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A common theme which runs through much of the investment literature is that private incentives may lead to sub-optimal levels of investment activity.[1] The idea has been extended casually to consideration of human capital investment as well. It is sometimes contended that decisions, made by parents, have adverse effects on their offspring, which could be prevented if intergenerational contracts could be struck. If so, a case can be made for government intervention or subsidization programs to alleviate these intergenerational externalities. Specifically, the sub-optimal investment in offspring human capital may take such obvious forms as poor clothing, too little health care, or too few resources devoted to the child's education.[2] Less obvious exernalities may result when parents underinvest in themselves because they fail to consider spill-over benefits to their children. Parental schooling, for example, may affect the child's ability (or desire) to learn. Dietary patterns established by parents for themselves may influence the child's eating habits and affect his health. More directly, healthy parents are less likely to transmit diseases to their offspring. This paper will examine the effects of these intergenerational externalities in greater detail.

Models of human capital formation have not been scarce in recent years. Starting with Ben-Porath (1967), a number of one-generation models have been forthcoming.[3] Although the literature most specific to human capital has tended to ignore intergenerational aspects[4], there is no dirth of theory which deals specifically with multi-generational questions in other contexts. Models of the sort presented by Samuelson (1958) and Diamond (1965) and most recently by Barro (1974) and Becker (1974) have considered intergenerational savings and consumption behavior and related issues. It seems natural, therefore, that these questions be extended to comprise human capital as well, especially given the amount of public attention devoted to schooling, health, food, and housing subsidization programs.

Before proceeding formally, it is useful to consider the way by which the intergenerational externality is actualized. If parental decisions affect the child's welfare, children should be willing to pay their parents to follow optimal investment schedules. That is, the child should be willing to compensate parents to obtain an amount of human capital which exceeds the parent's own wealth-maximizing level (but has spill-overs to their children), or to invest directly in the children even though they will not directly capture the returns to this investment. The problem, it is generally suggested, is that the parent has no certainty that his child will repay him. This is not quite correct, however, in the present institutional setting. Since the child is under parental jurisdiction and support during the first few years of his life, the parent has a good deal of discretion over the amount of resources the child receives. The parent could simply arrange to transfer a smaller amount to the child than he otherwise would, reflecting the cost of super-optimal investment. If the child were able to borrow on his future income, he would be perfectly willing to go along with his father's scheme since he would have higher wealth and a consumption set which is everywhere dominant. This notion has its origins in Coase (1960) and more directly in Becker (1974). If borrowing is difficult for young children, as must be the case, it would seem that the separation principle becomes inappropriate and that neither father nor child would prefer to first maximize the child's wealth and then his utility, subject to maximum wealth. The separation principle cannot be dismissed so easily, however. In the context of the family, the child can "borrow" from his father. Consider for simplicity, a three phase world -- childhood, adulthood, and retirement. Let X be the amount of resources used by the child in period zero which consists of his current consumption plus the wealth-maximizing level of investment in human capital. Let Y be defined as the expected present value of repayment from the child to the parent. X - Y is then the father's expected cost of a "loan" to the child. The father will be willing to bear cost X - Y if the child's utility enters his utility function in such a way that the equilibrium father/child transfer is greater than or equal to X - Y. If so, the father will "perfect" the capital market for the child and no externality will be present. Transfers from father to child may involve borrowing on the father's part. Higher borrowing costs to the father therefore make it more likely that X - Y will exceed the equilibrium transfer.

Externalities are more likely to show up then where borrowing costs are high, where the probability of repayment is low (making Y low), and where the level of parentally optimal parent-child transfers are low. Thus, for a given X - Y, poor children are more likely to suffer externalities than wealthy ones because the parent-child transfers are lower as the result of income effects. One might also expect externalities to be more important in geographically mobile societies since distance may reduce the probability of repayment. Finally, externalities are likely to be a greater problem during periods when liquidity difficulties are most serious.

It might well be argued that the family is itself an institution which reduces the extent of the externality problem. If social pressures make it sufficiently costly to "default" on one's parent (by not repaying him in old age), no resources would have to be witheld from the child to finance the investment. Repayment at the "market rate of interest" becomes more likely.

In what follows, a formal model is presented which makes explicit the nature of deviations between privately and socially optimal levels of human capital investment. The first part of the discussion will consider externalities which arise because the amount of parental human capital affects the child's ability to produce his own human capital. Although this type of externality seems intuitively less common than externalities which take the form of direct parental underinvestment in children, it turns out that the latter can be viewed as a specific application of the former. Thus, discussion follows from general to specific.

In the last two sections, an empirical model of schooling externalities is formulated and estimated. There, schooling as an efficient means of "upward mobility" and income redistribution is examined. Estimates of the optimal externality correcting schooling subsidy are provided.

A General Model:

The problem for society is to maximize the discounted value of wealth over all generations (since society can act as a guarantor on intergenerational transfers).[5] The present value of the first generation's income stream can be written as[6]

$$(1) \qquad P_1 = \int_0^T [RH_1(\tau) - C(\dot{H}_1(\tau), H_0(\tau))]e^{-r\tau}d\tau$$

where $H_1(\tau)$ is the stock of the parent's human capital in year τ and $C(\cdot)$ is the human capital cost function. R is the rental rate on a unit of human capital, and can be thought of as the amount by which the market or home productivity flow to the individual is augmented by the additional unit of human capital. It does not include spillover effects to other's productivity, but is simply the internal benefit. C depends on the rate at which the father acquires human capital as well as the stock of the grandparent's human capital in period τ. It is this latter effect through which intergenerational externalities operate. One can treat the externality between generations as one which reduces educational costs to children when parental education increases. We assume that there is no <u>direct</u> effect of grandparent's and previous generations' human capital stock on the child's cost function. The cost function also makes the standard assumption of neutrality with respect to own human capital.[7] Thus, it is reasonable that $C_1 > 0$, $C_2 < 0$, $C_{11} > 0$, $C_{22} \geq 0$ and $C(0,H_{i-1}) = 0$. If B is the length of a generation and one child is born each B years, then the second generation's wealth in period zero dollars is

$$(2) \qquad P_2 = \int_0^T [RH_2(\tau+B) - C(\dot{H}_2(\tau+B), H_1(\tau+B))]e^{-r(\tau+B)}d\tau \ .$$

The problem of maximizing wealth over all generations can then be written as

$$(3) \quad \text{Max } S = \int_0^T \sum_{i=1}^{\infty} [RH_i[\tau+(i-1)B] - C(\dot{H}_i[\tau+(i-1)B], H_{i-1}[\tau+(i-1)B])]e^{-r[\tau+(i-1)B]}d\tau$$

$$\begin{aligned} \text{s.t.} \quad & H_i[0 + (i-1)B] = 0 \\ & \dot{H}_i[T + (i-1)B] = 0 \\ & H_0(\tau) \text{ is given exogenously} \\ & H_i \geq 0 \end{aligned}$$

