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Financing Development: The Role of Information Costs

by

Jeremy Greenwood, Juan M. Sanchez and Cheng Wang

Abstract

How does technological progress in financial intermediation affect the economy? To address this question a costly-state verification framework is embedded into a standard growth model. In particular, financial intermediaries can invest resources to monitor the returns earned by firms. The inability to monitor perfectly leads to firms earning rents. Undeserving firms are financed, while deserving ones are under funded. A more efficient monitoring technology squeezes the rents earned by firms. With technological advance in the financial sector, the economy moves continuously from a credit-rationing equilibrium to a perfectly efficient competitive equilibrium. A numerical example suggests that finance is important for growth.

Keywords: financial intermediation, economic development, costly-state verification

JEL Nos: E13, O11, O16

Affliations: University of Pennsylvania, University of Rochester, and Iowa State University

1 Introduction

Financial development "accelerates economic growth and improves economic performance to the extent that it facilitates the migration of funds to the best user, i.e. to the place in the economic system where the funds will earn the highest social return," noted Goldsmith (1969, p. 400) some thirty five years ago. Ever since then, economists have been developing theories and searching for empirical evidence connecting financial and economic development together. Information production plays a key role in this process of steering of funds to the highest valued users in two ways. First, intermediaries collect and analyze information about potential investments before funds are committed by savers. Second, after savers have devoted their funds to investment, intermediaries monitor the activities of borrowers to ensure that the best return is attained. If the costs of information production drop, then financial intermediation should become more efficient with an associated improvement in economic performance. The current analysis provides a theoretical model of how reductions in the cost of information processing allow for more efficient capital allocation. A numerical example illustrates that the impact, of a reduction in the cost of information processing, on the efficacy of intermediation might be quite large.

1.1 Facts

Some stylized facts of the kind illustrated in Figure 1 were offered by Goldsmith (1969) to suggest that financial intermediation might be important: First, the ratio of business debt to GDP has risen. In 1952 business debt was 30% of GDP.[1] Today it is 65%. Second, the value of firms relative to GDP has also moved up. In 1951 firms were worth 50% of GDP, while today they are valued at 176%. Third, the size of the financial sector has increased. Output of the financial sector as a percentage of GDP rose from 2% in 1950 to 8% today. All of this may be evidence of improved intermediation; now it is easier for firms to enter stock and bond markets and raise funds.

[1] Data sources are provided in Appendix 11.2.

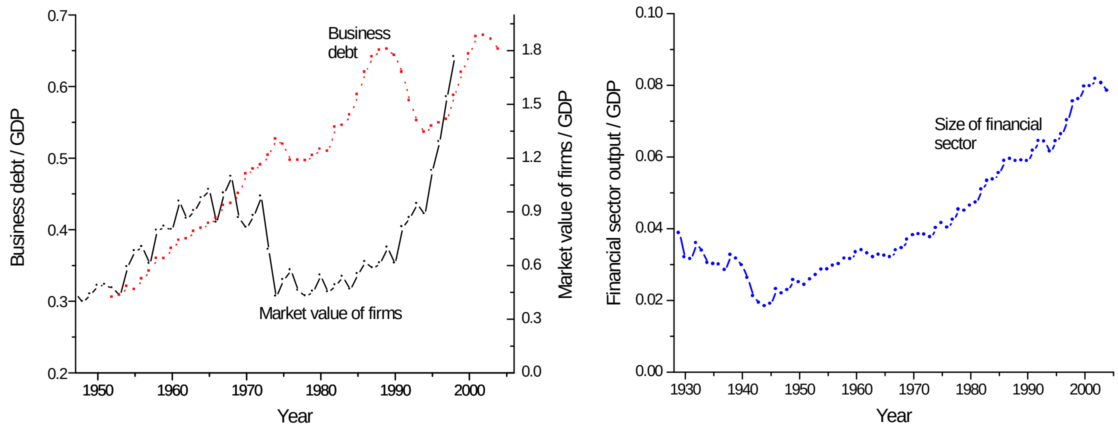

Figure 1: Trends in financial intermediation

Direct measures of the impact of improved efficiency in the financial system on the economy are hard to come by. Improvements in the efficiency of financial intermediation, due to improved information production, are likely to reduce the spread between the internal rate of return on investment in firms and the rate of return on savings received by savers. The spread between these returns reflects the costs of intermediation. This spread will include the costs of ex ante information gathering about investment projects, the ex post information costs of policing investments, and the costs of misappropriation of savers's funds by management, unions, etc. that arise in a world with imperfect information. One may observe little change in the rate of return earned by savers over time, because aggregate savings will adjust in equilibrium so that this return reflects savers' rates of time preference.

If the wedge between the internal rate of return earned by firms and the rate of return received by savers falls, due to more effective intermediation, then the capital stock in the economy should rise. Indeed, there is some evidence that this may be the case. Figure 2 plots the capital-to-GDP ratio for the business sector of the economy, where the capital stock includes intangible investments such as firms' research and development, following the lead

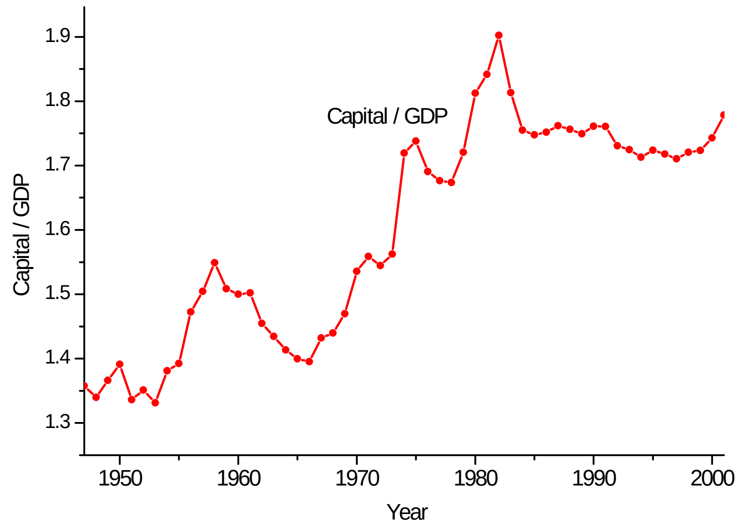

Figure 2: Capital-to-GDP ratio, 1947-2002

of Corrado et al. (2006). It has increased significantly over the postwar period. Of course, an economy's capital-to-GDP ratio may rise on other accounts, too. For example, lower taxes on capital income should increase it, as well as declines in the price of capital goods due to investment-specific technological advance. To the extent that capital stock measures exclude intangibles, or fail to incorporate improvements in quality due to embodied technological progress, then more effective intermediation will be picked up as increases in productivity, instead.

Empirical researchers have used increasingly sophisticated methods to tease out the relationship between financial intermediation and growth. This literature is surveyed masterfully by Levine (2005). The upshot is that financial development has a causal effect on economic development; specifically, financial development leads to higher rates of growth in income and productivity. Additionally, it may be of secondary importance whether the source of the development in financial systems arises from improvements in banks, stock markets or bond markets.

1.2 The Analysis

To address the impact that financial innovation has on economic development, a model is presented with four key features. First, output is produced by firms using capital and labor. The funds for capital must be raised outside of firms. The production technology is subject to idiosyncratic randomness, the realized state of which is private information to the firm. There is a distribution, across firms, over the distribution of these returns. That is, some firms may have investment projects that offer low-expected returns with little variance, while others may have projects that yield high-expected returns with a large variance.

Second, the production technologies used by firms are governed by constant returns to scale. This assumption is important. In the absence of financial market frictions no rents will be earned on production. Additionally, in a frictionless world only firms offering the highest expected return will be funded. With financial market frictions the cost of capital for the best projects is too high, and simultaneously inefficient projects are funded.

Third, there are competitive financial intermediaries in the economy that supply the capital to firms needed for production. These intermediaries write lending contracts with firms that mitigate the private information problem. An important feature of these contracts is the ability of the intermediary to monitor the revenue of the firm. Without the ability to monitor an intermediary cannot condition loan payments on the state of a firm's profits, since the latter always has an incentive to report the worst outcome in order to keep as much of the proceeds for itself as possible. Here, an equilibrium with credit rationing will prevail, and only one type of firm will be funded. This is the type that earns the most profits in its worst state of world. In a sense, only the safest type of firm is funded in the absence of monitoring.

Fourth, the monitoring technology improves over time due to technological innovation in the financial sector. At low levels of productivity in the financial sector, the economy rests in an equilibrium where loans are not monitored and credit is rationed. If technological improvement in the financial sector occurs at a faster pace than in the rest of the economy, then financial intermediation becomes more efficient. Loans are monitored more diligently and the rents earned by firms shrink. Additionally, lending activity will change along both extensive and intensive margins. Projects with high- (low-) expected returns will now receive more (less) funds. Those investments with the lowest expected returns will be cut. At high levels of efficiency in the financial sector the economy approaches the first-best equilibrium achieved in a world without informational frictions.

1.3 Literature Review

At the heart of the framework developed here is a costly-state verification model that has its roots in the work of Townsend (1979) and Williamson (1986). Bernanke and Gertler (1989) and Williamson (198) have used such a framework for business cycle analysis. The goal here is study economic development. The costly-state verification model employed here differs from the standard paradigm. The auditing technology is random, albeit in a different sense than Boyd and Smith (1994). Specifically, the probability of detecting malfeasance depends upon the amount of resources devoted to the activity and the efficiency of the monitoring technology. Additionally, unlike Townsend (1979) and Williamson (1986), the optimal contract is not a debt contract. As in Boyd and Prescott (1986) the contract is designed to induce truthful behavior by borrowers. The model connects the state of development in the financial system to economic activity, as do Bencivenga and Smith (1991), Greenwood and Jovanovic (1990), Levine (1991), Marcet and Marimon (1992), and Townsend and Ueda (2006). In the current analysis, the ability of the financial system to produce information about borrowers is continually evolving due to technological progress in information production. A numerical example presented here suggests that the state of financial system can have a large impact on the economy. Whether or not this implication will be robust to future extensions remains to be seen, but it appears to derive from the fact that in worlds with imperfect information the lure for rents by firms with less than best-practice technologies will have a detrimental impact on aggregate economic activity.

2 The Environment

Imagine a world made up of three types of agents: consumer/workers, firms and financial intermediaries. In a nutshell, firms produce output using capital and labor. The consumer/worker supplies the labor, and intermediaries the capital. Financial intermediaries raise the funds for capital from consumer/workers. They also use labor in their lending activity. Output is used for consumption by consumer/workers and for investment in capital by intermediaries. Each of these features of the economy will now be described in more detail.

2.1 The Representative Agent

The representative consumer/worker's goal in life is to maximize his lifetime utility as given by

$$\sum_{t=0}^{\infty} \beta^t \ln c_t, \text{ with } 0 < \beta < 1,$$

where c_t denotes period-t consumption. The individual has one unit of labor, which earns the wage rate w_t on the market. In any given period the consumer/worker has two additional sources of income. First, he earns interest on any financial assets, a_t, purchased in the previous period. A unit of consumption invested with a financial intermediary in period $t-1$ realizes the net return $\widehat{r}_t$ in period t. Second, he earns the profits or dividends, d_t, from the firms that he owns.

2.2 Firms

Firms produce output, o, in line with the production function

$$o = \theta k^{\alpha} l^{1-\alpha},$$

where k and l represent the inputs of capital and labor used in production. The variable θ gives the productivity level of the firm's production process. Productivity is a random variable drawn from a two-point vector $\tau \equiv (\theta_1, \theta_2)$ with $\theta_1 < \theta_2$. Let $\Pr(\theta = \theta_1) = \pi_1$ and $\Pr(\theta = \theta_2) = \pi_2 = 1 - \pi_1$. The mean and variance of θ are given by $\pi_1\theta_1 + \pi_2\theta_2$

and $\pi_1\pi_2(\theta_1 - \theta_2)^2$, respectively.[2] Thus, for a given set of probabilities these statistics differ in accordance with the values specified for θ_1 and θ_2. Now, the vector from which the productivities, τ, are drawn from differs across firms. In particular, suppose that firms in the economy are distributed over productivities in line with the distribution function $F : \mathcal{T} \rightarrow [0, 1]$, where $\mathcal{T} \sqsubseteq R_+^2$ and

$$F(x, y) = \Pr(\theta_1 \leq x, \theta_2 \leq y).$$

Think of this distribution as somehow specifying a trade-off between the mean and variance of project returns. Due to technological progress in the production sector of the economy, this distribution will evolve over time.

The firm borrows capital, k, from the intermediary *before* it observes the technology shock, θ. It does this with both parties knowing its type, τ. It can employ labor, at the wage rate w, *after* it sees the realization for θ. In order to finance its use of capital the firm must enter into a contract with a financial intermediary. Suppose that a firm can only borrow from one intermediary at a time.

2.3 Financial Intermediaries

There is a competitive intermediation sector that borrows funds from consumers and lends capital to firms. While the intermediary knows a firm's type it cannot observe the state of a firm's business either costlessly or perfectly.[3] That is, the intermediary cannot costlessly observe θ, o and l. The firm will make a report to the intermediary about its business situation. The intermediary can devote some resources in order to assess the veracity of this report. The payments, p, from a firm to the intermediary will be conditioned both upon the report made by former, and the outcome of any monitoring activity done by latter. Figure 3 illustrates the flow of funds in the economy. By channelling funds through financial intermediaries consumers avoid a costly duplication of monitoring effort that would occur

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[2] Observe that $\pi_1\theta_1^2 + (1-\pi_1)\theta_2^2 - [\pi_1\theta_1 + (1-\pi_1)\theta_2]^2 = \theta_1^2\pi_1 - \theta_1^2\pi_1^2 - 2\theta_1\theta_2\pi_1 + 2\theta_1\theta_2\pi_1^2 + \theta_2^2\pi_1 - \theta_2^2\pi_1^2 = (1-\pi_1)\pi_1\theta_1^2 + (1-\pi_1)\pi_1\theta_2^2 - 2\theta_1\theta_2\pi_1(1-\pi_1) = (1-\pi_1)\pi_1[\theta_1^2 + \theta_2^2 - 2\theta_1\theta_2] = (1-\pi_1)\pi_1(\theta_1 - \theta_2)^2$.

[3] One could potentially have a screening stage where the intermediary verifies the initial type of a firm.

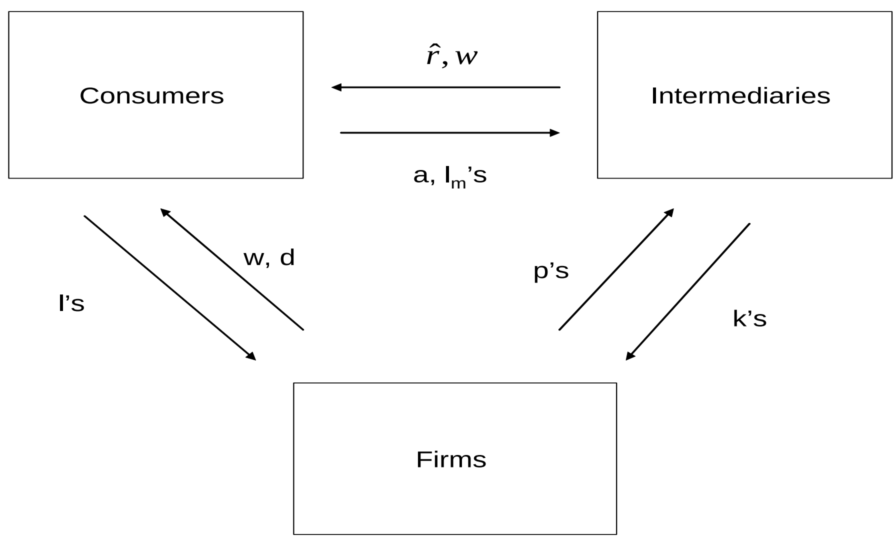

Figure 3: The flow of funds in the economy

in an equilibrium with direct lending between them and firms—see Williamson (1986) for more detail.

Suppose a firm reports that the productivity on its project in a given period is θ_j, which may differ from the true state θ_i. The intermediary can devote resources, m_j, to verify this claim. The probability of detecting fraud is increasing in the amount of resources devoted to this activity. In particular, let $P_{ij}(m_j/k)$ denote the probability that the firm is caught cheating conditional on: (1) the true realization of productivity is θ_i; (2) the firm makes a report of θ_j; (3) the intermediary spends m_j in monitoring; (4) the size of the loan is k. The function $P_{ij}(m_j/k)$ is assumed to be monotonically increasing in m_j/k. Additionally, let $P_{ij}(m_j/k) = 0$ if the firm truthfully reports that its type is θ_i.

