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A NOTE ON STOCHASTIC RATIONING MECHANISMS

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ABSTRACT

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There are a couple of well-known unsatisfactory properties in the notion of effective demand defined by Benassy and one by Drèze. This is why recent authors in disequilibrium analysis study the stochastic rationing mechanism. Douglas Gale proved the existence of the equilibrium with stochastic rationing mechanism. However, Gale's rationing mechanism requires an economic agent to know all the individual effective demands from the other agents. This creates the informational problem. Green examined a rationing scheme which depends only on the individual effective demand and the aggregate signals. However, he did not consider conditions on rationing mechanisms to show the existence of temporary equilibrium.

The purpose of this paper is to show a couple of sufficient conditions for the existence of temporary equilibrium preserving all properties Green considered on rationing mechanisms. We also discuss the possibility of balancing demand and supply in realization instead of in the mean.

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1. Introduction

The non-Walrasian (or quantity-constrained) equilibrium has been studied since Drèze (1975) and Benassy (1975). Since both of them have some difficulties in their concepts [explained, for example, by Grandmont (1977), Green (1978) and Svensson (1977)], stochastic rationing mechanisms have recently been the focus of attention.

Svensson (1977) shows, in a two-market example with a special rationing scheme, how the Dreze and Benassy effective demands are different from the effective demand with stochastic rationing. He did not proceed to examine whether there exists a consistent trade in a market under his stochastic rationing scheme. Douglas Gale (1978) gives the existence proof of market equilibrium (consistent trades) in an economy where continuum traders face a stochastic rationing scheme. He does not specify the stochastic rationing scheme an individual would face but just assumes that the expectation of rationing depends on a vector of all individual effective demands in an economy. He also gives a mathematical condition for non-trivial equilibrium, but it is not apparent what kind of economy would satisfy the condition. As for specifying the rationing mechanism, Green (1978) shows that with some reasonable conditions on rationing mechanisms, the individual ration can be represented as a product of the individual effective demand and the stochastic rationing proportion. The latter is a stochastic function of the individual demand and the aggregate signals of demand and supply in the market. Green does not have a market equilibrium, since an image set of the demand correspondence "may" be non-convex because of the effect of the individual effective demand through the stochastic rationing proportion. Since Green requires that a stochastic rationing mechanism is balancing demand and supply "in expectation (or in the mean)," his mechanism might violate "in realization." Honkapohja and Ito (1978) construct an economy to show the existence of non-trivial equilibrium with a set of assumptions similar to Green's and an additional assumption that the rationing proportion is dependent only on the aggregate signals. The last assumption is totally consistent if all the agents are identical (as they assume for firms in an industry). However, if there is no restriction on a vector of the individual effective demand, then it is impossible to construct a stochastic rationing mechanism which guarantees balancing in realization.

The purpose of this note is to further discuss the Green-type rationing mechanism (i.e., the individual effective demand times the stochastic proportion dependent only on the individual effective demand and the aggregate signals of demand and supply). Specifically, we will show the following: (i) to give an example of rationing mechanism which is stochastic to individuals and in accordance with Green's assumption but also satisfies balancing in realization with certainty in a case that all the individuals are identical; (ii) to discuss the possibility of requiring a condition of balancing in realization when individuals are not identical; and (iii) to give an example where the rationing proportion depends on the individual effective demand but still gives the convexity of an image set of demand correspondence. The example also satisfies balancing in realization.

The next two sections are adopted (with slight modifications) from Honkapohja and Ito (1978).

2. Framework and Notations

We will consider in the following an economy where there are H consumers; I different types of consumption goods; F firms in each of the consumption-good industries (no joint-production); and one type of labor. The production technology of each industry is characterized by a well-behaved neo-classical production function $f^i(\cdot)$. The prices of the consumption goods, $(p_1, p_2, \ldots, p_I)$, and the nominal wage, w, are fixed within a period.

