

Understanding fiscal and monetary policy in 2008-2009

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Abstract

I use the money demand equation and the valuation equation of government debt to understand fiscal and monetary policy in 2008-2009, to think about whether the US is headed for a fiscal inflation, and what that inflation will look like. I emphasize that inflation can come well before large deficits or monetization are realized.

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1 Introduction

I offer an interpretation of the Fall 2009 macroeconomic situation and outlook, focused on the fiscal stance of the U. S. government and its link to potential inflation. How do current policies work? Are we headed for inflation or deflation? Will the Fed be able to follow an “exit strategy?” Will large government deficits lead to inflation? If so, what will that event look like?

While the prognostication will undoubtedly be immediately dated (much of it already became dated through various drafts), I hope that the *method* of analysis and the clarification of channels by which stimulus, inflation and deflation may or may not occur will be more durable.

I base the analysis on two equilibrium conditions, which hold in almost every model of money and inflation: the valuation equation for government debt and a money demand equation,

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} (T_{\tau} - G_{\tau}) d\tau \quad (1)$$

$$M_t V(\cdot) = P_t Y_t, \quad (2)$$

where M is money, B is debt, Λ is a stochastic discount factor, T is tax revenue including seigniorage, and G is government spending. I will be more specific about the form of these equations below. Sargent and Wallace (1981) (also Sargent 1992) used these two equations to understand disinflation in the 1980s and hyperinflation following the two world wars. I use the same framework, with a few nuances, to interpret the current situation and outlook. (Monetary models also include some frictions by which inflation may affect output; I return to this question later.)

In “normal times” many monetary economists don’t pay much attention to fiscal issues. They suppose “fiscal backing” is not a serious constraint on monetary policy. This is a reasonable attitude. In “normal times” monetary influences on U.S. fiscal balance are minor. Seigniorage is small, and small slow changes in inflation have little impact on the real value of outstanding nominal debt.

But these are not normal times, and the fiscal equation may have more influence on affairs than usual. Massive fiscal deficits, credit guarantees, and Federal Reserve purchases of risky private assets raise the fiscal solvency question. At some point (rises in B_t , declines in $T_{\tau} - G_{\tau}$) fiscal constraints *must* take hold. There *is* a limit to how much taxes a government can raise, a top of a Laffer curve, a fiscal limit to monetary policy. At that point, inflation must result, no matter how valiantly the central bank attempts to split government liabilities between money and bonds. Long before that point, the government may *choose* to inflate and devalue outstanding debt rather than further raise distorting taxes. Argentina has found these fiscal limits. So far, the U. S. has not, at least recently. But unfamiliarity does not mean impossibility, and it’s worth at least asking how far we are from the fiscal limit and how it will work when and if it comes.

At the same time, conventional monetary policy may have less connection to economic affairs than usual. Conventional monetary policy affects only the *split* of government debt

holdings between “monetary” and “debt” assets. With interest rates on government bonds near zero, money and government bonds are nearly perfect substitutes, especially for the banks and financial institutions at the center of economic events. Therefore, conventional monetary policy is widely perceived to have much less connection to the price level or “aggregate demand” than usual. Wide and fluctuating credit spreads have further delinked conventional monetary policy from the prices that matter for real decisions.

Increasingly, commentators are discussing monetary events in fiscal terms. Below, I survey a range of oped writers on the inflation vs. deflation debate, from Krugman to Laffer, and find they all are really debating only how far from a fiscal inflation we are. However, they adduce vastly different mechanisms and warning signs, which I try to sort through.

Monetary models also include a description of dynamics, and price-stickiness or other mechanism that sometimes translates inflation into real output. I’ll be pretty vague about that for most of this essay. I’ll identify inflationary and deflationary forces, and allow for some output effects when thinking about events. I will also mostly abstract from dynamics, or explicit price stickiness. I return to this question at the end, where it is important, to assess the likely output effects of a fiscal inflation.

After a quick review of the theory underlying the fiscal equation, I analyze the current situation, common forecasts, and policy debates. I make the following points.

1. Fall 2008 saw a large increase in demand for both money *and* government debt, and a fall in “aggregate demand.” These events makes sense in the fiscal analysis as a deflationary decrease in the discount rate for government debt. Many of the Government’s innovative policies can be understood as ways to accommodate this demand, which a conventional swap of money for government debt does not address.
2. Winter 2009 saw the announcement of dramatic fiscal stimulus in the U. S. and U. K., along with academic controversy over their effectiveness. Will “fiscal stimulus” stimulate? In this analysis, deficits can generate inflation (the same thing as “stimulate” here) if people do not expect future taxes to pay off the increased debt. Unlike conventional “Ricardian equivalence,” we do not need irrationality or market failure for this expectation, since our government debt is nominal.
3. With interest rates at zero, “quantitative easing;” large additional purchases of government debt, is sometimes recommended instead. Bank of England governor Mervyn King is following this strategy explicitly; the US Federal Reserve’s remarkable expansion is often interpreted in this light.

I show that quantitative easing cannot inflate, especially in the zero-interest rate environment, without fiscal cooperation. If we want inflation, it must be by somehow convincing people that the government will not raise taxes or cut spending to pay off deficits. Both “fiscal” and “monetary” stimulus can only work if they change equation (1); if they are or are supported by increases in future primary deficits.

4. Much stimulus debate revolves around the fact that fiscal expenditures cannot happen quickly. In this analysis, *prospective* deficits are just as “stimulative” as current deficits. Perhaps this explains the Administration’s announcements of large future deficits.

I ask, what *are* current expectations about future surpluses? The Government's announcements can be read both ways, in some ways promising unsustainable deficits that can stimulate and inflate while at others promising fiscal discipline that cannot do so. However, we can *measure* the state of private expectations in the bond market. We do not have to argue about "Ricardian equivalence" as a matter of economic theory or intuitive plausibility. If the private sector does not expect higher surpluses in response to debt issue, the government cannot raise revenue by bond sales. It just raises interest rates instead. The relatively steady behavior of interest rates (so far) and the fact that the government is collecting a lot of revenue suggests "Ricardian" expectations and hence not much stimulus.

5. Conversely, many commentators tell us not to worry about fiscal inflation, because the US debt to GDP ratio is currently lower than that of other countries and than it has been in the past. I point out that the real issue in (1) is *prospective* deficits. We can suffer a fiscal inflation without a large debt/GDP ratio.
6. Others argue that we won't see inflation until we see large seigniorage operations. In this analysis, inflation can happen before seigniorage, and even without any seigniorage. Fiscal inflation, a "flight from the dollar," does not need to wait for monetary policy.
7. If a fiscal inflation comes, what will it look like? I extend the above equations to long-term debt, and analyze a stylized shock to expected surpluses. In the plausible scenario, interest rates rise, and inflation only comes slowly after a few years.
8. I examine the outlook for inflation. The easy question is, *will* the Fed soak up all the money it has issued? The harder question is, *can* the Fed do so? Or will we run in to the fiscal limits of monetary policy – will the Fed try to sell Treasuries just as everyone else is trying to do so, and no credible future surpluses can justify new debt issues? In this context, I make two points
 - (a) Credit guarantees make matters worse than actual deficits suggest.
 - (b) If taxes have any effect on growth, the 'Laffer limit' of taxation may come much sooner than static analysis suggests. The present value of taxes is strongly influenced by growth. The big inflation danger is a long period of slow growth.
9. Many economists think that a little inflation is ok and even desirable, because inflations historically come with booms. However, I point out that fiscal inflations, and in particular inflations that come from collapsing expectations of deficits, may have quite different output effects both in theory and in experience. If inflation always corresponded to "more demand" and a boom, Zimbabwe would be the richest economy in the world. A fiscal inflation may well lead to stagflation, not a boom.

The fiscal valuation equation can be seen as the "anchor" for monetary policy, or the "long-run expectation" that shifts the Phillips curve. A fiscal inflation is therefore likely to lead to the same stagflationary effects as any loss of "anchoring."

2 Fiscal review

2.1 The government debt valuation equation

Before applying it to the current situation, I quickly review how the government debt valuation equation works¹. This equation states that the real value of government debt must equal the present value of future primary surpluses. In the simplest case that the government issues floating-rate or overnight debt, it reads

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} \left(s_{\tau} + i_t \frac{M_t}{P_t} \right) d\tau \quad (3)$$

where M_t is money, B_t is government debt, Λ_{τ}/Λ_t is the real stochastic discount factor between periods t and τ , i_t is the nominal interest rate and $s_t = T_t - G_t$ denotes real primary surpluses. The appendix derives this and related equations. In particular, we can also discount at the ex-post real rate of return on government debt, i.e. substituting $1/R_{t,\tau}$ for Λ_{τ}/Λ_t . Seignorage iM_t/P_t is small for the U. S. economy, and I will ignore it in most application and discussion.

Suppose there is bad news about future primary surpluses. Equation (3) predicts inflation *now*, a rise in P_t . Loosely, a fall in surpluses means that current debt will be paid off by printing money in the future, meaning future inflation. If we all know there will be inflation in the future, we try to get rid of money and government debt now, leading to current inflation.

This fiscal price-equilibrating mechanism feels to participants exactly like “aggregate demand,” or a “wealth effect” of government debt. There is nothing unusual or unfamiliar about it. If the present value of future surpluses is less than the value of debt, people try to get rid of that debt, buying goods, services, and private assets instead. This extra “demand” raises prices to their equilibrium level.

More deeply, “aggregate demand” is really just the mirror image of demand for government debt. The household budget constraint says that after-tax income must be consumed, invested, or result in purchase of government debt,

$$C_t + I_t + \frac{dM_t + dB_t}{P_t dt} = Y_t - T_t$$

The only way to plan to consume *and* invest more is to plan to hold less government debt. Thus, we can think about “aggregate demand” and “supply vs. demand for government debt” interchangeably.

The government debt valuation equation (3) influences the price level in some unexpected ways. First, there is no first-order difference between money and bonds, so open market operations have very little effect on the price level. They only operate through usually small

¹Many of the points in this section are treated at more length in Cochrane (1998), (2001), (2005). These papers also contain bibliographic reviews, which more properly attribute credit for the ideas, which I will not attempt here.

(for the US) seignorage effects. Second, inside money is unimportant. Only government debt relative to the surpluses that will retire it matter for the price level. People can generate arbitrary inside claims to government debt with no inflationary pressure. Third, a version of the real-bills doctrine emerges: If the government issues money or debt in exchange for assets of equal value, which can retire that debt in time, no inflation results. Fourth, the valuation equation (3) can determine the price level even in a frictionless economy with $M_t = 0$. To some, this is a paradox. To others it offers a great simplification and an attractive frictionless benchmark for monetary theory in fiat money economies.

2.2 Monetary and fiscal policy

Money and debt appear symmetrically in (3). To understand monetary policy, which may affect the price level by the split of government liabilities between money and debt, we also need a money demand function, that captures the “special” nature of money,

$$M_t V(\cdot) = P_t Y_t. \quad (4)$$

The notation (\cdot) reminds us that many variables can affect velocity. Interest rates are a conventional argument of velocity $V(\cdot)$, but Fall 2008 emphasized that other arguments may belong as well, to capture a “precautionary” demand for money having nothing to do with interest rates.

Equations (3) and (4) each can determine the price level. Thus, government must arrive at a “coordinated policy” by which monetary and fiscal policy agree on that price level. It must choose $\{M_t, B_t, s_t\}$ in such a way that both (3) and (4) hold. Successful monetary policy needs an appropriate fiscal backing; successful fiscal stimulus needs monetary cooperation. We often specify stylized “regimes” to describe fiscal-monetary coordination. Most “normal times” monetary economics specifies that (4) with monetary control or interest rate rules² determines the price level, and then the treasury raises or lowers lump-sum taxes as necessary ex post so that (3) will hold. This is a “Ricardian” or “money-dominant” regime. For example, Woodford’s (2003) treatise on interest rate policy makes this assumption. Another possibility is that the fiscal equation (3) determines the price level, and then money demand (4) determines M_t . Money is passive, for example via an interest rate target, or providing an “elastic currency.” This is a “non-Ricardian” or “fiscal-dominant” regime. Real-world fiscal-monetary policy coordination is not so simplistic as these stylized theoretical treatments. History is full of interesting battles and hard-fought cooperation between monetary and fiscal authorities.

For my purposes, we do not have to take a stand on these issues or make a choice or diagnosis of “regime.” We are never “choosing which equation holds.” Both (3) and (4) hold in every equilibrium or regime, no matter how the government arrives at policy parameters to determine that equilibrium. In particular, even if one sticks firmly to a view that monetary

²Of course there is a good deal of controversy whether interest rate rules by themselves are enough to determine the price level, as there is some remaining controversy whether $MV(i) = PY$ can do so, and thus whether money-dominant regimes even work. Cochrane (2007) gives my take on these determinacy issues, but they need not concern us here.

policy chooses P , that choice must be consistent with a believable set of fiscal expectations. All my analysis can be framed, “is the fiscal backing of monetary policy adequate to properly anchor (or loosen the anchor of) inflation expectations?”

We also need not argue what is “exogenous” or “endogenous.” Using equation (3) does *not* require us to assume that surpluses are “exogenous” in any sense. We may easily take a view that they are the last thing to be determined – and then we can ask whether it’s at all plausible that the required surpluses will be produced.

2.3 Sargent, Wallace, seignorage and nominal debt

My analysis of (3) and (4) differs from Sargent and Wallace’s (1982) and many other joint fiscal-monetary analyses, in that I explicitly consider *nominal* government debt – a promise only to pay US dollars – where Sargent and Wallace implicitly consider real, indexed or foreign-currency debt.

To contrast this analysis with Sargent and Wallace’s, we can rewrite (3) (see the Appendix) as

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} \left(T_{\tau} - G_{\tau} + \frac{dM_{\tau}}{P_{\tau}} \right) d\tau, \quad (5)$$

counting seignorage by money creation rather than interest savings. Sargent and Wallace’s version of this equation is

$$b_t = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} \left(T_{\tau} - G_{\tau} + \frac{dM_{\tau}}{P_{\tau}} \right) d\tau, \quad (6)$$

where b_t denotes the *real* amount of debt, which does not change if the price level changes.

Sargent and Wallace argued that looming $T_{\tau} - G_{\tau}$ problems would have to be met by seignorage, dM_{τ}/P_{τ} . There is no other way to make their equation hold. That money creation, through $M_{\tau}V(\cdot) = P_{\tau}Y_{\tau}$ would create inflation at time τ . Finally, that future inflation might be brought back to the present, time t by hyperinflation dynamics $M_tV[E_t(P_{t+1}/P_t)] = P_tY_t$.

With nominal debt, in the first equation, inadequate future $T_{\tau} - G_{\tau}$ can cause the current price level P_t to rise directly. This rise lowers the outstanding value of nominal government debt, reestablishing equation (5). This channel is absent with real debt. (State-contingent debt or an explicit default can also accomplish such a revaluation, but Sargent and Wallace sensibly assumed that the U.S. government would not explicitly default.)

