. QED

There are, thus, some problems with continuity, which will be dealt with later.

The next step is to relate expected future signals ${}_t\alpha_{t+1}$ to current ones $\bar{\alpha}_t$. This is done in the usual way (Grandmont) (1977 or 1978): ${}_t\alpha_{t+1}$ has a probability distribution which depends continuously on α_t in the topology of weak convergence of probability measures. Let $\rho(\alpha_t)$ be the distribution of ${}_t\alpha_{t+1}$, given α_t. The problems of non-continuity, especially in lemma 5.2, necessitate the following assumptions.

(5.1) $\forall \bar{\alpha}_t \neq \theta$; $\rho(\alpha_t)$ is concentrated in a given compact set such that

$${}_t\alpha_{t+1} \gg \theta \ .$$

(5.2) $\underline{s}_{i,t+1}({}_t\alpha_{t+1}) > 0$ for ${}_t\alpha_{t+1} \neq \theta$.

(5.3) $\underline{s}_{\ell t} > 0$ for $\bar{\alpha}_t \neq \theta$ with $L^d > 0$.

(5.4) $V_i = \partial U_h / \partial \tilde{y}^h_{i,t+1} \to +\infty$ as $\tilde{y}^h_{i,t+1} \to 0$.

Then, we have

Lemma 5.3

Any solution to the young consumer's optimization problem is such that $\underline{m}^*_{ht} = w_t \ell^{*h}_t \underline{s}_{\ell t}(\alpha_t) - \sum_{i=1}^{I} p_{it} \bar{s}_{it}(\alpha_t) y^{*h}_{it} > 0$ when $\alpha_t \neq \theta$.

Proof: Assume the contrary. Then by construction $\tilde{y}^{*h}_{t+1} \equiv 0$, independently of ω_{t+1} . On the other hand, (5.1), (5.2) and (5.3) guarantee that the consumer can obtain with certainty a positive amount of some good in period t+1, if $\underline{m}_{ht} > 0$. This with (4) yield a contradiction.

QED

This proposition means that in considering $\beta_h(\alpha_t, {}_t\alpha_{t+1})$ we can assume that $\underline{m}_{ht} > 0$ so that the continuity problem has been overcome in lemma 5.2. By (5.3) we have that $\beta^h(\bar{\alpha}_t, \cdot)$ is continuous for all $\bar{\alpha}_t \neq \theta$. To complete the discussion of the young consumer's maximization problem we can appeal to the results of Grandmont (1972, 1977) and see that the correspondence

$$\delta_h(\bar{\alpha}_t)$$

giving current effective demands for goods and effective supply of labor is a compact- and convex-valued uhc correspondence, when $\alpha_t \neq \theta$. For $\alpha_t = \theta$

we can define $\delta_h(\alpha_t)$ arbitrarily, since realizations will always yield zeros to the consumer.

To complete the picture, consider the old consumer, k's problem. Since realizations of period t-1 have taken place, he has an endowment of money $m^k \geq 0$ and his problem is clearly,

$$\max \quad EU(\tilde{y}_t^k) \text{ subject to } \sum_{i=1}^{I} p_{it} y_{it}^k \leq m^k$$

$$\tilde{y}_{it}^k = y_{it}^k \tilde{s}_{it}(\alpha_t).$$

If $m^k > 0$, $p_{it} > 0 \ \forall_i$, this will yield a compact-, convex-valued and uhc correspondence $G_k(\bar{\alpha}_t)$.

6. Equilibrium and Non-triviality

So far we have not been very explicit about the range of values α_t (excluding the price components) can take. They are, however, important for the fixed-point argument that follows, so we consider them in detail.

Current-period prices and wages p_t and w_t are taken to be fixed and positive. Then it is evident that

$$0 \leq y^h_{it} \leq w_t \bar{L}^h_t / p_{it}, \quad i = 1, \ldots, I$$

$$0 \leq \ell^h_t \leq \bar{L}^h_t$$

give the bounds for a young consumer h, provided $\alpha_t \neq \theta$. Let K_h be this compact and convex set. Furthermore, if $\alpha_t = \theta$, then realizations are all zero independently of ω_t, the random element. Thus, in that case actions could be taken to satisfy the above inequalities as well. Thus, $\forall \alpha_t (y^h_t, p^h_t) \varepsilon K_h$.