The economic agents are asked to submit a vector of individual effective demands and supplies which are guaranteed by the purchasing power or the production technology and input quantity constraints. Let us denote by y_i^h the effective demand for the i-th consumption goods from household h; by y_i^f the effective supply of the i-th consumption goods from firm f; by ℓ^h the effective labor supply from household h and ℓ_i^f the effective labor demand from the f-th firm in the i-th industry.[1]

The government (or "lottery-auctioneer") assigns a vector of "realized trade" which an individual has to obey. The process of assignment is called "stochastic" if the assignment to an "individual" is stochastic. This does not preclude the possibility that the "realized trade" is non-stochastic in "aggregate."[2] Let us denote by $\tilde{y}_i^f$, the stochastic assignment of realized supply from firm f in the i-th industry. The rationing (or assignment) mechanism is known to all the economic agents.

<u>Assumption 2.1: [Rational Expectation of Rationing]</u>[3].

<u>An economic agent knows a mechanism of the stochastic realization of trade and calculates the probability distribution of stochastic realization for him.</u>

Now we are ready to describe how the economic agents decide their effective demands and supplies. Consumers are assumed to maximize their expected utility function with the "budget constraint in realization." That is, they have to end up with somewhere in the defined consumption set with probability one. Specifically they have to plan that they do not go bankrupt even in the worst case.

A firm is assumed to be risk-neutral and to maximize its expected profit. Its production plan has to be feasible with probability one. We will discuss the possibility of bankruptcy of firms later.

All the markets meet simultaneously.[4]

We did not so far specify a stochastic rationing mechanism. In general, the probability measure for an individual in one market, μ, depends on all other agents' effective demands in all other markets. Therefore, we have to consider the mapping from the space of all individual effective demands to the probability measures. Equilibrium with rationing is defined as the set of effective demands and supplies which "reproduce themselves" through the mapping of probability measures and optimal decisions of economic agents. This is a framework and a notion of equilibrium that Gale (1978) works out and proves the existence.[5]

Although Gale's economy preserves most generality, it is too vague to examine the rationing mechanism, and its effect on economic agents. Moreover, it contains the informational difficulty: how could an economic agent obtain all the information of other agents' individual effective demands?

At this point in our argument, one may recall the elegant general equilibrium theory a la Arrow-Debreu because it only requires an economic agent to know the market signals, i.e., the prices.

Green (1978) constructs an economy where aggregate signals in terms of quantity in addition to prices are available and it is enough for an agent to watch the aggregate signals to have the rational expectation. In the following we trace the avenue prepared by Green.

The government publishes the aggregate signals of demand and supply for each market, $\{\alpha\} \equiv \{(Y_i^d, Y_i^s), (L^d, L^s)\}$. The statistics are available to all economic agents free of charge. We introduce an assumption of signal-takers.

Assumption 2.2: [Signal-taker]

An economic agent takes the aggregate signals $\{\alpha\}$ as given and does not think his individual behavior changes the aggregate signals.

This assumption may be approximately justified in a large economy.

Assumption 2.3: [Independence Across Markets]

The rationing mechanism in a market is independent from the aggregate signals and effective demands (or supplies) in the other markets.

Assumption 2.4: [Rationing Mechanism]

The rationing mechanism is designed in such a way that the rationing probability to an economic agent depends only on his own effective demand and the aggregate signals.

Assumptions 2.1 and 2.4 imply that an economic agent is required to have only the signals and his own effective demand to know the probability distribution.

Since the probability distributions of rationing for an economic agent are dependent on the aggregate signals, we will have the correspondence from the properly defined subset of the signals space, $K \subset \mathbb{R}_+^{2I+2}$, to the space of individual effective demands. Let us denote this correspondence by $\Psi(\alpha) = \{\psi_i^h(\alpha), \psi_i^f(\alpha); \psi_\ell^f(\alpha), \psi_\ell^h(\alpha)\}$, where $\Psi: K \to 2^K$. Now we can define the temporary equilibrium if there exists a "self-reproducing" signal. Formally, it is defined as follows:

Definition 2.1: [Temporary Equilibrium with Stochastic Rationing]

An economy is said to be in a temporary equilibrium with stochastic rationing, if $\{\alpha^*\} = \{(Y_i^{d*}, Y_i^{s*}), L^{d*}, L^{s*}\}$,

$$Y_i^{d*} = \sum^H y_i^h \quad \text{and} \quad Y^{s*} = \sum^F y_i^f \quad , \qquad \forall_i$$

$$L^{d*} = \sum^I \sum^F \ell_i^f \quad \text{and} \quad L^{s*} = \sum^H \ell^h ,$$

where $y_i^f \in \psi_i^f(\alpha^*)$, $y_i^h \in \psi_i^h(\alpha^*)$, $\ell_i^f \in \psi_i^f(\alpha^*)$, $\ell^h \in \psi^h(\alpha^*)$.

