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ON THE THEORY OF PRODUCTIVE SAVING

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Considerable attention has been devoted in the economic literature to capital accumulation and savings. Little systematic attention, however, has been given to the actual process of saving, by which is meant the generation of nonwage income through the productive management of accumulated stocks of nonhuman assets. Such activity generally has been thought of as a "neutral" process which does not claim the use of scarce resources. At least, the returns from saving activity have been considered independent of the amount of resources foregone in its pursuit. The central thesis of this paper is that the management of portfolios incorporating a variety of investment assets does require the use of time and other scarce resources in searching for, collecting, interpreting, and applying relevant information. Accordingly, the returns on these assets would depend, in part, on managerial efforts and abilities and other related inputs.

The view that the generation of nonwage returns by individuals is a neutral process may be rooted in Irving Fisher's theory of interest which largely abstracted from differences in rates of interest at a point in time in order to focus attention on the determinants of an overall, unique market rate of interest. It has been indirectly defended in the contemporary economic literature as an implication of so-called "efficient" capital markets. In an ideal or efficient market, is is argued, all the relevant information concerning the productivity of assets is <u>fully</u> impounded in the market prices of these assets at any given point in time.[1] No gains are to be had by individual investors from search or from other management efforts in such a market. Specifically, expected nonwage income may vary across assets as a function of the "risk" indigenous to these assets but otherwise is expected to be strictly proportional to the amount of capital invested.

The condition that prices of all capital assets fully reflect all available information at every point in time could be fulfilled only if information were freely available or perfectly forecast by all investors and if all capital markets were free of transaction costs. Since none of these conditions literally are fulfilled in practice, search and management of portfolios could be considered unproductive to individual investors only if all had equally ready access to relevant information and the same opportunities to implement it. Because relevant information concerning the productivity of assets is costly to obtain and transaction costs slow the process of price adjustment, investors who can collect and apply new information at relatively low cost or stand to gain more through such activity would be willing to sacrifice sufficient resources to secure added returns. This hypothesis, though inconsistent with the notion that capital markets are "perfect," is not inconsistent with the notion that they are "efficient" in the sense of providing competitive rewards for superior ability and resource expenditures in the management of assets.

Although, in principle, the productive saving hypothesis applies to all capital markets, practically, its importance may vary widely across different markets. In the centralized markets for common stocks and other highly substitutable securities, the speed of price adjustments to newly available information may be much faster than in markets for real estate assets and unincorporated businesses, although even in the stock market adjustments are not instantaneous.[2] Furthermore, investors who secure superior information concerning the prospects of specific securities would have an incentive to appropriate its full value by selling it to others. The opportunities for selling superior information presumably are constrained, however, by added search and transaction costs to both seller and buyers. In addition, selling the information to all investors simultaneously would preclude differential returns to any one investor. Thus, one cannot rule out a priori differential returns to better-informed investors--especially those specializing in the collection of information--even in the markets for securities. Moreover, returns for superior management of capital assets may be more sizable in connection with the management of equity in unincorporated businesses (including farms), real estate operations, mortgage assets, and other loans. Markets for such assets are more segmented as the assets themselves are more differentiated, less divisible, and relatively costly to trade. Conceivably, then, the speed of adjustment of stock and rental values to changes in market conditions is relatively slow for these types of assets, and, consequently, the opportunities for gains from search and management efforts may be more abundant.[3] In contrast, in the markets for short-term government notes and other "safe" assets, where yields are relatively fixed or highly predictable, returns to management efforts may be low.

Whether saving activity pays or is important in explaining saving behavior ultimately is an empirical issue. Tests of the efficient markets hypothesis in its strong form, namely that no distinct investor or groups of investors have superior information and achieve differential returns, to our knowledge, have been conducted exclusively in connection with the markets for securities. Even there a number of tests yielded results clearly at odds with that hypothesis.[4] It may also be argued that the development and rapid growth of such occupations as portfolio managers and investment specialists is not inconsistent with the notion that opportunities for gains from productive management of assets do exist in practice. While the productive saving hypothesis has potential implications concerning the relative magnitude of returns from search in specific capital markets, our main interest is to derive a more general set of testable implications relating to the productive management of assets. These implications concern allocations of consumptive and productive resources throughout the life cycle and saving behavior in general.

The productive saving hypothesis extends a line of thought offered by Alfred Marshall in connection with his distinction between "gross" and "net" interest on capital. While recognizing that gross interest typically includes some insurance against risk, Marshall also stressed that it includes "earnings of management of a troublesome business" and that in some cases it may consist almost entirely of "earnings of a kind of work for which few capitalists have a taste."[5] But whereas Marshall ascribed the willingness to engage in the kind of work underlying the management of certain assets basically to peculiar individual preferences, we propose that an investor's willingness to devote resources to the management of capital assets is negatively related to his management costs and positively related to his expected gains determined, in part, by his specific training and experience. Details of these assumptions, along with some conventional assumptions concerning the utility of lifetime consumption and bequest, form the basis for the development of a life cycle model of consumption, work, and saving activity.

To simplify matters, the life cycle model developed abstracts from an explicit analysis of the joint accumulation of human and physical capital by treating human capital endowments as exogenously determined. It also abstracts from any analysis of nonneutral attitudes toward risk and from variations in the demand for leisure in the context of intertemporal decision-making. The basic implications of the model, however, are expected to hold under more general conditions. These implications concern the interdependencies between consumption and saving activities, on the one hand, and the allocation of individual working time between conventional work and productive saving on the other.

The plan of the paper is as follows. A life cycle model of consumption and productive saving without borrowing is developed in Section I. Borrowing is introduced into the model and its relationship to productive saving is explored in Section II. In Section III we attempt to elucidate the model's implications concerning capital accumulation paths and life cycle variations in resource allocations to productive activities. Implications regarding the determinants of the propensity to save are derived in Section IV and then briefly examined in light of some earlier theoretical and empirical findings.

I. A Life Cycle Model of Consumption, Work, and Productive Saving

To introduce systematically the novel implications of the productive-saving hypothesis, we consider a simple model of the allocation of time and goods over the life cycle in which time can be used in only two income generating activities: conventional work and capital management, or saving. We assume that the consumption unit's horizon consists of n equal "periods" over which it wishes to allocate its productive and consumptive resources so as to maximize the utility of the overall consumption plan:

$$U = U(X_1, X_2, \ldots, X_n, K_n) \,. \tag{1.1}$$

In equation (1.1), X_t represents the expected stock value of a nondurable composite market good consumed in period t, K_n is the terminal stock of physically nondepreciable nonhuman capital assets (measured in units of X) that are bequeathed or carried over beyond the planning horizon, and U is a function that converts stocks of dated goods and terminal wealth into consumption flows and utility. The consumption unit is assumed to have a neutral attitude toward risk. Consequently, it is assumed to behave as if all the expected variables pertinent to the production and consumption plans are known with certainty. For methodological simplicity, it is also assumed that the price of X is constant at unity throughout the planning horizon and that no direct costs are entailed in transforming X into K.

The basic constraints limiting the consumption unit's total consumption and bequest opportunities generally can be identified with its initial endowments of human and nonhuman capital, H_0 and K_0, and the total amount of time it can devote to the generation of wage and nonwage income in any period during the life cycle. We assume that productive pursuits are exhausted by work and saving. Therefore, abstracting from any effects of "aging" on productive capabilities (any depreciation of human resources), human capital is fixed at its endowed level H_0. Assuming that the rental value of one unit of time spent at work is a single-valued function of the stock of human capital, wage income in any period would depend only on working time,

$$E_t = w^0 \ell_t \qquad t = 1, \ldots, n , \tag{1.2}$$

where $w^0 = w(H_0)$ is a constant real wage rate defined in terms of the composite good X, and ℓ_t denotes the amount of time expended in wage-generating activities (work).

Expected nonwage income can be written as the product of the accumulated stock of nonhuman capital in the beginning of a given period K_{t-1} and the expected one-period gross rate of return: the expected interest, dividends, royalties, other rental income, and capital gains or losses yielded per unit of capital, net of the direct costs of <u>purchased</u> inputs including brokers' fees and other transaction costs but inclusive of the value added through own efforts. That is,

$$R_t(h_t, H_0, K_{t-1}) = K_{t-1}\, r(h_t, H_0) , \tag{1.3}$$

where h_t is time spent at saving in period t, with $r'(h) > 0$ and $r'(H_0) \geq 0$. Our basic thesis is that the expected gross rate of return to capital, r_t, or the expected private lending rate of interest, is not a fixed yield but rather a continuously increasing and twice differentiable function of saving time, h,[6] and the consumption unit's stock of human capital--at least that part of its human capital that is complementary to the management of assets.

The production function of nonwage income specified in equation (1.3) shows expected nonwage income to be strictly proportional to K_{t-1} at any given level of h_t. While this assumption may be largely valid with regard to the markets for securities on the grounds that individual portfolios are not large enough to affect the market prices of securities in their possession, it may not always hold in connection with the management of, say, mortgage assets and rentals of property. An increase in the amount of capital invested in these latter assets may require more search and own-supplied management efforts in order to maintain a given gross rate of return even though diminishing returns to managerial efforts may be suppressed to some extent through adjustments in the amounts of hired factors of production. Moreover, retaining such factors at relatively low levels of portfolio size may be uneconomical. More generally, then, the expected rate of returns may be written as $r(h_t, H_0, K_{t-1})$ with $r'(K_{t-1}) \lesseqgtr 0$. We assume, however, that capital and saving time are complements in the production of nonwage income (see n. 9) and that the magnitude of $r'(K_{t-1})$ at given levels of h_t is negligible over a wide range of portfolio size (see n. 10).

The total amount of resources available to a consumption unit for consumption and nonhuman capital holding in any given period is generally the sum of its nonhuman net worth in the beginning of the period and the stock value of the wage and nonwage income accruing to it during the period. Initially, the analysis will proceed on the assumption that the consumption unit engages in no borrowing. The capital constraint for a given period then can be stated in terms of the requirement that total outlays on goods plus the amount of capital accumulated at the end of the period just exhaust the amount of resources available. That is,

$$E_t + R_t + K_{t-1} = X_t + K_t \, , \tag{1.4}$$

or, substituting equations (1.2) and (1.3) in equation (1.4),

$$w^0 \ell_t + K_{t-1}\, \rho_t(h_t) = X_t + K_t \qquad t = 1, \ldots, n \, , \tag{1.5}$$

where

$$\rho_t(h_t) = 1 + r(h_t) \, .$$

The problem becomes that of maximizing equation (1.1) subject to the n one-period capital constraints given in equation (1.5), n time constraints,

$$T_0 = \ell_t + h_t \qquad t = 1, \ldots, n \, , \tag{1.6}$$

and the initial endowments of the individual's nonhuman and human capital,

$$K(0) = K_0 \, ; \qquad H(0) = H_0 \, . \tag{1.7}$$

A more illuminating formulation of the problem can be achieved through an inductive solution of the set of equations (1.5) in terms of the initial endowment of nonhuman capital given in equation (1.7). This is permissible on the assumption that no borrowing for consumption purposes is desired by the consumption unit so that none of the single period constraints is binding. Using equation (1.6), the resulting overall wealth constraint is[7]

$$\begin{aligned} W_n &\equiv \sum_{t=1}^{n} \ell_t w^0 \prod_{i=t+1}^{n+1} \rho_i(T_0 - \ell_i) + \prod_{t=1}^{n+1} \rho_t(T_0 - \ell_t) K_0 \\ &= \sum_{t=1}^{n} X_t \prod_{i=t+1}^{n+1} \rho_i(T_0 - \ell_i) + K_n \, , \end{aligned} \tag{1.8}$$

where $\rho_{n+1} \equiv 1$.

Equation (1.8) is just a variant of the Fisherian wealth constraint: it equates the total expenditures on goods and bequest evaluated in terms of period n "dollars" with the similarly evaluated future value of the consumption unit's wage and nonwage receipts. Unlike the Fisherian model, which assumes identical market discount rates for all individuals, this model allows the private lending rates of interest to vary across different consumption units and over time according to the extent of individuals' participation in and productivity at saving activities.