A convenient formulation for $P_{ij}(m_j/k)$ is

$$P_{ij}(m_j/k) = \begin{cases} 1 - \frac{1}{(\epsilon m_j/k)^{\psi}} < 1, \text{ with } 0 < \psi < 1, \\ \quad \text{for a report } \theta_j \neq \theta_i \text{ and } m_j/k > 1/\epsilon, \\ 0, \\ \quad\quad \text{for a report } \theta_j = \theta_i \text{ or } m_j/k \leq 1/\epsilon. \end{cases}$$

This specification requires that some threshold level of monitoring, $m_j > k/\epsilon$, must be exceeded to detect cheating.

Monitoring is a produced good, measured in units of consumption. The production of monitoring is project specific. Monitoring produced for detecting fraud in one project cannot be used in a different one. Let monitoring be produced in line with the production function

$$m = z l_m^{1/\gamma}, \text{ with } 0 \leq 1/\gamma \leq 1,$$

where l_m represents the amount of labor employed in this activity. The cost function, $C(m/z; w)$, associated with monitoring is given by

$$C(m/z; w) = w(m/z)^{\gamma}.$$

Costs are linear in wages, w. With diminishing returns to scale in the production ($1/\gamma < 1$) the cost function is increasing and convex in the amount of monitoring, m, and decreasing and convex in the state of the intermediation technology, z.

Now, exactly which firms are funded depends on three things: (1) the firm's type, τ; (2) the state of the monitoring technology in the financial intermediation sector, z; (3) the expense of monitoring effort as reflected by the wage, w. As will be seen, when the variance of a firm's project becomes larger, the informational problems associated with contracting become more severe. Therefore, high variance projects are less likely to get funded, ceteris paribus.

3 Utility Maximization by Consumers

The representative consumer/worker's goal in life is to maximize his lifetime utility. To do this, each period the consumer picks today's consumption, c, and assets to carry over for tomorrow, a'. This gives rise to the standard dynamic programming problem:

$$J(a) = \max_{c,a'}\{\ln c + \beta J'(a')\}, \tag{P1}$$

subject to

$$c + a' = w + (1+\widehat{r})a + d.$$

The value function implicitly depends upon wages, w, dividends, d, and the interest rate, $\widehat{r}$. In the deterministic model studied, these will all be functions of time, the model's primal state variable. This dependence of the value function upon time is implicitly indicated through the use of the prime symbol attached to J, which differentiates its functional form tomorrow from its form today. The first-order condition associated with this problem leads to the well-known Euler equation

$$\frac{1}{c} = \beta(1+\widehat{r}')\frac{1}{c'}. \tag{1}$$

4 Profit Maximization by Firms

As prelude to solving the contracting problem between a firm and a financial intermediary, consider the problem faced by a firm that receives a loan in terms of capital in the amount k. Recall that the firm hires labor after it sees the realization of its technology shock, θ. It will do this in a manner so as to maximize its profits. In other words, the firm will solve the maximization problem shown below.

$$R(\theta, w)k \equiv \max_{l}\{\theta k^{\alpha} l^{1-\alpha} - wl\}. \tag{P2}$$

The first-order condition associated with this maximization is

$$(1-\alpha)\theta k^{\alpha} l^{-\alpha} = w,$$

which gives

$$l = \left[\theta\frac{(1-\alpha)}{w}\right]^{1/\alpha} k. \tag{2}$$

Substituting the solution for l into the maximand and solving yields the unit return function, $R(\theta, w)$, or

$$r = R(\theta, w) = \alpha(1-\alpha)^{(1-\alpha)/\alpha} w^{-(1-\alpha)/\alpha} \theta^{1/\alpha} > 0.$$

Think about $r_i = R(\theta_i, w)$ as giving the gross rate of return on a unit of capital invested in the firm given that state θ_i occurs. The expected gross rate of return will be $\pi_1 r_1 + \pi_2 r_2$, while the variance reads $\pi_1 \pi_2 (r_1 - r_2)^2$.

5 The Financial Contract

A contract between a firm and an intermediary is summarized by the quadruple $\{k, p_j, p_{ij}, m_j\}$. Here k represents the amount of capital lent by the intermediary to the firm, p_j is the firm's payment to the intermediary if it reports θ_j and is not found cheating, p_{ij} is payment to the bank if the borrower reports θ_j and monitoring reveals that productivity is $\theta_i \neq \theta_j$, and m_j is the intermediary's monitoring effort when θ_j is reported. Denote the value of the firm's outside option by v.

The intermediary chooses the details of the financial contract, $\{k, p_j, p_{ij}, m_j\}$, to maximize its profits. The contract is designed to have two features: (1) it entices truthful reporting by firms; (2) it offers firms an expected return of v. The optimization problem is

$$I(\tau, v) \equiv \max_{p_1, p_2, p_{12}, p_{21}, m_1, m_2, k} \{\pi_1 p_1 + \pi_2 p_2 - \widetilde{r} k - \pi_1 w (m_1/z)^\gamma - \pi_2 w (m_2/z)^\gamma\}, \tag{P3}$$

subject to

$$p_1 \leq r_1 k, \tag{3}$$

$$p_2 \leq r_2 k, \tag{4}$$

$$p_{12} \leq r_1 k, \tag{5}$$

$$p_{21} \leq r_2 k, \tag{6}$$

$$[1 - P_{12}(m_2/k)](r_1 k - p_2) + P_{12}(m_2/k)(r_1 k - p_{12}) \leq r_1 k - p_1, \tag{7}$$

$$[1 - P_{21}(m_1/k)](r_2 k - p_1) + P_{21}(m_1/k)(r_2 k - p_{21}) \leq r_2 k - p_2, \tag{8}$$

and

$$\pi_1(r_1 k - p_1) + \pi_2(r_2 k - p_2) = v. \tag{9}$$

Note that the cost of capital, $\widetilde{r}$, is given by $\widetilde{r} = \widehat{r} + \delta$; i.e., the interest paid to investors plus the depreciation on capital. The first four constraints just say the intermediary cannot demand more than the firm earns; that is, the firm has limited liability. Equations (7) and (8) are the incentive-compatibility constraints. Take (8). This simply states that the expected return to the firm from reporting state one when it actually is in state two, as given by the left-hand side, must be less than telling the truth, as represented by the right-hand side. Observe that the constraint set is not convex due to the way that m_1 enters (8). Therefore, the second-order conditions for the maximization problem are important to consider. The last constraint (9) specifies that the contract must offer the firm an expected return equal to v, its option value outside. A firm's outside option is the expected return that it could earn on a loan from another intermediary. This will be determined in equilibrium. Finally, note the solution for $\{p_j, p_{ij}, m_j, k\}$ is contingent upon the firm's type, $\tau = (\theta_1, \theta_2)$. To conserve on notation, this dependence is generally suppressed.

The lemma below characterizes the solution to the above optimization problem.

Lemma 1 *The solution to problem (P3) is described by:*

1. *The size of the loan from the intermediary to the firm, k, is*

$$k = \frac{v}{\pi_2(r_2 - r_1)[1 - P_{21}(m_1/k)]}. \tag{10}$$

2. *The amount of monitoring per unit of capital when the firm reports a bad state, m_1/k, solves the problem*

$$I(\tau, v) \equiv \max_{m_1/k}\{(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})k - \frac{\pi_1 w}{z^\gamma} k^\gamma (\frac{m_1}{k})^\gamma - v\}, \tag{P4}$$

where k can be eliminated using (10) above.

(a) *Monitoring in the bad state is simply given by*

$$m_1 = (m_1/k)k,$$

where m_1/k solves (P4).

(b) The intermediary does not monitor when the firm reports a good state so that

$$m_2 = 0. \tag{11}$$

3. The payment schedule is

$$p_1 = r_1 k, \tag{12}$$

$$p_2 = r_2 k - v/\pi_2, \tag{13}$$

$$p_{12} = r_1 k, \tag{14}$$

$$p_{21} = r_2 k. \tag{15}$$

Proof. See Appendix 11.1. ■

It is intuitive that there are no benefits to the firm from claiming a better outcome than it actually realizes, since it will only have to pay the intermediary more. The intermediary would like to reduce the firm's incentive to report being in the low state. So, suppose the firm reports a low state. If cheating is not detected, then the firm pays all of its revenue (sans labor cost) that would be realized in the low state—see (12). If the firm is caught cheating, then it must surrender all of the revenue (sans labor cost) that it earns in the high state—see (15). Note that due to the incentive-compatibility constraints a false report will never occur so that the payments shown by (14) and (15) will never occur in equilibrium.

The contract specifies that the intermediary should only monitor the firm when it reports a bad outcome (state 1) on its project—see (11). Monitoring in the low state is done to maximize the intermediary's profits, subject to the incentive-cum-promising-keeping constraint (10), as problem (P4) dictates. Note that the higher is the value of the firm, v, the bigger must be the loan, k, to satisfy the incentive-cum-promising-keeping constraint (10). This constraint (10) ensures that the contract provides the firm an expected return equal to what it would earn if it misrepresented the outcome in the good state, $\pi_2(r_2 - r_1)[1 - P_{21}(m_1/k)]k$. Furthermore, this expected return is set equal to the firm's outside option, v. The size of the loan, k, is increasing in the amount of monitoring that occurs in the low state, m_1/k. This transpires since the probability of the firm not getting caught from misrepresenting its revenues, $1 - P_{21}(m_1/k)$, is decreasing in the intermediary's monitoring activity.

Now, a financial contract will be offered by an intermediary to a firm only if it yields the former nonnegative profits, $I(\tau, v) \geq 0$. It turns out that a necessary condition for a contract to yield nonnegative profits, when $r_1 < \widetilde{r}$, is for the firm to devote more than the minimal level of resources per unit of funds lent, $1/\epsilon$, to monitoring a report of a bad state. If this is not done then an incentive-compatible contract will not be viable, since it involves offering the firm too much of a reward in order to entice a truthful report in the good state.

Lemma 2 *$I(\tau, v) < 0$ in Problem (P4) when either $m_1/k = 0$ or $m_1/k = 1/\epsilon$, for all $v > 0$ and $r_1 < \widetilde{r}$.*

Proof. The argument is presented in Appendix 11.1. ■

5.1 Monitored Loans, $r_1 < \widetilde{r}$

Assume that $r_1 < \widetilde{r}$. The situation where $r_1 \geq \widetilde{r}$ will be analyzed later. It will be established that when $r_1 = \widetilde{r}$ a loan will not be monitored. This can only occur in an equilibrium with credit rationing. It will also be shown that an equilibrium with $r_1 > \widetilde{r}$ cannot exist. Returning to the maintained hypothesis of $r_1 < \widetilde{r}$, focus can be directed toward finding an interior solution for (P4) in light of the above lemma. To this end, substituting the constraint (10) into the problem's objective function, while making use of the formula $[1 - P_{21}(m_1/k)]^{-1} = (\epsilon m_1/k)^{\psi}$, yields

$$I(\tau, v) = \max_{m_1/k} \left\{ qv(\frac{m_1}{k})^{\psi} - \frac{sv^{\gamma}}{z^{\gamma}} \left(\frac{m_1}{k}\right)^{\psi\gamma+\gamma} - v \right\}, \tag{P5}$$

where

$$q = \frac{\epsilon^{\psi}(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})}{\pi_2(r_2 - r_1)},$$

and

$$s = \pi_1 w [\frac{\epsilon^{\psi}}{\pi_2(r_2 - r_1)}]^{\gamma}.$$

The first- and second-order conditions linked to this maximization problem are

$$\psi qv(m_1/k)^{\psi-1} - (\psi\gamma + \gamma)sv^{\gamma}(m_1/k)^{\psi\gamma+\gamma-1}/z^{\gamma} = 0, \tag{16}$$

and

$$(\psi-1)\psi qv(m_1/k)^{\psi-2}-(\psi\gamma+\gamma-1)(\psi\gamma+\gamma)sv^{\gamma}(m_1/k)^{\psi\gamma+\gamma-2}/z^{\gamma}<0.$$

The second-order condition is automatically satisfied since $\psi<1$ and $\gamma>1$.

Plugging in the expressions for q and s into the first-order condition leads to

$$m_1/k=\{\frac{\psi}{(\psi\gamma+\gamma)}\frac{(\pi_1 r_1+\pi_2 r_2-\widetilde{r})}{\pi_1 w}[\frac{\pi_2(r_2-r_1)}{\epsilon^{\psi}}]^{\gamma-1}\frac{z^{\gamma}}{v^{\gamma-1}}\}^{1/(\psi\gamma+\gamma-\psi)}. \tag{17}$$

Lemma 3 *The interior solution (17) for the optimal level of monitoring per unit of capital, m_1/k, satisfies the following properties (for a given wage rate, w):*

1. *m_1/k is:*
    - (a) *increasing in the state of the monitoring technology, z, the expected net return, $\pi_1 r_1+\pi_2 r_2-\widetilde{r}$, and the variance of the return, r_2-r_1;*
    - (b) *decreasing in the value of the firm, v.*
2. *$m_1/k\to\infty$, as $v\to 0$.*
3. *$m_1/k>1/\epsilon$ if and only if $v<\overline{v}$, where*

$$\overline{v}=\overline{V}(\tau)=\{[\psi/(\psi\gamma+\gamma)](\pi_1 r_1+\pi_2 r_2-\widetilde{r})[\pi_2(r_2-r_1)]^{\gamma-1}(\epsilon z)^{\gamma}/(\pi_1 w)\}^{1/(\gamma-1)}.$$

Proof. Again, see Appendix 11.1. ■

It's intuitive that when the state of the monitoring technology, z, improves more monitoring will be undertaken. When the variance of project returns, r_2-r_1, is big the benefits to a firm from under reporting income are large. Hence, it pays for the intermediary to do more monitoring. When expected net returns on a project, $\pi_1 r_1+\pi_2 r_2-\widetilde{r}$, move up there are more profits to be earned for a given level of investment. The marginal benefit from monitoring rises; i.e., q increases in the monitoring problem (P5). In fact, note from (10) that loan size, k, cannot increase without an increase in monitoring. Thus, without extra monitoring the intermediary would be unable to take advantage of any improvement in expected net returns, $\pi_1 r_1+\pi_2 r_2-\widetilde{r}$. This occurs because k is locked in by the incentive and promising-keeping constraints (8) and (9). More monitoring relaxes the incentive constraint. Finally, as the value of the firm, v, rises the size of the loan that the intermediary must make to fulfill the promising keeping constraint grows. As a result, the cost of monitoring rises. Monitoring per unit of loan drops as a consequence. At some point the threshold level of monitoring can no longer be met.

Finally, the size of the loan specified by the optimal contract can be uncovered by substituting (17) into (10). Doing so yields

$$k = [\frac{\epsilon^{\psi}}{\pi_2(r_2 - r_1)}]^{\gamma/(\psi\gamma+\gamma-\psi)}[\frac{\psi}{\psi\gamma+\gamma}\frac{(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})}{\pi_1 w}]^{\psi/(\psi\gamma+\gamma-\psi)} \times z^{\gamma\psi/(\psi\gamma+\gamma-\psi)} v^{\gamma/(\psi\gamma+\gamma-\psi)}. \tag{18}$$

As can be seen, the optimal contract specifies that size of the loan will be increasing in the firm's value, v, and the net return on its project $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r}$. More money will be lent the better is the state of technology in the financial sector, z. Last, loan size will be decreasing in the variance of the firm's project, $r_2 - r_1$. All of these statements take the wage rate, w, as given. The intuition underlying these results can be constructed along similar lines to that just provided for the monitoring equation.

6 Competitive Financial Intermediation

In the economy there is perfect competition in the financial sector. Consequently, an intermediary must offer a contract that maximizes a firm's value, subject to the restriction that the former does not incur a loss. If an intermediary failed to do so it would be undercut by others. The upshot is that intermediaries will make zero profits on each type of loan. The implications of perfect competition will now be analyzed. Key questions are: (i) Which firms will get funded? (ii) What will be the loan size?

6.1 Monitored Loans, $r_1 < \widetilde{r}$

Suppose that $r_1 < \widetilde{r}$ for some project type under consideration. By Lemma 2 any loan to a firm of this type will be monitored. The profit function for the intermediary, when it funds a firm of type τ, can be expressed with a little work as

$$I(\tau,v) = (\psi\gamma+\gamma-\psi)(\frac{1}{\psi})^{-\psi/(\psi\gamma+\gamma-\psi)}(\frac{1}{\psi\gamma+\gamma})^{(\psi\gamma+\gamma)/(\psi\gamma+\gamma-\psi)}q^{(\psi\gamma+\gamma)/(\psi\gamma+\gamma-\psi)}$$
$$\times s^{-\psi/(\psi\gamma+\gamma-\psi)}z^{\gamma\psi/(\psi\gamma+\gamma-\psi)}v^{\gamma/(\psi\gamma+\gamma-\psi)} - v. \tag{19}$$

Lemma 4 *The intermediary's profit function, $I(\tau,v)$, for monitored loans has the following properties on the domain $[0,\overline{V}(\tau)]$:*

1. $I(\tau,0)=0$,
2. $\lim_{v\to 0} dI(\tau,v)/dv = \infty$,
3. $d^2I(\tau,v)/dv^2 < 0$,
4. $I(\tau,\overline{V}(\tau)) < 0$.

