Duration structure of unemployment hazards and the natural rate of unemployment in the U.S.

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First draft: February 15, 2018

This version: March 16, 2018

Preliminary: Do not cite without permission.

Abstract

This paper proposes a new method to model monthly unemployment exit probabilities by duration with three time-varying factors—level, slope, and curvature. I refer to these three factors as the duration structure of unemployment hazards. The level represents the unemployment exit probability of newly unemployed individuals. The slope and curvature capture the influence of factors that could induce the changes in structural unemployment rate, such as skill depreciation or unobserved individual characteristics contributing to the duration dependence in unemployment hazards through which the hysteresis of unemployment is generated. The duration structure is incorporated into the dynamic accounting identity model of unemployment developed by Ahn and Hamilton (2016) to estimate the factors and factor loadings. Inflows and the level factor contribute almost equally to the rise of unemployment during economic downturns. The curvature factor also plays a crucial role in the rise of unemployment during the Great Recession and in hampering the unemployment rate from recovering after the recession was over. The proposed model can also be used to estimate the natural rate of unemployment (NRU) and allows us to decompose the NRU by duration as well as assess the contribution of each factor to the evolution of the NRU. I found that the long-term NRU continues to rise throughout the 2000s and reaches its highest level in 2011, and the inflows, level, and curvature all make an almost equal contribution to the recent decline of the NRU.

Keywords: business cycles, unemployment duration, unobserved heterogeneity, duration dependence, state space model, extended Kalman filter, Nelson-Siegel model, term-structure of interest rate.

[*] This version is still preliminary. The views in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1 Introduction

Duration dependence in unemployment hazards is a key to understanding the dynamic features in the unemployment rate and the distribution of unemployment duration. In particular, Hornstein (2012), Ahn (2016), and Ahn and Hamilton (2016) argue that the existing models (for example , Fujita and Ramey, 2009; Elsby, Michaels, and Solon, 2009; Shimer, 2012) have limitations in explaining the unprecedented rise of long-term unemployment during the Great Recession, and it is important to consider unobserved heterogeneity or genuine duration dependence– the two factors driving the duration dependence in unemployment hazards– in understanding the cyclical dynamics of unemployment. In the microeconometric literature, there is extensive research on the identification of unobserved heterogeneity and genuine duration dependence from the unemployment hazards by the duration of unemployment (for example, Elbers and Ridder, 1982; Heckman and Singer, 1984a,b,c; Ridder, 1990; Honoré, 1993; van den Berg, 2001; Alvarez, Borovičková, and Shimer, 2015). In this literature, however, the distribution of unobserved individual characteristics or the speed of genuine duration dependence is assumed to be fixed over time.

In fact, there is empirical evidence suggesting that the degree of duration dependence can vary over time. Many studies claim that the rise of the long-term unemployment rate during the Great Recession is associated with the mismatch (for example, Barnichon and Figura, 2013). The mismatch might be related to the unobserved individual characteristics such as specific skills sets that are not demanded anymore by firms due for instance to the skill-biased technical change, and firms might shed workers with those skillsets during a recession. This possibility implies that the distribution of unobserved heterogeneity could vary over time, and it might be crucial to consider time-varying distribution of unobserved heterogeneity to adequately address the role of worker characteristics in the rise of long-term unemployment. In addition, Kroft, Lange, and Notowidigdo (2013) show that the discrimination against the long-term unemployed is stronger when the local labor market is tighter, which implies that the genuine duration dependence could have cyclical features. Nonetheless, none of the previous studies attempted to model the time-varying duration dependence in unemployment hazards or to analyze how much it explains the cyclical dynamics of the unemployment rate.

In this paper, I develop a new model that characterizes the exit probabilities from unemployment by duration of unemployment with level, slope, and curvature. I refer to these three factors as the duration structure of unemployment hazards. In the model, the level, slope, and curvature components as well as the factor loading on each factor are time varying, so the model is designed to capture the time-varying profile of exit probabilities by duration of unemployment. The duration structure of unemployment hazards is incorporated in the dynamic accounting identity model of unemployment developed by Ahn and Hamilton (2016) for the estimation of three factors along with the inflows. This model is a parsimonious model of unemployment dynamics that jointly explains the changes in unemployment, unemployment hazards by duration, and the distribution of unemployment duration. The proposed model allows us to evaluate how important each factor is in driving the cyclical fluctuations of unemployment and, furthermore, to estimate the natural rate of unemployment.

What does the level, slope, and curvature mean? The level represents the exit probability of newly unemployed workers. The slope measures the difference between the exit probability of newly unemployed individuals and that of the very long-term unemployed, representing the overall size of deterioration in unemployment hazards. The curvature measures the relative speed of deterioration of unemployment hazards over unemployment duration. The influence of structural factors such as mismatch or skill depreciation that make an individual long-term unemployed is captured by the slope or curvature. In this context, the level essentially represents a more cyclical aspect of market tightness. In addition, the curvature is designed to capture the consequence of structural factors that drives a nonlinear relationship between the duration of unemployment and unemployment hazards. For example, the unemployment exit probability could deteriorate fast in the first six months of unemployment but then suddenly rise because unemployed workers decide to take a low-paying job as they exhaust their unemployment insurance claims (Krueger and Mueller, 2011). This possible nonlinearity or nonmonotonicity in the pattern of duration dependence can be captured by the curvature. To characterize the slope and curvature, I use the Laguerre function, a parsimonious one-parameter function that is also used in the Nelson-Siegel model of term structure of interest rates.[1]

The duration dependence in unemployment hazards characterized by the slope and curvature is closely related to the hysteresis of the unemployment rate. The hysteresis of the unemployment rate describes the phenomenon where changes in the natural rate of unemployment (hereafter, NRU) can be path dependent (Blanchard, 2017), and thus the structural unemployment rate is influenced by the history of the unemployment rate (Blanchard and Summers, 1986). The hysteresis implies that a rise in the unemployment rate—for instance, due to a negative shock to the labor market—can drive up the structural unemployment rate, which in turn can exert an upward pressure on the unemployment rate or hamper the unemployment rate from going down when the economy recovers.

In this context, the duration dependence is key channels through which the hysteresis is generated. One channel is genuine duration dependence. Examples include the following: as unemployed workers stay unemployed longer, they might lose their human capital more (Acemoglu, 1995; Ljungqvist and Sargent, 1998); jobless individuals may search less (Faberman and Kudlyak, 2014); and employers may statistically discriminate against those who have been unemployed for longer (Eriksson and Rooth, 2014; Kroft, Lange, and Notowidigdo, 2013). When the job-finding rate deteriorates during an economic downturn, unemployed individuals are more likely to become long-term unemployed and may not be rehirable due to loss of skills or discouragement after experiencing a long unemployment spell even though labor demand recovers.

A quite different channel is that there are important differences across job-seekers when they first become unemployed. The longer an individual is unemployed, the greater the

[1] The Nelson-Siegel model describes the yield curve and is used to forecast the bond yields by maturity. I also tried different specifications such as an exponential function to characterize the level, slope, and curvature, but the current specification yields the highest likelihood value.

chance that the individual is a member of a group whose unemployment exit probabilities were low to begin with. These workers who tend to stay unemployed longer might have lower job-finding rates than others due, for instance, to their skills becoming obsolete because of skill-biased technological changes. Firms might shed these workers whose skills are not crucially demanded any longer during an economic downturn. Therefore, their job-finding rates may not improve during an economic expansion, even though the overall labor market recovers. A larger fraction of these workers might flow into the unemployment pool in the inception of an economic recession and stay unemployed, which could also raise the structural unemployment rate during an economic recession. These two explanations suggest that the genuine duration dependence and unobserved heterogeneity could be important determinants of changes in the NRU.

In this context, the possible variations in degree of genuine duration dependence and in distribution of unobserved heterogeneity suggest that the magnitude and speed of hysteresis could continuously change . Therefore, the time-varying objects—slope and curvature—can tell us something about how much the hysteresis influences the evolution of unemployment rate over business-cycle phases or recession episodes.

The estimation results are broadly threefold. First, the level factor and the curvature factor both exhibit countercyclical movements. In particular, the countercyclicality in curvature implies that the unemployment exit probabilities deteriorate much faster in the short duration groups during an economic downturn. Second, the inflows recovered relatively quickly after the recession was over, while the level factor recovered very slowly, preventing the unemployment rate from falling further. Third, the curvature factor plays a non-negligible role in the cyclical dynamics of unemployment. In particular, the curvature factor hampered the unemployment rate from declining to the pre-recession levels during the recovery phases after the 1990 and 2007 recessions. The contribution of the slope and curvature factors implies that cyclical fluctuations of unemployment rate were importantly driven by structural factors such as skill depreciation and worker-specific characteristics.