Nominal debt works like equity: its price can absorb shocks to expected future cashflows. Real debt works like debt, which must be repaid or explicitly default. There is good sense in commentators’ view that exchange rates and inflation reflect “confidence” in the government, output, productivity and fiscal prospects, all having nothing to do with the central banks’ choice of how to arrange the maturity and liquidity structure of government debt.

With nominal debt, inflation can be induced by fiscal imbalance if there is no money at all, or no seignorage revenue at all, $dM = 0$. Future inflation is brought to the present directly, not via the interest-elasticity of money demand.

This point is important to thinking about our future. Most commentators assume that inflation will only come after money creation, whether induced by seignorage needs or by policy mistakes. In fact, with nominal debt, inflation does not require any current or past money creation at all. A “flight from the dollar” inflation can occur based only on expectations of future fiscal trouble.

2.4 Long term debt and inflation dynamics

Equation (3) assumes floating-rate or overnight debt. The dynamic relationship between debt, surpluses and inflation can be quite different with long-term debt. These differences are important to understand, at least at a qualitative level, in order to apply these ideas to the US economy. In particular, they suggest that a fiscal inflation will not consist of a one-time jump in the price level, but rather will consist of a smooth increase in inflation presaged by higher long-term interest rates.

As an extreme but simple example, suppose that debt consists of a single perpetuity, a constant coupon c is redeemed each period, with no other debt purchases and sales and no money. Then we have

$$\frac{c}{P_t} = s_t.$$

The price level is simply the ratio of the nominal coupon coming due each period to the real surpluses that can redeem it. In this case, inflation only happens when the actual poor surpluses s_{t+j} are realized, and not in anticipation of those surpluses.

With long-term debt, the present-value equation (3) still holds, in the form

$$\frac{B_t}{P_t} = \frac{\int_{j=0}^{\infty} Q_t^{(j)} B_t^{(j)} dj}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} s_{\tau} d\tau, \quad (7)$$

where $B_t = \int_{j=0}^{\infty} Q_t^{(j)} B_t^{(j)} dj$ denotes the market value of nominal government debt, $B_t^{(j)}$ denotes maturity j debt and

$$Q_t^{(j)} = E_t \left(\frac{\Lambda_{t+j} P_t}{\Lambda_t P_{t+j}} \right)$$

denotes the nominal price at t of j -year debt. With long term debt, the market value of debt as well as the price level can absorb expected-surplus shocks.

In the extreme coupon example, bad news about a future surplus s_{t+j} raises the future price level P_{t+j} . Future inflation lowers bond prices $Q_t^{(j)}$, so the numerator of (7) does *all* the adjusting at t rather than the denominator.

Tradeoffs

Long-term debt also gives the government the power to trade off current for future inflation, holding fixed the surplus stream. With floating-rate or overnight debt, the government can freely affect the *future* price level $\{P_{t+j}\}$ with no change in surpluses, by changing B_{t+j} . Changing debt without changing surpluses is the same thing as a currency reform. However

there is nothing it can do about the current price level P_t , without changing surpluses, as you can see in (3).

Long-term debt changes this picture. With long-term debt, the government can trade less inflation now for more inflation later, with no change in surpluses, by selling additional long-term debt. This new debt dilutes the claims of existing long-term debt, giving the government some resources to avoid inflation now. However, by increasing the stock of long-term debt it makes the eventual inflation worse. Thus, when it chooses a different future path $\{P_{t+j}\}$, that choice also affects P_t .

The maturity structure of long-term debt gives the “budget constraint” to the government’s options for trading inflation today for inflation at future dates by such debt sales and purchases. The more long-term debt outstanding, the better the tradeoff. This statement is easiest to digest in the case of a constant real rate so $\Lambda_t = e^{-rt}\Lambda_t$. Then (7) reads

$$\int_{j=0}^{\infty} E_t \left(\frac{1}{P_{t+j}} \right) e^{-rj} B_t^{(j)} dj = E_t \int_{\tau=t}^{\infty} e^{-rj} s_{t+j} dj. \quad (8)$$

By buying and selling debt at date t and later, after $E_t s_{t+j}$ is revealed, the government can achieve *any* sequence $E_t \left(\frac{1}{P_{t+j}} \right)$, consistent with this equation, with no change in surpluses. (For a proof, see Cochrane 2001 p. 88).

2.5 An inflation scenario

How will inflation react to a negative shock in expected surpluses

$$\Delta S \equiv (E_t - E_{t-\Delta}) \int_{\tau=t}^{\infty} e^{-rj} s_{\tau} d\tau?$$

This is our central question. If people lose confidence in the US government’s ability or willingness to run surpluses to pay off the debt, how will inflation develop? To answer this question, we have to measure the maturity structure of outstanding debt, and take a stand on how the government’s debt sales will respond, as follows.

If we only had overnight or floating-rate debt, there would have to be a sudden jump in the price level P_t , by (3). The government can choose any path $\{P_{t+j}\}$ after that, and might well choose no additional inflation. But with long term debt missing from (8), the government cannot avoid the price-level jump at time t .

Sudden jumps in the price level are not realistic, and they’re not desirable either. Long term debt is useful, because it allows the government to avoid price-level jumps, and that may be a good reason why our government issues such debt.

To analyze this question with long-term debt, suppose that the economy starts at a constant and expected future price level P , there is only a single expected-surplus shock ΔS at date t . Now, from (8), the price level paths $\{P_{t+j}\}$ must satisfy

$$\int_{j=0}^{\infty} e^{-rj} \left(\frac{1}{P_{t+j}} - \frac{1}{P} \right) B_t^{(j)} dj = \Delta S.$$

Dividing by S , we can rewrite this condition in a convenient dimensionless form as

$$\int_{j=0}^{\infty} \left(\frac{\frac{1}{P_{t+j}} - \frac{1}{P}}{\frac{1}{P}} \right) W_t^{(j)} dj = \frac{\Delta S}{S} \quad (9)$$

where

$$W_t^{(j)} \equiv \frac{e^{-rj} B_t^{(j)}}{\int_{j=0}^{\infty} e^{-rj} B_t^{(j)} dj}$$

denotes the fraction of the market value of debt due to maturity- j debt.

This equation does not give a unique solution for $\{P_{t+j}\}$ in response to the surplus shock, because the government can choose any path consistent with (9). Therefore, to display what will happen in response to a surplus shock, we have to take a stance on which kind of path the government will choose. Our government seems to prefer steady inflation to highly variable inflation and price level jumps, and for good reasons. Thus, suppose the government holds inflation to zero for T years, and then allows a constant inflation π_T , so that

$$\begin{aligned} P_{t+j} &= P; & j < T \\ P_{t+j} &= P e^{\pi_T(t-j)}; & j \geq T \end{aligned}$$

By (9) this policy is attractive, because a small steady inflation can add up to a large price level change.

Now, how large must inflation π_T be? We must have

$$\int_{j=T}^{\infty} \left(\frac{1}{e^{\pi_T(j-T)}} - 1 \right) W_t^{(j)} dj = \frac{\Delta S}{S} \quad (10)$$

It is simple enough to solve this equation numerically – obtain an estimate of the maturity structure of outstanding debt $W_t^{(j)}$ and then for each T find the value of π_T that solves this equation.

To make a calculation, I use a very simple estimate of the maturity structure of U. S. Federal debt, presented in Figure 1. I used every bond, bill, or note in the CRSP mbx database on Jan 31, 2009, and I assigned coupons to the month in which they come due. Thus, $B_t^{(j)}$ includes both principal and coupon payments coming due at time $t + j$. This is a very crude measure. I do not include Federal Reserve liabilities, nor offsetting government or Federal Reserve assets. I do not include credit guarantees, nor the nominal value of unused depreciation allowances and other nominal commitments. Ideally, we take all of the government's net real commitments and put them on the right, all the nominal ones on the left, and find price level (path) that equates the two. However, this is a useful starting place and allows some back-of-the-envelope calculations. It lets us begin to think about how much of a long-term debt cushion the US government has, and thus how quickly surplus shocks must feed in to inflation.

Figure 1 shows that a large fraction of US debt is short term, with a large spike at one year and less maturity. (Counting an additional \$1.5 trillion of currency and reserves adds to short term debt, though one must also subtract the Fed's short-term liabilities.) However,

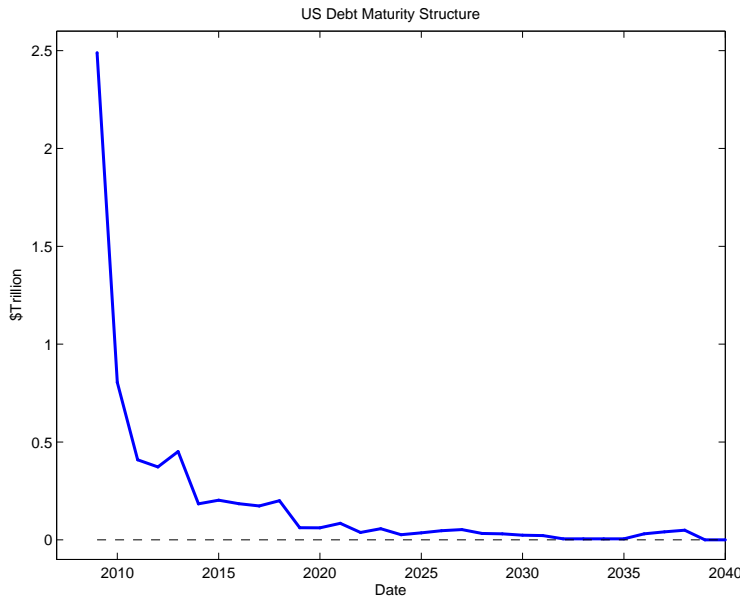


Figure 1: Maturity structure of US federal debt, zero-coupon basis. I use every bond, bill, or note in the CRSP mbx database on Jan 31 2009, and assign coupon payments to the date on which they come due.

there is also a substantial tail of long-term debt that may be able to cushion surplus shocks. The question is, how much?

I suppose a 10% negative shock to expected surpluses, $\Delta S = -10\%$. Using the U.S. Federal debt maturity structure plotted above, I calculate the π_T required to solve (10), and I plot the results in Figure 2. The leftmost point shows us that, rather than a 10% price level jump with no further inflation, the government can embark instead on a modest 2.75% steady inflation. It can put off inflation entirely for a few years, if it accepts higher values of the eventual inflation. For example, the government can sell enough debt to give us zero inflation from the time of the shock (2009) to 2012, and then embark on a steady 6.4% inflation starting in 2012. There isn't all that much long-term debt outstanding, however, so if the government puts off the inflation much longer, much higher values of inflation must result.

To bring these possibilities to life, Figure 3 plots three possible price-level paths. First, I plot (red triangles) a one-time 11% price level jump. This is the solution with no long-term debt, and it remains available in the presence of long-term debt; it is a solution of (9).

Next, I plot (blue circles) a steady 2.75% inflation starting immediately ($\pi_T, T = 0$, from Figure 10). This is a much more plausible path. To arrange it, the government sells long-term debt to meet the surplus shock, exactly as our government is doing right now. The valuation equation cares about the price *level*. This inflation path soon brings about higher future price levels than the one-time jump, which is how it still satisfies (9).

Finally, I plot (black triangles) a postponed inflation. Now the government sells even

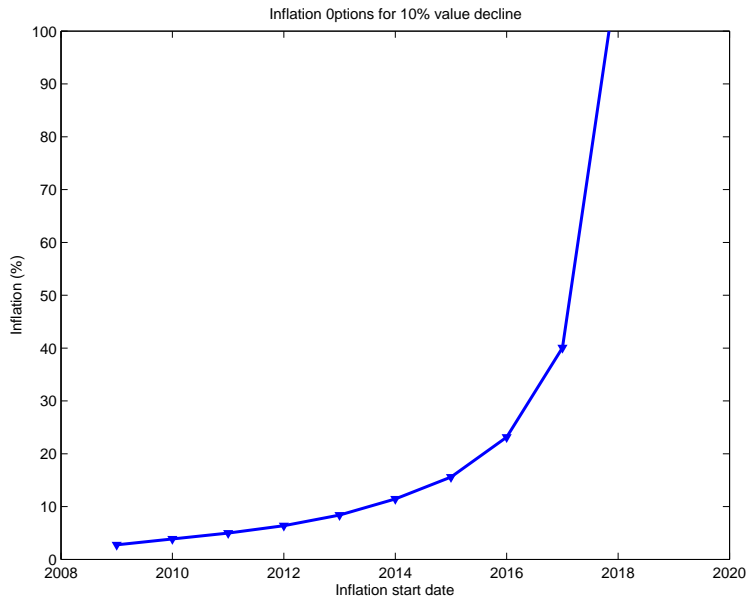


Figure 2: Steady inflation starting in the indicated year, required to meet an unexpected 10% decline in the value of future surpluses in 2009. Each point π_T solves $\int_{j=T}^{\infty} \left(\frac{1}{e^{\pi_T(j-T)}} - 1 \right) W_t^{(j)} dj = -0.1$.

more long term debt immediately, so as to have no inflation at all for four years. In the fifth year it allows the necessary inflation to emerge, but since there isn't that much long term debt left, the inflation is much larger and the cumulative price level increase is also much larger.

To further bring the postponed-inflation possibility to life, Figure 4 plots the corresponding time series of inflation and bond yields. The vertical line indicates the date of the surplus shock. The first reaction is a rise in interest rates on the long end of the yield curve. As the inflation approaches, shorter term rates rise as well. Finally, 5 years after the surplus shock, the inflation actually materializes.

The eventual price level response in all these cases is larger than if the government allowed an immediate 11% price jump. All of these responses also scale with the size of the surplus shock. A shock larger than 10% can result in a frightful inflation.

