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THE INFLUENCE OF PROBABILITY ON RISKY CHOICE: A PARAMETRIC EXAMINATION

P.K. Lattimore

J.R. Baker

A. Dryden Witte

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1050 Massachusetts Avenue
Cambridge, MA 02138
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ABSTRACT

The appeal of expected utility theory as a basis for a descriptive model of risky decision making has diminished as a result of empirical evidence which suggests that individuals do not behave in a manner consistent with the prescriptive tenets of EUT. In this paper, we explore the influence of probability on risky choice, by proposing and estimating a parametric model of risky decision making. Our results suggest that models which provide for probability transformations are most appropriate for the majority of subjects. Further, we find that the transformation differs for most subjects depending upon whether the risky outcomes are gains or losses. Most subjects are considerably less sensitive to changes in mid-range probability than is proposed by the expected utility model and risk-seeking behavior over "long-shot" odds is common

P.K. Lattimore
National Institute of Justice
633 Indiana Avenue, NW
Washington, DC 20531

J.R. Baker
Dept. of Management
Virginia Polytechnic Institute and State
Blackburg, VA 24060

A. Dryden Witte
Wellesley College and NBER
1050 Massachusetts Ave
Cambridge, MA 02138

1.0 INTRODUCTION

Results of experimental studies of risky decision making have diminished the descriptive appeal of expected utility theory (EUT; see von Neumann and Morgenstern 1947) and led to the development of a variety of alternative theories (see Kahneman, Slovic and Tversky 1982; Sugden 1986; Hogarth and Reder 1986; Machina 1987; and Fishburn 1987 for surveys). Development of methods for assessing preference models and, hence, describing risky decision making behavior has lagged however, remaining tied to the expected utility model as in the work by McCord and deNeufville (1986). Thus, while it is now well documented that a decision maker's sensitivity to probability levels and to changes in probability is not constant across the unit interval (e.g., Kahneman and Tversky 1979), little attention has been given to incorporating this knowledge in assessment methodology. It is strange that work estimating empirical models has continued under the maintained hypothesis of expected utility given the extensive experimental evidence at odds with EUT and the range of newer theories of risky decision making.

In this paper, we specify and estimate a parametric preference model that incorporates non-identity functions over both probability and outcome. The model allows, but does not require, the subjective valuation of both probabilities and outcomes. Our model is consistent with a set of alternative theories which modify some of the premises of EUT while maintaining the expectation-type structure of the preference functional.[2] The two-parameter form chosen for the probability function is a generalization of the frequency interpretation of probability. The form is sufficiently flexible to include (1) the EUT model (i.e., the probability function is identity); (2) a symmetrical form in which the sum of the weights equals one (a special case of models suggested, e.g., by Quiggin 1982, Yaari 1987, and Karmarkar 1978); and (3) a contin-

[2] For discussions of some alternative theories which depart from the expectation form see Machina 1987 and Becker and Sarin 1987.

uous approximation of the decision-weighting function of prospect theory (Kahneman and Tversky 1979). These various forms are derived by restricting the values of the parameters, as will be discussed in greater detail in the next section. The explanatory power of the resulting set of nested empirical models can be compared using traditional statistical tests to provide insight on the relevance of alternative theoretical models. Further, the estimated parameter values provide insight on the empirical influence of probability on risky decision behavior.

Briefly, our results show that models which allow for transformation of objective probabilities provide **significantly** better fits to our subjects' data than models which do not. Overweighting of "small" probabilities was common and the impact of changes in probability was disproportionately large when probabilities were "near" 0 or 1. For the majority of subjects, the "best-fit" model differed depending upon the domain (i.e., gains or losses). In particular, the properties of the probability function differed for the two domains with subcertainty found to be more common for losses than for gains. Finally, the results show that risk-seeking behavior over both gains and losses is common when the probability of the best outcome in "small," while risk avoidance is the norm when the probability of the best outcome is "large." These findings are consistent, for example, with an individual simultaneously gambling and purchasing insurance.

The following section presents the empirical model. Subsequently, we describe our estimation methodology and data. Section 5.0 presents our results and section 6.0 our discussion. In the final section, we present a summary and our conclusions.

2.0 EMPIRICAL MODEL

In this paper, we are interested in risky decision models of the following form:

$$S(Y) = \sum_{i=1}^{n} \pi(p_i)\, s(x_i), \tag{1}$$

where Y is a prospect which yields outcome x_i with probability p_i; $S(\cdot)$ is a function that converts a prospect Y to a level of satisfaction: $\pi(\cdot)$ is the subjective-probability or decision-weighting function: and $s(\cdot)$ transforms outcomes into levels of satisfaction.

A three-parameter general model is proposed. We assume that the satisfaction function, $s(\cdot)$, can be approximated by a power function, i.e.

$$s(x_i) = x_i^{\gamma}. \tag{2}$$

This form is concave, linear, or convex as $\gamma <=> 1$ and has been used extensively in utility function estimation. (See Fishburn and Kochenberger 1979 for a comparison of alternative forms for empirical utility functions.) Concavity, linearity, or convexity reflects diminishing, constant, or increasing marginal valuation of x and, if EUT is the theoretical construct, risk-avoiding, risk-neutral, or risk-seeking behavior. We estimate separate models for gains and losses, therefore permitting different forms in the two domains.

The decision-weighting function is a rational function as specified below:

$$\pi_i = \frac{\alpha p_i^{\beta}}{\alpha p_i^{\beta} + \sum_{k=1}^{n} p_k^{\beta}} \tag{3}$$

for i, k = 1, 2, ..., n, $k \neq i$, and $\alpha, \beta > 0$. In its most restricted form (i.e., $\alpha = \beta = 1$), equation 3 reduces to the frequency interpretation of probability, i.e. $\pi_i = p_i / \sum_{i=1}^{n} p_i$, where n = the number of outcomes. Further, for all values of α and β, $\pi(0) = 0$ and $\pi(1) = 1$. As can be seen, the weight given each event is a function of the entire probability distribution. This approach is consistent not only with the frequency interpretation of probability but also with recent theoretical models such as anticipated utility theory (AUT) proposed by Quiggin (1982) and Yaari (1987). AUT suggests, however, that transformations are applied to the cumulative (or the decumulative) probabilities rather than to individual probabilities, an approach which avoids violations of stochastic dominance implicit in the models of, for example, Karmarkar (1978) and Kahneman and Tversky (1979). As long as $\alpha = 1$, n = 2, our model is consistent with the special case of AUT in which $\pi(0.5) = 0.5$.

