NBER WORKING PAPER SERIES

RATIONAL EXPECTATIONS AND THE DYNAMIC STRUCTURE OF MACROECONOMIC MODELS: A CRITICAL REVIEW

Robert Shiller*

Working Paper No. 93

COMPUTER RESEARCH CENTER FOR ECONOMICS AND MANAGEMENT SCIENCE
National Bureau of Economic Research, Inc.
575 Technology Square
Cambridge, Massachusetts 02139

June 1975

Preliminary: not for quotation

*NBER Computer Research Center. Research supported in part by National Science Foundation Grant DCR 70-03456-A04 to the National Bureau of Economic Research, Inc.

Abstract

The recent literature on rational expectations in macroeconomic theory is surveyed here with the objective of distilling from the various papers useful suggestions for econometric methodology. The paper is not concerned with the empirical questions with which these models have been associated, but rather with the value and usefulness of the concept of rational expectations. The paper begins with a brief discussion of the theory of martingales as it has been applied to macroeconomic theory. Then, the general linear rational expectations model (of which most models discussed in the literature are, in terms of their structure, special cases) is developed and its properties, advantages and drawbacks discussed. The paper concludes with a discussion of the possibilities for estimation and application of such linear models.

Contents

I. Introduction

One of the most difficult problems which has confronted builders of macroeconometric models has been the need to model the mechanism by which the public forms its expectations of future economic variables. The dilemma has been that many of the most important theoretical macroeconomic behavioral relations depend critically on public expectations of future economic variables and yet we do not even have any data on what these expectations are. Even if we do have survey data or other data which purport to represent expectations, if these expectations are endogenous in our model then we still must model the determination of these expectations.

It is very easy to provide examples of macroeconomic behavioral relations which depend essentially on public expectations. In fact, if one looks at one of the major macroeconometric models one is impressed that most of the essential behavioral relations must incorporate assumptions about how expectations are formed. Public expectations of future inflation rates, interest rates, rental rates, demand, and particular components thereof influence current behavior in a fundamental way. Even the simplest IS-LM curve apparatus with its characterization of the liquidity preference curve and possible liquidity trap, the consumption function and the investment function has at its foundation some assumptions about expectations, and changes in expectations would shift all of the curves. It is, in fact, substantially for this reason that it has been such a tricky business to predict the macroeconomic effects of policy.

Builders of macroeconometric models interested in short term policy evaluation and forecasting have dealt with the problem of expectations modelling about the only way they could: by guessing at how individuals form their expectations in practice and trying to find some quantitative representation of this behavior. For instance, it has seemed reasonable that individuals might forecast future inflation rates by looking at past inflation rates. A common quantitative representation of this hypothesis has been that individuals expect future inflation rates to behave like a weighted average or "distributed lag" of recent past inflation rates. Such a weighted average, which we call an "expectations proxy", may then be included in our quantification of the behavioral relation in place of the actual expectation which may be unknown. Behavioral relations which rely on such expectations proxies usually work pretty well. They may also predict very badly if something happens which changes the way people form their expectations, e. g., if price controls are instituted or if there is a sudden hyperinflation. Macroeconometric modellers have attempted to deal with these problems as best they could by making a guess as to how expectations will respond to changes. Thus, U. S. macroeconometric model builders, when confronted with the Phase I price controls in 1971, were obliged to make some outright guess of their own as to how this policy would affect the mechanism which generates price expectations.

Recently, a number of macroeconomic theorists, Lucas [1973a], Sargent and Wallace [1974], [1975], Muench and Wallace [1974], Prescott and Kydland [1974] and others, dissatisfied with conventional macroeconometric models, have suggested a different approach to economic modelling. This we call the "rational expectations" approach. The approach differs in its objectives as well as its methods. The problem with the usual objectives of conventional macroeconometric models, it is argued, is precisely that they are concerned only with short run prediction of macroeconomic response to particular alternative policies. What is perhaps of greater interest, as Lucas [1973a] has stated most forcefully, is the ultimate effect of a proposed policy rule. A policy rule is a rule by which policy makers decide what to do in response to the economic situation. A policy rule can be described as a functional relationship from economic variables to variables policy makers control. What we want our models to tell us, then, is the ultimate behavior of the economy after individuals have learned how the policy rule affects the time paths and random properties of economic variables. It is conceivable, for example, that the high inflation rates which we are experiencing today is the result of a concentration of attention by policy makers in the past on the short run effects of their policy. The result of the "high pressure" economics in the United States in the 1960's was that individuals began to realize that higher inflation rates could be extrapolated into the future. Perhaps a change in inflationary expectations is the reason for the outward shift in the Phillips curve. What economists should have been asking was not "what will the proposed policy do over the next few quarters?" but "what would a policy rule of the proposed kind have achieved if we had followed it over the last 20 years?" In this context we might also ask, as did Lucas [1973c], "what has been the experience of other countries which have consistently followed different policy rules in response to inflation?"

Given that we are interested in long term, rather than short term, policy analysis, it is then possible to adopt a different approach to the modelling of expectations. If a policy rule is followed consistently over a long period of time, rational individuals will eventually learn how that policy rule has affected the random character of economic variables, and if they are truly rational, their expectations will not differ substantially from optimal forecasts. After a policy rule is adopted it may at first be difficult for individuals to make forecasts of future economic variables and they must rely on crude guesses. Since the nature of these guesses may in turn again affect the stochastic properties of the endogenous variables, the period immediately after the adoption of the policy rule may be a chaotic one for which it is hard even to define how variables should be forecasted. The essential assumption that rational expectations theorists then make, however, is that eventually the economy will converge on a dynamic path for which the expectations mechanisms which are involved in determining the path are indeed rational. The economy will then have reached a state in which "rational expectations" holds. The essential advice offered by the rational expectations theorists is that, in comparing alternative policies, we should compare their respective ultimate states of rational expectations.

A rational expectations model, as defined originally by Muth [1961], is defined loosely as any model in which economic behavior at time t depends on public or market expectations of future economic variables and in which these expectations are true mathematical expectations of the future variables conditional on all variables in the model which are known to the public at time t. The mechanism by which expectations are supposed to be generated in the model must be one that gives the true mathematical expectation of the future variables in view of the observed stochastic properties of the variables. If the structure of the model itself has implications for the functional relationship between true conditional expectations and the variables used for forecasting, then these implications must be consistent with the mechanism by which expectations are supposed to be generated.

Since the behavioral relations are not necessarily "rational" from the standpoint of individuals who make up the economy, Walters [1971] has suggested that we speak of "consistent expectations" rather than rational expectations. Cyert and DeGroot [1974] have suggested that "rational expectations" should refer to those models in which individuals are presumed to know the structure of the entire economy and to use this to form forecasts. They suggest then that individuals may exhibit what they call "consistent expectations" if their forecasts are true mathematical expectations even if individuals have an incorrect model of the economy. The difference between "consistent expectations" and "rational expectations" in this sense has empirical significance only in terms of the learning procedure by which we reach an equilibrium in which expectations are consistent. In this paper, however, we shall perpetuate the conventional use of the term "rational expectations".

The changes in econometric methodology proposed by these authors are radical departures from our customary way of doing things. To date, however, the theory of rational expectations has had little impact on practical econometric methodology. In part, this is because the concepts are still unfamiliar to most econometricians. More important, however, practicing econometricians may have the impression that the concept of a rational expectations model is too fraught with theoretical difficulties and complexities to have any relevance to practical policy.