Similarly, for the old consumer k it is seen that

$$0 \leq y^k_{it} \leq m^k / p_{it}, \qquad i = 1, \ldots, I.$$

Thus, his actions also lie in a given compact and convex set $\tilde{K}_k$, when $\alpha_t \neq \theta$. Finally, for $\alpha_t = \theta$ the above truncation argument holds again so $y^k_t \varepsilon \tilde{K}_k \forall \alpha_t$. To get the dimensions to conform let $K_k = \tilde{K}_k \times \{0\}$.

Let $M = \sum_{h=1}^{H} K_h + \sum_{k=1}^{H} K_k + \begin{bmatrix} g_t \\ 0 \end{bmatrix}$, where g_t in the vector of government demands for goods. By constructing M is a compact, convex set.

Next, consider the producers. From section 2 we recall that the firm's best-action correspondence is contained in a given compact and convex set $N^{if} \varepsilon \mathbb{R}^{I+1}_+$, where we have added zeros for those goods that the form doesn't produce (i.e. $j \neq i$) . Let $N = \sum_{i=1}^{I} \sum_{f=1}^{F} N^{if} + \begin{bmatrix} \theta \\ \ell g \end{bmatrix}$, which is compact and convex.

Finally, let us form K = M x N. Then, it is clear that

$$\alpha_t = (y_t^d, L_t^s, Y_t^s, L_t^d) \in K \subset \mathbb{R}_+^{2I+2} .$$

Now form the aggregate demand correspondence

$$\gamma(\alpha_t) = \sum_{h=1}^{H} \delta_h(\alpha_t) + \sum_{k=1}^{H} G_k(\alpha_t) + \begin{bmatrix} g_t \\ 0 \end{bmatrix}$$

where we have smoothed the individual correspondences for the case $\alpha_t = \theta$ (see e.g. Nikaido (1968, pp. 71-73)). Also, let $\Psi(\alpha_t)$ be the aggregate corresponder of the producers (see section 4).

And, finally,

$$\lambda(\alpha_t) \equiv \gamma(\alpha_t) \times \Psi(\alpha_t)$$

is the required uhc, compact- and convex-valued correspondence $\lambda : K \to K$ to which we apply the fixed-point theorem of Kakutani. Hence, there exists $\alpha_t^* \varepsilon K$ such that $\alpha_t^* \varepsilon \gamma(\alpha_t^*) \times \psi(\alpha_t^*)$. An easy decomposition argument shows that this is the required non-trivial equilibrium provided α_t^* is such that $L_t^{s*} > 0$. To ensure that, consider the following argument.

What would the rationing function for demand in a market look like when the signal of aggregate supply is equal to zero. The plausible answer is that the rationing degenerate at $\{0\}$. If there is any "rational" restriction in the rationing functions, this must be the one. Similarly, if the signals for aggregate demand in a market is equal to zero, then the rationing function for the supply side of the market should degenerate at $\{0\}$. Therefore, if all the aggregate signals are equal to zero $\{\alpha_t\} = \{0, 0, \ldots, 0\}$, then the induced aggregate demands and supplies are also zero, and this should be equilibrium. This is called the "trivial equilibrium." However, in our framework, the trivial equilibrium conflict with two assumptions we made earlier. First, the government which has the power to finance by itself printing money demands a positive amount of consumption goods and labor; Secondly, the older generation is at least indifferent between putting down the deposit for effective demands and carrying the money to a coffin. Therefore, Y^d cannot be a null-vector to begin with since Y^{*d} and L^{*d} are necessarily strictly positive. We have from assumption (5.3) that L^{*s} 0 which in turn induces firms to produce a positive amount, by proposition 2.3. Therefore, our economy with the government and the old generation who has money to spend and the well-behaved production function generates a "non-trivial equilibrium." These assumptions are compatible with Gale (1977: Theorem 3).

Since $L_t^{*d} > 0$, we see that the necessary smoothing (Lemmas 3.1-3.3) does not create any problems i.e, α_t^* does not belong to the exceptional set. Hence, we have proved the main theorem.

<u>Theorem 6.1</u> There exists a non-trivial temporary equilibrium with stochastic rationing.

7. Stability and Dynamics: Open Question

We have shown the existence of a temporary equilibrium with stochastic rationing. There are two steps in the research agenda we are going to proceed to in the near future. First, we have to show the short-run stability of expectations on the aggregate signals. We ought to devise some mechanism to reach the temporary equilibrium. This process is supposed to take place within a period to ensure "rational expectations on rationing." Secondly, we would like to show the long-run stability of price-adjustment. Although the prices are assumed to be fixed within a period, they may change over time. Therefore, we would like to show that the price dynamics in a sense does not "explode," or, moreover, stay near the properly defined equilibrium.