The β parameter determines whether there will be an inflection in the weighting functional and the direction of the inflection (e.g., concave-convex). Specifically, for all $0 < p_i < p^*$, $\pi_i >< p_i$ as $\beta <> 1$, where, at p^*, $\pi(p^*) = p^*$. Thus, the value of the β parameter determines whether "small" probabilities will be under- or over-weighted. If $\beta = 1$, $\pi_i >< p_i$ as $\alpha >< 1$ for all p_i.

The α parameter provides an additional weight on the outcome probability p_i. If $\alpha < 1$, p_i is downweighted, signifying perhaps a pessimistic view of the outcome occurring ("event or outcome pessimism"). Figure 1a shows a representative weighting function (n = 2) for the parameter values $\alpha = \beta = 0.5$; as can be seen, probabilities are over-weighted up to p = 0.2 and under-weighted thereafter. Generally, for n = 2, $p^* <=> 0.5$ as $\alpha <=> 1$. Additionally, if $\alpha < 1$, π is subcertain, i.e. $\sum_{i=1}^{n}\pi_i < 1$. Subcertainty might be viewed as "prospect pessimism" in the sense that the value of the prospect is reduced vis-a-vis certain outcomes. Kahneman and Tversky (1979) suggested subcertainty as a property of the prospect theory weighting function, although subcertainty may result in violations of stochastic dominance. Indeed, the property of subcertainty (as well as the parallel property of supra-certainty) was the basis of Fishburn's (1978) criticism of Handa's (1977) certainty equivalence theory (CET) model. Kahneman and Tversky also acknowledged the potential problem subcertainty posed for stochastic dominance and suggested that dominated prospects would be eliminated from consideration during an editing phase that they posit occurs before the evaluation of prospects.

If $\alpha = 1$, the weighting function is "certain" in that the sum of the weights attached to a complete prospect will equal to 1. Further, for distributions of the form $p_i = \frac{1}{n}$, $\forall\, i = 1, 2, \ldots, n$, inflection of the (certain) decision-weighting function occurs at $\frac{1}{n}$, so that $\pi_i = p_i$. Thus, for n = 2, $\pi(0.5) = 0.5$ and the weighting function is symmetrical about p = 0.5. Additionally, for the two-outcome case, the function is identical to that proposed by Karmarkar's (1978) subjective weighted utility theory (SWUT) and is consistent with Quiggin's (1982) AUT since $\pi(p) = 1 - \pi(1 - p)$.[3] See Figure 1b.

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[3] The restriction that $\alpha = 1$ generates a special case of the AUT model in which there is symmetry

Finally, if $\alpha > 1$, p_i is overweighted, signifying perhaps an optimistic view of the ith outcome occurring. In this configuration, the weighting function is supra-certain, i.e. the sum of the weights will be greater than 1.

In addition to being flexible enough to accommodate the subcertainty property of prospect theory's weighting function, the functional form can also generate decision weights which are subadditive and subproportional.[4] Thus, when $\alpha < 1$ and $\beta < 1$, equation 3 may be considered a continuous approximation of prospect theory's weighting function in that it accommodates the properties Kahneman and Tversky ascribe to this function. Of course, as a continuous approximation, the function does not allow investigation of discontinuities of π near $p = 0$ and $p = 1$, another property suggested for prospect theory's decision-weighting function.

Equation 3 provides a means to examine preference behavior vis-a-vis changes in probability. EUT, of course, suggests that decision makers are equally sensitive to changes in probability across the complete probability distribution (trivially, when $\pi_i = p_i$, $\forall$ i, $\frac{d\pi}{dp} = 1$). For more complex functionals, the derivative with respect to p is not constant. Thus, for certain parameter values, equation 3 suggests very little sensitivity to changes in mid-range probabilities versus considerable sensitivity to changes in probabilities near 0 and 1. Figure 2 shows a plot of the first derivative of equation 3 for $\alpha = \beta = 0.5$. As can be seen, a decision maker

between $\pi(p)$ and $\pi(1\text{-}p)$ of the form $\pi(p) = 1 - \pi(1\text{-}p)$. The AUT approach applies a transformation to the cumulative probability distribution and, thus, the appropriate AUT interpretation of the functions in Figure 1 would be that they are the weight, $\pi_2(p, 1\text{-}p)$, given to the worst outcome, x_2, when it has probability p. The weight associated with the best outcome, x_1, would be $\pi_1(1\text{-}p, p) = 1 - \pi_2(p, 1\text{-}p)$. We would like to thank an anonymous referee for insights on this issue. An additional point is that our parametric model, with α restricted to one, implies that $\pi(0.5) = 0.5$, as shown in Figure 1b. Although this was an early assumption of AUT (Quiggin 1982), this restriction was subsequently relaxed by Quiggin (1987). Thus, both forms shown in Figure 1 are consistent with AUT, but our full model (see equation 4) is not wholly compatible with AUT. Specifically, if $\alpha \neq 1$, $\pi_1(1\text{-}p, p) \neq 1 - \pi_2(p, 1\text{-}p)$.

4 Subadditivity and subproportionality are defined as follows. Given $0 < p, q, r < 1$, a weighting function is subadditive if $\pi(rp) > r\pi(p)$ (see Kahneman and Tversky 1979; also Segal 1987). If p = 0.002 and r = 0.5, subadditivity would suggest that $\pi(0.001)/\pi(0.002) > 0.5$. Kahneman and Tversky (1979) found that the majority of their subjects preferred a 0.001 chance of receiving 6000 to a 0.002 chance of receiving 3000. They interpreted these findings to support subadditivity over "small" probabilities (if the value function over outcomes is concave), as the results imply that $\pi(0.001)/\pi(0.002) > 0.5$. If $\alpha < 1$ and $\beta < 1$, equation 3 is subadditive for "small" probabilities. The decision-weighting function is subproportional if $\pi(pq)/\pi(p) < \pi(pqr)/\pi(pr)$. Again, if α and $\beta < 1$, equation 3 is subproportional for most probabilities. If $\alpha < 1$ and $\beta = 1$, the form is subproportional for all probabilities.

with such a weighting function would be extremely sensitive to changes in very small and large probabilities, but would be less sensitive to changes in mid-range probabilities. The doubling of "long-shot" odds would encourage such a decision maker to "disproportionately" increase the size of his bet. As α and β approach 1 (the EUT model), the first derivative also approaches 1.