Forming the Lagrangian function:

$$L(\underset{\sim}{X}, K_n, \ell) = U(X_1, \ldots, X_n, K_n) + \lambda\left(\sum_{t=1}^{n} \ell_t w^0 \prod_{i=t+1}^{n+1} \rho_i + \prod_{t=1}^{n} \rho_t K_0 - \sum_{t=1}^{n} X_t \prod_{i=t+1}^{n+1} \rho_i - K_n\right), \tag{1.9}$$

the set of first-order optimality conditions for internal solutions involving positive values of all the control variables is given by[8]

$$MU(X_t) - \lambda \prod_{i=t+1}^{n+1} \rho_i(h_i) = 0 \qquad t = 1, \ldots, n , \tag{1.10a}$$

$$MU(K_n) - \lambda = 0 , \tag{1.10b}$$

$$\lambda\left\{w^0 \prod_{i=t+1}^{n+1} \rho_i + \sum_{k=1}^{t-1} (\ell_k w^0 - X_k) \prod_{j=k+1}^{t-1} \rho_j \prod_{i=t+1}^{n+1} \rho_i [-r'(h_t)] + K_0 \prod_{k=1}^{t-1} \rho_k \prod_{i=t+1}^{n+1} \rho_i [-r'(h)]\right\} = 0 \qquad t = 1, \ldots, n , \tag{1.10c}$$

and

$$W_n - \sum_{t=1}^{n} X_t \prod_{i=t+1}^{n+1} \rho_i (h_i) - K_n = 0 . \tag{1.10d}$$

Given an optimal allocation of productive time between work and saving, the optimal plan of consumption and terminal capital must satisfy the set of $n + 1$ equations summarized above or, alternatively, the n independent equations

$$\frac{MU(X_t)}{MU(X_{t-1})} = \frac{1}{\rho(h_t)} \qquad t = 2, \ldots, n , \tag{1.11}$$

and

$$\frac{MU(K_n)}{MU(X_n)} = 1 . \tag{1.11a}$$

The latter set of equations reproduces the familiar result that the ratios of the marginal utilities of consumption goods in different time periods must be the same as the ratios of their respective marginal costs: the terms at which alternatively dated (otherwise identical) consumption claims can be traded between any two periods. If borrowing for consumption is undesired, these terms of trade are determined by "lending" or productive saving opportunities. Equation (1.11a) shows that the optimal level of terminal capital (measured in terms of the composite good X) must yield the same marginal utility as consumption in the final period of the planning horizon since this capital can be exchanged with similarly dated consumption claims at equal marginal costs.

In turn, given an optimal consumption-bequest plan, an optimal allocation of productive time between work and saving in any given period must satisfy the set of equations summarized in (1.10c). Dividing this equation through by the price of consumption in period t, $\prod_{i=t+1}^{n+1} \rho_i$, and noting that by definition (see n. 7), $\sum_{k=1}^{t-1} (\ell_k w^0 - X_k) \prod_{j=k+1}^{t-1} \rho_j + K_0 \prod_{k=1}^{t-1} \rho_k = K_{t-1}$, the necessary condition for optimal participation in work and saving involving positive values of ℓ_t and h_t can be rewritten as

$$w^0 = K_{t-1}\, r'(h_t) \qquad t = 1, \ldots, n . \tag{1.12}$$

Equation (1.12) is the familiar factor employment equation of general price theory. It states that an optimal allocation of time between work and saving at any given period in which there is positive participation in each activity can be achieved only if the marginal return to time is identical in both. The sufficient condition that such an allocation of productive time will maximize the total expected return from employment (given the level of initial capital and the constancy of the wage rate) is that there be diminishing marginal productivity of saving time, or $r''(h) < 0$. Figure 1 illustrates these conditions graphically. With the assumption that saving time and nonhuman capital are complements in the generation of nonwage income and given a constant opportunity cost of time, an immediate implication is that the optimal extent of self-employment in saving activities, except in cases of specialization in work or saving activities, would be an increasing function of the initial amount of accumulated capital that summarizes the results of earlier productive and consumptive resource allocations (see Figure 2).[9] *A fortiori*, given w and K, any decrease in the marginal productivity of management time due to exogenous factors operating in capital markets would increase individuals' demand for conventional market activities relative to time in productive saving.

These results may be modified to some degree if markets for capital provided opportunities for substitution between own management of portfolios and the services of hired specialists. However, we expect these opportunities to be imperfect in practice. In the first place, inasmuch as specialists differ in abilities and the information they possess regarding various assets at different points in time, investors would have an incentive to search for productive managers. In addition, transaction costs incurred in hiring specialists' services may make their retention uneconomical especially when the portfolio of capital and expected benefits to the owner are relatively small.

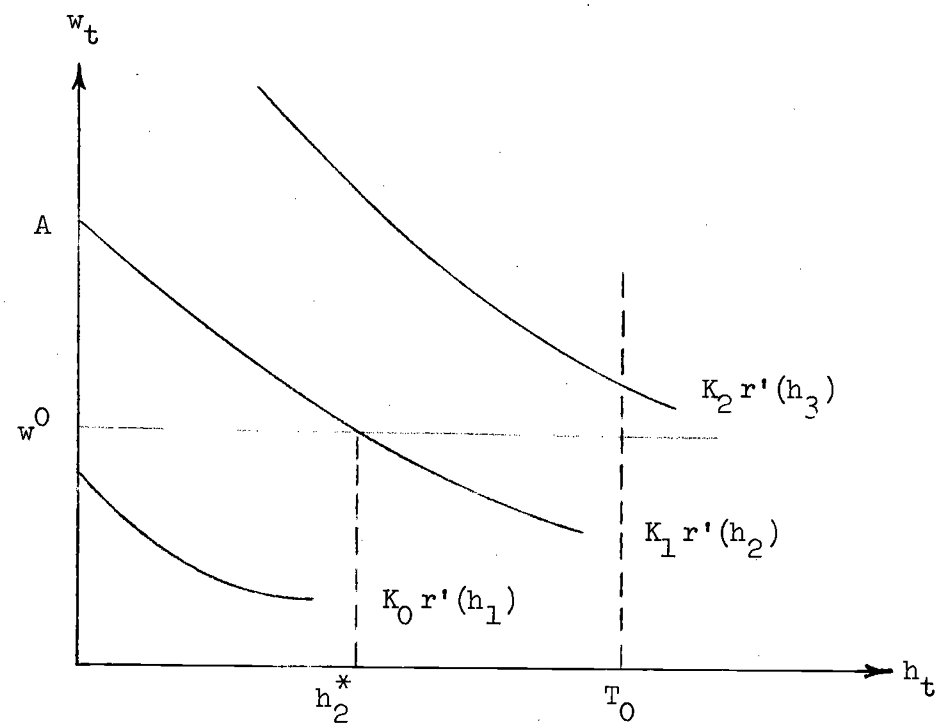

Figure 1

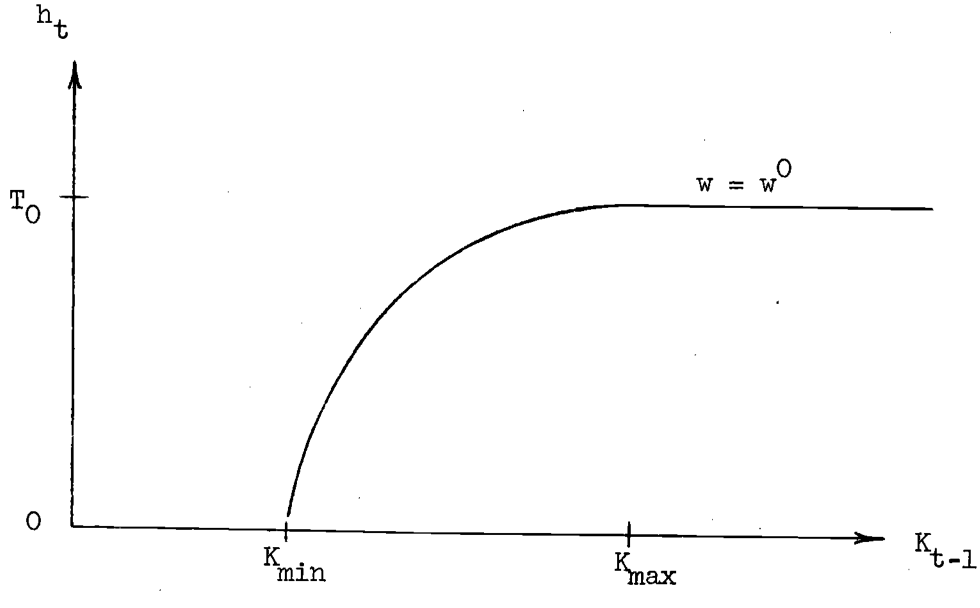

Figure 2

Moreover, a complete reliance on specialists generally would be optimal only if specialists' services could be considered as perfect substitutes to own productive saving in all relevant aspects of saving. But specialists need not have the same incentive for prudence in the management of owners' assets as have the owners themselves. Thus both factors of production may be used in managing individual portfolios in accordance with cost minimizing principles. As long as own entrepreneurial services and specialists' services were complementary in the production of nonwage returns, the employment of both may be expected to rise as individual portfolio sizes rise. Formally, the analysis of optimal employment of hired factors of production is similar to the analysis of borrowing for productive saving that is considered in the following section. Both are undertaken so as to maximize wealth. Since services rendered by hired factors are expected to raise the lending rate of interest at any given level of own saving activity, their optimal employment is expected to increase monotonically with portfolio size even when the consumption unit specializes in productive saving, i.e., when $h_t/T_0 = 1$.

The interaction between productive and consumptive allocations of resources and the stability conditions underlying these allocations can be illustrated graphically by means of conventional, two-period analysis. Let the consumption unit's horizon consist of two periods and let the desired bequest be zero. The endowment position represented by point C in Figure 3 represents the amounts of resources available for consumption in periods 1 and 2 in the absence of any intertemporal capital transfers. These are I_1 and $I_2 = w^0 T_0$, respectively. Consumption of goods can be traded between the two periods via the transformation curve:

$$X_2 = \rho_2(h_2)(I_1 - X_1) + w^0(T_0 - h_2) , \tag{1.13}$$

where the difference $I_1 - X_1 \equiv K_1$ denotes the amount of capital accumulated at the end of period 1. The slope of this curve is readily found to equal ρ_2.

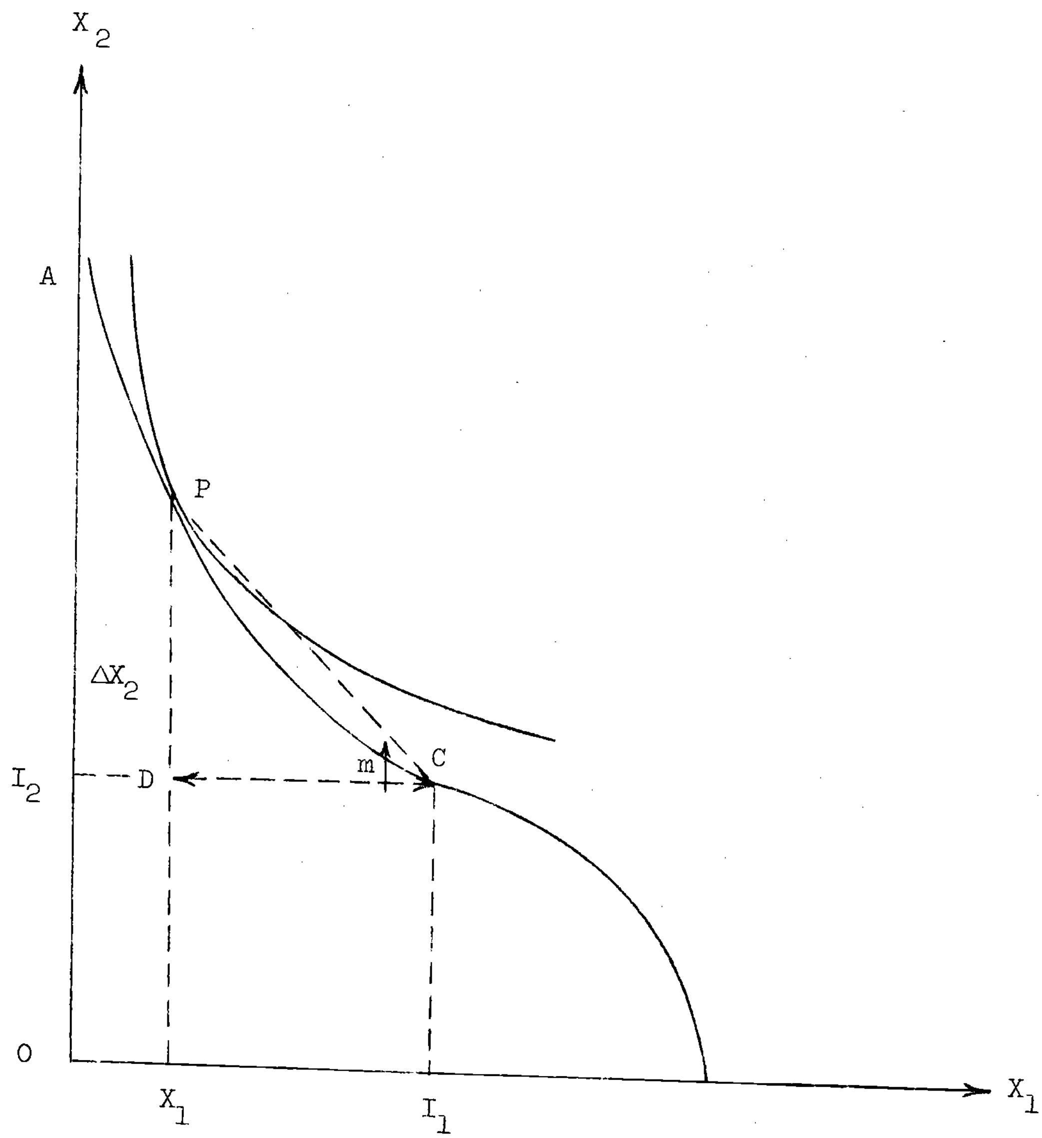

Figure 3

Formally:

$$- \frac{dX_2}{dX_1} = \frac{dX_2}{dK_1} = [\rho'(h_2)K_1 - w^0] \frac{dh_2}{dK_1} + \rho_2(h_2(K_1)) .$$