Proof. Refer to Appendix 11.1. ■

Figure 4 plots the $I(\tau,v)$ function, as shown by the solid line. The lemma establishes that this function both starts and grows from the origin (Points 1 and 2), is strictly concave (Point 3), and then eventually declines and cuts the axis (Points 3 and 4). The dashed line shows the profits that can be made if the intermediary never monitors ($m_1/k = 0$). Profits would always be negative, in accord with Lemma 2.[4] When $v = \overline{V}(\tau)$ the profit realized at the corner of the interior solution $m_1/k = 1/\epsilon$ must lie below that obtained at the corner $m_1/k = 0$. This is because in both solutions $P_{12}(m_1/k) = 0$, but the former involves paying monitoring costs while the latter does not. With a little effort, it can be shown that an increase in wages from w to w' will cause this function to shift down—see the dashed line in the Figure 5. The expected profits on any lending are reduced, in accordance with problem (P4), since $\pi_1 r_1 + \pi_2 r_2$ will drop. A rise in z to z' will result in $I(\tau,v)$ shifting upward—see the dashed and dotted line in Figure 5. This transpires because lending becomes more profitable when monitoring is less expensive.

[4] The proof of the lemma establishes that this line is linear.

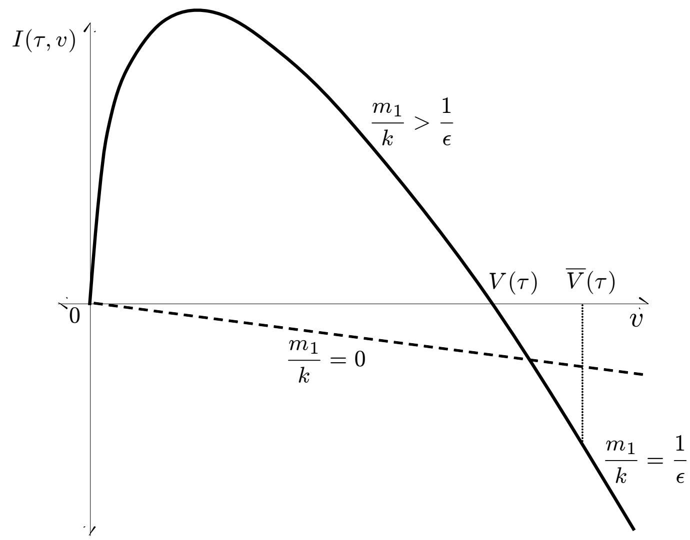

Figure 4: The profit function for the financial intermediary, $I(\tau, v)$

6.1.1 The Loan Portfolio

Which monitored projects will be funded and how much will they get? Let $\mathcal{A}(w)$ denote the set of funded, monitored projects. It turns out that the active set of monitored projects, $\mathcal{A}(w)$, is defined by $\mathcal{A}(w) = \{\tau : \pi_1 r_1 + \pi_2 r_2 - \widetilde{r} > 0\}$. A firm is funded if and only if it can make positive expected profits from any capital invested. From problem (P4), it is easy to see that

$$(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r}) = \alpha(1-\alpha)^{(1-\alpha)/\alpha} w^{-(1-\alpha)/\alpha}[\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}] - \widetilde{r} \geq 0.$$

Observe that profits are decreasing in wages. A type-τ firm will operate when $w \leq \overline{w}$, and will not otherwise, where the cutoff wage, $\overline{w}$, is specified by

$$\overline{w} = \overline{W}(\tau) \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)\left(\frac{\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}}{\widetilde{r}}\right)^{\alpha/(1-\alpha)}. \tag{20}$$

So, the set of active projects $\mathcal{A}(w)$ can be expressed equivalently as

$$\mathcal{A}(w) = \{\tau : \pi_1 r_1 + \pi_2 r_2 - \widetilde{r} > 0\} \text{ or } \mathcal{A}(w) = \{\tau : w < \overline{W}(\tau)\}. \tag{21}$$

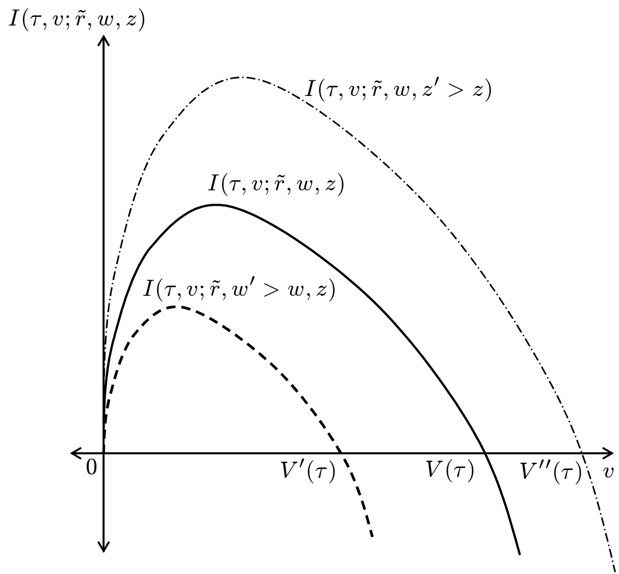

Figure 5: The impact of an increase in wages, w, and monitoring efficiency, z, on profits, $I(\tau, v; \widetilde{r}, w, z)$

The active set depends on the wage because $r_i = R(\theta_i, w)$. It contracts (expands) with a rise (decrease) in the real wage, since $R(\theta_i, w)$ is decreasing in w.

If the intermediation sector is competitive then a loan contract must maximize the value of the firm, v, subject to the fact that intermediary can't make a loss. If it didn't do this, the intermediary could be undercut by a competitor. Hence, v is simply given by

$$v = V(\tau) = \arg\max_{x}\{x : I(\tau, x) \geq 0\}.$$

By Figure 4, it is clear that for any firm in the active set: (i) $V(\tau) > 0$; (ii) $V(\tau)$ is unique; (iii) $V(\tau)$ solves $I(\tau, V(\tau)) = 0$. Note from (10) that $k(\tau) > 0$ if and only if $V(\tau) > 0$; a funded firm will have value and conversely so, too.

The lemma below takes stock of the discussion so far regarding the loan portfolio.

Lemma 5 *A necessary and sufficient condition for a type-τ firm to be active, or for $k(\tau) > 0$ and $V(\tau) > 0$, is that $\tau \in \mathcal{A}(w)$.*

Proof. Those interested should go to Appendix 11.1. ■

Intuitively, from equation (19) it can be seen that if $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} > 0$ (so that $q > 0$) then there will exist a $v > 0$ that gives positive profits for the intermediary. Hence, from Figure 4 it is apparent that the firm will be funded. When $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} < 0$ (implying $q < 0$) then from (19) there does not exist an interior solution that generates positive profits for any v. When a firm is in the active set, it can be deduced from (19) that v will be given by

$$\begin{aligned} v \quad = \quad & V(\tau) = (\psi\gamma + \gamma - \psi)^{(\psi\gamma+\gamma-\psi)/(\psi\gamma-\psi)}(\frac{1}{\psi})^{-\psi/(\psi\gamma-\psi)}(\frac{1}{\psi\gamma+\gamma})^{(\psi\gamma+\gamma)/(\psi\gamma-\psi)} \\ & \times q^{\frac{\gamma\psi+\gamma}{\gamma\psi-\psi}} s^{\frac{-\psi}{\gamma\psi-\psi}} z^{\frac{\psi\gamma}{\gamma\psi-\psi}}. \end{aligned} \tag{22}$$

Next, calculate the level of investment in a firm. To do so, plug the solution for v into formula (18), while making use of the definitions for q and s, to obtain

$$\begin{aligned} k \quad = \quad & (\psi\gamma + \gamma - \psi)^{\gamma/(\psi\gamma-\psi)}(\frac{1}{\psi})^{-\psi/(\psi\gamma-\psi)}(\frac{1}{\psi\gamma+\gamma})^{(\gamma+\psi)/(\psi\gamma-\psi)} \\ & \times(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})^{(\gamma+\psi)/(\psi\gamma-\psi)}(\pi_1 w)^{-\psi/(\psi\gamma-\psi)}[\frac{\epsilon^{\psi}}{\pi_2(r_2 - r_1)}]^{\gamma/(\psi\gamma-\psi)} z^{\psi\gamma/(\psi\gamma-\psi)}. \end{aligned} \tag{23}$$

By inspecting the above equation it is easy to characterize the level of investment in a project.

Lemma 6 *The level of investment in a firm, k, is increasing in its expected net return, $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r}$, and the state of technology in the financial sector, z, and is decreasing in the variance of the return, $r_2 - r_1$ (holding the wage rate, w, fixed).*

Figure 6 portrays the loan portfolio. Here, only projects with an expected return greater than 6.5 are funded. For projects in the active set (that is with a mean return greater than 6.5) the size of the loan increases with the expected return and decreases with the variance. Now, why should intermediaries fund more than one type of project? That is, why don't they invest solely in the projects that offer the highest rate of return? The reasoning is simple. The incentive for a firm to misrepresent its earnings in the good state is increasing in the size of the loan, k, holding monitoring effort m_1 fixed. Specifically, the expected gain from misrepresenting profits in the good state is $\pi_2[1 - P_{21}(m_1/k)](r_2 - r_1)k$—cf. (10). The contract sets this exactly equal to the expected benefit from telling the truth, $v = \pi_2(r_2 k - p_2)$. A bigger loan would violate this incentive constraint, which derives from (8), unless monitoring effort, m_1, is increased as well. From Problem (P4) it is apparent that marginal revenue for a given loan type is decreasing in monitoring effort, while marginal cost is increasing in it. Therefore, there is a unique loan size for each type of project.

6.2 Unmonitored Loans, $r_1 \geq \widetilde{r}$

Attention is now directed toward the situation where $r_1 \geq \widetilde{r}$ for some types of projects. Here the return on a loan to the firm in its worst state of nature is at least as large as the cost of capital, $\widetilde{r}$. Therefore, the intermediary does not have to monitor a loan in order to prevent a loss, as was necessary when $r_1 < \widetilde{r}$—recall Lemma 2. Indeed, it turns out that it is not optimal for the intermediary to monitor a loan.

Lemma 7 *The solution to Problem (P4) dictates that for $r_1 \geq \widetilde{r}$ and $v \geq 0$:*

1. $m_1/k = 0$,
2. $k = \frac{v}{\pi_2(r_2 - r_1)}$,
3. $I(\tau, v) = \frac{(r_1 - \widetilde{r})}{\pi_2(r_2 - r_1)} v \geq 0$.

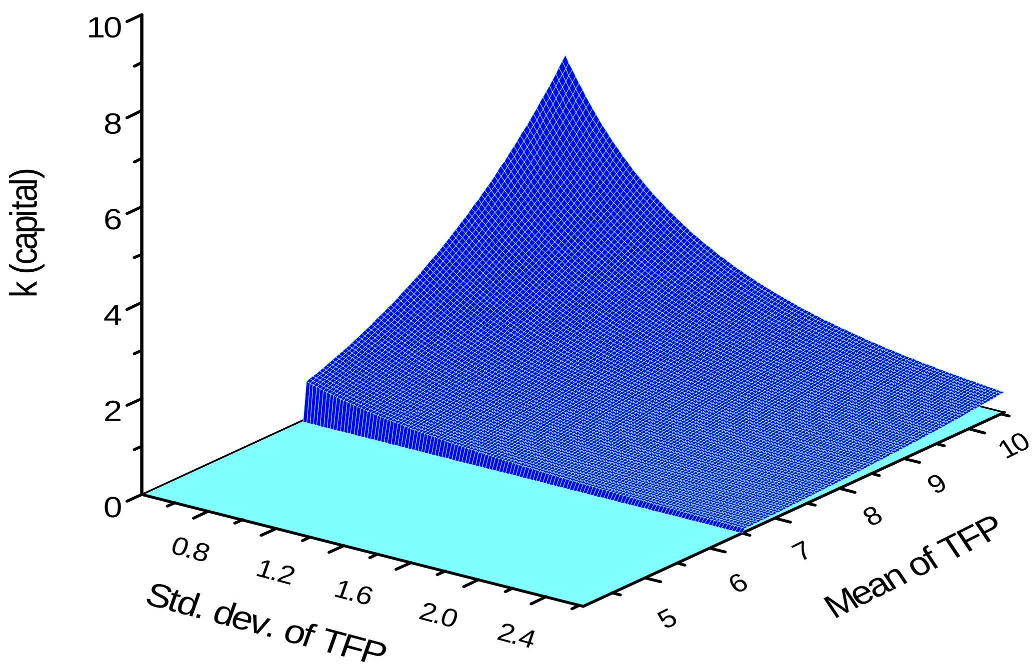

Figure 6: The loan portfolio

Proof. Contained in Appendix 11.1. ■

Take the situation where $r_1 = \widetilde{r}$. Here the intermediary makes zero profits on a loan of any size that it makes to the firm, because the lemma implies that $I(\tau, v) = (r_1 - \widetilde{r})k$. Recall that the dotted straight line in Figure 4 portrays the intermediary's profits when there is no monitoring. It becomes horizontal at the original when $r_1 = \widetilde{r}$. If $r_1 > \widetilde{r}$ unbounded profit opportunities exist for both the firm and intermediary. Both parties' profits are increasing in loan size, k. The dotted straight line in Figure 4 now moves up from the origin. Obviously, an equilibrium cannot exist here.

Corollary 1 *An equilibrium does not exist when $r_1 > \widetilde{r}$ for any project type.*

6.2.1 The Loan Portfolio

Which unmonitored projects will be funded and how much will they get? To answer this question, let $\overline{\theta}_1 \equiv \max\{\theta_1\}$, and assume that this occurs for a unique value of θ_2. Denote the measure of such projects by $\#(\overline{\theta}_1, \theta_2)$, which is presumed to be greater than zero. Now, suppose that the equilibrium wage rate w is set at $w = \omega$, where ω is determined by the equation

$$R(\overline{\theta}_1, \omega) = \widetilde{r}. \tag{24}$$

Observe that if $R(\overline{\theta}_1, \omega) = \widetilde{r}$ for some $\theta_1 = \overline{\theta}_1$, then $R(\theta_1, \omega) < \widetilde{r}$ for all $\theta_1 < \overline{\theta}_1$. Hence, *all* projects with $\theta_1 < \overline{\theta}_1$ will be monitored at the wage ω, if they are funded. Alternatively, suppose that $R(\theta_1, w) = \widetilde{r}$ for some $\theta_1 < \overline{\theta}_1$ and wage rate w. Then, $R(\overline{\theta}_1, w) > \widetilde{r}$. But, by Corollary 1 such an equilibrium cannot exist. Therefore, in equilibrium only one type of project will not be monitored, viz projects of type $(\overline{\theta}_1, \theta_2)$. This can only happen at the equilibrium wage $w = \omega$.

Next, using Lemma 1, when there is *no* monitoring a contract for a type-$(\overline{\theta}_1, \theta_2)$ firm will specify a loan size of $k = v/[\pi_2(r_2 - r_1)]$, where $r_1 = R(\overline{\theta}_1, \omega)$ and $r_2 = R(\theta_2, \omega)$. The firm's rents are linear in loan size since $v = k\pi_2(r_2 - r_1)$. The intermediary's profits are zero for any loan size. Therefore, the firm would like to obtain as big a loan as possible. As a consequence, any equilibrium where type-$(\overline{\theta}_1, \theta_2)$ firms are not monitored will involve the rationing of credit.

7 Stationary Equilibrium

The focus of the analysis will be on stationary equilibria, from here on out. First, the labor-market-clearing condition for the model will be presented. Second, a definition for a stationary equilibrium will be given. Third, it will be demonstrated that a stationary equilibrium for the model exists.