Given the assumptions that (1) π and s are monotonic and continuous, (2) there exists a certainty equivalent (CE) value for which an individual is indifferent between receiving the CE and prospect Y, and (3) we can append an additive error term, we can identify the model to be used in the estimations. Specifically, by substituting equations 2 and 3 into equation 1, for n = 2, we derive:

$$CE = \left[\frac{\alpha p^{\beta}}{\alpha p^{\beta} + (1-p)^{\beta}} x_1^{\gamma} + \frac{\alpha(1-p)^{\beta}}{p^{\beta} + \alpha(1-p)^{\beta}} x_2^{\gamma} \right]^{1/\gamma} + \varepsilon. \quad (4)$$

Table 1 shows the three "nested" models derived from our general model (equation 4). We will refer to the model with no restrictions on the parameter values as the **full model**. The model in which α is restricted to be equal to one will be referred to as the **certain-weight model**, to reflect the certainty property of the decision-weighting function when $\alpha = 1$. Finally, the model in which both α and β are restricted to one will be referred to as the expected utility or **EU model**. These three models were estimated for two scenarios using procedures described in section 3.0. These two scenarios, one over risky gains and the other over risky losses, are described in section 4.0.

3.0 ESTIMATION PROCEDURE

CE data, collected as described in the next section, were used to estimate for each subject six risky decision models, one for each of our empirical models (Table 1) with separate estimations for gains and losses. A nonlinear least squares regression procedure was used to solve for the model parameters. The solution technique was the multivariate secant or false position method developed by Ralston and Jennrich (1978). Given that search procedures of this sort can generate local rather than global optima, a variety of starting values were used.

Since quite different starting values did not generate different parameter estimates, we believe that the parameter estimates reflect a global rather than local optimum. The estimations over losses were conducted using the absolute values of x_1, x_2 and CE.

4.0 DATA COLLECTION METHODS

Certainty equivalent data were elicited for risky gains and losses from two study groups. The first group consisted of male and female undergraduates at the University of North Carolina at Chapel Hill; the second group consisted of 18-to-22-year-old male property offenders incarcerated in North Carolina. These two groups served as unskilled and skilled subjects for the choice scenarios which dealt with potential gains from a breaking-and-entering crime and potential losses of freedom associated with a plea bargain or jury trial. These somewhat unusual scenarios were selected as part of a larger study of economic models of criminal behavior (Lattimore 1987). Thus, our intent was not to "train" would-be burglars but rather to elicit responses for risky choices in an environment which would be more interesting to the subjects, particularly the offender group, many of whom had faced these types of choices. (All of the prisoners were incarcerated as a result of committing a property offense and half of these subjects had received their prison sentences as a result of plea bargains.) Our initial concerns that the student subjects would have difficulty responding to questions concerning an illegal activity were allayed when a number of students who pretested the questionnaire reported no difficulty in responding to the questions.

The data were collected using an interactive computer program that elicited approximate CE data. The assessments were framed as criminal choice problems where subjects were asked to provide CE information for a set of "lotteries" over gains from a breaking-and-entering crime and losses resulting from a prison sentence. Because it could not be assumed that subjects were familiar with the certainty equivalent concept, a tutorial was provided (see Appendix 1). The gains scenario was framed as a risky return to a breaking-and-entering crime; the loss scenario was framed within the context of a plea bargaining situation.

The illegal-gains scenario solicited CE's for risky gains to a hypothetical breaking-and-entering crime. Subjects were asked to indicate the smallest amount of cash there would have to be in one market before they would choose to break into that market rather than an identical market where the gains from the break-in were risky, i.e. a p chance of $\$x_1$ and a 1-p chance of $\$x_2$. The subjects were instructed to assume that the markets were identical with respect to the skill required to accomplish the crime and the chance of being caught. These restrictions were introduced to make potential illegal gains independent of other aspects of the situation--the skill required and the probability of capture--that might affect a subject's response. Thus, the restrictions serve to assure that the two choices differ only with respect to the payoff. The values of x_1 and x_2 ranged from $0 to $1000; x_1 was always the best outcome. The maximum gain of $1000 was chosen because for the subjects' age group, 18-to-22-year olds, it was felt that this amount was large enough to allow diminishing (or increasing) marginal valuation while at the same time being an amount to which the subjects could "relate."

To assess preference functions over the domain of losses, subjects were presented with scenarios which involved the potential loss of freedom as a result of incarceration. The subjects were asked to consider accepting a plea bargain rather than going to trial for a breaking-and entering crime. Specifically, subjects were asked to indicate the longest sentence they would accept and pass up going to trial where they faced a sentence of x_1 months with probability p or x_2 months with probability 1-p. The values of x_1 ranged from 0 to 6 months and the values of x_2 ranged from 24 to 36 months; again, x_1 was always the best outcome (i.e., the least prison time).

The order in which the subjects completed the two sets of assessments was randomly determined (i.e., some subjects did the breaking-and-entering assessments first, others did the plea-bargaining assessments first). The order in which the pairs of outcome levels was presented was also randomly determined for each subject. For both scenarios, the values of p ranged from 0.01 to 0.99 and the probability vector was randomly ordered for each outcome pair.[5] Subjects were instructed to provide an answer in the "feasible" region for each problem

[5] This approach was intended to prevent subjects from "anchoring" their responses, which might occur

(i.e., a value between x_1 and x_2) and were asked to verify each response.[6] At this point, a response could be changed. Subjects were not allowed to change earlier responses. Thirty-four CE's were provided for each scenario by each offender; 29 CE's were provided for each scenario by each student. Parameter estimates were obtained for 57 subjects for the two scenarios. Results are reported in the next section.

5.0 RESULTS

In this section, we present the results of the statistical tests comparing the fit of the three models for the two scenarios and discuss the "goodness of fit" of the most appropriate models. Subsequently, we present the values of the parameter estimates for gains and losses. Discussion of the implications of the results is reserved for the next section.