But since by equation (1.12), $\rho'(h_2)K_1 = w^0$, then

$$- \frac{dX_2}{dX_1} = \rho_2(K_1) , \tag{1.14}$$

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which duplicates the equilibrium condition for intertemporal allocation of consumption given in equation (1.11). The important implication of this result is that the relevant rate of interest affecting intertemporal consumption decisions is Marshall's <u>gross</u> interest. It is not the net yield. One plus the net yield per unit of capital, $\rho(h_t) - \frac{w^0 h_t}{K_{t-1}}$, is the slope, m, of the straight line connecting points P and C in Figure 3, or the average terms of trade between consumption in consecutive periods. What are relevant for consumption and savings decisions, however, are the marginal terms of trade that are determined by the (expected) gross rate of return inclusive of the value added through the investor's own efforts. Since, in general, the gross rate of return is expected to be positively related to the amount of capital accumulated, the transformation curve AC in Figure 3 is expected to be convex toward the origin (see n. 10). The sufficient condition for a stable equilibrium involving productive saving therefore requires that the marginal rate of substitution in consumption between X_2 and X_1 rise faster than the corresponding marginal rate of substitution in production as more of X_1 is exchanged for X_2 through productive saving. The shape of the segment of the consumption transformation curve that involves borrowing for consumption will be discussed in Section II below.

The behavioral content of the model can be sharpened at the expense of some simplifying assumptions concerning the form of the utility function. Let

U in equation (1.1) be a homothetic function. For illustration, we choose a specific form of a homothetic function such that

$$\frac{MU(X_t)}{MU(X_{t-1})} = \alpha_t\left(\frac{X_{t-1}}{X_t}\right)^{1/\sigma_t} , \qquad t = 2, \ldots, n , \tag{1.15}$$

where σ_t denotes a constant elasticity of substitution between X_t and X_{t-1}, and τ_t, which is to be defined by the identity $\tau \equiv 1 - \alpha_t$, is an index of time preference for X_{t-1} relative to X_t.11/ Similarly, it is assumed that

$$\frac{MU(K_n)}{MU(X_{t-1})} = \alpha_{kn}\left(\frac{X_n}{K_n}\right)^{1/\sigma_{kn}} , \tag{1.15a}$$

where σ_{kn} denotes the elasticity of substitution between K_n and X_n, and τ_{kn}, to be defined by the identity $\tau_{kn} \equiv 1 - \alpha_{kn}$, is an index of the consumption unit's preference bias in period n toward own consumption relative to its retirement fund or bequest. By introducing equations (1.15) and (1.15a) into equations (1.11) and (1.11a), it now can be shown that the optimal rate of growth of the composite good X over the life cycle and the corresponding optimal level of bequest relative to consumption in the last period of the planning horizon would be given by

$$\log \frac{X_t}{X_{t-1}} \approx \sigma_t(r(h_t) - \tau_t) \qquad t = 2, \ldots, n \tag{1.16}$$

and by

$$\log \frac{K_n}{X_n} \approx \sigma_{kn}(-\tau_{kn}) \ . \tag{1.16a}$$

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The homotheticity of the utility function for lifetime consumption and bequest generally implies that an increase in the gross rate of interest would raise the optimal ratio of later to earlier consumption claims in consecutive periods. More specifically, given the weakly separable C.E.S. utility function considered in the preceding illustration, the direction as well as the extent of the growth of consumption over time are shown by equations (1.16) and (1.16a) to be dependent essentially on the difference between the consumption unit's gross lending rate of interest and its time preference for "present" or own consumption relative to "future" consumption or bequest. Thus, if the consumption unit had neutral time preferences in all periods and the lending rate of interest were always positive, then consumption of goods would increase continuously over the planning horizon and peak at the final period. Moreover, since by assumption the lending rate of interest is positively related to the level of saving activity, so would be the optimal rate of growth of consumption over time, and thus, indirectly, the <u>absolute</u> size of bequest also would relate positively to saving activity.

The preceding analysis generally indicates the existence of interdependencies between consumption-saving decisions and work-saving decisions. Under given labor market opportunities, participation in saving activity is seen to be monotonically related to the amount of accumulated capital. In turn, the extent of participation in saving activities and the resulting rates of return on capital would, along with time preferences, determine the rate of growth of consumption over time and, consequently, the rate and direction of capital accumulation that are necessary to effect the desired consumption plan. The analysis indicates that when saving activity is viewed as a productive process, consumption decisions concerning the intertemporal allocations of goods cannot be separated from production decisions concerning the maximization of wealth and that the separation theorem of conventional models of intertemporal consumption and production decisions, asserting the independence of the two, cannot be considered valid.[13] Another apparent implication of this analysis, which is more fully developed in Section IV below, is that different consumption units--even equally wealthy ones--may have different consumption paths not necessarily because of differences in subjective discount rates or because of a systematic association between time preferences for consumption and wealth, but because of differences in their gross rates of return to savings due to differences in their initial endowments of nonhuman and human capital and abilities.

II. Productive Saving and Borrowing

For the sake of a simple yet general introduction of borrowing opportunities into our choice theoretic framework, we make the following assumptions concerning the market for borrowing. We assume that borrowing, like consumption and the augmentation of capital assets, occurs at the end of a standard period and that principal and interest payments mature and are planned to be paid at the end of the succeeding period. The length of a standard borrowing period is identified with the length of a single period of the consumption unit's planning horizon.[14] We further assume that the marginal borrowing cost of capital is a function of the amount borrowed, D, and the borrower's net worth, N. That is, in symbols, $MbC = MbC(D_{t-1}, N_{t-1})$, with $MbC'(D_{t-1}) > 0$ and $MbC'(N_{t-1}) \leq 0$.[15] The logic behind the productive saving hypothesis suggests that borrowing costs also should be influenced by resource expenditures by the borrower in search for less expensive sources of funds. We shall discuss this possibility later in this section. Initially, borrowing is viewed as an activity that does not consume resources.

A consumption unit can borrow for two fundamentally distinct purposes. One is to finance current consumption of nondurable goods.[16] The other is to augment the amount of income generating assets under its command. We call the second borrowing for productive saving. Clearly, borrowing for productive saving is inconsistent with the simple Fisherian model of saving which assumes that the lending and borrowing interest rates are identical. It also is inconsistent with differing, though constant, lending and borrowing rates since then the latter must always exceed the former. Under the assumptions of rising marginal borrowing costs, however, such borrowing is compatible with productive saving activities since the expected gross rate of return on assets is assumed to be an increasing function of saving activity.

Regardless of whether productive saving is own financed or funded by borrowing, borrowing for consumption is not expected to be undertaken simultaneously with productive saving activity. It would not pay the consumption unit to borrow funds to finance consumption purchases prior to tapping its own capital assets because the cost of borrowing funds at the margin necessarily exceeds the return from lending foregone if own, rather than (additionally) borrowed, capital is used. This is readily seen when no borrowing for productive saving is optimal. Then the marginal gross return on own capital must fall short of the minimum cost of borrowing funds. Moreover, if the consumption unit has already undertaken positive borrowings for productive saving, then borrowing additional funds for consumption cannot be optimal since, as will be shown later, optimal borrowing for productive saving requires equality in equilibrium between the marginal borrowing cost of capital and the gross rate of return on that capital.[17/] Thus, we derive the important implication that any borrowing can be considered borrowing for productive saving as long as the consumption unit has positive holdings of nonwage income generating capital assets (excluding emergency funds that may be subsumed under consumption expenditures). The following analysis indicates that the relevant rate of interest for intertemporal consumption decisions then would be identified with the gross (lending) rate of interest on own capital.

The preceding discussion suggests that when borrowing opportunities are available to the consumption unit, the resource constraint limiting consumption and capital holding in each period can be specified in either of two forms depending upon whether borrowing is done for the purpose of consumption or for productive saving. In the first case, the one-period resource constraint is given by

$$w^0 \ell_t + D_t = X_t + \delta_t D_{t-1} , \tag{2.1}$$

where $\delta_t \equiv 1 + b_t$ indicates the average return to the lending institution, $\ell = T_0$, and D_t may be positive, negative, or nil. The terms of trade between pairs of consumption goods in different periods would be determined by the marginal borrowing cost of capital, which is expected to rise with the scale of borrowings. Thus, the relevant transformation curve between future and present consumption would be concave toward the origin, as depicted by the arc CB in Figure 3. In the case of borrowing for productive saving, the one-period resource constraint is given by

$$w^0 \ell_t + N_{t-1} \rho_t + D_{t-1} \cdot (\rho_t - \delta_t) = X_t + N_t , \tag{2.2}$$

where $D_{t-1} \cdot (\rho_t - \delta_t) \equiv D_{t-1} \cdot (r_t - b_t)$ represents the net income generated through productive borrowing. By utilizing the equilibrium conditions for optimal)orrowing and saving activity, it can easily be shown that the slope of the ;ransformation curve between future and present consumption then would be lictated by the lending rate of interest as is the arc AC in Figure 3. In)rinciple, one cannot rule out the possibility that the consumption unit would ›e a net borrower in some periods and a net lender in other periods. By 'urther restriction of the utility function of lifetime consumption and bequest considered in Section I, however, one can show that erratic switching rom a position of a net borrower to one of a net lender is not likely to ccur from one period to another. Let the parameter α_t in equation (1.15) e constant throughout the relevant planning horizon and let the parameters of quation (1.15a) dictate any positive ratio of terminal capital to consumption 1 different periods. Clearly, then, as long as there were no exogenous increases in wage income over time, borrowings for consumption purposes would be inconsistent with a positive difference between objective (lending or borrowing) interest rates and subjective rates of time preference. The reason is that equation (1.16) then would imply a strictly rising consumption path over time, whereas the borrowing of capital for current consumption decreases the amount of resources that can be expended on bequest or the purchase of goods in at least one future period by the amount of debt service payments. This assertion must be modified, of course, if wage income grew over time at a rate exceeding the optimal rate of growth of planned consumption expenditures. In our deterministic model of intertemporal consumption and production decisions, however, the growth of wage income must be explained primarily as the result of human capital accumulation. Intensive investment in human capital is expected to take place early in the investor's life, during which time he might also resort to net borrowing of capital if his initial net worth is negligible. But with $r_t > \tau_t$, and with accumulation of human capital rapidly leveling off, as models of human capital accumulation invariably predict (see, in particular, Becker (1964) and Mincer (1974)), the only way to effect a plan of persistently rising consumption expenditures is by gradually building up, and then maintaining, a positive stock of own physical capital. Since the main interest in this paper is in explaining the behavior of consumption units with positive levels of capital assets, we henceforth focus on the implications of borrowing for productive purposes, assuming that $r(h_t) > \tau_t$ and that asset holdings are nonnegative throughout the relevant planning horizon, starting at $t = 1$.

