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A STOCHASTIC APPROACH TO DISEQUILIBRIUM MACROECONOMICS

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ABSTRACT

In this paper, our aim is to develop an alternative approach to analyzing a macroeconomic model where markets do not clear. Earlier approaches have had difficulties in interpreting effective demand, a key concept in disequilibrium macroeconomics. We propose a new definition of effective demand similar to that of Svensson, Gale, and Green. Given the states of the markets, there is in general uncertainty about the amount of trades individuals can complete. Considering this uncertainty, each individual has to make binding trade offers, i.e., effective demands, a fraction of which will be actually transacted. Using the newly-defined effective demand, we define the rationing equilibrium as a fixed point of disequilibrium signals.

We analyze various regimes of rationing equilibria. The most startling conclusion is the multiplicity of equilibria: (i) given wages and prices, there may exist more than one type of equilibrium and (ii) even at Walrasian prices there may exist non-Walrasian equilibria, and these are usually stable with respect to a quantity-adjustment mechanism while the Walrasian equilibrium is unstable. The comparative-static properties of policy we also considered, and they are comparable to those of the earlier approach.

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I. INTRODUCTION

In the 1970's, one of the centers of interest in the foundations of macroeconomics has been the disequilibrium or nonmarket-clearing approach. From the early influential contributions by Clower (1965), Leijonhufvud (1968) and Patinkin (1965), there developed a literature. Among others, some notable contributors are Barro and Grossman (1971, 1974, 1976), Benassy (1975), Drèze (1975), Grossman (1971), Malinvaud (1977), and Muellbauer and Portes (1978). Those authors have analyzed the determination of the short-run state of an economy when prices do not adjust to the market-clearing or Walrasian levels.

In this literature, the concepts of effective demand and supply-demand disequilibrium play an important role. Unfortunately, in the two standard approaches, usually associated with Benassy (1975) and Drèze (1975), respectively, these two desirata are not simultaneously achieved. Either agents are irrational or there is no natural and observable measure of market disequilibrium. To overcome this difficulty, Gale (1978), Green (1978), Honkapohja and Ito (1979), and Svensson (1977) have studied models with so-called trading uncertainty or stochastic rationing.[1]

In this paper, our aim is to utilize these recent developments to analyze further a new approach and contrast it with the standard one. To this end, we construct a simple macroeconomic model, based on the ideas of trading uncertainty which varies with disequilibrium signals. In this context, we study rational expectations equilibria with respect to the signals of the states of each market. An equilibrium is a situation in which the individuals' responses to the signals just reproduce the signals. For the sake of simplicity, we shall assume that the prices are exogenously given, which is in accordance with the earlier approach by Barro and Grossman (1976) and Malinvaud (1977). The most striking conclusion of the new approach is that non-Walrasian rational expectations equilibria often exist even when prices are at the Walrasian levels. Moreover, these equilibria are usually stable with respect to a quantity-adjustment mechanism, while the Walrasian equilibrium is unstable.

The paper is organized as follows. In Section II we formulate a simple microeconomic model with trading uncertainty. Section III is devoted to the analysis of the multiplicity and stability of various types of equilibria. Section IV relates the comparative-static properties of our model to those of the standard approach. Conclusions are offered in Section V.

II. THE FRAMEWORK

The model we shall consider is highly simplified and aggregated. There are three types of agents (consumers, firms, and the government), two commodities (labor and the composite consumption goods), and one asset (money). Consumers supply labor services and buy the goods for consumption. They have initial money balances. They save by carrying forward some money. Firms produce the goods with the aid of labor input. We shall abstract from capital and investments which are treated as constants. Furthermore, the distribution of profits by firms will play no role, for the sake of simplicity. There are two possible explanations: (i) profits of the current period are distributed only in the following period and consumers may not borrow against them; or (ii) the government has levied a 100% profits tax (for a discussion and examples of these facilitating assumptions, see Malinvaud [1977], Böhm [1978], Honkapohja [1979a,b] and others). The government demands both the good and labor services. Other government activities may be touched upon later in policy considerations, but otherwise we shall abstract from them.

The price of the goods, p, and the nominal wage, w, are exogenously given and unchanged during the period under consideration. Since they may not be at the general equilibrium values, consistency of individuals' plans, i.e., an equilibrium, must be achieved by some other means; in our framework, through quantity signals or <u>disequilibrium signals</u>. We shall assume that the relevant signals are given by

$$u = L^s/L^d$$

$$v = Y^s/Y^d ,$$

where L^s, L^d, and Y^s, Y^d are the aggregate demand and supply of labor and the good, respectively.[2] We assume that u and v lie in compact sets: $u \in [0,\bar{u}]$, $v \in [0,\bar{v}]$.

According to imbalance of demand and supply of the aggregate signals, we classify four regimes as follows.[3] We call an economy in:

(i) <u>Walrasian equilibrium</u> if $u = 1$ and $v = 1$

(ii) <u>Keynesian unemployment</u> if $u > 1$ and $v > 1$

(iii) <u>Repressed inflation</u> if $u < 1$ and $v < 1$

(iv) <u>Classical unemployment</u> if $7 > 1$ and $v < 1$

(v) <u>Under-consumption</u> if $u < 1$ and $v > 1$.

Given a pair of aggregate signals indicating disequilibria in the two markets, individual agents face uncertainty in trading in the sense that they do not fully know the proportion of their trade offers that will be completed.[4] A simple way to model this phenomenon is to use the following discrete distributions:

<u>Goods Market</u>

demand side: $$\text{actual trade} = \begin{cases} (s) \times (\text{offer}), \text{ with probability } \psi \\ \text{offer, with probability } 1 - \psi \end{cases}$$

supply side: $$\text{actual trade} = \begin{cases} (z) \times (\text{offer}), \text{ with probability } \lambda \\ \text{offer, with probability } 1 - \lambda \end{cases}$$

<u>Labor Market</u>

demand side: $$\text{actual trade} = \begin{cases} (r) \times (\text{offer}), \text{ with probability } \theta \\ \text{offer, with probability } 1 - \theta \end{cases}$$

supply side: $$\text{actual trade} = \begin{cases} (q) \times (\text{offer}), \text{ with probability } \pi \\ \text{offer, with probability } 1 - \pi . \end{cases}$$

The magnitudes s, z, ψ and λ depend on the goods market signal v, while r, q, θ and π depend on u. The mean-balance feasibility condition, the "short-side rule" and other considerations suggest the following restrictions on these functions.[5]

(II. 1) $$(1-\psi)Y^d + \psi s Y^d \equiv (1-\lambda)Y^s + \lambda z Y^s$$

(II. 2) $$(1-\theta)L^d + \theta r L^d \equiv (1-\pi)L^s + \pi q L^s$$

for all Y^d, Y^s, L^d and L^s.