2.5.1 Long term debt: Bottom line

In the presence of long-term debt, then, we expect dynamics of inflation and interest rates that are more subtle and more realistic than a simple one-time price-level jump envisioned by the simple case with floating-rate or overnight debt. Our government prefers smooth inflation, and will use long-term debt sales to offset sudden price-level jumps. The presence

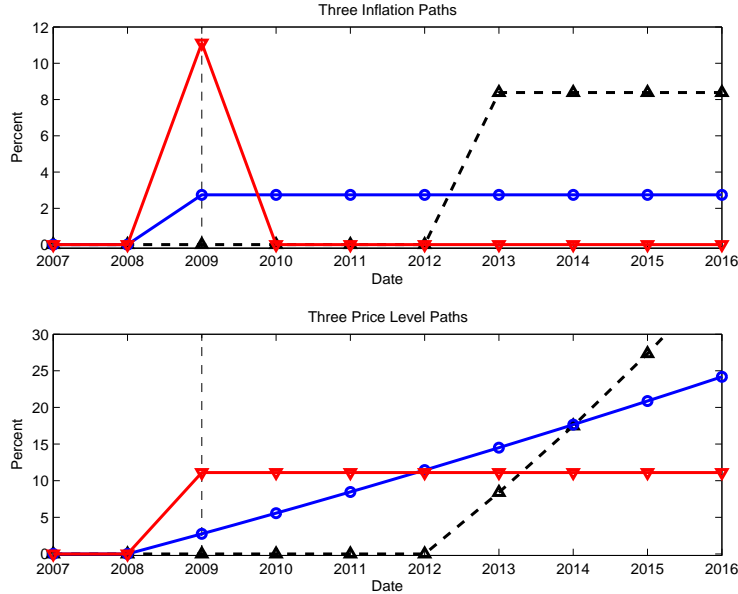


Figure 3: Three possible reactions to a 10% expected surplus shock $\Delta S = (E_t - E_{t-\Delta}) \int_{j=0}^{\infty} e^{-rj} s_{t+j} dj$. Red triangles display a time-t price level jump followed by no additional inflation. Blue circles display a steady inflation starting at time t. Black triangles display a steady inflation starting 4 years after the shock.

of outstanding long-term debt gives it that power.

Thus, a shock to expected future surpluses is likely to result in the first instance by a rise in long-term interest rates, with inflation emerging on a time scale roughly coincident with the average maturity of government debt, on the order of several years for the US. The longer the government puts off the inevitable inflation, the larger the cumulative price increase must be. The 4 year delay path in 4 and 3 seems to me a good qualitative guide to what to expect, given the maturity structure of US government debt and my sense of our Government’s desire to smooth inflation.

The simple floating-rate case remains a useful guide, however, if we remember to apply it on a scale of several years.

Nothing in my simulation mentioned the time path of the surplus shock – whether it was a shock to s_t or some long-dated s_{t+j} . Thus, expected *future* surpluses can still matter – inflation or the interest rate rises that precede it do not need to wait for actual large deficits, “crowding out,” or for monetization of those deficits. On the other hand, we can also see an inflation that responds slowly to a past deficit, with no plan to pay it off.

Keep in mind, all of these smooth dynamics come from a frictionless benchmark, without any price-stickiness. That consideration potentially adds a further source of inflation smoothing relative to the simplistic price level jump we started with.

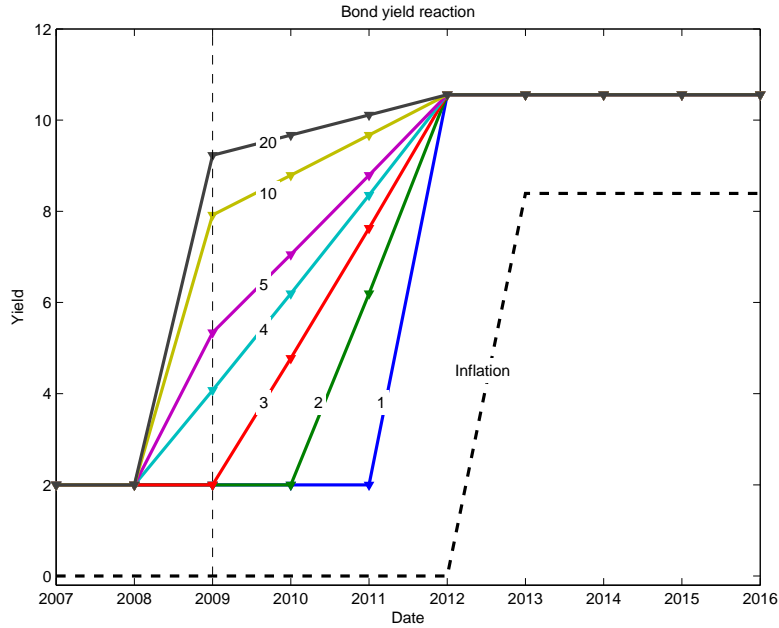


Figure 4: Bond yields and inflation, from a 10% shock to expected surpluses ΔS , when the government sells debt to postpone inflation for 5 years. Numbers indicate the maturity of the bond yields. The vertical line indicates the date of the surplus shock. I assume a 2% constant real rate.

3 2008-2009, and “more of both”

With this conceptual framework in mind, we can examine the 2008-2009 history and speculate about the future.

3.1 Money supply and demand

Figure 5 describes Federal Reserve assets and liabilities, which allow are the most direct measure of the Fed’s actions.

In Fall 2008, following the Lehman and Washington Mutual failures, the AIG bailout and secretary Paulson’s TARP speech, monetary aggregates exploded and the Fed Funds and Treasury bill rates dropped sharply. Deposits rose roughly \$600b, and excess reserves rose from \$6b to \$800b. Monetary policy is obviously at least very accommodative, and plausibly very expansionary.

There is an interesting question which it is – is the Fed just accommodating a huge spike in money demand, or is it running an expansionary, and, one might fear, eventually inflationary, policy in the face of stable demand? V is obviously not stable, but is $V(i)$ stable, or must we appeal to $V(i, \cdot)$? There are good arguments in favor of some “precautionary” increase in money demand. Firms, unsure whether they would be able to get short-term financing in the future, wanted to convert every possible asset into cash. They also drew down lines

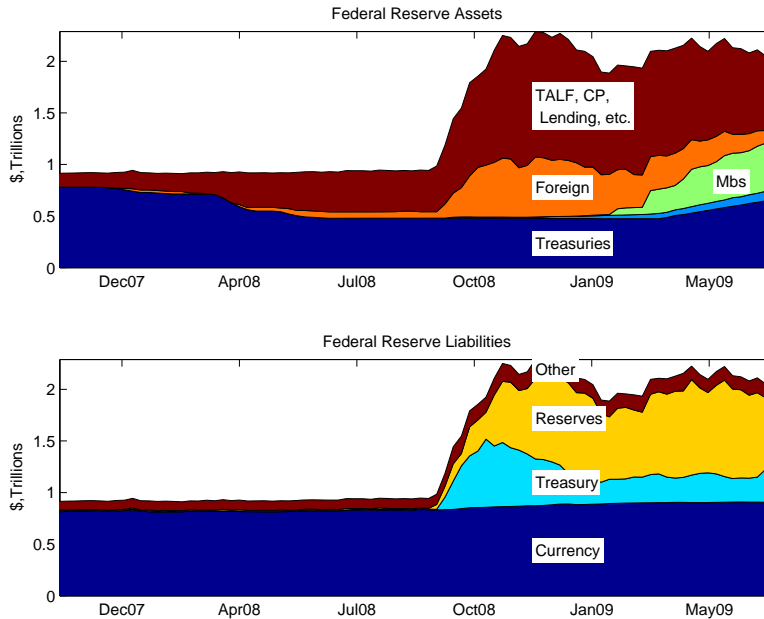


Figure 5: Federal reserve assets and liabilities. Source: Federal Reserve H.4.1 release, June 25, 2009.

of credit, often borrowing at relatively high rates in order to hold cash. (See Scharfstein and Ivashina 2009). Even well-endowed universities scrambled to hold cash. Since reserves now pay interest, their opportunity cost is much lower, and demand correspondingly higher. Quantitative evidence is harder, since we have little experience with an environment with nearly zero interest rates. Taylor (2009) argues that since Fed funds rates fell so far, policy went past “accommodative” to “expansionary.”

However, this argument is not particularly important for my purposes. Whether V decreases as interest rates rise, or as precautionary demands fade, the Fed will eventually have to drain extra money from the economy. It is clear that the Fed at least accommodated any shifts in money demand, learning from Friedman and Schwartz’s analysis of the Great depression. (In the Great Depression people wanted to pull money out of banks and put them in cash, while in 2009 people wanted to get rid of risky assets and put more money in banks, and banks wanted to hold more reserves, but the same principle applies.)

3.2 More of both; aggregate demand

However, conventional monetary policy only trades money for government debt. It can accommodate a demand for more money and *less* government debt. The events of the last few years suggested a large increase in demand for *both* money and government debt. All government bond interest rates declined sharply. Dramatic credit spreads opened. For example, high rated tax-free municipal bonds sold above treasuries. A large liquidity spread opened up between on-the-run and off-the-run government issues. The dollar rose, putting

a dramatic end to the “carry trade.” These events suggest a “flight to quality” or “flight to liquidity” from private assets to U. S. debt of all maturities.

As one micro motivation for the flight, government bonds became practically the only security one could repo. (Gorton and Metrick 2009). In normal times, if you own a corporate bond, you can sell it in a repurchase agreement or use it as collateral for a loan, thus financing the bond purchase. In the Fall of 2008, suddenly only government bonds were acceptable as collateral. A government bond was as good as a dollar, because if you had a government bond, you could borrow a dollar.

Interest rates on government bonds fell to dramatic lows, including some negative rates. In combination with reserves paying interest, the *distinction* between government bonds and money (reserves) was a third-order issue for financial institutions, especially compared to the very high interest rates, lack of collateralizability, and dramatic illiquidity of any instrument that carried a whiff of credit risk. The credit spread rather than the level of rates or the term spread was the dominant factor in the interest rates faced by the private sector.

In short, the “special” or “liquidity” services we usually associate with money apparently applied to all government debt for these central actors. Those services were related to liquidity, transparency on balance sheets, acceptability as collateral, and absolute security of nominal repayment, rather than the acceptability as means of payment that we usually emphasize in money demand theories.

Fall 2008 also corresponded to a sharp fall in “aggregate demand,” as people and business cut back on both consumption and investment purchases in order to build up stocks of government debt. Again, crucially, the distinction between cash and government debt was second order in this desire.

I have not been specific about the mechanism by which a decline in “aggregate demand” corresponds to a decline in output vs. prices. I turn to this question below. For the moment, let us look at the simple monetary and fiscal equations, think about deflationary scenarios, and allow some of that pressure to be reflected in lower output. I return to this question below where the question of the split matters.

$MV(\cdot) = PY$ does not really allow us to address this sort of event. We can understand it in terms of our fiscal equation however. A sudden demand for government debt, with no (good) news about surpluses, means that people are willing to hold that debt despite dramatically lower rates of return. In our fiscal framework,

$$\frac{M_t + B_t}{P_t} = E_t \int_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} dj, \quad (11)$$

a “flight to quality,” or increase in demand for government debt with no change in expected surpluses, is equivalent to a sharp decline in the discount rate $R_{t,t+j}$ for government debt. Such a decline is deflationary, just as a sudden improvement in surpluses would be.

This observation is an inspiring event for the project of understanding longer-term U. S. experience through fiscal eyes. Fluctuations in “aggregate demand” are somewhat mysterious. By definition, they correspond to fluctuations in demand for government debt. Accounting for fluctuations in demand for government debt in U. S. history by changes in

news of future surpluses has not been terribly successful. But accounting for the history of U. S. stock prices by changes in news about expected dividends has been an even more catastrophic failure. The asset pricing literature has concluded that time-varying discount rates account for essentially all stock market price fluctuations. This event suggests that we might similarly account for “aggregate demand” fluctuations by changes in the *discount rate* for government debt rather than (or as well as) changes in expectations of future surpluses. People fly to quality quite generally in recessions. Perhaps this flight is the central part of lower “aggregate demand.” If so, of course, it is completely missed by standard macroeconomic theorizing that focuses on “the” interest rate and ignores risk premiums.

Though I haven’t done it (yet), this view predicts that a variance decomposition of (11) will find that volatility in the value of government debt on the left will largely correspond to volatility in expected returns on the right rather than volatility in expected cashflows, just as Campbell and Shiller (1988), Cochrane (1992, 2008) and many others find for stocks, and even more analogously, as Gourinchas and Rey (2007) find for sovereign debt.

3.3 Accommodation and stimulus

The Treasury and Fed responded by accommodating the demand for government debt as well as accommodating the demand for money relative to bonds.

In the first phase of this accommodation, as shown in Figure 5, the Fed ran “open-market debt operations,” exchanging private debt for government debt. Between 2007 and September 2008, treasuries and agencies decline as a fraction of Fed assets (top graph), while the overall size of the Fed’s balance sheet does not change much. From Jan 3 2007 to Sept. 3 2008, for example, Fed holdings of Treasury securities declined from \$779b to \$480b while overall assets only increased from \$911b to \$946b. The Fed provided the private sector about \$300b of treasury debt in exchange for corresponding private debt.

The “Treasury” item in Federal Reserve liabilities, the bottom graph in Figure 5 represents a similar operation. The rapid rise here represents the Treasury Supplementary Financing Account. The Treasury sold additional debt and simply parked the proceeds with the Fed. Starting with \$4b on Sept. 9 2008, the total Treasury account hit a peak of \$621b on Nov. 11 and was \$502b on Dec. 12. The Fed turned around and lent this money or bought assets. On net, the government to issued Treasury debt in exchange for private debt, accommodating a switch in demand for the former.

How might an “open-market debt operation”; a switch of private for government debt without changing M , “stimulate” the economy? Let D_t denote private debt owned by the government. Our fiscal equation becomes

$$\frac{M_t + B_t - D_t}{P_t} = E_t \int_{j=0}^{\infty} \frac{1}{R_{t,t+j}(M + B, \cdot)} s_{t+j} \quad (12)$$

I write $R(M + B, \cdot)$ to capture the above idea that people are sometimes willing to hold government debt despite a low rate of return. (Krishnamurthy and Vissing-Jorgenson (2008) argue for a Treasury-debt liquidity demand of this sort.) Thus, by increasing the

supply of Government debt, the discount rate R rises (or the increased quantity offsets the deflationary effects of the flight to quality, captured in the \cdot terms), and aggregate demand increases, even if government holdings of private debt D_t offset greater government debt, so $B - D$ is unchanged, if money M is unchanged, and if there is no surplus news so s is unchanged.

The government also guaranteed large amounts of private debt, including Fannie and Freddie, guarantees of TARP bank credit, and guarantees of new securitized debt. The implicit guarantees of much larger amounts of debt – the widespread perception that no large financial institution will be allowed to fail – add to this list. To the extent that the private sector has a liquidity demand for debt with the government’s credit rating, at the expense of debt which does not carry that guarantee, issuing such guarantees is the same thing as explicitly issuing Treasury debt in exchange for private debt.

In the second phase of accommodation, starting in September 2008, the Fed rapidly expanded its balance sheet as well. For the Fed, this means printing money (creating reserves) to buy assets rather than just exchanging private for treasury assets. In conventional open-market operations, we would have seen Treasury debt in Fed assets rise in tandem. What is striking about this event is that the Fed took pains not to do this. Fed holdings of Treasury debt stay low through the winter of 2009. The Fed funded this entire near-doubling of its liabilities by buying private assets instead. We can think of this as a nearly \$1trillion conventional monetary expansion coupled with a \$1trillion “open-market debt operation,” designed so that the overall supply of government debt does not fall.