Our first objective was to determine which of the three empirical models (Table 1) provided the "best" fit to the data. A model was considered "best" (most parsimonious) for a subject's data if the restriction(s) on the parameter(s) did not degrade **significantly** the fit of the data as measured by F tests. (The asymptotic properties of nonlinear estimators are discussed in many references, including Judge et al. 1985. All tests were conducted at the 0.05 level of significance.)

The number of subjects for which each model was most appropriate is shown in Table 2. As can be seen, the best model for the majority of subjects differed depending upon whether the outcomes were gains or losses. Specifically, the certain-weight model was found to be most appropriate for 51 percent of the subjects when the outcomes were gains, while the full model was best for 60 percent of the subjects when the outcomes were losses. Additionally, the EU model was appropriate for only 26 and 16 percent of the subjects for the domains of gains and

if the probabilities had been presented in ascending (or descending) order. It should also be noted that this approach differs from the "fractile" method of utility assessment as described in, for example, Keeney and Raiffa (1976). Specifically, s was not normalized to values between [0,1] and CE_i was not used to elicit CE_j where $j = i + 1$.

6 After a C_L value was provided, a query of the following form appeared on the screen: "**$**__ is the SMALLEST certain payoff at Harry's for which you would pass up the __ percent chance of finding **$**__ and the __ percent chance of finding **$**__ at Jack's. Do you agree with this (Y/N)?".

losses, respectively. The distributions of best model over the two domains differed significantly (χ^2 statistic = 16.1154).

Table 3 shows the distribution of "best fit" by type of subject. (The estimation procedure failed to converge for one or more of the models for six subjects' data in each domain. These results were excluded from the analysis.) As can be seen for the gains models, the modal best fit for the offenders and female students was the certain-weight model (for 12 of 17 and 11 of 20 subjects, respectively), while the EU model was best for 9 of 20 male students. A different pattern is apparent for the comparison of best-fit loss models. Both groups of males (i.e., offenders and males students) were predominantly categorized by the full model, while the full and EU models were about equally likely to provide the best fit to the female students' data.

Our second objective was to examine the explanatory power of the best models. The value of R^2 ranged from 0.10 to 0.99. The median R^2 was 0.73 for the gain models and 0.74 for the loss models.

The final results we present are the values of the estimated parameters. Recall from section 2.0 that the parameters in the model determine the shape of the satisfaction and probability transform functions. Additionally, as will be discussed in the next section, the parameter values provide insight into attitudes towards risk. The parameter values reported are the estimates from each subject's best-fit model. Results for the satisfaction functions (gains and losses) are presented first, followed by those for the decision-weighting functions.

The satisfaction function, s, over illegal gains was defined by the parameter γ. The estimates for γ ranged from a low of 0.3335 to a high of 7.4726; the mean value was 1.3751 and the median was 0.9841. Table 4 summarizes these results. As can be seen subjects were about equally likely to have concave and convex satisfaction functions over gains. However, the hypothesis that $\gamma = 1$ could not be rejected for 31 of the 57 models (two-tailed test); for 15 of the remaining models, $\gamma < 1$ and for 11, $\gamma > 1$.

The value function component of the loss preference model measures the (dis-)value associated with "doing time," i.e. of "losing" free time. The estimates of the γ parameter ranged from

0.2781 to 3.7148; the mean was 1.3237 and the median was 0.9728. Results are included in Table 4. As with the gains models, most values were "close to" 1 so that the hypothesis that $\gamma = 1$ could not be rejected for 40 of the 57 subjects. For 15 of the 17 remaining subjects, our results indicate $\gamma < 1$, (significance level of 0.025) implying diminishing marginal valuation of prison time. These results are consistent with other reports suggesting that the value (or utility) function over losses is convex.[7] For only 2 subjects was $\gamma > 1$ -- a result which would be consistent with a globally concave utility function.

The α and β parameters define the shape (and properties) of π. Table 5 presents the results for the illegal gains model. The entry for $\alpha = \beta = 1$ corresponds to the 15 EU model subjects (see Table 2). The 29 subjects for whom $\alpha = 1$ (certain-weight model) had values of $\beta < 1$, implying a certain weighting function which overweights probabilities less than 0.50. Of the 13 subjects for whom the full model provided the best fit, α and β were both less than 1 for 10. Thus, for these subjects, π was subcertain with "small" probabilities overweighted. For two subjects, $\alpha < 1$ and $\beta > 1$ implying underweighting of "small" probabilities. For one subject, $\alpha > 1$ and $\beta < 1$ implying overweighting of "most" probabilities. The estimated values for α and β are plotted in Figure 3a.

We now consider the results for the decision-weighting function over losses -- finding considerable differences from those reported for gains. For 51 percent of the subjects, both α and β were less than 1 (see Table 6 and Figure 3b). These values suggest a decision weighting function similar to that shown in Figure 1a. For an additional 25 percent of the subjects, $\beta < 1$ and $\alpha = 1$, suggesting a (certain) weighting function consistent, for example, with Quiggin's AUT. For only 16 percent of the subjects was the decision-weighting function the identity function, as predicted by EUT.

Figure 4 shows the decision-weighting functions over gains and losses for two subjects. For the first subject (see Figure 4a), the certain-weight model was most appropriate for both scenarios and the decision-weighting functions were essentially identical. The estimated values

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[7] The estimations over sentence length were conducted for the absolute values of x_1, x_2, and CE.

of β were 0.2364 and 0.2559 for the gain and loss scenarios, respectively. Further, it can be seen that this subject's data do not indicate much sensitivity to changes in mid-range probabilities. Specifically, the value of the decision weight increases by only about 0.2 as p increases from 0.2 to 0.8. Figure 4b illustrates the probability transforms for a representative subject who used different probability transformations for the loss and gain scenarios. For this subject, the best-fit model was the certain-weight model when the outcomes were gains ($\beta = 0.5044$) and the full model when the outcomes were losses ($\beta = 0.3624$ and $\alpha = 0.5745$).