The first order conditions for optimal paths are given by the Euler equations:[8]

$$
\begin{aligned}
Re^{-r\tau} - C_2(\dot{H}_2, H_1)e^{-r(\tau+B)} &= -C_{11}(\dot{H}_1, H_0)\ddot{H}_1 e^{-r\tau} - C_{12}(\dot{H}_1, H_0)\dot{H}_0 e^{-r\tau} \\
&\quad + rC_1(\dot{H}_1, H_0)e^{-r\tau}
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
Re^{-r(\tau+B)} - C_2(\dot{H}_3, H_2)e^{-r(\tau+2B)} &= -C_{11}(\dot{H}_2, H_1)\ddot{H}_2 e^{-r(\tau+B)} - C_{12}(\dot{H}_2, H_1)\dot{H}_1 e^{-r(\tau+B)} \\
&\quad + rC_1(\dot{H}_2, H_1)e^{-r(\tau+B)} \\
&\quad \vdots
\end{aligned}
$$

or

$$
R - C_2(\dot{H}_{i+1}, H_i)e^{-rB} = -C_{11}(\dot{H}_i, H_{i-1})\ddot{H}_i - C_{12}(\dot{H}_i, H_{i-1})\dot{H}_{i-1} + rC_1(H_i, H_{i-1}) \tag{5}
$$

$$\text{for } i = 1, 2, \ldots, \infty .$$

Note that the structure of the system is quasi-recursive. Each state variable depends directly only upon the stock of human capital inherited from the previous generation, on the costs and within generation return to investment, and on the cost-savings to children which result from parental investment (the C_2 term on the left side of (5)). The C_{12} term on the right side of (5) reflects the fact that the grandfather's ability to affect costs of investment to the father depends upon the rate at which the father invests. Thus, when the father alters his level of investment, he must consider the effect of this change on cost-saving which results from his parent's investment (and the subsequent change in the grandfather's plan as the result of it). Additionally, since C_2 on the left side of (5) depends upon the child's rate of investment which in turn depends in part upon cost saving to the grandchild and so on, all generations are linked. If one makes the special assumption that $C_{12} = 0$, the system becomes truly recursive. If $C_{12} = 0$, C_2 is independent of the child's investment behavior so that the fact that children take grandchildren into account becomes irrelevant. That is, when the father's ability to save investment resources utilized by his child is independent of the child's level of investment, the father can totally ignore the child's actions. Furthermore, cost saving in the current generation as the result of the previous generation's investment becomes independent of the level of current investment, i.e., C_{12} drops out. More will be said on this below.

Consider the socially optimal path of human capital accumulation for the first generation. From (5), write

$$\ddot{H}_1(\tau) = \frac{-1}{C_{11}(\dot{H}_1,H_0)} [R - C_2(\dot{H}_2,H_1)e^{-rB} + C_{12}(\dot{H}_1,H_0)\dot{H}_0 - rC_1(\dot{H}_1,H_0)] \tag{6}$$

Since $C_{11} > 0$, $\ddot{H}_1 < 0$ when $[\cdot]$ is positive. If $\ddot{H}_1$ were positive, this would imply $H_1 \leq 0$ and $H_1 \leq 0$.[9] I.e., $\ddot{H}_1 > 0$ implies that it is optimal to disinvest. This makes sense. If $R - C_2 < rC_1 + C_{12}H_0$, the flow cost to investment in human capital exceeds the flow return to the individual plus the flow value of reduced cost to his child.[10] The individual is better off converting his stock of human capital into cash and earning the market rate on the physical asset. (If disinvestment cannot occur, his optimal strategy is to invest zero.)

It is now easily seen that investment in human capital by the first generation is everywhere greater when the social optimization rather than the private optimization rule is followed. If the first generation chooses human capital paths to maximize own wealth and no intergenerational transfer occurs, the problem simply becomes one of maximizing P in equation (1) subject to

$$H_1(0) = 0$$

$$\dot{H}_1(T) = 0$$

$H_0(\tau)$ is given exogenously

$$\dot{H}_i \geq 0$$

The Euler equation for this maximization is

$$R = -C_{11}(\dot{H}_1, H_0)\ddot{H}_1 - C_{12}(\dot{H}_1, H_0)\dot{H}_0 + rC_1(\dot{H}_1, H_0) \tag{7}$$

so that

$$\ddot{H}_1(\tau) = \frac{-1}{C_{11}(\dot{H}_1, H_0)}\left[R + C_{12}(H_1, H_0)\dot{H}_0 - rC_1(H_1, H_0)\right] . \tag{8}$$

The difference between (6) and (8) is $-C_2e^{-rB}$. Since this term is positive, $\ddot{H}_1^S(\tau) < \ddot{H}_1^P(\tau)$ for all τ (where the superscripts denote social and private optimal paths). This implies that $\dot{H}_1^S(\tau) > \dot{H}_1^P(\tau)\ \forall\ \tau \neq T$ and that $H_1^S(\tau) > H_1^P(\tau)$ for $0 < \tau \leq T$.[11] Thus, investment in human capital at each point in time is larger for social than for private optimization.

If investment were to be increased at time $\tau = 0$ on the promise that it would be repaid at a later date, it must be the case that the first generation is still alive during repayment in order to avoid the necessity of interpersonal utility comparisons. This implies a change in the interest rate which could cause an over- or understatement of the true difference between the social and private optimum. I.e., since the amount of current investment and savings is likely to change, r will change. The following diagram makes this clear:

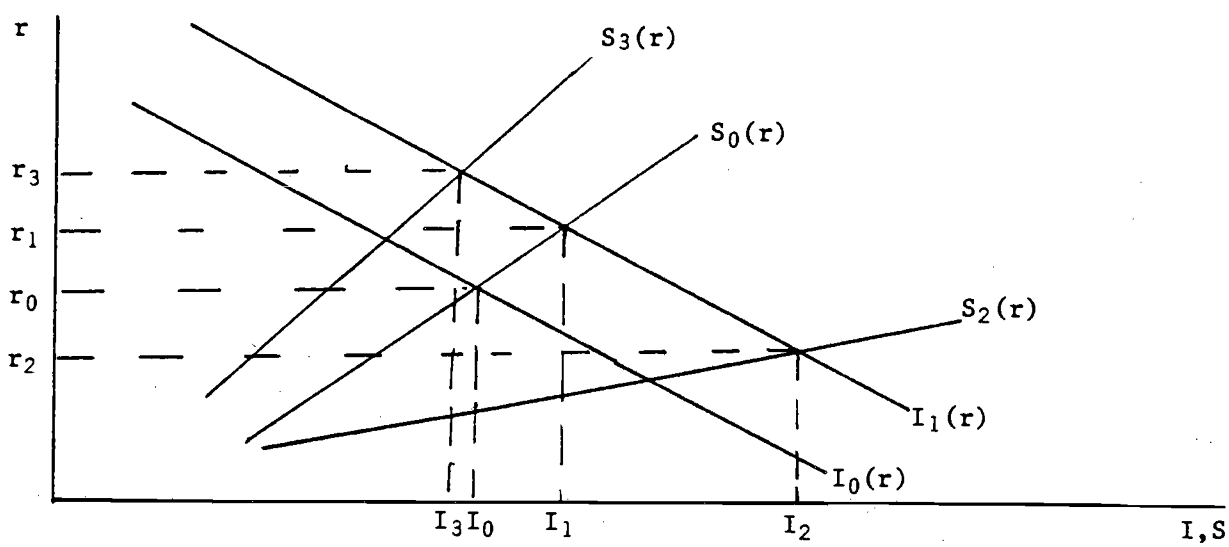

Figure 1

What has happened is that the economy has discovered an unexploited investment opportunity with a rate of return higher than r. (This takes the form of sub-optimal human capital investment.) If everything else were the same, total investment would increase from I_0 to I_1, and the interest rate would rise from r_0 to r_1 inducing individuals to take their consumption later in life. This increase in r would tend to choke off some of the increased investment in human capital so that estimated differences between social and private optima are overstated.

The society as a whole, however, has experienced an increase in real wealth as the investment schedule shifted from I_0 to I_1. This causes a shift in S_0, the direction of which is determined by relative wealth elasticities of present vs. future consumption. If the wealth effect on savings is sufficiently positive, the interest rate could actually fall (as with $S_2(r)$). Here the social-private difference is understated. At the other extreme, $S_3(r)$, the wealth effect induces so large a switch to current consumption that total investment and savings actually falls to I_3. It must be the case, however, that investment in human capital has increased since this is how the increased wealth was generated. (That is, a higher yield investment replaces a lower yield one). Again, r rises so the deviation is overstated.

With a bit of manipulation, estimates of the magnitude of the difference between private and social optima can be obtained. Start with the following simplifying assumptions: Let $C_{12} = 0$ and assume that C_{11} is a constant. This implies that $C_1 = C_{11}\, \dot{H}(\tau)$. Subtracting equation (8) from equation (6) then yields

$$(9) \qquad \ddot{H}^S(\tau) - \ddot{H}^P(\tau) = \frac{-1}{C_{11}} [-C_2 e^{-rB} - rC_{11}(\dot{H}^S(\tau) - \dot{H}^P(\tau))]$$

or, letting $X(\tau) \equiv H^S(\tau) - H^P(\tau)$,

(10) $$\ddot{X}(\tau) - r\dot{X}(\tau) = \frac{C_2 e^{-rB}}{C_{11}}$$

This is a second-order linear differential equation. Suppose that the cost-reducing externality effect is some fraction, ρ, of the internalized, within generation return so that $-C_2 e^{-rB} \equiv \rho R$ where C_2 is evaluated at its equilibrium level. The solution to (10) is then given by

(11) $$X(\tau) = A_1 e^{rt} + A_2 + \frac{\rho Rt}{rC_{11}}$$

where A_1 and A_2 are constants determined by two initial conditions. Since $H^P(0) = H^S(0) = 0$, $X(0) = 0$, and since $\dot{H}^S(T) = \dot{H}^P(T) = 0$, $\dot{X}(T) = 0$. Using these initial conditions, one obtains

$$A_1 + A_2 = X(0) = 0$$

$$rA_1 e^{rT} + \frac{\rho r}{rC_{11}} = \dot{X}(T_1) = 0$$

so that (10) becomes

(12) $$X(\tau) = \frac{\rho r}{rC_{11}} [\tau - \frac{e^{r(\tau - T)}}{r} + \frac{e^{-rT}}{r}].$$

This term is meaningless without perspective. It is therefore useful to evaluate $H^P(\tau)$ and to calculate $X(\tau)/H^P(\tau)$. Substituting the assumptions into (8), one obtains

(13) $$\dot{H}^P(\tau) = \frac{-1}{C_{11}} [R - rC_{11} H^P(\tau)] .$$

Thus (13) is written as

$$(14) \qquad \ddot{H}^P(\tau) - r\dot{H}^P(\tau) = \frac{-R}{C_{11}}$$

which is, again, a second-order differential equation. The solution to (14) is given by

$$(15) \qquad H^P(\tau) = B_1 e^{r\tau} + B_2 + \frac{R}{rC_{11}} .$$

B_1 and B_2 are obtained from the initial conditions that $H^P(0) = 0$ and $\dot{H}^P(T) = 0$. Equation (15) then becomes

$$(16) \qquad H^P(\tau) = \frac{R}{rC_{11}} [\tau - \frac{e^{r(\tau-T)}}{r} + \frac{e^{-rT}}{r}] .$$

Thus,

$$(17) \qquad \frac{X(\tau)}{H^P(\tau)} = \frac{H^S(\tau) - H^P(\tau)}{H^P(\tau)} = \rho$$

or

$$H^S(\tau) = (1 + \rho) H^P(\tau) .$$

This says that if ρR of the return to investment in human capital is passed on to the child and is not captured by the investor, the privately optimal stock of human capital will deviate from the socially optimal stock by exactly that proportion, ρ. Furthermore, this is true at all points in time as well as for the highest attained levels of human capital $H^P(T)$ and $H^S(T)$. If it can be argued that ρ is not trivial, (say, 10%), there will be a substantial potential difference between social and private optima. Nor is the nature of the externality restricted to intergenerational forms. Nothing in equations (9) through (17) required any parent-child linkage. If $-C_2e^{-rB}$ and the corresponding ρR are thought of as the within-generation spillover effect of human capital (the benefits that peers receive from having higher $H(\tau)$ friends), the argument still applies. Any estimate of ρ still provides an estimate of the deviation between private and social optima, irrespective of whether its causes are inter- or intragenerational.