The government has also simply increased the supply of government debt overall. Not only is $B + M - D$ rearranged, it’s much larger by the \$1.5 trillion fiscal deficit. This might represent fiscal stimulus, described next, but even if s_{t+j} rises enough that there is no such fiscal stimulus, this action also can be seen as helping to accommodate the large demand for government debt.

In sum, in this analysis, we can read the government’s actions as a much modified version of Friedman and Schwartz’s advice for the great depression. In that event, the Fed failed to accommodate a demand for money at the expense of government debt. In this one the government recognized and accommodated a massive demand for *both* money and government debt, at the expense of private debt.

The Fed view

This is not how the Fed thinks about its policy actions, at least as I interpret Fed statements. The first stage, trading private for government debt without increasing money, was, to the Fed, a way to support private credit markets without the inflationary effect that increasing M might have had. The Fed wanted to stimulate in a noninflationary way, an idea beyond my simple analysis.

A similar thought lies beyond the Fed’s recent asset purchases. Starting in October 2008, the Fed started buying commercial paper, reaching \$300b within a month. In early 2009, it started buying mortgage-backed securities, both directly and via agencies (the thin blue wedge in the top graph), and it started on an aggressive program of buying long-term treasuries, which you can see in the rise of the “treasury” component of Figure 5.

As I read Fed statements, the Fed is trying to attack interest rate spreads in these individual markets, not just to supply more government debt. The Fed sees somewhat “segmented” markets with liquidity premia higher than it thinks are appropriate, and it thinks that it can reduce the premiums in individual markets by buying securities. It hoped to do so by small purchases, a bit in analogy with $MV(i) = PY$, or through the act of trading – by becoming the “noise trader” that liquefies finance models, though in the event it often ended up being almost the whole market for new issues.

Whether the Fed was successful in affecting individual premiums in this way is an interesting question. Taylor (2009b) argues that the MBS, Term Auction, and Long-term Treasury purchase programs had very little effect on the associated spreads, though his “counterfactual” analysis is hard to do and easy to criticize. The opposite possibility is that the spreads on these assets represent credit risk and credit risk premiums; that the markets are not as segmented or liquidity-constrained as the Fed thinks, so that the Fed’s purchases can do little to lower spreads.

How might this action stimulate? As I read its statements, the Fed’s efforts are simply aimed at reducing interest rates faced by borrowers. In the Fed’s view of demand and inflation, lower interest rates raise demand, which in the first instance raises output and later leads to inflation unless stimulus is carefully removed.

This channel requires frictions absent in my analysis. In my analysis, “aggregate demand” must, in the end, be the mirror image of demand for government debt. I posited that the discount rate on government debt might be affected by its total quantity, so that debt operations which did not alter the amount of debt relative to future surpluses might still affect aggregate demand. One might extend this view: If the Fed’s liquidity operations in many individual markets are successful, they could make people more willing to hold that private debt rather than government debt, and this will again increase aggregate demand (the \cdot in $R(M + B, \cdot)$ represents liquidity of alternative private assets). But that’s a long way from a view that lower interest rates per se raise aggregate demand, and one that may suggest a quicker translation to inflation.

4 Fiscal - monetary stimulus

Fiscal stimulus

The government has also been engaged in a large “fiscal stimulus” designed to raise aggregate demand, with multi-trillion dollar deficits projected to last many years. Will these actually “stimulate” as promised?

The fiscal valuation equation

$$\frac{M_t + B_t}{P_t} = E_t \int_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} dj. \quad (13)$$

offers a standard view of this issue, with two important twists: If additional debt $M + B$ corresponds to expectations of higher future taxes or lower spending, it has no “stimulative”

effect. (Again, I am for the moment looking just for inflationary pressure, leaving the nominal/real split for later.). If larger short-term deficits (first few j) are financed by promising larger long-term surplus we again have no stimulative effect. If, however, additional debt and short-term deficits correspond to expectations that future surpluses will *not* be raised, then indeed the the debt issue can raise aggregate demand.

This sounds like fairly standard “Ricardian equivalence” analysis. However, standard Ricardian equivalence presumes that the government issues *real* debt, so that some irrationality or market incompleteness or failure is needed for any stimulative effect. Here, we realize that the government issues *nominal* debt. It can be perfectly rational for long-lived agents to expect that the government does not plan to raise future surpluses, but that it plans instead to monetize debt when the debt comes due. If you know debt will be inflated away in the future, you try to dump it today, causing inflation right away.

Will the spending come too late?

The Administration has been criticized that fiscal stimulus won’t stimulate in time, because the spending will come “too late,” after the recession is over. Equation (13) suggests the opposite conclusion. In order to get stimulus (inflation) now, *future* deficits (s_{t+j} for large j) are just as effective as current deficits (s_t, s_{t+1}). What matters is to communicate effectively how large the future deficits will be, and that they will be pursued regardless of the stock of debt. In fact, in this analysis, expected future deficits are even more important than current deficits. Large short-run deficits combined with the standard expectation that large short term deficits will be paid off by larger future surpluses have no “stimulative” effect.

Future deficits, current deficits, and expectations.

This analysis suggests how hard a fiscal stimulus/inflation attempt really is. Government debt sales are deliberately set up to engender expectations that the debt *will* be paid off. Most of the time, governments do not sell debt to inflate; they sell debt to raise real resources that they can use for temporary expenditures like wars. If a debt sale comes with no change in expected future surpluses, it only raises interest rates and raises no revenue. Governments are very careful to communicate that this is *not* the case.

As an extreme contrast, consider a currency reform in which the government redeems the old currency and issues new currency with three zeros missing. This operation is *exactly* a debt rollover in which $B_t = B_{t-\Delta}/1,000$. But a currency reform is designed to communicate expectations that real surpluses will not change, precisely so that it *will* move the price level at $t + 1$ and raise *no* real revenue. The *only* difference between a currency form and a debt rollover is that these expectations are communicated.

Since the institution of government debt sale is designed to convey the expectation that deficits will eventually be paid off, engendering the opposite expectations may be quite difficult. Everyone is used to fairly meaningless long-term budget projections.

Quantitative easing; Joint Monetary/Fiscal policy; Japan

We're thinking about fiscal stimulus at all because of the widespread view that monetary policy can do no more once interest rates hit zero. My analysis agrees with that view, perhaps with the asterisk that the central bank can still engage in "financial policy" affecting credit spreads and aggregate demand. But M vs. B is done.

The Fed can still pursue quantitative easing, continuing to buy Treasury debt and increase the money supply. People who think in terms of monetary aggregates rather than interest rates advocate such easing. The Bank of England has explicitly engaged in a massive quantitative easing program, and many commentators view the expansion of reserves in the US in this light. But it's hard to see how quantitative easing can have any effect here. The Fed *can* increase reserves M and decrease B , but nobody cares if it does so. Agents are happy to trade perfect substitutes at will. Velocity V simply absorbs any further changes.

What about a "helicopter drop?" Wouldn't this increase M and inflate? A helicopter drop is at heart a *fiscal* operation. To implement a drop in the U. S., the Treasury would borrow money, issuing more debt. It would spend the money as a government transfer. Then the Federal Reserve would buy the debt, so that the money supply increased. A real drop of cash from real helicopters would be recorded as a transfer payment.

Even a helicopter drop would not be "stimulative" if everyone knew that the money would be soaked up the next day in higher taxes, or by the Fed, i.e. by future taxes. Milton Friedman's helicopters are a brilliant device to dramatically communicate that this cash *does not* correspond to higher future fiscal surpluses; this money will be left out in public hands. As in the contrast between a currency reform and a treasury refinancing, expectations are the only difference.

To be effective, then, a monetary expansion at near zero rates *must* be accompanied by a non-Ricardian fiscal expansion as well. People must understand that the new debt or money does *not* correspond to higher future surpluses.

The last time these issues came up was Japanese monetary policy in the 1990s, to escape its long period of stagnation, low inflation and near-zero interest rates. Money creation, quantitative easing, and huge fiscal deficits were all tried, and did not lead to inflation even though these policies are the accounting equivalent of helicopter drops. Why not? The fiscal equation *holds*, so the answer *must* be that people were simply not convinced that the government would fail to pay off its debts. Breaking the expectation that debts will eventually be paid is not so easy once that expectation is formed. (Critics of the Japanese government essentially point out their statements sounds pretty lukewarm about the project, perhaps wisely.)

In sum, what matters, especially in an environment of near-zero rates, is the expectation of future surpluses. If you can convince people that these are lower than the real value of outstanding debt, including money, then you can get inflation. If you cannot persuade them, then exchanges of money for debt have no effect, and increases in money or debt have no effect. If you can persuade them – perhaps by inventing a new institution such as helicopter drops – then you can stimulate inflation. But in that event, but whether you drop money or treasury bills from the helicopter makes little difference.

Identification

This analysis implies that historical evaluation of fiscal multipliers suffers a (an additional) deep identification problem. What were expectations in previous events? If people expected eventual inflation, i.e. that the debt would not be paid off, we should see stimulus. That experience would *not* inform us about the effects of a stimulus package that *did* come with a commitment not to inflate and therefore the expectation that future tax revenues would rise.

The usual arguments about Ricardian equivalence, which apply to real debt – market failure, liquidity constraints, or just plain lack of foresight – are perhaps more “structural,” so estimates of past behavior might be more likely to apply to the current situation. But expectations whether each debt event will be paid or inflated can vary arbitrarily with the circumstances of the event. Wars are plausibly quite different from recession-fighting stimulus packages, and stimulus packages come with different fiscal backgrounds. For example, Chile, with a large positive net asset position, is likely to face different expectations about long-run fiscal solvency of a large stimulus plan than is the U. S., with a fairly large outstanding debt. Italy or Greece might well face even more sceptical expectations.

4.1 What are expectations?

With this perspective in mind, will or did the current package stimulate aggregate demand? What *are* expectations of future surpluses and deficits?

Government announcements

On one hand, we can take the Government’s dramatic deficit projections and small tax policy proposals surrounding the stimulus bill in January and February 2009 as loud announcements “you’d better spend the money now, because we’re sure not raising taxes or cutting spending enough to soak it up.”

More recently, long-term budget projections remain bleak. On March 20 2009 OMB director Peter Orszag was quoted to say “Over the medium to long term, the nation is on an unsustainable fiscal course.” “Unsustainable” literally means that the right hand side of the fiscal equation is lower than the left. The normally staid Congressional Budget Office’s (2009) Long Term Budget Update echoes the sentiment: “Over the long term ... the budget remains on an unsustainable path,” complete with graphs of exponentially exploding debt.

On the other hand, the main problem in long-term budget projections are Social Security and Medicare. We’ve known that these programs are on an unsustainable course for years, this is not news this winter, and it’s a reasonable expectation that sooner or later the government would get around to doing something about them.

Furthermore, by spring, the tone of government statements had changed completely from “stimulus” to concern over long-term budget deficits and a desire to lower them, not commit to them. OMB director Orszag’s March 20 “unsustainable” comment was followed quickly by “to be responsible, we must begin the process of fiscal reform now,” and emphasized controlling medicare costs. It was delivered at a “Fiscal Responsibility Summit.”

Most of the Administration’s defense of fiscal stimulus (for example, Bernstein and Romer 2009) cites simple Keynesian flow multipliers, not the sort of fiscal-monetary inflation I have described as “stimulus.” And by May, these had also turned in to a worries about fiscal sustainability that can be read as dramatically negative multipliers. For example, the CEA’s (2009) health policy analysis states that “slowing the growth rate of health care costs will prevent disastrous increases in the Federal budget deficit” and will raise the level of GDP by 8%, permanently. These kinds of statements don’t suggest a commitment to permanent deficit. (There’s a bit of fiscal stimulus *and* deficit reduction in Administration statements, but the case for stimulus from tax-supported spending is weak enough I won’t stop here to think about it.) Chairman Bernanke’s June 3 (2009b) testimony worries about long-term deficits, and thus whether the fiscal backing to *contain* rather than to *produce* inflation will be present.

Furthermore, Chairman Bernanke and the other Federal Reserve Governors are loudly saying the Fed can and will control inflation. Whether the Fed will be *able* to do so is another question, but at least we hear determination to fight and win any game of chicken with the Treasury. Secretary Geithner went out of his way to assure the Chinese that the dollar will not be inflated (Cha 2009).

In sum, government statements do not paint a clear picture. This may reflect an understandable indecision on the part of the government facing a Catch-22: In my analysis, the *only* way to “stimulate” is to commit forcefully and credibly to an unsustainable fiscal path, so that people will try to get rid of their government debt including money, and in so doing drive up demand for goods, services, and real assets. But the government clearly understands that such an action trades modest stimulus today for financial and economic chaos when the inflation really comes. Faced with that stark decision, it is not surprising that the government settles for half-measures and contradictory statements – as the Japanese were accused of doing for a decade.

Measuring Ricardian expectations

Fortunately, we don’t have to argue about Ricardian or non-Ricardian expectations. The bond market and the fiscal equation let us *measure* private expectations. If the government sells additional debt and the private sector does not believe that debt will correspond to more taxes, then the government raises no revenue from the debt sale. It merely raises interest rates. Thus, *the revenue from additional debt sales and the behavior of interest rates allow us to measure the state of Ricardian-equivalence expectations.*

Alas, of course, economics is never easy because supply and demand both move. Some of the current rise in long-term rates may simply reflect a reversal of flight to quality and news of lower future surpluses (Λ, R, s news). Thus, it’s not immediately easy to see how much extra “stimulus” bond sales are driving up nominal interest rates over what they would otherwise be. Also, the accounting is trickier with outstanding long-term debt. However, government interest rates are still quite low, so a good guess is that the massive deficits have not raised interest rates much. The government is certainly raising revenue from its bond sales.

In sum, the fact that bond markets are absorbing so much debt with surprisingly low

interest rates is a direct *measure* that expectations are “Ricardian,” so the stimulus is not yet having its desired (?) effect.

5 Inflation or deflation?

5.1 Money worries

Aggregates

We have certainly seen a dramatic monetary expansion. The increase in reserves, peaking around \$800b and the roughly \$1 trillion expansion of the Fed’s balance sheet is seen in Figure 5. Figure 6 presents M1, currency and deposits, more standard measures of money. The left hand panel presents dollar increases, which show a \$250b rise in M1, a \$100b rise in currency, and a \$200b spike in deposits during the panic, leveling off at about \$120b. The right hand panel shows percentage increases. Currency has risen 15% and M1 has risen 20%, despite a fall in GDP.