7.1 Labor Market

There are two cases to consider. First, suppose that all loans are monitored. Only firms with $\tau \in \mathcal{A}(w)$ will be producing output. Recall that the economy has one unit of labor in aggregate. The labor-market-clearing condition will then appear as

$$\int_{\mathcal{A}(w)} [\pi_1 l_1(\theta_1, \theta_2) + \pi_2 l_2(\theta_1, \theta_2) + \pi_1 l_{m1}(\theta_1, \theta_2)] dF(\theta_1, \theta_2) = 1, \text{ for } w > \omega. \tag{25}$$

Second, suppose that some firms are credit rationed. When there is credit rationing, the wage rate is given by $w = \omega$. In this case, the aggregate amount of labor hired by type-$(\overline{\theta}_1, \theta_2)$ firms, or $\#(\overline{\theta}_1, \theta_2)[\pi_1 l_1(\overline{\theta}_1, \theta_2) + \pi_2 l_2(\overline{\theta}_1, \theta_2)]$, is given by

$$\begin{aligned} &\#(\overline{\theta}_1, \theta_2)[\pi_1 l_1(\overline{\theta}_1, \theta_2) + \pi_2 l_2(\overline{\theta}_1, \theta_2)] = \\ &1 - \int_{\mathcal{A}(\omega)/\overline{\theta}_1} [\pi_1 l_1(\theta_1, \theta_2) + \pi_2 l_2(\theta_1, \theta_2) + \pi_1 l_{m1}(\theta_1, \theta_2)] dF(\theta_1, \theta_2), \text{ for } w = \omega, \end{aligned} \tag{26}$$

where $\mathcal{A}(\omega)/\overline{\theta}_1$ refers to the active set of monitored loans excluding type-$(\overline{\theta}_1, \theta_2)$ firms. From the firm's problem (P2) it is apparent that $\pi_1 l_1(\overline{\theta}_1, \theta_2) + \pi_2 l_2(\overline{\theta}_1, \theta_2)$ is a function of the size of the loan to a type-$(\overline{\theta}_1, \theta_2)$ firm, $k(\overline{\theta}_1, \theta_2)$. Therefore, when there is credit rationing, condition (26) is implicitly determining the quantity of capital that is rationed out to type-$(\overline{\theta}_1, \theta_2)$ firms. Equivalently, (26) can be thought of as determining the value of a credit-rationed firm by using the relationship $v = k\pi_2(r_2 - r_1)$.

7.2 Definition of a Stationary Equilibrium

It is now time to take stock of the situation so far by presenting a definition of the equilibrium under study. It will be assumed that the economy rests in a stationary state.

Definition 1 *A stationary competitive equilibrium is described by a set of labor allocations, l and l_m, a financial contract, $\{p_1, p_2, p_{12}, p_{21}, k, m_1, m_2\}$, a set of active monitored firms, $\mathcal{A}(w)$ and firm values v, and rental and wage rates, $\widetilde{r}$ and w, such that:*

1. *Firms hire labor, l, so as to maximize their profits in accordance with (P2), given prices and the contract offered by intermediaries.*
2. *The financial intermediary offers a contract, $\{p_1, p_2, p_{12}, p_{21}, k, m_1, m_2\}$, which maximizes its profits in accordance with (P3), given prices and the value of firms. The labor used in monitoring is given by $l_m = (m/z)^\gamma$.*
3. *The set of active monitored firms, $\mathcal{A}(w)$, and their values, v, are specified by the fact that the intermediary makes zero profits on each type of loan, so that (21) and (22) hold. the value for a type-$(\overline{\theta}_1, \theta_2)$ credit-rationed firm, v, is pinned down by (26).*
4. *The wage rate, w, is determined so that $w \geq \omega$ and the labor market clears in line with either (25) or (26). Here ω is defined by (24).*
5. *The rental rate, $\widetilde{r}$, is given by $\widetilde{r} = 1/\beta - 1 + \delta$.*

7.3 Existence

In a stationary equilibrium (with no growth) the interest rate facing consumers will be locked in at $\widehat{r} = 1/\beta - 1$, a fact evident from examining the consumer's Euler equation (1) in a steady state. The cost of capital for the intermediary will then be $\widetilde{r} = 1/\beta - 1 + \delta$. Therefore, demonstrating that a stationary equilibrium exits is equivalent to showing that either there is a $w > \omega$ that solves (25), or a value of $\#(\overline{\theta}_1, \theta_2)[\pi_1 l_1(\overline{\theta}_1, \theta_2) + \pi_2 l_2(\overline{\theta}_1, \theta_2)] \geq 0$ that is consistent with (26) when $w = \omega$. The situation is portrayed in Figure 7, which graphs the demand and supply for labor. Some features of the demand schedule for labor will now be discussed.

The demand for labor is portrayed by the solid line. The properties of this demand schedule are established during the course of the proof for Lemma 8. Above the wage rate ω there is no credit-rationing. Here the aggregate demand for labor is given by the left-hand side of (25), denoted by $\mathbf{L}^d$. In this region ($w > \omega$) the demand schedule is downward sloping. As the wage rate approaches the upper bound that maintains profitability for firms, $\widehat{w} = \max_{\tau \in \mathcal{T}}\{\overline{W}(\tau)\}$, the demand for labor goes to zero. When wages drop to ω the economy enters into the situation where there is credit rationing. Recall that wages cannot fall below the level ω. If they did, then $r_1 = R(\overline{\theta}_1, w) > R(\overline{\theta}_1, \omega) = \widetilde{r}$. This would imply that infinite profits could be reaped from loans to type-$(\overline{\theta}_1, \theta_2)$ firms. Such an equilibrium cannot exist—recall the discussion regarding Corollary 1. Therefore, the demand for labor schedule becomes perfectly elastic at the wage rate, $w = \omega$. Note that when there is credit rationing the demand schedule starts to the left of $\mathbf{L}^d$, evaluated at the point $w = \omega$. To understand why, let $\mathbf{L}^d_{/\overline{\theta}_1}$ represent the aggregate demand for labor when there is no credit rationing and the use of labor for type-$(\overline{\theta}_1, \theta_2)$ projects has been subtracted out.[5] This curve obviously lies to the left of $\mathbf{L}^d$. In a situation with credit rationing the intermediary is indifferent about the size of the loan it makes to a type-$(\overline{\theta}_1, \theta_2)$ firm. Hence, the demand for labor schedule starts at the point $\mathbf{L}^d_{/\overline{\theta}_1}$; i.e., at point showing the demand for labor by all the other projects in the active set excluding the type-$(\overline{\theta}_1, \theta_2)$ ones. Note that there will be two equilibria when the supply schedule, $\mathbf{L}^s$, lies in the range between $\mathbf{L}^d_{/\overline{\theta}_1}$ and $\mathbf{L}^d$ at the wage rate $w = \omega$: the equilibrium with $w > \omega$ will not involve credit rationing, while the other one with $w = \omega$ will. Finally, as will be discussed, the demand for labor shifts rightward with an increase in z. Hence, credit rationing will be more likely at low levels of financial development.

Once a solution for the wage rate is found other variables of interest can be recovered, such as k, m_1/k, p_1, p_2, and v using (10), (17), (12), (13), (22), etc.

Lemma 8 *There exists a stationary equilibrium for the economy.*

Proof. See Appendix 11.1. ∎

[5] I.e., $\mathbf{L}^d_{/\overline{\theta}_1} = \int_{\mathcal{A}(\omega)/\overline{\theta}_1} [\pi_1 l_1(\theta_1, \theta_2) + \pi_2 l_2(\theta_1, \theta_2) + \pi_1 l_{m1}(\theta_1, \theta_2)] dF(\theta_1, \theta_2)$, where $\mathcal{A}(\omega)/\overline{\theta}_1$ refers to the active set of monitored loans expunging type-$(\overline{\theta}_1, \theta_2)$ firms.

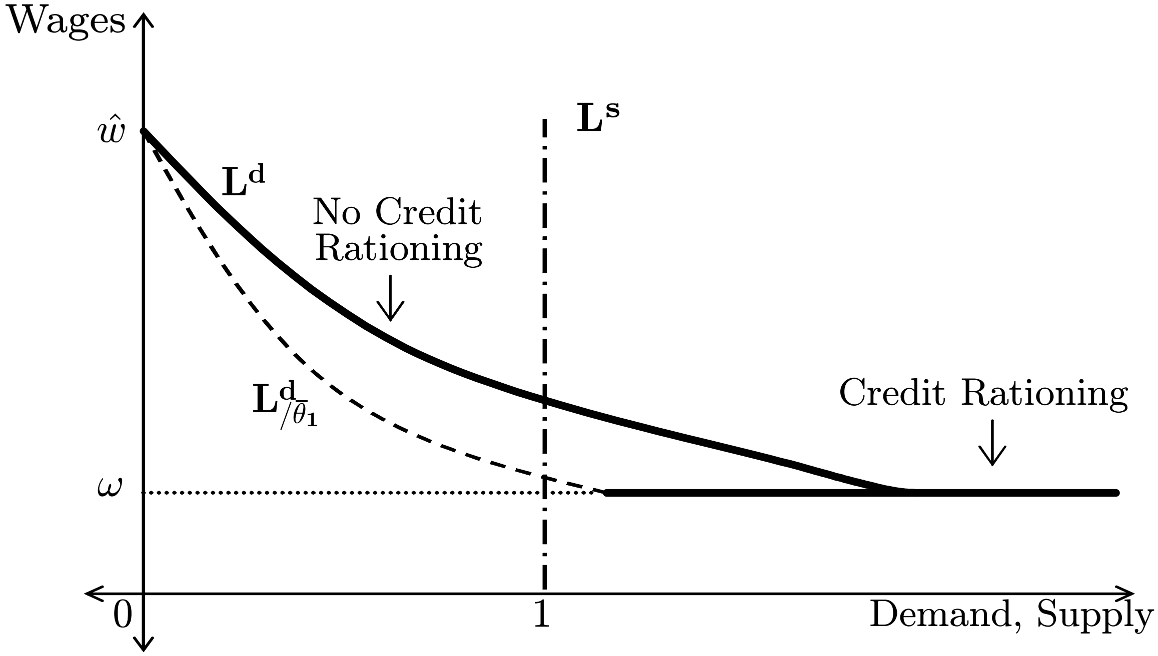

Figure 7: Existence, demand and supply for labor

8 The Impact of Technological Progress on the Economy

The primary goal of the analysis is to understand how technological advance in the financial sector affects the economy. To this end, the impact that technological progress, in either the financial or production sector, has on the portfolio of funded projects will be characterized. To develop some intuition for the economy under study, three special cases will be examined.

8.1 Balanced Growth

In the first special case, technological progress in the financial sector proceeds in balance with the rest of the economy. Specifically, assume that the economy is moving along a balanced growth path where the $\theta_i^{1/\alpha}$'s grow at the common rate $g^{1/\alpha}$ and z grows at rate $g^{1/(1-\alpha)}$. Therefore, $F_{t+1}(\theta_1, \theta_2) = F_t(\theta_1/g, \theta_2/g)$. The salient features of this case are summarized by the proposition below.

Proposition 1 *(Balanced growth) Along a balanced growth path the capital stock, k, wages, w, and rents, v, will grow at rate $g^{1/(1-\alpha)}$ with the cost of capital given by $\widetilde{r} = g^{1/(1-\alpha)}/\beta - 1 + \delta$. The amount of resources devoted to monitoring per unit of capital, m_1/k, will remain constant.*

Proof. Refer to Appendix 11.1. ■

In this situation the financial sector is not becoming more efficient over time, relative to the rest of the economy. The amount of monitoring done per unit of capital invested remains constant over time. Thus, the probability of a firm getting caught by misrepresenting a high level of earnings, $P_{21}(m_1/k)$, is constant over time too. For any particular project type, the spread between the return on capital (net of labor costs) and the interest earned by investors, $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r}$, is fixed over time.

8.2 Efficient Finance

The above case suggests that for technological progress in the financial sector to have an impact it must outpace advance in the rest of the economy. Suppose that this is the case.

Then, one would expect that as monitoring becomes more efficient those projects offering the lowest expected return will be cut.

Proposition 2 *(Technological progress in financial intermediation) Consider z and z' with $z < z'$. Let w and w' be the wage rates associated with z and z', respectively. Then, $\mathcal{A}(w') \sqsubset \mathcal{A}(w)$. Additionally, if $\tau = (\theta_1, \theta_2) \in \mathcal{A}(w) - \mathcal{A}(w')$ and $\tau' = (\theta_1', \theta_2') \in \mathcal{A}(w')$ then $\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha} < \pi_1(\theta_1')^{1/\alpha} + \pi_2(\theta_2')^{1/\alpha}$.*

Proof. See Appendix 11.1. ■

An increase in z makes financial intermediation more efficient, proved that all loans are not credit rationed. For any given wage rate, w, the aggregate demand for labor will increase for two reasons. First, more capital will be lent to each funded project. Second, more labor will also be hired by the intermediary to monitor the project. Since the demand for labor rises, the wage rate must move up to clear the labor market. This increase in wages causes the set of active projects, $\mathcal{A}(w)$, to shrink, with the projects offering the lowest expected return being culled.

An extreme example of the above proposition would be to assume that z grows forever. Then, the financial sector will become infinitely efficient relative to the rest of the economy. This leads to the second special case.

Proposition 3 *(Efficient finance) Suppose $\mathcal{T}$ is a compact and countable subset of R_+^2, with a positive measure of projects for each type, $\tau = (\theta_1, \theta_2)$. Then,*

1. $\lim\limits_{z\to\infty} m_1/z = 0$,
2. $\lim\limits_{z\to\infty} m_1/k = \infty$ *and* $\lim\limits_{z\to\infty} P_{ij}(m_i/k) = 1$ *for* $i, j = 1, 2$,
3. $\lim\limits_{z\to\infty} p_i = r_i k$ *and* $\lim\limits_{z\to\infty} p_{ij} = r_i k$ *for* $i, j = 1, 2$,
4. $\lim\limits_{z\to\infty} A(w) = A^* \equiv \underset{\tau=(\theta_1,\theta_2)\in\mathcal{T}}{\arg\max}[\,\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}]$,
5. $\lim\limits_{z\to\infty} v = 0$,
6. $$\lim_{z\to\infty} w = w^* \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)\{\max_{\tau=(\theta_1,\theta_2)\in\mathcal{T}}[\,\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}]/(1/\beta - 1 + \delta)\}^{\alpha/(1-\alpha)}, \tag{27}$$

7.

$$\lim_{z\to\infty}\int_{\mathcal{A}(w)} kdF = \mathbf{k}^* \equiv [\frac{\alpha}{(1/\beta-1+\delta)}]^{1/(1-\alpha)}\{\max_{\mathcal{T}=(\theta_1,\theta_2)\in\mathcal{T}}[\pi_1(\theta_1)^{1/\alpha}+\pi_2(\theta_2)^{1/\alpha}]\}^{\alpha/(1-\alpha)}. \tag{28}$$

Proof. Refer to Appendix 11.1. ■

As the cost of monitoring borrowers drops, the intermediation sector becomes increasingly efficient. The financial intermediary can then perfectly police loan payments without devoting a significant amount of resources in terms of labor to this activity, as points (1) and (2) in the proposition make clear. Since firms are operating constant-returns-to-scale production technologies no rents will accrue on their activity—see point (5). Firms must pay the full marginal product of capital to the intermediary—point (3). In this world only projects with the highest return are financed, as point (4) states, even though they may be the most risky. In the aggregate any idiosyncratic project risk washes out. Therefore, in the absence of a contracting problem, only the mean return on investment matters. And, with constant-returns-to-scale technologies everything should be directed to the most profitable opportunity. The wage rate, w^*, and aggregate capital stock, $\mathbf{k}^*$, in the efficient economy are determined in standard fashion by the conditions that the marginal product of capital for the most profitable projects must equal the user cost of capital, $1/\beta-1+\delta$ and the fact that the labor market must clear. These two conditions yield (28) and (27). (By comparison, consider the standard deterministic growth model with the production technology $o=\theta k^\alpha l^{1-\alpha}$ and one unit of aggregate labor. Here $w^*\equiv\alpha^{\alpha/(1-\alpha)}(1-\alpha)[\theta^{1/\alpha}/(1/\beta-1+\delta)]^{\alpha/(1-\alpha)}$ and $\mathbf{k}^*\equiv[\alpha/(1/\beta-1+\delta)]^{1/(1-\alpha)}(\theta^{1/\alpha})^{\alpha/(1-\alpha)}$. The differences in the formulae are due to two facts that pertain to the current setting: (i) the best projects from a portfolio $\mathcal{T}$ are chosen; (ii) there is uncertainty in θ.)

8.3 Equilibrium Credit Rationing

Alternatively, technological advance could occur in the production sector and not the financial one. Here, the lack of development in the financial sector will hinder growth in the rest of the economy. Specifically, technological advance in the production sector of the economy will drive up wages. This leads to the costs of monitoring rising. Therefore, less is done. This lack of scrutiny by intermediaries now allows firms with marginal projects offering low-expected returns to receive funding.

Proposition 4 *(Technological progress in production) Suppose all the $\theta_i^{1/\alpha}$'s increase by the factor $g^{1/\alpha}$, holding z fixed. Then, the set of active projects, $\mathcal{A}(w)$, expands with the new projects offering lower expected returns than the old ones.*

Proof. See Appendix 11.1. ■

An extreme case of the above proposition would be to consider a primitive economy where there has been no financial development. In particular, shut down the monitoring technology by letting z become arbitrarily small. As z drops it becomes more expensive to monitor to loans. Capital accumulation in the economy falls and wages drop. Eventually, wages hit the lower bound ω. At this point, it no longer pays to monitor type-$(\overline{\theta}_1, \theta_2)$ loans. As z continues to drop capital is redirected away from the fixed set of monitored loans, $\mathcal{A}(\omega)$, toward credit-rationed projects. Eventually, all loans are made to credit-rationed type-$(\overline{\theta}_1, \theta_2)$ firms. The capital stock in the credit-rationed economy lies below what would obtain in the economy with efficient finance.

