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THE TIME-VARYING-PARAMETER MODEL AS AN ALTERNATIVE TO ARCH FOR MODELING CHANGING CONDITIONAL VARIANCE: THE CASE OF LUCAS HYPOTHESIS

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ABSTRACT

The main econometric issue in testing the Lucas hypothesis (1973) in a times series context is the estimation of the variance conditional on past information. The ARCH model, proposed by Engle (1982), is one way of specifying the conditional variance. But the assumption underlying the ARCH specification is ad-hoc. The existence of ARCH can sometimes be interpreted as evidence of misspecification. Under the assumption that a monetary policy regime is continuously changing, a time-varying-parameter (TVP) model is proposed for the monetary growth function. Based on Kalman filtering estimation of recursive forcast errors and their conditional variances, the Lucas hypothesis is tested for the U.S. economy (1964.1 - 1985.4) using monetary growth as an aggregate demand variable. The Lucas hypothesis is rejected in favor of Friedman's (1977) hypothesis: the conditional variance of monetary growth affects real output directly, not through the coefficients on the forcast error term in the Lucas-type output equation.

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"... a person's uncertainty about the future arises not simply because of future random terms but also because of uncertainty about current parameter values and of the model's ability to link the present to the future." [Harrison and Stevens, 1976, p. 208]

Section I. Introduction

The Lucas hypothesis (1973) predicts a negative relationship between the variance of nominal shocks and the magnitude of the output-response to nominal shocks. Lucas (1973), Froyen and Waud (1980), Alberro (1980), Kormendi and Meguire (1984) and others have examined the Lucas hypothesis using the cross-country data, assuming that policy regimes did not change within countries, different policy regimes being represented by different countries. However, the assumption of a constant variance of nominal shocks within a country over time is not realistic. The measure of variance conditional on information available at the time of forecasting may be time-varying due, for example, to a continuously changing policy regime which is represented by evolutionary regression coefficients. This conditional variance, rather than the unconditional variance which is based on the whole sample, is what really matters for the behavior of economic agents, as Engle (1982) has pointed out. A more powerful test of the Lucas hypothesis may therefore result from modeling variation in conditional variance through time.

This paper discusses some of the implications of a continuously changing monetary policy regime (of the Federal Reserve) on tests of the Lucas hypothesis and proposes an alternative test in a time series context using money growth (M1) as an aggregate demand variable. The key issue involved in testing the Lucas hypothesis in the time series context is that of estimating the conditional variance which changes over time. Calculating a moving variance based on several past observations (for example, Froyen and Waud, 1984, and Lawrence, 1983) is one way of capturing the changing conditional variance of time series data, and the Autoregressive Conditional Heteroscedasticity (hereafter, ARCH) modeling introduced by Engle (1982) is another way. The former method usually gives biased estimates of the conditional variance, as the mean of the series is misspecified due to the failure to incorporate the explanatory variables. On the contrary, a problem with Engle's ARCH model is that the specification of the conditional variance is ad-hoc, in the sense that it is arbitrarily assumed to depend on past squared innovations. Besides, neither method specifies the source of changing conditional variance.

In many cases existence of ARCH can be interpreted as evidence of misspecification. If we suspect that a changing policy regime is the cause of an ARCH effect (or the source of misspecification), then, a time-varying-parameter model (hereafter, a TVP model) may be preferred to the ARCH model. The empirical results presented in this paper suggest that the existence of ARCH in a monetary growth function is mainly due to the evolutionary regression coefficients of the model.

Kalman filtering, which was first introduced in the Engineering literature, is applied to estimate the time-varying coefficients of the TVP model. As by-products of the Kalaman filtering estimation of the TVP model, recursive forecast errors and their conditional variances are obtained, which can be used to test the Lucas hypothesis. A nice thing about Kalman filtering is that it gives us insight into how a rational economic agent would revise his estimates of the coefficients of the model in a Bayesian fashion when new information is available, especially under the changing policy regime.

An alternative to the Lucas hypothesis is Milton Friedman's (1977) hypothesis which states that the increased variability of the inflation rate causes a reduction in the allocative efficiency of the price system, causing a reduction in natural level of output. By allowing the natural level of output to depend on the conditional variance of the monetary forecast error,[1] Friedman's hypothesis is also tested.

The organization of the paper is as follows. In section II, based on the stability test results of the regression coefficients, a monetary growth function is specified and estimated assuming the regression coefficients follow random walks. Section III provides further justification for TVP modeling of the monetary growth function by showing that the existence of ARCH in the OLS regression is mainly due to the time-varying property of the coefficients of the model. In section IV, empirical tests of the Lucas hypothesis and Friedman's hypothesis are performed based on the Kalman filtering estimation of forecast errors and their conditional variance from section II. Section V concludes this paper.

Section II. Kalman Filtering and TVP Estimation of the Monetary Growth Function

McNees (1986) states, "Policy reaction function is likely to be a fragile creature. Over time, ... the importance attached to conflicting objectives (of the policy) may change, (policy makers') views on the structure of the economy may change, ..." In this section, the TVP model is applied to a monetary growth function. Stability tests for the

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[1] In money oriented Phillips-curve models such as Barro (1976), unanticipated inflation is posited as an intermediate link in the causal chain connecting unanticipated money growth to real variables in the system. So, the conditional variance of monetary forecast errors is used as a proxy for the variability of inflation rate.

regression coefficients reported in Appendix 1 provide a rationale for the TVP modeling. Two different stability tests were performed on the monetary growth function specified in (2.1) below, one against the alternative hypothesis of 'unstable regression coefficients,' and the other against the alternative hypothesis of 'random walk coefficients.' The null hypothesis of constant regression coefficients is rejected for both of the tests.

Based on the stability test results, it is assumed that each of the regression coefficient of the model follows a random walk. The TVP model[2] with estimmated innovation variances for U.S. quarterly data 1964.1 - 1985.4 is

$$DM1_t = \beta_{0t} + \beta_{1t}DINT_{t-1} + \beta_{2t}INF_{t-1} + \beta_{3t}SURP_{t-1} + \beta_{4t}DM1_{t-1} + e_t \quad (2.1)$$

$$\beta_{it} = \beta_{it-1} + v_{it}, \ (i=0,1,..,4)$$

$$\sigma_e^2 = 0.126284$$

$$\sigma_{v0}^2 = 0.012133$$

$$\sigma_{v1}^2 = 0.000896$$

$$\sigma_{v2}^2 = 0.074544$$

$$\sigma_{v3}^2 = 0.000683$$

$$\sigma_{v4}^2 = 0.001184,$$

where DM1, DINT, INF, and SURP stand for the quarterly M1 growth rate, the change in the interest rate on Treasury bills, inflation measured by the CPI, and the full-employment budget surplus, respectively. In estimating the above non-time-varying parameters of the model, the likelihood value

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[2] The specification of the model (i.e., the choice of the right hand side variables) is motivated by Mishkin (1981) and Weintraub (1981). The OLS regression results of the model are very similar to theirs, except that lagged inflation term is added in our model.

obtained from the Kalman filter algorithm was maxmized using the scoring method[3] proposed by Engle and Watson (1981, 1983). The Kalman filtering algorithm and the estimation of the TVP model are discussed in Appendix 2.