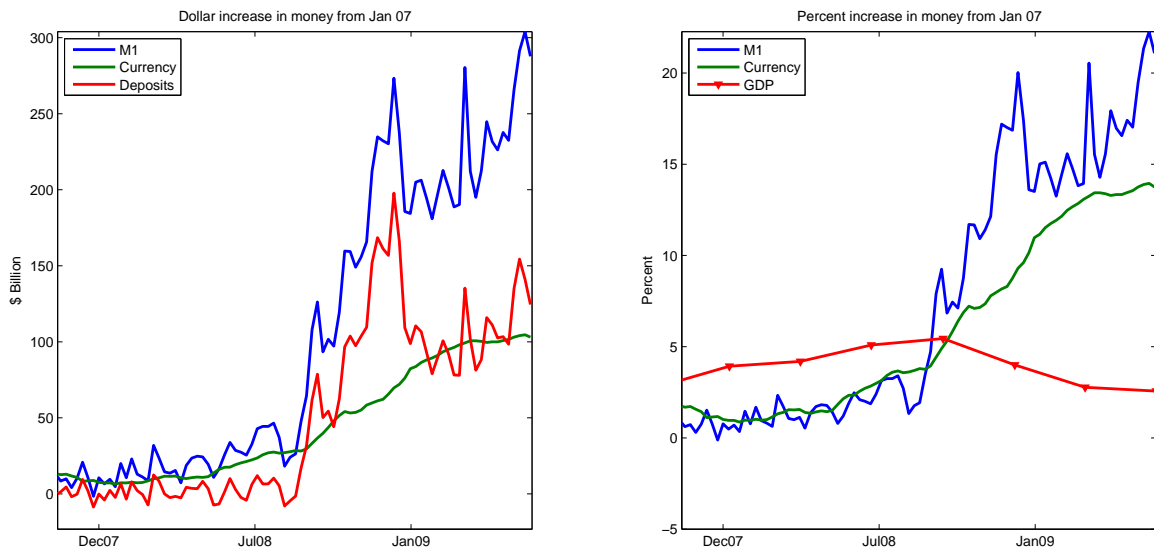


Figure 6: Money stock

For those who like to think in terms of interest rates, Figure 7 emphasizes what we all know, they are very low.

This stance leads to the obvious question, is a large inflation on the way? When the time comes to reverse course, will the Fed be willing to do so? More troubling, will the Fed be *able* to do so, or will we discover the fiscal limits to monetary policy? Will mounting fiscal deficits instead force the Fed to monetize even more debt? Will we in fact see a fiscal inflation without current monetization, but based on a flight from the dollar, a fear of future monetization, as (3) describes?

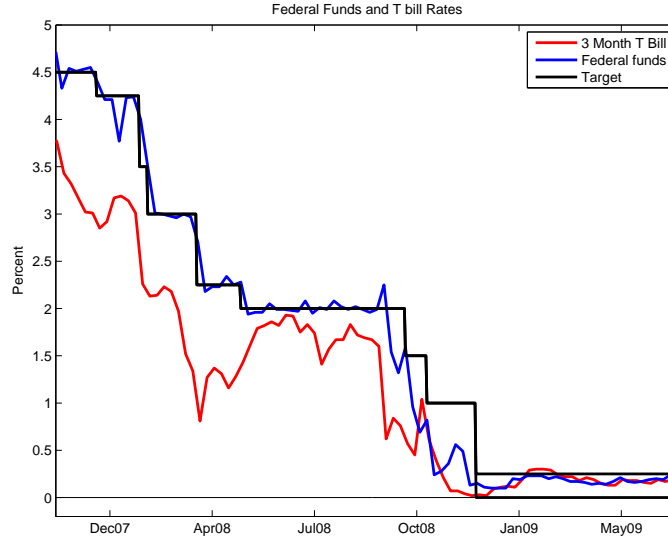


Figure 7: Federal Funds and 3 month Treasury bill rates

Opinions are certainly mixed. Paul Krugman (2009) argues that “Deflation, not inflation, is the clear and present danger.” Of course, Fed officials have given many comforting speeches on their “exit strategy.” But Niall Ferguson (2009) Martin Feldstein (2009) and Anna Schwartz (Satow 2009) think inflation is on its way. Arthur Laffer (2009) thinks something like hyperinflation is on the way. Nonetheless, when I look at these opinions through the eyes of our fiscal (3) and monetary (4) equations, I see a surprising (though not perfect) consensus that the fiscal situation is really the underlying worry, and primarily a different assessment of the probabilities.

$$MV = PY$$

Some inflation hawks simply look at the vast amount of reserves and the smaller but substantial increase in M1 and currency, and infer that inflation must follow. Some of these observers, I think are simply stuck in a view that in $M_t V = P_t Y_t$, velocity is stable, but “long and variable lags” transmit money to inflation, so that past money must imply future inflation. This view is simplistic; I think we’ve moved on to realize that velocity does shift, especially at near-zero rates, and that today’s money need not mean tomorrow’s inflation if the Fed soaks that money up fast enough. What the Fed giveth, the Fed can taketh away.

The argument centers on how $M_t V_t = P_t Y_t$ works. For example, Laffer thinks M1 is the right aggregate; he worries that the huge expansion in reserves means more M1 expansion to come. Moreover, he worries that this process will then be difficult to reverse. If the Fed tries to soak up reserves, he thinks it will require a massive contraction in bank lending in order to reduce the relevant M1, which will require a sharp recession that the Fed will not be willing to countenance.

In the dove’s view, we are still in a “liquidity trap” so the extra reserves aren’t going anywhere. I have argued above for this view of our current situation. Banks are just as happy

to hold reserves as to hold government bonds, especially now that reserves pay interest; their lending activity is disconnected from their reserve holdings. Similarly, one can argue whether *M1* is in fact the relevant aggregate. Finally, one can argue how difficult it will be for the Fed in fact to soak up aggregates. Loans are redeposited in the banking system, so there is no connection between the amount of lending and the stock of any monetary aggregate. A cashless economy will still have lots of loans. The Fed was able to expand in response to a sudden velocity shock, I don't see why it cannot contract just as quickly.

The Fed's balance sheet

Feldstein (2009) points out that the Fed no longer has much Treasury debt, as you can see in Figure 5. If it wants to soak up reserves, it may be very hard to sell all the illiquid, long-dated and risky private securities that the Fed has accumulated, and impossible to sell direct loans. Feldstein writes “..the commercial banks may not want to exchange their reserves for the mountain of private debt that the Fed is holding and the Fed lacks enough Treasury bonds with which to conduct ordinary open market operations..”

I think this is less of a worry—or rather it's an internal political worry not an economic worry. There is nothing that stops the Fed and Treasury together from simply issuing new Treasury debt to soak up the trillion dollars or so of reserves, even if the Fed has nothing left on its balance sheet. The Treasury can issue new debt, and simply deposit the proceeds with the Fed.

Political will

The remaining worry is whether the Fed will have the political will to start soaking up reserves or raising short-term interest rates quickly enough. The “credit crunch” and “financial crisis” are already over – short-term debt spreads have returned if not to normal, at least to functioning levels. The “flight to quality” will soon fade as well, and long-term rates will rise all on their own. This is the time to begin the “exit strategy.” Yet we will still be in a serious recession for some time. Commercial real estate, state debt, and some pension funds are still in trouble. Mortgage foreclosures are continuing, and unemployment will be high. Many financial institutions will still be on the edge, and many of them make a lot of money by borrowing low and short and lending long. In this environment, will the Fed really engage in hundreds of billions of dollars of open-market operations, and start worrying about inflation before it breaks out?

More to the conceptual and economic theme of this essay, will the Fed be seduced by house and stock prices below their peaks in to thinking that “asset price deflation” counteracts good and services inflation? Will its potential GDP” estimates, as in the 1970s, suggest large and illusory “gaps” remaining to be filled? The Fed seems focused on “managing expectations” by announcements rather than direct open market operations in order to control inflation. Will it continue too long to trust in that ability? These options will all be tempting.

Unwinding the new lending facilities and asset purchase programs will pose their own challenges. There is a downside to any policy action. To the extent that the Fed's purchases were effective in lowering spreads, this means lower profits for private investors who would otherwise enter these markets, or to develop new less leveraged institutions to bring them

funding. Unwinding the Fed's support will have to mean higher interest rates in those markets.

5.2 Fiscal constraints on a monetary exit

In sum, I conclude that no substantial *monetary* problems stop the government from selling Treasury debt to soak up whatever assets constitute the M in $MV = PY$ and remove monetary stimulus, if it wants to do so and can suffer the higher short-term interest rates that this action may provoke.

The remaining question is *fiscal* backing – whether the government will be *able* to undo monetary expansion. For the next several years, the Treasury will still be selling trillions of additional debt to finance deficits. If investors and the Treasury are also trying to sell, can the Fed sell additional trillions as well? To sell additional debt, the government must convince investors there will be additional surpluses to pay off that debt. At some point, the government runs out of this capacity: additional debt sales just lower bond prices and do not raise more real revenue. Future s does not rise and the real discount rate R does rise, adding a credit spread to US government debt. At the Laffer limit of truly fixed s , there is literally nothing the Government can do to raise additional resources; the elasticity of demand is one. Long before that point, as bond sales drive up long-term rates, the Fed is likely to stop trying. Keep in mind that it is now still aggressively buying long-term treasuries to try to lower mortgage rates.

In sum, what really matters to a monetary exit strategy is the government's ability to issue new debt, by credibly promising higher future surpluses. Central banks do not need reserves if the government can borrow them; conversely large reserves or even a currency board will not stem a currency collapse if the government's borrowing ability is gone.

Now, how close is the U.S. to this point? I think “not very.” First, the Fed's main task now is to reverse the massive expansion of reserves, and perhaps some of the expansion of currency. But in (3) or better (12), reserves and currency enter symmetrically with government debt. It takes *no* additional resources to unwind a reserve or currency expansion. Prospective investors in new government debt were already holding currency or reserves, which are just a different maturity of government debt. Additional resources, new debt issues matched by higher future surpluses, are important to a government that needs *foreign* reserves, gold reserves, etc. in order to unwind a monetary expansion, but not to a government that wants to unwind an expansion of domestic reserves.

The main difficulty could come if the Fed runs out of treasuries, cannot sell its portfolio of private assets, (or if that portfolio turns out to be worthless), so the Treasury has to issue new debt, in essence to bail out the Fed. But, as seen in Figure 5, the absolute worst this problem could be is about \$1 trillion, and more likely on the order of a few hundred billion. That's still a lot of money, but it's less than one year's deficit, which the US government is running so far without enormous fiscal difficulty. A few hundred billion, spread out over a few years, are not enough to push the US government beyond its borrowing capacity.

I conclude that the US has both the ability and fiscal capacity to rapidly unwind its

monetary expansion, should the government choose to do so.

5.3 Fiscal inflation

A fiscal inflation, the consequence of current and future deficits, are therefore, in this analysis, a greater inflation danger than the Fed's recent monetary policy. Our worry is not an exit strategy for currently loose monetary policy. Our worry is directly fiscal; a default engineered through inflation; a sharp drop in expectations of future surpluses s , or rise the risk premium R , that forces inflation directly in (3).

I think there is widespread agreement on this danger. Even Krugman (2009) admits “others claim that budget deficits will eventually force the U.S. government to inflate away its debt... [This claim] could be right but isn't” “Could be right” means that the view is logically possible. “But isn't” means Krugman just doesn't think we're close enough to the fiscal limit to worry about it. “[inflation hawks] say that America will eventually have to inflate away that debt — that is, drive up prices so that the real value of the debt is reduced. Such things have happened in the past. For example, France ultimately inflated away much of the debt it incurred while fighting World War I.” He doesn't think it will happen, but the danger is well described by (3).

How exactly does this work, what are the dangers and warning signs? Here again, I think looking at (3) clarifies some issues and points out some common traps.

Debt/GDP ratios

Krugman and other doves point out that the U. S. debt/GDP ratio is below that of many other countries, and our own experience. Figure 8, for example, shows our current debt/GDP at 40%, and projected to rise to 60% by spending during the current recession. This is small compared to the 110% debt/GDP ratio at the end of WWII, and the ratios over 100% that several European countries and Japan now experience.

The long-term US budget outlook is much more bleak, as shown in Figure 9. It is unusual that even the CBO's 10 year forecast shows no reduction in deficit as we exit the recession. However, the long-term “unsustainable” trends are driven by social security and health expenditures not recession-fighting stimulus, and even under the more plausible “alternative fiscal scenario” we only reach 100% of GDP in about 2022 and 200% of GDP in 2035. What's to worry about?

Most of all, the fiscal equation (3) does not point to a “sustainable” debt/GDP ratio – say 100% – and “everything will be fine until you cross this point.” Equation (3) says that you get inflation *now* as soon as people think that *future* debt/GDP ratios will grow uncontrollably, i.e. the left hand side is greater than the right.

If anyone believed the CBO's long-term forecasts of Figure 9, the inflation *would have already happened*. People are expecting that eventually the government will take heed of absolutely everyone's advice and do something about social security and medicare expenditures. We could borrow 120% of GDP at the end of WWII because everyone understood war expenditures were temporary, and that huge deficits would end once that temporary

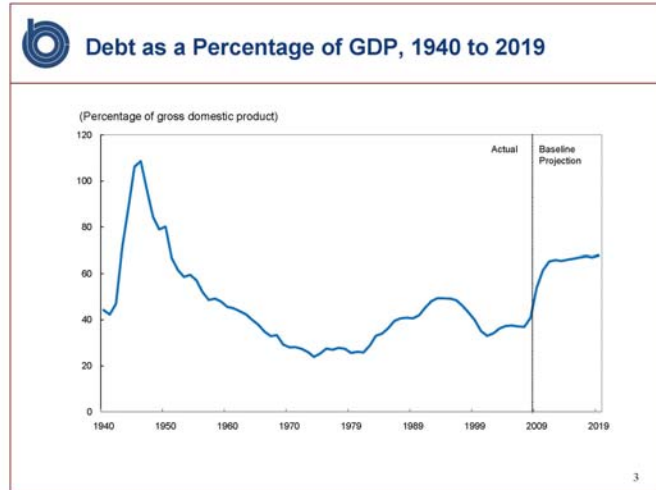


Figure 8: Debt/GDP. Source: Elmendorf’ (2009)

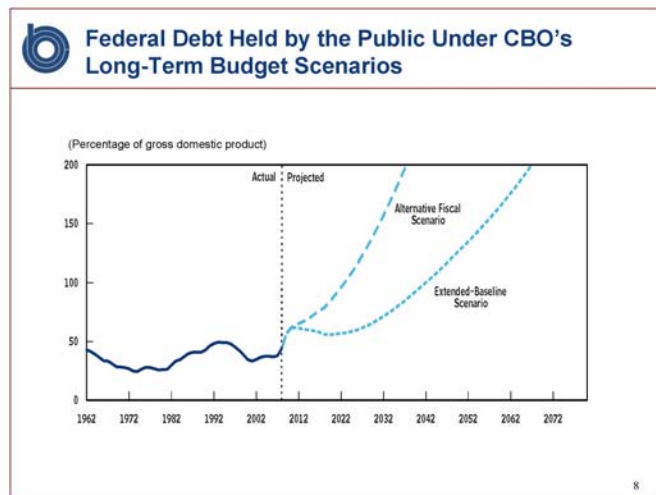


Figure 9: Long Term CBO forecast. Source: Elmendorf (2009)

exigency passed. Other countries have experienced exchange rate collapses – meaning, their governments were unable to pledge enough real resources to borrow foreign exchange reserves – with much lower than 100% current debt/GDP ratios, when markets saw unsustainable *prospective* deficits. Burnside, Eichenbaum, and Rebelo (2001) are a good example, and survey the international literature’s conclusion that *current* debt/GPD ratios are not a good forecast of currency crashes – in either direction. And we’re not yet sure that high debt/GDP ratios accumulated from intractable social spending will not result in a global inflation.