Furthermore, the proposed model can be used to estimate the NRU. As mentioned above, the estimated inflows and the factor loadings on the three factors exhibit both low-frequency and business-cycle frequency movements. If we extract the trend from each component, we can recover the unemployment rate that is formed only by the trend components. I treat this unemployment rate as the NRU in this paper. One merit of this approach is that the estimation does not rely on inflation that did not provide us meaningful signal about the cyclical position of the economy in past decades.

The estimated NRU moves pretty closely with the Congressional Budget Office's (CBO) NRU, staying a quarter to a half percentage point above our estimate of the natural rate between 1990 and 2010. It continues to go up until the end of the Great Recession, flattens out for about three years after the recession, and then declines, reaching 4.7 percent by the third quarter of 2017. The NRU exhibits a moderate downward trend, and the low-frequency decline captures demographic changes such as the aging of the population (for example, the decreased inflows of young workers who tend to have frequent short-term unemployment spells). The substantial rise of NRU from the mid-2000s to the end of the Great Recession and its sustained high level for a few years even after the end of the Great Recession suggest a rise in the structural unemployment rate due to increased mismatch.

The proposed method of estimating the NRU allows us to analyze how much each trend component contributes to the path of NRU, which is entirely new in the literature. I found that the NRU has fallen a little over 1 percentage point from 2012 to the third quarter of 2017.

The trend component in inflows, the outflow probability of newly unemployed individuals, and the curvature factor all contributed almost equally to the 1 percentage point decline. In particular, the slope and curvature factor could reflect the direct role of positive hysteresis in the changes of NRU.

In addition, another novel aspect of the proposed method is to allow us to decompose the NRU by the duration of unemployment. Since the mid-1980s, the share of long-term unemployment in NRU increased over time. The size of NRU with a duration of three months or less exhibits a downward trend, while that with a duration longer than six months shows a secular rise from the late 1990s to 2012 before moving down slightly afterwards. The importance of the long-term NRU is that the factors that make individuals long-term unemployed are the key to understanding the evolvement of the NRU.

This paper is organized as follows. Section 1 introduces the empirical method. Sections 2 and 3 discuss the empirical result and the contribution of each factor. Section 4 introduces the method to estimate the NRU based on the proposed model, to decompose the NRU by duration, and to analyze how each component contributed to the changes in the NRU.

2 Empirical Method

The unemployment exit probabilities by the duration of unemployment are characterized by three factors—level, slope, and curvature—and the factor loadings. The factor loading on each factor serves as the weight on each factor in shaping the profile of unemployment hazards by duration. The three factors and factor loadings are time-varying objects that can capture changes in the profile of exit probabilities from unemployment. Since we do not directly observe the three factors and the factor loadings, they can be treated as dynamic latent variables. This implies that the duration structure of unemployment hazards is incorporated in the nonlinear state space model that casts the dynamic accounting identity developed by Ahn and Hamilton (2016) for the estimation of the factors and the factor loadings.

2.1 Nelson-Siegel Model

The yield curve is the relationship of the yield to maturity of bonds or spot rate to the time to maturity. The spot rate of maturity τ is determined by the average of the forward rate curve up to maturity. Nelson and Siegel (1987) specified the forward rate of maturity τ, $f(\tau)$, as follows,

$$f(\tau) = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} 1 \\ e^{-\tau/\lambda} \\ (\tau/\lambda)e^{-\tau/\lambda} \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix} \tag{1}$$

where $\beta_0, \beta_1, \beta_2$ and λ are coefficients, with $\lambda > 0$.

The first factor, f_0, is a constant and sets the level of the long-term interest rate in the yield curve. The second factor, f_1, is an exponential decay function and determines the difference between the short-term and long-term interest rates. If $\beta_1 > 0$, the function is downward sloping; otherwise, it is upward sloping. The third factor, f_2, is a Laguerre function that is the product of an exponential with a polynomial and determines the curvature of the yield curve. The higher the absolute value of β_2, the more pronounced the hump or trough is. The coefficient λ is the shape parameter and determines both the steepness of the slope factor and the location of the maximum or minimum of the Laguerre function. Figure 1 documents how the forward rate by duration changes depending on the value of λ.

2.2 Model of duration structure of unemployment hazards

The Nelson-Siegel (1987) model is a parsimonious way to model the term structure of the interest rate. A similar idea can be applied to the unemployment hazards by duration in time t.

What does each parameter represent? First, the parameter, λ_t, determines the basic shape of the slope and curvature, as well as the location of the maximum point of the curvature function. See figure 2 for an example where $\lambda_t = 2.8$. The level captured by constant 1 represents the exit probability of newly unemployed workers. The slope measures the difference between the exit probability of newly unemployed individuals and the very long-term unemployed, capturing the overall size of deterioration in unemployment hazards. Discrimination against the long-term unemployed and human capital depreciation that makes a job loser long-term unemployed are the factors that influence the slope. The curvature measures the relative speed of deterioration of unemployment hazards over unemployment duration. Possible nonlinearity or nonmonotonicity in the pattern of duration dependence is captured in the curvature.

Not only the market tightness determining the overall unemployment-exit probability but also the degree of duration dependence changes over time. These realistic features of the labor market are parsimoniously captured by the time-varying factor loadings, β_{0t}, β_{1t}, and β_{2t}. The factor loading β_{0t} pins down the level of newly unemployed individuals at each point in time. In addition, factor loading β_{1t} determines how steep the slope is, and factor loading β_{2t} determines how pronounced the curvature is.

In this set-up, the component that determines the unemployment continuation probability of those who have been unemployed for τ months,

$$\begin{aligned} x_t(\tau) &= \beta_{0t} + \beta_{1t} e^{-(c-\tau)/\lambda_t} + \beta_{2t}((c-\tau)/\lambda_t) e^{-(c-\tau)/\lambda_t} \qquad 1 \leq \tau \leq 12 \\ &= \beta_{0t} + \beta_{1t} e^{-(c-12)/\lambda_t} + \beta_{2t}((c-12)/\lambda_t) e^{-(c-12)/\lambda_t} \qquad \tau > 12 \end{aligned} \tag{2}$$

where c is a sufficiently big number. In the model, I set $c = 12$. The reason for using $\frac{-(c-\tau)}{\lambda_t}$ in equation (2) instead of $\frac{-\tau}{\lambda}$ in equation (1) is that we want the first factor, β_{0t}, to represent the exit probability from unemployment of newly unemployed individuals.

To ensure that the probability falls between 0 and 1, I use a double exponential function to characterize the exit probability from unemployment of those who have been unemployed for τ consecutive months as

$$p_t(\tau - 1) = \exp[-\exp(x_t(\tau))] \quad \text{for} \quad \tau = 1, 2, 3, ... \tag{3}$$

2.3 Dynamic accounting identity with the duration structure of unemployment hazards

The exit probability by duration in equation (3) is incorporated into the state-space model for the dynamic accounting identity of unemployment by Ahn and Hamilton (2016). The model is estimated with the observed numbers of unemployed individuals with a certain observable characteristic j whose duration of unemployment is 1 month, 2-3 months, 4-6 months, 7-12 months and longer than 1 year, $y_t = (U_t^1, U_t^{2.3}, U_t^{4.6}, U_t^{7.12}, U_t^{13.+})'$. Suppose that these numbers are observed with measurement errors $r_t = (r_t^1, r_t^{2.3}, r_t^{4.6}, r_t^{7.12}, r_t^{13.+})'$.

Let w_t be the true number of people newly unemployed at time t, where we interpret

$$U_t^1 = w_t + r_t^1. \tag{4}$$

I assume smooth variation over time for w_t with each assumed to follow an unobserved random walk,

$$w_t = w_{t-1} + \epsilon_t$$

where ϵ_t is the innovation term which drives the dynamics of w_t. As noted by Baumeister and Peersman (2013), a random walk on parameters is often used as a general approach that can pick up structural changes. Random walk specifications allow the inflows and the continuation probabilities to track structural breaks in the duration data of CPS, which might come from the 1994 redesign of the questionnaire, changes in the definition of words, changes in the classification of industries and occupations and so on and then to just move on after the break to adapt to whatever comes next.[2]

I define $P_t(k)$ as the fraction of individuals who were unemployed for one month or less

[2] I do not adjust the number of individuals unemployed for 1 month after the CPS redesign in 1994 in this paper. Previous studies increased the number of individuals unemployed for 1 month, because it can affect the contribution of inflows and outflows to unemployment dynamics. Since the main focus of this paper is not to analyze the role of inflows and outflows to unemployment dynamics but to investigate the role of unobserved heterogeneity in shaping the distribution of unemployment duration, I take the duration data as it is.

as of date $t-k$ and are still unemployed and looking for work at time t. Note that in order for someone to have been unemployed for 2-3 months at time t, they either must have been newly unemployed at time $t-1$ and looking for a job at t, or they were newly unemployed at $t-2$ and still looking at $t-1$ and t. Thus $U_t^{2.3}$ can be written as follows

$$U_t^{2.3} = [w_{t-1}P_t(1) + w_{t-2}P_t(2)] + r_t^{2.3}. \tag{5}$$

Likewise $U_t^{4.6}, U_t^{7.12}$ and $U_t^{13.+}$ are

$$U_t^{4.6} = \sum_{k=3}^{5} [w_{t-k}P_t(k)] + r_t^{4.6} \tag{6}$$

$$U_t^{7.12} = \sum_{k=6}^{11} [w_{t-k}P_t(k)] + r_t^{7.12} \tag{7}$$

$$U_t^{13.+} = \sum_{k=12}^{23} [w_{t-k}P_t(k)] + r_t^{13.+}, \tag{8}$$

where I terminate the calculations after 4 years of unemployment following Hornstein (2012) and Ahn and Hamilton (2016).