Given estimates of the parameters, the next step is to estimate the evolutionary coefficients of the model based on past information. ($\beta_{it/t-1}$, i=0,1,..,4).[4] At this step, the Kalman filter is run again with the above estimates of σ_e^2 and σ_{vi}^2's, for given initial values of $\beta_{i0/-1}$(i=0,...,4) and their variance-covariance matrix.[5] The estimates of $\beta_{it/t-1}$ (i=0,...,4) are shown in Figures 6--10. As by-products of running the Kalman filtering algorithm, the one step ahead forecast errors ($\eta_{t/t-1}$) and their conditional variances (H_t) can be estimated as in (A2.5) and (A2.6) of Appendix 2. These are shown in Figures 11-12.

Once we estimate the model, we need to see whether the model is correctly specified. One useful way of checking the appropriateness of the specified model is to check for whiteness or lack of serial correlation in the one period ahead forecast errors ($\eta_{t/t-1}$) of equation

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[3] A nice feature of the scoring method is that we need only the first derivatives. The computer program was written in the GAUSS programming language, and the numerical derivatives were used.

[4] Benjamin Friedman (1979) argues that "what is typically missing in rational expectation mechanisms is a clear outline of the way in which economic agents derive their knowledge which they use to formulate expectation...." By referring to the informational availability assumption of rational expectations hypothesis, he proposed least-squares-learning as an optimal learning process. Under a continuously changing polic regime, however, the least-squares-learning is no longer optimal. In this circumstance, an application of the Kalman filter is an appropriate way of proxying the rational agents' learning process.

[5] These initial values were estimated at the expense of first 16 observations.

(A2.5), as suggested by Watson and Engle (1981). The null hypothesis of no serial correlation in the forecast errors was not rejected at the 5% significance level, which shows the appropriateness of the model specification.[6]

Section III. A TVP Model and ARCH: The Existence of ARCH as Evidence of Misspecification.

The ARCH model introduced by Engle (1982), explicitly models varying conditional variances by relating them to variables known from the past periods. For example, the varying conditional variances are assumed to be dependent upon past squared innovations. Thus, Engle argues that by considering the heteroscedastic conditional variances, one could get more efficient estimates of the coefficients. But a problem with the ARCH model is that this specification of the conditional variances is ad-hoc. If one knows the factors that may cause conditional variances to vary, more efficient estimates could be obtained than with the ARCH specification by explicitly considering these factors. It is argued in this paper that one very likely source of ARCH is a continuously changing policy regime as represented by time-varying regression coefficients of a model.

There are two channels through which a TVP model and ARCH are related. First, in financial markets in which the risk premium plays an important role, the mean in the regression model could be time varying due to ARCH. Engle, Lilien, and Robins (1987), for example, postulated that

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[6] Box-Pierce test statistics for the one period ahead forecast error from the TVP specification are Q(12)=15.8, Q(24)=26.7, and Q(36)=33.9.

the expectation of the excess holding yield on a long term bond is dependent upon its conditional variance, and proposed a TVP model. In their model, the evolutionary coefficient (in their case, a time-varying risk premium) of the model is dependent upon the conditional variance. (ARCH-M model). [7] This is the case in which ARCH induces a time varying parameter model.

The second channel is one in which a TVP model induces an ARCH effect, which is of current concern. Engle (1982) proposed a test of ARCH based on the OLS regression of the model. But if the coefficients of the true model are continuously changing due to a continuous change in the policy regime, the ARCH test based on OLS regression tends to show existence of ARCH, even when the true disturbance term is not heteroscedastic. In other words, the existence of ARCH from OLS regression could be interpreted as evidence of misspecification of the model. If this is the case, though the ARCH model gives us more efficient estimates than OLS regression does, a time-varying parameter (TVP) model is superior to the ARCH model, in the sense that one need not make an ad-hoc assumption about the structure of varying conditional variance as in the ARCH specification.

When the ARCH tests were performed based on the OLS regression of the

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[7] Their model can be summarized as follows:

$$R_t = a_0 + a_1 h_t + r_t + v_t$$

$$v_t \text{ --- } N(0, h_t)$$

$$h_t = b_0 + b_1 \hat{v}_t^2 + \dots + b_p \hat{v}_{t-p}^2,$$

where R_t is is ex-post one period holding yield on long term bond and r_t is the certain return one period treasury bills. If the coefficient a_1 is constrained to be zero the model reduces to the usual ARCH model.

monetary growth function (2.1), very strong evidence of an ARCH effect was found. In order to determine if the source of the ARCH effect is the varying coefficients of the model, we need to check if the serial correlation still remains in the squared error terms after taking the evolutionary coefficients into account.

In doing so, prediction errors based on the whole sample, $\eta_{t/T}$, were estimated using Kalman filtering and the TVP model, where

$$\eta_{t/T} = DMI_t - x_t\beta_{t/T}.$$

Here, x_t is a vector of regressors in (2.1) and $\beta_{t/T}$ is the estimate of β_t based on the whole sample (sometimes called 'SMOOTHING').[8] In Table 3, the ARCH test results based on OLS errors and those based on the errors from TVP estimation with the whole sample are shown. After the time varying property of the regression coefficients is accounted for, the null hypothesis of no serial correlation in the squared error terms was not rejected. This shows that the existence of the ARCH in the monetary growth function is mainly due to the changing regression coefficients of the model.

The estimates of conditional variances of monetary forecast errors based on the ARCH specification and those based on the TVP specification are shown in Figure 12. The conditional variances from the ARCH estimation reasonably approximate those from the TVP estimation. But it is interesting to note in Figure 12 that the TVP estimation results show higher uncertainty about future monetary policy during the oil shock (around 1974), while the ARCH estimation results barely pick up this

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[8] The smoothing algorithm, from which we can get estimates of the coefficients based on the whole sample, is defined in Appendix 2. The idea of the smoothing is running the filter backwards.

point. This is because, in the ARCH model, in so far as forecast errors in the near past are small, the uncertainty about the future is assumed to be small (regardless of the current state of the economy.). Under a continuously changing policy regime, however, the uncertainty about a future policy variable is very closely related to the current state of the economy, and this is explicitly taken into account in the TVP estimation of the model, without any ad-hoc assumption about the structure of the conditional variance as in the ARCH model. This argument is apparent from the following equation for conditional variance of forecast errors, which is part of the Kalman filtering algorithm in Appendix 2:

$$H_t = x_t P_{t/t-1} x_t' + \sigma_e^2, \tag{3.1}$$

where $P_{t/t-1}$ is variance-covariance matrix of $\beta_{t/t-1}$ and σ_e^2 is variance of the disturbance term e_t in (2.1). Under the changing poliy regime, there are two sources of uncertainty: uncertainty that arises because of future random terms and uncertainty that arises because of evolutionary regression coefficients.