Proposition 5 *(Equilibrium Credit Rationing)*

1. $\lim\limits_{z \to 0} \int_{\mathcal{A}(w)} k dF(\tau) = 0,$
2. $\lim\limits_{z \to 0} \mathbf{k} = \underline{\mathbf{k}} \equiv 1/\{(\pi_1(\overline{\theta}_1)^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}) \left[(1-\alpha)/\omega\right]^{1/\alpha}\} < \mathbf{k}^*.$

Proof. For the last time, go to Appendix 11.1. ■

9 Numerical Example

The theoretical mechanisms developed above are now illustrated with a numerical example. Before proceeding a caveat is in order. The example suggests that the impact of financial intermediation on economic activity may be large. In truth, the model would need to be refined further to make such statements with any confidence. Some potential refinements are discussed in the conclusions.

To simulate the model, values must be assigned to its parameters. The parameter values used are presented in Table 1. Capital's share of income, α, is chosen to be 0.30, a very standard number. Likewise, the depreciation rate, δ, is set to 0.06, again a very common number. The chosen value for the discount factor, β, implies that the interest rate earned by savers is 7.5 percent. This is a bit higher than Cooley and Prescott's (1995, p. 19) estimate of 6.9 percent for the real return to capital over the postwar period. Note that the concept used for the capital stock is much narrower here, though; i.e., it is just the stock of business capital. Therefore, hitting the low observed capital-output ratio is harder. Nothing is known about an appropriate choice for the parameters governing monitoring by the intermediary, viz ϵ, ψ, and γ. The selection of a value for ϵ amounts to a normalization (relative to some baseline level of z). The other two parameters are more important, but the features of the model reported below turn out to be robust for a wide number of choices. In the example 1,000 values for firm type, or $\tau \equiv (\theta_1, \theta_1)$, are drawn from the distribution F. Let μ_m be the mean *across firms* of expected total factor productivity (TFP); i.e., $\mu_m = \int(\pi_1\theta_1 + \pi_2\theta_2)dF$. Likewise, μ_v will denote the mean *over firms* of the *logarithm* of the volatility of TFP; i.e., $\mu_v = \int \ln[\pi_1\pi_2(\theta_2 - \theta_1)^2]dF$. In a similar vein, ρ will denote the correlation between the means and (ln) volatilities of *firm-level* TFP, while σ_m^2 and σ_v^2 will denote the variance of these firm-level variables. Assume that these means and (ln) volatilities of firm-level ln(TFP) are distributed according to a bivariate normal, $N(\mu_m, \mu_v, \sigma_m^2, \sigma_v^2, \rho)$. The parameters selected for $\mu_m, \mu_v, \sigma_m^2, \sigma_v^2$, and ρ imply that the standard deviation of TFP across active firms lie in the 0.264 to 0.284 range, at least for values of z that result in the model exhibiting capital-to-output ratios that are in accord with the postwar data. (This latter property is discussed more below.) This is a bit higher than the 0.20 value reported by Foster, Haltiwanger and Syverson (2005, Table 1, p. 38). The slightly higher value used here helps the model fit the data in the cross-country analysis conducted in Section 9.1.

Table 1: Parameter Values

Parameter	*Definition*	*Basis*
$\alpha = 0.3$	Capital's share of income	Standard value
$\delta = 0.06$	Depreciation rate	Standard value
$\beta = 0.93$	Discount factor	Cooley and Prescott (1995)
$\epsilon = 1$	Pr of detection, constant	Normalization
$\psi = 0.95$	Pr of detection, exponent	Arbitrary
$\gamma = 1.57$	Monitoring cost function	
$\mu_m = 5$, $\mu_v = 0$	means	Foster et al. (2005)
$\sigma_m^2 = 2.56$, $\sigma_v^2 = 0.1225$, $\rho = 0.95$	variances and correlation	
$\#(\mathcal{T}) = 1,000$	# of feasible projects	Arbitrary

To illustrate the impact that financial intermediation has on the economy, the model is simulated for various levels of the state of technology in the financial sector, or z. Look at the upper panel of Figure 8, first. Note that the capital-to-output ratio for the economy mimics the increase observed in the data over the postwar period for $z \in [400, 1300]$. In particular, the model hits the observed capital-to-output ratios for 1962 and 2000. To match this increase, z must rise at approximately 3.2% per year. This may be reasonable, if one believes this period coincides with an information technology revolution. Additionally, other factors may also have been at work in causing the economy's capital-to-output ratio to rise, such as declines in the rate of capital income taxation or drops in the prices of capital goods. The goal here is not to decompose the rise in the capital-to-output ratio into its underlying causes. Instead, it is merely to illustrate how improvements in financial intermediation will have this effect.

Table 2 details the results. Here, the aggregate value for a variable is indicated in bold, so that $\mathbf{x} = \int_{\mathcal{A}(w)} x dF$ for $x = m_1$, $w\pi_1(m_1/z)^\gamma$, k, etc. The expected value for x is given by $\mathbf{x} \div \int_{\mathcal{A}(w)} dF$. Monitoring becomes less expensive as z rises. This results in the amount of monitoring per unit of capital rising, as reflected in the larger values for $\mathbf{m}_1 \div \mathbf{k}$. As a consequence, the likelihood of intermediaries detecting fraud increases. The fraction of a

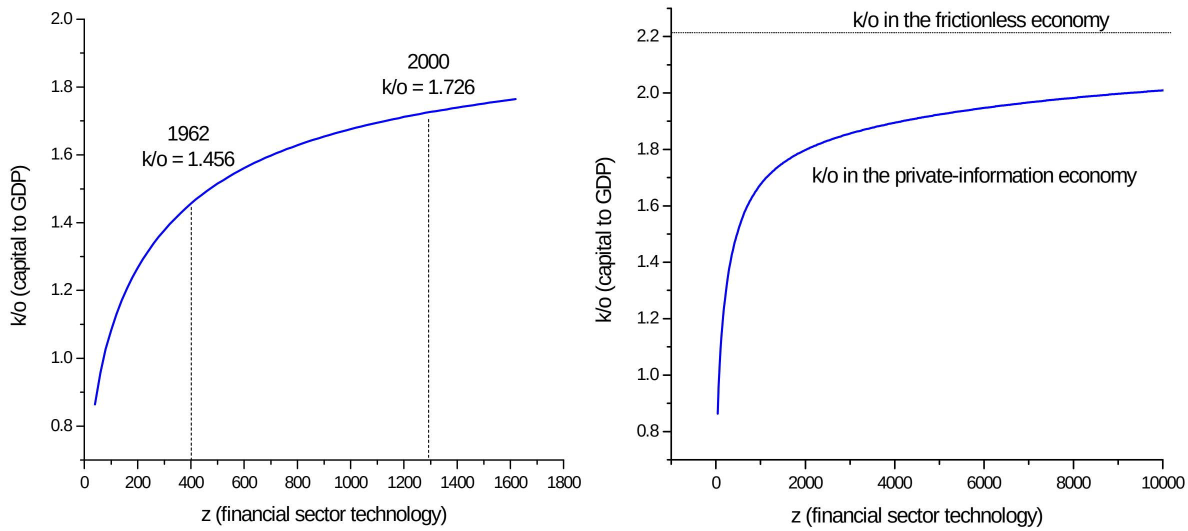

Figure 8: The impact of technological improvement in the financial sector on the capital-to-output ratio

firm's output dissipated in pure rents, $\mathbf{v} \div \mathbf{o}$, declines. This can be seen another way. The internal rate of return, i, earned by a firm on its investment is given by $i = \pi_1 r_1 + \pi_2 r_2 - \delta$. The average internal return earned by firms, weighted by their level of investment, will then be defined by $\mathbf{i} \equiv \int_{\mathcal{A}(w)} kidF \div [\mathbf{k} \times \int_{\mathcal{A}(w)} dF]$. Likewise, denote the average rate of return earned by the intermediary on its lending activity by $\widetilde{\mathbf{i}} \equiv \int_{\mathcal{A}(w)} (\pi_1 p_1 + \pi_2 p_2 - \delta k) dF \div [\mathbf{k} \times \int_{\mathcal{A}(w)} dF]$. The gap between these two returns, $\mathbf{e} \equiv \mathbf{i} - \widetilde{\mathbf{i}}$, measures the average excess return earned by firms due to rents. This excess return is squeezed as rents shrink. In similar fashion, the average spread between the rates of return that intermediaries and savers earn, $\mathbf{s} \equiv \widetilde{\mathbf{i}} - \widehat{r}$, reflects the costs of intermediation incurred by the necessity to monitor borrowers. This interest rate spread declines as the costs of intermediation fall due to technological progress in information production. Rousseau (1998, Figure 4) presents evidence suggesting that financial innovation reduced loan-deposit spreads in the U.S. between 1872 to 1929.

As rents get squeezed, firms offering the lowest expected return are culled. In particular, the number of active firms, $\int_{\mathcal{A}(w)} dF$, is reduced. This results in the average level of expected TFP across firms, $(\boldsymbol{\pi}_1 \boldsymbol{\theta}_1 + \boldsymbol{\pi}_2 \boldsymbol{\theta}_2) \div \int_{\mathcal{A}(w)} dF$, rising. On this, Levine (2005, Table 4) reports that the (exogenous component of) financial development is associated empirically with improved productivity. A rise in the probability of detecting fraud relaxes the incentive constraint (8), and makes it easier to lend more capital to firms. The result is an increase in the amount of capital invested per firm, as reflected by an upward movement in the economy's capital-to-output ratio, $\mathbf{k} \div \mathbf{o}$. Denote the levels of capital and output that would obtain in the first-best economy by $\mathbf{k}^*$ and $\mathbf{o}^*$. As can be seen, capital and output steadily rise, relative to their first-best outcome, as z moves up. Note finance is important in the model.

TABLE 2: IMPACT OF TECHNOLOGICAL PROGRESS IN THE FINANCIAL SECTOR

Technology, z	$z = 40$	$z = 400$	$z = 1300$
Monitoring-to-capital, $\mathbf{m}_1 \div \mathbf{k}$	1.818	3.312	5.363
Monitoring cost to output, $\mathbf{w}\boldsymbol{\pi}_1(\mathbf{m}_1/\mathbf{z})^{\gamma} \div \mathbf{o}$	0.057	0.032	0.021
Monitoring labor share, $\mathbf{l}_i \div 1$	0.075	0.044	0.029
Pr of detecting fraud, $\mathbf{P}_{12} \div \int_{\mathcal{A}(w)} dF$	0.416	0.673	0.791
Rents to output, $\mathbf{v} \div \mathbf{o}$	0.126	0.071	0.046
Measure of active firms, $\int_{\mathcal{A}(w)} dF$	109	14	5
TFP, $(\boldsymbol{\pi}_1\boldsymbol{\theta}_1 + \boldsymbol{\pi}_2\boldsymbol{\theta}_2) \div \int_{\mathcal{A}(w)} dF$	7.491	8.699	9.134
Std ln(TFP), $\sqrt{\boldsymbol{\pi}_1\boldsymbol{\pi}_2}(\ln\boldsymbol{\theta}_2 - \ln\boldsymbol{\theta}_1) \div \int_{\mathcal{A}(w)} dF$	0.247	0.264	0.284
Internal return (weighted), $\mathbf{i}$	0.287	0.146	0.114
Lending rate, $\widetilde{\mathbf{i}}$	0.141	0.097	0.087
Excess return, $\mathbf{e} = \mathbf{i} - \widetilde{\mathbf{i}}$	0.146	0.049	0.027
Interest rate spread, $\mathbf{s} = \widetilde{\mathbf{i}} - \widehat{r}$	0.066	0.022	0.012
Return to savers, $\widehat{r} = 1/\beta - 1$	0.075	0.075	0.075
Capital-to-output ratio, $\mathbf{k} \div \mathbf{o}$	0.864	1.456	1.726
Capital relative to first best, $\mathbf{k} \div \mathbf{k}^*$	0.218	0.511	0.670
Output relative to first best, $\mathbf{o} \div \mathbf{o}^*$	0.561	0.778	0.860

The upshot of all this is that as z moves up the financial system becomes more efficient.

Figures 8 and 9 summarize the situation well. The lower panel of Figure 8 illustrates how the model economy's capital-to-output ratio monotonically increases with z. As z becomes large it approaches the capital-to-output ratio that would occur in the economy without informational frictions. The model is consistent with the upward trend in the capital-to-output ratio displayed in Figure 2. Figure 9 shows how at low levels of z many inefficient projects are funded. As z moves up inefficient projects are culled and capital is redirected toward projects that have higher expected returns. The importance of such reallocation effects for cross-country income differences has been noted by Restuccia and Rogerson (2004), although they emphasize inefficiencies due to policy distortions. To paraphrase Goldsmith (1969), technological progress in the financial sector promotes economic development by facilitating the migration of funds to the best user, i.e., to the place in the economy where the funds will earn the highest social return. Since all investment funds are borrowed in the model economy, the ratio of business debt to GDP rises, too. Likewise, the value of firms to GDP increases as well. Thus, the model is congruent, in a qualitative sense, with the first two facts presented in Figure 1.

9.1 Cross-Country Evidence

Ever since Goldsmith (1969), economists have been interested in the cross-country relationship between financial structure and economic development. Levine (2005) provides an up-to-date review of this evidence. An implication of the current model is that as the state of technology in the intermediation sector advances the spread between borrowing and lending rates in an economy will shrink, while its capital-to-output ratio and level of aggregate output increases. Figure 10 plots the relationship between interest-rate spreads and output for a sample of 49 countries.[6] As can be seen, there is a negative association. Capital-to-output ratios and output are positively related, in a sample of 48 countries.

For the cross-country analysis assume that production in an economy is now undertaken

[6] Some of the data in this section is taken from Beck, Demirguc-Kunt and Levine (2000, 2001), who kindly make their data publically available.

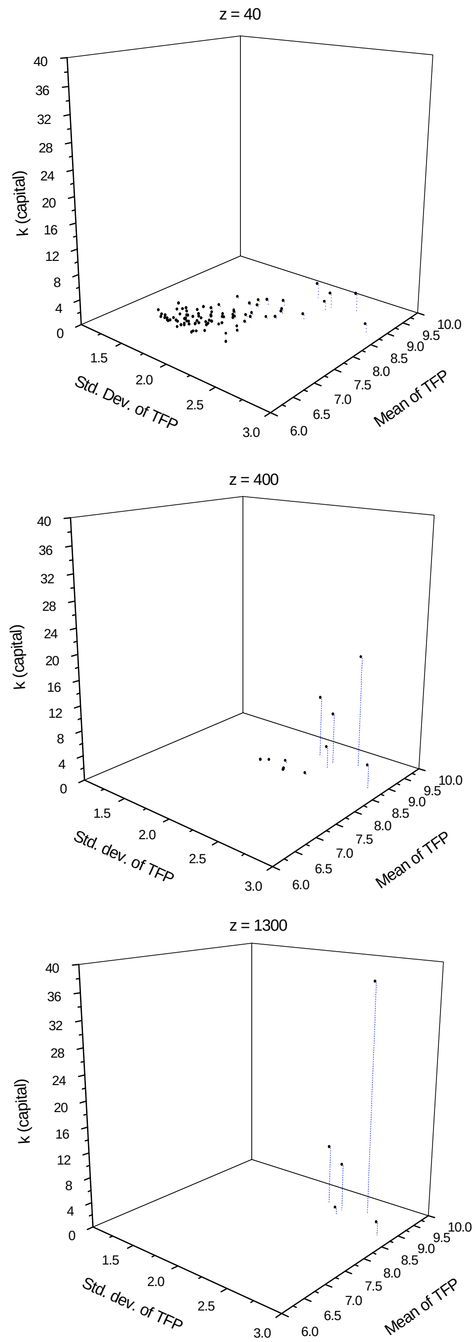

Figure 9: The impact of technological progress in the financial sector on investment in firms

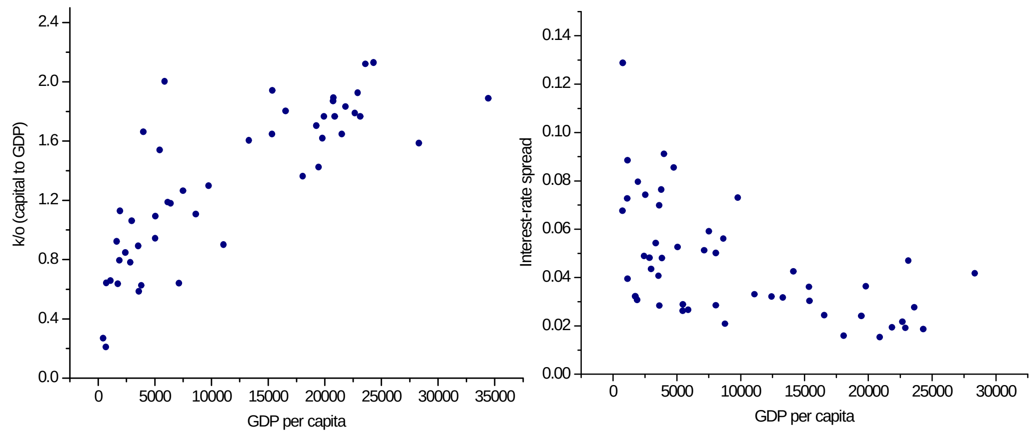

Figure 10: The cross-country relationship between per-capita GDP, on the one hand, and interest-rate spreads and capital-output ratios on the other

in line with the production function

$$o = x\theta k^{\alpha} l^{1-\alpha},$$

where x is a country-specific productivity factor. The model provides a mapping between a country's level of output, $\mathbf{o}$, and its capital/output ratio, $\mathbf{k}/\mathbf{o}$, on the one hand, and the state of technology in its production and financial sectors, x and z, on the other. Represent this mapping by $(\mathbf{o}, \mathbf{k}/\mathbf{o}) = M(x, z)$. Now, while the state of a country's financial technology is unobservable directly, this mapping can be used to make an inference about (x, z), given an observation on $(\mathbf{o}, \mathbf{k}/\mathbf{o})$, by using the relationship $(x, z) = M^{-1}(\mathbf{o}, \mathbf{k}/\mathbf{o})$. This is done for a sample of 40 countries, using the parameter values listed in Table 1. The results are reported in Table 5 in in Appendix 11.2. By construction the model explains all the variation in output and capital/output ratios across countries. Still, one can ask how well the model explains cross-country differences in interest-rate spreads, $\mathbf{s}$, and aggregate productivity levels, TFP. Additionally, one could ask how well the measure of the state of technology in the financial sector that is backed out using the model correlates with independent measures of financial intermediation. Here, take the ratio of private credit by deposit banks and other financial institutions to GDP as a measure of financial intermediation, as reported by Beck et al. (2001). (Other measures produce similar results but reduce the sample size too much.)