Our purpose here will be to review the literature on rational expectations models with the specific objective of sorting out what it is that the literature really has to offer, if anything, for practical macroeconometric modelling. Since our objective is to find those suggestions which might immediately be implemented, we will confine our attention to the simple linear structure which has characterized the less theoretically advanced discussions.

A more general definition of rational expectations was given by Lucas and Prescott [1974] and Grossman [1972],[1973]. In models in which human behavior at time t is supposed to depend on the subjective distribution held by market participants of future economic variables (not just its mean) rational expectations requires that this subjective distribution be the same as the true distribution conditional on all information available at time t. Literature which makes use of this more general definition would take us into the literature on optimal decision under uncertainty or optimal search theory and is beyond the scope of this paper.

The concentration of attention on the mean of the subjective distribution of future economic variables rather than other parameters of the distribution has been a characteristic of most of the papers (especially empirically oriented papers) which make use of the concept of rational expectations. There is a good reason for this: if models are linear then models involving simple expectations of variables are easily handled. The theory of martingales, already developed by probability theorists, is readily applied to them. Moreover, most of the characterizations of human behavior in the practical macroeconomic literature which preceded the development of the rational expectations models depended on simple expectations of future variables, rather than other moments. The new assumption that these expectations are true mathematical expectations is then a natural outgrowth of this literature.

Since we are concerned here with methodology we shall not discuss the particular empirical questions raised by these models, except insofar as they shed light on the realism of the assumptions regarding the generation of expectations. We emphasize in this context that the characteristic assumption made in many rational expectations macroeconomic models that variations in aggregate economic activity are due primarily to errors made by economic agents in forecasting prices has no necessary connection to the methodology implied by the concept of rational expectations. The validity of this assumption about fluctuations in aggregate economic activity is a bigger question than we can deal with here.

In the next section of this paper we shall discuss simple martingale models. This section will introduce some basic concepts which precede a discussion of the more general linear rational expectations models. Some of the properties of martingales discussed here have already seen application in the estimation and refinement of macroeconometric models.

In the third section we will discuss the general linear rational expectations model. This is a simultaneous equations model which involves as variables expectations of endogenous variables. This general model then represents, a special cases, the structures of most of the models in the rational expectations literature. We can write down in matrix form, using forward operators, the true conditional expectations of the endogenous variables given the structure of the model and hence we can describe the behavior of the model under alternative policies. We can then discuss in general terms the estimation and use of these models.

II. Simple Martingale Models

The concept of a martingale, originated by Levy and Doob [1953], has had an important place in the development of the theory of stochastic processes. Economists were late to appreciate the possibilities of martingale models, and it was apparently not until the concepts were introduced to economists by Samuelson [1965] and Mandelbrot [1966], that these models saw widespread use. The obvious application of these models was to the theory of the pricing of financial assets, and the theory of martingales has become the groundwork for the theory of efficient markets. The literature on efficient markets has been surveyed elsewhere (Fama, [1970]). Our interest here will instead be the use of martingale theory in the construction of macroeconomic models.

For our purposes, we may define a martingale as follows. (For a more theoretical discussion see Kemeny et al., 1966.) A vector stochastic process $x_t = [x_{1t}, x_{2t}, \ldots x_{nt}]$ is a martingale for $t \geq t_0$ with respect to a vector of information I_t if the expectation of x_t conditional on I_t equals x_{t-1}:

$$E(x_t|I_t) = x_{t-1} \qquad t = t_0, t_0+1, \ldots \qquad (1)$$

where $I_{t_0-1} = 0$ and I_t includes I_{t-1}, x_{t-1}, and possibly other random variables u_t, v_t,.... It follows immediately from the definition also that:

$$E(x_{t+n}|I_t) = x_{t-1} \qquad \begin{array}{l} n = 1, 2, 3, \ldots \\ t = t_0, t_0+1, \ldots \end{array} \qquad (2)$$

The term "martingale" arises because in a fair game (fair with respect to information I_t), the martingale betting system of increasing the stakes every time there is a loss to recoup the loss and returning to the original stakes after a win results in a sequence of accumulated winnings x_t which satisfies (1). In fact, an important property of fair games is the impossibility of systems: accumulated winnings x_t by any betting strategy will satisfy (1). The accumulated winnings need not, however, be a random walk, since the distribution of future increments in accumulated winnings may depend on previous winnings even though the expected value of the increment is always zero.

Martingales arise in economics as public or market expectations of future economic variables. If $\hat{y}(k,t)$ is the expectation at time t of an n element column vector random variable y_{t+k} conditional on a vector of public information I_t available at time t, $\hat{y}(k,t) \equiv E(y_{t+k}|I_t)$ where I_t includes y_{t-1}, I_{t-1} and perhaps other variables, then:

$$E[\hat{y}(k,t)|I_{t_0}] = \begin{cases} \hat{y}(k+t-t_0,\ t_0) & t_0 \leq t \\ \hat{y}(k,t) & t_0 \geq t \end{cases} \tag{3}$$

and $x_t = \hat{y}(k-t,\ t)$ satisfies (1) and is hence a martingale with respect to I_t.

One consequence, emphasized by Samuelson and Mandelbrot, is that the vector stochastic process Δx_t is spectrally white, i.e. serially uncorrelated. Sargent [1971] extended this in a way that can be summarized in terms of the array:

$$\begin{array}{lll} [\hat{y}(j,t)-\hat{y}(j+1,t-1)] & [\hat{y}(j+1,t-1)-\hat{y}(j+2,t-2)] & [\hat{y}(j+2,t-2)-\hat{y}(j+3,t-3)],\ldots \\ [\hat{y}(j-1,t)-\hat{y}(j,t-1)] & [\hat{y}(j,t-1)-\hat{y}(j+1,t-2)] & [\hat{y}(j+1,t-2)-\hat{y}(j+2,t-3)],\ldots \\ [\hat{y}(j-2,t)-\hat{y}(j-1,t-1)] & [\hat{y}(j-1,t-1)-\hat{y}(j,t-2)] & [\hat{y}(j,t-2)-\hat{y}(j+1,t-3)],\ldots \\ \vdots & \vdots & \vdots \end{array} \tag{4}$$

The expectation of each element in the array is zero, and rows and diagonals (but not columns) are spectrally white.

It is well known that martingale properties have been attributed to stock prices. This assertion was discussed most concisely by Samuelson [1973]. Other macroeconomic variables which have been characterized as martingales with respect to public information are forward interest rates.

Sargent [1974] has also asserted that long-term interest rates should resemble martingales since, by the expectations theory of the term structure of interest rates, long-term interest rates are averages of expected future interest rates. Finally, it has been pointed out that the popular simple rational expectations model which asserts that variations in aggregate economic activity are due to errors in forecasting prices would imply that certain measures of aggregate economic activity (such as the unemployment rate) should resemble first differences of a martingale with respect to public information. This last assertion, incidentally, is demonstrably false and has been used to criticise these simple rational expectations models.