The empirical results in this section, in addition to the stability test results in Appendix 1, further justify TVP modeling of the monetary growth function.

Section IV. An Empirical Test of the Lucas Hypothesis and Friedman's Hypothesis

This section presents empirical results from a Lucas-type reduced form output equation which also allows conditional uncertainty to have direct effect on output as hypothesized by Friedman. The recursive forecast errors ($\eta_{t/t-1}$) and the conditional variance (H_t) estimated in section II are used to test the Lucas and Friedman hypotheses. Consider the following model.

$$cy_t = \alpha_0 + \alpha_{1t}SHOCK_t + \alpha_2 CV_t + \alpha_3 cy_{t-1} + \mu_t \quad (4.1)$$

$$\alpha_{1t} = f(CV_t), \quad (4.2)$$

where cy_t is time-detrended output,[9] $SHOCK_t$ is a recursive forecast error in monetary growth, and CV_t is the conditional variance of the forecast error. The functional form of the time-varying coefficient α_{1t} in equation (4.2) was chosen in such a way to minimize the multicolinearity problem in OLS regression of equation (4.1). Thus, the specification $\alpha_{1t} = \gamma_0 + \gamma_1 \ln(CV_t)$ was chosen. Testing the Lucas hypothesis is equivalent to testing $\gamma_0 > 0$ and $\gamma_1 < 0$, and testing the Friedman's hypothesis is equivalent to testing $\alpha_2 < 0$. Test results based on the OLS regresion of equation (4.1) are reported in Table 4 and Table 5.

In most of the regressions, γ_0 is positive and significantly different from zero. The estimated values of γ_1, on the contrary, have the wrong signs and are not significantly different from zero. (When a $SHOCK_t * \ln(CV_t)$ term is added to the regression, there is little improvement in R^2.) So the Lucas hypothesis is not supported by the data. But when the conditional variance term (CV_t) is included directly in the regression equation, the R^2 increased significantly, and its coefficient

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[9] In order to test Friedman's (1977) hypothesis as well, it is assumed that the natural level of output is affected by CV_t term, a proxy for inflation uncertainty. That is, following Froyen and Waud (1984), the following is assumed:

$$y_t = y_{nt} + y_{ct}$$

$$y_{nt} = \delta_0 + \delta_1 t + \delta_2 CV_t,$$

where y_t, y_{nt}, y_{ct} represent log of real output, natural output, and cyclical output, respectively. Therefore, the time-detrended output, cy_t is the sum of cyclical output and the portion of natural output which is dependent upon the inflation uncertainty.

(α_2) is significantly different from zero with a negative value.[10] This suggests that the conditional variance of forecast errors affects the output level directly, and not through the coefficient on the SHOCK term.

Coefficients on lagged detrended output in Table 4 are close to one (ranges from 0.9188 to 0.9368). This could be due to downward bias in the coefficients when there is a unit root, as explained by Dickey and Fuller (1979). So the regressions were performed by using the first difference of detrended output as a dependent variable (Table 5). The test results were not significantly different from those using detrended output in levels. The R^2 from regression using differenced data ranges from 0.1456 to 0.2775 after correcting for serial correlation. This suggests that monetary shocks and their conditional variance alone cannot provide a complete picture of short term business fluctuations.

Section V. Conclusion

A conventional fixed coefficient model of a monetary growth function understates the degree of learning by economic agents. A typical test of rational expectations hypothesis is performed based on the whole sample, but at time t, agents do not have information on t+1,...,T. Especially, when the policy regime changes continuously, the correct specification of the learning process by agents is crucial to the test result and its interpretation.

A time-varying-parameter model was proposed in modeling a monetary

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[10] When the tests were performed using the conditional variances from the ARCH specification, both γ_1 and α_2 were not significantly different from zero. The test results based on the ARCH specification is sometimes inconclusive because of the ad-hoc assumption about the conditional variance.

growth function, and the Kalman filtering technique was applied to estimate the model. The Kalman filter shows how rational economic agents would combine past information and new information to form a new expectation. The algorithm also provides recursive forecast errors and their conditional variance at each point in time.

Based on the estimated forecast errors and conditional variances from the Kalman filtering estimation of a monetary growth function, the Lucas hypothesis (in a time-series context) was tested. Test results reject the hypothesis. But the conditional variance of monetary shocks itself plays a very important role in explaining the business cycle in the U.S. economy between 1964.1--1985.4. That is, the conditional variance affects real output directly, not through the coefficient on the shock. This result supports Friedman's (1977) view that natural output is negatively dependent upon inflation uncertainty.

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Appendix 1. Testing the Constancy of Regression Coefficients in a Monetary growth Function.

Various methods of testing the stability of regression coefficients have been proposed by, for example, Lamotte and McWhorter, Jr. (1978), Breusch and Pagan (1979), Brown, Durbin and Evans (1975), and others. Among those, the simplest technique which is based on the moving regression proposed by Brown et. al (1975) and the Lagrange multiplier test proposed by Breusch and Pagan (1979) were applied to the monetary growth function in (2.1).

With the alternative hypothesis of unstable regression coefficients, the first step in the moving regression method is to calculate confidence intervals for each of the coefficients from successive moving regressions. If all the confidence intervals overlap with one another, we may accept the null hypothesis that the regression coefficients are stable. This procedure at least gives us an insight into the periods in which significant structural changes have occurred. Next, a significance test for the constancy of coefficients, sometimes called a homogeneity test, is derived from the results of regressions based on non-overlapping subsamples, using analysis of variance. By splitting the entire sample into non-overlapping subsamples, the 'between groups over within groups' ratio of mean squares is calculated. Under the null hypothess of stable coefficients, this test statistic is distributed as $F(kp-k,T-kp)$, where k is number of regressors, p is number of non-overlapping subsamples, and T is the whole-sample size.

Confidence intervals for the coefficients from the moving regressions based on 12 observations are shown in Figures 1-5. There is strong evidence that regression coefficients are not stable, for most of the coefficients, we can pick up the cases in which conficence intervals (95%)

do not overlap. F-statistics for the homogeneity tests are reported in table 1. The null hypothesis is rejected at the 5% significance level in all cases.[11]

Having rejected the stability of regression coefficients, of further interest is the actual structural form of the time-varying regression coefficients. Engle and Watson (1985) suggest unit roots for the coefficients in cases of structural change in which agents adjust their estimation of the state only when new information becomes available. Therefore, a stability test can be performed against the alternative hypothesis that the coefficients follow random walks.