The fraction of individuals who were unemployed for one month or less as of date $t-k$ and are still unemployed and looking for work at time t, $P_t(k)$, can be written as a product of monthly fractions $p_{t-j+h}(h)$ for $h = 1, 2, ..., j$ as follows

$$P_t(j) = p_{t-j+1}(1)p_{t-j+2}(2)...p_t(j). \tag{9}$$

As shown in the previous section, the monthly probability of individuals unemployed for $\tau - 1$ months to stay unemployed next month is characterized by equations (2) and (3).

We can arrive at the likelihood function for the observed data $\{y_1, ..., y_T\}$ by assuming that the vector of measurement errors r_t is independent Normal,

$$r_t \sim N(0, R),$$

$$\underbrace{R}_{5\times 5} = \begin{bmatrix} (R^1)^2 & 0 & 0 & 0 & 0 \\ 0 & (R^{2.3})^2 & 0 & 0 & 0 \\ 0 & 0 & (R^{4.6})^2 & 0 & 0 \\ 0 & 0 & 0 & (R^{7.12})^2 & 0 \\ 0 & 0 & 0 & 0 & (R^{13.+})^2 \end{bmatrix},$$

where R^1, $R^{2.3}$, $R^{4.6}$, $R^{7.12}$ and $R^{13.+}$ are the standard deviations of $r_t^1, r_t^{2.3}, r_t^{4.6}$, $r_t^{7.12}$ and $r_t^{13.+}$ respectively. Let ξ_t be the vector $(w_t, \beta_{0t}, \beta_{1t}, \beta_{2t}, \lambda_t)'$ and $\epsilon_t = (\epsilon_t^w,\ \epsilon_t^{\beta_0},\ \epsilon_t^{\beta_1},\ \epsilon_t^{\beta_2},\ \epsilon_t^{\lambda})'$. Our assumption that the latent factors evolve as random walks would be written as[3]

$$\underbrace{\xi_t}_{5\times 1} = \xi_{t-1} + \underbrace{\epsilon_t}_{5\times 1} \tag{10}$$

$$\underbrace{\epsilon_t}_{5\times 1} \sim N(\underbrace{0}_{5\times 1}, \underbrace{\Sigma}_{5\times 5})$$

$$\underbrace{\Sigma}_{5\times 5} = \begin{bmatrix} (\sigma_w)^2 & 0 & 0 & 0 & 0 \\ 0 & (\sigma_{\beta_0})^2 & 0 & 0 & 0 \\ 0 & 0 & (\sigma_{\beta_1})^2 & 0 & 0 \\ 0 & 0 & 0 & (\sigma_{\beta_1})^2 & 0 \\ 0 & 0 & 0 & 0 & (\sigma_\lambda)^2 \end{bmatrix}.$$

Since the measurement equations (4)-(8) are a function of $\{\xi_{jt}, \xi_{j,t-1}, ..., \xi_{j,t-23}\}$, the state equation should describe the joint distribution of ξ_{jt}'s from $t-23$ to t, where I and 0 denote a (5×5) identity and zero matrix, respectively:

[3] The shock could be contemporaneously correlated and can be captured with a factor structure of Σ. Ahn and Hamilton (2016) found that imposing a factor structure did not change the results.

I assume that $\beta_{0t}, \beta_{1t}, \beta_{2t}$, and λ_t evolve as random walks as follows:

$$\beta_{0t} = \beta_{0,t-1} + \epsilon_t^{\beta_0} \tag{11}$$

$$\beta_{1t} = \beta_{1,t-1} + \epsilon_t^{\beta_1} \tag{12}$$

$$\beta_{2t} = \beta_{2,t-1} + \epsilon_t^{\beta_2} \tag{13}$$

$$\lambda_t = \lambda_{t-1} + \epsilon_t^{\lambda}. \tag{14}$$

$$\underbrace{\begin{bmatrix} \xi_t \\ \xi_{t-1} \\ \xi_{t-2} \\ \vdots \\ \xi_{t-22} \\ \xi_{t-23} \end{bmatrix}}_{120\times 1} = \underbrace{\begin{bmatrix} \underbrace{I}_{5\times 5} & \underbrace{0}_{5\times 5} & 0 & 0 & ... & 0 & 0 & 0 \\ I & 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & I & 0 & 0 & ... & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & ... & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & ... & I & 0 & 0 \\ 0 & 0 & 0 & 0 & ... & 0 & I & 0 \end{bmatrix}}_{120\times 120} \underbrace{\begin{bmatrix} \xi_{t-1} \\ \xi_{t-2} \\ \xi_{t-3} \\ \vdots \\ \xi_{t-22} \\ \xi_{t-23} \end{bmatrix}}_{120\times 1} + \underbrace{\begin{bmatrix} \underbrace{\epsilon_{jt}}_{5\times 1} \\ \underbrace{0}_{5\times 1} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{120\times 1}. \tag{15}$$

2.4 Estimation

The model takes the form of a nonlinear state space model in which the state transition equation is given by (15) and observation equation by (4)-(8) where $P_t(\tau)$ is given by (9) and $p_t(\tau)$ by (2) and (3). Our baseline model has 10 parameters to estimate, namely the diagonal terms in the variance matrices Σ and R.

Because the observation equation is nonlinear in the latent variables of interest, the extended Kalman filter can be used to form the likelihood function for the observed data $\{y_1, ..., y_T\}$ and form an inference about the unobserved latent variables $\{\xi_1, ..., \xi_T\}$, as detailed in the appendix. Inference about historical values for ξ_t provided below corresponds to full-sample smoothed inferences, denoted $\hat{\xi}_{t|T}$.

3 Estimation results

Figure 3 shows the estimated inflows, λ_t, and the factor loadings. The number of newly unemployed individuals, w_t, (panel A of figure 3) shows clear countercyclicality. In this context, the movement of inflows is a timely indicator of recessions. It is notable that the estimated w_t declines further than the lowest levels in the inflows observed in the previous two expansions. There is a sudden decrease in the estimated w_t between 1993 and 1994 due to the redesign in the Current Population Survey. Before the redesign, respondents' duration of unemployment is asked in the first month they are reported as unemployed, and after 1994 this information is automatically updated by either four or five weeks if they remain unemployed in the subsequent month. Studies (for example, Polivka and Miller, 1998) show that this change lowered the count of unemployed individuals whose duration of unemployment is less than five weeks, which is also shown in the dramatic fall in the estimated w_t.

As previously mentioned, λ_t determines the monthly duration where the maximum of curvature is located. In other words, the month in unemployment when the speed of deterioration in unemployment exit probabilities changes abruptly is pinned down by λ_t. As shown in panel B, λ_t does not really change over time. The estimated λ_t implies that the relationship between unemployment exit probability and unemployment duration changes substantially when individuals stay unemployed for longer than nine months. This result is consistent with the finding of Kroft, Lange, and Notowidigdo (2013) that the marginal effect of additional months of unemployment is essentially very small after eight months of unemployment.

The factor loading on the level factor, β_{0t}, determines the unemployment exit probability of newly unemployed individuals. β_{0t} tends to go down a few years before the recession begins, and it plunged during recessions since the late 1980s. This result suggests that β_{0t} can be an early indicator of economic recessions. In addition, it is notable that β_{0t} shows a downward trend since the late 1980s. Ahn (2016) discusses factors that can account for this phenomenon, including the increased labor force attachment of women and the decline in the number of young workers who tend to have short-duration jobs and thus make frequent transitions between employment and unemployment.

The factor loading on the slope factor, β_{1t}, determines the slope in the profile of unemployment hazards. β_{1t} does not really show a particular trend or cycle. It goes down during the 1981 and 2007 recessions but moves up during the 1990 and 2001 recessions. The estimates imply that the slope becomes steeper during the 1981 and 2007 recessions but flatter during the 1990 and 2001 recessions. In addition, β_{1t} fluctuates within a pretty narrow range between -1.5 and -1.8.