In sum, the fiscal limit is not determined by *current* debt and deficits, but by expectations of long-term *future* deficits. Current debt/GDP is only a warning when it points to unsustainable future debt/GDP.

Crowding out

Much discussion of the downside of deficits focuses on the *flow* of spending, and interest rates through a “crowding out” mechanism. Higher deficits pose a danger by adding to “spending,” which either raises interest rates, or if interest rates are kept low by the Fed, leads to additional inflation.

Nothing like this mechanism is mirrored in the fiscal equation (3). One can even potentially have high inflation with no current deficits, if expected future deficits are high. “Demand” in (3) refers entirely to the balance between total private spending vs. accumulation of government debt.

In the fiscal analysis, high long-term rates are a sign of trouble to come, as highlighted by the above inflation scenarios, but not because deficits have crowded out private investment; rather they simply reflect expected future inflation.

Seigniorage, monetization

Finally, most writing about the dangers of deficits focuses on the idea that the Fed will have to monetize deficits, this action will raise the money stock, and only *then* will inflation break out. Equation (3) emphasizes that we can have inflation *now* when people expect future monetization. We do not have to wait for seigniorage; there doesn’t even have to be any seigniorage; and inflation certainly does not have to wait until after seigniorage revenues work their way through the economy.

Now, when $MV = PY$ operates, it is also true that a fiscal inflation has to be accommodated by monetary authorities. If P in the fiscal equation rises, but the Fed adamantly refuses to raise M , we have an “uncoordinated policy.” One side must give way. This happens automatically when the Fed follows an interest rate target or otherwise passively adjusts money in response to liquidity needs, as it has been explicitly and aggressively doing for the past year. Even if the Fed were to be more resistant, it’s not obvious that it will win a game of chicken with a Treasury in extremis, or that markets will not find a way to satisfy liquidity demands (find alternatives to M , raise V) were it to attempt one.

The picture

In sum, the fiscal valuation equation

$$\frac{M_t + B_t - D_t}{P_t} = E_t \int_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} dj.$$

and experience of past fiscally-induced collapses paints a far different picture of a fiscal inflation than in most commentator’s scenarios. This equation looks (and is) a lot like the valuation equation for a stock. Hence, a fiscal inflation may well look like a stock market collapse. The tipping point, where investors change expectations of long-term future surpluses s , valuations of government-held assets D , or require larger real risk premiums R to hold them, can come quickly and unpredictably, without necessarily large current debt/GDP, large current deficits, large current monetization; without strong “demand” and small “gaps.” It can come as a surprise to a Federal Reserve and to economists unused to thinking about fiscal limits to monetary policy.

Where is the fiscal limit? I don’t know. But there *is* a fiscal limit, and wherever it is,

we are a few trillion dollars closer to it than we were last year, and we will be another few trillion dollars closer next year.

In this debate, I have two points to add: Credit guarantees affect the calculation far beyond on-the-books debt to GDP ratios, and weak *growth* is the central danger.

5.4 Credit guarantees and the fiscal limit

If official debt to GDP ratios are “only” headed to 100% or so, credit guarantees are quite large. The government has explicitly guaranteed Fannie and Freddie debt and underlying mortgages, the TARP bank debt, and many others. Pittman and Ivry (2009) added guarantees up to 13 trillion, another 100% of GDP, though of course one can argue with their methodology and the chance that everything defaults with zero recovery is remote. Implicit guarantees are much larger. Fed Chairman Ben Bernanke³ has pretty much guaranteed that no financial firm will fail, and bailouts of more industrial firms, state and local governments, defined-benefit pension plans, and sovereign debt either directly or via the IMF loom.

Credit guarantees have two effects. First, and most obviously, having to make good on these guarantees on top of large budget deficits can be the piece of poor surplus news that kicks us against the fiscal limit. Again, all we need is investors to *expect* large payouts in order to get an immediate flight from the dollar.

Second, nominal credit guarantees mean that government finances are much worse if the price level goes down, and much better if there is inflation. Higher nominal real estate prices in particular will make the government’s guarantees much easier to maintain. Surpluses are *not* independent of the price level. Our equation is really

$$\frac{M_t + B_t}{P_t} = E_t \int_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}(P_{t+j}) dj$$

with $s'(P) > 0$. (In an analogous way, Burnside, Eichenbaum and Rebelo 2006 find that the Korean devaluation helped government finances largely by lowering the real value of nominal wages paid to government workers, rather than devaluing domestically-denominated nominal debt – $s(P)$ not B/P .)

This consideration has a good and a bad implication. On the “good” side, this means that a smaller inflation can solve a larger budget problem. Since a rise in P makes the right side larger as well as the left side smaller, a lower rise in P is necessary than would otherwise be the case. (“Good” is in quotes, because I don’t want to label what is basically a creditor default as an unequivocally good thing. It just means inflation will be lower than otherwise.)

On the bad side, this fact makes it much more likely that the government will choose inflation. One should not think of surpluses as exogenous in this fiscal analysis. They result from the Government’s taxing and spending decisions, and raising taxes and lowering

³See Bernanke (2009a), and in particular, “.government assistance to avoid the failures of major financial institutions has been necessary to avoid a further serious destabilization of the financial system, and our commitment to avoiding such a failure remains firm.”

spending (particularly the latter) are physical possibilities, up to the Laffer limit at least. Really we should think of the Government’s *decision* to inflate, trading off distorting taxes, useful or politically popular spending, and other considerations in this decision. $s'(P) > 0$ means that a smaller inflation can help to restore budget balance, but it also means that the government is more likely to choose that inflation rather than less pleasant measures, and measures for which it can be more directly held accountable.

5.5 The dynamic Laffer curve and the fiscal limit

One absolute fiscal limit is the point that higher taxation simply cannot raise any more revenue — the top of the “Laffer curve.” At this point, any government must follow a “non-Ricardian regime” and the fiscal equation determines whatever is left of the price level⁴.

Since *present values* matter, small effects of tax rates on growth can put us at the fiscal limit much sooner than static analysis suggests. Thus, a high marginal tax and interventionist policy which stunts growth can be particularly dangerous for setting off a fiscal inflation.

We are used to thinking of the static Laffer curve, in which tax revenue T_t is generated by a tax rate τ_t from income Y_t as

$$T_t(\tau_t) = \tau_t Y_t.$$

The marginal revenue generated from an increase in taxes is

$$\frac{\partial \log T_t}{\partial \log \tau_t} = 1 + \frac{\partial \log Y_t}{\partial \log \tau_t}$$

The second term is negative – higher taxes lower output, so the elasticity of tax revenues with respect to tax rates is less than one. The top of the Laffer curve is where the elasticity is equal to zero, so higher tax rates raise no revenue.

Many economists think the U.S. is comfortably below that point. For example, a rise in the tax rate from $\tau = 0.30$ to $\tau = 0.35$ is a 15% ($\log(0.35/0.30) = 0.15$) increase, so it would have to result in a 15% decline in output before it generates no additional revenue. (Yes, this calculation is too simple, and the tax system is graduated. The point is to contrast this calculation with the dynamic calculation below, not to assess realistically the U.S. tax system.)

More people voiced concern that the UK’s recent move to a 50% marginal rate plus VAT put it above the top, especially since high-wealth people can leave. When tax rates are already high, the same percentage point tax rate rise is a smaller percentage (log) rise, so smaller output effects of each percentage point tax rise are necessary to offset the tax rate increase.

The *present value* of future tax revenues is what matters for the fiscal valuation equation, however. For a simple calculation, suppose growth is steady at rate g (this is growth of total

⁴See Piergallini and Rodano (2009) for a model of the Laffer limit in fiscal theory.

income, not growth of income per capita) and the interest rate is constant at r . Then, the present value of future tax revenues is

$$PV_t = \int_{s=0}^{\infty} \frac{1}{e^{rs}} \tau Y_s e^{gs} ds = \frac{\tau Y_t}{r - g}$$

Taking the same derivative,

$$\frac{\partial \log PV}{\partial \log \tau} = 1 + \frac{\partial \log Y}{\partial \log \tau} + \frac{1}{r - g} \frac{\partial g}{\partial \log \tau}$$

We see there is an additional term, which is also negative.

Since $r - g$ is a small number, small growth effects can have big effects on the fiscal limit. For example, if $r - g = 0.02$, then $\partial g / \partial \log \tau = -0.02$ puts you at the fiscal limit immediately. Thus, if a rise in τ from 30% to 35% only has a $0.02 \times 0.15 = 0.003 = 0.3\%$ reduction in long term growth, then we're at the fiscal limit already, with no level effect at all.

I do not digress here to the economics by which marginal tax rates lower the level or growth rate of output. The disincentive effects of working, saving or investing are widely discussed. Migration of high-wealth people and businesses is perhaps even more important, especially to small countries: Even if growth per capita is not affected by distorting taxes, fewer capitas mean less tax revenue. Growth theory points to accumulation of knowledge as the main driver of long run per-capita growth rates, but I don't want to stop here to model how distorting taxes interfere with that process, nor tie the calculation to one particular such model.

6 Phillips curves—Will inflation “stimulate?”

The point of stimulus is not to inflate, of course, but to boost output in the short run. Many economists argue that a little inflation isn't such a bad thing in the current circumstance, as they argued for deliberate inflation in Japan in the 1990s. (For example, Greg Mankiw and Ken Rogoff are quoted in Miller (2009) as being in favor of inflation, both to bail out borrowers and on Phillips curve grounds to raise output.) I have not described a particular mechanism for output effects, in part because both the theory and experience of Phillips curves under fiscal inflations is unexplored territory. But I do want to point out that not all inflations come with output booms either in theory or in practical experience. There is no guarantee that inflation will “stimulate” the real economy. Inflation with real stagnation is a possibility too.

We have many precedents in traditional monetary analyses and historical experience. The 1970s had inflation with recession or stagnation. This experience is captured in two ideas: “aggregate supply” shifted adversely, and inflation expectations rose or its “anchoring” disappeared, shifting the Phillips curve. Of course we all understand that currency reforms (exchanging old currency for new, with fewer zeros, or moving to the Euro) change the price level with no output effects at all.

Certainly, the history of fiscal inflations and currency collapses does not inspire hope that a fiscal inflation will result in prosperity. The hyperinflations that follow wars (Sargent 1992), as the current hyperinflation in Zimbabwe, were associated with horrible economic conditions, not the spectacular booms that a simple Phillips curve might predict. Currency collapses have a similar history. In fact, I cannot think of a single fiscal inflation that is associated with a boom.

In the fiscal context,

$$\frac{M_t + B_t}{P_t} = E_t \int_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} dj,$$

we can distinguish different sources of inflation. There can be “inflation tomorrow” from issuing more money M_t or debt B_t , $B_t^{(j)}$ without changes in surpluses. There can be shocks to prospective deficits s_{t+j} , causing a flight from debt, or a rise in the risk premium $R_{t,t+j}$.

It’s not at all obvious from theory or experience that all of these inflationary changes would be accompanied by a boom or by the same boom. We have some sense that unexpectedly printing up a lot of money – a fiscal helicopter drop – might give a short-term output boost, especially if it were done as a surprise and with clear statements that the money would not be soaked up by taxes any time soon. However, the experience of fiscal inflations caused by current and prospective deficits – currency collapses – is not comforting. And of course currency reforms are neutral.

As I look at our money and fiscal equations (3) and (4), and begin to think about how they work with less than frictionless markets, the fiscal equation (3) seems to provide the “anchoring” of inflation expectations necessary for successful monetary policy. In this way, a fiscal inflation could correspond to a “Phillips curve shift”, which would lead to inflation without expansion. Fiscal inflations may also correspond to poor output through an “aggregate supply shift.” Governments resort to horribly distorting taxes before they “default” through inflation.

One way to approach this question is to include the government debt valuation equation (3) in a model of sticky prices and output effects, for example the standard New-Keynesian Phillips curve.

$$\pi_t = \beta E_t \pi_{t+1} + \gamma(y_t - \bar{y}) + v_t.$$

Woodford (2003) briefly considers this regime, and uses it to think about the interest-rate pegging episode in the early postwar U.S. A fiscal inflation, along the lines of the scenarios in Figure 3 and Figure 4, means more current inflation but also higher expected future inflation. This standard forward-looking Phillips curve model then predicts *lower* current output, i.e. “stagflation.”

More generally, as Woodford points out, once one adds (3) back to the New-Keynesian model, one has to switch to a Fed policy rule that violates the Taylor principle – interest rates react less than one-for-one with inflation – so in a technical sense, the debt valuation equation (3) really does take on the role of “anchoring” long run expectations. (Obviously, there is much left to do in analyzing a model that includes an active government debt valuation equation (3), long term debt and a role for long term debt to smooth inflation shocks, an interest rate rule by the Fed, and a rule for buying and selling long-term debt to smooth

inflation over time.)

Most economists view the postwar U. S. experience as one that comes from a regime in which the fiscal constraint was not important. Our future may not be drawn from this same experience, and in particular may come from bad news about deficits. If that happens, we may find our comfortable experience of booms associated with inflations will vanish once again.

Here I part company with most of the inflation/deflation commenters. All of them link inflation always and everywhere to increased “demand” and hence tighter markets. Krugman (2009) writes “[in ordinary times]...banks, flush with reserves, would increase loans, which would drive up demand, which would push up prices.” Laffer (2009) describes the same mechanism. Feldstein (2009) describes a more general “demand” based mechanism: “The key fact is that inflation rises when demand exceeds supply. A fiscal deficit raises demand when the government increases its purchase of goods and services or, by lowering taxes, induces households to increase their spending...” In a statement very revealing of what mechanisms he has in mind, Bernanke (2009b) said to Congress,

Even after a recovery gets under way, the rate of growth of real economic activity is likely to remain below its longer-run potential for a while, implying that the current slack in resource utilization will increase further. ...In this environment, we anticipate that inflation will remain low. The slack in resource utilization remains sizable, and, notwithstanding recent increases in the prices of oil and other commodities, cost pressures generally remain subdued. As a consequence, inflation is likely to move down some over the next year relative to its pace in 2008. That said, improving economic conditions and stable inflation expectations should limit further declines in inflation.