6.0 DISCUSSION

The results are consistent with the large body of experimental literature which suggests, in contrast to the tenets of EUT, that most decision makers transform probabilities in the course of assessing the value of a risky prospect. Specifically, "small" probabilities are overweighted and "large" probabilities are underweighted ($\alpha \leq 1$, $\beta < 1$) by 68 percent (gains) and 79 percent (losses) of our subjects. Additionally, the results suggest that risky losses are treated differently than risky gains, as suggested by Kahneman and Tversky (1979). However, in contrast to Kahneman and Tversky's work, we find that the primary distinction between models of risky gains and losses is in the character of the probability transform rather than the outcome transform. As different outcomes were used for the gains and losses, it may be that the differences in the most appropriate model in the two domains are attributable to the characteristics of the outcomes and not to any universal difference in assessing gains and losses. Whether these findings are peculiar to the scenarios used or will be found through additional experimentation to be more generally applicable, they are interesting to the extent that most theoretical models considered do not suggest that the weighting function is scenario dependent.

Probability transformation (α, $\beta \neq 1$) was more common when the outcomes were losses than when they were gains (84 percent versus 74 percent of the models). Further, the nature of the transformation differed significantly between the two domains. The certain-weight model was best for 51 percent of the subjects when outcomes were gains in contrast to 25 percent when the outcomes were losses. The more complex, full model was appropriate for 60 percent of the subjects when outcomes were losses and only 23 percent when outcomes were gains. The distinction between these two models is, of course, whether the value of α is 1. When $\alpha = 1$, the function is "certain" in that the sum of weights over a prospect total to 1. If in addition, as we found, β is less than 1, individuals will overweight probabilities up to 0.5 and underweight larger ones. Such a π function is consistent with models suggested by Quiggin (1982) and, for the two-outcome case, is equivalent to that proposed by Karmarkar (1978).

When both α and β are less than 1, as was true for the majority of subjects when the outcomes were losses, the weighting function acquires the rather "disagreeable" property of subcertainty. Subcertainty is disagreeable to the extent that it can result in violations of stochastic dominance in choices between prospects. Simplistically, subcertainty would suggest that a decision maker would prefer a certain \$10 to a 50-50 chance of receiving \$10 or \$10. More seriously, subcertainty would also imply \$10 is preferred to a 50-50 chance of $\$10 + \varepsilon_1$ or $\$10 + \varepsilon_2$, for arbitrarily small, positive ε_1, ε_2. As was noted in section 2.0, subcertainty is among the properties ascribed to the prospect theory weighting function by Kahneman and Tversky (1979), who suggested that individuals would edit dominated prospects from consideration prior to evaluation. Our results suggest that subcertainty is common -- particularly over risky losses. We suggested that subcertainty of the weighting function could imply event and prospect pessimism by the decision maker vis-a-vis a certain outcome (the CE). These pessimism concepts are consistent with the interpretation of decision weights by Kahneman and Tversky (1979, p. 280) as measuring "the impact of events on the desirability of prospects and not merely the perceived likelihood of these events." Another interpretation is that given an undesirable prospect (be it *any* risky situation or prison sentences), the "rational" decision maker may "reserve" part of the cumulative weighting function for some unspecified and, definitionally, impossible event. Thus, for example, when contemplating the choice between a distribution of sentence at trial and a plea bargain, which yields a certain prison term, the decision maker may consider and assign probabilities to other unspecified and impossible outcomes such as, for example, the prosecutor's key witness will fail to appear, the judge will throw the case out of court, or "Providence" will intervene. The discrepancy between the cumulative densities over probability and weights then may be interpreted as a measure of attitude towards risky situations in general and towards the prospects in particular. As prison sentences are less desirable than ill-gotten gains, it would then not be surprising to find that subcertain models were more prevalent for our loss situations.

A curvilinear decision weighting function has implications both for a decision maker's sensitivity to changes in probability and attitudes towards risk. Consider the simplest case. Let $x_1 = 1$ and $x_2 = 0$. Then, $CE = \pi_1^{1/\gamma}$ and $\frac{\partial CE}{\partial p} = \frac{1}{\gamma} \pi^{(1-\gamma)/\gamma} \frac{d\pi}{dp}$. If γ is also equal to 1, $\frac{\partial CE}{\partial p} = \frac{d\pi}{dp}$. Thus, for $\alpha = \beta = 0.5$, the value of the CE can be read directly from Figure 1a and the rate of change in the value of the CE is as shown in Figure 2. As can be seen, small increases in very small and large probabilities result in disproportionate increases in the willingness to pay for the prospect, whereas changes in mid-range probabilities generate disproportionately small increases. Of course, the EUT model would suggest a constant rate of change.

We turn now to a consideration of the impact of curvilinear weighting functions on attitudes towards risk. Risk attitudes were measured using the risk premium (RP). Our results indicate that for (at least) some probabilities 87 percent of the subjects are risk seeking when the outcomes are gains and 89 percent are risk seeking when the outcomes are prison sentences. The risk-seeking behavior is attributable primarily to transformations of objective probabilities rather than to convexity of the satisfaction functions. Both the shape of the probability (π) and the satisfaction functions affect the magnitude of RP. For purposes of these analyses, the value p^R will be defined as the value of p_1 at which the RP = 0. The value of p^R is not unique, but is dependent upon x_1, x_2, α, β and γ. A unique p^R does exist, however, for the special case in which one of the risky outcomes is 0. The RP $>=<$ 0 as the expected value is greater than, equal to, or less than the CE. Let $x_2 = 0$, then RP $>=<$ 0 as

$$px_1 >=< [\pi(p)x_1^{\gamma}]^{1/\gamma},$$

or as $p >=< \pi(p)^{1/\gamma}$. Thus, the parameters α, β, and γ define a unique transition probability, p^R, where $[\pi(p)]^{1/\gamma} = p$. If $\gamma = 1$, the RP will be positive for all $p < p^R = p^*$, where, as before, p^* is the value at which $\pi(p) = p$.[8] A search algorithm was used to identify the value p^R for each subject and model. These p^R are appropriate for all $[x_1, p; 0, (1-p)]$ events. For gains, therefore, p is the probability of the best outcome (e.g., $1000) occurring and for losses p is the probability of the worst outcome (e.g., 36 months in prison) occurring.

For the illegal gains scenario, we found that 11 subjects were risk averse and 6 subjects were risk seeking for all probabilities and outcomes (see Table 7). The remaining 40 subjects were risk seeking for some values of p, the probability of the best outcome. Estimated values of p^R varied from 0.02 to 0.89. Individuals were risk seeking, therefore, for all p **less than** these values. On average, these 40 subjects were risk seeking for all probabilities less than about 0.44 (standard deviation = 0.2294).