The factor loading on the curvature factor, β_{2t}, pins down how curvy the profile of unemployment hazards is. β_{2t} shows a clear countercyclical feature, going down during an economic downturn and recovering during an expansion. In particular, β_{2t} recovers immediately after the end of recession following the two recessions in the 1980s but continues to go down or stays at a low level even after the recession is over since the 1990s. The decline in β_{2t} implies that the unemployment exit probability falls much faster as a worker stays unemployed longer and thus reaches that of long-term unemployed relatively quickly. To the contrary, a rise in β_{2t} suggests that the exit probability deteriorates slower in the short-duration groups. The estimation result suggests that the faster deterioration in unemployment exit probabilities of those unemployed for a relatively short period of time is an important feature in understanding the unemployment exit probabilities during an economic downturn. In fact, this estimation result is contradicted by the experimental finding in Kroft, Lange, and Notowidigdo (2013) that potential employers pay less attention to applicants' duration of unemployment when the labor market is weaker. This finding suggests that factors other than the discrimination of employers could be more crucial in understanding aggregate unemployment hazards over business cycles.[4]

How are the changes in factor loadings translated into the actual exit probabilities from

[4] Ahn and Hamilton (2016) also found similar evidence using polynomial functions to characterize the duration dependence in unemployment hazards.

unemployment? Panel B of figure 3 shows that the value of β_{0t} is about 0.1 in January 1985, while it is around -0.5 in January 2009. These values make the unemployment continuation probabilities of newly unemployed individuals in January 1985 and January 2009 0.33 and 0.55, respectively, as plotted in panel A of figure 4. Panel C of figure 3 shows that the value of β_{1t} is about -1.45 in January 1981, while the value is around -1.85 in January 1985. The drop of 0.4 in the slope factor's loading is pretty big given that the range of fluctuations of β_{1t} generates small difference in the slope, raising the unemployment continuation probability of those who have been unemployed for one year by 0.05. Panel D of figure 3 shows that the value of β_{2t} is about -4.0 in January 2005 and below -5.5 in January 2009. The drop of 1.5 in the curvature factor's loading makes the curvature more pronounced, raising the peak of curvature by about 0.1 in the unit of unemployment continuation probability.

Figure 5 shows the model-implied monthly unemployment continuation probabilities by duration of unemployment over the sample period. As suggested by the estimated slope and curvature and the loadings on these two factors, the unemployment continuation probability goes up as the unemployment duration increases up to nine months, after which it begins to decline.

4 Contribution analysis

4.1 Historical decomposition

One benefit of the dynamic statistical model is that it allows us to quantify how much of the realized variation over some historical episode came from particular structural shocks. In the case of a linear VAR, we can decompose the historical time path for y between some date t and $t+s$ into the component that would have been predicted at time t and the part that is due to innovations in each of the shocks. Ahn and Hamilton (2016) demonstrate how we can adopt an approach similar to a linear VAR to analyze how much each structural shock contributed to the changes in the unemployment rate during the period of interest.

In the model, the vector of five latent variables evolves as follows,

$$\xi_{t+1} = \xi_t + \epsilon_{t+1},$$

from which

$$\begin{aligned} \xi_{t+s} &= \xi_t + \epsilon_{t+1} + \epsilon_{t+2} + \epsilon_{t+3} + ... + \epsilon_{t+s} \\ &= \xi_t + u_{t+s}. \end{aligned}$$

Letting $y_t = (U_t^1, U_t^{2.3}, U_t^{4.6}, U_t^{7.12}, U_t^{13.+})'$ denotes the (5×1) vector of observations for date t, our model implies that in the absence of measurement error $y_t = h(\xi_t, \xi_{t-1}, \xi_{t-2}, ..., \xi_{t-23})$ where $h(\cdot)$ is a known nonlinear function. Hence

$$y_{t+s} = h(u_{t+s} + \xi_t, u_{t+s-1} + \xi_t, ..., u_{t+1} + \xi_t, \xi_t, \xi_{t-1}, ..., \xi_{t-23+s}).$$

We can take a first-order Taylor expansion of this function around $u_{t+j} = 0$ for $j = 1, 2, ..., s$,

$$y_{t+s} \simeq h(\xi_t, ..., \xi_t, \xi_t, \xi_{t-1}, ..., \xi_{t-23+s}) + \sum_{j=1}^{s} [H_j(\xi_t, \xi_t, ..., \xi_t, \xi_t, \xi_{t-1}, ..., \xi_{t-23+s})] u_{t+j}$$

for $H_j(\cdot)$ the (5×5) matrix associated with the derivative of $h(\cdot)$ with respect to its jth argument. Using the definition of u_{t+j}, this can be rewritten as

$$y_{t+s} \simeq c_s(\xi_t, \xi_{t-1}, ..., \xi_{t-23+s}) + \sum_{j=1}^{s} [\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-23+s})] \epsilon_{t+j} \tag{16}$$

for $\Psi_{s,j}(\cdot)$ a known (5×5)-valued function of $\xi_t, \xi_{t-1}, ..., \xi_{t-23+s}$.

The smoothed inferences satisfy

$$\hat{\xi}_{t+s|T} = \hat{\xi}_{t|T} + \hat{\epsilon}_{t+1|T} + \hat{\epsilon}_{t+2|T} + \hat{\epsilon}_{t+3|T} + ... + \hat{\epsilon}_{t+s|T}$$

where $\hat{\epsilon}_{t+s|T} = \hat{\xi}_{t+s|T} - \hat{\xi}_{t+s-1|T}$. Let $\mathbf{1}$ denote a 5 by 1 vector of ones. For any date $t+s$ we then have the following model-inferred value for the number of people unemployed:

$$\mathbf{1}'h(\hat{\xi}_{t+s|T}, \hat{\xi}_{t+s-1|T}, \hat{\xi}_{t+s-2|T}, ..., \hat{\xi}_{t+s-23|T})$$

For an episode starting at some date t, we can then calculate

$$\mathbf{1}'h(\hat{\xi}_{t|T}, \hat{\xi}_{t|T}, \hat{\xi}_{t|T}, ..., \hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t+s-23|T}).$$

The above expression represents the path that unemployment would have been expected to follow between t and $t+s$ as a result of initial conditions at time t if there were no new shocks between t and $t+s$. Given this path for unemployment that is implied by initial conditions, we can then isolate the contribution of each separate shock between t and $t+s$. Using the linearization in equation (16) allows us to represent the realized deviation from this path in terms of the contribution of individual historical shocks:

$$y_{t+s} \simeq c_s(\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t-23+s|T}) + \sum_{j=1}^{s} [\Psi_{s,j}(\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t-23+s|T})]\hat{\epsilon}_{t+j|T}. \tag{17}$$

From the above equation, we get a contribution for example of $\epsilon^w_{t+1}, \epsilon^w_{t+2}, ..., \epsilon^w_{t+s}$ (the shocks to w_t between $t+1$ and $t+s$) to the deviation between the level of unemployment at $t+s$ from the value predicted on the basis of initial conditions at t:

$$\mathbf{1}' \sum_{j=1}^{s} [\Psi_{s,j}(\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t-23+s|T})] e_1 \hat{\epsilon}'_{t+j|T} e_1.$$

4.2 Results of historical decomposition

Figure 6 shows the contribution of each component to the realized unemployment rate in the past five recessions, and figure 7 documents it in the past four expansions.[5] In each

[5] Because of the length and severity of the 2007–09 recession, the linearization (17) around the January 2007 values on which the last panel is based becomes poorer as we try to predict values for 2010. This is why

panel, the solid line (labeled U_{base}) gives the change in the unemployment rate relative to the value at the start of the episode that would have been predicted on the basis of initial conditions.

There are broadly three noticeable features. First, the inflows, w_t, are as important as the factor loading on the level, β_{0t}, in explaining the rise in the unemployment rate during economic downturns. However, in the expansions, the contribution of w_t and β_{0t} to the evolution of the unemployment rate differs across expansions. In the expansions after the 1981 and 2007 recessions, β_{0t} is more important than w_t in the decline of the unemployment rate. In the expansions after the 1990 recession, w_t is the main force bringing down the unemployment rate, while both w_t and β_{0t} are important in the expansion after the 2001 recession.

Second, it is notable that β_{0t} is the main driver of the sluggish recovery of the unemployment rate in the past three recessions. While w_t tends to recover relatively quickly along with the decline of the unemployment rate after the recession was over, β_{0t} goes down very slowly, preventing the unemployment rate from falling. In this context, β_{0t} is the most important component that drives the persistent decline in the unemployment rate during economic expansions.