The first problem corresponds to the tatonnement process in the Walrasian (temporary) equilibrium. The only difference is that our tatonnement process is done in the quantity terms instead of the prices space. The second problem has been studied in a similar framework. Grandmont and Hildenbrandt (1974) used an over-lapping generations model without rationing, i.e., p_t is adjusted to clear the spot markets, but the endowments are random every period. They showed that the price dynamics becomes the Markov process. Green and Majumbar (1975) considered the model that prices are determined before the stochastic endowments are revealed so that there may be the excess demands in a period, which, with the current price, determines the price in the following period. They showed the existence of a "stochastic equilibrium" as invariant probability distributions of prices. Green and Majumdar allow the excess demand to prevail. However, they did not provide any theory of how to carry out, if any, transactions when there is excess demand. Therefore, our model can be regarded as giving a rationale for their model. But, note also that the quantity signals as well as the prices should be considered as the variables to describe dynamics in a proper framework. This is the topic we are going to explore. An alternative way to handle the above-mentioned two concepts of stability is that we allow the temporary discrepancy between the aggregate signals announced at the beginning of the period and the aggregate effective demand and supply. All other features are the same within a period. The question to be asked then is whether the dynamics of the quantity signals, the prices and the expectation formations on the both signals (in the sense of Fuchs (1977)), are "stable." This seems to be very interesting but complicated.

Finally, there is one more source of dynamics in the model, namely that of savings, when one imposes a government budget constraint to the system. Böhm (1978) and Honkapohja (1978) considered this issue in the context of a macro-economic model of the Benassy type. It would be interesting to analyze similar problems with the present framework, in particular, since expectation formation is now explicit.

8. Conclusions

The purpose of this paper was to develop a workable framework for the analysis of temporary equilibrium with stochastic rationing, which, so far, is the most convincing framework about the formation of demands and supplies in the context of non-clearing markets. Though perhaps the most interesting problems of dynamics still await an answer, we were able to analyze the determination of a temporary solution in some detail. In particular, an analysis shows that the problem of non-triviality of equilibria can be overcome in a relatively simple way: government expenditures provide the necessary signal for producers to produce and hire inputs.

The example of a non-Walrasian equilibrium at the Walrasian equilibrium prices raises some fundamental issues. The Keynesian multiplier story is more than a "deviation-amplifying" process at "wrong" prices. Indeed, even at the Walrasian equilibrium prices, the pure pessimism about trading opportunities can make agents reduce their effective demands and hence create a recession. This should be a subject of futher investigation.

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Appendix

We will give an example of a non-Walrasian equilibrium with stochastic rationing at the Walrasian equilibrium prices, which is reported in Ito (1979).

Example

Suppose the representative consumer with the following utility function,

(1) $$u = 2\sqrt{\bar{L} - \ell} + 2\sqrt{m + y}$$

where ℓ and L denote the actual labor supplied and the disposable time endowment; y denotes the actual consumption goods bought; m denotes the end-of-period money balance. The consumer maximizes (1) with respect to ℓ, y and m subject to the budget constraint and the non-negativity constraint:

(2) $$m = \bar{m} + w\ell - py$$
$$m \geq 0, \quad \ell \geq 0, \quad y \geq 0$$

where $w \geq 0$ and $p \geq 0$ denote the nominal wage and the price of the consumption goods; $\bar{m}$ denotes the initial money balance. The Walrasian demand for the consumption goods, y^d (which is multi-valued at p = 1 in this example) and the Walrasian supply of labor, ℓ^s, are obtained as functions (or correspondences to be precise) of the wage and the price.(2)

The representative firm maximizes its profit,

(3) $$v = py - w\ell$$

subject to the feasibility constraint,

(4) $$y \leq f(\ell)$$

where f is a neoclassical production function dependent on the actual employment of the labor force, ℓ. This gives the Walrasian demand for labor, ℓ^d, and the Walrasian supply of the consumption goods, y^s, as functions of the price and the wage.