Definition 2.2: [Temporary Equilibrium Signals]

A set of aggregate signals $\{\alpha^*\}$ is called a set of temporary equilibrium signals if the economy is in a temporary equilibrium with stochastic rationing.

It is an immediate observation that we need (i) the upper-lower continuity and convex-valuedness for each image set of ψ's and (ii) K is a compact and convex set to assert the existence of a temporary equilibrium with stochastic rationing. The purpose of this paper is to show what kinds of stochastic rationing mechanisms satisfying a certain set of axioms give the above conditions for existence. A set of axioms for the stochastic rationing mechanism is explained in the next section.

3. Stochastic Rationing

Let us focus on a single market, say, the i-th market of the consumption goods. Other markets, including the labor market will be treated symmetrically because of Assumptions 2.3 and 2.4.

We will discuss the axioms that a stochastic rationing mechanism should satisfy, following Green (1978):

From the assumptions in the preceding section, we have the rationing to economic agents h and f in the i-th market, μ_i^h and μ_i^f, respectively depends on his effective demand and the market signals in that market:

$$\tilde{y}_i^h = \mu_i^h(y_i^h, Y_i^d, Y_i^s) \tag{3.1}$$

$$\tilde{y}_i^f = \mu_i^f(y_i^f, Y_i^d, Y_i^s) \ ,$$

where $y_i^h \geq 0$, $y_i^f \geq 0$, $Y_i^d \geq 0$, $Y_i^s \geq 0$; μ's are stochastic functions; $\tilde{y}$'s are realized (or assigned, rationed) trade; y's are effective demands (and supplies).

From the definition of equilibrium, we have the following equilibrium condition:

$$\sum^H y_i^h = Y_i^d \tag{3.2}$$

$$\sum^F y_i^f = Y_i^s \quad .$$

Let us omit a subscript i in the following:

Axiom 1: [Anonimity]

The distribution of μ^h is the same if the effective demands are the same:

$$\mu^h = \mu^{h'} \quad \underline{\text{if}} \quad y^h = y^{h'}$$

(3.3)

$$\mu^f = \mu^{f'} \quad \underline{\text{if}} \quad y^f = y^{f'} \quad .$$

Axiom 2: [Voluntary Exchange]

Rationing does not change the side of the market, i.e., a demander remains buying the goods, and a supplier selling the goods. An agent is not forced to buy (or sell) more than the effective demand or supply he announces:

$$0 \leq \tilde{y}^h \leq y^h \quad \text{with probability one}$$

(3.4)

$$0 \leq \tilde{y}^f \leq y^f \quad \text{with probability one.}$$

Axiom 3: [Balancing in the Mean]

The rationing mechanism balances in the mean, i.e.,

(3.5) $$\sum^F E(\tilde{y}^f) = \sum^H E(\tilde{y}^h) \quad .$$

Axiom 4: [Weak Continuity]

The distribution of μ is weakly continuous in its arguments.

Axiom 3 is a weaker requirement than an alternative axiom of balancing in realization.

Axiom 3A: [Balancing in Realization or Feasibility]

The rationing mechanism balances in realization, i.e.,

(3.6) $$\sum^F \tilde{y}^f = \sum^H \tilde{y}^h \quad \text{with probability one.}$$

As Green noted Axiom 3A implies that the rationing mechanism for individuals is functionally related to each other. Green commented that "this restriction might come into conflict with the first condition." However, it is important to ask what kind of rationing mechanism would satisfy or not satisfy Axiom 3A in addition to Axiom 3, which is a main focus in the next section of this paper. As also noted by Green, the difference in Axioms 3 and 3A becomes approximately negligible in a large economy.

Now we are ready to state Green's theorem (1978):

Theorem 3.1: [Green (1978)]

With Assumptions 2.1 - 2.4, and Axioms 1, 2, 3, and 4, equation (3.1) can be written in the following form:

$$\text{(3.7)}\quad \begin{aligned} \mu_i^h(y_i^h, Y_i^d, Y_i^s) &= y_i^h \tilde{s}_i^h(y_i^h, Y_i^d, Y_i^s) \\ \mu_i^f(y_i^f, Y_i^d, Y_i^s) &= y_i^f \tilde{s}_i^f(y_i^f, Y_i^d, Y_i^s) \quad , \end{aligned}$$

where $\tilde{s}_i^h$ (or $\tilde{s}_i^f$) is a random function whose mean is independent of y_i^h (y_i^f, respectively).