Third, it turns out that the factor loading on the curvature factor, β_{2t}, plays a non-negligible role in the cyclical dynamics of unemployment. One noticeable feature is that β_{2t} is a crucial variable that hampers the unemployment rate from declining to its pre-recession levels in a recovery phase following the economic recessions since the 1990s. In the 1980, 1981, and 2001 recession, β_{2t} exerts about $\frac{1}{2}$ percentage point of upward pressure on the unemployment rate during the recovery phase. The contribution of β_{2t} is much greater in the 1990 and 2007 recessions. It explains around 1 percentage point of the rise in the

the "U_{all}" line in panel D falls below the actual path of unemployment in the case of this recession. If I also calculate the exact nonlinear contribution of each component in isolation of the others and compare the sum of contribution of each factor to the actual observed unemployment rate, the picture looks very similar to the current result. The advantage of the linear decomposition is that the sum of the individual contributions exactly equals the aggregate, whereas the same is not true in a nonlinear dynamic representation.

unemployment rate relative to the pre-recession level one year after the recession is over, hampering the faster recovery of the unemployment rate. Unlike the recessions, β_{2t} plays a more important role in driving down the unemployment rate. In the past two expansions, β_{2t} is as important as w_t and β_{0t}. In the expansion after the 1981 and 1990 recessions, β_{0t} is more important than w_t in the recovery of the unemployment rate.

Considering that β_{2t} captures some structural changes in the labor market that create a dispersion in the job-finding rates among unemployed workers or that shift the share of workers who particularly have greater difficulty being re-employed than others among job losers, the contribution of β_{2t} might represent the changes in the unemployment rate driven by structural factors. The estimation results essentially tell us that β_{2t} is a crucial factor in the recovery of the unemployment rate during economic expansions and plays a more important role during expansions than recessions.

Meanwhile, the role of β_{1t} and λ_t are negligible in both recessions and expansions.

5 Implications on the natural rate of unemployment

In this section, I define the NRU as the unemployment rate at which the components do not have both cyclical and high-frequency movements. As shown in figure 3, the estimated inflows and factors that determine the unemployment hazards by duration of unemployment show the business-cycle fluctuations, trends, and high-frequency movements. For example, the inflows, w_t, can be decomposed into the following,

$$w_t = \tau_t^w + c_t^w + \epsilon_t^w, \tag{18}$$

where τ_t^w is the trend, c_t^w is the cyclical component, and ϵ_t^w is the high-frequency measurement error. The other factors can also be decomposed similar ways,

$$
\begin{aligned}
\beta_t^0 &= \tau_t^{\beta^0} + c_t^{\beta^0} + \epsilon_t^{\beta^0}, &(19)\\
\beta_t^1 &= \tau_t^{\beta^1} + c_t^{\beta^1} + \epsilon_t^{\beta^1}, &(20)\\
\beta_t^2 &= \tau_t^{\beta^2} + c_t^{\beta^2} + \epsilon_t^{\beta^2}, &(21)\\
\lambda_t &= \tau_t^{\lambda} + c_t^{\lambda} + \epsilon_t^{\lambda}. &(22)
\end{aligned}
$$

As shown in the previous section, for any date $t+s$ we then have the following model-inferred value for the number of people unemployed:

$$\mathbf{1}' h(\xi_{t+s}, \xi_{t+s-1}, \xi_{t+s-2}, ..., \xi_{t+s-23}),$$

where $\xi_t = [w_t, \beta_t^0, \beta_t^1, \beta_t^2, \lambda_t]'$. If we let $\tau_t = [\tau_t^w, \tau_t^{\beta^0}, \tau_t^{\beta^1}, \tau_t^{\beta^2}, \tau_t^{\lambda}]'$, then the natural rate of unemployment only composed of the trend component of each latent variable, n_t, is expressed into

$$n_t = \mathbf{1}' h(\tau_{t+s}, \tau_{t+s-1}, \tau_{t+s-2}, ..., \tau_{t+s-23}).$$

I estimate the trend components, $\tau_t^w, \tau_t^{\beta^0}, \tau_t^{\beta^1}, \tau_t^{\beta^2}$ and τ_t^{λ}, based on the following procedure. First, I remove the cyclical component by regressing the factor on the current value and four lags of GDP gap as follows,

$$w_t = c_0 + c_1 g_t + c_2 g_{t-1} + c_3 g_{t-2} + c_4 g_{t-3} + c_5 g_{t-4} + r_t^w \tag{23}$$

where g_t is the GDP gap and r_t^w is the first-stage residual that does not have the cyclical contents correlated with the GDP gaps. Second, as the estimate of the first-stage residual, $\hat{r}_t^w$, can still have the remaining cyclical features that are not controlled by the current and past values of GDP gap, I eliminate the residual cyclicality using the asymmetric Christiano-Fitzgerald filter with the range of durations to pass through between 6 and 32 years to obtain the second-stage residuals, $\hat{r}^w_{2t}$.[6] Assuming that the cyclical component of w_t is teased out by the previous two procedures, $\hat{r}^w_{2t}$ is now composed of the trend of w_t, τ^w_t, and the unexplained high-frequency movements, ϵ^w_t,

$$\hat{r}^w_{2t} = \tau^w_t + \epsilon^w_t.$$

I get rid of the unexplained high-frequency fluctuations and recover the trend using a parametric AR model.

$$\hat{r}^w_{2t} = b_0 + b_1 \hat{r}^w_{2,t-1} + b_2 \hat{r}^w_{2,t-2} + \cdots + b_p \hat{r}^w_{2,t-p} + \epsilon^{rw}_t.$$

The number of lags, p, is determined by AIC, Schwarz or Hanna-Quinn. The estimates of τ^w_t and ϵ^w_t are

$$\begin{aligned} \hat{\tau}^w_t &= \hat{b}_0 + \hat{b}_1 \hat{r}^w_{2,t-1} + \hat{b}_2 \hat{r}^w_{2,t-2} + \cdots + \hat{b}_p \hat{r}^w_{2,t-p} \\ \hat{\epsilon}^w_t &= \hat{\epsilon}^{rw}_t. \end{aligned}$$

Note that $\hat{\tau}^w_t$ is a trend demeaned in the equation (23). I use the same procedure to estimate the trend component of $\beta^0_t, \beta^1_t, \beta^2_t$, and λ_t. Finally, I feed the trend components and the means of inflows and four other factors into the dynamic accounting identity model of unemployment (equation (4)-(8)) and recover the natural unemployment rate.

5.1 Estimated natural unemployment rate

Figure 8 shows the estimates of the NRU from 1986:Q1 to 2017:Q2. I used the GDP gaps published by the CBO as g_t. The estimates are compared with the CBO's estimates of the NRU.

Overall, the estimated NRU moves pretty closely with the CBO's NRU. However, it tends

[6] I also considered the band-pass filter and the Christiano-Fitzgerald filter with a fixed window for robustness checks, but the results are very similar to the baseline case.

to stay a quarter to a half percentage point above the CBO's natural rate between 1990 and 2010. During the 1990s recession, the structural unemployment rate begins to diverge from the CBO's estimate of the natural rate. The structural unemployment rate declines more slowly than the CBO's estimate. The gap between the two estimates narrows after 2010.

The estimated NRU declines gradually from the mid-1980s to the mid-2000s and then starts to rise a few years before the Great Recession. It continues to go up until the end of the Great Recession, flattens out for about three years after the end of the recession, and then declines, reaching 4.7 percent by the third quarter of 2017.

The low-frequency decline in the estimated NRU captures demographic changes such as the aging of the population (for example, the decreased inflows of young workers who tend to have frequent short-term unemployment spells). The substantial rise from the mid-2000s to the end of the Great Recession and its sustained high level for a few years even after the end of the Great Recession suggests a rise in the NRU due to increased mismatch.

One notable merit of the proposed approach is that it allows us to decompose the structural unemployment rate by the duration of the unemployment rate—something that previous research has not attempted to do.

Figure 9 shows the duration components of the estimated NRU. The estimated NRU with a one-month duration exhibits a clear downward trend. A more subdued downward trend is observed in the NRU with a duration of two to three months, and the NRU with a duration of four to six months does not exhibit low-frequency movements. Meanwhile, the NRU with a duration longer than six months—the long-term component of NRU—shows a secular rise from the late 1990s to 2012 and then moves down continuously.

The difference in the trend across duration components implies changes in the distribution of unemployment duration within the NRU. In the mid-1980s, a little less than half of the NRU is explained by the NRU with a one-month duration, and about 10 percent is accounted for by the long-term component. Since 2015, the one-month duration component and long-term component both take a quarter of the NRU. The growing share of long-term component in the NRU suggests that factors that make individuals stay unemployed long are the key to understanding the evolution of structural unemployment rate.

5.2 Historical decomposition

From 2012 to the end of 2017:Q3, the estimated NRU fell a little over 1 percentage point. How much did each component contribute to the drop in the NRU? Like the historical decomposition portrayed in section 5, we can also analyze how much each trend component explains the changes in the NRU. Figure 10 documents the decomposition results. There are three notable features in the results.

First, the downward trend in inflows explains 0.3 percentage point of the decline. Second, another 0.3 percentage point of the drop is accounted for by the improvement in the trend outflows probability of newly unemployed individuals. The outflows probability exhibits a downward trend, but the downward trend slows somewhat since 2015, putting downward pressure on the estimated NRU. This result implies that the unemployment exit probability of newly unemployed individuals improves more than what is consistent with the historical relationship between the business cycles and the unemployment exit probabilities of newly unemployed workers and the downward trend combined. If we are willing to interpret that the slowed downward trend in the exit probability is the outcome of a tight labor market, then the recent improvement in β_{0t} might be one aspect of positive hysteresis. In this context, the role of β_{0t} in the decline of the structural unemployment rate might represent the positive hysteresis from this margin.