The use of martingales in economic theory is relatively new. However, the idea of representing expectations by optimal linear forecasts has a longer history and yields similar results. Defining the optimal forecast $\bar{y}(k,t)$ of y_{t+k} which is linear in the vector I_t information at time t as the linear forecast with smallest expected squared error we get:

$$\bar{y}(k,t)' = I_t \xi_{t,k} \tag{5}$$

where the matrix ξ_{t+k} is defined by:

$$\xi_{t,k} = E(I_t' I_t)^{-1} E(I_t' y_{t+k}')$$

Now the process $\bar{x}_t$ defined by $\bar{x}_t = \bar{y}(k-t,t)$ is not necessarily a martingale with respect to I_t but $\bar{x}_t$ has the analogous property that the optimal linear forecast of $\bar{x}_{t+n}$ for $n \geq 0$ is $\bar{x}_t$. If we replace elements $\hat{y}(i,j)$ of the array (4) with $\bar{y}(i,j)$, then rows and diagonals (but not columns) are again spectrally white.

Two important principles regarding optimal linear forecasts of stationary processes have had important uses in economic theories: the error learning principle and the chain principle of forecasting. Both have simple forms only under the assumption that I_t consists only of the entire history of the forecasted variable y_t, so that the k-period forecast of y can be written:

$$\bar{y}(k,t) = \sum_{i=1}^{\infty} \phi_i^{(k)} y_{t-i} \tag{6}$$

where each ϕ_i, i=1, ∞, is an n x n matrix. On this assumption, then, we have the error learning principle:

$$\bar{y}(k,t+1) - \bar{y}(k+1,t) = \phi_1^{(k)}[y_t - \bar{y}(0,t)] \tag{7}$$

or, more generally:

$$\bar{y}(k,t+m) - \bar{y}(k+m,t) = \sum_{i=1}^{m} \phi_i^{(k)}[y_{t+m-i} - \bar{y}(m-i,t)] \tag{7'}$$

Meiselman [1961] was the first to assert that changes in economic forecasts might be related to the most recently discovered forecast error. He did not perceive, however, that such forecast behavior is a property of optimal linear forecasts, a fact which was later pointed out by Diller [1969] and Nelson [1970]. Benjamin Friedman [1975] has also derived a relation of the form (7) under the assumption that $y_t = x_t\beta + \varepsilon_t$ where x_t is a known vector of variables, β is an unknown vector of constants and ε_t is an error term with zero mean. If the forecast $\bar{y}(k,t) = x_{t+k}b_t$ where b_t is the ordinary least squares estimate of β based on $x_{t_0},\ldots x_t$ and $y_{t_0},\ldots y_t$ then (7) holds except that the coefficient on the right hand side is not $\phi_1^{(k)}$ but is instead a function of $x_{t_0},\ldots x_{t+1}$ and x_{t+k}.

The chain principle of forecasting states that the optimal linear forecast of y_{t+k} given (6) is also given by:

$$\bar{y}(k,t) = \sum_{i=1}^{k} \phi_i^{(0)}\hat{y}(k-i,t) + \sum_{i=1}^{\infty} \phi_{k+i}^{(0)} y_{t-i} \qquad k \geq 0 \tag{8}$$

so that the same relation that is used to produce the zero-period forecast may be used to produce the k-period forecast. To produce a k-period forecast one begins by producing a zero-period forecast. This forecast is then used in the zero-period forecasting equation to produce a one-period forecast, which in turn is used to produce a two-period forecast, and so on. In other words, the k-period forecasting weights may be defined by the recursive relation:

$$\phi_i^{(k)} = \phi_{k+i}^{(0)} + \sum_{j=1}^{k} \phi_j^{(0)} \phi_i^{(k-j)} \qquad i = 1,\ldots\infty \tag{9}$$

If we accept the proposition, then, that market expectations are optimal linear forecasts based on the entire history of certain known variables, then these principles can be used to provide structure to our models. Some papers that have used these results in an essential way are those by Sutch [1968], Modigliani and Sutch [1967], Diller [1969], Nelson [1970,a,b], Sargent [1972], Shiller [1972], and Modigliani and Shiller [1973]. For example, Sutch [1968] wished to test the proposition that the yield $y_t^{(m)}$ on m-period bonds is an m-period weighted average of expected future one-period interest rates $y_t^{(1)}$ at time t which are in turn linear forecasts based on lagged short rates only:

$$y_t^{(m)} = \frac{1}{m} \sum_{j=0}^{m-1} \bar{y}^{(1)}(j,t) \tag{10}$$

so that, by (6):

$$y_t^{(m)} = \sum_{k=0}^{m-1} \frac{1}{m} \sum_{i=0}^{\infty} \phi_i^{(k)} y_{t-i-1}^{(1)} = \sum_{i=1}^{\infty} \beta_i y_{t-i}^{(1)} \tag{11}$$

where the $\beta_i = \frac{1}{m} \sum_{k=0}^{m-1} \phi_i^{(k)}$. Sutch then computed the $\phi_i^{(0)}$ two different ways: (1) by regressing $y_t^{(1)}$ on (a finite number λ of) lagged values of $y^{(1)}$: $y_{t-1}^{(1)}, y_{y-2}^{(1)}, \ldots y_{t-\lambda}^{(1)}$ to get the vector of coefficients $\phi_i^{(0)}$, $i = 0,\ldots\lambda$ directly and (2) by regressing $y_i^{(m)}$ on the same number of lagged values

$y_{t-1}^{(1)}$, $y_{t-2}^{(1)}, \ldots y_{t-\lambda}^{(1)}$ to obtain estimates of β_1, $\beta_2, \ldots$ and then using (9) and (11) to solve for $\phi_i^{(0)}$, $i = 1, \ldots \lambda$. Except for sampling error and if all true $\phi_i^{(0)} = 0$ for $i > \lambda$, the two estimates of the vector $\phi_i^{(0)}$, $i = 1, \ldots \lambda$ should be the same (in fact his two estimates were remarkably close). Essentially the same analysis was extended by Shiller [1972] and Modigliani and Shiller [1973] to the case in which I_t includes both the history of inflation rates as well as the history of one-period interest rates.

Another example of the application of these principles is afforded by the tests of the assumption of rationality in forward interest rate determination performed by Diller [1969] and Nelson [1970]. These authors reproduced Meiselman's regression of changes in forward interest rates (computed from data on the term structure of bond yields) onto the difference between the current one-period interest rate and previous period's one-period forward rate which applied to the current period (i.e. the "forecast error"). They then effectively compared the coefficients with the leading coefficients $\phi_1^{(k)}$ of an estimated optimal k-period forecasting equation. The coefficients, incidentally, turned out to be very close, which would seem to suggest that actual forward rates may indeed be optimal linear forecasts.

The success of the comparison made by Sutch, Diller, and Nelson may actually seem quite surprising since one would hardly have expected that forward interest rates actually arise by a forecasting equation which is linear in lagged interest rates only. In fact, business forecasters use a variety of information in forming their expectations, information that may often not even have a quantitative nature. This leads one to suspect that the tests that these authors ran are also tests of a more believable martingale model in which market expectations are mathematical expectations of future variables conditional on an information set which includes other variables as well as lagged interest rates. In fact, this supposition is valid for the Sutch test but not for the Diller and Nelson test.