An estimate of the size of transfer necessary to induce an individual to move to the socially optimal level of investment can also be easily obtained. If the individual received $R + \rho R$ rental on his human capital rather than simply R, he would move to the social rather than private optimum. Thus, the required subsidy in period τ is

$$(18) \qquad F_\tau = \rho R(H^S(\tau)) = \rho R(1 + \rho)(H^P(\tau))$$

so that the present value of the lifetime transfer is

$$(19) \qquad F = \int_0^T \rho R(1 + \rho)H^P(\tau)e^{-r\tau}d\tau \; .$$

Substituting from (16),

(20) $$F = \frac{\rho R^2(1+\rho)}{rC_{11}} \int_0^T [\tau e^{-rt} - \frac{e^{-r(2\tau-T)}}{r} + \frac{e^{-r(\tau+T)}}{r}]d\tau \ .$$

Thus,

(21) $$F = \frac{\rho R^2(1+\rho)}{r^3C_{11}} [re^{-rT}(e^{-rT} - 1 - T - 1/r) + 1]$$

If $T = 60$, $r = .1$ and $\rho = .1$,

(22) $$F = (108.06)\frac{R^2}{C_{11}} \ .$$

R and C_{11} are both determined exogenously (the former being market determined, the latter being technologically determined), but are in theory, estimable.[12] It is clear that as C_{11} falls toward zero, the required transfer rises. This makes sense. As the marginal cost of investment rises less steeply, the difference between $H^S(\tau)$ and $H^P(\tau)$ increases (although its proportion to $H^P(\tau)$ does not) so that the required subsidy becomes larger.

So far, two extreme cases have been discussed. In the first case, the parent is assumed to take all generations into account (he maximizes S) and in the second case, he takes only himself into account when compiling his life-time investment strategy. An intermediate case is now considered. Suppose that the parent ignors all future generations beyond his direct offspring. He takes only his child into account. Assume further that all generations are similar so that the child only considers his child and so forth. The question is whether or not the optimal path in this situation differs from paths obtained when S is maximized. Under the assumption that $C_{12} = 0$, it is clear that the two-generation problem is the same as maximizing S. Here, all the Euler equations in (5) are two-generational. The individual's investment depends only upon an exogenously given stock of parental human capital and his own level of investment. Nothing is affected by his child's investment level, so whether or not the child takes grandchildren into account becomes irrelevant.

Even when $C_{12} \neq 0$, it turns out that the two-period maximization is the same as the multi-period problem as long as each child also performs the two-period maximization problem. The reasoning is straightforward. The two-period problem is

$$\text{(23)} \qquad \text{Max } (P_1+P_2) = {}_0\int^T \sum_{i=1}^{2} [RH_i - C(\dot{H}_i, H_{i-1})] e^{-r[\tau+(i-1)B]} \, d\tau$$

$$\begin{aligned} \text{s.t.} \quad & H_i(t) = 0 \ \forall \ i \quad \text{s.t.} \quad t \leq (i-1)B \\ & H_i(t) = 0 \ \forall \ i \quad \text{s.t.} \quad t < (i-1)B \\ & \dot{H}_i(T+(i-1)B) = 0 \end{aligned}$$

where $H_2(t)$ is determined by

$$\text{Max } (P_2+P_3) = {}_0\int^T \sum_{i=2}^{3} [RH_i - C(\dot{H}_i, H_{i-1})] e^{-r[\tau+(i-2)B]} \, d\tau$$

$$\begin{aligned} \text{s.t.} \quad & H_i(t) = 0 \ \forall \quad \text{s.t.} \quad t \leq (i-1)B \\ & H_i(t) = 0 \ \forall \quad \text{s.t.} \quad t > (i-1)B \\ & \dot{H}_i(T+(i-1)B) = 0 \end{aligned}$$

where $H_3(t)$ is determined by

$$\text{Max } (P_3+P_4) = {}_0\int^T \sum_{i=3}^{4} [RH_i - C(\dot{H}_i - C(\dot{H}_i, H_{i-1})]e^{-r[\tau+(i-3)B]} \, d\tau$$

s.t. . . .

The solution to each of the subsidiary constraints generates a system of Euler equations of the form

$$R - C_2(\dot{H}_{i+1}, H_i)e^{-rB} = -C_{11}(\dot{H}_i, H_{i-1})\ddot{H}_i - C_{12}(\dot{H}_i, H_{i-1})\dot{H}_{i-1} + rC_1(\dot{H}_i, H_{i-1}) \tag{24}$$

$$\text{for } i = 1,2,\ldots,\infty$$

The distinction between this system and the one in (4) is that in each problem all non-present period state variables are treated as exogenous. However, since the system must still be solved simultaneously, all equations are linked as they are in (4). In other words, the fact that the parent cares about his child forces him also to care about his child's actions. Since the child consideres the grandchild, the parent is linked to future generations exactly as if he took them explicitly into account. This would not be the case if the parent's stock of human capital affected the grandchild's cost function, but the parent took only the child into account. Then, a term equal to $-C_2(\dot{H}_{i+2}, H_i)e^{-2rB}$, present in the Euler equations for maximization of S, would be absent in each Euler equation corresponding to maximization of $(P_i + P_{i+1})$.

The preceding few paragraphs make one point: If individuals maximize wealth over as many generations as are directly affected by their own investment, private decisions will be socially optimal. One should not infer from this, however, that the intergenerational externality has been rendered ineffective. It is clear that two-generational transfers do occur. However, for reasons already discussed, there may still be impediments to free transfer of funds from the child to the parent. The fact that one observes transfers from child to parent and from parent to child is in no way sufficient to establish that the means exist for intergenerational payment for parental schooling.[13]

Direct Underinvestment:

Let us now consider intergenerational externalities which manifest themselves as direct underinvestment in the child. This second type of externality, it will be seen, is easily treated within the above framework. The question of direct underinvestment is not an entirely new one. In an interesting paper, Ishikawa (1975) discusses the problem, as specific to education, in a family context. He suggests that if parents are selfish, they underinvest in the child because they only consider his earnings until the time of the child's "independence." The child may make adjustments after independence, but this is necessarily worse than optimal investment undertaken throughout the entire lifetime. This story is somewhat unrealistic, however, and results in a perhaps misleading conclusion (underinvestment in education). It seems better to think of transfers from the child to parent as being a function of the child's net income, which can be affected by investment in human capital or by direct monetary transfers from the parent. The previous model can be modified to consider this question.

