The Welfare Cost of Uncertainty in Policy Outcomes

Appendix

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This appendix outlines the derivation of the primary results in the paper. It is organized using the two cases considered in our paper: (a) treating the environmental quality resulting from policy as a random variable; and (b) treating the baseline level of quality and the level resulting from policy as independent random variables. Within each case we present the alternative welfare definitions.

A. Policy Determined Environmental Quality as Random Using Certainty Equivalent

Definition of certainty equivalent ($C(t)$)

$$(1A) \quad \int V(q_0 + t\varepsilon, p, m) dF_1(\varepsilon) = V(q_0, p, m + C(t))$$

where: q_0, p, m are the baseline level of environmental quality, a vector of prices for market goods, and income respectively

$\tilde{\varepsilon}$ is random with distribution function, $F_1(\varepsilon)$, $E(\varepsilon) = \bar{q}_1 - q_0$, and a variance of σ_ε^2.

Differentiating (1A) with respect to *t* yields:

$$(2A) \quad \int V_q \varepsilon dF_1(\varepsilon) = V_m C'(t)$$

Differentiating (2A) with respect to *t* yields

$$(3A) \quad \int V_{qq}\, \varepsilon^2 dF_1(\varepsilon) = V_m C''(t) + V_{mm} C'(t)$$

$$(4A) \quad E(\varepsilon^2) = \sigma_\varepsilon^2 + (\bar{q}_1 - q_0)^2$$

Solving (2A) and (3A) for $C'(\mathrm{t})$and $C''(t)$ and evaluating at *t*=0. By evaluating the partial derivatives of $V(q, p, m)$ at *t*=0 we assure consistency in the point of evaluation of the functions on each side of the equations defining our welfare measures. As a result we will not identify in each case the specific point of evaluation in each derivation below.

$$(5A)\ \ C'(0) = \frac{V_q}{V_m}(\ \bar{q}_1 - q_0)$$

$$\text{(6A)}\ \ C''(0) = \frac{V_{qq}}{V_m}(\sigma_\varepsilon^2\ +\ (\bar{q}_1 - q_0)^2)\ - \frac{V_{mm}}{V_m}\ \frac{V_q}{V_m}(\ \bar{q}_1 - q_0)$$

The second expansion for $C(t)$ with $C'$and C'' evaluated at *t*=0 and with *t*=1 is:

$$(7A)\ C(t) \approx \frac{V_q}{V_m}(\ \bar{q}_1 - q_0)t + \frac{1}{2}\ (\frac{V_{qq}}{V_m}(\sigma_\varepsilon^2\ +\ (\bar{q}_1 - q_0)^2) - \frac{V_{mm}}{V_m}\ \frac{V_q}{V_m}(\ \bar{q}_1 - q_0))t^2$$

Using the definition for $b = \frac{V_q}{V_m}$ and (9a) and (9b) in the paper yields (8A)

$$(8A)\ C(t) \approx b(\bar{q}_1 -\ q_0)t\ +\ \frac{1}{2}\ (\bar{q}_1 -\ q_0)^2\big(b_q + b\cdot\ b_m\big)t^2 + \frac{1}{2}\ b\frac{V_{qq}}{V_q}\ \sigma_\varepsilon^2 t^2$$

B. Policy Determined Environmental Quality as Random Using Willingness to Pay

Definition of willingness to pay for this problem ($W(t)$)

$$(9A)\quad \int V(q_0 + t\varepsilon\,, p, m - W(t))dF_1(\varepsilon)\ \ = V(q_0, p, m)$$

Differentiating with respect to *t*

$$(10A)\ \int(\,V_q\varepsilon - V_m W'(t))dF_1(\varepsilon)\ = 0$$

Differentiating (10A) with respect to *t*

$$(11A)\ \int(\,V_{qq}\varepsilon^2 - V_{qm}\varepsilon W'(t) -\ V_m W''(t) - V_{mq}\varepsilon W'(t) + V_{mm}\,(W'(t))^2)dF_1(\varepsilon)\ = 0$$