Table 3 reports the findings. Take the results for the benchmark calibration first; i.e., the first column of numbers. As can be seen, the interest-rate spreads predicted by the model are positively associated with those in the data. The correlation is reasonably large. The correlation between the imputed state of technology in the financial sector and the independent measure of financial intermediation is quite high. Interestingly, Finland and Peru both have a capital-to-output ratio of about 1.6. The model predicts that the former's z is 594, compared with 124 for the later—again, see Table 5 in Appendix 11.2. Why? Finland has a much higher level of income per worker and hence TFP than does Peru ($40,603 versus $10,200). Therefore, given the higher wages, monitoring will be more expensive in Finland. To give the same capital/output ratio, efficiency in Finland's financial sector must be higher. Also interestingly, the three countries with the lowest z's, Bolivia, India and Sri Lanka, have credit-rationing equilibriums where some loans are not monitored. That these two correlations aren't perfect, should be expected. There are other factors, such as the big differences in public policies discussed in Parente and Prescott (2000), which may explain a large part of the cross-country differences in capital/output ratios. Additionally, there is a lot of noise in these numbers given the manner of their construction—see Appendix 11.2. The model's predictions for aggregate total factor productivity display a high correlation. Finally, the last columns report the results of some sensitivity analysis. Here ψ, γ, and ρ are changed, one at a time, away from their benchmark value. The results do not change much with moderate departures away from the benchmark parameterization.

Table 3: Cross-Country Evidence

Parameters values				
	Benchmark	Sensitivity analysis		
		ψ	γ	ρ
		(change to)		
ψ	0.95	0.75	0.95	0.95
γ	1.57	1.57	1.4	1.57
ρ	0.95	0.95	0.95	0.75
	Correlation(model, data)			
interest-rate spread	0.461	0.449	0.418	0.683
TFP	0.971	0.970	0.971	0.959
financial intermediation	0.446	0.426	0.447	0.418

9.1.1 How Much Does Financial Development Matter?

It is now possible to gauge how important efficiency in the financial sector is for economic development, at least in the model. To this end, note that the best financial and industrial practices in the world are given by $\overline{x} = \max\{x_i\}$ and $\overline{z} = \max\{z_i\}$, respectively. Represent country i's output, as a function of the efficiency in its industrial and financial sectors, by $\mathbf{o}_i = \mathbf{O}(x_i, z_i)$. If country i could somehow adopt the best financial practice in the world it would produce $\mathbf{O}(x_i, \overline{z})$. Similarly, if country i used the best practice in both sectors it would attain the output level $\mathbf{O}(\overline{x}, \overline{z})$. The shortfall in output from the inability to attain best practice is $\mathbf{O}(\overline{x}, \overline{z}) - \mathbf{O}(x_i, z_i)$.

The percentage gain in output for country i by moving to best financial practice is given by $100 \times [\ln \mathbf{O}(x_i, \overline{z}) - \ln \mathbf{O}(x_i, z_i)]$. The results for this experiment are plotted in Figure 11. As can be seen, the gains are quite sizeable. On average a country could increase its GDP by 11%. The country with the worst financial system, Sri Lanka, would experience a 23% rise in output. While sizeable, these gains in GDP are small relative to the increase that is needed to move a country onto the frontier for income, $\mathbf{O}(\overline{x}, \overline{z})$. The percentage of the

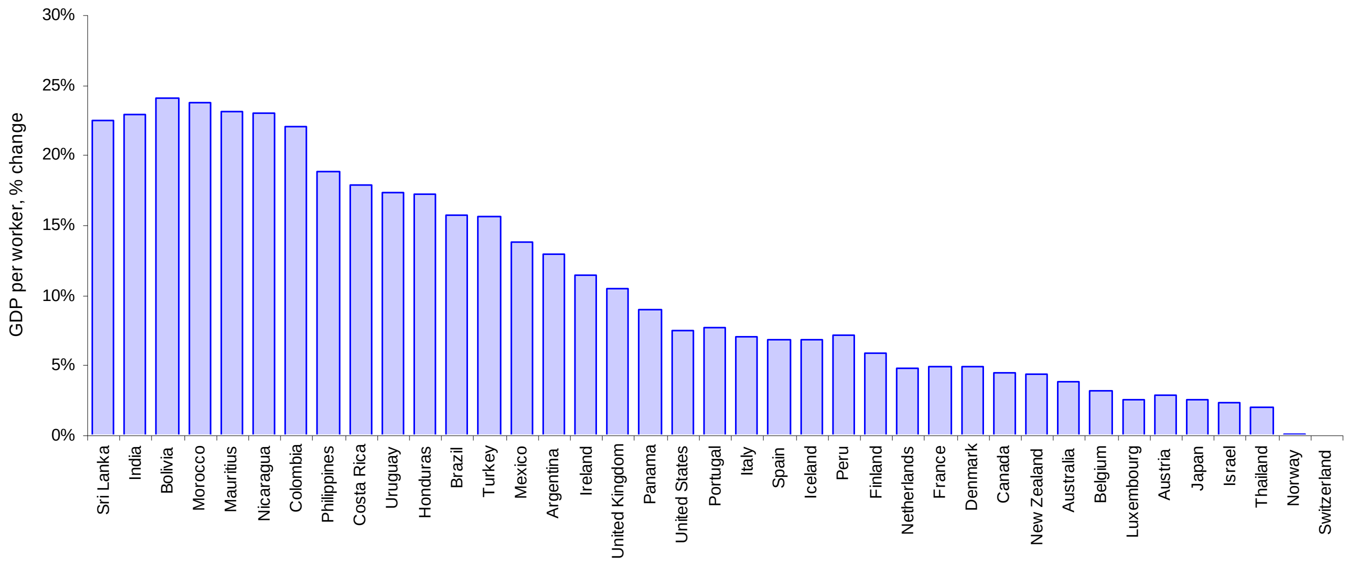

Figure 11: The impact of a move to financial best practice on GDP per worker

gap that is closed by a movement to best financial practice is measured by $100 \times [\mathbf{O}(x_i, \overline{z}) - \mathbf{O}(x_i, z_i)]/[\mathbf{O}(\overline{x}, \overline{z}) - \mathbf{O}(x_i, z_i)]$. Figure 12 plots the reduction in this gap for the countries in the sample. The average reduction is this gap is only 14%. For most countries the shortfall in output is accounted for by a low level of total factor productivity in the non-financial sector.

Therefore, the importance of financial intermediation for economic development depends on how you look at it. World output would rise by 21% by moving all countries to the best financial practice—see Table 4. This is a sizeable gain. Still, it would only close 9% of the gap between actual and potential world output. Dispersion in cross-country output would fall by 8 percentage points from 77% to 64%. Financial development explains about 19% of the cross-country dispersion in output by this metric.

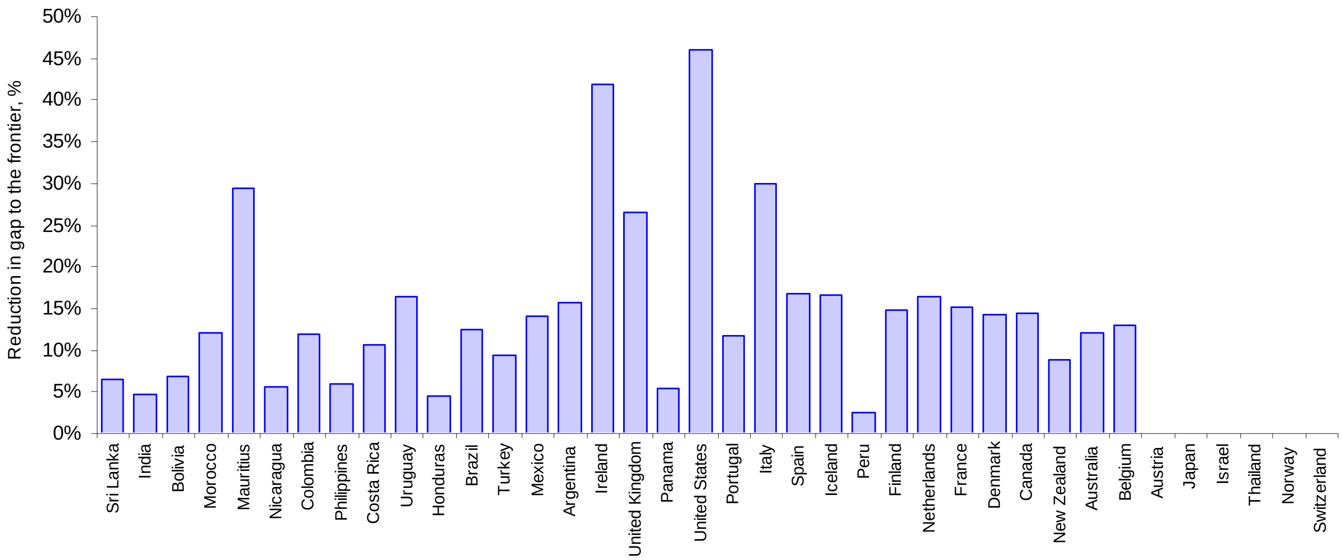

Figure 12: The impact of a move to financial best practice on the gap in GDP per worker

Table 4: World-Wide move to financial best practice, $\overline{z}$

Increase in world output (per worker)	21%
Reduction in gap between actual and potential world output	9%
Fall in dispersion of output across countries, std(ln($\mathbf{o}_i$))	13% = (77% -64%)

10 Conclusions

What is the link between state of financial intermediation and economic development? This question is explored here by embedding a costly-state verification model into the standard neoclassical growth paradigm. The model had several key features. There is a distribution of firm types offering different combinations of risk and return. Firms raise capital from financial intermediaries. Intermediaries borrow from savers. The ex post return on a project is private information. The terms of a loan are determined by an optimal incentive-compatible contracting scheme between the firm and intermediary. The firm repays the intermediary an amount that is contingent upon the return it reports. An intermediary can audit the reported return. The likelihood of a successful audit depends upon both the amount of resources devoted to monitoring and the technological state of the auditing technology. The inability to audit perfectly, and therefore the necessity to rely on incentive-compatible contracts, implies that firms can earn rents. As a result, deserving firms are underfunded and undeserving ones are overfunded. As the efficacy of auditing increases due to technological progress in the financial sector the efficiency of financial intermediation improves. This is manifested in several ways. First, rents are squeezed. This is reflected in a narrowing of the wedge between a firm's internal rate of return on investment and the effective lending rate at which it borrows from the intermediary. Second, as the costs of monitoring fall the spread between the return received by the intermediary on monitored loans and what it pays to savers shrinks. Third, over time unproductive projects are winnowed out of the intermediary's loan portfolio and capital is redirected toward the more profitable ones. This is reflected by both an increase in the economy's level of TFP and a rise in its capital-to-output ratio. A numerical example suggests that the mechanism outlined could have quantitative significance.

Extensions of the above framework are easy to envision. The most natural one would be to allow for long-term contracts. Gertler (1992) is an early example of this approach. More recently, Smith and Wang (2006) embed a long-term contracting framework into a model of financial intermediation. Clementi and Hopenhayn (2006) and Quadrini (2004) have examined the properties of dynamic contracting for firm finance in worlds with private information and obtain relatively simple solutions. Whether or not a simple solution to the above costly-state verification contracting problem exists is an open question.[7] If the framework does not admit a tractable theoretical solution, the work could proceed numerically. It would be nice to combine this analysis with a model of entry and exit by firms, along the lines of Hopenhayn and Rogerson (1992). Here, firms face diminishing returns to scale in production and consequently earn profits. These profits are whittled away, ex ante, by free entry into production, subject to an entry cost. On the one hand, one might expect that long-term contracts would mitigate the informational problem. On the other hand, profits will now be larger, ceteris paribus, due to diminishing returns in production and the fact that they will be capitalized over a firm's lifetime. Therefore, free entry combined with the opportunity to capture larger profits, may lead to more inefficient firms being financed in equilibrium. How all of this will play out, is anyone's guess.

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[7] In a different context, Wang (2005) analyzes theoretically a dynamic version of the standard costly-state verification framework. So, there is some hope that theoretical results can be obtained.

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11 Appendix

11.1 Theory

Proof for Lemma 1. First, substitute the promise-keeping constraint (9) into the objective function to rewrite it as

$$(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})k - \pi_1 w(m_1/z)^\gamma - \pi_2 w(m_2/z)^\gamma - v.$$

Next, it is almost trivial to see that optimality will dictate that $p_{12} = r_1 k$ and $p_{21} = r_2 k$, since this costlessly relaxes the incentive constraints (7) and (8). Next, drop the incentive constraint (7) from problem (P3) to obtain the auxiliary problem now displayed:

$$\widetilde{I}(\tau, v) \equiv \max_{p_1, p_2, m_1, k} \{(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})k - \pi_1 w(m_1/z)^\gamma - v\}, \tag{P6}$$

subject to

$$p_1 \le r_1 k, \tag{29}$$

$$p_2 \le r_2 k, \tag{30}$$

$$[1 - P_{21}(m_1/k)](r_2 k - p_1) \le r_2 k - p_2, \tag{31}$$

and

$$\pi_1(r_1 k - p_1) + \pi_2(r_2 k - p_2) = v. \tag{32}$$

The strategy will be to solve problem (P6) first. Then, it will be shown that (P3) and (P6) are equivalent. Problem (P6) will now be solved. To this end, note the following facts:

1. The incentive constraint (31) is binding. To see why, suppose not. Then, reduce m_1 to increase the objective.

2. The constraint (30) is not binding. Assume, to the contrary, it is. Then, (31) is violated. This transpires because the right-hand side is zero. Yet, the left-hand side is positive, given that $p_1 \le r_1 k < r_2 k$, so that $[1 - P_{21}(m_1/k)](r_2 k - p_1) > 0$.

3. The constraint (29) is binding. Again, suppose not, so that $p_1 < r_1 k$. It will be shown that exists a profitable feasible deviation from any contract where this constraint is slack. Specifically, consider increasing k very slightly by $dk > 0$ while adjusting p_1 and p_2 in the following manner so that (31) and (32) still hold. Also, hold m_1 fixed. The implied perturbations for p_1 and p_2 are given by

$$\begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} = \begin{bmatrix} -[1 - P_{21}(m_1/k)] & 1 \\ \pi_1 & \pi_2 \end{bmatrix}^{-1} \times \begin{bmatrix} r_2 P_{21}(m_1/k) - (r_2 k - p_1)(m_1/k^2) dP_{21}/d(m_1/k) \\ \pi_1 r_1 + \pi_2 r_2 \end{bmatrix} dk.$$

Note that such an increase in k will raise the objective function.