4. <u>On Green's Theorem</u>

Green's theorem gives the convenient characterization of the individual rationing probability distribution. However, he does not show a condition that the demand correspondence has the characteristic that each image set is convex, so that the fixed point theorem works. It is of great interest to investigate possible restrictions on $\tilde{s}$, to give this property.

In the following we are going to discuss (i) the sufficient conditions on $\tilde{s}$ to preserve the convexity of each image set; (ii) the possibility of imposing the axiom of balancing in realization instead of balancing in the mean; and in the next section we will give examples which give the convexity of each image set and the axiom of balancing in the mean with the finite number of economic agents.

Although the mean of $\tilde{s}_i^h$ is independent of y_i^h, the individual effective demand, the probability distribution of $\tilde{s}^h$ may change preserving the mean constant when y_i^h changes. Therefore, in general "the objective function may be non-concave in the y_i^h," as noted by Green. Let us explore further restrictions on the rationing mechanism to preserve that the objective function is concave in the y_i^h.

The first thought would be that $\mu_i^h(\cdot)$ has properties not only that the mean is linear in y_i^h but also that the additive noise is independent of y_i^h. Because this preserves the desired feature of concavity, let us consider this case in the following. μ_i^h can be decomposed into two parts with properly defined stochastic functions δ^1 and δ^2:

$$\mu_i^h = \delta_i^1(Y_i^d,\ Y_i^s)\cdot y_i^h + \delta_i^2(Y_i^d,\ Y_i^s)$$

(4.1)

$$E\delta_i^2(\cdot) = 0$$

(similarly for μ_i^f omitted hereafter).

However (3.7) and (4.1) imply that

(4.2) $$\tilde{s}_i^h = \delta_i^1(Y_i^d,\ Y_i^s) + \delta_i^2(Y_i^d,\ Y_i^s)/y_i^h \quad .$$

We remind you of some of the axioms in the preceding section in terms of $\tilde{s}_i^h$.

Voluntary exchange requires

(4.3) $0 \leq \tilde{s}_i^h \leq 1$ with probability one for all $y_i^h \geq 0$.

Balancing in the mean and anonimity requires

(4.4) $$H\cdot E(y_i^h \cdot \tilde{s}_i^h) = Y_i^s \,.$$

We are going to examine (4.2) in two cases:

<u>Case 1</u>: $\delta_i^2 = \{0\}$.

Suppose $\tilde{s}_i^h$ is independent of y_i^h, i.e., a stochastic function δ_i^2 in (4.2) is degenerate at $\{0\}$. Then

(4.3) $$\tilde{s}_i^h(\cdot) = \delta_i^1(Y_i^d,\ Y_i^s) \quad .$$

There are three subcases to consider:

(1-a). If individuals are identical, then the rationing scheme (4.3) is consistent with the rest of assumptions and axioms. We are also able to give an example of a rationing mechanism which balances in realization. This will be given in the next sections.

(1-b). $\tilde{s}_i^h(\cdot)$ degenerates at a point, i.e., δ_i^1 is not a stochastic function. In this case, it is easy to have an example satisfying all the assumptions and axioms. See Green's Corollary.

(1-c). If individuals are not identical and δ_i^1 is not a deterministic, then it is impossible to construct a rationing mechanism which satisfies the axiom of balancing in realization. Suppose Y_i^d and Y_i^s are given and $(\hat{s}_i^1, \hat{s}_i^2, \ldots, \hat{s}_i^H)$ is a vector of realization of stochastic proportion. It is still stochastic for individuals and the realization is deterministic in aggregate because of random ordering of households and sampling without replacement from an urn where the sum of numbers on tickets is constant. It is impossible to find a vector, $\{\hat{s}_i\}$, which is <u>not constant</u> (because it is created from the non-degenerate process) and satisfy

$$\sum^H \hat{s}_i^h \cdot y_i^h = \text{constant} \quad , \quad \forall \quad y_i^h \geq 0, \quad \sum^H y_i^h = Y_i^d$$

and

$$\hat{s}_i^h \quad \text{is independent of} \quad y_i^h \ .$$

The examples in the next section will make it easier to understand this point.