(II. 3) $$r'(u) \geq 0, \quad \theta'(u) \leq 0, \quad r(u) = 1 \quad \text{for} \quad u \geq 1$$
$$q'(u) \leq 0, \quad \pi'(u) \geq 0, \quad q(u) = 1 \quad \text{for} \quad u \leq 1,$$

(II. 4) $$s'(v) \geq 0, \quad \psi'(v) \leq 0, \quad s(v) = 1 \quad \text{for} \quad v \geq 1$$
$$z'(v) \leq 0, \quad \lambda'(v) \geq 0, \quad z(v) = 1 \quad \text{for} \quad v \leq 1.$$

To this market framework we introduce firms and households by postulating that each agent is a signal-taker as well as a price-taker and, as a simplifying assumption, that he fully knows the probability distributions in trading uncertainty. The individual optimization problems are briefly described in the two ensuing subsections.

II. 1. Firm's Behavior

The production technology of the representative firm is given by $y \leq f(\ell)$, with the properties $f(0) = 0$, $f' > 0$, $f'' < 0$, where y and ℓ are the actual output and input, respectively. The firm is assumed to maximize its expected profit subject to the condition that realizations must satisfy the technology constraint with probability one. Therefore, the problem is

(II.5) $\quad \text{Max } p\bar{z}y - w\bar{r}\ell$ subject to $f(r\ell) \geq y$,

where $\bar{z}$ and $\bar{r}$ are the means: $\bar{z} = \lambda z + (1-\lambda)$, $\bar{r} = r\theta + (1-\theta)$. Solving (II.5) yields the effective demand for labor $\ell^d(u,v)$ and the effective supply of goods $y^s(u,v)$.[6] As special cases, we get the behavioral functions in the regimes of Keynesian unemployment (denoted by superscript k) and of Repressed inflation (denoted by superscript R), by the short-side rule that the firm faces trading uncertainty only in the goods market or, respectively, only in the labor market.

$$\text{(II.6)} \quad \begin{aligned} &\ell^d(u,v) \to \ell^{kd}(v) \text{ and } y^s(u,v) \to y^{ks}(v) \text{ as } u \to 1^- \\ &\ell^d(u,v) \to \ell^{Rd}(u) \text{ and } y^s(u,v) \to y^{Rs}(u) \text{ as } v \to 1^+ . \end{aligned}$$

Within the regimes of Classical unemployment and the Walrasian equilibrium, ℓ^d and y^s are independent of quantity signals.

Finally, for the later purposes, we introduce new terminology. The locus $\{\ell^d(u,v),\ y^s(u,v) | u \in [0,\bar{u}],\ v \in [0,\bar{v}]\}$ is called <u>the offer curve of the firm</u>. This curve represents the trace of the expected-profit-maximizing firm's offer on the labor-output plane in response to changes of the aggregate signals. When a market is not in the Walrasian equilibrium, the offer is not always transacted. The mean transaction locus $\{(\bar{r}\ell^d,\ \bar{z}y^s) | u \in [0,\bar{u}],\ v \in [0,\bar{v}]\}$ is called <u>the trade curve of the firm</u>.[7] Their location will become apparent in the discussion of the types of equilibria below in Section III.

II.2. <u>Household's Behavior</u>

In the household problem, there is the complication that the solution to the maximization of the expected value of a concave utility function with multiple agruments leaves ambiguous signs of the derivatives of the demand function. Moreover, since our analysis is a short-run analysis, we have the real money balance, m, as an argument in the utility function. It is supposed that the household maximizes the expected value of the utility function $U(y,\bar{\ell}-\ell,m)$, where $\bar{\ell}$ is an endowment of time. The utility function is monotone increasing with respect to each argument, and strictly concave. In analogy to the problem of the firm, we require that the household never becomes bankrupt. Therefore, the maximization problem becomes

$$\text{(II.7)} \qquad \text{Max } EU = (1-\psi)(1-\pi)U(y,\bar{\ell}-\ell,m_1) + \psi(1-\pi)U(sy,\bar{\ell}-\ell,m_2) + (1-\psi)\pi U(y,\bar{\ell}-q\ell,m_3) + \psi\pi U(sy,\bar{\ell}-q\ell,m_4)$$

subject to

$$m_1 = \bar{m} + w\ell - py \geqslant 0$$
$$m_2 = \bar{m} + w\ell - psy \geqslant 0$$
$$m_3 = \bar{m} + wq\ell - py \geqslant 0$$
$$m_4 = \bar{m} + wq\ell - psy \geqslant 0 .$$

The resulting effective supply of labor $\ell^s(u,v)$ and the effective demand for goods $y^d(u,v)$ are assumed to have the following properties:

$$\text{(II.8)} \qquad \begin{aligned} &\ell^s = \ell^s(u,v), \quad u\ell_1/\ell < 1, \quad \ell_2 > 0 \quad \text{for } u \geqslant 1 \text{ and } 1 \geqslant v; \\ &y^d = y^d(u,v), \quad y_1^d < 0, \quad vy_2^d/y^d > -1 \quad \text{for } u \geqslant 1 \text{ and } 1 \geqslant v. \end{aligned}$$

Keynesian unemployment and Repressed inflation emerge as boundary cases of Classical unemployment:

$$\text{(II.9)} \qquad \begin{aligned} &\ell^s(u,v) \to \ell^{ks}(u) \text{ and } y^d(u,v) \to y^{kd}(u) \text{ as } v \to 1^- \\ &\ell^s(u,v) \to \ell^{Rs}(v) \text{ and } y^d(u,v) \to y^{Rd}(v) \text{ as } u \to 1^+ . \end{aligned}$$

Finally, we define the offer curve and the trade curve of the household in analogy with those of the firm: $\{\ell^s(u,v), y^d(u,v) \mid u \in [0,\bar{u}], v \in [0,\bar{v}]\}$ and $\{\bar{q}\ell^s(u,v), \bar{s}y^d(u,v) \mid u \in [0,\bar{u}], v \in [0,\bar{v}]\}$. They are, respectively, the utility maximizing and mean-realization points in the (ℓ, y)-space, parametrized by (u,v).