All of these analyses ignore the stagflation experience of the 1970s, in which inflation was high even with “slack” markets and little “demand.” They ignore the experience of hyperinflations and currency collapses, which happen in economies well below “potential.” The Phillips curve does shift, and a fiscal inflation may well correspond to a shift, not a movement along that curve.

7 Intellectual Casualties

As I have suggested, the current situation yields to an interesting analysis in which the fiscal valuation equation (3) takes center stage. It strikes me, however, that the current experience will leave two classic modes of analysis behind.

First, our old friend $MV = PY$ with constant velocity (“stable money demand”) and long and variable lags seems a likely casualty. The Fed has pretty clearly accommodated a large shift in money demand. When that shift reverses, the Fed *can* (subject to a fiscal limit) reverse course and soak up that money. Simply looking at current aggregates is not a serious sign of future inflation.

Second, most monetary policy analysis has settled into the interest rate target doctrine of central banking. The Taylor rule by which interest rate targets will rise if inflation rises, and “managing expectations” of future inflation are thought to be the key to controlling inflation. Of course, this analysis can only hope to work far from the fiscal limit. (Formal models are explicitly Ricardian with lump-sum taxes.) If a fiscal stagflation overcomes the U. S. , those who focus only on the standard doctrine will be puzzled at an inflation that seems to come from nowhere, just as many economists in the 1970s were puzzled that familiar relations were breaking down. Already, we see that most analysis is focused on the fiscal constraints on monetary policy or direct fiscal links rather than expectations about short-rate policy or simple open-market operations. The Fed’s model of inflation – Fed sets interest rates (now on a large variety of assets), lower interest rates increase demand, demand reduces gaps, low gaps push prices up – may describe a *mechanism* that affects inflation, but it does not describe all variation in inflation. A fiscal inflation would be a large error term in this model.

More generally, now that we see an event in which the *split* between federal funds and short term debt is essentially irrelevant compared to other events in the credit market, we may see that split as much less important in the future as well.

My analysis has been extremely simplistic of course. In places, the algebraic complexity of more realistic models didn’t bring me more intuition, as with long-term debt. In many other places, we simply don’t have well-tested theories on which to rely. One important item is to explicitly incorporate some sort of nominal rigidity along with a fiscal-dominant analysis, in such a way as to integrate the standard experience of the U. S. with small inflations, as well as the typical stagflation associated with fiscal currency collapses. My hunch that the fiscal equation provides the “anchoring” or lack of it for monetary policy is only a hunch.

8 Conclusion

The government debt valuation equation

$$\frac{M_t + B_t}{P_t} = E_t \int_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} dj$$

is clearly at the center of macroeconomic events right now, from stimulus, to monetary policy, to the inflation/deflation debate. As I boil it down, conventional monetary policy – the split between M and B – is essentially irrelevant for current macroeconomic events

Will we get inflation? I am not a forecaster, but I can outline scenarios. The “nightmare scenario” for inflation starts with growth much poorer than the administration’s forecasts, possibly due to larger government distortions and higher tax rates. Lower growth is the single most important danger to the Federal budget. Then, the government may have to make good on its many credit guarantees, and continue its string of bailouts. If this happens, prospective deficit to GDP ratios will rise much further than current projections suggest. A failure to resolve entitlement programs that everyone sees lead to wildly unsustainable deficits will not help.

When investors see that path coming, they will quite suddenly bail out of the dollar; we will see a dramatic rise in interest rates, and a fall of the dollar. A large inflation will follow—and quite possibly “stagflation” not inflation associated with a boom. A rise in R – loss in the “quality” of the dollar – will combine with bad news about $T - G$ to make matters worse. The interest rate rise and inflation will come long before the worst of the deficits and monetization. Economists in and out of the Fed will scratch their heads at the “loss of anchoring” or “Phillips curve shift.”

This is not a forecast. It need not happen. Whether it does depends on the actions of our public officials, which are very hard to forecast.

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10 Appendix

This appendix establishes the basic present value forms of the government debt valuation equation.

10.1 Discrete time, one period debt

10.1.1 One period, discount debt

$B_{t-1}(t)$ is the face value of debt sold at the end of period $t - 1$, repaid at period t . M_{t-1} is issued at period $t - 1$ and held overnight to period t . In this case, we have

$$\frac{M_{t-1} + B_{t-1}(t)}{P_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \left[(T_{t+j} - G_{t+j}) + \frac{M_{t+j}}{P_{t+j}} \frac{i_{t+j}}{1 + i_{t+j}} \right]$$

Derivation: Start with the nominal flow budget constraint

$$M_{t-1} + B_{t-1}(t) = P_t (T_t - G_t) + M_t + Q_t^{(1)} B_t(t + 1)$$

where $Q_t^{(1)}$ is the one-period nominal bond price,

$$Q_t^{(1)} = E_t \left(m_{t,t+1} \frac{P_t}{P_{t+1}} \right) = \frac{1}{1 + i_t}.$$

Then substitute for $Q_t^{(1)}$,

$$\begin{aligned} \frac{M_{t-1} + B_{t-1}(t)}{P_t} &= (T_t - G_t) + \frac{M_t}{P_t} \frac{i_t}{1 + i_t} + \frac{M_t}{P_t} E_t \left(m_{t,t+1} \frac{P_t}{P_{t+1}} \right) + E_t \left(m_{t,t+1} \frac{B_t(t + 1)}{P_{t+1}} \right) \\ \frac{M_{t-1} + B_{t-1}(t)}{P_t} &= (T_t - G_t) + \frac{M_t}{P_t} \frac{i_t}{1 + i_t} + E_t \left(m_{t,t+1} \frac{M_t + B_t(t + 1)}{P_{t+1}} \right) \end{aligned}$$

and iterate forward.

10.1.2 Seignorage as a flow

An equivalent expression has only debt on the left, and counts seignorage as the flow proceeds from money creation rather than the interest spread,

$$\frac{B_{t-1}(t)}{P_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \left[(T_{t+j} - G_{t+j}) + \frac{M_{t+j} - M_{t+j-1}}{P_t} \right].$$

Derivation: Again start with the flow constraint,

$$M_{t-1} + B_{t-1}(t) = P_t (T_t - G_t) + M_t + Q_t^{(1)} B_t(t + 1)$$

$$Q_t^{(1)} = E_t \left(m_{t,t+1} \frac{P_t}{P_{t+1}} \right) = \frac{1}{1+i_t}.$$

$$\frac{B_{t-1}(t)}{P_t} = (T_t - G_t) + \frac{M_t - M_{t-1}}{P_t} + E_t \left(m_{t,t+1} \frac{B_t(t+1)}{P_{t+1}} \right)$$

and again iterate forward.

10.1.3 The discount rate

We can discount by a rate of return rather than the stochastic discount factor m .

$$B_{t-1}(t) = P_t s_t + \frac{1}{1+i_t} B_t(t+1)$$

$$\frac{B_{t-1}(t)}{P_t} = s_t + \left(\frac{1}{1+i_t} \frac{P_{t+1}}{P_t} \right) \frac{B_t(t+1)}{P_{t+1}}$$

$$\frac{B_{t-1}(t)}{P_t} = \sum_{j=0}^{\infty} \left[\prod_{k=0}^j \left(\frac{1}{1+i_{t+k}} \frac{P_{t+k}}{P_{t+k-1}} \right) \right] s_{t+j}$$

$$\frac{1}{1+i_t} \frac{P_{t+1}}{P_t} = \frac{1}{R_{t+1}}$$

is the ex-post real return on nominal bonds. So, we can “discount” by this return. It is not the real or nominal interest rate, however. The real interest and nominal interest rates are

$$\frac{1}{1+i_t} = E_t \left(m_{t+1} \frac{P_t}{P_{t+1}} \right)$$

$$\frac{1}{1+r_t} = E_t(m_{t+1})$$

10.1.4 Real debt

Real debt $b_{t-1}(t)$ promises to pay P_t dollars. Equivalently, debt denominated in foreign currency or gold must be paid off with P_t dollars. With real debt, the flow constraint is

$$\frac{M_{t-1}}{P_t} + b_{t-1}(t) = (T_t - G_t) + \frac{M_t}{P_t} + \frac{1}{1+r_t^f} b_t(t+1)$$

Iterating forward, the “present value” relations are

$$b_{t-1}(t) = \sum_{j=0}^{\infty} \left[\prod_{k=0}^j \left(\frac{1}{1+r_{t+k}^f} \right) \right] \left[(T_{t+j} - G_{t+j}) + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right]$$

$$b_{t-1}(t) = \sum_{j=0}^{\infty} E_t(m_{t,t+j}) \left[(T_{t+j} - G_{t+j}) + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right]$$

$$b_{t-1}(t) = E_t \left\{ \sum_{j=0}^{\infty} m_{t,t+j} \left[(T_{t+j} - G_{t+j}) + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right] \right\}$$

These expressions hold *ex-post* as well as *ex-ante*. $b_{t-1}(t)$ places a constraint on subsequent actions, if the debt is not to default. Also, in this case we can use the real riskfree rate to discount.

10.1.5 Interest-paying debt

Suppose that bonds pay interest i_t at time $t + \Delta$. It's i_t because the nominal interest rate is fixed one period in advance. This case is useful also as the discrete-time counterpart to the continuous-time formulas. It also is more realistic to think of a steady stock of floating rate debt rather than continual rollover of explicit one-period discount debt, though the two arrangements are of course economically equivalent.

Debt B_t is outstanding at time t , and pays interest i_t at time $t + \Delta$. Thus, the time $t + \Delta$ flow constraint is

$$P_{t+\Delta}s_{t+\Delta} + (B_{t+\Delta} - B_t) - i_t B_t = 0$$

I show in this case that the “present value” formula holds with slightly different timing.

$$\frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \frac{\Lambda_{t+j\Delta}}{\Lambda_t} s_{t+j\Delta}.$$

Again, we can also discount at the *ex-post* real return on nominal bonds,

$$\frac{B_t}{P_t} = \sum_{j=1}^{\infty} \prod_{k=1}^j \left(\frac{1}{1 + i_{(t+k-1)\Delta}} \frac{P_{t+k\Delta}}{P_{t+(k-1)\Delta}} \right) s_{t+j\Delta} = \sum_{j=1}^{\infty} \left(\prod_{k=1}^j \frac{1}{R_{(t+k-1)\Delta, (t+k)\Delta}} \right) s_{t+j\Delta} = \sum_{j=1}^{\infty} \frac{1}{R_{t, (t+k)\Delta}} s_{t+j\Delta}$$

Derivations: Write the flow constraint as

$$\begin{aligned} s_{t+\Delta} + \frac{B_{t+\Delta}}{P_{t+\Delta}} - (1 + i_t) \frac{P_t}{P_{t+\Delta}} \frac{B_t}{P_t} &= 0 \\ \frac{1}{1 + i_t} \frac{P_{t+\Delta}}{P_t} s_{t+\Delta} + \frac{1}{1 + i_t} \frac{P_{t+\Delta}}{P_t} \frac{B_{t+\Delta}}{P_{t+\Delta}} &= \frac{B_t}{P_t} \end{aligned}$$

Iterating forward, we obtain

$$\frac{B_t}{P_t} = \sum_{j=1}^{\infty} \prod_{k=1}^j \left(\frac{1}{1 + i_{(t+k-1)\Delta}} \frac{P_{t+k\Delta}}{P_{t+(k-1)\Delta}} \right) s_{t+j\Delta}$$

The *ex-post* real return on one period nominal bonds is of course

$$R_{t, t+\Delta} = (1 + i_t) \frac{P_t}{P_{t+\Delta}}.$$

To arrive at the stochastic discount representation, multiply the flow constraint by $\Lambda_{t+\Delta}/\Lambda_t$ and take expectations,

$$E_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} s_{t+\Delta} \right) + E_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \frac{B_{t+\Delta}}{P_{t+\Delta}} \right) - (1 + i_t) E_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \frac{P_t}{P_{t+\Delta}} \right) \frac{B_t}{P_t} = 0.$$

But of course the price of one period bonds is

$$\frac{1}{1+i_t} = E_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \frac{P_t}{P_{t+\Delta}} \right)$$

so the term in front of B_t/P_t vanishes, and, iterating forward we have again

$$\frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \frac{\Lambda_{t+j\Delta}}{\Lambda_t} s_{t+j\Delta}.$$

10.2 Continuous time

10.2.1 Real debt

The flow constraint says that interest payments equal primary surplus plus new debt sales,

$$r_t b_t dt = s_t dt + db_t.$$

The flow constraint means that debt sales must also be of order dt , so we don't have to worry about db_t^2 terms. In this case the debt valuation equation takes the form

$$b_t = E_t \int_t^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} s_{\tau} d\tau.$$

We can also discount by the real interest rate, which is of course also the ex-post return on government debt.

$$b_t = \int_t^T \frac{V_t}{V_{\tau}} s_{\tau} d\tau$$

where V_t is the value process corresponding to the real interest rate, i.e.

$$\frac{V_t}{V_0} = e^{\int_{\tau=0}^t r_{\tau} d\tau}.$$

This relation holds ex-post as well as ex-ante.

Derivations. Start with the flow constraint,

$$db_t - r_t b_t dt = -s_t dt.$$

Note

$$\begin{aligned} E_t \left[\frac{d(\Lambda_t b_t)}{\Lambda_t} \right] &= E_t \left[\frac{d\Lambda_t}{\Lambda_t} \right] b_t + E_t [db_t] \\ E_t \left[\frac{d(\Lambda_t b_t)}{\Lambda_t} \right] &= -r_t b_t dt + db_t \end{aligned}$$

Thus, we can write the flow constraint

$$E_t [d(\Lambda_t b_t)] = -\Lambda_t s_t dt.$$

Integrating,

$$E_t(\Lambda_T b_T) - \Lambda_t b_t = -E_t \int_t^T \Lambda_\tau s_\tau d\tau.$$

and imposing the transversality condition,

$$\Lambda_t b_t = E_t \int_t^\infty \Lambda_\tau s_\tau d\tau.$$

To express the present value relation discounted by bond returns, start from the same flow constraint,

$$r_t b_t dt = s_t dt + db_t.$$

Define the value corresponding to the return on government debt as

$$V_t = V_0 e^{\int_0^t r_t d\tau}.$$

Now,

$$d\left(\frac{b_t}{V_t}\right) = \frac{db_t}{V_t} - r_t \frac{b_t}{V_t} dt$$

Thus, divide the flow constraint by V_t and write

$$\begin{aligned} r_t \frac{b_t}{V_t} dt &= \frac{1}{V_t} s_t dt + \frac{db_t}{V_t} \\ \frac{db_t}{V_t} - r_t \frac{b_t}{V_t} dt &= -\frac{1}{V_t} s_t dt \\ d\left(\frac{b_t}{V_t}\right) &= -\frac{1}{V_t} s_t dt \\ \frac{b_T}{V_T} - \frac{b_t}{V_t} &= -\int_t^T \frac{1}{V_\tau} s_\tau d\tau \end{aligned}$$

and applying the transversality condition,

$$b_t = \int_t^T \frac{V_t}{V_\tau} s_\tau d\tau = \int_t^T \left[e^{-\int_{\sigma=t}^\tau r_\sigma d\sigma} \right] s_\tau d\tau.$$

10.2.2 Nominal debt and money

The nominal flow constraint is

$$P_t s_t dt + dB_t + dM_t - i_t B_t dt = 0. \quad (14)$$

The government gains money from primary surpluses, debt sales, money issue, and spends money on interest payments. Note here that $dB_t + dM_t$ must be of order dt , since both of the other terms are of order dt . To keep the analysis simple I will assume that each of dB and dM is of order dt rather than assume offsetting Ito terms or jumps.