Let $H_i^{Pj}(\tau)$ be individual j's privately optimal plan for individual i's human capital stock (j = i in the case of the individual's own private optimum). Let $M(\tau)$, the expected present value of transfer from child to parent, be a function of the child's discounted income,[14]

$$M(\tau + B) = f(RH_2(\tau + B)e^{-r\tau}) . \tag{25}$$

This $M(\tau + B)$ should be thought of as the present value of "repayment" that results from income earned in period $(\tau + B)$. (If the child were able to immediately "repay" his parent, he might just as well finance his own investment directly.) It is assumed here that the parent bears the full cost of the child's investment. This assumption is more than necessary, but not completely superfluous. If the child commanded his own resources, he would obtain human capital up to the wealth maximizing level, irrespective of parental actions. Parental subsidies would be regarded as lump sum transfers and would in no way change optimal human capital paths because it would not alter the child's marginal cost of investment. Nor would exceeding the wealth-maximizing level of human capital ever be worthwhile from the parent's point of view. A lump sum cash transfer would be preferable to both parties. Thus, it must be assumed that the child faces a liquidity constraint, which prevents him from unilaterally achieving wealth maximization. For simplicity, and without loss of generality, this is assumed to occur at zero dollars, the child is unable to finance any of his own investment. Further, for simplicity, assume $C_2 = 0$. The "selfish" parent will then maximize

(26) $$\max P_2^1 = {}_0\int^T [M(\tau + B) - C(\dot{H}_2^{P_1}(\tau + B))e^{-r\tau}]d\tau$$

or

$$\max P_2^1 = {}_0\int^T [f(RH_2^{P_1}(\tau + B)e^{-r\tau}) - C(\dot{H}_2^{P_1}(\tau + B)e^{-r\tau}]d\tau$$

s.t. $H_2^{P_1}(B) = 0$

$\dot{H}_2^{P_1}(T + B) = 0$

$\dot{H}_2^{P_1} \geq 0$

The Euler equation is then

(27) $$Rf' = rC_1(\dot{H}_2^{P_1}(\tau + B)) - C_{11}(\dot{H}_2^{P_1}(\tau + B))\ddot{H}_2^{P_1}$$

or

(28) $$\ddot{H}_2^{P_1} = \frac{-1}{C_{11}}[Rf' - rC_1] .$$

The social maximization is given by

(29) $$\max P_2^S = {}_0\int^T [RH_2^S(\tau + B) - C(\dot{H}_2^S(\tau + B))]e^{-r\tau}d\tau$$

s.t. $H_2^S(B) = 0$

$\dot{H}_2^S(T + B) = 0$

$\dot{H}_2^S \geq 0$

which implies

$$(30) \qquad \dot{H}_2^S = \frac{-1}{C_{11}} [R - rC_1]$$

The relationship between $\dot{H}_2^S$ and $\dot{H}_2^{P_1}$ depends upon whether f' is greater or less than one. If $f' < 1$, $\dot{H}_2^{P_1} > \dot{H}_2^S$ so that $\dot{H}_2^{P_1}(\tau + B) < \dot{H}_2^S(\tau + B)$ and $H_2^{P_1}(\tau + B) < H_2^S(\tau + B)$ for all $(\tau) > 0$.[15] The reverse does not hold. If $f' > 1$, $H_2^{P_1}(\tau + B) = H_2^S(\tau + B)$, but the former never will exceed the latter. The reason is that both child and parent would prefer direct cash transfers from the latter to the former rather than investment in the child's human capital beyond the wealth-maximizing level.[16] f' should be interpreted as the child's marginal propensity to transfer resources to his parent. Since the income elasticity of child-to-parent transfers equals $(f')(\frac{1}{S_T})$ where S_T (< 1) is the share of the child's wealth spent on parental transfers, f' will exceed 1 only if child-to-parent transfers are sufficiently luxurious. Yet nothing rules this out *a priori* so that even the "selfish" parent may invest optimally for his child.

To the extent that sub-optimal investment occurs, one might expect situations to arise which reduce the magnitude of the problem. Again, the family might be exactly that sort of institution. If generation 2 invests optimally for generation 3 because generation 1 did so for generation 2, reliance on transfers from child to parent become less necessary. Social stigmas associated with failure to invest optimally in one's child can be thought of as an enforcement mechanism in this scheme to insure efficient intergenerational resource allocation.

An Empirical Methodology:

In this section, a technique is sketched by which the size of the father-child externality can be estimated. Let us confine our discussion to the externality which results in underinvestment by the parent in his own education because he ignores spillovers to his child. It is this manifestation of the externality that relates to questions of intergenerational upward mobility. It is no secret that there is a positive correlation between an individual's income and his parents' educational attainments. If an externality exists in that parents do not invest enough in their own education, their children may be "doomed" to a life of poverty. Lip service at least, is paid to this factor as a justification of subsidized schooling, especially to disadvantaged groups. It is therefore important to consider the size of the "upward mobility" externality.

The formulation above states that the perceived rental rate is R while the actual social rate is $R + \rho R$. Define one year of education as $\dot{H} = 1$ and assume that P, the present value of the flow of the private rental rate over the lifetime is the same for all units of education. The prices, P, can vary across individuals depending upon their productivity at school. If individuals invest in education at $\dot{H} = 1$ per year then write the cost of education function for individual i as

$$\text{Cost} \equiv K^i(E, X \mid \dot{H} = 1) \tag{31}$$

where E is number of years of schooling and X is a vector of personal characteristics, one of which is the parent's level of education. On the margin the <u>perceived</u> marginal return to a unit of education, P, must equal the marginal cost so that

$$(32) \qquad P = K_1^i(E, X|\dot{H} = 1) \ .$$

If the cost function is monotonic in E, equation (32) can be inverted with respect to E, so in equilibrium,

$$(33) \qquad E^P = K_1^{-1}(P, X|\dot{H} = 1) \ .$$

The difference between socially and privately optimal levels of E is then approximately

$$(34) \qquad \alpha E = \frac{\partial K_1^{-1}(P, X|\dot{H} = 1)}{\partial P} \, dP$$

where $dP = \rho^*P$, i.e., the difference between social and private returns. Note that ρ^*P is not the total amount of intergenerational spillover, but merely the amount that the parent chooses to ignore. So

$$(35) \qquad E^S - E^P = \frac{\partial K_1^{-1}(P, X|\dot{H} = 1)}{\partial P} (\rho^*P) \ .$$

Estimation:

To obtain estimates of ρ^* and thereby infer $E^S - E^P$, it is necessary first to determine the cost function, $K(E, X|\dot{H} = 1)$, (henceforth written as $K(E,X)$). Data from the Michigan Income Dynamics Study (1966-1974) will be used for this purpose. This study contains information on the education, family background expenditure patterns and earnings of about 5500 individuals. For this analysis, a subsample of 1455 individuals was selected to meet the following criteria: First, all were male heads of households above 30 years of age (to insure that their highest level of schooling attained reflected lifetime schooling). Second, information on relevant background variables and current wage rates was available.

Given these data, it is necessary to choose a functional form for the cost function. Following the method developed in Lazear (1976a), a Cobb-Douglas function should be sufficiently general for the purpose. Thus, let

$$K(E,X) = \eta E^{\gamma} F^{\alpha} N^{\delta} A^{\theta} M^{\varepsilon} \exp(\lambda S + \mu t + \nu Y + \xi Z + \sigma D + \pi V) \quad (36)$$

where E is the highest grade of schooling completed by the individual

F is the highest grade of schooling completed by the father

M is the highest grade of schooling completed by the mother

N is the number of siblings

A is the individual's age (a vintage effect)

S is a dummy set equal to one if the individual was raised in the South

D is a dummy set equal to one for whites

L is a dummy set equal to one if the individual was raised on a farm

Y is a dummy for being Jewish

V is a dummy for being Catholic

Z is a dummy for having a foreign born mother.