III. STOCHASTIC RATIONING EQUILIBRIUM

III. 1. Definition and Classification

In the preceding section, we discussed behavior of economic agents in response to the disequilibrium signals. The offer and trade curves represent those responses. We will define and analyze a stochastic rationing equilibrium in the following. A stochastic rationing equilibrium is a pair of self-fulfilling disequilibrium signals or, in other words, those signals which are confirmed by economic agents' response.

Definition [Stochastic Rationing Equilibrium]. A stochastic rationing equilibrium is defined as a pair of disequilibrium signals which induce economic agents to express effective demands and supplies that reproduce the signals.

It is easy to see that a stochastic rationing equilibrium is obtained as an intersection of the trade curves of the firm and the household for the same regime.

Depending upon how trade curves intersect, we have four possibilities (from now on, the "boundary" cases will be excluded). Figures III.1, III.2, III.3, and III.4 correspond to Keynesian unemployment, Repressed inflation, Classical unemployment, and Under-consumption, respectively. (Disregard letters with subscript 1 in Figures III.1 and III.2 until Section III.3.) Points F and H represent the notional points for the firm and the household, respectively.

Insert Figures III.1 - III.4 about here

Suppose that the firm and the household receive the disequilibrium signals which imply Keynesian unemployment, i.e., there is excess aggregate supply in each market, so that some of the firms have difficulties in selling their output and some of the households find themselves unemployed. In Figure III.1, the trade curve of the firm in the case of selling uncertainty is shown as a curve, F^k, from F to the origin. The corresponding offer curve is a broken line, F^{kk}. The trade and offer curves of the household in the case of employment uncertainty are shown as H^k and H^{kk}, respectively. The curve of H^k has a positive intercept on the vertical axis, since consumers are assumed to buy some consumption goods from some of initial money balance, even if they have difficulties in being employed. The intersection of the two trade curves, K, represents a stochastic rationing equilibrium with Keynesian unemployment. To see this, suppose that both disequilibrium signals for the consumption goods and for labor are larger than one and more specifically,

$$\hat{v} = y^s/\bar{y}$$

$$\hat{u} = \ell^s/\bar{\ell}.$$

The firm responds to $(\hat{v},\hat{u})$ by offering (ℓ^d, y^s) and the household responds to $(\hat{u},\hat{v})$ by offering (ℓ^s, y^d). Since $y^s > y^d$, some of the firms suffer from excess production, but on average the firm can sell the amount $\bar{y}$, where $\bar{y} = y^s/\hat{v}$. Since $\ell^s > \ell^d$, some of the households are rationed in their labor supply. By construction of the trade curve, we know that on average the household can sell $\bar{\ell}$ of labor. The firm does not have difficulties in hiring, since $\bar{\ell} = \ell^d$; and the household does not have difficulties in buying the consumption good since $\bar{y} = y^d$. Thus the disequilibrium signals $(\hat{v},\hat{u})$ are confirmed by economic agents' responses.

That is, a point K and associated signals $(\hat{v}, \hat{u})$ constitute a Keynesian unemployment equilibrium.

A parallel argument explains why a point R in Figure III.2 represents a stochastic rationing equilibrium in the repressed inflation regime. The disequilibrium signals for repressed inflation are:

$$v^* = \bar{y}/y^d < 1$$

$$u^* = \bar{\ell}/\ell^d < 1 .$$

In the classical unemployment equilibrium with stochastic rationing, the firm can realize whatever trade it wants, while the household is constrained in both the markets. If the offers in each market are independently determined with respect to the stochastic rationing of each market,[8] then we can figure out the disequilibrium signals which make the household respond by the offers whose mean realization is equal to the firm's offer. Since the level of the offer which corresponds to $\bar{\ell}$ is ℓ^s by the H^{kk} curve and also the level of the offer which corresponds to $\bar{y}$ is y^d by the H^{kk} curve, we find the point F in Figure III.3 and

$$v = \bar{y}/y^d$$

$$u = \ell^s/\bar{\ell}$$

is the classical unemployment equilibrium.

A parallel argument shows that Figure III.4 explains the underconsumption equilibrium.

Considering that the two trade curves of the firm reach the origin, and that the two trade curves of the household have positive intercepts on the axes, the existence of some kind of stochastic rationing equilibrium is guaranteed.[9] In general, the proof is seen in Figure III.5.A where, given F, the relative position of H would cause at least one type of stochastic rationing equilibrium. It may be helpful to understand the figure

Insert Figure III. 5. A about here

by transforming it into Malinvaud's diagram where the axes are the price and the nominal wage. Since the notional points F and H are determined by the prices, it is easy to translate Figure III. 5. A into Figure III. 5. B. Starting from an arbitrary point in Figure III. 5. B, note that once the prices are announced, the notional points F and H are determined. Depending upon the relative position of H to F, Figure III. 5. A, we know which type(s) of equilibria we can have. Then classify the point of Figure III. 5. B into a region of the type(s) learned from Figure III. 5. A. Since a point F moves along the production function as the price-wage combination moves, such a figure as Figure III. 5. A is redrawn for each point of Figure III. 5. B.