For the loss scenario, $x_1 > 0$ is the **worst** outcome and p is the probability of the worst event happening. (The best outcome is, of course, 0, realized with probability 1-p.) The estimated values of p^R ranged from 0.03 to 0.77. On average subjects were risk seeking for all p greater than about 0.30 (standard deviation = 0.1772) -- or, equivalently, for all $0 < (1 - p) < 0.70$. Subjects were risk seeking for all probabilities **greater than** these p^R values.. Although this result may seem counter-intuitive, it is consistent with the findings with respect to gains, i.e. individuals are risk seeking when the probability of the **best** outcome is "small." Consider the following example. For one subject, $\gamma = 0.9728$, $\beta = 0.5532$, $\alpha = 1$, and $p^R = 0.48$. If $x_1 = 24$ and $p = 0.25$, the EV = 6 and the CE = 8.22. Thus, this individual's plea bargaining model suggests that he would accept a plea bargain sentence about 2 months longer than the expected value of the trial outcome. As he is willing to "pay" a higher certain "price" than the EV of the trial prospect, he can be deemed risk avoiding. On the other hand, if $p = 0.75$ (implying a 0.25 chance of 0 months, the best outcome), the EV = 18 and the CE = 15.35, suggesting risk-seeking behavior with respect to the best outcome.

The results suggest that risk-seeking behavior is common when the probability of the best outcome is relatively "small" for both gains and losses. These findings are, of course, con-

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[8] If $\alpha = \beta = 1$, the RP $> = < 0$ as $\gamma < = > 1$ for all probabilities, consistent with EUT risk measures.

sistent with an individual simultaneously gambling and purchasing insurance. Risk seeking appears, however, to "persist" for a much larger probability range when the outcomes are losses.

7.0 SUMMARY AND CONCLUSIONS

Preference models over gains and losses were estimated that explicitly allowed for subjective valuation of outcome levels and probabilities. This methodological approach extends the recent literature by beginning the search for an appropriate parametric model of risky decision making that will reflect empirical regularities and recent theoretical work. Our findings are consistent with the results reported in the literature which suggest that transformations of probabilities are common in risky decision making. However, our work extends this literature in that it demonstrates that behavior is **significantly** affected by the transformation of probabilities. Specifically, our results indicate that, for most subjects, models which allow for transformation of objective probabilities provide significantly more explanatory power than models such as EUT which do not allow such a transform. The EUT model provided the "best" explanation of the decision making behavior of only about 20 percent of the subjects (26 and 16 percent for gains and losses, respectively). The full model (with both parameters assuming values less than 1) was more likely to be appropriate for losses and the certain-weight model ($\alpha = 1$; $\beta < 1$) was more likely to be appropriate for gains. In Section 6.0, we considered the implications of the parametric estimates for behavior towards risk. The deviation of π from identity was shown to imply variation in responsiveness to changes in p, with individuals exhibiting greater sensitivity to changes in small and large probabilities than to mid-range probabilities. Additionally, the results showed that risk seeking over "small" probabilities is common, while risk avoidance is the norm for larger probabilities. Risk seeking was found to be appropriate for a wider probability range when outcomes were losses than gains. Two major differences in gain and loss models were thus identified. First, the difference in results for the two domains suggested that subcertainty ("pessimism" towards the risky prospects) is more common for losses than gains. Secondly, risk-seeking behavior appears to be more persistent with respect to probabilities for losses.

The intent of our research was to provide a systematic empirical test of some of the more recent theoretical and empirical insights regarding risky decision making. We were particularly interested in providing insights on the influence of probability on risky choice. In pursuit of this objective, we extended the methodology of preference functional assessment to accommodate non-linearity of preferences with respect to probability and proposed a quite flexible parametric form for the probability function. We believe that assessment of preference functionals which accommodate nonlinearities in probability provides more information than examination of pair-wise choices. The benefit of the estimation of a parametric model is the ability to separate the influence of outcomes and probabilities on risky choice behavior, yielding insight on (1) the extent of over- and underweighting of probabilities, (2) sensitivity to changes in probability, (3) the impact of decision weights on risk-taking benavior, and (4) the impact of outcomes on risk-taking behavior

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APPENDIX 1

This appendix provides a description of the interactive computer program that was used to elicit the CE's. The initial screen provided a general introduction to the task. This introduction noted that the subject is going to be shown some choices "similar to those that might be faced by an individual who is trying to decide which store to break into or whether to accept a plea bargain" and that his/her responses should be "what you would do if you were faced with the choices that will be shown." The introductory screen also noted that responses to the questions "in no way imply that you would ever break into a building or engage in any other criminal activity." This caveat was included to assure the subjects, particularly the offenders, that their responses would not be used against them in any way.

The situation for the breaking-and-entering scenario was described as follows:

> Suppose that you had decided to break into one of two markets--Jack's or Harry's. Further, suppose that you knew it would take the same skill to break into either market and that the risk of capture was the same. Further, suppose that you know that Jack has $900 in his register half the time and $100 the other half, while Harry always has some cash in his register.

An example of the types of choices that would be shown was then given. The possible payoffs at Jack's in the example above is a 50 percent chance of $900 and a 50 percent chance of $100. The explanation accompanying the example (shown next) provided instruction as to how they should think about the choice problems.

> If you knew Harry's register contained $110, you would probably choose to break into Jack's. On the other hand, if you knew Harry had $890 in his register, you would probably choose his market. What if Harry had $300? Or $700? Which would you choose?
>
> As you can see, there is some number of dollars less than $900 and more than the $100 payoff at Jack's for which it would be difficult for you to decide which market to break into. Your task is to decide the S[illegible]T amount of certain cash at Harry's for which you would choose to break into Harry's rather than Jack's:

The instructions concluded by asking the subjects to "Consider each question carefully and tell us how you would act if you were really faced with each choice."

The structure of the plea bargaining scenario introduction is identical to that of the breaking-and-entering scenario shown above. An example of the plea-bargaining scenario is given below.