Under the alternative hypothesis of random walk coefficients, residuals from an an OLS regression have a particular heteroscedasticic form, which depends upon $t*x_t^2$. (x_t is a vector of regressors.) Breusch and Pagan (1979) have shown that one half of the explained sum of squares from a regression of $\hat{u}_t^2/\sigma_u^2$ on $t*x_t^2$, where $\hat{u}_t^2$ is the vector of residuals from an OLS regression of (2.1), is distributed as $\chi^2(k)$ under the null hypothesis. The test results are shown in Table 2. First, under the assumed constancy of other coefficients, the test was performed for each of the coefficients. The test results show that the null hypothesis of stable coefficients is rejected for coefficients on $DINT_{t-1}$, INF_{t-1}, and the intercept term. But the joint hypothesis that all the coefficients are stable (against the alternative hypothesis of random walks) is rejected at the 1 % significance level.

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[11] The test of regression stability based on moving regression suffers from a low power, because the test assumes that the regression coefficients are stable within a subsample.

Appendix 2. Kalman Filtering and Estimation of a Time-Varying-Parameter Model

In this Appendix, a state-space representation of a TVP model and the Kalman filtering estimation of the coefficients are introduced. The state-space model and Kalman filtering has a wide range of applications in econometrics and time series analysis. Two of the examples are the TVP model and the unobserved component model.[12]

The basic linear regression model with a changing coefficient vector β_t is represented in terms of state space model by :

$$\text{Measurement equation: } y_t = x_t\beta_t + e_t \qquad \text{(A2.1)}$$

$$\text{Transition equation: } \beta_t = A\beta_{t-1} + v_t, \qquad \text{(A2.2)}$$

$$\begin{bmatrix} e_t \\ v_t \end{bmatrix} \sim N\left[0 \quad \begin{pmatrix} \sigma_e^2 & 0 \\ 0 & Q \end{pmatrix}\right] ,$$

where Q is is a positive definite matrix. The coefficients β_t of the above TVP model can be estimated by Aitken's GLS procedure, assuming that the parameters of the transition equation A and Q and σ_e^2 are known. The GLS estimation of (A2.1), however, requires an enormous amount of computation, as we need inverse of very large matrices. Thus, this procedure sometimes is inoperational.

But by applying Kalman filtering recursively to equations (A2.1) and (A2.2), the estimation procedure becomes dramatically easier. The equivalence of Kalman filter estimates of β_t in (A2.1) and GLS estimates is well explained in Los (1984). The main difference between GLS

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[12] See Clark (1987), Watson (1986), and Nelson (1987) for the application of the state space model and the Kalman filter to unobserved component model, see Rosenberg (1973) and Chow (1984) for recent surveys of TVP model.

estimates and Kalman filtering estimates of β_t is that in the former all the observations up to time t are simultaneously taken account, while in the latter, we are interested in 'what a new observation contribute to our existing knowledge.'

The Kalman filtering algorithm is described below; for a detailed derivation of the algorithm, see Los (1984) and Andrerson and Moore (1979).

<u>Prediction</u> :

$$\beta_{t/t-1} = A\beta_{t-1/t-1} \tag{A2.3}$$

(estimate of β_t based on information up to t-1)

$$p_{t/t-1} = Ap_{t-1/t-1}A' + Q \tag{A2.4}$$

(variance covariance matrix of $\beta_{t/t-1}$)

$$\eta_{t/t-1} = y_t - x_t\beta_{t/t-1} \tag{A2.5}$$

(forecast error based on information up to t-1)

$$H_t = x_t p_{t/t-1} x_t' + \sigma_e^2 \tag{A2.6}$$

(conditional variance of forecast error, $\eta_{t/t-1}$)

<u>Updating</u> :

$$\beta_{t/t} = \beta_{t/t-1} + K\eta_{t/t-1} \tag{A2.7}$$

$$p_{t/t} = (I_k - Kx_t)p_{t/t-1}, \tag{A2.8}$$

where $K = p_{t/t-1}x_t'H_t^{-1}$ (Kalman gain).

<u>Smoothing</u> :

$$\beta_{t/T} = \beta_{t/t} + p_t^*(\beta_{t+1/T} - A\beta_{t/t}) \tag{A2.9}$$

$$p_{t/T} = p_{t/t} + p_t^*(p_{t+1/T} - p_{t+1/t})p_t^{*\prime} \tag{A2.10}$$

where $p_t^* = p_{t/t}A'p_{t+1/t}^{-1}$

Notice that the estimate of β_t based on information up to t, $\beta_{t/t}$, is an optimal combination of the prior on β_t, i.e., $\beta_{t/t-1}$, and the forecast error $\eta_{t/t-1}$, the weight being K, which is sometimes called the 'Kalman gain.' Thus, for given starting values of $\beta_{0/-1}$ and $p_{0/-1}$, if the values of σ_e^2, Q, and A are known, the above filter recursively produces estimates of the mean and variance of the state vector β_t using observations on y_t and x_t.

In the engineering literature, the values of the time-invariant parameters of the TVP model(A, σ_e^2, and Q) are sometimes known a priori. In economics, however, these values are seldom known, and need to be estimated.[13] To estimated these parameters, the following log likelihood function, which is based on the forecast errors and their conditional variance from the Kalman filtering algorithm, can be maximized:

$$\ln L = \text{constant} - 0.5*\Sigma^{T}_{t=1}\{\ln|H_t| + \eta_{t/t-1}'H_t^{-1}\eta_{t/t-1}\} \qquad (A2.11)$$

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[13] In estimating the monetary growth function in Section II, the parameter A is constrained to be an identity matrix, based on the stability test result in Appendix 1.

TABLE 1. Testing constancy of regression coefficients in monetary reaction function (3 year moving regressions)

$$F=[(T-pk)*\{(S(1,T)-W\}]/[(pk-k)*W] \text{ ----} F(kp-k,T-kp), \text{ where}$$

$W=S(1,n)+S(n+1,2n)+....+S(pn-n+1,T)$
n= number of observation in subsamples=12
p= number of subsamples=8
k= number of regressors=5
$S(r_1,r_2)$= RSS of regression from observation r_1 to r_2.