Insert Figure III. 5. B about here

III. 2. Uniqueness and Multiple Equilibria for Given Prices

Looking at a region, U (Under-consumption), one realizes that this region is also covered by K and R. This means that, given the prices which would cause Under-consumption, it is always possible that an economy turns into other regimes, depending on the quantity signals. In general, a price-wage vector does not give a unique correspondence to the type of stochastic rationing equilibrium in which the economy comes to rest.[10] This gives, on the one hand, some difficulty in analysis (especially in econometric formulation). On the other hand, it leads to very interesting interpretations of an economy. Salop (1978) discusses some characteristics of multiple equilibria in a similar context. Let us take the interesting case that the price-wage vector is "correct," in that the Walrasian equilibrium is possible if economic agents act without considering quantity constraints. Is it still possible that they may be stuck at a non-Walrasian equilibrium? In general, the answer is yes. It is clear that neither Classical unemployment nor Under-consumption is possible if the prices are the Walrasian equilibrium prices. However, Keynesian unemployment and/or Repressed inflation are possible even at the Walrasian prices (i.e., the firm and household having the common notional points), depending on the nature of the trade curves. Suppose that points F and H coincide in Figure III.1 (Figure III.2, respectively), keeping other qualitative characteristics intact. Then it represents the case where Keynesian unemployment (Repressed inflation, respectively) is possible in addition to the Walrasian equilibrium at the Walrasian equilibrium prices.[11]

These multiple equilibria may be confusing for some readers. Let us explain what is happening in a case of multiple equilibria at the Walrasian equilibrium prices. Take a case where Keynesian unemployment is possible in addition to the Walrasian equilibrium. Suppose now that an economy has been enjoying a Walrasian equilibrium. Suppose that the consumer and the firm suddenly become pessimistic. Specifically, the firm becomes pessimistic about the demand for their products, so that they reduce their production and employment (F^{kk} curve). The household becomes pessimistic about their employment possibility.

Facing the uncertainty in income, the household reduces its demand for the consumption goods (H^{kk} curve), which reinforces the firm's pessimism. This "multiplier effect" goes on to the point where the two trade curves intersect each other (point K).

Note that this multiplier effect works even though prices are "correct." Therefore, it is not only a "deviation-amplifying" process (emphasized by Leijonhufvud [1968]) but also a "pure pessimistic multiplier process." Since the stochastic rationing equilibrium is defined as a solution of a simultaneous equation system, the exact start of this pessimism is irrelevant as long as it creates the common belief among economic agents; for example, a mere government announcement of predicting a recession without exercising any real policies becomes a self-fulfilling prophecy. A numerical example is provided in Ito (1979b).

There is a criticism of the conventional disequilibrium theory, saying that the price stickiness is crucial to the theory without explaining why prices are sticky. However, our example of multiple equilibria shows that the "correct" price does not guarantee the Walrasian equilibrium so that price stickiness is no longer crucial to disequilibrium macroeconomics.

Let us briefly turn to a specific question of government policies. We consider a Keynesian unemployment equilibrium at the Walrasian equilibrium prices. Since the cause of unemployment is a lack of confidence in successful trading, all that the government has to do in order to bring back the economy to the Walrasian equilibrium is a mere announcement of a willingness to buy, without limit, the labor force and the consumption good at the Walrasian equilibrium prices. Suppose that the government also promises that it neither resells the consumption goods nor puts the labor force in the productive purpose, in order to avoid crowding out private activity. Then the effect of this policy is that economic agents on the supply side expect that they can sell as much as they want, i.e., they offer the Walrasian supply levels. Accordingly, they modify their demand to the Walrasian levels because of the disappearance of the spill-over effect. After all, by the definition of the Walrasian equilibrium, the private economy balances by itself. Therefore, the government does not have to exercise any transactions which it guarantees to carry out if there is excess supply.

Proposition 1. [The Placebo Effect]: Suppose that the economy is in a Keynesian unemployment equilibrium at the Walrasian equilibrium prices. If the government announces that it would buy any excess supply, the economy is brought back to the Walrasian equilibrium, and there is no additional trade by the government.

We name this phenomenon the placebo effect, since the policy does not involve any medical ingredient (i.e., ex post expenditure) and still cures the disease (i.e., unemployment) through psychological belief.[12]

In the real world, the policies to cause the placebo effect are the following: (i) The price support program (of agricultural goods), which guarantees to buy the remains, if any, of the supply side of a market. (ii) A government program of job creation to hire the unemployed. However, the unemployment compensation itself would not create the placebo effect, since it changes the utility of unemployment but does not guarantee that one is fully employed. The effect of the latter will be analyzed later in Section IV.2. (iii) As Baily (1978) pointed out, the Federal Deposit Insurance Corporation is an example to illustrate the placebo (or "confidence" by Baily) effect. The fact that deposits are insured changes the behavior of depositors. A self-fulfilling expectation equilibrium of a run on the bank is broken by the existence of the F.D.I.C. Therefore, as a result, it is a rare event that the F.D.I.C. has to pay the cost of bankruptcy.

III. 3. Nonrational Expectations and Learning

The next question is how, if ever, the expectations of rationing converge to the rational (correct or objective) ones. In other words, what would happen if the disequilibrium signals economic agents perceive are different from the true aggregate supply-demand ratios?

Let us go back to Figure III. 1 to consider the case of a unique Keynesian unemployment equilibrium.

Suppose that the firm perceives the (subjective) expectation of the disequilibrium signals:

$$v_1 = y_1^s/\bar{y}_1 > 1$$
$$u > 1.$$

Although a firm does not perceive any constraints in hiring labor, it faces a certain stochastic rationing in selling the consumption good. Perceiving v_1, the firm produces and offers y_1^s hiring $\hat{\ell}_1$ in Figure III. 1.

Suppose that the household perceives the (subjective) expectation of the disequilibrium signals,

$$v_1 > 1$$
$$u = \ell_1^s/\bar{\ell}_1 > 1.$$

The household perceives that it faces the possibility of unemployment, but no constraint in buying the consumption goods. It offers the labor supply at ℓ_1^s and buys $\hat{y}_1$.

Both the firm and the household have the correct hunch about which regime they are in, but the perceptions of the disequilibrium signals are incorrect. Although the representative firm offers y_1^s, only $\hat{y}_1$ is the average demand from consumers; therefore, the realized disequilibrium signal is $y_1^s/\hat{y}_1$, which is larger than v_1, the perceived disequilibrium signal. At the same time, the representative household finds that its perceived disequilibrium index of the labor market was wrong, since the realized disequilibrium signal is $\ell_1^s/\hat{\ell}_1$, which is larger than u_1, the perceived disequilibrium index. In short, the perceived signals are not rational.

Now, suppose that the firm and the household learn from this fact and revise their expectations adaptively.