Taking the log of both sides of (36) produces

$$(37)\ \ln K(E,X) = \ln \eta + \gamma \ln E + \alpha \ln F + \delta \ln N + \theta \ln A + \varepsilon \ln M + \lambda S$$
$$+ \sigma D + \mu L + \nu Y + \xi Z + \pi V$$

so that given data on $K(E,X)$ for each individual, OLS identifies η, γ, α, δ, θ, ε, λ, σ, μ, ξ, and π. With these estimates, and attained E, P may be calculated for each individual. Since in equilibrium $P = K_1(E^P,X)$ where E^P is the attained and therefore privately optimal level of schooling an estimate of P is obtained as

$$(38)\qquad \hat{P} \equiv \gamma E^{\gamma-1} N^{\delta} A^{\theta} M^{\varepsilon} \exp(\lambda S + \mu L + \nu Y + \xi Z + \sigma D + \pi V) \ .$$

P is determined by inserting the actual values of the attained level of education and of endowment variables into (38). Then $\hat{E}^P = K_1^{-1}(P,X)$ is

$$(39)\qquad \hat{E}^P = [\gamma \hat{P}^{-1} N^{\delta} A^{\theta} M^{\varepsilon} \exp(\ldots)]^{\frac{1}{1-\gamma}}$$

so

$$(40)\qquad \frac{\partial E^P}{\partial P} = \frac{1}{1-\gamma}\, [\gamma \hat{P}^{-1} N^{\delta} A^{\theta} M^{\varepsilon} \exp(\ldots)]^{\frac{1}{1-\gamma} - 1}\ [-\gamma N^{\delta} A^{\theta} M^{\varepsilon} \exp(\ldots)]\hat{P}^{-2}$$

or equation (35) now becomes

$$(41)\qquad E^S - E^P = \frac{1}{1-\gamma}\, [\gamma \hat{P}^{-1} N^{\delta} A^{\theta} M^{\varepsilon} \exp(\ldots)]^{\frac{1}{1-\gamma} -1}\ [-\gamma N\ A\ M\ \exp(\ldots)]\hat{P}^{-1}\ \rho^* \ .$$

In order to estimate $E^S - E^P$, all that is needed is information on left-hand-variable of equation (36) (the cost of a given level of schooling for an individual with attributes X), and an estimate of $\rho*$. Estimates of the first can be obtained by employing a transformation of foregone earnings. The method is described in detail in Lazear (1976a) and will only be sketched here. The method is to estimate what the wage rate would have been during the jth year of school based upon endowment characteristics and accumulated schooling to that point. The wage rate multiplied by the hours of school then provides an estimate of the foregone earnings component of schooling. Direct costs are assumed to be zero through grade 12 and then one-half of foregone earnings thereafter. The result is that

$$(39) \qquad K(J,X) \equiv \begin{cases} \sum_{j=1}^{J} (830 + 70j)W_j^*\left(\frac{1}{1+r}\right)^{j+5} & \text{for } J \leq 12 \\ \sum_{j=1}^{12} (830 + 70j)W_j^*\left(\frac{1}{1+r}\right)^{j+5} + 1.5 \sum_{j=13}^{J} (830 + 70j)W_j^*\left(\frac{1}{1+r}\right)^{j+5} & \text{for } j > 12 . \end{cases}$$

$(830 + 70j)$ is the hours of school functionl W_j^* is the predicted wage in year j and is obtained by estimating the following wage function:

$$(40) \qquad W_{72} = a_0 + a_1E + a_2N + a_3A + a_4A^2 + a_5F + a_6M + a_7S + a_8L + a_9Y + a_{10}V + a_{11}Z + a_{12}D + a_{13}D \cdot M + a_{14}D \cdot F$$

where W_{72} is the individual's wage rate in 1972. The results of that estimation by OLS and contained in table 1. W_j^* is then defined as

$$(41) \qquad W^*_j = a_0 + a_1(j) + a_2N + a_3(j+5) + a_4(j+5)^2 + a_5F + a_6M + a_7S + a_8L + a_9Y + a_{10}V + a_{11}Z + a_{12}D + a_{13}D\cdot M + a_{14}D\cdot F \ .$$

Now all variables are specified so that the parameters of equation (37) may be estimated. The results are contained in Table 1.[17]

From estimation of (36), all the relevant parameters have been obtained to solve for $E^S - E^P$ except for ρ^*. It should be recalled that ρ^*P is the unaccounted for proportion of the return to education that is captured by the individual's child, rather than by the individual himself. Thus, ρ^*P is not the total spillover to the following generation. However, since $\rho^* \leq \rho$ where ρP is the total intergenerational spillover, an upper bound to the difference between $E^S - E^P$ is given by $\frac{\partial K_1^{-1}(P,X)}{\partial P}(\rho P)$. Estimates of ρP can be obtained. Estimation of equation (40) reveals a significant effect of parental education on an individual's wage rate. If the relationship is in fact a causal one (rather than merely reflecting unobserved ability differences), and if the effect of parental education on work is the same as its effect on non-worked time (neutrality in the sense of Michael [1972]), then the value of a year of a male's education to his (male) child is

$$(42) \qquad (\rho P) = \int_{30}^{90} \frac{\partial W_{72}}{\partial F} (8760) e^{-r\tau} \, d\tau$$

and of a female to her (male) child is

$$(43) \qquad (\rho P)' = \int_{30}^{90} \frac{\partial W_{72}}{\partial M} (8760) e^{-r\tau} \, d\tau$$

Table 1

Regression Results

Variable	Eq. (41)	Eq. (37)
E	.2787 (.0278)	
N	-.0200 (.0372)	
A	.2966 (.0599)	
A^2	-.0028 (.0006)	
F	.1593 (.0582)	
M	.0423 (.0486)	
S (South dummy)	-.3612 (.2183)	-.3513 (.0320)
L (Farm dummy)	-.4477 (.2003)	-.4028 (.0300)
Y (Jewish dummy)	1.320 (.524)	.7518 (.0685)
V (Catholic dummy)	.4950 (.2419)	.3880 (.0329)
Z (Foreign mother dummy)	.6771 (.2981)	.5258 (.0402)
D (white dummy)	.8839 (.5248)	.1563 (.0377)
D·F	-.1205 (.0664)	
D·M	.0080 (.0570)	
ln E		4.7866 (.0609)
ln F		.3630 (.0353)
ln M		.3572 (.0329)
ln N		-.0169 (.0088)
ln A		-.6224 (.0523)
Constant	-6.407 (1.57)	-3.442 (.273)
R^2	.193	.884

Number of observations equals 1455. Figures in parentheses are standard errors.

where 8760 is the number of hours in a year. Using the estimates, ρP and $(\rho P)'$ are \$168 and \$219 respectively for white individuals. Thus, at most, the difference between E^S and E^P evaluated at the point of means is

$$(44) \qquad E^S - E^P \leq \frac{\partial K_I^{-1}(P,X)}{\partial P}(\rho P) = (.00168)(168) = .282 \text{ years for white fathers}$$

and

$$(45) \qquad E^S - E^P \leq (.00168)(219) = .369 \text{ for white mothers.}$$

If the parent takes none of the spill-over into account, he will underinvest in his own education by at most one-third year. A few comments are in order.