With this restriction in mind, it is easily shown that the overall wealth constraint for consumption units that are net lenders throughout their planning horizon would be

$$W^{(n)} \equiv \sum_{t=1}^{n} \ell_t w^0 \prod_{i=t+1}^{n+1} \rho_i + \sum_{t=1}^{n-1} (\rho_{t+1} - \delta_{t+1}) D_t \prod_{j=t+2}^{n} \rho_j + N_0 \prod_{t=1}^{n} \rho_t$$

$$= \sum_{t=1}^{n} X_t \prod_{i=t+1}^{n+1} \rho_i + N_n \tag{2.3}$$

where N_0 and $N_n \equiv K_n$ denote, respectively, the consumption unit's own initial and terminal physical capital. Optimal consumption and production decisions now can be derived by differentiating the utility function of lifetime consumption and bequest with respect to the relevant choice variables, subject to the wealth constraint given in equation (2.3). The optimality conditions for consumption decisions are formally the same as those derived in Section I. Production decisions, however, involve the simultaneous determination of optimal values of ℓ_t (or h_t) and D_{t-1}. Given the value of D_{t-1}, the optimal allocation of time between work and saving must satisfy the set of optimality conditions

$$w^0 = (N_{t-1} + D_{t-1})\, r'(h_t) \tag{2.4}$$

$$(N_{t-1} + D_{t-1})\, r''(h_t) < 0\ , \tag{2.4a}$$

provided that h_t and ℓ_t are positive; $t = 1, \ldots, n$. These conditions are formally identical to the conditions for optimal work and saving decisions derived in Section I except that here K_t is comprised of both own and borrowed assets. In turn, the optimal scale of borrowing in any period where $D > 0$ must satisfy the set of equations

$$r(h_t) = b(D_{t-1}, N_{t-1})(1 + \epsilon_{bD}) \equiv MbC_t \ \ 18/ \tag{2.5}$$

$$\partial MbC_t/\partial D_{t-1} \equiv -b'(D_{t-1})(1 + \epsilon_{bD}) - b(D_{t-1}, N_{t-1}) \frac{\partial \epsilon_{bD}}{\partial D_{t-1}} > 0\ , \tag{2.5a}$$

where $\epsilon_{bD} = \frac{\partial b_t}{\partial D_{t-1}} \cdot \frac{D_{t-1}}{b_t}$. The expression on the left-hand side of equation (2.5) is the marginal rate of return on lending and the expression on the right is the marginal borrowing cost of capital. In equilibrium, of course, they must be the same. The sufficient conditions for optimal values of ℓ_t and D_{t-1} to maximize the consumption unit's wealth, in addition to (2.4a) and (2.5a), require that

$$\Delta \equiv -(D_{t-1} + N_{t-1})\, r''(h_t)\, \partial MbC_t/\partial D_{t-1} - [r'(h_t)]^2 > 0 \;. \qquad (2.6)$$

Diagrammatically, equation (2.6) is satisfied if the marginal borrowing cost of capital becomes increasingly adverse to the borrower, relative to his rate of return on nonhuman capital, as he expands the scale of his borrowing.[19] These optimality conditions are portrayed in Figure 4.

The central behavioral implications of the foregoing analysis involve potential interdependencies between N and D. At low levels of net worth, saving activity may be negligible, and so $r(h)$ is expected to be lower than the marginal borrowing cost of capital. Little borrowing for the purpose of acquiring additional income generating assets is expected. But as N grows and participation in saving activity is enhanced, $r(h)$ also is expected to rise sufficiently to make borrowing for productive saving optimal. Moreover, an increase in own assets is expected to promote a greater <u>absolute</u> amount of borrowing as well as greater participation in productive saving activity.[20] This is demonstrated graphically in Figure 4, where an increase in N from N_0 to N_1 is associated with a greater absolute increase in K from K_0 to K_1. This analysis does not rule out borrowing for productive saving at low levels of asset holdings. It implies, however, that such borrowing is more likely to be undertaken by those who specialize in the management of their as-

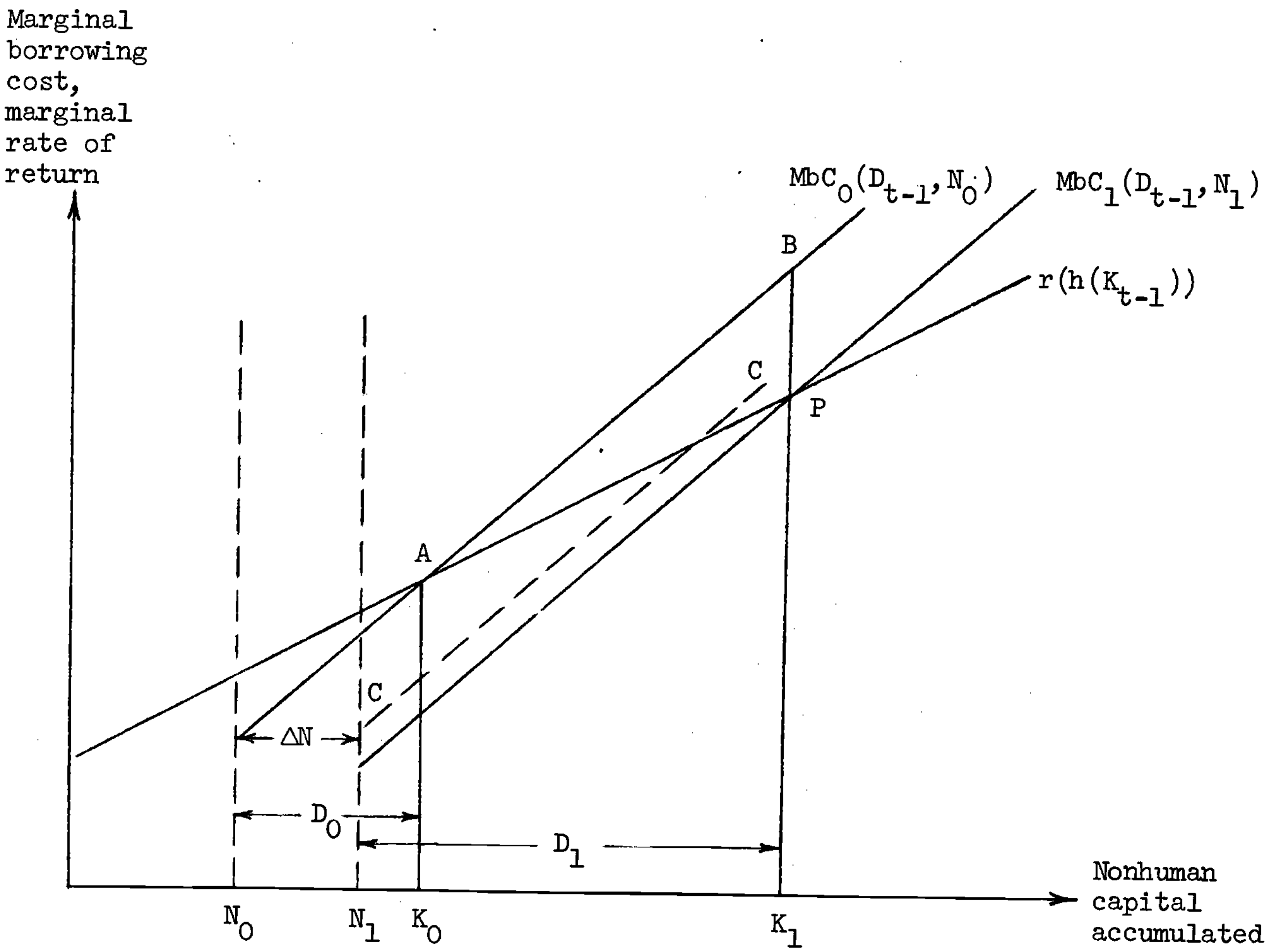

Figure 4

(The linear curves are only illustrative.)

sets since their lending rates of interest are expected to be relatively large at given levels of asset holdings. Thus we expect that borrowing for productive saving will be more prevalent among unincorporated business families than among families of wage earners. Indeed, at low levels of asset holdings entrepreneurial savings consequently may be more negative than those of wage earners. The analysis generally implies that saving activity and productive borrowing can be considered "complements" in the sense that an increase in one activity due to exogenous factors will promote both.

The analysis so far has ignored the possible role of time and other resources in borrowing costs. In the generally segmented market for borrowing, careful search among lenders may reduce the borrowing costs of capital to the consumption unit at any level of borrowing and net worth. Optimal borrowing activity would then be achieved when the marginal return from time devoted to shopping for funds, a, is equal to the opportunity cost of time in other pursuits. The necessary condition for an optimal allocation of time between work, borrowing, and saving activities, if all are positive, is given by

$$w^0 = (N_{t-1} + D_{t-1})\, r'(h_t) = -D_{t-1}\, b'(a_t) , \tag{2.7}$$

where $b(a_t)$ is a continuously decreasing, twice differentiable, and convex function of a_t.

Productive borrowing and saving activities generally are expected to be complementary in affecting nonwage returns. Indeed, these may be joint activities from the investor's point of view. In general, active borrowing for productive saving is likely to take place at a relatively high level of borrowing which, in turn, is expected to increase with net worth. Moreover, the higher is N at any given level of opportunities for work, the greater will be the amount of time devoted to both saving and borrowing. Thus, the formal incorporation of productive borrowing activity in the model strengthens the implications of the preceding analysis of productive saving. The main significance is in pointing to the potential existence of an additional source of productivity in the process of generating nonwage income.

III. Capital Accumulation Paths and Life Cycle Variations in Productive Resource Allocations

While the allocation of goods over time, the allocation of time between work and saving, and the optimal level of productive borrowing in each period are the "control variables" of our model, the accumulation of nonhuman capital over the planning horizon is the "state variable" that summarizes all previous production and consumption decisions. Since changes in the level of accumulated capital, in turn, indicate the magnitude of savings in each period, optimal savings paths can be derived through analysis of capital accumulation paths. As in Section II, the analysis here will focus on consumption units with positive capital assets. For methodological convenience, we assume that on the average life cycle expectations are fulfilled so that the planned resource allocations of a representative consumption unit are realized in practice.

The one-period resource constraint given by equation (1.5) can alternatively be defined by

$$X_t = w^0 \ell_t + K_{t-1} r_t - (N_t - N_{t-1}) \qquad t = 1, \ldots, n \qquad \ell_t > 0 . \qquad (3.1)$$

Without loss of generality, we now ignore the separate role of productive borrowing and identify K_t with N_t. We also assume that $\partial r(h_t)/\partial K_{t-1} = 0$.[21] If human capital and the wage rate from work are constant, the change in the consumption of goods over time can be approximated by

$$\dot{X}_t = X_{t-1} - X_t = [w^0 - N_{t-1} r'(h_{t+1})] \dot{\ell}_t + r(h_{t+1}) \dot{N}_{t-1} - \dot{N}_t + \dot{N}_{t-1} \text{ [22]}$$

$$t = 1, \ldots, n - 1 . \qquad (3.2)$$

The term $\dot{N}_t = N_{t+1} - N_t$ represents the magnitude of savings in period t. Since an optimal allocation of time between work and saving insures the equality of the marginal returns on all productive uses of time, the first term on the right-hand side of equation (3.2) in equilibrium is equal to zero (provided that specialization in saving activities implies that h_t is constant), and equation (3.2) reduces to

$$\dot{X}_t = r(h_{t+1})\dot{N}_{t-1} - (\dot{N}_t - \dot{N}_{t-1}) \qquad t = 1, \ldots, n-1 . \tag{3.3}$$

Equation (3.3) defines a direct relation between the rate of change in the allocation of goods and the rate of capital formation over time. For example, if the gross lending rate of interest exceeds the subjective rate of time preference in all the relevant periods, as was assumed in the preceding section, then the optimal path of the allocation of goods will be rising throughout the planning horizon, or $\dot{X}_t > 0$ in all t. If, in addition, relative preferences between own consumption and bequest in the final period of the planning horizon are such that the optimal ratio $\frac{X_n^*(N_{n-1}^*)}{N_n^*}$ implies $N_n^* > N_{n-1}^*$, then $\dot{N}_t$, and hence the algebraic value of savings in period t, must have been positive in all preceding periods. Alternatively, if the consumption unit is initially a net borrower, then its debt must be reduced in part or in full following the initial period(s) of borrowing when net worth is either zero or negative. To prove this, note that if $\dot{N}_{t-1}$ were negative, then $\dot{N}_t$ necessarily has the same algebraic sign to insure that $\dot{X}_t > 0$, since by rearranging terms, equation (3.3) can be written as

$$\dot{X}_t = [1 + r(h_{t+1})]\,\dot{N}_{t-1} - \dot{N}_t \qquad t = 1, \ldots, n-1 . \tag{3.4}$$

The path of capital accumulation would be falling continuously over time once the consumption unit starts dissaving. But such a path would be inconsistent with $\dot{N}_{n-1} = N_n - N_{n-1} > 0$. The optimal path of capital accumulation must then be continuously rising over time. Possibly, however, relative preferences between own consumption and bequest in the typical case are such that the ratio $\frac{X_n^*(N_{n-1}^*)}{N_n^*}$ implies $N^* < N_{n-1}^*$ (see the discussion in n. 12). If $N_n \geq N_0$ in this case, the optimal path of capital accumulation must rise initially over some range of the planning horizon, attain a peak, and then continuously decline over the final range (see Figure 5). This path of capital accumulation, of course, may be consistent with $N_n < N_0$ also. The main implications of this analysis can be summarized by the following theorem.

Theorem: Given a homothetic utility function of lifetime consumption as considered in Section I, with the gross rate of interest always exceeding the rate of time preference, and with a constant wage rate, the capital accumulation path is either continuously rising (with a possible net indebtedness being first settled) or is initially rising and then falling, provided that initial capital does not exceed terminal capital.