(III. 1)
$$v_{j+1} - v_j = \alpha(v_j - \hat{v}_j)$$
$$u_{j+1} - u_j = \beta(u_j - \hat{u}_j).$$

Unless the adjustment speeds α and β are so large that an adjustment process overshoots, the adjustment process of expectations converges to Keynesian unemployment of K.

By the same argument, if H^k has a steeper slope than F^k at a Keynesian unemployment, then that Keynesian unemployment equilibrium is unstable with respect to the expectation adjustment of (III. 1). Therefore, it is important to know the slopes of H^k and F^k.

The slope of H^k is calculated as the ratio of the changes of mean realizations, Y^{kd} and (L^{ks}/u), in responses to the disequilibrium signal, u. Similarly, the slope of F^k is calculated as the ratio of the changes of mean realizations, (Y^{ks}/v), and L^{kd} in responses to the disequilibrium signal, v. Therefore, the condition that F^k is steeper than H^k becomes,

$$\text{(III. 2)} \qquad \frac{dY^{kd}}{du}\frac{dL^{kd}}{dv} - \left[\frac{dL^{ks}}{du}u - L^{ks}\right]\left[\frac{dY^{ks}}{dv}v - Y^{ks}\right]\frac{1}{u^2v^2} < 0 .$$

This condition means that the ratio of the decline in the purchase of consumption goods to the decline in the average realized supply of labor in response to an increase in the unemployment ratio is smaller than the ratio of a decline in the average realized sale of consumption goods to the decline in the labor demand in response to the excess supply signal in the consumption good market.

Now we are ready to summarize the results on the stability of equilibrium with respect to the quantity expectational adjustment of (III. 1).

Proposition 2 [Stability Conditions]. Evaluating H^k and F^k at a Keynesian unemployment equilibrium, or at the Walrasian equilibrium, the equilibrium is stable if and only if F^k has a steep slope than H^k.

From this proposition, with the fact that H^k has the positive intercept on the horizontal axis while F^k intersects the origin,

Theorem 1 [Existence of Stable Keynesian Unemployment Equilibrium]. Suppose that (III. 2) holds with a reverse inequality at the Walrasian equilibrium. Then the Walrasian equilibrium is unstable and there exists at least one Keynesian unemployment equilibrium.

In a different model, Varian (1977) proved the coexistence of non-Walrasian and Walrasian equilibrium and the instability of the latter. However, his analysis is very different since the underlying dynamical system involves real wage changes. Recently, Gourieroux et al. (1978b) analyzed in an econometric context a quantity adjustment process similar to ours and showed that stability implies the uniqueness of equilibria. This is so because their model is a piecewise linear system which may be justified by Cobb-Douglas utility functions and a deterministic rationing scheme.

It is easy to work out an analogous theorem for the Repressed inflation regime. Suppose a case where the Walrasian equilibrium (W) and a Repressed inflation equilibrium (R) are possible at the Walrasian prices. By the same argument as the one in the Keynesian unemployment regime, the nonrational expectations of rationing between W and R results in more rationing than expected. This leads to the Repressed inflation equilibrium by adaptive expectations.

Proposition 3 [Stability Conditions]. Evaluating H^R and F^R at the Repressed inflation equilibrium, or at the Walrasian equilibrium, then the equilibrium is stable if and only if H^R has a steeper slope than F^R, with respect to the adaptive adjustment of rationing expectations of (III.1).

Theorem 2. Suppose F^R has a steeper slope than H^R at the Walrasian equilibrium. Then the Walrasian equilibrium is unstable, and there exists a stable Repressed inflation equilibrium at the Walrasian equilibrium prices.

IV. POLICY CONSIDERATIONS

In this section, we shall analyze some of the short-run effects of economic policies that aim at changing the current state of the economy. The analysis is limited to being short run, since it is assumed that prices and asset stocks remain constant throughout. First, we shall focus on fiscal policy, i.e., on the effects of changing government demands for goods and labor services. Second, we present the other comparative-statics of the model.

IV.1. Fiscal Policy

In the model, there are two instruments which can be classified as fiscal policy, namely government expenditure on goods and on labor services. Thus, in addition to general effectiveness of expenditure policies in different regimes, it is possible to raise the question of allocating expenditures: should the government spend on goods produced by firms or create directly new employment opportunities in order to cure unemployment and increase output?

Let us work out in detail the analysis in the Keynesian unemployment case and then just summarize the results for other regimes.

In the Keynesian situation, the equilibrium is characterized by the equations

$$Y^d(u) + g = Y^s(v)/v \tag{IV.1}$$

$$L^d(v) + \ell^g = L^s(u)/u\,, \tag{IV.2}$$

where g and ℓ^g denote the government expenditures on the goods and labor. Since Y^d, L^s do not depend on v, Y^s, L^d do not depend on u, and the mean fulfillments fractions of Y^s and L^s are $\bar{z} = 1/v$, $\bar{q} = 1/u$.

A standard implicit function technique gives the results

$$\frac{\partial u}{\partial g} = \frac{-(dL^d/dv)}{\Delta}, \quad \frac{\partial u}{\partial \ell^g} = \frac{-1}{\Delta}\frac{1}{v^2}\left[v\frac{dY^s}{dv} - Y^s\right]$$

$$\frac{\partial v}{\partial g} = \frac{-1}{\Delta}\frac{1}{u^2}\left[u\frac{dL^s}{du} - L^s\right], \quad \frac{\partial v}{\partial \ell^g} = \frac{-(dY^d/du)}{\Delta},$$

where

$$\Delta = \frac{dY^d}{du}\cdot\frac{dL^d}{dv} - \frac{1}{v^2u^2}\left[v\frac{dY^s}{dv} - Y^s\right]\left[u\frac{dL^s}{du} - L^s\right].$$

Since $dY^s/dv < 0$ and $(dL^s/du)u/L^s < 1$, the signs of partial derivatives are the same as the sign of Δ. One immediately recognizes that the stability condition of (III. 2) gives $\Delta < 0$. In the following, we focus on a stable Keynesian equilibrium. (For an unstable equilibrium, just reverse the signs.) Now we have the following intuitively appealing results.