Whole sample period	F-statistic*
1959.3--1983.2	1.929
1959.4--1983.3	1.961
1960.1--1983.4	1.975
1960.2--1984.1	1.834
1960.3--1984.2	1.806
1960.4--1984.3	1.862
1961.1--1984.4	1.774
1961.2--1985.1	1.772
1961.3--1985.2	1.772
1961.4--1985.3	1.787
1962.1--1985.4	1.793

* Critical values for F(35,56)=2.03(1% significance level)
=1.65(5% significance level)

TABLE 2. Testing constancy of regression coefficients in the monetary growth function (Lagrange multiplier test)

EQUATION:

$$\hat{u}_t^{\,2}/\sigma_u^{\,2} = \hat{c}_0 + \hat{c}_1 t + \hat{c}_2 DM1_{t-1}^{\,2} t + \hat{c}_3 DINT_{t-1}^{\,2} t + \hat{c}_4 INF_{t-1}^{\,2} t + \hat{c}_5 SURP_{t-1}^{\,2} t \;, \quad (t=1,2,\ldots T),$$

where $\hat{u}_t$ is residuals from OLS regression of equation (3.2.4), and $\sigma_u^{\,2}$ is variance of $\hat{u}_t$.

coefficients					k	$\chi^2(k)$**	R^2
$\hat{c}_1$	$\hat{c}_2$	$\hat{c}_3$	$\hat{c}_4$	$\hat{c}_5$			
.017 (1.96)	--	--	--	--	1	9.77	.04
--	-0.00015 (-.167)	--	--	--	1	.008	.0003
--	--	.003 (2.79)	--	--	1	19.2	.07
--	--	--	.0069 (7.52)	--	1	92.7	.37
--	--	--	--	-.0023 (-.54)	1	.79	.003
-.018 (-1.47)	-.006 (-.73)	-.00062 (.64)	.0081 (6.40)	.0071 (1.49)	5	99.6	.38

** One half of the explained sum of squares is distributed as $\chi^2(k)$ under the null hypothesis of stable coefficients.

TABLE 3. ARCH Tests

REGRESSION EQUATION:

$$\hat{e}_t^{\,2} = a_0 + a_1\hat{e}_{t-1}^{\,2} + a_2\hat{e}_{t-2}^{\,2} + \ldots\ldots + a_p\hat{e}_{t-p}^{\,2}$$

Null hypothesis: $a_1 = a_2 = \ldots\ldots = a_p = 0$

Under the null hypothesis, $T*R^2$ is distributed as $\chi^2(p)$, where T is number of observations.

1) Existence of ARCH from OLS regression of monetary growth function function.

$\hat{e}_t = DM1_t - x_t\hat{b}$, where $\hat{b}$ is OLS estimate based on the whole sample.

p	$\chi^2(p)$
1	10.98
2	12.65
3	23.39
4	24.36
8	24.51

2) Serial correlation in the squared error terms from TVP estimation (based on the whole sample.)

$\hat{e}_t = DM1_t - x_t\beta_{t/T}$, where $\beta_{t/T}$ is TVP estimate based on the whole sample. (smoothing)

p	$\chi^2(p)$
1	1.49
2	1.57
3	3.17
4	3.19
8	4.82

TABLE 4. Test of the Lucas Hypothesis

EQUATION:

$$cy_t = \alpha_0 + (\gamma_0 + \gamma_1 \ln CV_t) SHOCK_t + \alpha_2 CV_t + \alpha_3 cy_{t-1} + u_t$$

cy_t = time-detrended output

$SHOCK_t$ = recursive forecast error

CV_t = conditional forecast error variance

1) Uncorrected for serial correlation

α_0	γ_0	γ_1	α_2	α_3	R^2	DH
.00053	.0026	--	--	.954	.9188	3.29
(.485)	(2.049)			(31.944)		
.00053	.0026	-.00009	--	.954	.9198	3.28
(.418)	(2.030)	(-.009)		(31.761)		
.0069	.0035		-.013	.967	.9334	2.20
(3.823)	(2.962)		(-4.23)	(35.098)		
.00798	.0038	.0027	-.014	.968	.9355	2.20
(4.200)	(3.199)	(1.67)	(-4.59)	(35.489)		

2) Corrected for serial correlation

α_0	γ_0	γ_1	α_2	α_3	R^2	DH
.00009	.0027	--	--	.912	.9292	-.520
(.538)	(2.592)			(18.022)		
.00087	.0026	.00056	--	.910	.9293	-.546
(.547)	(2.559)	(.39)		(17.821)		
.00674	.0036		-.012	.949	.9368	-.546
(3.208)	(3.425)		(-3.56)	(25.623)		
.00798	.0037	.0025	-.014	.951	.9355	-.112
(3.641)	(3.575)	(1.72)	(-4.01)	(26.240)		

* t-values are in parentheses.

TABLE 5. Test of the Lucas Hypothesis

EQUATION:

$$Dcy_t = \alpha_0 + (\gamma_0 + \gamma_1 \ln CV_t)SHOCK_t + \alpha_2 CV_t + u_t$$

$Dcy_t = cy_t - cy_{t-1}$

cy_t = time-detrended output

$SHOCK_t$ = recursive forecast error

CV_t = conditional forecast error variance

1) Uncorrected for serial correlation

α_0	γ_0	γ_1	α_2	R^2	DW
.00015	.00276	--	--	.0502	1.3649
(.137)	(2.179)				
.00014	.00276	-.00013	--	.0502	1.3673
(.134)	(2.158)	(-.078)			
.00684	.00364	--	-.0131	.2186	1.5788
(3.778)	(3.102)		(-4.38)		
.00793	.00392	.00277	-.0151	.2436	1.5750
(4.169)	(3.342)	(1.70)	(-4.74)		

2) Corrected for serial correlation

α_0	γ_0	γ_1	α_2	R^2	DW
.00017	.0029	--		.1456	2.0623
(.113)	(2.640)				
.1768	.0029	.00048	--	.1465	2.0647
(.119)	(2.640)	(.3155)			
.00668	.0038		-.012	.2535	2.0132
(3.164)	(3.499)		(-3.76)		
.00788	.0039	.0026	-.015	.2775	2.0109
(3.577)	(3.639)	(1.712)	(-4.16)		

* t-values are in parentheses.

FIGURE 1. Confidence intervals from moving regression (DINTt-1)

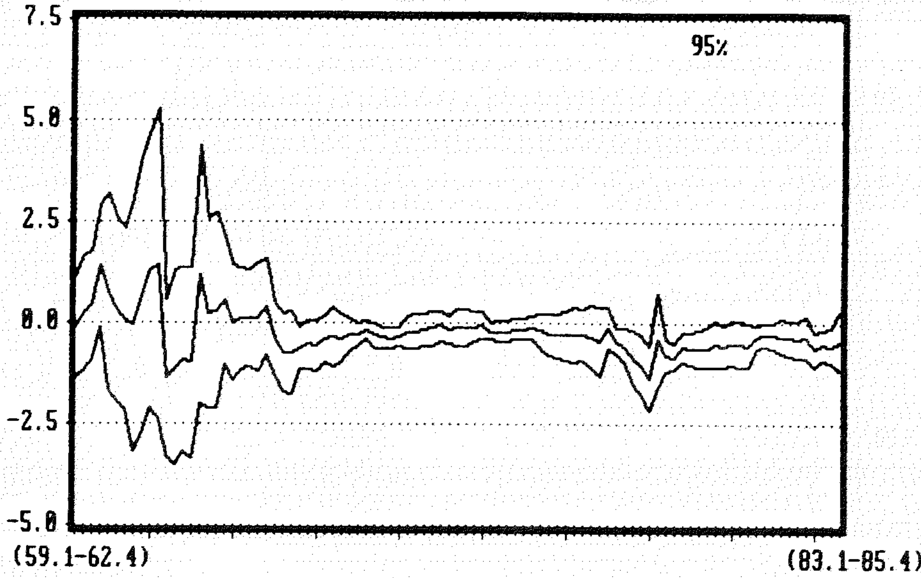

FIGURE 2. Confidence intervals from moving regressions (INFt-1)