The preceding analysis depends crucially on the assumption that the wage rate $w^0 = w(H_0)$ is constant throughout the planning horizon. If it were subject to uncontrollable variations over the life cycle, equation (3.4) would reflect such variations as follows:

$$\dot{X}_t - \ell_t \dot{w}_t = [1 + r(h_t)] \dot{K}_{t-1} - \dot{K}_t . \quad (3.5)$$

Clearly, then, any systematic exogenous decreases in the wage rate would strengthen the above theorem, whereas systematic exogenous increases in the wage rate may render it invalid, unless the difference $\dot{X}_t - \ell_t \dot{w}_t$ were always positive. Under the deterministic model of productive saving, however, changes

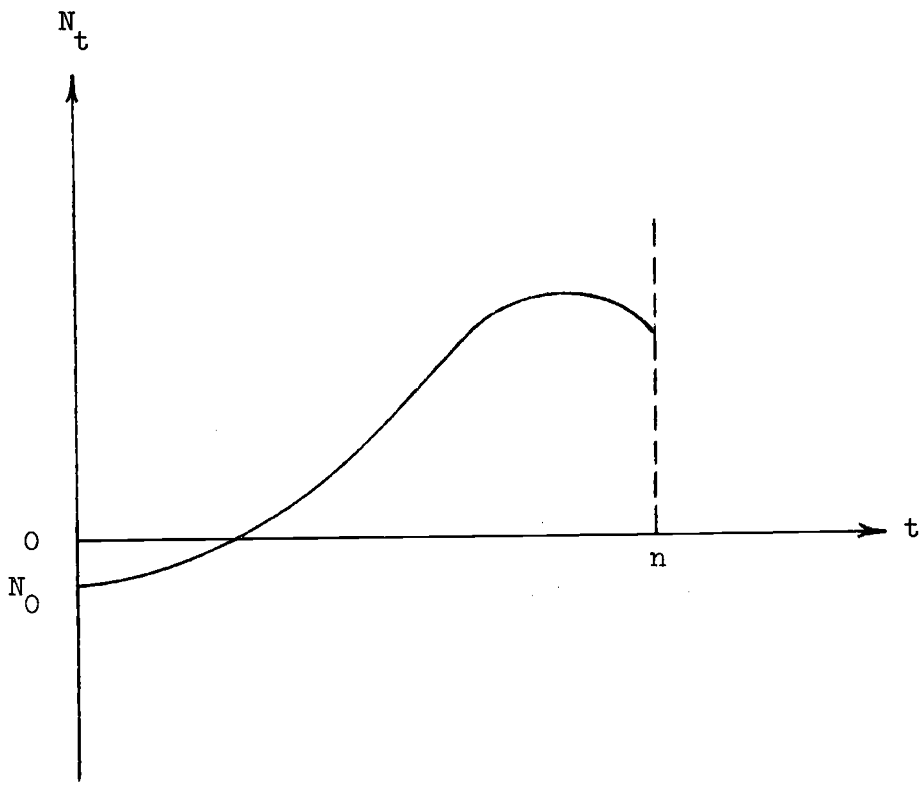

Figure 5

Capital Accumulation with $\dot{X}_t > 0$,

$N_0 \leq N_n < N_{n-1}$, $w(H) = w^0$

in the wage rate over time are expected to occur mainly as a result of positive investments in human capital, so far assumed to be endowed at a constant level. Investment in human capital is clearly an alternative means of accumulating capital assets or engaging in productive saving. And, as has already been pointed out in our analysis in Section II, the accumulation of human capital, due to its embodiment in the investor and its nonmarketability in general, is expected to take place early in the life cycle. Following the termination of investments in human capital, $w = w(H)$ is expected to remain constant or decline as a result of depreciation of human capital. The theorem expounded in the preceding discussion would then hold unambiguously. But even during the period of accumulation of human capital, the theorem may remain valid if human and nonhuman capital were complements in the production of wealth, because planned persistent increases in consumption expenditures, in the absence of expectations for purely exogenous increases in wage income, must be effected through some positive accumulation of future income-generating assets--human or nonhuman--as long as initial (nonhuman) capital is less than or equal to terminal capital $(N_0 \leq N_n)$. Indeed, human and nonhuman capital assets are likely to be acquired simultaneously, at least throughout the latter part of the human capital investment period, if only because an increase in human capital may enhance productivity in saving activities.[23]

The paths of capital accumulation derived above generally appear compatible with empirical evidence relating to asset holdings in a cross section of age groups at a point in time (see, e.g., Projector and Weiss (1966), Tables A8 and A10). These paths might also be used to predict life cycle variations in hours of work and labor force participation rates. If wage rates and the marginal productivity of saving time were constant over the planning horizon, the amount of working time devoted to labor market activities would tend to decline as capital was continually accumulated and the amount of time devoted to self-employment in the management of capital assets would gradually increase, peaking at the peak of nonhuman asset accumulation. The latter result follows from equation (1.12) since, with a constant market price of time and an unchanging function $r(h, k)$, there exists a unique relationship between the size of capital accumulated and the expected absolute amount of time devoted to saving. This prediction remains valid even if human capital changed systematically over the life cycle, provided that it had a neutral effect on the productivity of time devoted to saving and to work, as well as to other potential pursuits. Specifically, if $w_t = wH_{t-1}$ and $r_t = r(h_t \cdot H_{t-1})$, an increase in H_{t-1} would not affect the equilibrium condition for an optimal allocation of time between work and saving since the ratio $w_t/r'(h_t)$ would be invariant to changes in H_{t-1}. It is possible, of course, that human capital acquired through general schooling and training is biased in favor of conventional labor market activities. Persons with such training would tend to specialize in labor market activities at least through the period of intensive training and other related investments. Still, our basic prediction concerning the effect of physical capital accumulation on the allocation of time between work and saving is expected to hold at any given level of human capital accumulation.[24/] And it is expected to hold even more strongly during the period of net human capital depreciation as long as there was no comparable deaccumulation of nonhuman capital. Indeed, empirical evidence controlling for family income indicates that the share of nonhuman assets among all assets increases continuously with age: controlling for family income, Projector and Weiss (op. cit., pp. 7, 8) report a positive correlation between net asset holdings and age across the entire age range. A general implications of the analysis is that as age rises the fraction of all working time devoted to asset management also rises. Evidence derived from the same data analyzed by Projector and Weiss is consistent with this implication. It shows that among all household heads the proportion of those who are self-employed increases monotonically with age. Even retirement may be explained, in part, as a consequence of nonhuman capital accumulation inducing specialization in saving activities. If, however, asset holdings substantially decrease later in life, then the allocation of working time between saving and work may once again shift in favor of the latter. Reentry into the labor market after a period of specialization in self-employment might be explained as a consequence of deaccumulation of assets which decreases the marginal productivity of saving time relative to labor market earning power.

IV. The Average Propensity to Save

In the preceding section we discussed expected capital accumulation paths over the life cycle. Of course, realized paths may vary among consumption units of like characteristics. Consumption units also differ in their initial endowments of human and nonhuman assets and in their chosen occupational careers. In this section we discuss the effects of these differences on the propensity to save in light of the productive saving hypothesis.

Consider, first, the effect of an increase in wealth due to an exogenous increase in the endowment of nonhuman capital or the corresponding magnitude of nonwage income. In the Fisherian model this would not affect the relative intertemporal allocation of goods if the utility function of lifetime consumption and bequest were homothetic. This implication of the Fisherian model underlies the well-known theories of the consumption function by Friedman (1957) and by Modigliani and Brumberg (1954). In terms of the permanent income hypothesis one can write

$$X_t = k_t(r)\ Y_{pt}\ , \qquad (4.1)$$

where Y_{pt} denotes permanent income (or wealth) as evaluated by the consumption unit at age t, and k_t measures its average propensity to consume out of permanent income. If r were constant, so would be k. [25/] In constrast, our model suggests that an exogenous increase in net worth or nonwage income would increase simultaneously the desired amount of asset holdings, the optimal amount of productive resources allocated to saving activity (including own time, hired factors, and borrowing activity), and the rate of return on savings. Consequently, we expect an exogenous increase in net worth to decrease the average propensity to consume out of permanent income. This major implication of the analysis can be demonstrated graphically via the two-period consumption model discussed in Section I. An increase in K_0 (as in Section I, we here identify K with N) implies that the initial endowment point E in Figure 6 shifts out horizontally to point F. Since the consumption unit's wage rate is unaffected, the consumption transformation curve passing through F, BFB', is just a horizontal translate of the initial transformation curve, AEA'. [26/] Clearly, the slope of the curve BFB' is steeper than the slope of AEA' along the ray OP' passing through the initial equilibrium position P. Thus, if the utility function is homothetic the new equilibrium position must lie to the left of the ray OP', say at point Q. Both the optimal level of net worth in period 1 and the optimal ratio X_2/X_1 increase as a result. That an increase in the desired level of asset holdings increases the optimal ratio of future to current consumption can be verified mathematically by differentiating equation (1.16) with respect to K_1 since

$$\frac{d \log(X_2/X_1)}{dK_1} = \sigma_2 dr_2/dK_1 > 0 , \tag{4.2}$$

as long as $r'(K_1) \equiv dr_2/dK_1 > 0$. If desired bequest were nil, an increase in X_2/X_1 necessarily implies that the percentage increase in wealth, defined at the new equilibrium position Q by $W_2 = X_1\, p(h_2) + X_2$, exceeds the percentage increase in X_1. The ratio of current consumption to permanent

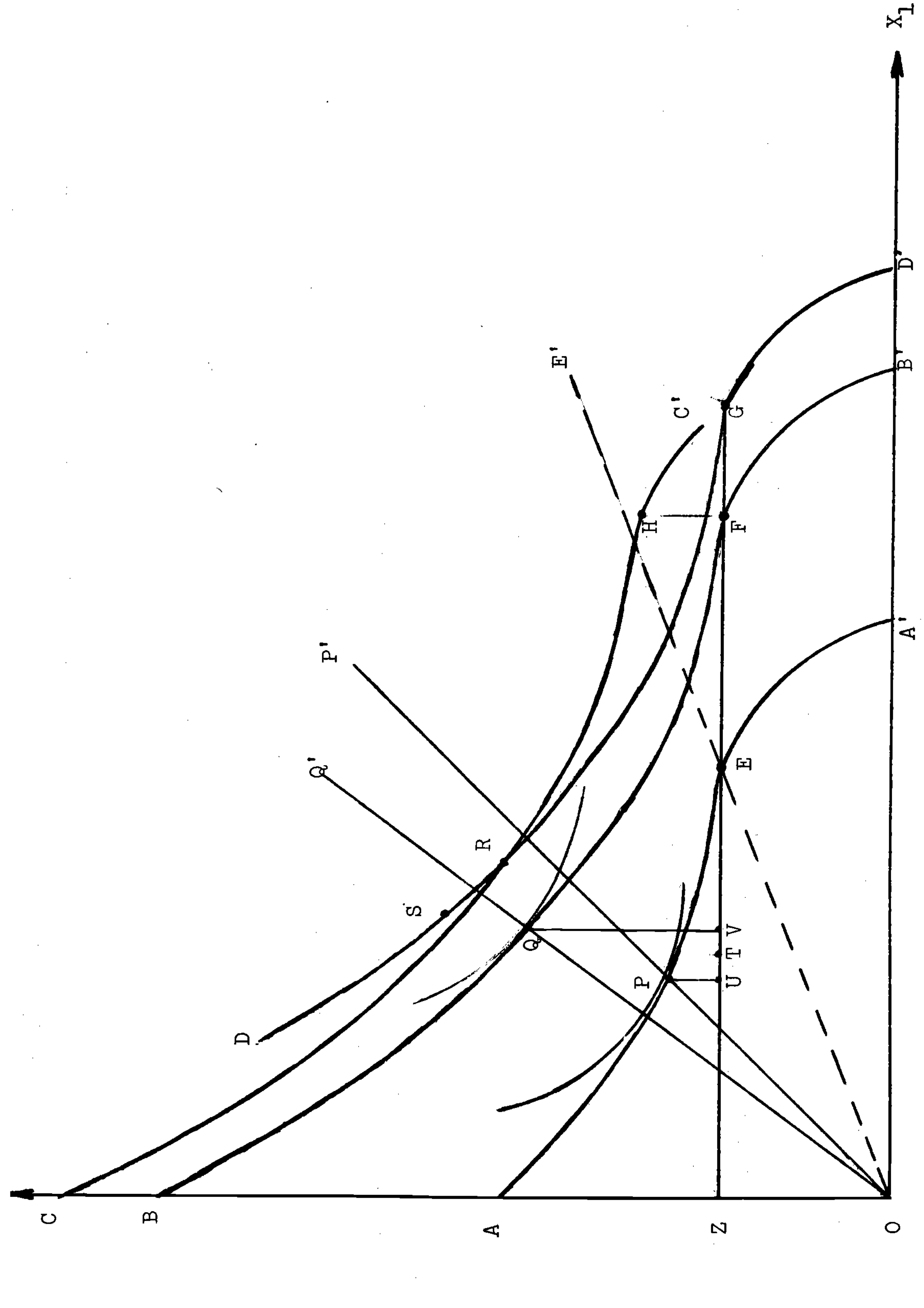

Figure 6

income, k_1, then is expected to fall. The same result obtains if the analysis is generalized to n periods. Furthermore, since by equation (1.16a) the ratio of bequest to the planned consumption expenditures in period n is independent of wealth, planned bequest, if positive, would rise by the same proportion as X_n. Consequently, the ratio of the discounted, as well as undiscounted, life time consumption to life time income would necessarily fall.27/ Regardless of whether bequest is optimal, the decline in the ratio of current consumption to wealth as nonhuman wealth rises also implies that the ratio of current consumption to current income falls. That is, the analysis implies that both X_t/Y_{pt} and X_t/Y_t decline as wealth rises.28/

Next, consider an increase in wealth due to a greater "endowment" of human capital. Since the corresponding increase in earning power implies an increase in current wage income as well as in potential wage income in all relevant future periods, the endowment position in Figure 6 shifts upward and to the right of point E, but below the ray OE'.29/ Point H illustrates an increase in wage income in period 1 by an amount equal to the absolute increase in nonwage resources considered in the preceding illustration. If the increase in human capital does not affect the consumption unit's rate of return at given levels of nonhuman capital, that is, if $dr(K)/dH = 0$, then the new consumption transformation curve passing through H, CHC', again would be a horizontal translate of curves AEA' and BFB' after adjusting for the vertical distances between the different endowment positions. It can easily be shown in this case that the new optimal ratio of X_2 to X_1 would be higher than their initial ratio at point P, but lower than the corresponding ratio associated with an equal increase in nonwage income or nonhuman wealth (compare points R and Q in Figure 6). A permanent increase in wage income here is found to raise the optimal savings to wealth ratio by less than would an equal once and for all increase in nonwage income or nonhuman wealth. Moreover, this conclusion is strengthened if the effect of an increase in wage income is compared with the effect of an increase in nonwage income or nonhuman wealth that led to an equal increase in the consumption unit's overall wealth. This easily can be verified by comparing the equilibrium positions R and S associated with the consumption transformation curves CHC' and DGD', respectively. By construction these two curves intersect at point R.