Equation (10A) implies a result similar to (5A)

$$(12A)\ \ W'(0) = \frac{V_q}{V_m}(\bar{q}_1 -\ q_0)$$

Solving (11A) for $W''(0)$ using equation (12A) we have:

$$(13A)\ W''(0) = \frac{V_{qq}}{V_m}(\sigma_\varepsilon^2 + (\bar{q}_1 - q_0)^2) - 2\frac{V_{qm}}{V_m}\frac{V_q}{V_m}(\bar{q}_1 - q_0)^2 + \frac{V_{mm}}{V_m}(\frac{V_q}{V_m})^2 (\bar{q}_1 - q_0)^2$$

Using the definition for $b = \frac{V_q}{V_m}$ and (9a) and (9b) in the paper yields (14A)

$$(14A)\ \ W''(0) = \frac{V_{qq}}{V_m}(\sigma_\varepsilon^2 + (\bar{q}_1 - q_0)^2) + (b_q - bb_m)(\bar{q}_1 - q_0)^2$$

The second expansion for $W(t)$ with $W'$and W'' evaluated at t=0 is:

$$(15A)\ \ W(t) \approx b(\bar{q}_1 - q_0)t + \frac{1}{2}(\bar{q}_1 - q_0)^2\big(b_q - b \cdot b_m\big)t^2 + \frac{1}{2}b\frac{V_{qq}}{V_q}\sigma_\varepsilon^2 t^2$$

With substitutions this yields an equivalent expression (in the last term of (15A)) as the adjustment for adaptation to uncertainty in realized environmental quality as with the certainty equivalent measure in equation (8A).

C. Policy Determined Environmental Quality as Random With Baseline Environmental Quality as Random Using Equivalent Definition for Option Price

$$(16A)\ \int V(\bar{q}_0 + t\varepsilon, p, m)dF_1(\varepsilon) = \int V(\bar{q}_0 + \nu t, p, m + OP(t))dF_0(\nu)$$

Where ν is a random variable with distribution function $F_0(\nu)$, $E(\nu) = 0$, and variance = σ_ν^2. Now we assume $E(\varepsilon) = \bar{q}_1 - \bar{q}_0$.

Differentiating (16A) with respect to t yields (17A)

$$(17A)\ \int V_q\, \varepsilon dF_1(\varepsilon) = \int (V_q \nu + V_m OP'(t))dF_0(\nu)$$

Differentiating (17A) with respect to *t*

$$(18A)\ \int V_{qq}\varepsilon^2\ dF_1(\varepsilon) = \int (V_{qq}\nu^2 + V_{qm}\ \nu OP'(t) + V_{mq}\nu OP'(t) + V_{mm}(OP'(t))^2 + V_m OP''(t))dF_0(\nu)$$

Solving (17A) for $OP'(0)$ we have:

$$(19A)\ OP'(0) = \frac{V_q}{V_m}(\bar{q}_1 - \bar{q}_0)$$

Using the properties of ε and ν along with (19A) we can solve (18A) for OP''

$$(20A)\ OP''(0) = \frac{V_{qq}}{V_m}(\sigma_\varepsilon^2 + (\bar{q}_1 - \bar{q}_0)^2) - \frac{V_{qq}}{V_m}\sigma_\nu^2 - \frac{V_{mm}}{V_m}(\frac{V_q}{V_m})^2(\bar{q}_1 - \bar{q}_0)^2$$

Using the definition for $b = \frac{V_q}{V_m}$ and (9a) and (9b) in the paper yields (21A)

$$(21A)\quad OP''(0) = \frac{V_{qq}}{V_m}(\sigma_\varepsilon^2 - \sigma_\nu^2) + \quad (\bar{q}_1 - \bar{q}_0)^2(b_q + b \cdot b_m)$$

The second expansion for $OP(t)$ with OP'(t)and $OP''(t)$ evaluated at *t*=0 and with *t*=1 is:

$$(22A)\ OP(t) \approx b(\bar{q}_1 - \bar{q}_0)t + \frac{1}{2}(\bar{q}_1 - \bar{q}_0)^2(b_q + b \cdot b_m)t^2 + \frac{1}{2}\ b\frac{V_{qq}}{V_q}(\sigma_\varepsilon^2 - \sigma_\nu^2)t^2$$

D. Policy Determined Environmental Quality as Random With Baseline Environmental Quality as Random Using Compensating Definition for Option Price (labeled as W in Text)

$$(23A)\ \int V(\bar{q}_0 + t\,\varepsilon, p, m - OP(t))dF_1(\varepsilon) = \int V\,(\bar{q}_0 + \nu t, p, m)dF_0(\nu)$$