It is easily shown that if $\hat{y}(k,t)$ is the expectation of y_{t+k} conditional on the vector of information I_t then the coefficient $E(U_t'U_t)^{-1}E(U_t'\hat{y}(k,t)')$ of a regression of $\hat{y}(k,t)'$ on a vector U_t which is a part of I_t (i.e. $I_t = [U_t|V_t]$) are the same as the coefficients $E(U_t'U_t)^{-1}E(U_t'y_{t+k}')$ of a regression of y_{t+k}' onto U_t. This is just another property of our basic martingale model. It follows then (Shiller [1973a]) that under these assumptions we still have a sort of chain principle of forecasting: if a regression of $\hat{y}(0,t)$ onto y_{t-1}, y_{t-2},... produces coefficients $\phi_1^{(0)}$, $\phi_2^{(0)}$,... then the coefficients of a regression of $\hat{y}(m-1,t)$ onto y_{t-1}, y_{t-2},... should be related to the coefficients of the first regression by the relationship (9). But we cannot show that a regression of $\hat{y}(k,t+1) - \hat{y}(k+1,t)$ onto $y_t - \hat{y}(0,t)$ should give us the coefficient $\phi_1^{(k)}$. Thus the error learning principle is specific to the narrow assumption that forecasts are linear in lagged y_t only.

The proposition that a regression of $y(k,t)$ onto U_t yields the same coefficients as a regression of y_{t+k} onto U_t has also provided the framework for other recent studies of forward rate determination: Rutledge [1974], Lucas [1975]. Lucas tested the hypothesis that interest rates at time t equal a constant real rate of interest plus a rational expectation $\hat{y}(1,t)$ of inflation y_{t+1} by regressing y_t onto the lagged interest rate and other variables (as well as a constant). Our proposition would imply that the coefficient of the lagged interest rate be one and all other coefficients (except the constant) be zero. Along similar lines, deMenil [1975] regressed direct observations of price expectations derived from survey data onto publicly available data and compared the coefficients with those obtained by regressing actual prices onto the same data.

The general impression one gets from this literature is that the assumption that public expectations of future variables are indeed mathematical expectations conditional on public information is borne out far better than one would have expected. Thus, the structure obtained by assuming martingale properties for expectations is likely to be valuable in helping us to model the dynamic structure of the economy.

III. The General Linear Rational Expectations Model

In the preceding section we saw that the theory of martingales helps us to put some structure into our econometric models. The theory has already been used to refine macroeconometric models. For example, the relation (11) between one-period interest rates and m-period interest rates was used as an equation in earlier versions of the MIT-Penn-SSRC Econometric Model of the United States. Information on the coefficient vector β_i can be obtained not only from the observed relation between m-period interest rates and one period interest rates but also from the observed stochastic properties of the one-period interest rate itself. The additional information in the observed random behavior of the one-period rates enables us to improve our estimates of the coefficients β_i or to check the reasonableness of our model.

Once econometric model builders make explicit reference to such expectations mechanisms in their derivation of their models, however, they become vulnerable to a couple of important criticisms.

A first criticism is that while the expectations in the individual equations which make up the model may indeed be rational in the sense we have described, they may not, as was suggested in the introduction to this paper, be rational in terms of the model as a whole. Given the stochastic structure of the exogenous variables, the model implies a stochastic structure for the endogenous variables. The resultant stochastic behavior may be different from that which was observed in the original data, and so the expectations mechanism hypothesized by the model may not be rational in terms of the model itself. Thus, in a sense, the model may be internally inconsistent. Perhaps, it has been suggested, it may be possible to estimate econometric models which are constrained to be internally consistent in this sense. This idea is clearly due originally to Muth [1961]. We will discuss in this section the kind of structure that such internal consistency may impose on our models and the possibilities for econometric models.

A second criticism, also noted in the introduction, applies to the usual use of most econometric models, which is to evaluate the effects of alternative policies by simulation of these policies with the model. The criticism is that if the policies have any effect at all, they are likely to change the stochastic structure of the policy variables and also of the endogenous variables which will in turn affect the coefficients of the model. For example, suppose the monetary authority wishes to use a macroeconometric model to gauge the effects of a proposed policy of stabilizing short term interest rates. If they had not been following the policy in the past, then the new policy will change the stochastic structure of one-period interest rates $y_t^{(1)}$ and, in general, of the optimal linear autoregressive forecasting coefficients $\phi_1^{(0)}, \phi_2^{(0)}, \ldots$ The effect will be, via expression (9), to change the coefficients β_i in (11). Thus, the model is not stable under alternative policy rules and thus cannot be used to evaluate the effects of these rules.

Most of the rational expectations models in the literature which attempt to handle these criticisms have been linear models which can be represented in the general form:

$$Ay_t + B(F_1)\hat{y}(0,t) + CZ_t = U_t \tag{12}$$

where y_t is an n x 1 column vector of endogenous variables at time t, Z_t is an m x 1 vector of exogenous variables and U_t is an n x 1 vector of errors with zero mean, which is uncorrelated with Z_t for all t and is itself serially uncorrelated. The vector $\hat{y}(k,t)$ is the market or public expectation at time t of y_{t+k}. A and C are coefficient matrices of order n x n and n x m respectively, while $B(F_1)$ is an n x n order matrix whose elements are polynomials in the operator F_1. The operator F_1 is here defined by $F_1^j\hat{y}(u,v) = \hat{y}(u+j,v)$, that is, F_1 is the forward operator on the first argument of the function it is applied to. We then write the ijth element of B as $b_{ij}^{(0)} + b_{ij}^{(1)}F_1 + b_{ij}^{(2)}F_1^2 + \ldots b_{ij}^{(d_{ij})}F_1^{d_{ij}}$ where d_{ij} is the degree of the polynomial.

The ith equation represented by (12) can also be written

$$\sum_{j=1}^{n} a_{ij}y_{j_t} + \sum_{j=1}^{n}\sum_{k=0}^{d_{ij}} b_{ij}^{(k)}\hat{y}_j(k,t) + \sum_{j=1}^{m} c_{ij}Z_{j_t} = U_{i_t}$$

where y_{j_t} is the jth element of y_t, etc. Each of the n equations is linear in today's expectation of future values of the n endogenous variables $y_{1_t},\ldots y_{n_t}$. It is important to note the model does not include lagged expectations. Expectations held by the public in previous time periods do not enter into the determination of current endogenous variables. Although such lagged expectations would probably enter into any realistic model, except in certain degenerate cases they pose difficult mathematical problems, as we shall see below. Expectations of future exogenous variables may enter as elements of Z_t. They may be treated as ordinary exogenous variables for our purposes and hence we do not deal with them separately.

The model (12) becomes a rational expectations model when the public expectations $\hat{y}(k,t)$ are assumed to be mathematical expectations of $\hat{y}_{t+k}$ conditional on the information publicly available at time t:

$$\hat{y}(k,t) = E(y_{t+k}|I_t) \tag{13}$$

where I_t is the vector of information publicly available at time t. We assume I_t includes Z_t, y_{t-1}, U_{t-1}, and I_{t-1} but does not include y_t and U_t.

If we then define $\hat{Z}(k,t) \equiv E(Z_{t+k}|I_t)$ and $\hat{U}(k,t) = E(U_{t+k}|I_t)$, we have:

$$\begin{aligned} &\hat{y}(k,t) = y_{t+k},\ \hat{U}(k,t) = U_{t+k} && k < 0 \\ &\hat{Z}(k,t) = Z_{t+k} && k \leq 0 \end{aligned} \tag{14}$$

In addition, we assume that $\hat{U}(k,t) = 0$ for $k \geq 0$.