First, this calculation has taken as given current levels of educational subsidy. The estimation of the socially optimal level of education assumes that current subsidies occur for reasons which do not include, but are additive to intergenerational spillovers. Even under this assumption which yields the largest possible estimate of additional efficient schooling, the efficient amount is less than one-third of a year.

Second, the calculation is for the "average" white individual, It might be interesting to consider groups of individuals who normally acquire fewer years of schooling than the population as a whole. Non-whites and individuals from low income households immediately come to mind. Since schooling subsidies are often directed at these groups (especially with respect to higher education), it is useful to consider them separately.

By segmenting the sample, the calculation performed in (44) can be repeated for blacks and low income individuals. For blacks, equation (14) yields estimates of ρP and $(\rho P)'$ of \$693 and \$184 respectively. (Note that color interacts with the F and M effects.) Using these estimates and the conditional mean values of the endowment variables for black individuals, one obtains

(46) $E^S - E^P \leq (.0033)(693) = 2.297$ years for black fathers and

and

(47) $E^S - E^P \leq (.0033)(184) = .610$ years for black mothers.

A question was included on the survey that asked whether or not the individual was from a poor home. The variable of questionable interest for two reasons. First, about 45% of the sample replied that they did in fact come from poor families. Second, the variable did not interact significantly with either F or M in the wage equation. Nevertheless, it is informative to segment the sample to see whether $E^S - E^P$ for poor whites is similar to $E^S - E^P$ for blacks. Again, for whites (ρP) and $(\rho P)'$ are \$168 and \$219 respectively. So for poor white males,

(48) $E^S - E^P \leq (.00244)(168) = .409$ years

and for poor white females,

(49) $E^S - E^P \leq (.00244)(219) = .534$ years .

For "rich" white males,

(50) $E^S - E^P \leq (.001556)(168) = .261$ years

and for rich white females,

(51) $$E^S - E^P \leq (.001556)(219) = .341 \text{ years}$$

Thus, maximum estimates of the deviation between social and private optimal levels of schooling for "poor" whites resemble those for rich whites much more than those for blacks. The primary reason is that costs of schooling differ so substantially between blacks and whites as the result of differential foregone earnings.

One additional caveat is in order. An implicit assumption in the above analysis is that parental education is a private rather than a public good. That is, education used to further the productivity of one child cannot then be used again to affect a second one. This makes sense if one thinks of the benefits from education as being realizable only when combined with parental time. If, on the other hand, education is a pure public good, the spillover to the next generation would approximately equal

$$N \int_{30}^{90} \frac{\partial W_{72}}{\partial F} (8760) e^{-r\tau}\, d\tau$$

where N is the number of children (this calculation ignores the fact that children are usually born at different times). The truth probably lies somewhere in the middle; education is neither a purely public nor purely private good.[18]

Let us recap. This section started by asking, what is the value of and desired level of subsidization associated with the "upward mobility" externality produced by education. Specifically, how much of a case can be made for subsidized schooling on the grounds that it is an efficient way to improve the long-run standard of living of low income individuals. A lump-sum transfer of income to the poor has the advantage that the recipient's own utility is maximized in this manner. However, it has the disadvantage that it does not correct intergenerational inefficiencies generated by education spillover effects. An alternative is to tax the present generation and invest these resources. On the margin this returns rate r which could then be transferred to the future generation if desired. This is the justification for discounting spillovers to children at rate r: it is the opportunity cost of those funds to them as well as to individuals currently alive.

To this point, the findings are that no additional subsidy of any size to white individuals is warranted. However, the upper bound of $E^S - E^P$ for black males is substantial. It suggested that these blacks underinvest in own education by as much as 2.3 years because they fail to consider spillover effects. Thus, if the average black male's marginal years of schooling were subsidized by \$693 per year in present value terms or \$4192 in age 18 dollars, the required subsidy of (2.3)(\$4192) = \$9641 would induce a move to the socially optimal level of schooling. This number is based upon the assumption that none of the spillover is accounted for by the parent and that education is a purely private good.

Summary and Conclusion:

This paper considers intergenerational externalities produced either when parental human capital affects an individual's human capital cost function or when parental decisions about direct investment in a child fail to account for the total benefits of that investment. The model is presented in terms of general human capital stocks, but the analysis can be made specific to apply to schooling, health care, "productive" recreation, on-the-job training, and so forth. The following points are made:

1) In the absence of impediments to borrowing by young children, no intergenerational externality could be effective. Parents could withold resources from their young children as payment for privately super-optimal investment undertaken by the parent on the child's behalf. Both parent and child would be better off as the latter could borrow freely on his higher future income to finance smooth lifetime consumption.

2) If intergeneration transfers cannot occur and parental utility calculations totally ignore the offspring's wealth, the fact that parental human capital affects an individual's human capital cost function will result in a deviation between private and socially optimal levels of investment. If R is the internalized (rental) return to a human capital and ρR is the inter- or intragenerational spillover, the privately optimal stock of human capital will, at each point in time, be $1/(1 + \rho)$ of the socially optimal stock. Furthermore, the subsidy necessary to correct the underinvestment varies directly with R and ρ and inversely with C_{11} (the steepness of the marginal cost of $\dot{H}$).

3) It is only necessary that the present generation maximize wealth over as many generations as are directly affected by its investment. The recursive nature of decisions guarantees that the social optimum will be reached if this rule is followed. Thus, in the context of a family, if a father cares sufficiently about his own offspring, he will invest optimally

with respect to <u>all</u> future generations even though they enter his utility functions nor transfer any resources to him.

4) If the parent receives transfers from his child in accordance with the child's wealth, the socially optimal level of the child's human capital will exceed the parent's privately optimal level if the child's propensity to transfer is less than one. The private optimum equals and does not exceed the social optimum when this propensity is greater than one. All of the relevant parameters are empirically obtainable.