Third, the remaining 0.4 percentage point drop in the estimated NRU is explained by the further improvements in the slope (0.1 percentage point) and curvature (0.3 percentage point) factors compared to what is predicted by the business cycles captured by the GDP gaps and the CF filter. The contribution of β_{2t} suggests that the deterioration in unemployment hazards due, for instance, to unobserved individual characteristics, employers' discrimination, or skill loss becomes slowed more than what is accounted for by the slowing consistent with the historical relationship between the business cycles and the factor loading on curvature. Considering that the slope and curvature factors are related to the channel through which the structural unemployment rate is generated, the contribution of these factors could also reflect the direct role of positive hysteresis in the changes of the NRU.

To summarize, the sizable decline in the estimated NRU is driven not only by the downward trend in inflows, but also by the positive hysteresis that shows through the level, slope, and curvature factors. The historical decomposition suggests that positive hysteresis might have contributed more to the recent decline than other secular changes in the labor market.

6 Conclusion

In this paper, I propose a new model of monthly unemployment exit probabilities by duration of unemployment with three time-varying factors—level, slope, and curvature, similar to the three factors that are used to model the yield curve. I call the three time-varying factors the duration structure of unemployment hazards. The level captures the unemployment exit probability of newly unemployed individuals. The slope and curvature capture the effects from, for example, skill depreciation, employers' discrimination against the long-term unemployed, and unobserved individual characteristics that can generate the duration dependence in unemployment hazards. The duration structure is incorporated into the dynamic accounting identity model of unemployment developed by Ahn and Hamilton (2016) to estimate the factors and factor loadings, and to analyze the contribution of each factor in the fluctuations of unemployment rate.

Inflows and the level factor contribute almost equally to the rise of unemployment during economic downturns. The curvature factor is also crucial in the rise of unemployment during the Great Recession and in hampering the unemployment rate to recover after the recession was over. Considering that the duration dependence can be understood as the channel through which the hysteresis of unemployment is generated, the importance of the curvature factor suggests that the hysteresis could be important in understanding the cyclical variation in the unemployment.

The duration structure of unemployment model can also be used to estimate the NRU and allows us to decompose the NRU by duration as well as to assess the contribution of each factor in the evolution of the NRU. I found that the long-term NRU continues to rise throughout the 2000s and reaches its highest level in 2011, and the trend component of inflows, level, and curvature all make an almost equal contribution to the decline of the NRU.

There is a strand of future research that can be done with the proposed model. This model can also be estimated with the number of individuals with specific characteristics by duration—for instance, race, gender, and education—to see which group tends to show faster deterioration in their unemployment hazards, which will help us identify the groups of workers who get discriminated more by potential employers due to being unemployed longer. Furthermore, this estimation of the model with disaggregated data will allow us to analyze how much different groups constitute the NRU and what each group's contribution is in the changes of the NRU. This decomposition can help us to think about how socioeconomic changes would affect the NRU.

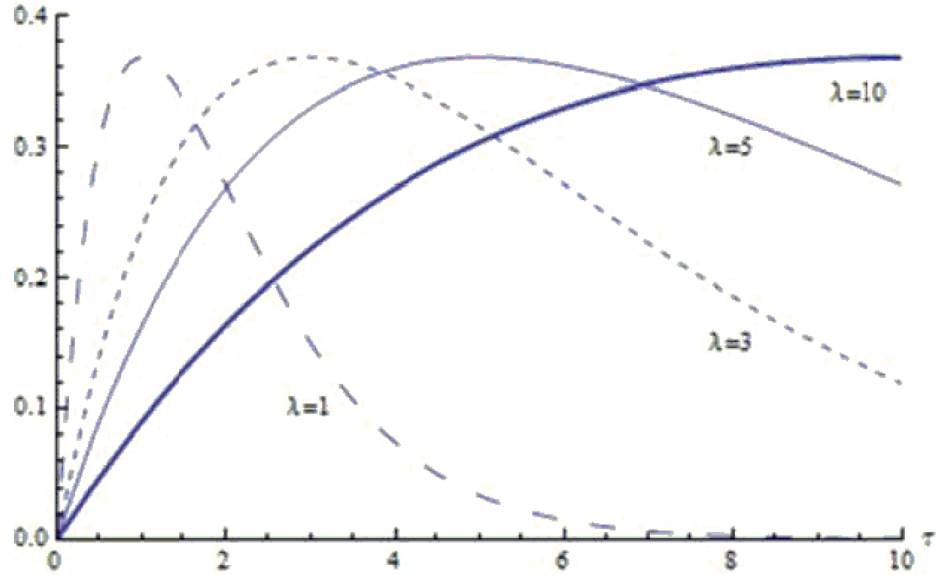


Figure 1. Shape of forward rate curves by λ (Annaert et al., 2013).

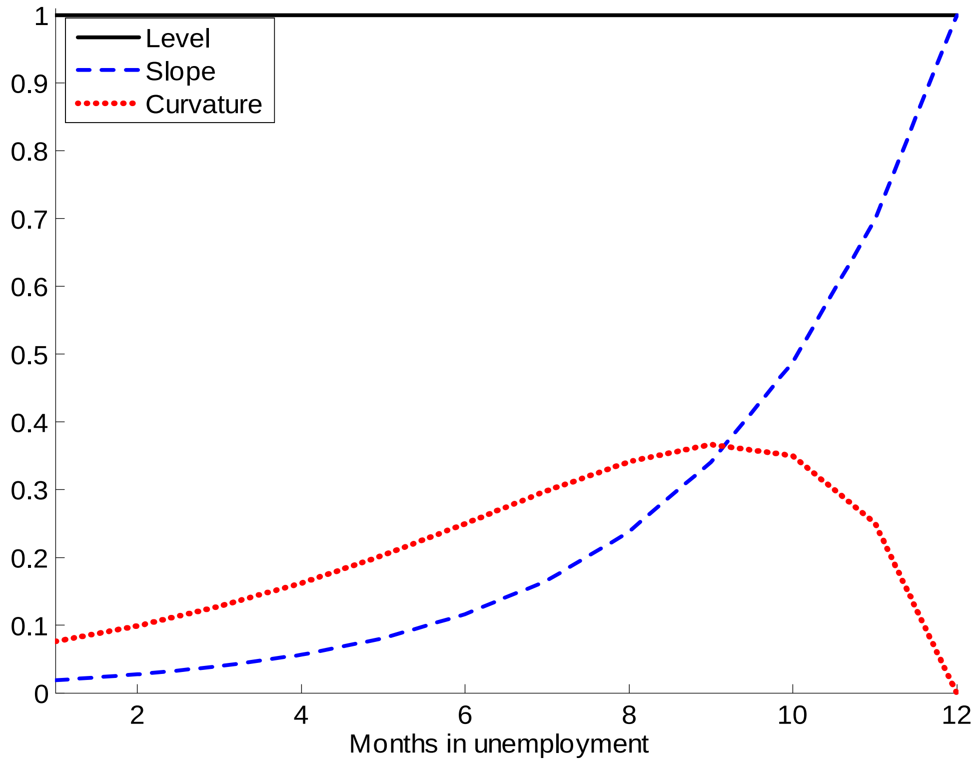
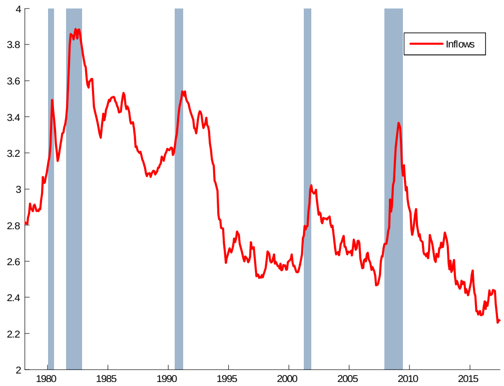
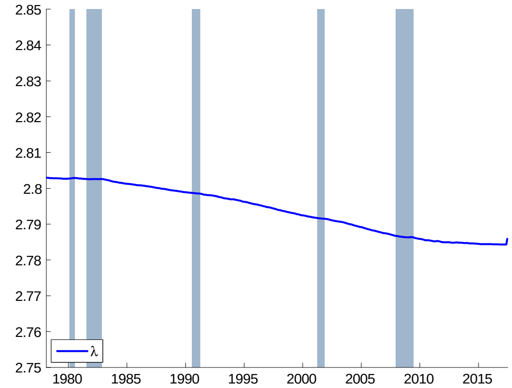