It is worth pointing out that it is also possible to define a "partly rational" expectation model in which certain expectations in certain equations are either exogenous or have an imposed relationship with other variables. For instance, one could include an "irrational expectation" which is exogenous among the variables Z. One could even hypothesize that the behavior of half of the individuals in the economy shows rational expectations and half of the individuals have some sort of irrational expectations, so that what appears in the equations which represent their aggregate behavior might be a weighted average of the two. All of these possibilities can then be represented as special cases of (12).

The original discussion of rational expectations by Muth [1961] developed a degenerate case of (12) in which $B(F_1)$ contained only one non-zero element which was a constant (i.e. $d_{ij} = 0$). Also in this category are papers by Walters [1971], Lucas [1973c], Cyert and DeGroot [1974], and Roll [1974]. Papers by Lucas [1970], Sargent and Wallace [1974,1975] and Barro [1975] involved $B(F_1)$ matrices which had non-zero elements which were not constant but were linear in F_1.

The essential property that rational expectations models of the form (12) share is the dependence (except in certain degenerate cases) of the expectations of future y on the exogenous variables comprising Z and on the form of the stochastic process Z. Thus, if policy makers interfere with elements of Z, then the stochastic process $B(F_1)\hat{y}(k,t)$ may also change in character. The strength of models of the form (12) is that they enable us to evaluate the effects of any policy rule by defining a "final form" for (12) which depends only expectations of Z, not y.

Replacing t with t+k in (12) and taking expectations conditional on I_t we get:

$$A\hat{y}(k,t) + E(B(F_1)\hat{y}(0,t+k)|I_t) + C\hat{Z}(k,t) = \hat{U}(k,t) \tag{15}$$

If $k < 0$, we see, using (3) and (14), that (15) is identical to (12). However, if $k \geq 0$, using (3) we instead get:

$$A\hat{y}(k,t) + B(F_1)\hat{y}(k,t) + C\hat{Z}(k,t) = 0 \tag{16}$$

Expression (16) is a system of linear difference equations in k which may be solved to yield an expression for $\hat{y}(k,t)$, $k \geq 0$ in terms of the forcing function $\hat{Z}(k,t)$. To find the solution we need a terminal condition for each root of the determinantal equation $|A + B(x)| = 0$. If all the roots of $|A + B(x)| = 0$ lie outside the unit circle, however, then the condition that $\hat{y}(k,t)$ does not explode as $k \to \infty$ is sufficient to define the solution. The terminal condition is rarely mentioned in the literature. In one instance, Sargent and Wallace [1975] described this terminal condition as "ruling out speculative bubbles". Of course, for Muth and others who assumed zero-order difference equations, no terminal conditions are necessary. In another instance, Sargent and Wallace [1975] built a macroeconomic model for which oen of the roots of $|A + B(x)| = 0$ lay on the unit circle. In this model, which was a macroeconomic model with the interest rate exogenous because the monetary authority was assumed to stabilize it, the authors concluded that the solution (for the endogenous variable which was the price level) was "indeterminate" because the terminal condition required was "a much stronger terminal condition" than they had to impose on a previous convergent model. However, the terminal condition which is in fact routinely placed on convergent models is also very strong. Since the economy has not had this structure forever, there must have been some first date at which the structure (12) held. On this date, knowing (12) alone would not enable us to forecast y, since one would not know that the solution would not explode. If somehow the public initially (or even at some later date) got the idea that y would explode, then their expectations would be borne out.

In any event, under these assumptions about roots and terminal conditions, we may then write

$$\hat{y}(k,t) = -[A + B(F_1)]^{-1}C\hat{Z}(k,t) \tag{17}$$

where $[A + B(F_1)]^{-1}$ is a matrix whose elements are power series in F_1 so that $[A + B(F_1)]^{-1}[A + B(F_1)] = I$. It is easily verified that $x_t = \hat{y}(k-t,t)$ is then a martingale in t.

Substituting (17) into (12) yields, after simplifying:

$$y_t = -[A + B(F_1)]^{-1}C\hat{Z}(0,t) + A^{-1}U_t \tag{18}$$

whish is the desired final form for our model. It shows that y_t depends on the expectation at time t of all future values of the exogenous variable, but not at all on the expected future values of the endogenous variables. The expected values of the endogenous variables are themselves functions of the expected values of the exogenous variables, so they have been solved out. Once we know how $\hat{Z}(k,t)$ depends on I_t, we can also write (18) in terms of I_t. We may constrast (18) with the reduced form of (12):

$$y_t = -A^{-1}B(F_1)\hat{y}(k,t) - A^{-1}C\hat{Z}(k,t) + A^{-1}Ut \tag{19}$$

The final form (18) resembles the reduced form (19) and reduces to it if $B(F_1) = 0$. If $B(F_1) \neq 0$ and we wish to consider the ultimate effects of alternative policy rules which can be translated into changes in parameters or changes in the stochastic structure of Z, then according to the criticisms made by Lucas, Sargent, Wallace and others it is (18) and not (19) we must use. Expression (19) cannot be used to forecast the ultimate effects of policy because $B(F_1)\hat{y}(0,t)$ may change when any parameter or element of Z is changed.

There has been little discussion in the literature as to how an economy reaches the equilibrium (18) after a policy change which changes $\hat{Z}(k,t)$ or after a change in one of the parameters A, B, or C. Cyert and DeGroot [1974] proposed a model in which individuals learn about the parameters in A, B, or C by updating an initial prior distribution in a Bayesian learning procedure. Their model did converge ultimately to the rational expectations solution. However, the model they analyzed was Muth's original model for which no terminal conditions need to be determined. Taylor [1974] examined a simple process which converged on another simple rational expectations equilibrium. It would be interesting, however, to see a learning process which converges on the rational expectations equilibrium of a more general model for which it was necessary to provide terminal conditions in the solution, i.e. in models which allow other explosive solutions.

Until economic theorists come up with a believable story as to how an economy converges on such a rational expectations path in a short period of time these models will be of rather limited interest. There are a lot of other questions we will want to answer before we can rely on such models. We will want to know how sensitive are the forecasts to small errors in the parameters. If they are very sensitive, our rational expectations solution will not be reliable. We will have to confront the obvious possibility that all roots need not lie outside the unit circle. As regards the process to the equilibrium, we will need to have some idea how long this process will take and what may be the cost, in terms of economic distress, on the way to equilibrium.

To use the model, it is important to note that the model (18) can be rewritten to eliminate the expectations of variables extending into the infinite future if we can provide a forecasting rule for Z. Suppose, then, that we know that the forecasts of Z are given by a linear auto-regressive forecasting rule:

$$\hat{Z}(k,t-1) = \sum_{i=1}^{\infty} \phi_i^{(k)} Z_{t-i} \tag{20}$$

where each $\phi_i^{(k)}$, $i = 1,\ldots\infty$ is an m x m matrix. Now premultiply (17) where $k = 0$ by $A + B(F_2)$ where F_2 is the forward operator which applies to the second argument of the function it is applied to. We then have:

$$[A + B(F_2)]\hat{y}(0,t) = -[A + B(F_2)][A + B(F_1)]^{-1}C\hat{Z}(0,t)$$

Now since we know that, formally, the substitution of $F_2 = F_1$ in the expression $[A + B(F_2)][A + B(F_1)]^{-1}$ yields the identity matrix, it must be the case that if we expand the expression $[A + B(F_2)][A + B(F_1)]^{-1}$

in powers of F_2 and F_1, then the sum of the (matrix) coefficients of all rth degree terms $F_2^s F_1^{r-s}$ must be zero if $r > 0$ (and I if $r = 0$). We can write the sum of all rth degree terms T_r, $r \geq d$ as:

$$T_r = R_0^{(r)}F_1^r + R_1^{(r)}F_1^{r-1}F_2 + R_2^{(r)}F_1^{r-2}F_2^2 + \ldots R_d^{(r)}F_1^{r-d}F_2^d$$

where d is the degree of the highest order polynomial in B. Thus, $\sum_{i=0}^{d} R_i^{(r)} = 0$ if $r > 0$. For $0 < r < d$ there will be fewer terms in the expansion, but the sum of the coefficients will remain zero. Since the sum of the coefficients is zero, we can rewrite the T_r as:

$$T_r = R_1^{(r)}F_1^{r-1}(F_2 - F_1) + R_2^{(r)}F_1^{r-2}(F_2^2 - F_1^2) + \ldots R_d^{(r)}F_1^{r-d}(F_2^d - F_1^d)$$