$$\partial u/\partial g, \quad \partial u/\partial \ell^g, \quad \partial v/\partial g, \quad \partial v/\partial \ell^g < 0,$$

i.e., government spending on goods or labor services reduces excess supply in both markets. For the mean output, $\bar{Y}$, and the mean employment levels, $\bar{L}$, we obtain

$$\partial\bar{L}/\partial g = (dL^d/dv) \times (\partial v/\partial g) > 0$$

$$\partial\bar{L}/\partial \ell^g = 1 + (dL^d/dv) \times (\partial v/\partial \ell^g) > 1$$

$$\partial\bar{Y}/\partial g = 1 + (dY^d/du) \times (\partial u/\partial g) > 1$$

$$\partial\bar{Y}/\partial \ell^g = (dY^d/du) \times (\partial u/\partial \ell^g) > 0,$$

since

$$\bar{Y} = Y^d(u(g,\ell^g)) + g \quad \text{and} \quad \bar{L} = L^d(v(g,\ell^g)) + \ell^g.$$

One may observe that an increase in g has a familiar multiplier effect on output, which is greater than one. On the other hand, its effect on employment may very well be less than that of an increase of spending on direct labor services, unless $\partial v/\partial g$ is considerably bigger in magnitude than $\partial u/\partial \ell^g$.

Geometrically, we have that, given $dg > 0$, the curve H^k shifts upwards, while F^k remains unchanged. In the stable case, this increases both $\overline{Y}$ and $\overline{L}$.

Insert Figure IV.1 about here

A similar calculation for the case of Repressed inflation gives the plausible results:

$$\partial\overline{Y}/\partial g = (dY^s/du) \times (\partial u/\partial g) < 0$$

$$\partial\overline{Y}\partial \ell^g = (dY^s/du) \times (\partial u/\partial \ell^g) < 0$$

$$\partial\overline{L}/\partial g = (dL^s/dv) \times (\partial v/\partial g) < 0$$

$$\partial\overline{L}/\partial \ell^g = (dL^s/dv) \times (\partial v/\partial \ell^g) < 0 ,$$

provided we have stability in the sense that the slope of H^R is steeper than that of F^R. Geometrically, $dg > 0$ means that the curve H^R shifts upwards, thereby decreasing output and employment as in Figure IV.2.

Insert Figure IV.2 about here

In an equilibrium of classical unemployment, the mean output and employment levels are given by

$$\bar{Y} = Y^s$$

$$\bar{L} = L^d + \ell^g ,$$

and since Y^s and L^d do not depend on the signals u and v, fiscal policy is relatively powerless. Changes in g are totally ineffective while changing ℓ^g the level of employment will rise by the same amount, but there will be no further repercussions. Similarly, in an equilibrium with Under-consumption, government expenditure on labor services has no effect whatsoever, while spending on goods will increase output by the same amount, but without any further repercussions on employment. The same amount of labor is then just used more effectively. These two cases are illustrated as follows. Looking back at Figure III. 3 in the preceding section, the effect of the increase in governmental purchase of the labor services in classical unemployment is the change of point F rightward. Similarly, the increase in governmental purchase of the consumption good in Under-consumption is the shift of point H upward in Figure III. 4.

IV. 2. Other Comparative Statics

To facilitate the comparison of this approach to the standard one, we shall give in this section the dependence of the different types of equilibria on the exogenous parameters which are initial nominal balances M, the money prices p, and the money wage w. For a discussion of the comparative statics in the standard model, see Malinvaud (1977, pp. 53-75) and Muellbauer and Portes (1978, pp. 812-817).

To begin with, let us make the following plausible assumptions:

$$\partial Y^d/\partial M > 0\ , \quad \partial L^s \partial M < 0\ , \quad \partial Y^s/\partial M = 0\ , \quad \partial L^d/\partial M = 0$$
$$\partial Y^d/\partial p < 0\ , \quad \partial L^s/\partial p < 0\ , \quad \partial Y^s/\partial p > 0\ , \quad \partial L^d/\partial p > 0$$
$$\partial Y^d/\partial w > 0\ , \quad \partial L^s/\partial w > 0\ , \quad \partial Y^s/\partial w < 0\ , \quad \partial L^d/\partial w < 0\ .$$

With these we can summarize the dependence of mean output and employment on nominal balances, the price level and the money wage in the following tables.

TABLE IV.1

Dependence of the Mean Output on Exogenous Parameters

	K	I	C	U
$dM > 0$	+	-	0	+
$dp > 0$	?	?	+	-
$dw > 0$	?	?	-	+

TABLE IV.2

Dependence of the Mean Employment on Exogenous Parameters

	K	I	C	U
$dM > 0$	+	-	0	-
$dp > 0$	?	?	+	-
$dw > 0$	?	?	-	+

It is evident that the results are not very different from the standard approach, such as Malinvaud (1977).

V. CONCLUDING REMARKS

The purpose of this paper has been to work out a new short-run disequilibrium macroeconomic model which has a different description of the rationing mechanism but otherwise shares the basic features of the earlier models. The striking conclusion of the approach is the probable multiplicity of different types of equilibria and the fact that the Walrasian equilibrium is usually unstable with respect to a fairly natural quantity-adjustment mechanism at fixed Walrasian prices. On the other hand, the policy conclusions and comparative-static properties of our model are not inconsistent with the standard models by Barro and Grossman (1976) and Malinvaud (1977).

Two other implications of this new approach may be noted. First, the analysis seems to shed some light on the recent controversy of rational expectations in macroeconomics. The standard analyses, e.g., Sargent and Wallace (1975), S. Fischer (1977), Phelps and Taylor (1977) have been conducted in market-clearing frameworks so that quantity expectations play no role. The discussion in Section III.2 clearly points out their importance. The possible coexistence of Walrasian equilibria and of Keynesian unemployment equilibria, both in which expectations are rational, suggests that the idea of an equilibrium with rational expectations is subtle and that the current macroeconomic literature has not yet digested all of its implications. In the same vein, the placebo or promise-to-buy policy options clearly need further analysis, in particular, beyond the short run.

Second, methods of estimating the Malinvaud-type disequilibrium macro model have been available: Ito (1978, 1979a) and Gourieroux, Laffont and Monfort (1978a). Our model suggests that there is a mix of overbidding or the discouragement effects within the market under disequilibrium, in addition to the spill-over effect from other markets to affect the effective demand. It would not be difficult to formulate an econometric model incorporating them. However, multiple equilibria would cause a serious problem in estimating effective demand and supply functions, unless we make an assumption to separate samples into different regimes without losing the consistency of estimates.