It will now be demonstrated that the optimization problems (P3) and (P6) are equivalent. First, note that $\widetilde{I}(\tau, v) \geq I(\tau, v)$, because problem (P6) does not impose the constraint (7). It will now be established that $\widetilde{I}(\tau, v) \leq I(\tau, v)$. Consider a solution to problem (P6). It will be shown that this solution is feasible for (P3). On this, note that Fact 1 implies that

$$r_2 k - p_2 = [1 - P_{21}(m_1/k)](r_2 k - p_1) \leq r_2 k - p_1,$$

so that

$$p_1 \leq p_2.$$

Now, set $m_2 = 0$ in (P3), which is feasible but not necessarily optimal. Then, constraint (7) becomes $p_1 \leq p_2$, which is satisfied by the solution to (P6). Therefore, $\widetilde{I}(\tau, v) \leq I(\tau, v)$.

Last, with the above facts in hand, recast the optimization problem as

$$I(\tau, v) \equiv \max_{p_2, m_1, k} \{(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})k - \pi_1 w (m_1/z)^\gamma - v\},$$

subject to

$$r_2 k - p_2 = (r_2 - r_1)k[1 - P_{21}(m_1/k)],$$

and

$$r_2 k - p_2 = v/\pi_2.$$

The above two constraints collapse in the single constraint (10), by eliminating $r_2k - p_2$, that involves just m_1 and k. The problem then appears as (P4). ■

Proof for Lemma 2. Suppose that the solution dictates that $m_1/k \leq 1/\epsilon$. Then, from (P4) it is clear that the optimal solution will dictate that $m_1/k = 0$. This transpires because $P_{21}(m_1/k) = 0$ for all $m_1/k \leq 1/\epsilon$, yet monitoring costs are positive for all $m_1/k > 0$. Next, by substituting (10) into (P4) it is easy to deduce that the intermediary's profit function can be written as

$$\begin{aligned} I(\tau, v) &= [\frac{\pi_1 r_1 + \pi_2 r_2 - \widetilde{r}}{\pi_2(r_2 - r_1)} - 1]v = \frac{r_1 - \widetilde{r}}{\pi_2(r_2 - r_1)}v, \quad \text{when } m_1/k \leq 1/\epsilon, \qquad (33) \\ &\lesseqgtr 0 \text{ as } r_1 \lesseqgtr \widetilde{r}. \end{aligned}$$

Therefore profits are negative if $r_1 < \widetilde{r}$ and $v > 0$. Hence, a contract will not be offered when $m_1/k \leq 1/\epsilon$. (The above equation gives the dashed straight line plotted in Figure 4.) ■

Proof for Lemma 3. By inspecting (17), the first two results follow immediately. The last result follows from evaluating (17) at $m_1/k = 1/\epsilon$ and solving for the threshold v, or $\overline{v}$. Above this level no monitoring will occur since the interior solution for m_1/k is decreasing in v. ■

Proof for Lemma 4. The first result is obvious. For the second point, differentiate with respect to v. Note, that the exponent on the first v will be $\gamma/(\psi\gamma + \gamma - \psi) - 1 < 0$. Hence, this term will go to ∞ as v becomes small. Simple differentiation establishes the third point. By employing the line of reasoning used in the proof of Lemma 2, it is easy to deduce that

$$\lim_{v \uparrow \overline{V}(\tau)} I(\tau, v) = [\frac{\pi_1 r_1 + \pi_2 r_2 - \widetilde{r}}{\pi_2(r_2 - r_1)} - 1]\overline{V}(\tau) - \frac{\pi_1 w}{z^\gamma}[\frac{\overline{V}(\tau)}{\pi_2(r_2 - r_1)}]^\gamma (\frac{1}{\epsilon})^\gamma < 0. \qquad ■$$

Proof for Lemma 5. *Necessity*: From (17) it is clear that an interior solution cannot exist when $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} < 0$ and $v > 0$, for any $\tau \in \mathcal{T}$. If $v = 0$ then $k = 0$ by (10), which implies no funding. When $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} = 0$ then $\overline{v} = 0$ for finite θ's. Again, an interior solution will not exist, because $k = 0$ when $v = 0$.

Sufficiency: Suppose that $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} > 0$ for some $\tau \in \mathcal{T}$. By equation (19) it is clear that there will exist a $v > 0$ such that $I(\tau, v) > 0$, since $q > 0$ when $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} > 0$. The issue is whether or not there will be an interior solution associated with this. By Lemma 4 it is clear that there will exist a $V(\tau) < \overline{V}(\tau)$ such that $I(\tau, V(\tau)) = 0$; i.e., for $v = V(\tau)$ the solution to the contracting problem will be interior. ■

Proof for Lemma 7. Suppose $r_1 \geq \widetilde{r}$. By condition (33) when there is no monitoring $I(\tau, v) = \{(r_1 - \widetilde{r})/[\pi_2(r_2 - r_1)]\}v$ so that the intermediary makes nonnegative profits. From Lemma (1) it is easy to deduce that when there is no monitoring a contract will specify a loan size of $k = v/[\pi_2(r_2 - r_1)]$. Will the intermediary monitor the project? Note that when there is an interior solution to the contracting problem, k and v will be determined in accordance with (22) and (23). In particular, both k and v will be bounded. The firm will earn rents in amount $v = \pi_2(r_2 - r_1)[1 - P_{12}(m_1/k)]k$ and the intermediary makes zero profits. Now, without monitoring the intermediary could offer the firm a loan of the same size k. The intermediary earns nonnegative profits with no monitoring, but the return for the firm will be higher because $v = \pi_2(r_2 - r_1)k > \pi_2(r_2 - r_1)[1 - P_{12}(m_1/k)]k$. If the intermediary tried to offer a contract with monitoring that offered this higher level of v (or an even larger one) it would earn negative profits. ■

Proof for Lemma 8. To prove Lemma 8, three facts about aggregate labor demand when all loans are monitored, or the left-hand side of (25), will be established. First, labor demand is a continuous and decreasing function in wages, w. Second, as $w \to \widehat{w} \equiv \max_{\tau \in \mathcal{T}}\{\overline{W}(\tau)\}$ the left-hand side of (25) converges to 0. Third, as $w \to 0$ labor demand approaches ∞. Therefore, by the intermediate value theorem there will exist a (single) value of w that sets the left-hand side to 1, or labor supply.

If the value of w that sets the left-hand side of (25) equal to one occurs below ω, then the equilibrium wage is ω. In this situation there is credit rationing and the demand for labor by unmonitored type-$(\overline{\theta}_1, \theta_2)$ projects is determined residually by (26). When $w = \omega$ the intermediary doesn't care about the size of the loan that it makes to type-$(\overline{\theta}_1, \theta_2)$ firms. Therefore, the amount of labor hired by these firms can be anything depending on the size of the loans that are made in equilibrium. Consequently, the aggregate demand for labor with credit rationing is a horizontal line emanating from the point $w = \omega$ that starts at $\int_{\mathcal{A}(\omega)/\overline{\theta}_1}[\pi_1 l_1(\theta_1,\theta_2) + \pi_2 l_2(\theta_1,\theta_2) + \pi_1 l_{m1}(\theta_1,\theta_2)]dF(\theta_1,\theta_2)$, where $\mathcal{A}(\omega)/\overline{\theta}_1$ is the active set of projects with type-$(\overline{\theta}_1,\theta_2)$ firms purged. This starting point must lie to the left of aggregate demand for labor when all loans are monitored, evaluated at the point $w = \omega$, since $\mathcal{A}(\omega)/\overline{\theta}_1 \sqsubset \mathcal{A}(\omega)$. Figure 7 illustrates the situation. As can be seen, if when $w = \omega$ it happens that

$$\begin{aligned}
\mathbf{L}^d_{/\overline{\theta}_1} &= \int_{\mathcal{A}(\omega)/\overline{\theta}_1}[\pi_1 l_1(\theta_1,\theta_2) + \pi_2 l_2(\theta_1,\theta_2) + \pi_1 l_{m1}(\theta_1,\theta_2)]dF(\theta_1,\theta_2) < 1 \\
&< \int_{\mathcal{A}(\omega)}[\pi_1 l_1(\theta_1,\theta_2) + \pi_2 l_2(\theta_1,\theta_2) + \pi_1 l_{m1}(\theta_1,\theta_2)]dF(\theta_1,\theta_2) = \mathbf{L}^d,
\end{aligned}$$

then two equilibria will exist. In the first equilibrium $w = \omega$ and there is credit rationing. Here, the aggregate amount of labor used by type-$(\overline{\theta}_1,\theta_2)$ firms will be determined residually by (26). In the second there is no credit rationing. The wage rate $w > \omega$ clears the labor market in line with (25).

The above three facts about the aggregate demand for labor when all loans are monitored will now be established. Before proceeding, use (2) to substitute out for l_1 and l_2 in equation (25) to obtain

$$\int_{\mathcal{A}(w)}\{(\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})\left[\frac{(1-\alpha)}{w}\right]^{\frac{1}{\alpha}}k + \pi_1(m_1/z)^{\gamma}\}dF = 1. \tag{34}$$

This equation will determine the equilibrium wage rate, w, when $w > \omega$.

To begin with, let $L^d(w;\tau,z)$ represent the demand for labor by both the firm and intermediary for an active project of type τ. The function $L^d(w;\tau,z)$ is defined by

$$L^d(w;\tau,z) = (\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})\left[\frac{(1-\alpha)}{w}\right]^{\frac{1}{\alpha}}k + \pi_1(m_1/z)^{\gamma}. \tag{35}$$

The first term in $L^d(w;\tau,z)$ represents the demand for labor by a type-τ firm. It will be decreasing in w if k is. On this, rewrite equation (23) as

$$k = \Upsilon(\pi_1\Theta_1 + \pi_2\Theta_2 - \widetilde{r}w^{(1-\alpha)/\alpha})^{(\gamma+\psi)/(\psi\gamma-\psi)}[\frac{\epsilon^{\psi}}{\pi_2(\Theta_2-\Theta_1)}]^{\gamma/(\psi\gamma-\psi)}z^{\gamma/(\gamma-1)}w^{-1/[\alpha(\gamma-1)]}, \tag{36}$$

where $\Theta_i \equiv \alpha(1-\alpha)^{(1-\alpha)/\alpha}\theta_i^{1/\alpha}$ and $\Upsilon \equiv (\psi\gamma+\gamma-\psi)^{\gamma/(\psi\gamma-\psi)}(\frac{1}{\psi})^{-\psi/(\psi\gamma-\psi)}(\frac{1}{\psi\gamma+\gamma})^{(\gamma+\psi)/(\psi\gamma-\psi)}$. Note the following things about this solution for k: (i) The level of investment in a firm, k, is continuously decreasing in w; (ii) $k \to 0$ as $w \to \overline{w} = \overline{W}(\tau) = [(\pi_1\Theta_1 + \pi_2\Theta_2)/\widetilde{r}]^{\alpha/(1-\alpha)}$ (when z is finite); (iii) $k \to \infty$ as $w \to 0$; (iv) k is increasing in z. Therefore, it is easy to see that the demand for labor by the firm shares these properties.

Now, switch attention to the second term in $L^d(w;\tau,z)$. This represents the demand for labor by the intermediary for monitoring a project of type τ. After a little work on (16), while making use of (22), an expression for m_1/k can be obtained:

$$m_1/k = (\frac{\psi\gamma+\gamma-\psi}{\psi\gamma+\gamma})^{-1/\psi}q^{-1/\psi}. \tag{37}$$

Note that $m_1/z = m_1/k \times k \div z$. Therefore, equations (36) and (37), in conjunction with the definition for q, imply

$$m_1/z = \Delta(\pi_1\Theta_1 + \pi_2\Theta_2 - \widetilde{r}w^{(1-\alpha)/\alpha})^{(\psi+1)/(\psi\gamma-\psi)}w^{-1/[\alpha(\gamma-1)]}z^{1/(\gamma-1)}, \tag{38}$$

where $\Delta \equiv \Upsilon[(\psi\gamma+\gamma-\psi)/(\psi\gamma+\gamma)]^{-1/\psi}\{\epsilon^{\psi}/[\pi_2(\Theta_2-\Theta_1)]\}^{1/(\psi\gamma-\psi)}$. Note that following things about this solution for m_1/z: (i) m_1/z is continuously decreasing in w; (ii) $m_1/z \to 0$ as $w \to \overline{w} = \overline{W}(\tau) \equiv [(\pi_1\Theta_1 + \pi_2\Theta_2)/\widetilde{r}]^{\alpha/(1-\alpha)}$ (when z is finite); (iii) $m_1/z \to \infty$ as $w \to 0$; (iv) m_1/z is increasing in z. Therefore, it is easy to see that the demand for labor by intermediaries shares these properties.

Thus, demand for labor by a type-τ active project has the following properties: (i) $L^d(w;\tau,z)$, is continuously decreasing in w; (ii) $\lim_{w\to\overline{w}} L^d(w;\tau,z) = 0$; (iii) $\lim_{w\to 0} L^d(w;\tau,z) = \infty$. Now, define the function

$$\widetilde{L}^d(w;\tau,z) = \begin{cases} L^d(w;\tau,z), & \text{for } w \leq \overline{w} = \overline{W}(\tau), \\ 0, & \text{for } w > \overline{w} = \overline{W}(\tau). \end{cases} \tag{39}$$

This function inherits the properties of $L^d(w;\tau,z)$. The aggregate demand for labor can be expressed as $\int_{\mathcal{A}(w)} L^d(w;\tau,z)dF(\tau) = \int_{\mathcal{T}} \widetilde{L}^d(w;\tau,z)dF(\tau)$. To summarize, the aggregate demand for labor when all loans are monitored, $\int_{\mathcal{T}} \widetilde{L}^d(w;\tau,z)dF(\tau)$, has the following properties:

1. $\int_{\mathcal{T}} \widetilde{L}^d(w;\tau,z)dF(\tau)$ is continuously decreasing in w;

2. $\lim_{w\to\widehat{w}} \int_{\mathcal{T}} \widetilde{L}^d(w;\tau,z)dF(\tau) = 0$, where $\widehat{w} = \max_{\tau\in\mathcal{T}} \overline{W}(\tau)$;

3. $\lim_{w\to 0} \int_{\mathcal{T}} \widetilde{L}^d(w;\tau,z)dF(\tau) = \infty$;

4. $\int_{\mathcal{T}} \widetilde{L}^d(w;\tau,z)dF(\tau)$ is continuously increasing in z. ■

Proof for Proposition 1. Before proceeding, use equation (10) in (34) to rewrite the labor-market-clearing condition as

$$\int_{\mathcal{A}(w)} \{(\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha}) \left[\frac{(1-\alpha)}{w}\right]^{\frac{1}{\alpha}} \frac{v(\epsilon m_1/k)^{\psi}}{\pi_2(r_2-r_1)} + \pi_1(\frac{v}{z})^{\gamma}\frac{\epsilon^{\psi\gamma}(m_1/k)^{\gamma(1+\psi)}}{[\pi_2(r_2-r_1)]^{\gamma}}\}dF = 1. \quad (40)$$

Now, if consumption grows a rate $g^{1/(1-\alpha)}$ the Euler equation (1) dictates that the interest rate will given by $\widehat{r} = \widetilde{r} - \delta = g^{1/(1-\alpha)}/\beta - 1$. Next, suppose at some point in time that a solution, k, m_1/k, w, and v and, has been found to (10), (17), (22) and (40). Then, $k' = g^{1/(1-\alpha)}k$, $w' = g^{1/(1-\alpha)}w$, $v' = g^{1/(1-\alpha)}v$, and $(m_1/k)' = m_1/k$ will solve this equation system for the subsequent period. Take equation (10). Note that the r_i's will remain constant under the conjectured solution. Hence, the proposed solution solves this equation. Next, turn to (17). Again, the conjectured solution will solve this equation. The active set $\mathcal{A}(w)$ will not change—equation (21). Therefore, it is easy to see from (40) that the labor-market clearing will still hold. Last, equation (22) will still hold—note that q is constant while s grows that the same rate as w and z. ■

Proof for Proposition 2. First, point 4 in the proof of Lemma 8 established that the aggregate demand for labor is continuously increasing in z. Therefore, at a given wage rate the demand for labor rises as z moves up. In order for equilibrium in the labor market to be restored, wages must increase, since the demand for labor is decreasing in wages—Point 1. Last, recall from (20) that a type-τ project will only be funded when $w \leq \overline{W}(\tau) = \alpha^{\alpha/(1-\alpha)}(1-\alpha)[(\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})/(1/\beta - 1 + \delta)]^{\alpha/(1-\alpha)}$. It's trivial to see that as w rises the set of $\tau \in \mathcal{T}$ satisfying this restriction, or $\mathcal{A}(w)$, shrinks; if $\tau = (\theta_1, \theta_2)$ fulfills the restriction for some wage it will meet it for all lower ones too, yet there will exit a higher wage that won't satisfy it. Furthermore, observe that $\overline{W}(\tau)$ is strictly increasing in $\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha}$. Therefore, those τ's offering the lowest expected return will be cut first as w rises. ■

Proof for Proposition 3. The set of projects in $\mathcal{T}$ offering the highest expected return is given by

$$\mathcal{A}^* = \underset{\tau=(\theta_1,\theta_2)\in\mathcal{T}}{\arg\max}[\,\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}].$$

By assumption $\int_{\mathcal{A}^*} dF > 0$. Take *any* equilibrium wage w. From (21) it is immediate that if $\tau \in \mathcal{A}^*$ then $\tau \in \mathcal{A}(w)$, since $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} = \alpha(1-\alpha)^{(1-\alpha)/\alpha} w^{-(1-\alpha)/\alpha}(\pi_1\theta_2^{1/\alpha} + \pi_2\theta_2^{1/\alpha}) + 1 - 1/\beta - \delta$ is increasing in $\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha}$. Hence, $\mathcal{A}^* \sqsubseteq \mathcal{A}(w)$ for all w. In equilibrium the wage will be a function of z, so denote this dependence by $w = W(z)$. Now, let $z \to \infty$. It will be shown that $w = W(z) \to w^*$, where

$$w^* \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)\{\max_{\tau=(\theta_1,\theta_2)\in\mathcal{T}}[\ \pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}]/(1/\beta - 1 + \delta)\}^{\alpha/(1-\alpha)}. \tag{41}$$

To see why, suppose alternatively that $w \to \widetilde{w} \neq w^*$. First, presume that $\widetilde{w} < w^*$. Then, by (20) all projects of type $\tau \in \mathcal{A}^*$ will be funded since their cutoff wage is $\overline{W}(\tau) = w^* > \widetilde{w}$. From equations (35), (36) and (39) it is clear that $\lim_{z\to\infty} \widetilde{L}^d(W(z);\tau,z) = \infty$, for $\tau \in \mathcal{A}^*$. Since, $\int_{\mathcal{A}^*} dF > 0$, this implies that $\lim_{z\to\infty} \int_{\mathcal{T}} \widetilde{L}^d(W(z);\tau,z)dF = \infty$. Therefore, such an equilibrium cannot exist because the demand for labor will exceed its supply. Second, no firm can survive at a wage rate bigger than w^*, by (20). Here, $\lim_{z\to\infty} \int_{\mathcal{T}} \widetilde{L}^d(W(z);\tau,z)dF = 0$. This establishes (27). Last, note that $\mathcal{A}(w^*) = \mathcal{A}^*$.