From the generalized error learning principle (7') the product of T_r with $C\hat{Z}(0,t)$ can thus be written in terms of d forecast errors:

$$T_r C\hat{Z}(0,t) = \sum_{i=1}^{d} W_i^{(r)}[Z_{t+d-i+1} - \hat{Z}(d-i+1,t)] \qquad r > 0$$

so that, in turn, the product $[A + B(F_2)]\,[A + B(F_1)]^{-1}CZ(0,t)$ can be written in terms of $\hat{Z}(0,t)$ and the d forecast errors. Then, since $y(t) = \hat{y}(0,t) + A^{-1}U_t$ we can write:

$$[A + B(F)]y_t = -CZ_t + \sum_{i=1}^{d} V_i(Z_{t+d-i+1} - \hat{Z}[d-i+1,t)] + [A + B(F)]A^{-1}U_t \qquad (18')$$

where the coefficients V_i depend on the parameters of A, B, C, and $\phi_i^{(0)}$, $i = 1,\ldots\infty$. Thus y_t is described by a dth order linear difference equation driven by a forcing function which is given in terms of d forecast errors in Z and d error terms U. In other words, one may replace the expectations of future y in (12) by their realizations if one adds the additional variables consisting of forecast errors. The error term then acquires d + 1 order moving average serial correlation.

As a special case, we note that if all forecast errors are zero in (18') then the model reduces to a straightforward rational distributed lag model of y on <u>future</u> values of Z. This result could be seen directly from (18) by noting that, if there are no forecast errors, then $\hat{Z}(k,t) = \hat{Z}_{t+k}$. Incidentally, one should not confuse this case with a case in which all future y's are known. In that case, substituting y_{t+k} for $\hat{y}(k,t)$ onto (12), we get a different result, different in that the error term is not autocorrelated.

How might we estimate the parameters of such a model? There appears to be, surprisingly enough, very little discussion in the literature on the estimation of these rational expectations models. Most of the literature has been purely theoretical. What empirical work has been done has generally merely tested some isolated consequences of the models.

Suppose now that we wish to estimate the parameters of A, $B(F_1)$ and C given the structure (12), some identifying coefficient restraints, and a characterization of the random character of U_t (say that it is spherically normal). If we have data on y_t, $\hat{y}(k,t)$ and Z_t one could estimate (12) directly using standard simultaneous equation methods. As a rule, though, we do not generally have data on $\hat{y}(k,t)$, so this route is generally not of much interest. If we did have survey data purporting to give $\hat{y}(k,t)$ then, on estimating (12), one would of course run the risk that the model would imply that the observed expectations $\hat{y}(k,t)$ are, through (17), irrational. Another possible procedure would be to attempt to find optimal forecasts of future y first, using Box Jenkins' techniques or the like and then substitute these forecasts into (12) as expectations proxies. This procedure is already commonplace. If one does indeed come close to the optimal forecasts first before estimating the model, then the estimate of A, B, and C should be valid. Again, however, we run the serious risk that the optimal forecasts implied by these parameters through (18) will contradict the optimal forecasts with which we started out.

Since we generally wish to assure internal consistency for our model, estimation would better proceed via expression (18) or (18'). Clearly, one must first obtain some characterization of $\hat{Z}(k,t)$, and our model provides no help with this. Suppose, then, that somehow (through Box Jenkins' techniques, for instance, or through other modelling of the exogenous variables or even from survey data) we have obtained the optimal forecasts of future Z, $\hat{Z}(k,t)$. Then the problem of estimating the parameters of (18) or (18') is relatively straightforward. One could derive an expression for the full information maximum likelihood estimate (assuming of course that the parameters are identified).

It should be noted that another procedure, which is essentially one suggested by Sargent and Wallace, will not yield the maximum likelihood estimate. This procedure is to form initial guesses of $\hat{y}(k,t)$, substitute these into (12), and then estimate A, $B(F_1)$, and C. From these estimates, using (17) we could then get revised guesses of $\hat{y}(k,t)$, substitute these into (12) and repeat the process until it converges. Although such a procedure does not yield the maximum likelihood estimate, it will, if it converges, yield a consistent model, so the procedure may have some merit.

Expression (18') suggests another simple strategy. This would be merely to substitute actual $y(t+k)$ into (12) in place of $\hat{y}(k,t)$ and add the additional variables corresponding to the forecast errors in (18').

We might then estimate the parameters A, $B(F_1)$, C, and V_i, i = 1,...d using conventional simultaneous equations methods, and then disregard the coefficients V_i, i = 1,...d. Since the error term in (18') is serially correlated and since lead values of the dependent variables are included in the equations, conventional methods which do not take into account the serial correlation properties of U will be inconsistent. However, if the error terms U_t are small such procedures may be satisfactory.

An important generalization of the model (12) would be a model in which lagged dependent variables occur. Such a model would be especially important for determining the effects of a policy rule which depends on lagged values of the endogenous variables. Alternative policy rules of this nature can then be evaluated by making the policy variable an endogenous variable in the model and considering alternative parameters in the rule. Suppose, then, that our model is:

$$Ay_t + B(F_1)\hat{y}(0,t) + CZ_t + D(L)y_t = U_t \qquad (21)$$

where D(L) is a matrix whose elements are polynomials in the lag operator L which do not include constant terms. Substituting t+k for t and taking expectations with respect to y_t we get:

$$[A + B(F_1) + D(L_1)]\hat{y}(k,t) + C\hat{Z}(k,t) = 0 \qquad k > 0$$

where L_1 is the lag operator on the first argument of the function it is applied to and $L_1F_1 = 1$. This equation is again a difference equation in k which may be solved for $\hat{y}(k,t)$. This time, however, we have initial conditions that $\hat{y}(k,t) = y_{t+k}$ for k < 0.

Another important generalization of the model (12) would be a model in which lagged expectations enter:

$$Ay_t + D(L_2)B(F_1)\hat{y}(0,t) + CZ_t = U_t \tag{22}$$

where L_2 is the lag operator on the second argument of the function it is applied to so that $L_2F_2 = 1$. $D(L_2)$ is a matrix whose elements are polynomials in L_2. As before, we can then replace t in (22) with t+k and take expectations conditional on I_t:

$$A\hat{y}(k,t) + E[D(L_2)B(F_1)\hat{y}(0,t+k)|I_t] + C\hat{Z}(k,t) = \hat{U}(k,t) \tag{23}$$

As before, if $k < 0$, (23) is identical to (22). If k is greater than or equal to the degree d of the highest order polynomial in $D(L_2)$ then using (3), the system (22) reduces to:

$$A\hat{y}(k,t) + D(L_1)B(F_1)\hat{y}(k,t) + C\hat{Z}(k,t) = 0 \tag{24}$$

On the other hand, for values of k greater or equal to zero but less than d, a different system of partial difference equations holds.