Case 2: $\delta_i^2 \neq \{0\}$ but δ_i^1 is deterministic.

There are two subcases.

(2-a). If there is the lower bound on y_i^h such that $y_i^{min} \geq \varepsilon > 0$, maybe from the definition of the consumption set, such that the minimum subsistence level of nutrition, then it is possible to construct an example to satisfy all the assumptions and axioms including balancing in realization. This is illustrated in Example 2 in the next section.

(2-b). If y_i^h is not bounded away from zero, it is clear from (4.2) that as $y_i^h \to 0$, $\tilde{s}_i^h \to \infty$ and violates the axiom of voluntary exchange, (4.3).

We do not have any examples for non-linear cases. Although it may be possible to derive general conditions on rationing mechanisms to preserve the desired property, it will be left for further research.

5. Examples

Example 1

We will give examples which we promised in the preceding section. We will consider firm f in the i-th industry. We will explain the individual rationing expectation $\tilde{s}_i^f$ first and give the mechanism which guarantees balancing in realization with probability one and creates the explained $\{\tilde{s}_i^f\}$ for individuals through rational expectation.

We will give an example of a rationing scheme in a market where all agents are identical and the $\tilde{s}_i^f$ are independent of y_i^f. The following rationing scheme preserves rational expectations on the quantity rationing scheme, the randomness for individuals; and balances of the aggregate demand and supply not only in the mean but also in realization with probability one.

Take the representative firm in the i-th consumption good market: his stochastic realization of supply, $\tilde{y}_i^f$, is now the effective demand multiplied by a stochastic "rationing number" $\tilde{s}_i^f$ which depends only on the market signals. $\tilde{s}_i^f$ is distributed uniformly between $\underline{s}_i^f$ and $\bar{s}_i^f$ in the following manner: (we omit the obvious subscript i hereafter):

$$\bar{s}^f = 1 \qquad \text{if} \quad Y^d/Y^s \geq 1$$

$$= 1/2 + (Y^d/2Y^s) \qquad \text{if} \quad (1/2) \leq Y^d/Y^s \leq 1$$

$$= 3Y^d/2Y^s \qquad \text{if} \quad 0 \leq Y^s/Y^s \leq 1/2$$

$$s_m^f = \min(Y^d/Y^s,\ 1) \qquad Y^d/Y^s \geq 0$$

where s_m is the mean of $\{s\}$;

$$\underline{s}^f = 1 \qquad \text{if} \qquad Y^d/Y^s \geq 1$$

$$= -(1/2) + 3Y^d/2Y^s \qquad \text{if} \qquad (1/2) \leq Y^d/Y^s \leq 1$$

$$= Y^d/2Y^s \qquad \text{if} \qquad 0 \leq Y^d/Y^s \leq 1/2 \ .$$

$\{\tilde{s}_i^h\}$ are defined in the symmetrical manner, i.e., substituting the superscript f by h and Y^d/Y^s by Y^s/Y^d. The idea is illustrated in Figure 1.

Observe the axiom of voluntary exchange, i.e., $0 \leq s \leq 1$ with probability one is preserved.

Here the short side agents in the market is assumed to realize their effective demands (or supplies) with probability one. The long side divides the amount in the short side in such a manner that agents draw the lottery of "rationing proportion" without replacing lottery tickets, where the sum of the lottery is equal to the disequilibrium index Y^d/Y^s (or Y^s/Y^d). The assumption of identical agents guarantees that the sum of realization is equal to the amount in the short side. The following is the formal description of how to set up the lottery.