I derive the following forms of the debt valuation equation. Let $s_t = T_t - G_t$ denote primary surpluses without seignorage. First, with seignorage counted as the flow from money creation,

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left(s_\tau + \frac{dM_\tau}{P_\tau} \right) d\tau.$$

Second, with seignorage counted as the interest savings on money and then with money on the left hand side,

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left(s_\tau + i_\tau \frac{M_\tau}{P_\tau} \right) d\tau.$$

We can also discount at the ex-post real return on nominal government debt, yielding

$$\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{V_t}{V_\tau} \left(s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right)$$

and

$$\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{V_t}{V_\tau} \left(s_\tau + i_\tau \frac{M_\tau}{P_\tau} \right) d\tau$$

where V_t is the ex-post real cumulative value process from investment in nominal government debt,

$$V_t = e^{\int_{\tau=0}^t i_\tau d\tau} \frac{P_0}{P_t}$$

i.e, it has a rate of return

$$\frac{dV_t}{V_t} = i_t dt - \frac{dP_t}{P_t}. \quad (15)$$

This is not the real interest rate.

Derivations Start with the flow constraint,

$$\begin{aligned} s_t dt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t} - i_t \frac{B_t}{P_t} dt &= 0 \\ \Lambda_t s_t dt + \Lambda_t \frac{dM_t}{P_t} + \Lambda_t \frac{dB_t}{P_t} - \Lambda_t i_t \frac{B_t}{P_t} dt &= 0 \end{aligned}$$

Use the definition of the nominal interest rate

$$i_t dt = -E_t \left[d \left(\frac{\Lambda_t}{P_t} \right) / \left(\frac{\Lambda_t}{P_t} \right) \right]$$

to write

$$\begin{aligned} \Lambda_t s_t dt + \Lambda_t \frac{dM_t}{P_t} + \Lambda_t \frac{dB_t}{P_t} + \Lambda_t E_t \left[d \left(\frac{\Lambda_t}{P_t} \right) / \left(\frac{\Lambda_t}{P_t} \right) \right] \frac{B_t}{P_t} &= 0 \\ \Lambda_t s_t dt + \Lambda_t \frac{dM_t}{P_t} + E_t \left[\frac{\Lambda_t}{P_t} dB_t + d \left(\frac{\Lambda_t}{P_t} \right) B_t \right] &= 0 \\ \Lambda_t s_t dt + \Lambda_t \frac{dM_t}{P_t} + E_t \left[d \left(\frac{\Lambda_t}{P_t} B_t \right) \right] &= 0 \\ \Lambda_t s_t dt + \Lambda_t \frac{dM_t}{P_t} + E_t \left[d \left(\Lambda_t \frac{B_t}{P_t} \right) \right] &= 0 \end{aligned} \quad (16)$$

Now we can integrate, and impose the consumer transversality condition to obtain

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left(s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right)$$

To express seignorage it in terms of interest cost, I proceed analogously. From (16), I add and subtract $E_t \left[d \left(\frac{\Lambda_t}{P_t} \right) M_t \right]$ and rearrange to

$$\begin{aligned} \Lambda_t s_t dt - E_t \left[d \left(\frac{\Lambda_t}{P_t} \right) M_t \right] + E_t \left[\frac{\Lambda_t}{P_t} dB_t + \frac{\Lambda_t}{P_t} dM_t + d \left(\frac{\Lambda_t}{P_t} \right) B_t + d \left(\frac{\Lambda_t}{P_t} \right) M_t \right] &= 0 \\ \Lambda_t s_t dt + \frac{\Lambda_t}{P_t} i_t M_t dt + E_t \left[\frac{\Lambda_t}{P_t} d(B_t + M_t) + d \left(\frac{\Lambda_t}{P_t} \right) (B_t + M_t) \right] &= 0 \\ \Lambda_t s_t dt + \Lambda_t i_t \frac{M_t}{P_t} dt + E_t \left[d \left(\frac{\Lambda_t}{P_t} (B_t + M_t) \right) \right] &= 0 \\ \Lambda_t \left(s_t + i_t \frac{M_t}{P_t} \right) dt + E_t \left[d \left(\Lambda_t \frac{B_t + M_t}{P_t} \right) \right] &= 0 \end{aligned}$$

Integrating again, and imposing the consumer's transversality condition,

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left(s_\tau + i_\tau \frac{M_\tau}{P_\tau} \right) d\tau.$$

To discount with the ex-post return (15), start again with the nominal flow constraint (14),

$$\begin{aligned} dB_t - i_t B_t dt &= -(P_t s_t dt + dM_t) \\ \frac{dB_t}{P_t} - \left(\frac{dP_t}{P_t} + i_t dt - \frac{dP_t}{P_t} \right) \frac{B_t}{P_t} &= - \left(s_t dt + \frac{dM_t}{P_t} \right) \\ \frac{1}{V_t} \frac{dB_t}{P_t} - \left(\frac{dP_t}{P_t} + \frac{dV_t}{V_t} \right) \frac{B_t}{V_t P_t} &= - \left(\frac{s_t}{V_t} dt + \frac{dM_t}{V_t P_t} \right) \\ \frac{1}{P_t V_t} dB_t + d \left(\frac{1}{V_t P_t} \right) B_t &= - \frac{1}{V_t} \left(s_t dt + \frac{dM_t}{P_t} \right) \\ d \left(\frac{1}{V_t} \frac{B_t}{P_t} \right) &= - \frac{1}{V_t} \left(s_t dt + \frac{dM_t}{P_t} \right) dt \\ \frac{B_t}{P_t} &= \int_\tau^\infty \frac{V_t}{V_\tau} \left(s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right) \end{aligned}$$

To do the same thing expressing seignorage as an interest expense,

$$\begin{aligned}
dB_t + dM_t - i_t B_t dt &= -P_t s_t dt \\
\frac{d(B_t + M_t)}{P_t} - i_t \frac{(B_t + M_t)}{P_t} dt &= -\left(s_t + i_t \frac{M_t}{P_t}\right) dt \\
\frac{d(B_t + M_t)}{P_t} - \left(\frac{dP_t}{P_t} + i_t dt - \frac{dP_t}{P_t}\right) \frac{(B_t + M_t)}{P_t} &= -\left(s_t + i_t \frac{M_t}{P_t}\right) dt \\
\frac{d(B_t + M_t)}{V_t P_t} - \left(\frac{dP_t}{P_t} + \frac{dV_t}{V_t}\right) \frac{(B_t + M_t)}{V_t P_t} &= -\frac{1}{V_t} \left(s_t + i_t \frac{M_t}{P_t}\right) dt \\
\frac{d(B_t + M_t)}{V_t P_t} + d\left(\frac{1}{V_t P_t}\right) (B_t + M_t) &= -\frac{1}{V_t} \left(s_t + i_t \frac{M_t}{P_t}\right) dt \\
d\left[\frac{1}{V_t P_t} (B_t + M_t)\right] &= -\frac{1}{V_t} \left(s_t + i_t \frac{M_t}{P_t}\right) dt \\
\frac{B_t + M_t}{P_t} &= \int_t^\infty \frac{V_t}{V_\tau} \left(s_\tau + i_\tau \frac{M_\tau}{P_\tau}\right) d\tau
\end{aligned}$$

Note that as a result of (15), the second order terms cancel in

$$V_t P_t d\left(\frac{1}{V_t P_t}\right) = -\frac{dV_t}{V_t} - \frac{dP_t}{P_t} + \frac{dV_t^2}{V_t^2} + \frac{dP_t^2}{P_t^2} + \frac{dV_t dP_t}{V_t P_t}.$$

10.3 Long term debt

Here I derive the present-value formula

$$\frac{\int_{j=0}^\infty Q_t^{(j)} B_t^{(j)} dj}{P_t} = E_t \int_{\tau=0}^\infty \frac{\Lambda_{t+\tau}}{\Lambda_t} s_{t+\tau} dt$$

from the flow budget constraint.

It helps to start with a discrete-time approach. $B_t^{(j)}$ is the stock of zero coupon bonds of j maturity outstanding at the beginning of t , and $Q_t^{(j)}$ is its price. $B_t^{(j)}$ is determined at the end of period $t - \Delta$ and is in in the $t - \Delta$ information set.

The flow constraint thus states that surplus, plus revenue from bond sales of all maturities, equals the cost of redeeming debt that comes due between time t and time $t + \Delta$,

$$P_t s_t \Delta + \int_{j=\Delta}^\infty Q_t^{(j)} (B_{t+\Delta}^{(j-\Delta)} - B_t^{(j)}) dj - \int_{j=0}^\Delta Q_t^{(j)} B_t^{(j)} dj = 0.$$

This constraint is equivalent to

$$P_t s_t \Delta + \int_{j=0}^\infty Q_t^{(j+\Delta)} B_{t+\Delta}^{(j)} dj - \int_{j=0}^\infty Q_t^{(j)} B_t^{(j)} dj = 0,$$

which states that surpluses, plus the revenue of selling new debt equals the cost of buying back and redeeming the outstanding stock of debt.

To iterate forward in this discretization we use the standard bond pricing formula, the value of a $j + \Delta$ period bond today is the discounted value of a j period bond tomorrow,

$$\Lambda_t \frac{Q_t^{(j+\Delta)}}{P_t} = E_t \left[\Lambda_{t+\Delta} \frac{Q_{t+\Delta}^{(j)}}{P_{t+\Delta}} \right].$$

Hence,

$$\Lambda_t s_t \Delta + E_t \int_{j=0}^{\infty} \Lambda_{t+\Delta} Q_{t+\Delta}^{(j)} \frac{B_{t+\Delta}^{(j)}}{P_{t+\Delta}} dj - \int_{j=0}^{\infty} \Lambda_t Q_t^{(j)} \frac{B_t^{(j)}}{P_t} dj = 0.$$

Iterating forward, we have

$$\frac{\int_{j=0}^{\infty} Q_t^{(j)} B_t^{(j)} dj}{P_t} = E_t \sum_{k=0}^{\infty} \frac{\Lambda_{t+k\Delta}}{\Lambda_t} s_{t+k\Delta} \Delta.$$

Now, we can take the limit,

$$\frac{\int_{j=0}^{\infty} Q_t^{(j)} B_t^{(j)} dj}{P_t} = E_t \int_{\tau=0}^{\infty} \frac{\Lambda_{t+\tau}}{\Lambda_t} s_{t+\tau} d\tau.$$

Doing the same thing directly in continuous time takes a bit more work. In continuous time, the flow constraint is

$$P_t s_t dt + \int_{j=0}^{\infty} \left[Q_t^{(j)} dB_t^{(j)} + B_t^{(j)} \frac{\partial Q_t^{(j)}}{\partial j} dt \right] dj = 0. \quad (17)$$

The $\partial Q_t^{(j)} / \partial j$ term compensates for the fact that $dB_t^{(j)} = \lim_{\Delta \rightarrow 0} B_{t+\Delta}^{(j)} - B_t^{(j)}$ is not the difference in quantity of the same bond, but is a bond maturing at $t + \Delta$ less a bond maturing at t . To derive this formula, break up the discrete time flow constraint as follows, and take the limit

$$\begin{aligned} P_t s_t \Delta + \int_{j=0}^{\infty} \left[Q_t^{(j+\Delta)} B_{t+\Delta}^{(j)} - Q_t^{(j+\Delta)} B_t^{(j)} + Q_t^{(j+\Delta)} B_t^{(j)} - Q_t^{(j)} B_t^{(j)} \right] dj &= 0 \\ P_t s_t \Delta + \int_{j=0}^{\infty} \left[Q_t^{(j+\Delta)} \left(B_{t+\Delta}^{(j)} - B_t^{(j)} \right) + \left(Q_t^{(j+\Delta)} - Q_t^{(j)} \right) B_t^{(j)} \right] dj &= 0 \end{aligned}$$

Next, in continuous time the bond pricing relationship is

$$\frac{\Lambda_t}{P_t} \frac{\partial Q_t^{(j)}}{\partial j} dt = E_t \left[d \left(\Lambda_t \frac{Q_t^{(j)}}{P_t} \right) \right]. \quad (18)$$

Again, the derivative with respect to j appears because $Q_t^{(j)}$ is the value of $Q_{t+\Delta}^{(j-\Delta)}$ not of $Q_{t+\Delta}^{(j)}$. To show this, write

$$\begin{aligned} \frac{\Lambda_t}{P_t} \frac{\partial Q_t^{(j)}}{\partial j} \Delta &= \frac{\Lambda_t}{P_t} \left(Q_t^{(j+\Delta)} - Q_t^{(j)} \right) \\ &= E_t \left[\Lambda_{t+\Delta} \frac{Q_{t+\Delta}^{(j)}}{P_{t+\Delta}} \right] - \Lambda_t \frac{Q_t^{(j)}}{P_t} \end{aligned}$$

Now, using (18) in (17),

$$\Lambda_t s_t dt + \int_{j=0}^{\infty} \left[\Lambda_t \frac{Q_t^{(j)}}{P_t} dB_t^{(j)} + \Lambda_t \frac{B_t^{(j)}}{P_t} \frac{\partial Q_t^{(j)}}{\partial j} dt \right] dj = 0$$

$$\Lambda_t s_t dt + \int_{j=0}^{\infty} \left[\Lambda_t \frac{Q_t^{(j)}}{P_t} dB_t^{(j)} + B_t^{(j)} E_t \left[d \left(\Lambda_t \frac{Q_t^{(j)}}{P_t} \right) \right] \right] dj = 0$$

$$\Lambda_t s_t dt + E_t \int_{j=0}^{\infty} d \left[\Lambda_t \frac{Q_t^{(j)} B_t^{(j)}}{P_t} \right] dj = 0$$

(As before, I restrict $B_t^{(j)}$ not to have Ito terms). Integrating, we obtain the present value result

$$\frac{\int_{j=0}^{\infty} Q_t^{(j)} B_t^{(j)} dj}{P_t} = E_t \int_{\tau=0}^{\infty} \frac{\Lambda_{t+\tau}}{\Lambda_t} s_{t+\tau} dt$$