Suppose the following numerical values for the parameters:

(5) $$\bar{m} = 200,\ \bar{\ell} = 400,\ \text{and}\ f(\ell) = 20\sqrt{\ell}$$

The Walrasian equilibirum in this economy is defined as the price vector (p^*, w^*) such that

$$y^d = y^s \qquad \text{and} \qquad \ell^d = \ell^s.$$

Observe that the Walrasian equilibrium is obtained at $(p^*, w^*) = (1, 1)$, which gives

$$y = y^d = y^s = 200$$
$$\ell = \ell^d = \ell^s = 100$$

Now keep the economic environment and the Walrasian prices intact, i.e., (1), (4), (5) and $(p, w) = (1, 1)$. However, the consumer and the firm suddenly become pessimistic.(3) They believe (i) that they are in the Keynesian unemployment regime, i.e., the aggregate excess supply in the both markets; (ii) that the short-side of a market (i.e., demanders) can fulfill their offers; (iii) that $100\pi\%$ of workers are fully employed at their offers, and $100(1-\pi)\%$ of workers are employed at a half of their offers;

(iv) that $(100\theta)\%$ of the firms manage to sell everything they produce to offer, while the rest can sell only $(100\underline{s})\%$ of their products, in such a way that the mean of successful-sales proportion is

$$\bar{s} = \theta + (1-\theta)\underline{s}\ , \qquad 0 \leq \bar{s} \leq 1.$$

Both markets meet simultaneously at the beginning of a period and decide randomly who are lucky to fulfill their supply and who are rationed.(4)

Taking into these constraints, the consumer maximizes the expected utility to find the <u>effective</u> supply of labor, ℓ^{es}, and the <u>effective</u> demand for the consumption goods, y^{ed}:

$$\text{Max} \quad Eu = \pi(2\sqrt{\bar{\ell} - \ell} + 2\sqrt{m_1 + y}) + (1 - \pi)(2\sqrt{\bar{\ell} - \ell/2} + 2\sqrt{m_2 + y})$$

subject to $m_1 = \bar{m} + w\ell - py \geq 0$

$m_2 = \bar{m} + w\ell/2 - py \geq 0$

$\ell \geq 0 , \qquad y \geq 0$

where $\bar{m}$ and $\bar{\ell}$ are given in (5); p = 1, w = 1; and

$$(6) \quad \pi = \frac{(1/2\sqrt{255} - 1/2\sqrt{345})}{(1/2\sqrt{255} - 1/2\sqrt{345} + 1/\sqrt{290} - 1/\sqrt{300})}$$

$$\approx .70$$

The firm is assumed to be risk-neutral, wherefore they maximize the expected profit with respect to the effective supply of the consumption goods, y^{es}, and the effective demand for labor, ℓ^{ed}. The average proportion of successful sales becomes the "effective price" of products. Therefore the firm would reduce their production plan. Moreover, the production plan should be feasible not only on average but also in its offer, the (ℓ^{ed}, y^{es}) vector.

$$\text{Max} \quad Ev = \bar{s}py - w\ell$$

$$\text{subject to} \quad y \leq f(\ell),$$

where f is given in (5); p = 1, w = 1; and

$$(7) \qquad \bar{s} = (11(1+\pi)/20)^{1/2}$$

$$\approx .97$$

Since we assume the rational expectations on rationing, i.e., the economic agents know an actual rationing scheme, we have the mean balance condition,

$$(\text{E}) \qquad \begin{aligned} \ell^{ed} &= \pi\ell^{es} + (1 - \pi)\ell^{es}/2 \\ y^{ed} &= \bar{s}y^{es} \end{aligned}$$

A non-Walrasian equilibrium with stochastic rationing is defined as a self-reproducing aggregate quantity signals (Y^d, Y^s, L^d, L^s).[5] Take $Y^d = 200(\bar{s})^2 N$, $Y^s = 200(\bar{s})N$, $L^d = 100(\bar{s})^2 N$, $L^s = 110N$, where N is the number of firms or consumers. It is easy to see that if (E) is satisfied then (Y^d, Y^s, L^d, L^s) is a non-Walrasian equilibrium.

In order to confirm that (E) is satisfied by the values given in (6) and (7), we take the first order condition of the expected utility maximization problem, i.e., $Eu_\ell = 0$, where

$$Eu_\ell = \pi(1/\sqrt{\bar{m} + \ell} - 1/\sqrt{\bar{\ell} - \ell}) + (1 - \pi)(1/2\sqrt{\bar{m} + \ell/2} - 1/2\sqrt{\bar{\ell} - \ell/2})$$

Observe that this is satisfied if $\ell^{es} = 110$. The firm's expected maximization yields the followings:

$$\ell^{ed} = 100(\bar{s})^2$$
$$y^{es} = 200(\bar{s}).$$

Given the values of (6) for π and (7) for $\bar{s}$, (E) is satisfied

$$\ell^{ed} \approx 93 \ , \quad e^{es} = 110$$
$$y^{ed} \approx 186 \ , \quad y^{es} \approx 193 \ .$$