> Suppose you were captured breaking into a market and are discussing your alternatives with your attorney. Your attorney feels that if your case goes to court you will face a 50% chance of receiving a 6-month sentence and a 50% chance of receiving a 36-month sentence.
>
> The district attorney has offered to plea bargain: You will get a sentence shorter than 36 months if you agree to plead guilty.
>
> What is the LONGEST sentence the DA could offer before you would just forego the trial and accept the shorter sentence?

More details of the breaking-and-entering and plea-bargaining questions are shown in Figure 5.

TABLE 1. EMPIRICAL MODELS

MODEL	FREE PARAMETERS	RESTRICTION(S)
Full Model	α, β, γ	none
Certain-Weight Model	β, γ	$\alpha = 1$
EU Model	γ	$\alpha = \beta = 1$

TABLE 2. BEST-FIT MODEL RESULTS*

MODEL	FREE PARAMETERS	GAINS	LOSSES
Full Model	α, β, γ	13	34
Certain-Weight Model	β, γ	29	14
EU Model	γ	15	9

* Entries are number of subjects.

TABLE 3. BEST-FIT MODEL RESULTS BY SUBJECT TYPE*

TYPE	FULL	CERTAIN-WEIGHT	EU	TOTAL
GAINS				
Offenders	4	12	1	17
Male Students	5	6	9	20
Female Students	4	11	5	20
Total	13	29	15	57
LOSSES				
Offenders	15	5	0	20
Male Students	12	7	3	22
Female Students	7	2	6	15
Total	34	14	9	57

* Entries are number of subjects.

TABLE 4. ESTIMATED VALUES OF γ*

MODEL	VALUE OF γ $0 < \gamma \leq 1$	$1 < \gamma \leq 2$	$\gamma > 2$	Total
GAINS				
Full Model	3	8	2	13
Certain-weight	12	14	3	29
EU Model	10	3	2	15
Total	25	25	7	57
LOSSES				
Full Model	11	14	9	34
Certain-weight	11	2	1	14
EU Model	4	4	1	9
Total	26	20	11	57

* Entries are number of subjects.

TABLE 5. VALUES OF DECISION WEIGHT PARAMETERS: ILLEGAL GAINS*

	$\alpha < 1$	$\alpha = 1$	$\alpha > 1$	Total
$\beta < 1$	10	29	1	40
$\beta = 1$	0	15	0	15
$\beta > 1$	2	0	0	2
Total	12	44	1	57

*Entries are number of subjects.

TABLE 6. VALUES OF DECISION WEIGHT PARAMETERS: PLEA BARGAINING*

	$\alpha < 1$	$\alpha = 1$	$\alpha > 1$	Total
$\beta < 1$	29	14	2	45
$\beta = 1$	0	9	0	9
$\beta > 1$	3	0	0	3
Total	32	23	2	57

*Entries are number of subjects

TABLE 7. RISK ATTITUDES BY MODEL*

MODEL	GAINS			LOSSES		
	RS	RS/RA	RA	RS	RS/RA	RA
Full Model	1	11	1	4	29	1
Certain-Weight	0	29	0	4	10	0
EU Model	5	-	10	4	-	5
Total	6	40	11	12	39	6

* Entries are number of subjects.
Subjects were classified as risk seeking (RS) and risk averse (RA) vis-a-vis the **best** outcomes for both scenarios.

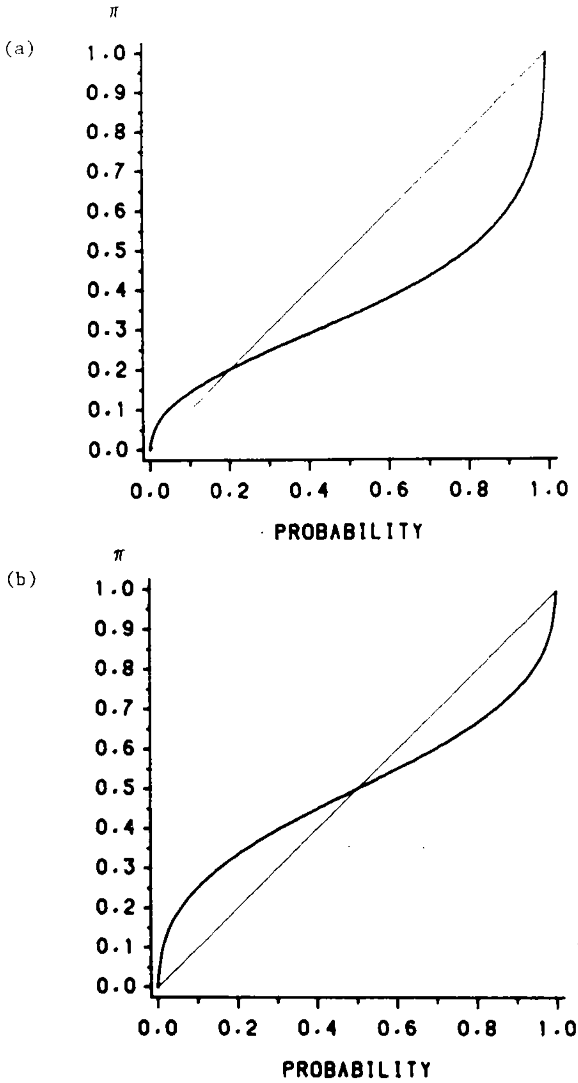

Figure 1. Decision-weighting function π for example parameter values: Decision-weighting functions for (a) $\alpha = \beta = 0.5$ and (b) $\alpha = 1$ and $\beta = 0.5$

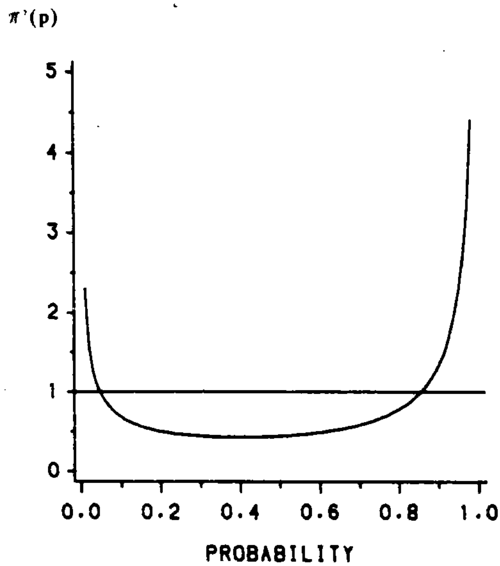

Figure 2. **First derivative of the π function:** First derivative of π for $\alpha = \beta = 0.5$