Differentiating (23A) with respect to *t*

$$(24A)\ \int (V_q\varepsilon - V_m\, OP'(t))dF_1(\varepsilon) = \int V_q\, \nu dF_0(\nu)$$

Differentiating (24A) with respect to *t* yields:

$$(25A)\ \int (V_{qq}\varepsilon^2 - V_{qm}\varepsilon OP'(t) - V_{mq}\varepsilon OP'(t) + V_{mm}(OP'(t))^2 - V_m OP''(t))dF_1(\varepsilon)$$
$$= \int V_{qq}\nu^2\, dF_0(\nu)$$

Using the properties of $\tilde{\varepsilon}$ and $\tilde{\nu}$ as well as the definition for $b = \frac{V_q}{V_m}$ yields:

$$(26A)\ OP''(0) = \frac{V_{qq}}{V_m}(\sigma_\varepsilon^2 + (\bar{q}_1 - \bar{q}_0)^2) - \frac{V_{qq}}{V_m}\sigma_\nu^2\ - 2b(\bar{q}_1 - \bar{q}_0)^2)\frac{V_{mq}}{V_m} + \frac{V_{mm}}{V_m}b^2(\bar{q}_1 - \bar{q}_0)^2$$

With equations (9a) and (9b) in the paper we can re-write (26A) as

$$(27A)\ \ OP''(0) = \frac{V_{qq}}{V_m}(\sigma_\varepsilon^2 - \sigma_\nu^2) + \left(b_q - b \cdot\ b_m\right)(\bar{q}_1 - \bar{q}_0)^2$$

Using the second order expansion for $OP(t)$ around $t = 0$ we have

$$(28A)\ OP(t) \approx b(\bar{q}_1 - \bar{q}_0)t + \tfrac{1}{2}(\bar{q}_1 - \ \bar{q}_0)^2\left(b_q - b \cdot\ b_m\right)t^2\ + \tfrac{1}{2}\, b\frac{V_{qq}}{V_q}(\sigma_\varepsilon^2 - \sigma_\nu^2)t^2$$

E. Indirect Utility Provides Lower Bound for Risk Aversion in q

The indirect utility function assumes that the consumer can adjust the consumption of all private goods in response to different realizations of q. To verify the claim, consider the other extreme where we assume that the consumption of private goods does not vary at all in response to different realizations of *q*. In this case aversion to risk in *q* is:

$$-\frac{u_{qq}(x,q)}{u_q(x,q)}$$

where: $u(x,q)$ is the direct utility function. Let $\bar{x} = d(m,p,\bar{q})$ describe demand for x when $q = \bar{q}$. It follows that (suppressing price and income arguments):

$f(q) = V(q) - u(\bar{x},q) \geq 0$ for every $q > 0$ with equality when $q = \bar{q}$.

It follows that the first and second-order necessary conditions for a minimum must hold

at $q = \bar{q}, f'(\bar{q}) = 0$ and $f''(\bar{q}) \geq 0$. By the envelope theorem these are equivalent to

$V_q(\bar{q}) = u_q(\bar{x}, \bar{q})$ and $V_{qq}(\bar{q}) \geq u_{qq}(\bar{x}, \bar{q})$. The inequalities, taken together imply:

$$-\frac{V_{qq}}{V_q} \leq -\frac{u_{qq}}{u_q}$$

The same logic can be applied when we consider sets of private goods with some of them fixed. Using the example in note#13 , let $x = (y, z)$, with y a single commodity and z a vector of $N + 1$ goods. If y is allowed to vary in response to q and $\bar{y}$ is demanded for $q = \bar{q}$ then with $z = \bar{z}$ we can establish that:

$$-\frac{V_{qq}}{V_q}(\bar{q}) \leq -\frac{U_{qq}}{U_q}(\bar{q}, \bar{z}) \leq -\frac{u_{qq}}{u_q}(\bar{x}, \bar{q})$$

Where: $U(\bar{q}, \bar{z}) = max_{p_y y + p_z \bar{z} \leq m}\, u(y, \bar{z}, \bar{q})$

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