Expanding $D(L_2)$ into $D(L_2) = D_0 + D_1L_2 + D_2L_2^2 + \ldots D_dL_2^d$ we get, using (3):

$$A\hat{y}(k,t) + [D_0 + D_1L_1 + D_2L_1^2 + \ldots D_kL_1^k + D_{k+1}L_1^kL_2 + D_{k+2}L_1^kL_2^2 + \ldots D_dL_1^kL_2^{d-k}] \times$$
$$B(F_1)\hat{y}(k,t) + C\hat{Z}(k,t) = 0 \tag{25}$$

The introduction of both L_1 and L_2 in the matrix D is due to the inequality in (3). Expressions (24) and (25) then constitute a system of linear partial difference equations in k and t with variable coefficients. They may be solved for $\hat{y}(k,t)$ and the result substituted into (22) to yield the final form for the model. The solution requires, of course, the specification of terminal conditions in terms of entire functions, not just a finite number of points. The difficulties posed by this problem are sufficiently great, however, that it has generally been ignored in the literature, except for the simple cases discussed by Sargent [1973] and Sargent and Wallace [1973]. In these cases, the authors set d = 1 and made $B(F_1)$ a constant times F_1 so that the problems caused by (25) did not arise. It was then sufficient to solve the ordinary difference equation (24) and then substitute the result into (22) to yield the final form.

A further generalization that would be desirable if we are to discuss believable models in these terms would be to allow for nonlinear models. In some cases it is possible to handle models which are nonlinear in certain variables along lines discussed above. As a general rule, however, nonlinear models do not enable us to reduce the problem to that of a system of difference equations at all - nonlinear difference equations or otherwise. This is another defect of the model, insofar as it has been developed, since clearly most realistic macroeconomic models are likely to have nonlinear terms.

Sargent and Wallace [1973a] have proposed dealing with the problem of nonlinearity simply by linearizing our macroeconomic models. They have, in fact, been devising a log linear econometric model of the U.S. economy for the purpose of applying rational expectations estimation techniques to it. This clearly seems like the wrong approach. Since the real economy is not log linear, not much purpose would be served by deliberately misspecifying the true model so that we can apply our techniques to it. This is especially true of rational expectations estimation techniques, since the concept of a rational expectations equilibrium is such a slippery one. Obviously, the misspecified model will not produce good forecasts of future endogenous variables, so what is the point of requiring internal consistency in the expectations hypothesized by the model?

There are, it should be noted, some cases in which we have simple models which can be represented as linear without doing too much violence to the facts. The best example is probably Sargent and Wallace's model of hyperinflation [1973b]. In general, however, we will have to accept the fact that our models must be nonlinear.

With nonlinear models, the best we can probably do in the spirit of the rational expectations models at the present time is to assure that hypothesized expectations formation mechanisms are not grossly irrational. Thus, we may estimate a "limited information rational expectations" model by using as expectations proxies forecasts of future variables based on Box Jenkins' techniques or other simple forecasting techniques. As was noted above, some existing macroeconometric models have used such procedures. These existing macroeconometric models, however, have not been used as rational expectations theorists would require. In the spirit of the rational expectations theories, we should use these models to compare alternative policy rules without assuming that expectations mechanisms do not change when we change the policy rule. A possible procedure to follow then would be first to simulate the model under the proposed policy rule under the assumption that the expectations mechanism is not changed by the policy rule. One could then use the simulated values of the endogenous variables to check that the expectations mechanism hypothesized is still reasonably rational. For instance, if the hypothesized expectations mechanism was autoregressive, one could then regress the simulated values of the endogenous variables on their own lagged values to see if the coefficients had changed dramatically. If not, then the first simulation would appear to be acceptable. If the coefficients do change a lot, then we might try substituting the new autoregressive scheme estimates from the simulated variables for the original expectations generation mechanism and then run another simulation. If we repeat the procedure and it finally converges so that the auto-regressive scheme estimated from the simulated data is the same as that assumed in generating the data, then we will have found a "limited information rational" expectations equilibrium associated with the different policy rule. This procedure is analogous to the iterative procedure discussed above in connection with linear models.

IV. Conclusion

The literature reviewed here has made criticisms of conventional econometric methodology which can be translated into some concrete suggestions. The suggestions would relate first to the use of existing macroeconometric models and second to the improved estimation of macroeconometric models.

If we assume first that existing macroeconometric models correctly represent the manner by which individuals formed their expectations over the sample period, then we can use these models to evaluate what would have been the effect of alternative policy rules over the sample period under rational expectations assumptions, by replacing the expectations proxies in these models with optimal forecasts. If the existing models are linear of the form (12) where $\hat{y}(k,t)$ are expectations proxies, then this is a simple matter. The evaluation of alternative policies which can be described either in terms of parameter changes in the model or changes in the stochastic structure of Z can then be evaluated through (18). In attempting such a policy evaluation with an existing linear model one runs the risk that, in evaluating the policy which was actually followed over the sample period, the model (18) will not track well, i. e., will not reproduce well the actual values of the endogenous variables. This may mean that the expectations proxies used by the model builders were not close to the true optimal forecasts in terms of the model. Theoretically, such an outcome shouldn't occur if the original model with its expectations proxies tracked well and if the model builders saw to it, as was discussed in the second section of this paper, that their expectations proxies were indeed close to the optimal forecasts. In practice, however, we suspect that many existing models do not use expectations proxies which resemble optimal forecasts, The other suggestions that have been made for estimation of rational expectations models would take into account the relationship of the expectations to the true structure of the model, and would eliminate the possibility that hypothesized expectations proxies are inconsistent with optimal forecasts implied by the model.

Unfortunately, most existing macroeconometric models are nonlinear, and this then leaves us in a theoretical vacuum regarding rational expectations. The best we can do then to meet the objectives outlined by the rational expectations theorists would be to run "limited information rational expectations" policy simulations of the kind described in the preceding section. This amounts to checking to see that the hypothesized expectations proxies do not become grossly irrational under alternative policy rules. We may then compare the long run behavior of alternative policy rules in the manner described in the preceding section without the assumption that expectations mechanisms are unchanging. Such "limited information rational expectations" simulations may be practically valuable to help us to evaluate policy rules, and they can be done with existing macroeconometric models.

We conclude with a couple of caveats.

Firstly, in building econometric models, we should not routinely assume that all expectations are optimal forecasts. We can, in fact, probably identify which behavioral relations involve rational expectations and which do not. For instance, expectations which affect corporate investment are more likely to be optimal forecasts than are expectations which affect consumer durable investment. The tendency to ascribe too much "rationality" to individuals has been a common error in this literature.

If we need a justification for the failure of individuals to forecast optimally, this can be had by noting that forecasting itself has costs. Feige and Pierce [1974] have formalized this idea with the concept of "economically rational" expectations. In their view, forecasts by individuals confronted by big decisions (e. g., whether to build a new plant) will be more nearly optimal than forecasts by individuals making small decisions (e. g., whether to buy a new television set). As was noted above, we can incorporate "irrational expectations" within the structure outlined in this paper, or we can incorporate "partly rational" expectations which are weighted averages of optimal forecasts and suboptimal forecasting rules.