It is immediate that $\mathcal{A}^* \sqsubseteq \lim_{w\uparrow w^*} \mathcal{A}(w)$, because $\tau \in \mathcal{A}^*$ is viable for *all* wages $w \leq w^* = \overline{W}(\tau)$ by (20). It is also true that $\lim_{w\uparrow w^*} \mathcal{A}(w) \sqsubseteq \mathcal{A}^*$, since from (20) any project $\tau \notin \mathcal{A}^*$ requires an upper bound on wages $\overline{W}(\tau) < w^*$ to survive; that is, for any $\tau \notin \mathcal{A}^*$ there will exist some high enough wage w such that $\overline{W}(\tau) < w < w^*$. Therefore, $\lim_{w\uparrow w^*} \mathcal{A}(w) = \mathcal{A}^* = \mathcal{A}(w^*)$. This establishes Point 4 of the Proposition.

To have an equilibrium it must be the case that $k < \infty$ for $\tau \in \mathcal{A}^*$. From equation (36) this can only happen when $\widetilde{r}w^{(1-\alpha)/\alpha} \to \pi_1\Theta_1 + \pi_2\Theta_2$ at a rate of no slower than $O(z^{-\psi/(\psi/\gamma+\psi)})$. This implies that $\lim\limits_{z\to\infty}(m_1/z)^\gamma = 0$. For $0 < \lim\limits_{z\to\infty}(m_1/z)^\gamma < \infty$ it must happen that $\widetilde{r}w^{(1-\alpha)/\alpha} \to \pi_1\Theta_1 + \pi_2\Theta_2$ at rate $O(z^{-\psi/(1+\psi)})$. Since $O(z^{-\psi/(1+\psi)}) < O(z^{-\psi/(\psi/\gamma+\psi)})$,

$\lim_{z\to\infty}(m_1/z)^\gamma = 0$. Hence, $\lim_{z\to\infty} m_1/z = 0$. Using this result and (41), in conjunction with the labor-market-clearing condition (34), then leads to (28). Likewise, equation (22) implies that $\lim_{z\to\infty} v = 0$. Hence, it is easy to see that $p_i = r_i k$ and $p_{ij} = r_i k$ solve the contracting problem (P3) when $v = 0$. Using equation (37) it is apparent that $\lim_{z\to\infty} m_1/k = \infty$. Consequently, a false report by a firm will be caught with certainty, or $\lim_{z\to\infty} P_{ij}(m_1/k) = 1$. ■

Proof for Proposition 4. Let the $\theta_i^{1/\alpha}$'s increase by the common factor $g^{1/\alpha} > 1$. Suppose that wages grow by $g^{1/(1-\alpha)}$. Now, focus on the labor-market-clearing condition (40). Take the first term behind the integral, which gives the demand for labor by a firm. Under this conjectured solution for wages, the r_i's will remain constant. It is easy to see from (37) that m_1/k will remain constant, because q is fixed. The term $(\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})/w^{1/\alpha}$ grows by the factor $g^{-1/(1-\alpha)}$. Therefore, for the demand for labor by firms and intermediaries to remain constant v must grow by $g^{1/(1-\alpha)}$. From equation (22) it is easy to calculate that it grows by less than this; specifically, it grows at rate $g^{-1/[(1-\alpha)(\gamma-1)]}$—note that s grows at the same rate as w, and q is constant. Hence, the demand for labor by firms drops. The demand for labor by intermediaries also decreases. To see this, turn to the second term. All variables in this term are constant, except for v, which declines as just mentioned. Therefore, wages must rise by less than $g^{1/(1-\alpha)}$, since the demand for labor is decreasing in w (as was established in the proof of Lemma 8). The active set, $\mathcal{A}(w)$, will therefore expand, because $\pi_1 r_1 + \pi_2 r_2$ increases. ■

Proof for Proposition 5. Let $z \to 0$. It will be shown first that $\lim_{z\to 0} w = \omega$. To demonstrate this point, recall from the proof of Lemma 8 that the wage rate, w, will decline as z falls. Suppose instead that w approaches $\underline{w} > \omega$; i.e., a wage rate for which there is no credit rationing. From (35) to (39), it is easy to see that $\lim_{z\to 0} \widetilde{L}^d(\underline{w};\tau,z) \to 0$. Labor demand will fall short of labor supply as $z \to 0$, the desired contradiction.

When there is credit-rationing the labor-market-clearing condition is

$$\int_{\mathcal{A}(\omega)/\overline{\theta}_1} L^d(\omega;\tau,z)dF(\tau) + \#(\overline{\theta}_1,\theta_2)L^{dn}(\omega,k;\overline{\theta}_1,\theta_2,z) = 1. \tag{42}$$

In this expression $L^d(\omega;\tau,z)$ represents the demand for labor by a type-τ monitored project at the wage rate ω. This function is determined by (35), (36) and (38). Recall that $\mathcal{A}(\omega)/\overline{\theta}_1$

refers to the active set of projects excluding type-$(\overline{\theta}_1, \theta_2)$ firms. The term $L^{dn}(k; \overline{\theta}_1, \theta_2, z)$ represents the demand for labor by an unmonitored type-$(\overline{\theta}_1, \theta_2)$ project. Note that $L^{dn}(\omega, k; \overline{\theta}_1, \theta_2, z)$ $= (\pi_1 \overline{\theta}_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}) [(1-\alpha)/\omega]^{1/\alpha} k$. Thus, the above condition represents one equation in one unknown, k. A type-$(\overline{\theta}_1, \theta_2)$ firm earns rents in the amount $v = \pi_2(r_2 - r_1)k$. Therefore, the credit-rationed firm's value is determined by the labor-market-clearing condition. As z falls in the credit-rationing regime the active set $\mathcal{A}(\omega)/\overline{\theta}_1$ remains constant because wages are fixed at the lower bound ω. Funding for a project in the active set will go to zero using (36), while assuming that $w = \omega$.

From (42) it is easy to deduce that the aggregate capital stock, $\mathbf{k}$, will be given by

$$\mathbf{k} = \frac{[1 - \int_{\mathcal{A}(\omega)/\overline{\theta}_1} L^d(\omega; \tau, z) dF(\tau)]}{(\pi_1 (\overline{\theta}_1)^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}) [(1-\alpha)/\omega]^{1/\alpha}}.$$

Next, it is apparent from (35) that $L^d(\omega; \tau, z) \to 0$ as $z \to 0$, because $k \to 0$ and $m/z \to 0$ by (36) and (38). Thus, $\mathbf{k} \to \underline{\mathbf{k}}$ as $z \to 0$. Finally, to establish that $\underline{\mathbf{k}} < \mathbf{k}^*$, observe that

$$\begin{aligned} \underline{\mathbf{k}} &\equiv 1/\{(\pi_1 (\overline{\theta}_1)^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}) [(1-\alpha)/\omega]^{1/\alpha}\} \\ &< [\frac{\alpha}{(1/\beta - 1 + \delta)}]^{1/(1-\alpha)} [\pi_1 (\overline{\theta}_1)^{1/\alpha} + \pi_2 (\theta_2)^{1/\alpha}]^{\alpha/(1-\alpha)} < \mathbf{k}^*. \end{aligned}$$

■

11.2 Data

- *Figure 1*: The numbers represent total debt outstanding for nonfederal businesses (excluding financial business) relative to gross domestic business value added (excluding gross farm value added). The source of the data for the value of firms relative to GDP is Hobijn and Jovanovic (2001, Figure 1, p. 1204). Last, the numbers on the size of the financial intermediation sector represent the gross value added of financial corporate business taken from the National Income and Product Accounts.

- *Figure 2*: To construct this figure, a series for the intangible stock of capital is constructed by backing out the implied data series on investment in intangibles that is reported in Corrado, Hulten and Sichel (2006, Figure 1). Specifically, a capital stock series for intangibles is constructed by iterating on the law of motion $k_i' = (1-\delta_i)k_i + i_i$, where k_i is the current stock of intangible capital, i_i is investment in intangibles, and δ_i is the depreciation rate on intangibles. Two issues arise with this procedure. First, what is the depreciation rate on intangible capital? A weighted average of the rates reported in Corrado, Hulten and Sichel (2006, p. 23) suggests that it should be 33%. McGrattan and Prescott (2006, p. 782) feel that an upper bound of 11% is appropriate. Taking a simple average of these two numbers gives 22%, the value used here. Second, what starting value for the intangible stock of capital should be used? Along a balanced growth path the stock of intangible capital is given by $k_i = i_i/(g + \delta_i)$, where g is the growth rate of GDP. This formula is used to start the capital stock off in 1947, where g is assigned a value of 0.015. The stock of intangible capital is then simply added to private nonresidential nonfarm fixed assets. The resulting series is divided through by nonfarm business GDP.

- *Section 9.1:* The data for the interest-rate spread is taken from Beck, Demirguc-Kunt and Levine (2000, 2001). It is defined as the accounting value of bank's net interest as a share of their interest-bearing (total earning) assets. The numbers for the ratio of private credit to GDP are also reported there. The other numbers derive from the Penn World Tables, Version 6.1—see Heston, Summers and Aten (2002). The capital stock for a country, k, is computed for the 1990-2000 sample period using the formula $k = i/(g + \delta)$, where i is gross investment, g is the growth rate in investment, and δ is rate of depreciation. This formula heroically assumes that an economy is on a balanced growth path. The depreciation is taken to be 0.06. The growth rate for investment is calculated from the investment data reported in the tables for the period 1950-2000. Investment is recovered by using data on investment's share of GDP and GDP. The average capital-to-GDP ratio over the period for each country is used. A country's total factor productivity, θ, was computed using the formula $\theta = (y/l)/(k/l)^{\alpha}$, where y is GDP, l is aggregate labor, and α is capital's share of income. A value of 0.30 was picked for α. Aggregate labor is backed out using data on per-capita GDP, GDP per worker, and population. The numbers in the analysis are reported in Table 5. In this table an asterisk attached to a country indicates that the assumption $r_1 < \widetilde{r}$ for all project types is not fulfilled. Here the economy has a credit-rationing equilibrium where some projects are monitored and others are not.

Table 5: Cross-country numbers, data and model

Country	Data					Model					
	GDP p.w.	k/o	TFP	Spread	fin. dev.	z	Spread	x	ΔGDP p.w.	gap	g
Sri Lanka*	7013	0.774	491	0.051	0.339	3.4	0.019	0.245	0.225	73689	0.065
India*	5121	0.787	394	0.030	0.512	2.8	0.026	0.198	0.229	75582	0.047
Bolivia*	6779	0.839	468	0.035	0.441	4.9	0.049	0.246	0.241	73923	0.068
Morocco	11419	0.884	654	0.036	0.576	9.9	0.063	0.353	0.238	69284	0.120
Mauritius	23705	0.892	1091	0.032	0.683	21.2	0.063	0.587	0.232	56997	0.294
Nicaragua	5923	0.915	435		0.234	5.7	0.060	0.220	0.230	74779	0.055
Colombia	12332	0.935	693	0.064	0.402	12.9	0.058	0.363	0.221	68371	0.120
Philippines	7864	1.054	481	0.042	0.890	12.3	0.046	0.251	0.189	72838	0.059
Costa Rica	13913	1.085	721	0.052	0.158	24.2	0.044	0.369	0.179	66790	0.106
Uruguay	20251	1.099	901	0.056	0.266	36.9	0.043	0.477	0.174	60451	0.165
Honduras	6823	1.120	439	0.069	0.282	13.3	0.041	0.221	0.173	73879	0.045
Brazil	18001	1.170	836	0.120	0.538	41.5	0.038	0.427	0.157	62701	0.125
Turkey	14340	1.179	720	0.094	0.336	34.0	0.037	0.363	0.156	66363	0.094
Mexico	22100	1.257	952	0.053	0.556	67.4	0.032	0.479	0.138	58603	0.141
Argentina	25056	1.291	1016	0.082	0.335	85.2	0.030	0.517	0.130	55646	0.157
Ireland	46945	1.355	1573	0.016	0.977	195.7	0.027	0.786	0.114	33758	0.418
UK	39908	1.416	2018	0.020	2.481	202.5	0.024	0.689	0.104	40794	0.266
Panama	15255	1.532	705	0.020	0.692	114.9	0.019	0.341	0.090	65448	0.054
US	57151	1.578	1720	0.039	3.297	508.7	0.017	0.849	0.075	23551	0.460
Portugal	30350	1.596	1111	0.035	1.046	289.3	0.016	0.542	0.077	50352	0.116
Italy	50569	1.610	1559	0.036	1.104	510.3	0.016	0.773	0.071	30133	0.299
Spain	40138	1.639	1332	0.038	1.279	455.9	0.015	0.652	0.068	40564	0.169
Iceland	39834	1.639	1321		0.940	453.3	0.015	0.649	0.068	40868	0.166
Peru	10200	1.655	503	0.072	0.312	124.0	0.014	0.249	0.072	70503	0.026
Finland	40603	1.695	1319	0.016	1.551	593.6	0.013	0.648	0.059	40099	0.148
Netherlands	46929	1.758	1435	0.015	2.430	949.6	0.011	0.706	0.049	33773	0.164
France	45317	1.758	1407	0.035	1.592	917.4	0.011	0.689	0.049	35385	0.152
Denmark	44024	1.759	1374	0.049	1.741	894.0	0.011	0.675	0.049	36678	0.143
Canada	45933	1.781	1424	0.018	1.706	1057.9	0.010	0.691	0.045	34769	0.145
New Zealand	36422	1.794	1195	0.025	1.363	911.6	0.010	0.586	0.044	44280	0.089
Australia	45907	1.824	1405	0.019	1.470	1387.7	0.009	0.683	0.038	34795	0.122
Belgium	50839	1.862	1490	0.023	1.487	1985.5	0.008	0.727	0.032	29863	0.130
Luxembourg	80702	1.881	2048	0.007	2.064	3610.8	0.008	1.000	0.025	0	1.000
Austria	45560	1.884	1385	0.019	1.143	2085.4	0.007	0.670	0.029	35142	0.000
Japan	37061	1.917	1212	0.018	3.043	2155.0	0.007	0.575	0.026	43641	0.000
Israel	40777	1.933	1272	0.033	0.979	2672.0	0.006	0.613	0.023	39926	0.000
Thailand	11632	1.995	543	0.030	1.786	1230.1	0.005	0.251	0.021	69071	0.000
Norway	47845	2.112	1374	0.031	1.403	18840.1	0.002	0.662	0.001	32857	0.000
Switzerland	45706	2.122	1335	0.016	3.464	21532.3	0.002	0.640	0.000	34996	0.000