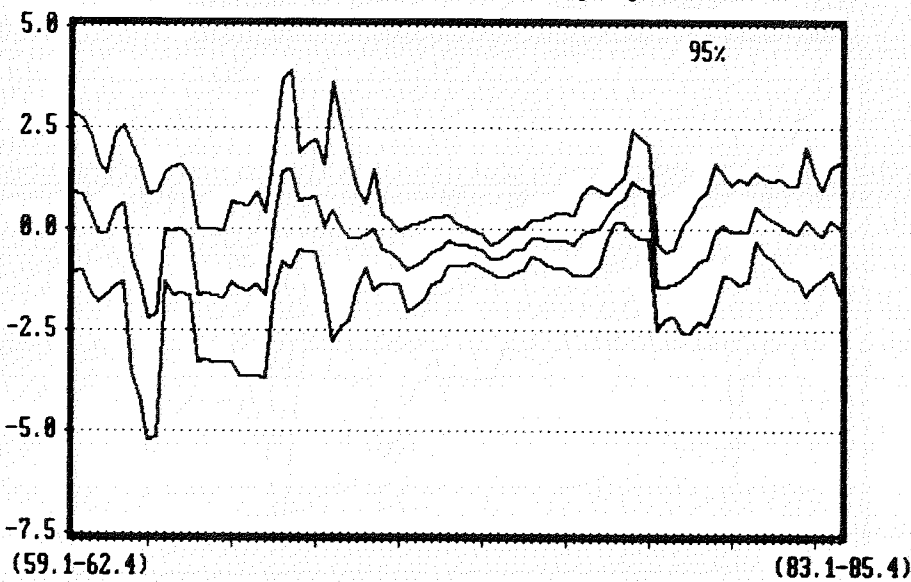

FIGURE 3. Confidence intervals from moving regressions (SURPt-1)

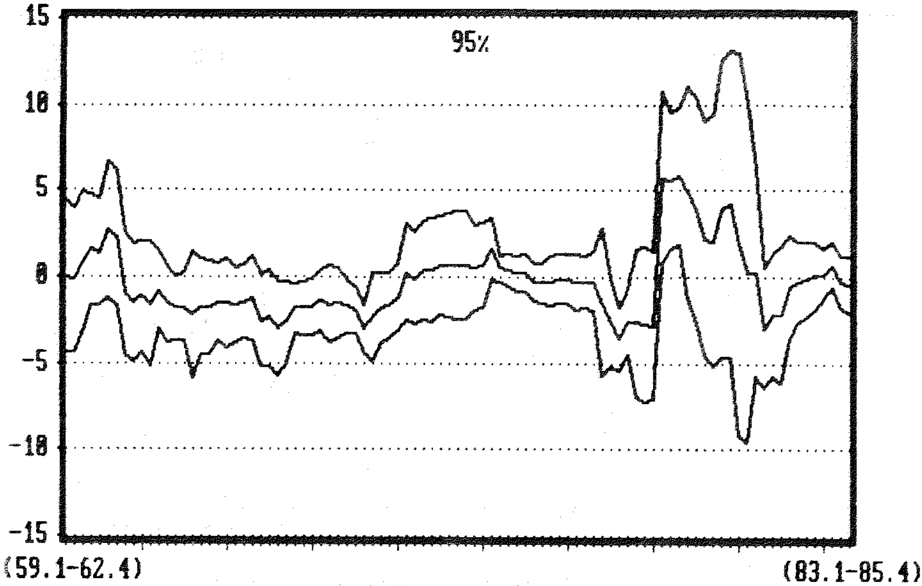

FIGURE 4. Confidence intervals from moving regressions (DM1t-1)

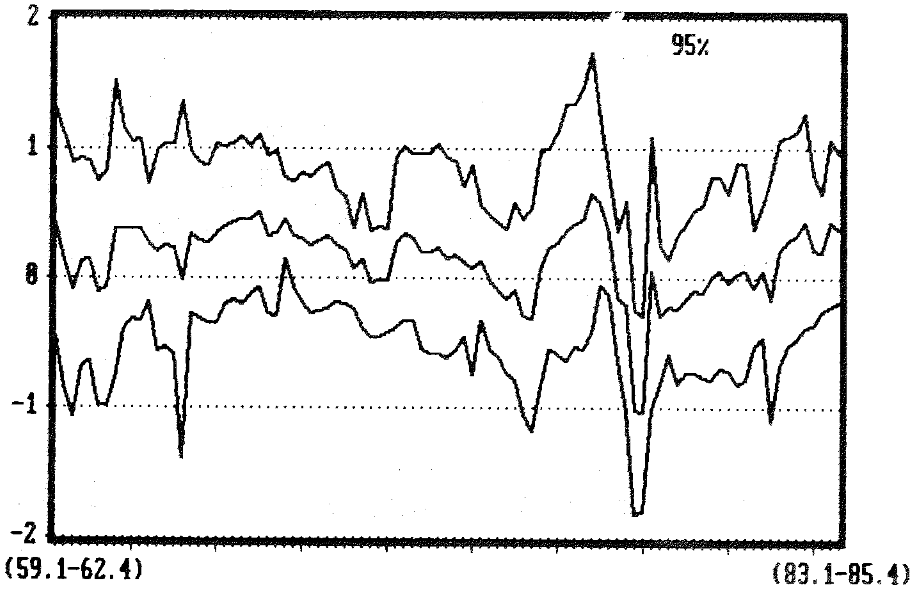

FIGURE 5. Confidence intervals from moving regressions (INTERCEPT)