Secondly, model builders should not become excessively concerned with the constraints that internal consistency in rational expectations imposes on their models. The problems we have modelling expectations are only a part of the problem we face in trying to gauge the ultimate effects of our policy rules. The internal consistency constraint for expectations has probably received undue attention simply because it produces a neat mathematical structure of the kind economic theorists like to work with. Moreover, there are still deep problems that have been brushed aside by theorists on their way to rational expectations models. How does an economy reach a rational expectations equilibrium? How long does it take to reach it? What are the costs in terms of economic disturbances of a transition to rational expectations? How are the terminal conditions necessary for a solution to the difference equations determined? How likely is it that the roots of the characteristic equation lie outside the unit circle, and what do we conclude if they do not? How sensitive are the optimal forecasts to certain small parameter changes in the model? All these problems together mean that, for the present at least, rational expectations models, except possibly for the "limited information rational expectations" models, are properly regarded as interesting additions to our ways of viewing the economy, and are not ready to replace our conventional models.

References

Barro, Robert J., "Rational Expectations and the Role of Monetary Policy" unpublished manuscript, University of Chicago, 1975.

Cyert, Richard M. and Morris H. DeGroot, "Rational Expectations and Bayesian Analysis" Journal of Political Economy 82:521-36, 1974.

deMenil, George, "The Rationality of Popular Price Expectations", unpublished paper, Princeton University, 1974.

Diller, Stanley, "Expectations and the Term Structure of Interest Rates", in J. Mincer, ed., Economic Forecasts and Expectations, New York, 1969.

Doob, J. L., Stochastic Processes, Wiley, New York, 1953.

Fama, Eugene F., "Efficient Capital Markets: A Review of Theory and Empirical Work", Journal of Finance 25:383-417, 1970.

_____, "Short-Term Interest Rates as Predictors of Inflation", American Economic Review 65:269-82, 1975.

Feige, Edgar L and Douglas K Pierce, "Economically Rational Price Expectations" unpublished paper, University of Houston, 1974.

Friedman, Benjamin M., "Rational Expectations are Really Adaptive After All", unpublished manuscript, Harvard University, 1975.

Gordon, Robert H., "Can Econometric Policy Evaluation be Salvaged?" forthcoming in a volume edited by Karl Brunner, North Holland.

Grossman, Sanford, "A Bayesian Approach to Static Stochastic Equilibria", unpublished paper, University of Chicago, 1973.

_____, "Self-Fulfilling Expectations and the Theory of Random Markets" unpublished paper, University of Chicago, 1973.

Kemeny, John G., J. Laurie Snell and Anthony Knapp, Denumerable Markov Chains, 1966.

Lucas, Robert E., "Econometric Policy Evaluation: A Critique" 1973 (a) forthcoming in a volume edited by Karl Brunner, North Holland.

_____, "Econometric Testing of the Natural Rate Hypothesis" 1970 in Otto Eckstein, ed., The Econometrics of Price Determination Conference Board of Governors of the Federal Reserve System, Washington, 1972.

_____, "Expectations and the Neutrality of Money" Journal of Economic Theory 4:103-4, 1973 (b).

_____, "Some International Evidence on Output Employment Tradeoffs", American Economic Review, 1973 (c).

Lucas, Robert E. and Edward C. Prescott, "Equilibrium Search and Unemployment", Journal of Economic Theory, 7:188-209, 1974.

Mandelbrot, Benoit, "Forecasts of Future Prices, Unbiased Markets and Martingale Models", Journal of Business 39:242-55, special supplement, 1966.

Meiselman, David, The Term Structure of Interest Rates, Englewood Cliffs N. J. 1962.

Mincer, Jacob, "Models of Adaptive Forecasting" in Mincer, ed., Economic Forecasts and Expectations, New York, 1969.

Modigliani, Franco, and Robert Shiller, "Inflation, Rational Expectations and the Term Structure of Interest Rates", Economica 40:12-43 1973.

Modigliani, Franco, and Richard Sutch, "Debt Management and the Term Structure of Interest Rates", Journal of Political Economy 75: 569-89, 1967.

Muench, Thomas, and Neil Wallace, "On Finding a Good Stabilization Policy: Models and Methods", unpublished manuscript, University of Minnesota, 1974.

Muth, J. F., "Rational Expectations and the Theory of Price Movements", Econometrica 29:315-35 1961.

Nelson, Charles R., "A Critique of Some Recent Empirical Research in the Explanation of the Term Structure of Interest Rates: Comment" in Journal of Political Economy 78: 764-7, 1970.

_____, "Testing a Model of the Term Structure of Interest Rates by Simulation of Market Forecasts", Journal of the American Statistical Association, 65:1163-90, 1970.

Prescott, Edward C., and Finn Kydland, "Optimal Stabilization Policy: A New Formulation" unpublished paper, Carnegie Mellon University, 1974.

Roll, Richard, "Rational Response to the Money Supply", Journal of Political Economy 587-97, 1974.

Rutledge, John, A Monetarist Model of Inflationary Expectations, Lexington B ooks, 1974.

Samuelson, Paul A., "Proof that Properly Anticipated Prices Fluctuate Randomly" 1965, in R. C. Merton, ed., Collected Scientific Papers of Paul A. Samuelson, vol. 3, ch. 98, pp. 782-90, M. I. T. Press, Cambridge, 1972.

_____, "Proof that Properly Discounted Present Values of Assets Vibrate Randomly", Bell Journal of Economics and Management Science, 1973, 368-74.

Sargent, Thomas J.,"Rational Expectations and the Term Structure of Interest Rates", Journal of Money, Credit and Banking, 1972.

Sargent, Thomas J., "Rational Expectations, the Real Rate of Interest and the Natural Rate of Unemployment" Brookings Papers on Economic Activity, 1973, pp. 429-72.

_____, "Testing for Rationality and Neutrality", unpublished paper, University of Minnesota, Minneapolis, 1974.

Sargent, Thomas J., and Neil Wallace, "Notes on Loglinear Macroeconomic Models", unpublished paper, University of Minnesota, 1973 (a).

_____, "Rational Expectations and the Dynamics of Hyperinflation", International Economic Review 14:328-50, 1973 (b).

_____, "Rational Expectations and the Theory of Economic Policy", unpublished manuscript, University of Minnesota, 1974.

_____, "Rational Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule", Journal of Political Economy 83: 241-55, 1975.

Shiller, Robert J., "Omitted Variables in an Expectations Model of the Term Structure of Interest Rates", unpublished paper, University of Minnesota, 1973 (a).

_____, Rational Expectations and the Structure of Interest Rates, unpublished Ph.D. dissertation, Massachusetts Insititute of Technology, 1972.

_____, "Rational Expectations and the Term Structure of Interest Rates" Journal of Money, Credit and Banking, 1973 (b).

Sutch, Richard C., "Expectations, Risk and the Term Structure of Interest Rates, unpublished Ph.D. dissertation, Massachusetts Insitute of Technology, 1968.

Taylor, John, "Monetary Policy during Transition to Rational Expectations" unpublished manuscript, Columbia University, 1974.

Walters, A. A., "Consistent Expectations, Distributed Lags and the Quantity Theory", Economic Journal 81:273-81, 1971.