FOOTNOTES

[1] The basic idea is that in disequilibrium there is uncertainty about the amount of trades individuals can complete. Considering this, each individual has to make trade offers which are binding in the sense that no recontracting is allowed and they cannot be contingent on the result of rationing. (This has much in common with the partial equilibrium models of markets under incomplete information; see Rothschild [1973] for a survey.)

[2] This is an ad hoc simplification of an axiomatic framework developed by Green (1978) where the signals for the two markets are the pairs (L^s, L^d) and (Y^s, Y^d), respectively. Note, however, that some of Green's (1978) examples actually give u and v as the relevant variables. Furthermore, it is worth observing that positive government demands for labor and the good are needed to insure that u and v are always well-defined. The analysis of Honkapohja and Ito (1979) also shows that aggregate demands and supplies always lie in compact sets, so that $u \in [0, \bar{u}]$, $v \in [0, \bar{v}]$ for some $\bar{u}, \bar{v}$.

[3] These names for the classical regions are common among the usual disequilibrium macroeconomic models; e.g., Barro and Grossman (1976), Malinvaud (1977), Portes and Muellbauer (1978), and Ito (1978). However, keep in mind that in this paper we have a definition of the effective demand which is different from the above-mentioned works.

[4] This idea of trading uncertainty or stochastic rationing has been studied in a general equilibrium contaxt by Gale (1978), Green (1978), Ito (1979a), and Honkapohja and Ito (1979), while earlier Foley and Hellwig (1975), Howitt (1978) and Svensson (1977) discussed its implications for individual agents. In this paper, we adopt the approach originated by Green (1978) and utilized by Honkapohja and Ito (1979) with further simplifications.

[5] For the mean-balance condition, see Gale (1978), Green (1978), Honkapohja and Ito (1979), Ito (1979a). The short-side rule is familiar from Clower (1965), Barro and Grossman (1971, 1976), Benassy (1975), Malinvaud (1977) and others. The nonnegativity and nonpositivity conditions on the derivatives follow from the hypothesis that rationing of demanders becomes less severe when u or v increases, and conversely for suppliers.

[6] These functions are assumed to have the properties $\ell_1^d > 0$, $\ell_2^d < 0$, $y_1^s > 0$, $y_2^s < 0$ for $1 \geq u$, $v \geq 1$. These follow if it is stipulated that $r(0) = 0$ and $r'u/r < 1$.

[7] For $u < 1$, $v > 1$, there are in fact two-parameter surfaces in (ℓ, y)-space, but in the situations of Keynesian unemployment ($u \geq 1$, $v > 1$) and Repressed inflation ($u < 1$, $v \leq 1$), they become curves, which give the boundaries of the surface. In classical unemployment, both of them reduce to a single point, given by the Walrasian notional demand and supply.

[8] This is valid only as the first approximation. The effects of the stochastic rationing in the labor market are different depending on the level of disequilibrium in the consumption goods, through the income effect, even if there is no substitution effect.

[9] Gale (1979) and Honkapohja and Ito (1979) have proved the existence of stochastic rationing equilibrium.

[10] The idea of a unique correspondence has been asserted in Malinvaud (1977) and used in econometric models by Ito (1978) and Gourieroux et al. (1978a). Hildenbrand and Hildenbrand (1978) and Hahn (1978) criticized Malinvaud (1977) by showing the possibility of multiplicity.

[11] Sometimes it may be the case that the Walrasian equilibrium is the only equilibrium the economy can obtain at the Walrasian equilibrium prices. Neither the Classical unemployment nor the Under-consumption is possible at the Walrasian equilibrium prices. Suppose that the F^k curve is steeper than the H^k curve at the Walrasian equilibrium and the latter is above the former (noting that H^k has the positive intercept on the vertical axis while F^k goes to the origin). Then there is no Keynesian unemployment at the Walrasian prices. Suppose also that the H^R curve is steeper than the F^R curve at the Walrasian equilibrium point, and the latter is always left of the former. Then there is no Repressed inflation either. Therefore, the Walrasian equilibrium is the only equilibrium at the Walrasian equilibrium prices.

[12] Kenneth Arrow pointed out to us that the essence of the placebo effect was argued by Pierson (1944).