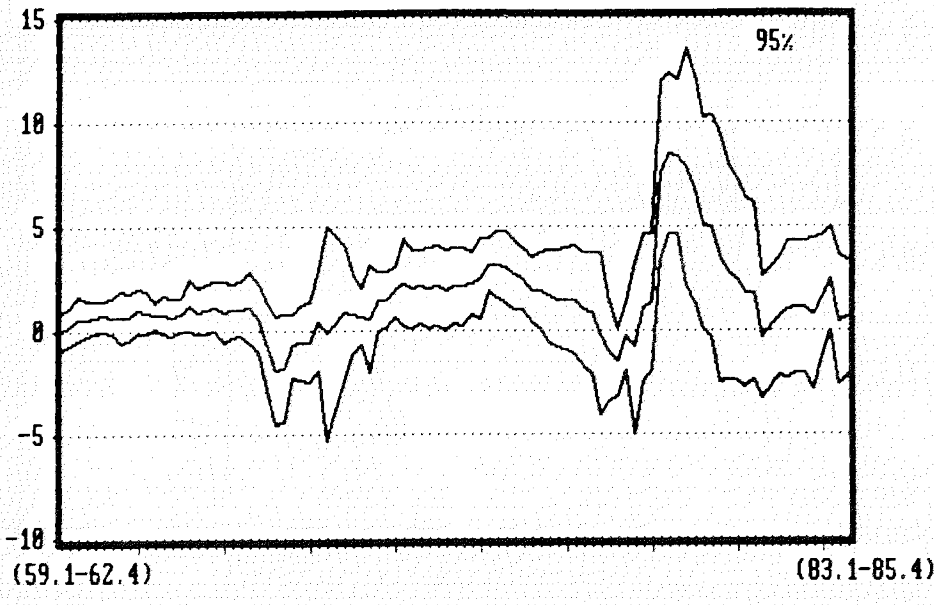

FIGURE 6. Estimate of time-varying coefficient (INTERCEPT)

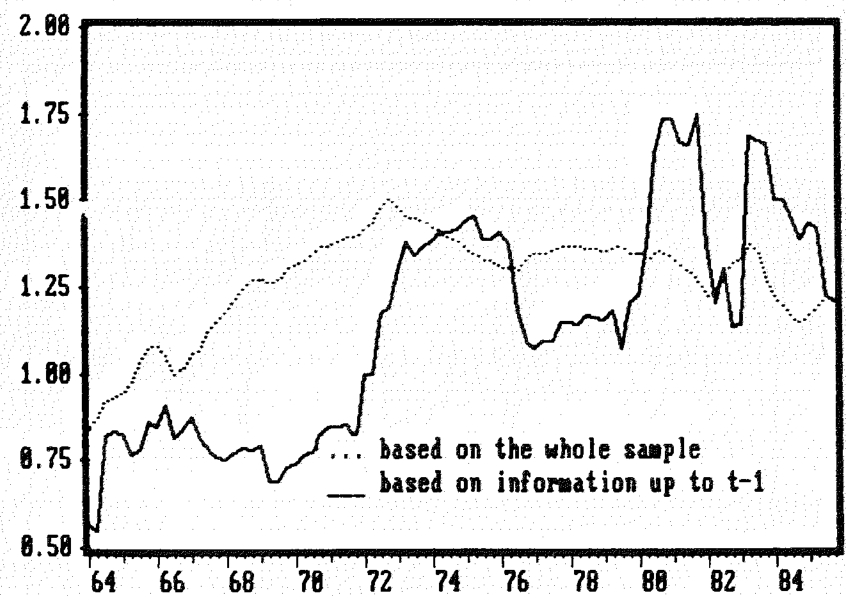

FIGURE 7. Estimate of time-varying coefficient (DINTt-1)

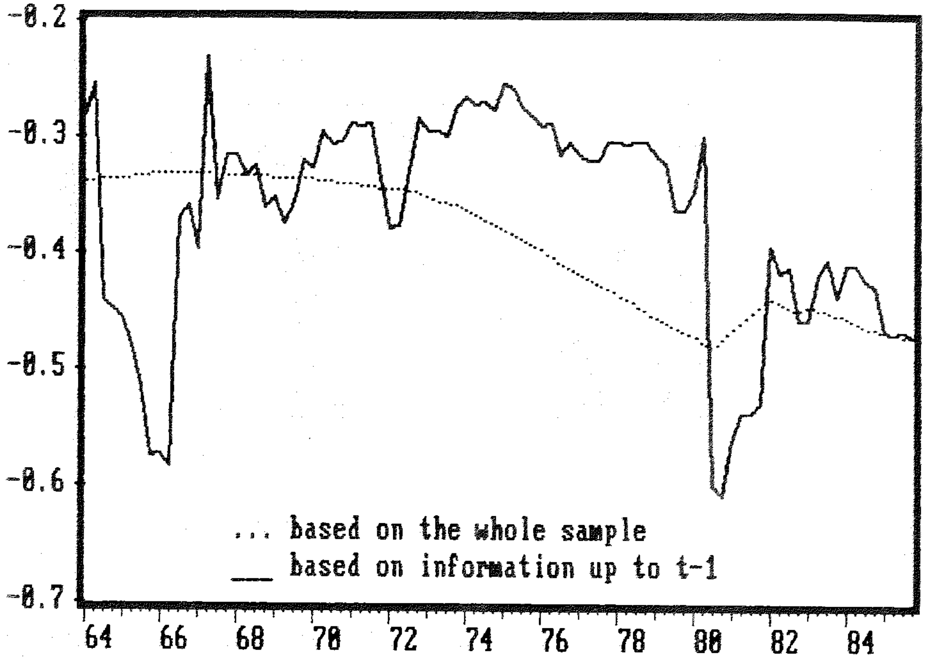

FIGURE 8. Estimate of time-varying coefficient (INFt-1)

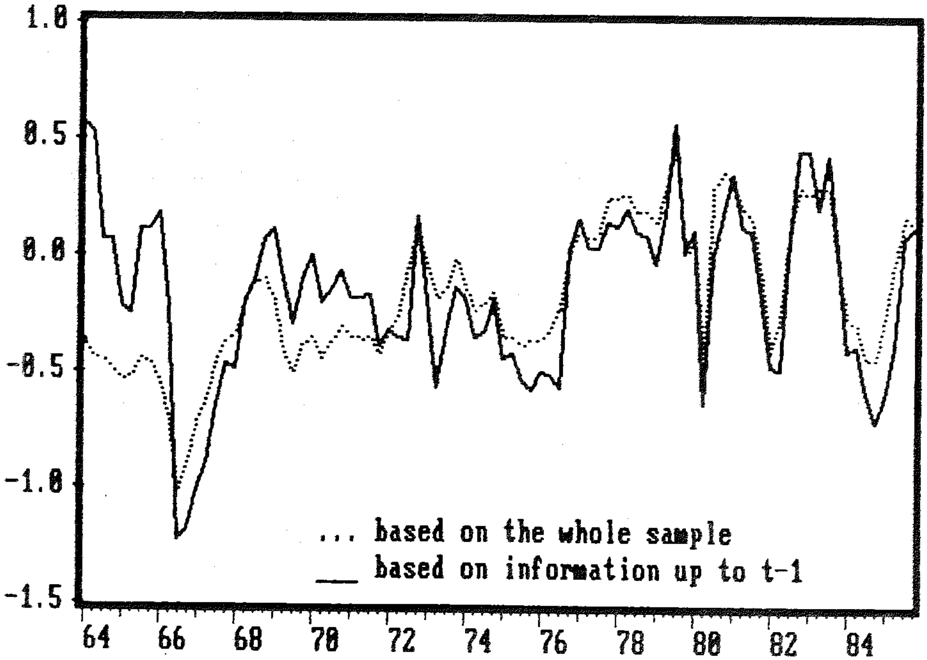

FIGURE 9. Estimate of time-varying coefficient (SURPt-1)