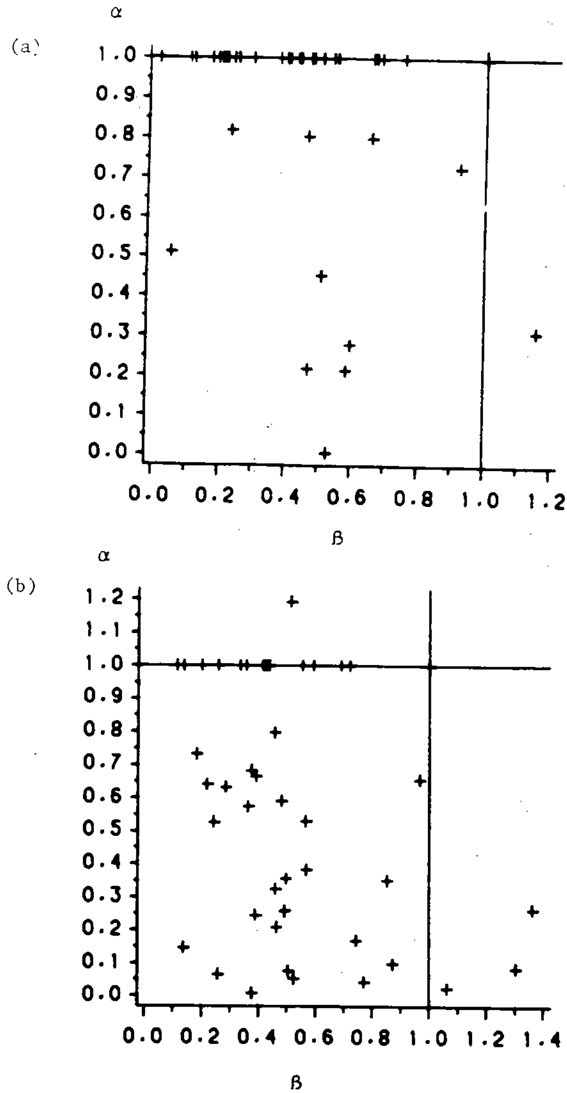

Figure 3. Estimated parameter values for the π function: Estimated parameter values for (a) the illegal-gain models and (b) the plea-bargaining models.

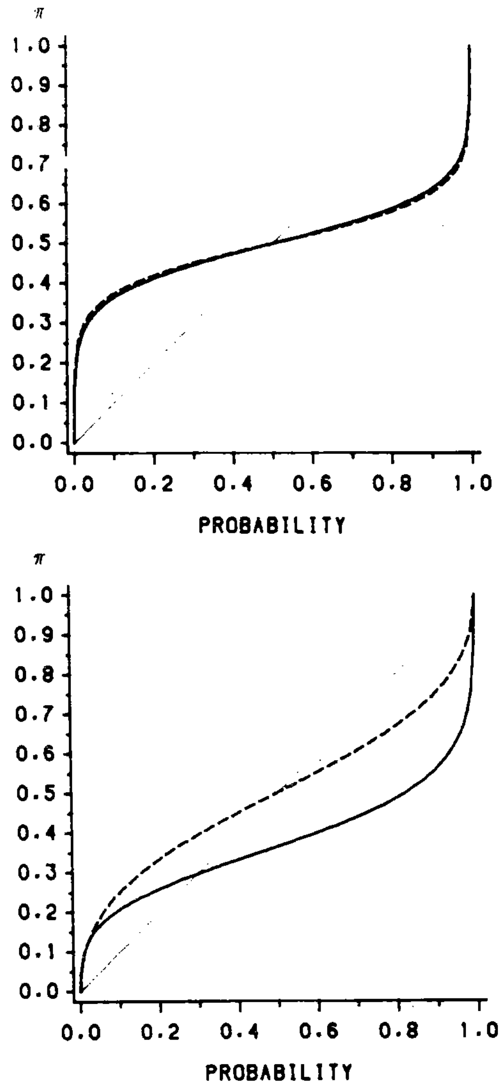

Figure 4. Example decision-weighting functions for two subjects: The dashed and solid lines show the estimated decision-weighting functions for the gains and loss models, respectively for the subjects for whom (a) $\alpha = 1$ and $\beta = 0.2364$ (illegal returns model) and $\beta = 0.2559$ (sentence length model); and (b) $\alpha = 1$ and $\beta = 0.5044$ (illegal returns model) and $\alpha = 0.5745$ and $\beta = 0.2559$ (sentence length model).

PAYOFF FROM JACK'S
25% chance of $900
75% chance of $100

PAYOFF FROM HARRY'S
$? for sure

What is the SMALLEST number of dollars there would have to be in Harry's register before you would choose Harry's rather than Jack's?

YOUR RESPONSE SHOULD BE AN AMOUNT BETWEEN $900 AND $100.

ENTER YOUR RESPONSE $

(after response -- say 300)

$300 is the SMALLEST certain payoff at Harry's for which you would pass up the 25% chance of finding $900 and the 75% chance of finding $100 at Jack's.

Do you agree with this (Y/N)?

(if no)
RECONSIDER THE CHOICE AT JACK'S AND ENTER YOUR RESPONSE $

(if yes, to next choice at Jack's)

TRIAL OUTCOMES
50% chance of 6 months
50% chance of 36 months

PLEA BARGAIN SENTENCE
? months for sure

What is the LONGEST sentence the DA could offer before you would take the chance of going to trial?

YOUR RESPONSE SHOULD BE A NUMBER OF MONTHS BETWEEN 6 AND 36.

ENTER YOUR RESPONSE IN MONTHS

(after response -- say 18)

You would go to trial where there is a 50% chance of receiving a 36-month sentence and a 50% chance of receiving a 6-month sentence if the sentence offered by the DA was MORE than 18 months.

Do you agree with this (Y/N)?

(if no)
RECONSIDER THE TRIAL OUTCOMES AND
ENTER YOUR RESPONSE IN MONTHS

(if yes, to next trial outcomes)

Figure 5. Examples of breaking-and-entering and plea-bargaining questions.