Since, in general, a change in human capital may alter the allocation of resources to productive saving activity, the preceding illustration, assuming no such effects, may serve only as a benchmark for analysis of more general cases. A neutral effect of human capital on the productivity of resources at work and saving, as analyzed in Section III, would not affect the allocation of working time and other resources between the two activities but would raise the productivity of time and other resources spent saving at given levels of K_{t-1}. The result could be an increase in $r(K)$, hence in the steepness of the arc CH along the curve CHC'. The implication of the preceding analysis concerning the positive effect of an increase in wage income on the optimal saving to wealth ratio then would be fortified. In constrast, if human capital were "biased" in favor of conventional labor market activities, which is likely if it were comprised mainly of "general" schooling and labor market experience, then an increase in human capital, by shifting the allocation of resources away from productive saving toward "work," might decrease the level of $r(K)$, hence the steepness of the transformation curve CH. An exogenous increase in wage earnings due to an improvement in labor market opportunities is an obvious example of a shift in opportunities that may "bias" the allocation of resources away from productive saving. In these cases the optimal savings to wealth ratio resulting from an increase in wage income may be decidedly lower than the one resulting from an equal increase in nonwage income or nonhuman wealth.

This analysis may be used to reinterpret the positive association between the level of measured income and the average propensity to save as reported in numerous budget studies without exclusive resort to explanations drawing on the effect of variations in transitory components of income on the propensity to save. Households or spending units of given age, schooling, and labor market experience and with larger endowments of nonhuman capital may have higher propensities to save out of permanent income because of their relatively higher rates of return on savings. A similar conclusion may also apply to the propensities to save of consumption units of a given age and net worth and with higher levels of human capital, provided the character of their human capital were not "biased" significantly against saving activities.[30] Inasmuch as cross-sectional variations in income reflect variations in permanent components of income, an observation of a systematic association between the propensity to save and income may be explained, at least in part, as a consequence of a positive association between permanent income and the productive management of assets. That is, one may write

$$X_t = k_t(Y_{pt})\,Y_{pt} \quad , \tag{4.3}$$

with $\frac{dk_t}{dY_{pt}} < 0$.[31] Equation (4.3) is compatible with Keynes' (second) "law" of consumption (see Keynes (1961), pp. 28, 126). In our analysis the law is conditional, however, upon the interaction between the optimal level of accumulated assets and productive saving activity and does not rely on any systematic association between time preference for consumption and wealth. It also should be noted that our result is stated in terms of a negative association between permanent income and the <u>average</u>, but not the <u>marginal</u> propensity to consume.

The preceding analysis may be applied more directly in interpreting empirical evidence on the average propensities to consume of different occupational and racial groups. Assuming (as does Friedman (1957), pp. 64, 80) that transitory components of measured income and consumption expenditures tend to average out to zero across all income brackets of specific occupational and racial groups, the average consumption to income ratios of these groups may serve as unbiased estimators of their average propensities to consume out of permanent income.[32] Indeed, Table 1 contains evidence on average consumption to average income ratios of independent business, farm, and other spending units in the United States in 1948-50 which appears to be highly compatible with the productive saving hypothesis. The groups of farmers and independent business units can be distinguished from others in that they presumably are more "specialized" in direct management of their assets and engage less in conventional "work" relative to other groups. Not only are independent business units and farmers expected to allocate more resources into productive saving activities relative to those who specialize in wage earnings, also their specific human capital--training and job experience--is likely to be more complementary to the management of their business assets. Consequently, we would expect the rate of return on their nonhuman capital (especially on equity in own business), and hence their average propensity to save out of permanent income (especially in the form of business assets), to be relatively high. Moreover, since independent business units in the 1948-50 sample on the average have more income and presumably larger portfolios of capital assets than farmers, they are expected to allocate relatively more resources, own and hired, into productive saving than do farm units (many of which are farm laborers). Consequently, the average propensity to save of independent nonfarm business units may indeed be expected to be higher than that of farm units.[33]

The well-known permanent income hypothesis provides an effective and systematic explanation for cross-sectional differences between least squares regression estimates of marginal propensities to consume out of measured income and income elasticities of consumption of different occupational groups by

TABLE 1

RELATION BETWEEN CONSUMPTION AND INCOME FOR INDEPENDENT BUSINESS, FARM, AND OTHER SPENDING UNITS

(1948-1950)

Occupational Group	Average Disposable Income		Average Propensity to Consume	Income Elasticity of Consumption
	Current Prices $	1935-59 Prices $		
Independent business	4,789	2,795	.77	.70
Farmers	2,404	1,403	.88	.69
Others	3,038	1,773	.95	.86

Note: Figures are for money consumption and money disposable income.

Source: Friedman (1957, p. 71)

virtue of the different degrees to which variations in the total income of these groups are accounted for by variations in transitory components of income. However, it does not appear to offer similarly powerful explanations of systematic differences in the average consumption to income ratios of these groups. The main consideration raised by Friedman in connection with the evidence reported in Table 1 is that the relatively larger variance in transitory income obtained by business and farm units makes it optimal for them to save more than nonfarm, nonbusiness units for the purpose of building up reserves for emergencies. By similar reasoning, however, the average propensity to save out of permanent income is also expected to be negatively related to the ratio of nonhuman capital to permanent income, as the discussion in n. 25 indicates. Since the income of business and farm families derives to a large extent from their nonhuman capital assets, the ratio of their nonhuman assets to their permanent incomes should be markedly higher than that of other families as an occupational prerequisite. Indeed, evidence based on the Survey of Financial Characteristics of Consumers in 1963 shows that the ratio of mean nonhuman wealth (including business assets) to mean income of self-employed units is about 7 as compared to 2 for salaried units. Even when business assets are excluded, the nonhuman capital to income ratio of the self-employed is about 4, as compared to 1.8 for the salaried. (See Projector and Weiss (1966), Tables A8 and A34.) The higher ratio of nonhuman assets to permanent income of business families might lessen the incentive in these groups to save for emergencies. The productive saving hypothesis thus offers an independent and consistent interpretation for differences in the <u>average</u> propensities to consume of these different occupational groupings (also see Friedman (1957), pp. 69, 78).

A second application of this analysis concerns the apparently different average propensities to consume of black and white families as reported in Friedman (1957, Table 7). In each of ten comparisons of average propensities to consume of black and white families in various city sizes in

Northern and Southern states, the ratio is higher for blacks than for their white neighbors. Reference to systematic differences in the relative dispersion of transitory components of measured income across the two groups fails to provide a consistent explanation for the evidence just mentioned: the relative dispersion of transitory components of income of black families is estimated to be higher than or equal to that of white families in three of the four Southern communities included in the sample. Yet the average propensity to consume of black families still appears higher than that of whites for all income classes. On the productive saving hypothesis the relatively lower propensity to save of black families can be consistently interpreted as a consequence of objective opportunities. Discrimination in real estate markets and in markets for certain unincorporated businesses may limit the range of investment options available to blacks just as labor market discrimination reduces their wage income opportunities. Even if discrimination in labor markets affected earning opportunities more than discrimination in capital markets, any presence of the latter implies that $r(h)$ would be lower for blacks than for whites at any given value of h. Moreover, blacks generally possess smaller endowments of assets and acquire less (specific) human capital than whites. Thus, the average propensity to save from permanent income for black families might be lower than that for whites not because of unique motivation or diferent time preference, but becaues of inferior opportunities for producing returns on nonhuman assets.

Although the productive saving model developed in this paper can be applied in explanation of cross-sectional differences in average propensities to consume, it is not directly applicable in explanation of secular trends in the aggregate consumption to income ratios over time. The productive saving hypothesis links cross-sectional differences in average propensities to consume to differences among consumption units in their private rates of return on savings. But while the theory identifies the basic factors affecting the distribution of private rates of return at a point in time, it has no direct implications for movements in the level of the distribution over time. The latter essentially depends on market forces determining the real rates of interest in the economy. Moreover, trends in the allocation of resources to productive saving activities depend on the secular growth of labor-market-specific human capital relative to other human and nonhuman capital assets as well as on structural trends in occupational composition in the economy. Thus, the cross-sectional implications of the theory would be consistent with a virtually zero correlation between permanent income per capita and the aggregate propensity to consume out of permanent income over time if, as seems to be the case, the proportion of business and farm families in the economy decreased relative to the proportion of "wage earners" while the average per capita portfolio sizes increased over the long haul. A comprehensive analysis of secular trends in the aggregate propensity to consume in light of this model further requires consideration of secular trends in institutional and technological factors bearing upon the degree of segmentation or "efficiency" of various capital markets and, thus, indirectly on private returns available from search and related asset management efforts.

Conclusion

The thesis developed in this paper is that insofar as opportunities for gains from saving activities by individuals exist in capital markets, the rates of return on capital assets will depend, in part, on the amount and efficiency of resources devoted to such activities. Accordingly, private investment in information concerning the prospects of capital assets and other related management efforts may be thought of as a special asset commanding a unique market return. Since opportunities for productive saving activities depend on the degree of segmentation in capital markets and the magnitude of relevant transaction costs, the importance of the productive saving hypothesis in explaining saving behavior may vary across different economies or over time according to the degree of "imperfection" inherent in capital markets.

Differences in rates of return on portfolios of capital assets, or private lending rates of interest, are traced exclusively to differences in resource allocations to productive saving activities since the model is developed on the simplifying assumption that individual attitudes toward risk are neutral. In practice, much of the variation in private rates of return is the result of differences in the degree of objective risk associated with portfolios. The basic implications of the simple model developed here nonetheless are general since they apply to rates of return achieved on capital assets classified in the same objective "risk class." A systematic study of the full nature of the interdependence between productive saving, risk, and return, as well as other generalizations of the model, are set aside for future work.

The set of behavioral implications that are obtained follows essentially from the hypothesized interactions between allocations of resources to saving activities and the level of human and nonhuman capital assets possessed by consumption units. The main results concern the allocations of productive and consumptive resources throughout the life cycle and the interdependencies among these allocations. The analysis indicates that the magnitude of own, hired, and borrowed resources generally is an increasing function of net worth. It also implies that the paths of human and nonhuman capital accumulation together determine the allocation of productive resources between wage and nonwage earnings generating activities. Some of the observed variation in the extent of participation in conventional "work" over the life cycle might be explained through consideration of nonhuman capital accumulation paths as derived in Section III. The model generally provides a framework for analysis of the determinants of self-employment in the management of assets relative to participation in conventional labor market activities.