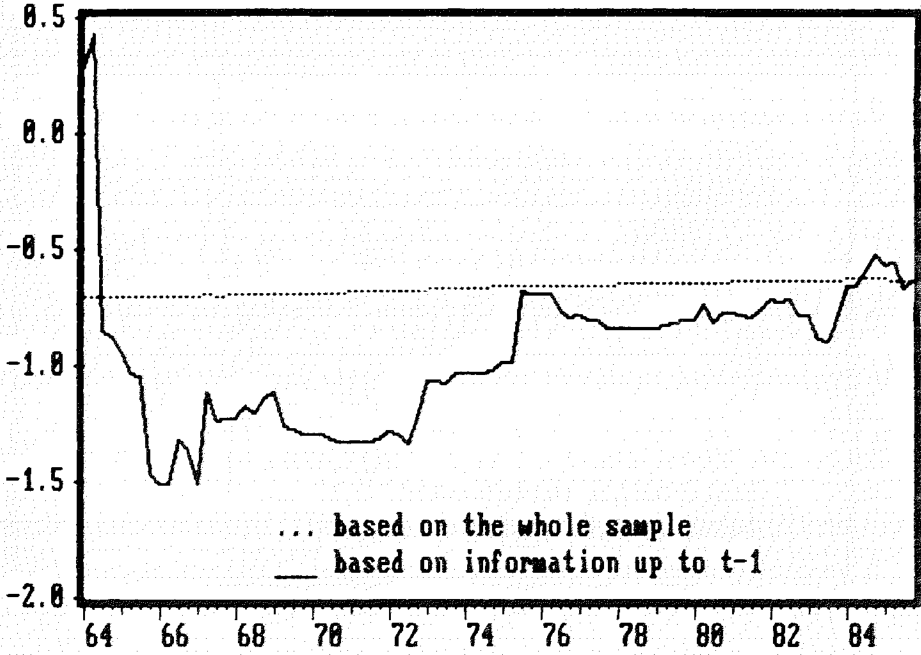

FIGURE 10. Estimate of time-varying coefficient (DM1t-1)

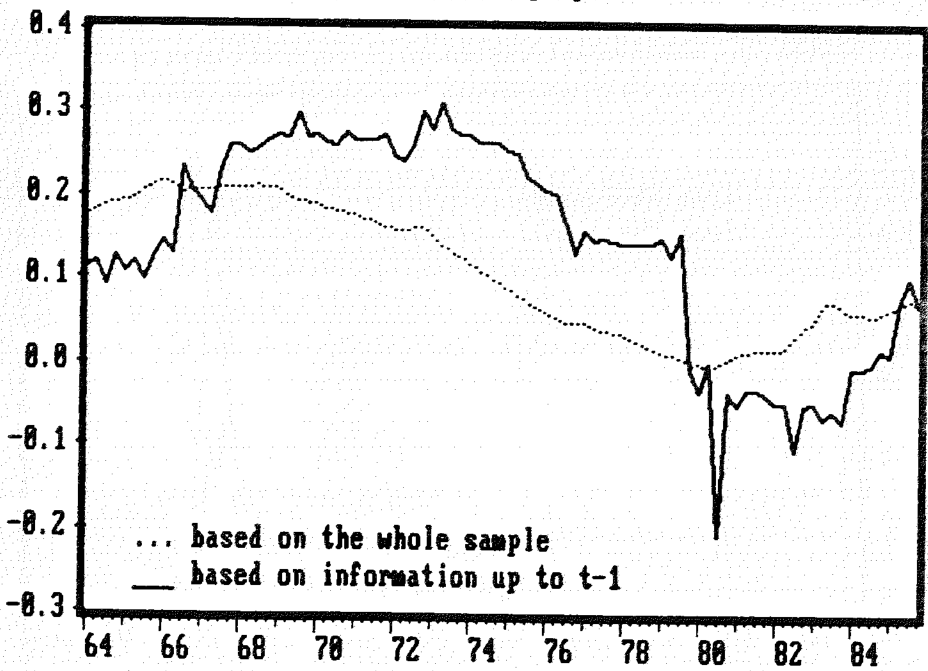

FIGURE 11. Forecast errors from TVP model.

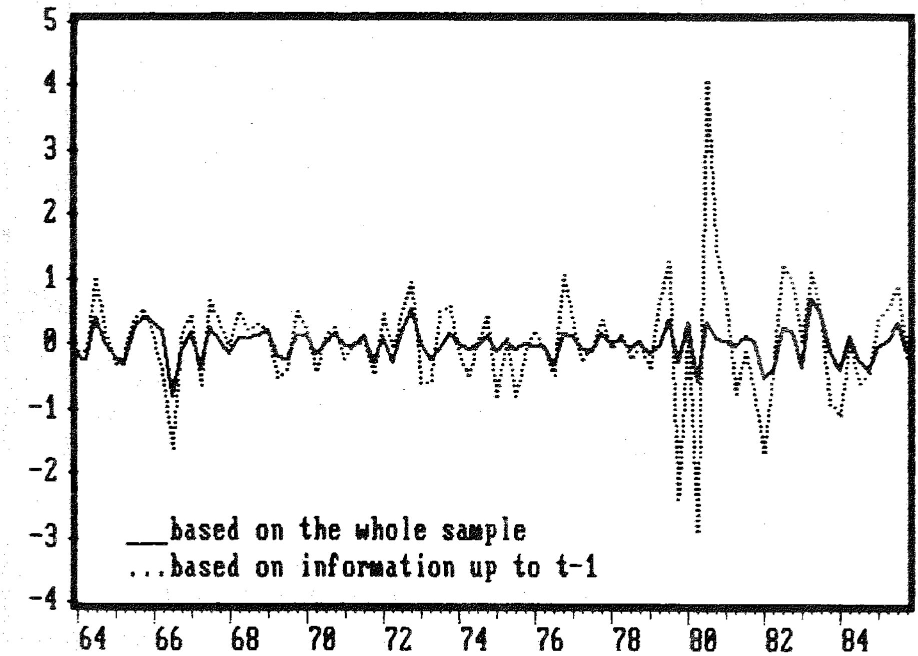

FIGURE 12. Conditional variances of forecast errors.