We also need one technical assumption. There is an even number of firms. Now we explain the procedure of rationing. Suppose the market signal Y^d/Y^s is less than one. (I) First stage is to make the lottery tickets stochastically: We are going to create F (an even number of firms) tickets. For the first ticket, draw a random number between $\underline{s}$ and $\bar{s}$. The second ticket is $s_m + (s_m - n_1)$ where n_1 is the number of the first ticket. Repeat this process $F/2$ times. (II) The second stage is the drawing to decide the ordering of firms to draw the prepared tickets. (III) The third stage is that the first firm decided by the second stage draw randomly one ticket from the urn prepared by the first stage, and the drawn ticket is

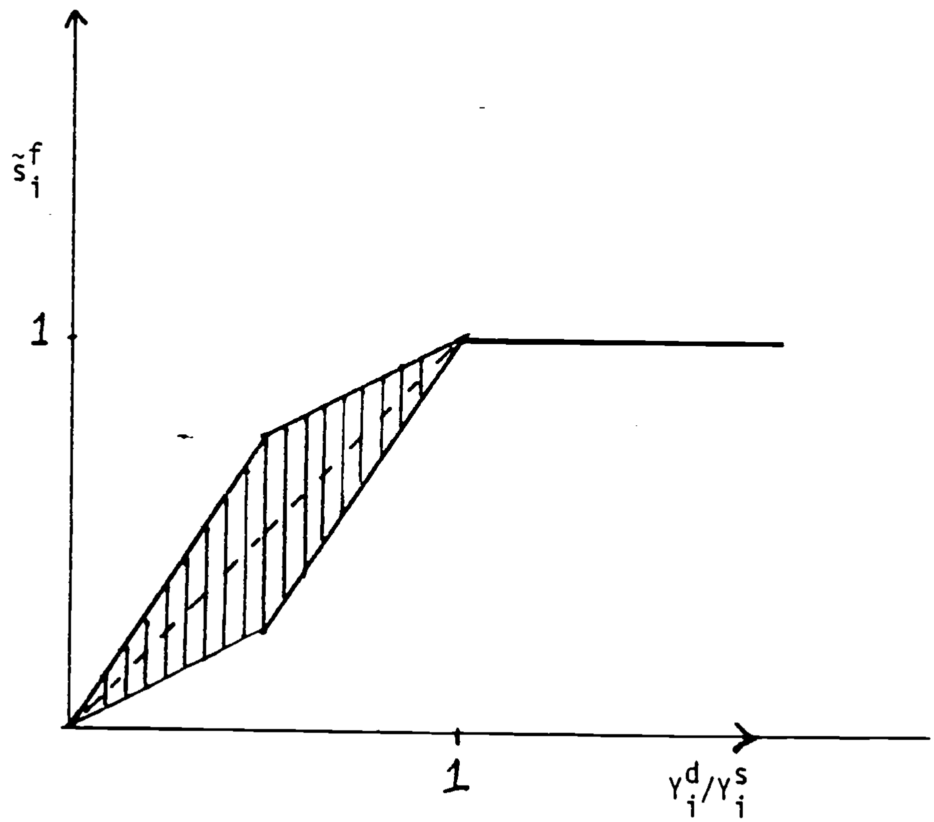

Figure 1

removed from the urn. Repeat the process F times to exhaust firms and tickets.

By the first stage, the sum of the rationing tickets add up to $s_m F \equiv F\, Y^d/Y^s$ with certainty. Each firm is assumed to submit the same effective demand y_f^i. That means that the actual rationing is $Fy_i^f\, Y^d/Y^s$. From the definition of temporary equilibrium with stochastic rationing, $Fy_i^f = Y^s$. Therefore, the aggregate realization of rationing is always Y^d. That is, the rationing scheme balances the aggregate demand and supply in realization with probability one. However, from the individual point of view, the rationing is stochastic and the distribution of rationing number is exactly explained above. Although the third stage of drawing means that a distribution of rationing tickets is dependent on other drawings, the second stage erases this problem. Hence the identical distribution of rationing with anonymity as a whole system is preserved.

Example 2

Consider consumer k in the i-th market. In this example $\tilde{s}_i^h$ are not independent of y_i^h. It is clear from the argument in the preceding section that

$$\tilde{s}_i^h = \delta_i^1 + \delta_i^2/y_i^h \quad .$$

Now, the trick is similar to Example 1. Consider the following bounds within which y_i^{min} denotes that the minimum order that the i-th agent is required to order, $y_i^h \geq y_i^{min}$, $y_i^{min} \geq \varepsilon > 0$. This may come from the restriction on the consumption set. Suppose the short side rule holds again. Then

$$\tilde{s}^h = 1 \qquad \text{when} \quad 1 \leq Y^s/Y^d$$

$$\tilde{s}^h = Y^s/Y^d + \tilde{\delta}(Y^d, Y^s)(y^{min}/y^h) \qquad \text{when} \quad 0 \leq Y^s/Y^d \leq 1$$

where

$$0 < \tilde{\delta}(Y^d, Y^s) < 1 - Y^s/Y^d \qquad \text{with probability one}$$

$$\sum^H \hat{\delta}(Y^d, Y^s) = 0 \quad , \quad \text{where } \hat{\delta} \text{ are real realizations.}$$