Since the magnitude of resources devoted to productive saving, in turn, affects the private rates of return on capital assets pertinent to consumption decisions, differences among consumption units in their allocations of consumptive resources over the life cycle may result largely from differences in their private rates of interest rather than in their subjective time preferences or attitudes toward risk. Perhaps the most intriguing behavioral implications developed in this paper concern the association between levels of human and nonhuman wealth and the average propensity to consume out of permanent income. Under certain conditions the analysis can be used to reformulate Keynes' postulates concerning a positive association between the propensity to save and wealth without resort to any systematic association between time preference for consumption and wealth. The analysis also provides consistent explanations for evidence reported in the literature concerning differences in average propensities to consume across occupational and racial groups without reference to variations in transitory components of measured income. Our analysis complements the permanent income hypothesis in one important sense. Whereas the permanent income hypothesis provides a systematic explanation for differences in statistical estimates of marginal and average propensities to consume out of measured income, the productive saving hypothesis provides a framework for analyzing differences in average propensities to consume out of permanent income across different wealth and occupational groups. Since our behavioral propositions are based upon systematic variation in expected rates of return from capital assets in the same risk class, the empirical implementation of the theory should begin by estimation of nonwage earnings generating functions by relating risk adjusted levels of portfolio returns to the basic determinants of the allocation of resources to productive saving.

Footnotes

[1]For an excellent survey of the literature pertaining to the "efficient markets"hypothesis, see Fama (1970). Although the presentation of the hypothesis in the literature is general and may be interpreted as applicable to all markets for capital assets, the illustrations usually have related to the markets for securities.

[2]For example, Scholes (1972) has presented evidence indicating that the total adjustment of a price of a stock to a large secondary distribution of that stock takes approximately six days. The evidence also indicates that the sale period itself is too short to account for the entire length of the adjustment period in the market.

[3]For theoretical and empirical analyses bearing upon the role of education in the efficient management of farm assets, see Welch (1970), Shultz (1974), and references therein. Differences in efficiency of management of agricultural enterprises have been recognized in these studies in regard to the speed of adoption of innovations and the rapidity of adjustments in resource allocations to changing market conditions.

[4]Neiderhoffer and Osborne (1966), Lorie and Niederhoffer (1968), Scholes (1969), and Jaffee (1973) present evidence suggesting that officers of corporations consistently utilize superior information regarding the performance of stocks. A similar finding concerning specialists in major security exchanges also is presented in Neiderhoffer and Osborne (1966). Specialists and corporation officers constitute a small fraction of all investors, but the question as to whether deviations from the strong form conditon of the efficient markets model reach further through the investment community has not yet been adequately explored.

[5]See Marshall (8th edition, 1949, pp. 588, 599). Marshall noted in this context that

> "A pawnbroker's business involves next to no risk; but his loans are made at the rate of 25 percent per annum or more; the greater part of which is really earnings of management of a troublesome business. Or to take a more extreme example, there are men in London, and Paris, and probably elsewhere, who make a living by lending money to costermongers . . . at a profit of ten percent [per day]: there is little risk in the trade, and the money is seldom lost . . . .. But no one can become rich by lending to costermongers . . . .. The so-called interest on the loans really consists almost entirely of earnings of a kind of work for which few capitalists have a taste."

We are indebted to Lawrence Fisher for pointing out this reference to us.

[6]The overall, expected rate of return on nonhuman capital generally can be thought of as consisting of two parts

$$r_t = r(0) + \int_0^{h_t} r'(h)\, dh \qquad 0 < h \leq h_t ,$$

where $r(0)$ is the expected rate of return with no saving activity, and $r'(h)$ is an additional expected yield achieved through efficient selection and management activities. It may be noted that the expected rate of return is defined in equation (1.3) as a function of current saving activity only. A more general formulation of the productive saving hypothesis would relate r to past periods' allocation of time and other resources to productive saving as well. For simplicity, (the effect of) past experience is subsumed under the "endowment" of human capital H_0.

[7]The terminal stock of capital is given by the equation

$$K_n \equiv \ell_n w^0 + \rho_n K_{n-1} - X_n ,$$

where

$$K_{n-1} = \ell_{n-1} w^0 + \rho_{n-1} K_{n-2} - X_{n-1}$$

$$\vdots$$

$$K_1 = \ell_1 w^0 + \rho_1 K_0 - X_1 .$$

Substituting the values of $K_{n-1}, \ldots, K_1$ in the first equation, one obtains

$$K_n = \prod_{t=1}^{n+1} \rho_t K_0 + \sum_{t=1}^{n} (\ell_t w^0 - X_t) \prod_{i=t+1}^{n+1} \rho_i(h_i) ,$$

where $\rho_{n+1} \equiv 1$. By rearranging terms, the wealth constraint given in equation (1.8) is easily derived.

[8]Equation (1.10a) is derived on the assumption that $\partial r/\partial K_{t-1} = 0$. For a simple illustration of the solution when r_t is defined as a direct function of K_{t-1}, see n. 10. The sufficient conditions for values of the control variables satisfying equations (1.10a-1.10d) to maximize equation (1.9) require that the principal minors of the relevant bordered Hessian matrix of second-order derivatives alternate in sign.

[9]The effect of an exogenous increase in K_{t-1} on the optimal value of h_t, for $h_t > 0$, can be found by differentiating equation (1.12) with respect to K_{t-1}. Assuming that $\frac{\partial r'(h)}{\partial K_{t-1}} = 0$,

$$-\frac{d\ell_t}{dK_{t-1}} = \frac{dh_t}{dK_{t-1}} = \frac{-r'(h_t)}{r''(h_t)\, K_{t-1}} = \frac{(-)}{(-)} > 0 .$$

Even if $\frac{\partial r'(h_t)}{\partial K_{t-1}} < 0$,

$$\frac{dh_t}{dK_{t-1}} = \frac{-r'(h_t)[1 - \epsilon_{r'K}]}{r''(h_t)K_{t-1}} = \frac{(-)}{(-)} > 0 ,$$

where $\epsilon_{r'K} = -\frac{\partial r'(h_t)}{\partial K_{t-1}} \cdot \frac{K_{t-1}}{r'(h_t)}$, since by the assumption that h_t and K_{t-1} are complements in the generation of nonwage income,

$$r'(h_t)[1 - \epsilon_{r'K}] = \frac{\partial R}{\partial r_t\, \partial K_{t-1}} > 0 .$$

Since $r''(h) < 0$, it is plausible to assume that $\frac{d^2h_t}{dK_{t-1}^2} < 0$ provided that

$r''(h)$ were either invariant to K_{t-1} or became more negative as K_{t-1} increased. Figure 2 portrays a relationship between h_t and K_{t-1} which is compatible with these assumptions.

[10]If $\frac{\partial r(h_t, K_{t-1})}{\partial K_{t-1}}$ were negative, differentiation of equation (1.13) with respect to K_1 would modify equation (1.14) as follows

$$-\frac{dX_2}{dX_1} = \rho_2(K_1)(1 + \epsilon_{\rho_2 K_1}) \equiv MR\rho_2(K_1) \ ,$$

where $\epsilon_{\rho_2 K} = \frac{\partial \rho_2}{\partial K_1}\frac{K_1}{\rho_2} = \frac{\partial r_2}{\partial K_1}\frac{K_1}{r_2} \cdot \frac{r_2}{1 + r_2}$ and $MR\rho_2$ replaces ρ_2 as the gross marginal return. In this case, if $\epsilon_{\rho_2 K_1}$ were constant and h_t were positive, then the consumption transformation curve AC depicted in Figure 3 would be convex toward the origin if, and only if,

$$\frac{dr}{dK_{t-1}} = \frac{\partial r}{\partial h_t}\frac{dh_t}{dK_{t-1}} + \frac{\partial r}{\partial K_{t-1}} > 0 \ .$$

We do not expect the direct negative effect of K on r to dominate its indirect effect on r through its positive effect on h except, perhaps, at very high levels of asset holdings. In that case, $\frac{dr}{dK_{t-1}}$ would be negative in sign and the transformation curve AC in Figure 3 might be concave toward the origin.

[11]Specifically, $\tau_t = 0$ (or $\alpha_t = 1$) implies zero time preference for X_{t-1} relative to X_t. This form of the utility function and the following analysis leading to equation (1.16) are entirely based upon that of Becker (1971, pp. 192-193). For a related analysis, see Modigliani and Brumberg (1954).

[12]An alternative formulation of equations (1.11a), (1.15a), and (1.16a) can be achieved by redefining the choice variable K_n in terms of a "perpetual" sequence of one-period endowments of the composite good $\bar{X}$ that can be generated by the terminal stock of capital beginning at period $n + 1$ and extending indefinitely: $\bar{X} = dK_n$, where $0 < d < 1$ is, say, an "institutional" yield

per unit of bequeathed capital. This approach is an application of the general treatment of the problem of intertemporal decision-making as a choice between current consumption and perpetual or permanent future income flows (see Hirshleifer (1970), p. 69). If this definition of the choice variable implicit in K_n is valid, equation (1.16a) can be rewritten as

$$\log \frac{\bar{X}}{X_n} = \sigma_{\bar{X}n}(\log d - \tau_{\bar{X}n}) \ . \tag{1.16b}$$

Equation (1.16b) indicates that $\bar{X}$ would be smaller than X_n if the consumption unit had neutral (time) preference for own consumption in period n relative to an annuity of consumption opportunities bequeathed to heirs, since $\log d < 0$. If wage earnings and saving activity in period n were nil, the condition $\bar{X} < X_n$ would also imply that $K_n < K_{n-1}$. Savings in the final period of the planning horizon then would necessarily be negative.

[13] Hirshleifer (1958) showed that the separation theorem implicit in the assumption of "perfect capital markets" collapses whenever there is a divergence between the market's borrowing and lending rates of interest. The preceding statement can be viewed as a generalization of Hirshleifer's famous result; the separation theorem collapses even in the case of net lenders, since the gross lending rates of interest are not constant.

[14] Since the length of a period in any discrete time analysis is arbitrary, there is no loss of generality in this assumption. Alternatively, funds borrowed for a duration of more than one period can be looked upon as two separate loans taken on successive marketing dates, the latter designated to pay off the principal and interest on the previously maturing loans.

[15] The marginal as well as the average borrowing cost of capital can be expected to rise with the amount borrowed if only because of segmentation in

capital markets that arises from transaction costs and legal restrictions on certain kinds of borrowing. In addition, the probability of default, as perceived by the lending institution, is likely to rise with the consumption unit's debt-equity ratio even if the true risk per dollar of borrowing were constant or nil (for a discussion of similar arguments in the context of corporate finance, see Fisher (1959) and Stiglitz (1972)). However, the effect of an increase in D_{t-1} need not be symmetrical to that of a decrease in N_{t-1}. In the following analysis, "b" denotes the average borrowing rate of interest.

[16] Borrowing in order to finance the purchase of durable goods including own houses does not belong in this category since it is essentially determined according to whether it is more economical to rent the services yielded by durable goods or to own them. Thus it is largely independent of time preference for consumption and can be considered as special borrowing for productive saving that does not involve future management costs.

[17] Let the investor's initial net worth and his initial asset holdings be given by N_1 and K_1 in Figure 4 below and assume that he then finds it optimal to increase his current outlays on consumption by ΔC. He can follow one of two policies to accomplish that. Policy 1 requires a reduction in his net worth from N_1 to N_0, where $N_1 - N_0 \equiv \Delta N = \Delta C$. By the forthcoming analysis of optimal borrowing for productive saving, the investor's optimal asset holdings then would fall by an even greater amount from K_1 to K_0 (see n. 20). Policy 2 requires additional borrowings in the amount of ΔC. That policy thus amounts to a decision to keep intact the initial level of asset holdings at K_1, notwithstanding a decrease in N from N_1 to N_0. Clearly, Policy 1 is superior to Policy 2. In comparison to 1, Policy 2 generates higher returns but even higher borrowing costs. The net loss associated with 2 relative to 1 is represented by the triangular area APB in Figure 4.

[18] Differentiating the wealth constraint given in equation (3.3) partially with respect to D_{t-1} and equating the result to zero, we obtain

$$\frac{\partial W^{(n)}}{\partial D_{t-1}} = (\rho_t - \delta_t - D_{t-1}\frac{\partial \delta_t}{\partial D_{t-1}}) \prod_{i=t+1}^{n+1} \rho_i = 0 ,$$

from which equation (2.5) is easily derived. This differentiation assumes that r is not a direct function of K. If it were, the term $r(h)$ in equation (2.5) would be replaced by $MRR_t = r(h_t, K_{t-1})(1 + \epsilon_{rk})$, where $\epsilon_{rk} = \frac{\partial r_t}{\partial K_{t-1}} \cdot \frac{K_{t-1}}{r_t}$.

[19] Equation (3.6) implies that

$$\partial MbC_t/\partial D_{t-1} > -[r'(h)]^2/[(N_{t-1} + D_{t-1})\, r''(h_t)] \, . \tag{2.6a}$$

The left-hand side of equation (2.6a) is the slope of the supply curve of borrowed funds in Figure 3, MbC, and the right-hand side can easily be shown to be equal to $dr(h_t^*)/dK_{t-1} = r'(h_t^*)\, dh_t^*/dK_{t-1}$ (see n. 9). The latter expression represents the slope of the demand curve for productive capital depicted by $r(h(K_{t-1}))$ in Figure 4. The curve r must then cut the relevant MbC curve from above as Figure 4 illustrates.