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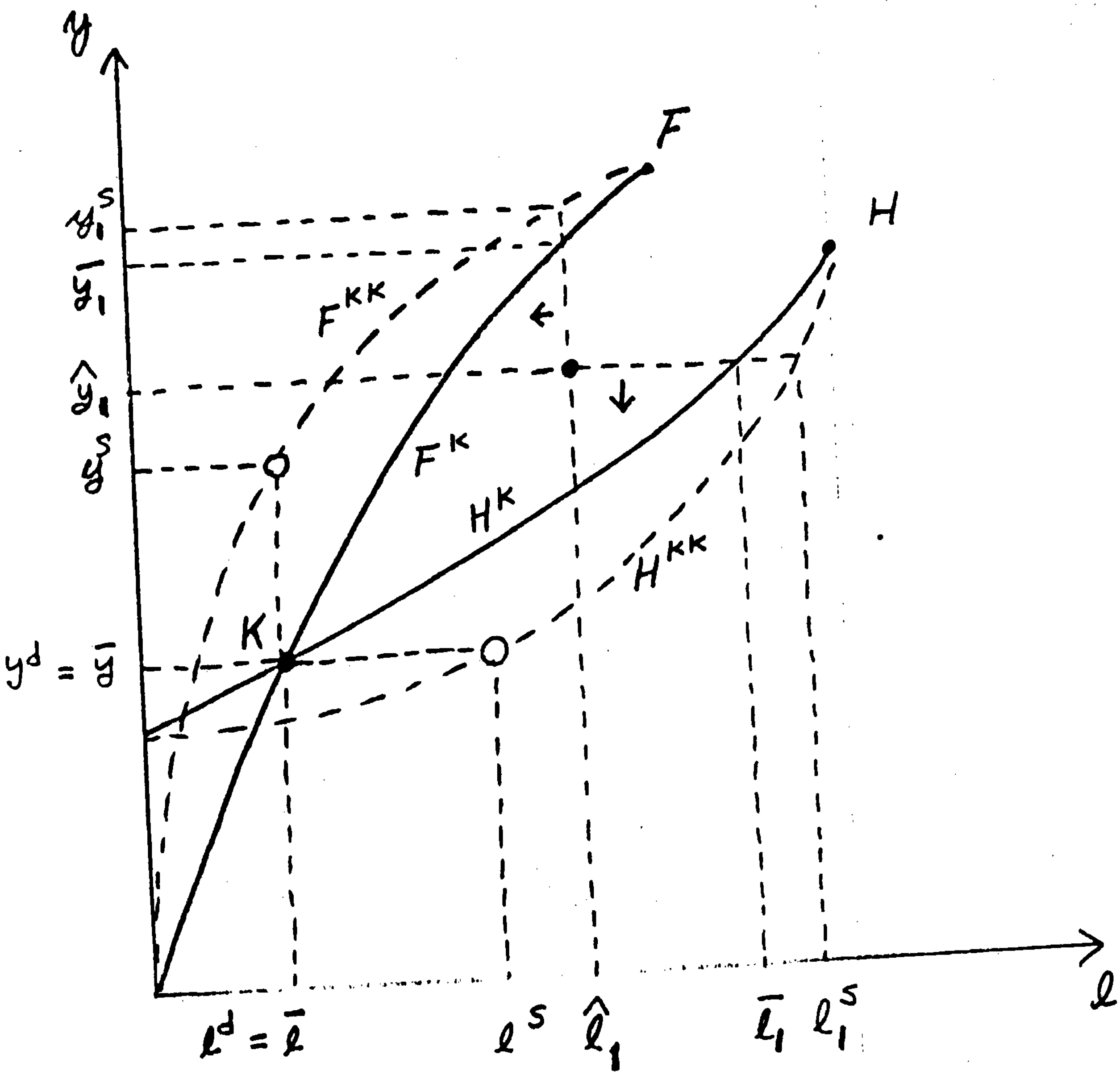

y
F
H
y_1^s
$\bar{y}_1$
F^{KK}
$\hat{y}_1$
F^K
y^s
H^K
H^{KK}
K
$y^d = \bar{y}$
ℓ
$\ell^d = \bar{\ell}$
ℓ^s
$\hat{\ell}_1$
$\bar{\ell}_1$
ℓ_1^s

Figure III.1

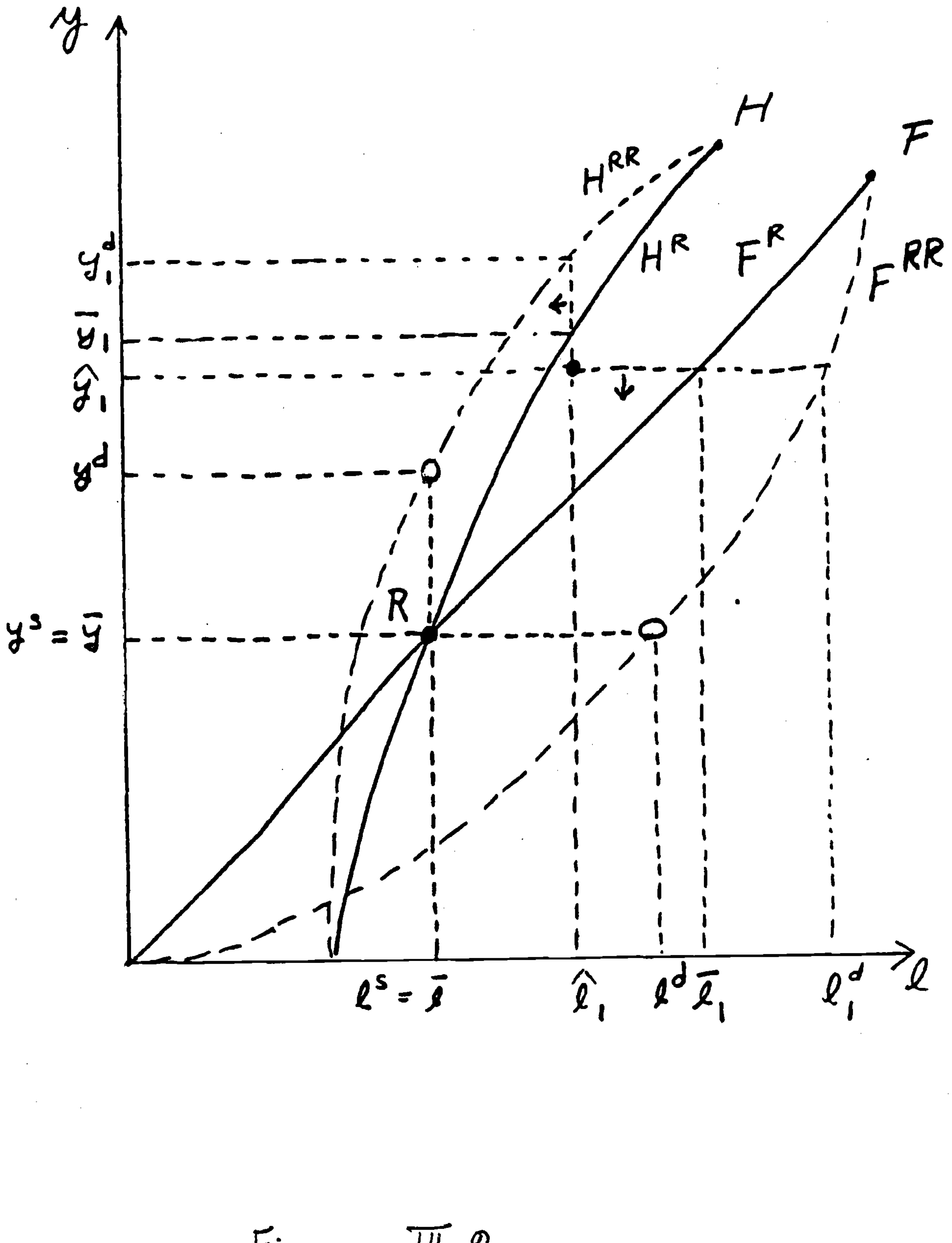

Figure III.2