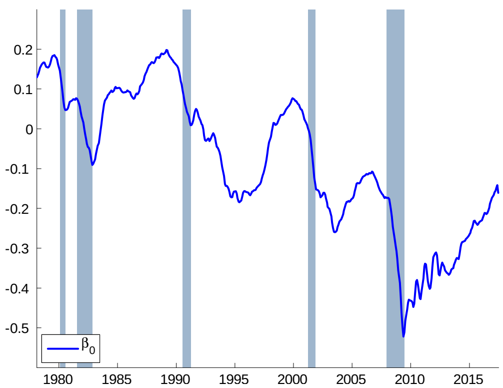
Figure 2. Estimates of level, slope and cuvature with the average value of λ



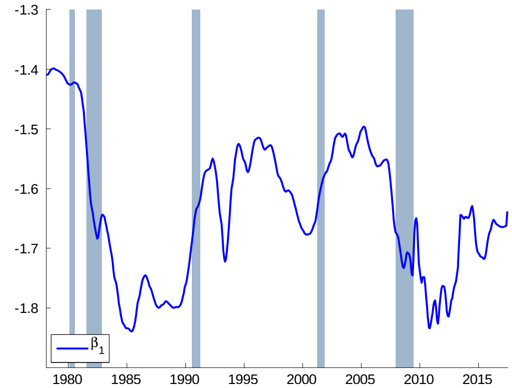
Panel A: w_t



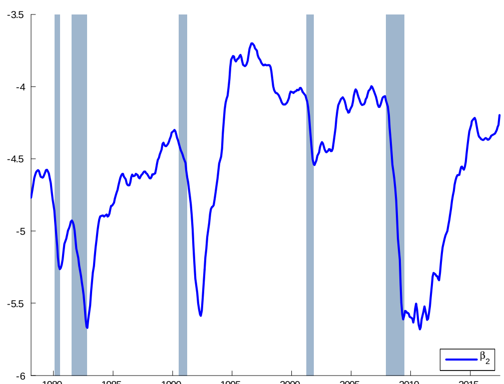
Panel B: λ



Panel C: β_{0t}



Panel D: β_{1t}



Panel E: β_{2t}

Figure 3. Estimates of $w_t, \lambda_t, \beta_{0t,}\beta_{1t,}$ and β_{2t}.
Notes to Figure 3. The shaded areas denote NBER recessions.

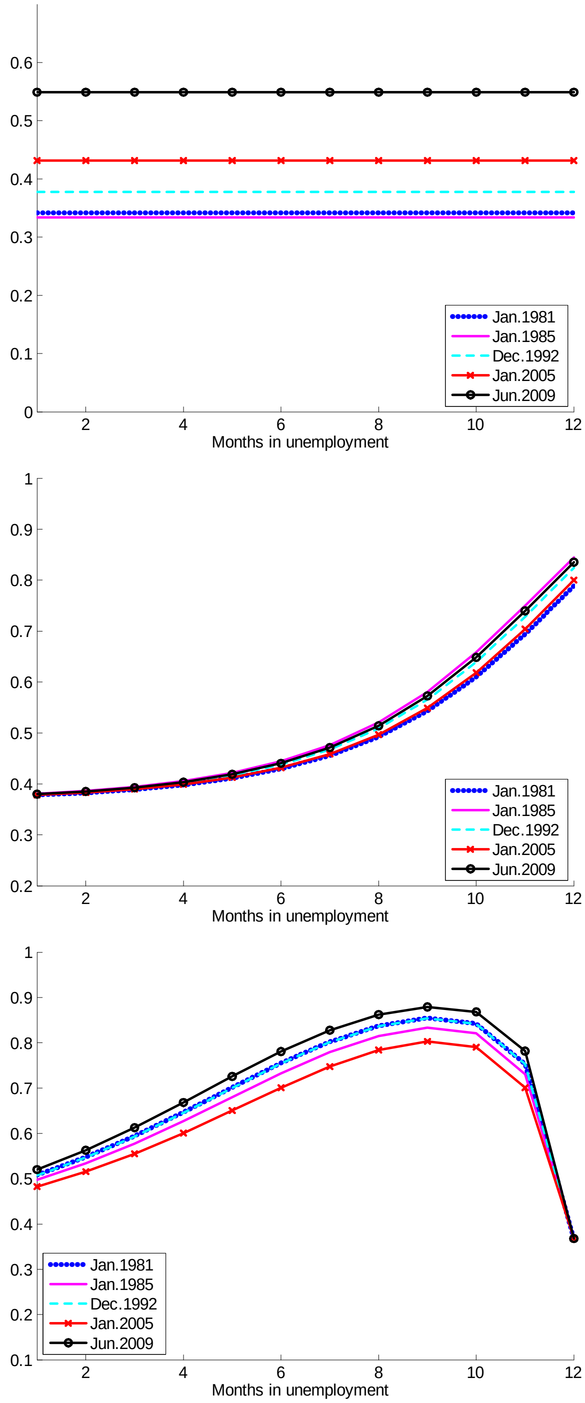


Figure 4. Estimated level, slope and curvature and changes over business cycles.

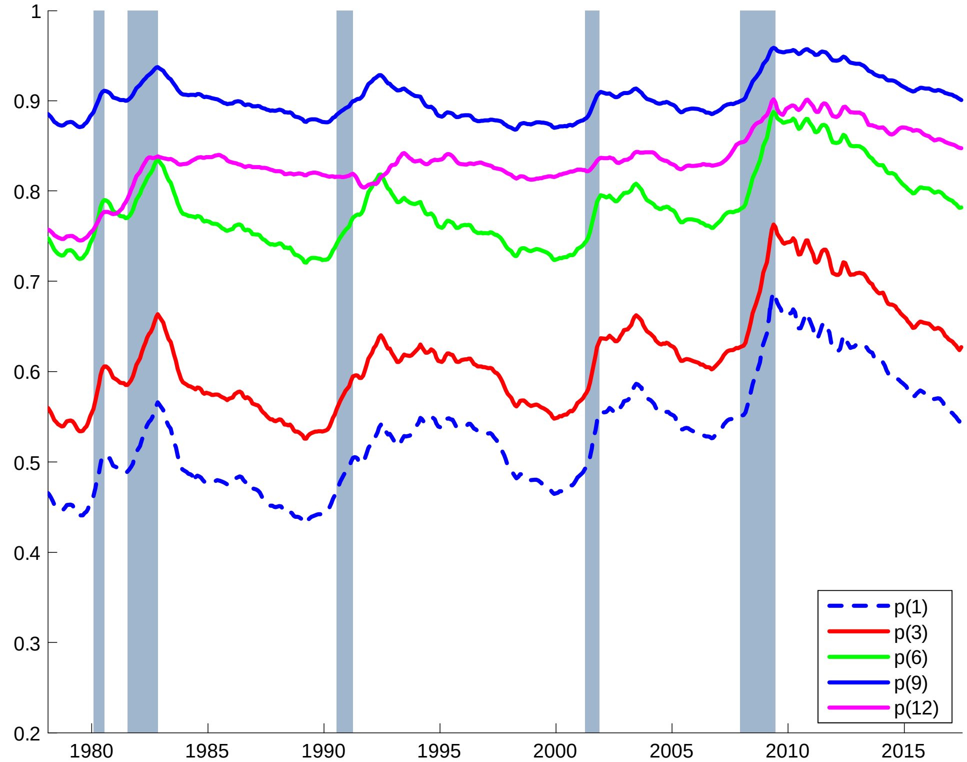


Figure 5. The model-implied monthly unemployment continuation probabilities by duration of unemployment.

Notes to Figure 5. The shaded areas denote NBER recessions.