5) Estimates of the difference between social and private optimal levels of schooling reveal that for the mean individual, an upper bound of .282 years of underinvestment occurs. If some intergenerational spillovers are taken into account by the investor, the magnitude will be smaller. The distribution of underinvestment is important. One finds that low income individuals and especially black males are more likely to underinvest in their education from a social point of view than are whites and individuals from wealthier homes. An educational subsidy of about $9600 would be required to induce the average black male to move to the socially optimal level of schooling if he currently takes none of the intergenerational spillover into account. To the extent that education has public good attributes, the optimal subsidy is understated by these estimates. There does seem to be a case for educational subsidies to the poor in order to efficiently achieve "upward mobility."

FOOTNOTES

[1]Stigler (1968), Scherer (1970) and Hirschleifer (1971), for example, consider optimal levels of technological innovations.

[2]A number of studies have examined the effect of parental income on the ability to obtain education. Jencks (1972), et. al., Bowles (1972) and a preliminary paper by Parsons (1974) are just a few.

[3]See, for example, Rosen (1972), Rosen (1973), Haley (1973), and Heckman (1974).

[4]Ishikawa (1974) is an exception and will be discussed below.

[5]Wealth-maximization is a weak criterion. In the context of this paper, only intertemporal efficiency is considered. It is the fact that generations overlap that allows us to concern ourselves with wealth rather than utility maximization. Since fathers are retired when sons are working, repayment can occur during this stage and both generations will agree that their position has been improved. This requires that consumption be delayed, and a change in the equilibrium interest rate will therefore result. More is said on this below.

[6]The notation is similar to that in Rosen (1973). The model assumes for simplicity that there is only one parent and one child. Also, at this point, the assumption that there are no intragenerational externalities is maintained. This will be relaxed below.

[7]Lazear (1975) finds support for the assumption that own human capital is neutral while parental human capital affects human capital returns to a greater extent than costs. In the context of this model, this implies $C_2 < 0$.

See Brown (1976), Heckman (1974) and Lazear (1976) for more detailed discussion of the neutrality question.

[8]The Euler equation states that $\frac{\partial I}{\partial H_i} = \frac{d}{dt}(\frac{\partial I}{\partial \dot{H}_i})$ where I is the expression inside the integral. For $H_i = H_1$,

$$\frac{\partial I}{\partial H_1} = Re^{-r\tau} - C_2(\dot{H}_2, H_1)e^{-r(\tau+B)}$$

and

$$\frac{\partial I}{\partial \dot{H}_1} = -C_1(\dot{H}_1, H_0)e^{-r\tau}$$

Then

$$\frac{d}{dt}(\frac{\partial I}{\partial \dot{H}}) = re^{-r\tau}C_1(\dot{H}_1, H_0) - [\frac{\partial C_1}{\partial \dot{H}_1}\cdot\frac{\partial \dot{H}_1}{\partial t} + \frac{\partial C_1}{\partial H_0}\cdot\frac{\partial H_0}{\partial t}]e^{-r\tau}$$

$$= re^{-r\tau}C_1(\dot{H}_1, H_0) - C_{11}(\dot{H}_1, H_0)\ddot{H}_1e^{-r\tau} - C_{12}(\dot{H}_1, H_0)\dot{H}_0e^{-r\tau} .$$

Thus, a necessary condition is

$$Re^{-r\tau} - C_2(\dot{H}_2, H_1)e^{-r(\tau+B)} = -C_{11}(\dot{H}_1, H_0)\ddot{H}_1e^{-r\tau} - C_{12}(\dot{H}_1, H_0)\dot{H}_0e^{-r\tau} + rC_1(\dot{H}_1, H_0)e^{-r\tau}$$

and so forth for each H_i.

[9]If $\ddot{H} > 0$, then $\dot{H}(\tau + \varepsilon) > \dot{H}(\tau)$ for $\varepsilon > 0$. But $\dot{H}(T) = 0$ so $\dot{H}(T - \varepsilon) < 0$. This implies that $H(\tau)$ is negatively sloped everywhere, but at T. Since $H(0) = 0$, $H(\tau) < 0$ for $\tau > 0$.

[10] Note the distinction between flow returns and costs to a unit of human capital and to a unit of investment. On the margin, the cost of additional $\dot{H}$ must equal the marginal returns. This is the logic of the Euler equation. However, the total within period returns to the increased stock of human capital may clearly exceed total costs as the result of inframarginal rents on $\dot{H}$.

[11] It is sufficient to show that given $f(x)$, $g(x)$, the conditions $f(0) = g(0)$, $f'(T) = f'(T)$, and $f''(x) < g''(x)$ together imply that $f'(x) > g'(x)$ for $0 \leq x < T$ and $f(x) > g(x)$ for $0 < x \leq T$. This is easily seen: Since $f''(x) < g''(x)$ for all x,

$$\int_{T-\varepsilon}^{T} f''(x)dx < \int_{T-\varepsilon}^{T} g''(x)dx \qquad \text{for } \varepsilon > 0 .$$

Thus,

$$f'(T) - f'(T-\varepsilon) < g'(T) - g'(T-\varepsilon) .$$

Since

$$f'(T) = g'(T), \quad f'(T-\varepsilon) > g'(T-\varepsilon) \quad \text{so that} \quad f'(x) > g'(x)$$

$$\text{for } x < T$$

Similarly, since $f'(x) > g'(x)$ for $x < T$

$$\int_{0}^{\varepsilon} f'(x)dx > \int_{0}^{\varepsilon} g'(x)dx .$$

Since $f(0) = g(0)$, $f(\varepsilon) > g(\varepsilon)$ for $\varepsilon > 0$.

[12] See Lazear (1975), for estimates of R and the parameters of the parameters of a non-quadratic cost function.

[13] Potential exists for ascertaining whether or not the parent takes children into account in his investment plan. If, other things constant, parents with children obtain higher levels of education than those without (say, unmarried adults), it can be argued that some parental investment occurs on the child's behalf.

[14] In work currently in progress, I am investigating the relationship between child-to-parent transfers and demographic variables as parental wealth, child's wealth, number of siblings, level of child's schooling, marital status, race, etc.

[15] The proof is identical to that in footnote 9 and need not be repeated here.

[16] In fact, as long as $f' > 1$, the parent will keep transferring dollars to his child because the present value of the return to the parent exceeds the cost. In order for transfers to stop short of exhausting total parental resources, f' must eventually sink below 1.

[17] In fact, the concept of deviations from socially optimal investment levels can be usefully extended to other family problems as well. In current work, I define "child-neglect" as existing when less than the socially optimal amount of parent-to-child transfers (for health, housing, education...) occurs. Thus, a wealthy child may in a real sense be "neglected" to a greater extent than one in much poorer condition on an absolute scale.

[18] Even if parental education were purely public, $E^S - E^P$ would be less than N times previous estimates. The reason is that the convex nature of the cost function means that $\frac{\partial K_1^{-1}(P,X)}{\partial P} dP$ overstates dE by more as dP increases.

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