We create the tickets in the same manner as Example 1. Observe $0 \leq \tilde{s}^h \leq 1$ with probability one

$$\sum^H y^h \tilde{s}^h = \sum^H (Y^s/Y^d)y^h + \hat{\delta}(Y^d, Y^s)$$

$$= (Y^s/Y^d) \sum^H y^h$$

$$= Y^s \quad .$$

The random element δ plays the role of tickets whose sum is equal to zero in the urn with certainty.

The idea in this example is that when an agent draws this ticket, the absolute amount the agent is assigned is that the disequilibrium rate in aggregate times his effective demand plus the random noise whose mean is equal to zero. Obviously, a large demander is facing a smaller deviation relative to his size of his demand but this is required to preserve the axiom of balancing in realization.

6. Individual Behavior and the Existence of Non-trivial Equilibrium

In the previous sections, we have shown that there exists a rationing mechanism in such a way that the utility function is concave in the individual effective demand and it balances in the realization. Therefore, the convexity of the image set in demand correspondence should be preserved, unlike Green commented in the last section of his paper. Therefore, we have the following existence theorem.

Theorem 6.1:

(i) Suppose there is the positive minimum bound in the consumption set, $y_i^{min} \geq \varepsilon > 0$. Other characteristics of the consumer are the same as those in Green (1978). Suppose a stochastic rationing mechanism which satisfies all the assumptions in the previous section and Axioms 1, 2, 3A and 4; and which preserve the concavity of utility function in y_i^h; for example, the rationing mechanism in Example 2 in Section 5.

(ii) Suppose all the firms are identical in an industry. Suppose also a stochastic rationing mechanism which satisfies all the assumptions in the previous sections, and Axioms 1, 2, 3A and 4; for example, the rationing mechanism in Example 1 in Section 5. This is the same with the firm division problem in Hankapohja and Ito (1978).

Then there exists the temporary equilibrium with stochastic equilibrium.

Proof: The upper-hemi continuity in consumers in demand correspondence is explained in Green (1978). The convex-valuedness is explained above in this section of the paper. The upper-hemi continuity and convex-valuedness for firm's demand correspondence is explained in Honkapohja and Ito (1978).

Gale (1978) gives the condition that there exists a non-trivial equilibrium, which includes positive trades. Honkapohja and Ito (1978) showed a sufficient condition for the existence of non-trivial equilibrium which is essentially the same with Gale's argument but in terms of utility functions and production functions.

Theorem 6.2

(i) Suppose the consumer has enough initial money holdings so as to be able to buy the minimum vector of consumption sets even in a case where he is totally unemployed.

(ii) The production function in each industry is "well-behaved," i.e., $f'(0) = \infty$, $f'(\infty) = 0$, $f(0) = 0$, and $f'(\ell) > 0$, $f''(\ell) \leq 0$ for $\ell \geq 0$.

(iii) If $Y_i^d > 0$, then $s_i^f > 0$. Then, there exists a non-trivial temporary equilibrium with stochastic rationing.

Proof: Honkapohja and Ito (1978).

7. Applications to Disequilibrium Macroeconomics

It is of interest to note how our concept of effective demands will be applied to a macroeconomic model in the near future. In this section we will review the existing disequilibrium macroeconomic models and point out some of their difficulties which would not arise in our framework. Although this section is not a complete argument, this will give some directions we are aiming at after proving the existence of temporary equilibrium.

Clower (1965), Barro and Grossman (1976), and Malinvaud (1977) give a rationale and importance of the concept of effective demands as a tool of (Keynesian or Keynes') macroeconomics. They show the classification of fixed price vectors into several regimes where the impacts of changes in policy variables are different. Since they have used the Clower-type effective demands, as opposed to the Drèze-type ones, it is not too difficult to trace the dynamic path of price adjustments responding to the excess effective demands. However, we have to hastily add that the Clower-type effective demands are not jointly feasible. Therefore it is not obvious why prices will respond to the Clower-type excess effective demands. On the other hand, it is well known that the Drèze-type equilibrium provides us with no excess effective demand or supply.