[20] The effect of an exogenous increase in N_{t-1} on the optimal values of D_{t-1} and ℓ_t can be determined by differentiating the first-order optimality conditions given in equations (2.4) and (2.5) with respect to N_{t-1}. It is then seen that if $\partial r(h_t)/\partial K_{t-1} = 0$, then

$$\frac{dD_{t-1}^*}{dN_{t-1}} = \frac{\frac{\partial MbC}{\partial N_{t-1}}(N_{t-1} + D_{t-1})\, r''(h_t) + [r'(h)]^2}{\Delta} > 0 ,$$

where Δ is defined in equation (2.6). Similarly,

$$\frac{d\ell_t^*}{dN_{t-1}} = \frac{-r'(h_t)\ \frac{\partial MbC}{\partial D_{t-1}} - \frac{\partial MbC}{\partial N_{t-1}}}{\Delta} < 0 \ ,$$

since $r''(h_t) < 0$ and $\frac{\partial MbC}{\partial N_{t-1}}$ is assumed negative in sign. The geometrical proof illustrated in Figure 4 is based on the following reasoning. If MbC were just a function of D_{t-1}, then MbC_1 would be a horizontal translate of MbC_0, indicated by the curve CC, with the horizontal distance between the two being equal to $\Delta N = (N_1 - N_0)$. Even then, $\Delta K = (K_1 - K_0)$ would exceed ΔN. Moreover, since MbC is assumed to be a decreasing function of N, MbC_1 would lie below the auxiliary curve CC at any level of K_{t-1}. Clearly, then, the increase in K would exceed the increase in N. It should be noted, however, that if the marginal rate of return on assets were a decreasing function of the size of asset holdings (see the discussion in n. 10), then the curve $r(K_{t-1})$ in Figure 1, interpreted as indicating the marginal gross rate of return on capital, would be a downward-sloping curve. It might then be possible for an exogenous increase in N to lead to a reduction in the amount borrowed for productive saving. For evidence consistent with the proposition that the individual portfolio sizes are positively correlated with debt secured by investment assets, see Projector and Weiss (1966), Table A14.

[21]Given the one-period resource constraint as summarized by equation (2.2), the modified equation of motion (3.3) can be derived by an analysis similar to the one pursued in n. 22 and shown equal to

$$\dot{X}_t = [1 + r(h_{t+1}) - D_{t-1}\ b'(N_{t-1})]\ \dot{N}_{t-1} - \dot{N}_t \ , \qquad (3.3a)$$

where $-D_{t-1}\ b'(N_{t-1})$ is, by assumption, positive if $D_{t-1} > 0$. Clearly, the qualitative implications of equation (3.3a) are the same as those inferred from (3.3). Note that these implications would not be affected even if it were assumed that $\partial r(h_t, N_{t-1})/\partial N_{t-1} \neq 0$. In such a case, $r(h_{t+1})$ in equation (3.3a) would be replaced by MRR_{t+1}, where MRR is defined in n. 18.

[22]More specifically,

$$\Delta X_t = X_{t+1} - X_t = w^0 \Delta\ell_t + K_{t-1}[r(h_{t+1}) - r(h_t)] +$$

$$(K_t - K_{t-1}) r_{t+1} - \Delta K_t + \Delta K_{t-1} .$$

By a Taylor expansion of $r(h_{t+1})$ about $r(h_t)$ we obtain

$$r_{t+1} = r(h_t) + r'(h_t)\Delta h_t + r''(h_t)\frac{\Delta h_t^2}{2!} + \ldots .$$

Assuming that Δh_t is sufficiently small, $\Delta h_t^2 \approx 0$, the difference $r(h_{t+1}) - r(h_t)$ can be approximated by $r'(h_t)\Delta h$, and the second term on the right-hand side of equation (4.2), $K_{t-1}[r(h_{t+1}) - r(h_t)]$ can be approximated by $-K_{t-1}\, r'(h_t)\Delta\ell_t$. By collecting terms, we obtain equation (3.2).

[23]In addition, as investments in human capital accumulate, they become more specific and thus more risky to the investor. Relaxing our simplifying assumption concerning neutral attitudes toward risk, we may conclude that a risk-averse consumption unit would then seek to diversify its asset holdings through the building of a positive stock of nonhuman assets preceded, perhaps, by a period of initial dissaving or net borrowing to finance investment in human capital.

[24]This prediction is compatible with evidence reported by Becker concerning the association between the mean number of hours worked in the labor market by male cohorts of given educational and racial groups, the mean hourly earnings, and the nonwage family income of the same cohorts. The evidence based on the 1/1000 sample from the 1960 Census shows that an increase in hourly earnings

increases the number of hours worked and that an increase in nonwage family income reduces it at given average family size and age of the respective male cohorts (see Ghez and Becker (1972, Ch. III). In Becker and Ghez's analysis, changes in hours of work over the life cycle are expected to be independent of anticipated changes in nonwage income (or initial net worth) since their model does not recognize saving activity as an alternative to work and consumption activities.

[25]Friedman argues, however, that k may increase with the ratio of nonhuman capital to permanent income K/Y_p, because nonhuman capital, being more "liquid" and a better collateral against loans than human capital, provides a superior means of insurance against emergency consumption needs. An increase in K/Y_p thus reduces the need to save for emergency funds at any given level of permanent income (see Friedman (1957), p. 16). With neutral attitudes toward risk our analysis below implies that the absolute level of nonhuman wealth and possibly even its ratio to permanent income may be positively related to the propensity to save. Empirical evidence compatible with this latter prediction thus would indicate the relative importance of the productive saving hypothesis in explaining observed behavior.

[26]This result holds even if working and leisure time were not assumed to be constant in each period but were assumed to vary with wealth. Given the market price of time, w^0, then by equation (1.12) or (2.4) the absolute magnitudes of h and r, hence the slope of the consumption transformation curve, are uniquely related to the level of accumulated net worth as long as labor market time were positive. Furthermore, insofar as productive saving involved employment of hired factors, r(K) would generally be invariant to changes in wealth. As our discussion in Section I indicates,

the optimal employment of hired factors, being determined through wealth maximizing principles, is an increasing function of K and is independent of the consumption unit's wealth constraint as well as its wage earnings.

[27]Utilizing the preceding two-period analysis and assuming that optimal bequest is positive, the ratio of the discounted life time consumption to total wealth is given by

$$m_1 = \frac{W_2 - K_2}{W_2} < 1 \quad ,$$

and the ratio of the undiscounted life time consumption to life time income is given by

$$\hat{m}_1 = \frac{X_1 + X_2}{Y_1 + E_2 + rK_1} \equiv \frac{X_1 + X_2}{W_2 - rX_1} = \frac{X_1 + X_2}{X_2 + K_2 + X_2} < 1 \quad ,$$

where Y_1 denotes total income receipts, including the initial endowments obtained, in period 1. Clearly, any change in W_2 that increases the ratios X_2/X_1 and K_2/X_1 by the same proportion necessarily decreases both m_1 and $\hat{m}_1$.

[28]In the two period-no-bequest model, consumption in the second period equals the total amount of resources available in that period: $X_2 = K_1\rho(h_2) + E_2$. By definition, $X_2/W_2 = [K_1\rho(h_2) + E_2]/[Y_1\rho(h_2) + E_2]$, where Y_1 is defined in n. 27. Clearly, X_2 and W_2 are positive linear transformations of K_1 and Y_1. Thus $X_1/Y_1 \equiv 1 - K_1/Y_1$ is a decreasing function of X_2/W_2. The same result obtains if bequest is positive, since the ratio K_2/X_2 is assumed to be independent of wealth.

[29]Future income endowments consist of potential wage income only. Consequently, the endowment position E could shift along ray OE' following an increase in H only if current income endowments also consisted solely of wage income.

[30]Solmon (1971) reported a positive zero-order correlation between mean savings to income ratios of families classified by schooling attainments of their heads and the head's educational ranking. However, he has not controlled for the families' net worth or their age distribution.

[31]Some evidence in support of increasing average propensities to consume out of permanent income is reported by Crocket and Friend (1967). They have attempted to estimate permanent or "normal" income from cross-sectional data containing observations of the same household for more than one year. A principal finding of their regression analysis is that the ratio of net worth to normal income increases with income when the age of the household's head is held constant. They suggest that this finding is inconsistent with saving theories which contend that permanent savings is a constant fraction of permanent income.

[32]It should be noted that the statistics reported in Table 1 have been derived by summing data on consumption and income over all income and age brackets sampled in the nationwide surveys of consumer finances covering 1948, 1949 and 1950 incomes. One interpretation of these statistics is that they approximate the undiscounted ratios of life time consumption to life time income of consumption units in different occupational groups. Given a stationary equilibrium with no bequest, these ratios would be unitary across all groups. However, with optimal bequest being positive, the average undiscounted life time consumption to life time income would be less than unitary and our theoretical analysis would be compatible with the ranking of the statistics reported in Table 1 as our discussion in n. 27 indicates.

An alternative interpretation of the ratios reported in Table 1 is that they approximate, on the average, "current" consumption to income ratios of consumption units of age brackets in which savings is positive (say, age 40;

an assumption of this kind is made in a similar context in Friedman (1957, p. 92)). Given this interpretation, our theoretical analysis, again, is compatible with the ranking of the consumption to income ratios in Table 1, as the discussion in n. 28 indicates.

33 Other data concerning the ranking of propensities to save of different occupational groups also are consistent with this analysis. For example, Klein (1960) reports relatively higher rates of saving among entrepreneurial groups. He also shows that the self-employed save more than other families principally because of their business savings. More generally, Friend and Kravis (1957) show that saving rates of different occupational groups are closely correlated with the average income of these groups.

References

Becker, G. S. Economic Theory. New York: Alfred A. Knopf, 1971.

__________. Human Capital. New York: Columbia University Press for National Bureau of Economic Research, 1964.

Crockett, J., and Friend, I. "Consumer Investment Behavior," Determinants of Investment Behavior. Edited by R. Ferber. New York: Columbia University Press for National Bureau of Economic Research, 1967.

Fisher, I. The Theory of Interest. London: The Macmillan Company, 1930. Reprinted Kelly (1955).

Fisher, L. "Determinants of Risk Premiums on Corporate Bonds," Journal of Political Economy, 67 (June, 1959).

Friedman, Milton. A Theory of the Consumption Function. Princeton, New Jersey: Princeton University Press for National Bureau of Economic Research, 1957.

Friend, I., and Kravis, I. B. "Consumption Patterns and Permanent Income," American Economic Review, Proceedings 47 (May, 1957).

Ghez, G., and Becker, G. S. "A Theory of the Allocation of Time and Goods Over Time," Report No. 7217, Center for Mathematical Studies in Business and Economics, University of Chicago, 1972.

Hirshleifer, J. "On the Theory of Optimal Investment Decision," Journal of Political Economy, 66 (August, 1958).

__________. Investment, Interest and Capital. Englewood Cliffs, New Jersey: Prentice-Hall, 1970.

Jaffe, J. F. "Special Information, Security Regulations and Insider Trading." Unpublished Ph.D. thesis, University of Chicago, 1973.

Keynes, J. M. The General Theory of Employment, Interest and Money. New York: Harcourt, Brace and World, 1961.

Klein, L. R. "Entrepreneurial Saving." Proceedings of the Conference on Consumption and Saving. Vol. 2. Edited by I. Friend and R. Jones, Philadelphia, Pennsylvania, 1960.

Lorie, J. H., and Niederhoffer, V. "Predictions and Statistical Properties of Inside Trading," Journal of Law and Economics, 11 (April, 1968).

Marshall, A. Principles of Economics. 8th ed. London: The Macmillan Company, 1952.

Mincer, J. Schooling, Experience, and Earnings. New York: Columbia University Press for National Bureau of Economic Research, 1974.

Modigliani, F., and Brumberg, R. E. "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," Post Keynesian Economics. Edited by K. K. Kurihara. New Brunswick, New Jersey: 1954.

Neiderhoffer, V., and Osborne, M. F. M. "Market Making and the Reversal on the Stock Exchange," Journal of the American Statistical Association, 61 (December, 1966).

Projector, D. S., and Weiss, G. S. Survey of Financial Characteristics of Consumers. Board of Governors of the Federal Reserve System, 1966.

Scholes, M. S. "A Test of the Competitive Hypothesis: The Market for New Issues and Secondary Offerings." Unpublished Ph.D. thesis, University of Chicago, 1969.

__________. "The Market for Securities: Substitution versus Price Pressure and the Effects of Information on Share Prices," Journal of Business, 45 (April, 1972).

Solmon, L. "The Relationship between Schooling and Saving Behavior: An Example of the Indirect Effects of Education." New York: National Bureau of Economic Research, 1971. (Mimeographed.)

Schultz, T. W. "The Value of the Ability to Deal with Disequilibria." Unpublished manuscript, University of Chicago, 1974.

Stiglitz, J. E. "Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Take-Overs," The Bell Journal of Economics and Management Science, 1972.