Our approach using the concept of temporary equilibria with stochastic rationing suggests a new framework for disequilibrium macroeconomics. It gives us the excess demands in signals and in realization (they coincide if we take the assumptions of rational expectations and of a rationing mechanism with balancing in realization). It would be possible to trace the dynamics of prices in our framework, if we show the uniqueness of temporary equilibrium signals given the price vector. If there are multiple equilibria, then we have a comparable situation to Hahn's idea (1978) of Non-Walrasian equilibria at the Walrasian prices. Even if the prices are "correct," there are also non-Walrasian equilibria in addition to the Walrasian equilibrium. A story goes as the following: If people expect rationing in their supply of labor, then they cut down expenses. This in turn can create such a signal of effective demands that the firms shrink their employment plan, which confirms the expectation of rationing in labor supply. This story can be applied to our stochastic rationing model as well as Hahn's model, which is based on the concept of the Drèze-type effective demands. It is difficult that one extends his model to a dynamic framework.

Varian (1977) gives a simple macroeconomic model with expectations on the effective demand. His treatment of the (point) expectation and its effect on the production decision is ad hoc.

Our framework may give better understandings than the existing models. However, the detailed investigation of applications of stochastic rationing to disequilibrium macroeconomic models is left for further research.

8. Concluding Remarks

We have shown examples of stochastic rationing mechanisms to prove the existence of temporary equilibrium. Non-triviality comes from positive money balance, positive lower bound in rationing and constraints on consumption and production sets. One may wonder if this economy continues to exist since a firm "may" go deficit from hoarding too much labor. Since the origin is included in the production possibility set, an expected profit maximizer would not plan the production plan with expected profit loss, but it does not prevent some firms from suffering from deficit with positive probability. Honkapohja and Ito (1978) assumed an economy where the government owns the firms; therefore the government does not suffer from losses with probability one if we assume Axiom 3A. In the same manner, we can think of an economy where each firm is shared by all consumers with an equal proportion, and the profit is distributed at the end of the period. Then with Axiom 3A, consumers always receive the positive aggregate profit, and an economy continues.

We conclude this note with suggesting several topics to be explored in the near future:

I) [Short-run Stability]

Although the existence of temporary equilibrium signals is proved, the uniqueness convergence (stability) of an equilibrium signal has not been discussed. This is an interesting question to be studied.

II) [Long-run Dynamics]

We may be able to trace the disequilibrium dynamics or a sequence of temporary equilibria over time in our economy by some stochastic processes.

FOOTNOTES

[1]In Honkapohja and Ito (1978), there is the government as a demander of consumption goods.

[2]A simple example is a sampling without replacement. Suppose two individuals are drawing an assignment of household works through drawing two tickets: one of them says one hour of scrubbing the floor and the other says two hours of laundry and ironing. The individuals are facing the stochastic rationing mechanism, although the mechanism assigns three hours labor with certainty.

[3]The name of "rational expectation" is inherited from Gale (1978).

[4]Futia (1977) considers a model with sequential markets.

[5]Although Gale (1978) works in a framework of continuum traders, his economy can be translated in our framework as follows: Let us denote the mapping $\mu\colon \mathbb{R}^r \to M(\mathbb{R}^r_+)$, $r = 2(H+IF)$ or

$$\mu\colon (y_i^h, y_i^f, \ell_i^f, \ell^h) \mapsto (y_i^h, y_i^f, \ell_i^f, \ell^h)$$

where $M(\cdot)$ represents the set of all probability measures on the measurable space $(\mathbb{R}^r, B(\mathbb{R}^r))$ where $B(\mathbb{R}^r)$ denotes the family of Borel sets. There is a natural topology on $M(\mathbb{R}^r)$ known as the topology of weak convergence. An equilibrium is defined as (y,ℓ) such that

$$(y^*,\ell^*) \in \mathbb{R}^r$$

$$(y^*,\ell^*) \in \psi(\mu(y^*,\ell^*))$$

where ψ represents the correspondence of effective demand or supply decisions.

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