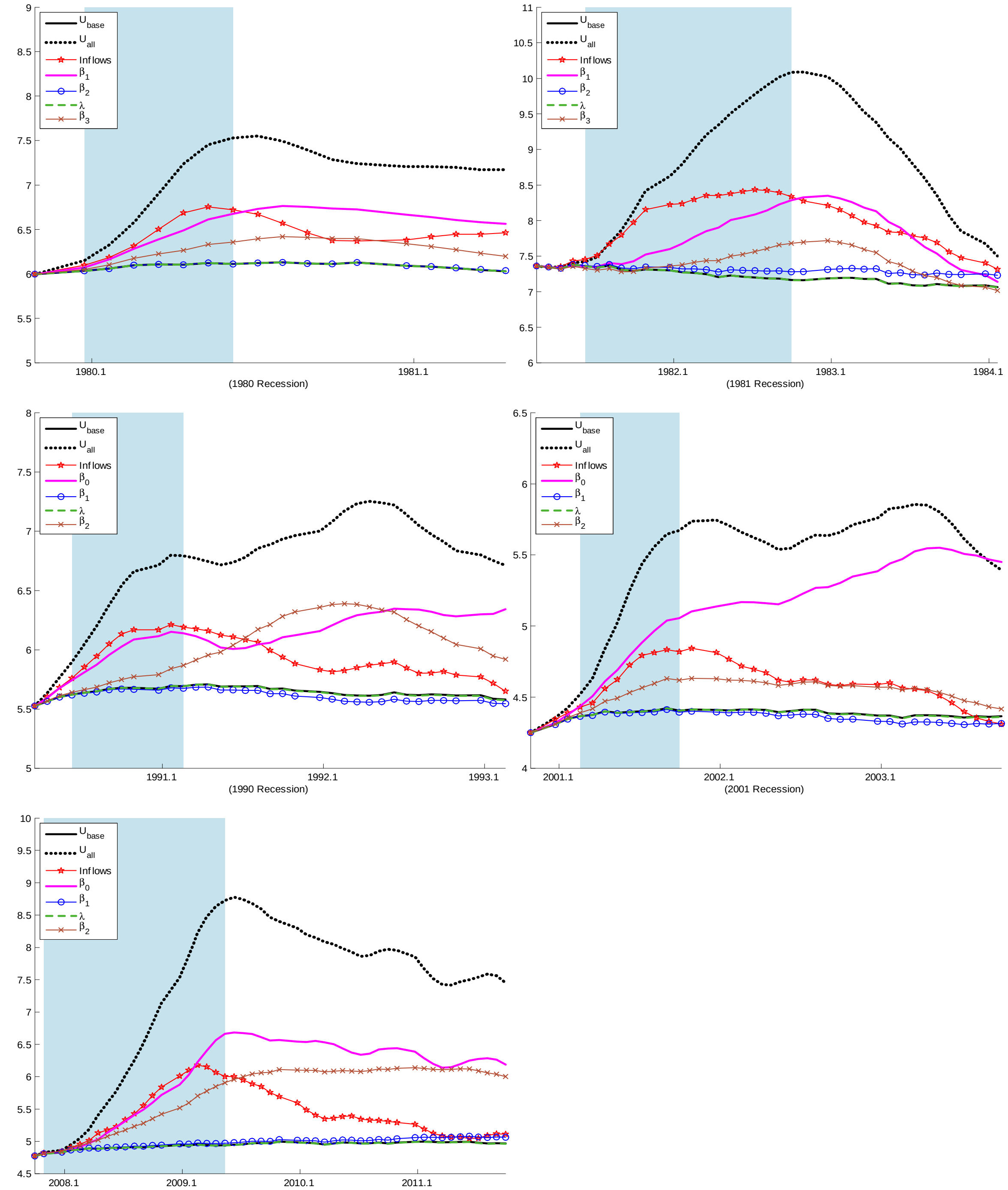


Figure 6. Historical decompositions of U.S. recessions

Notes to Figure 6. The shaded areas denote NBER recessions.

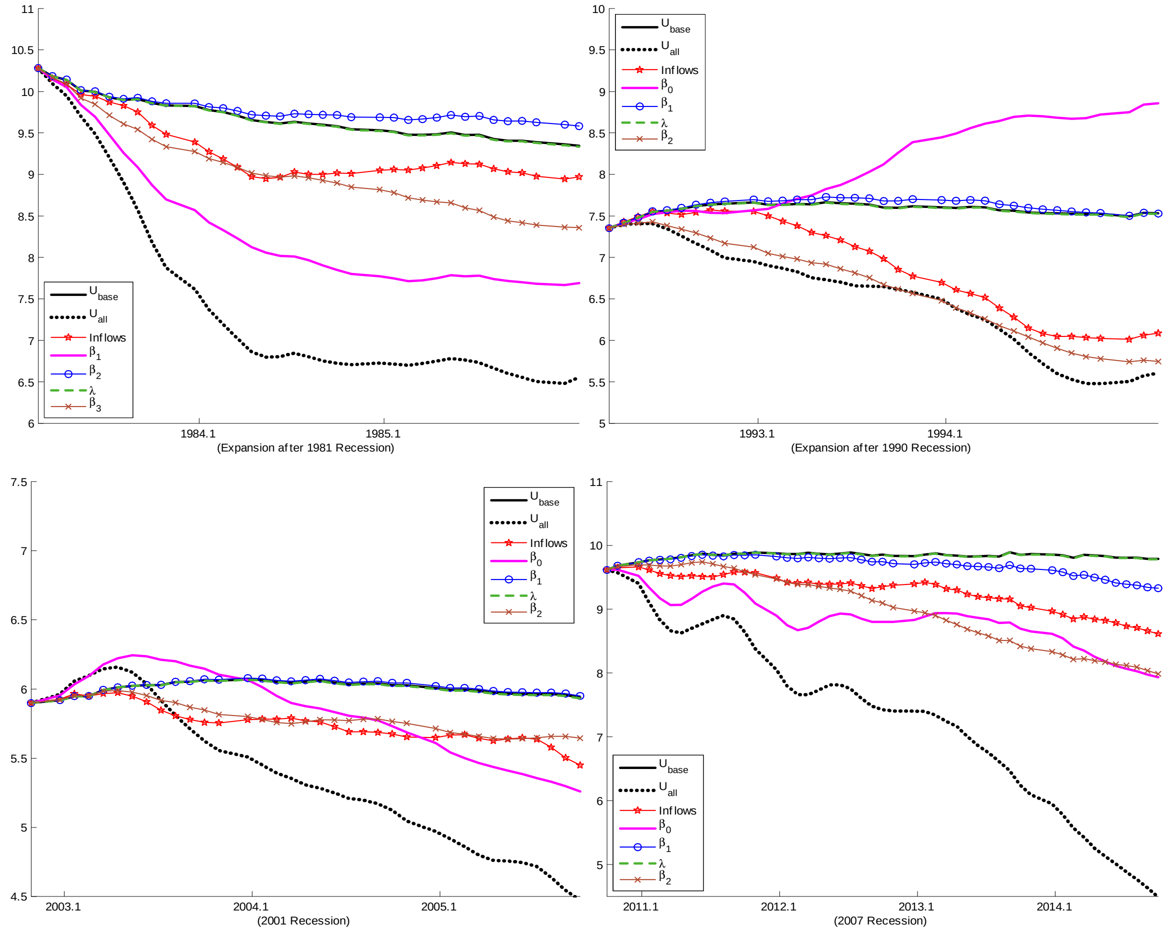


Figure 7. Historical decompositions of U.S. expansions.

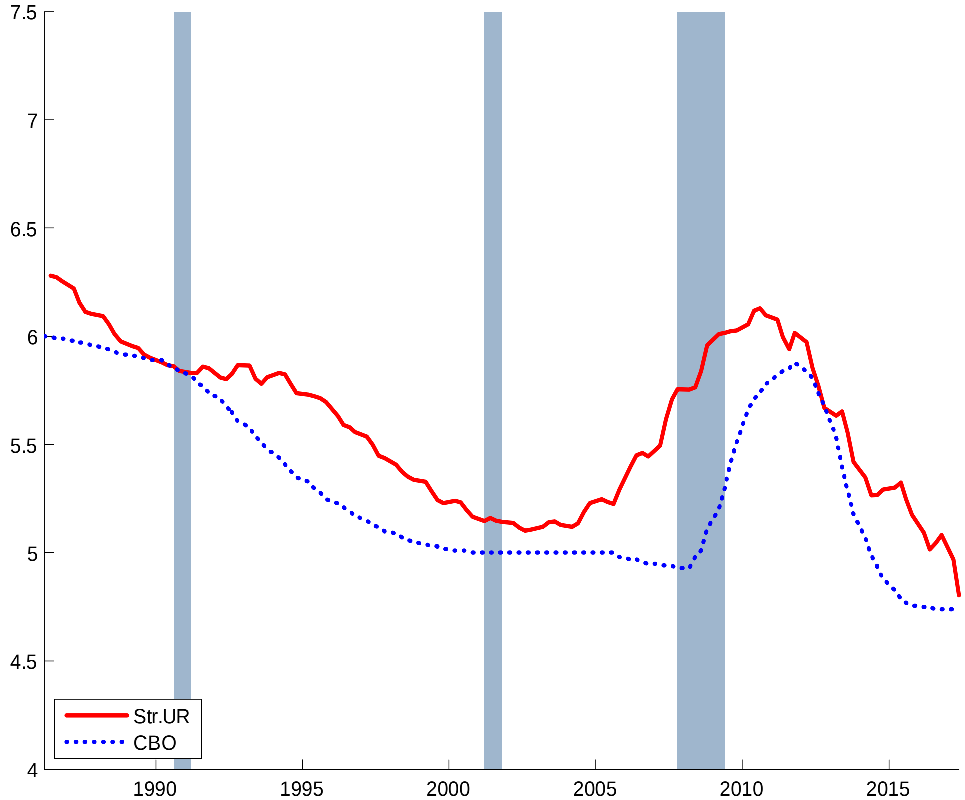


Figure 8. The estimates of structural unemployment rate
Notes to Figure 8. The shaded areas denote NBER recessions.

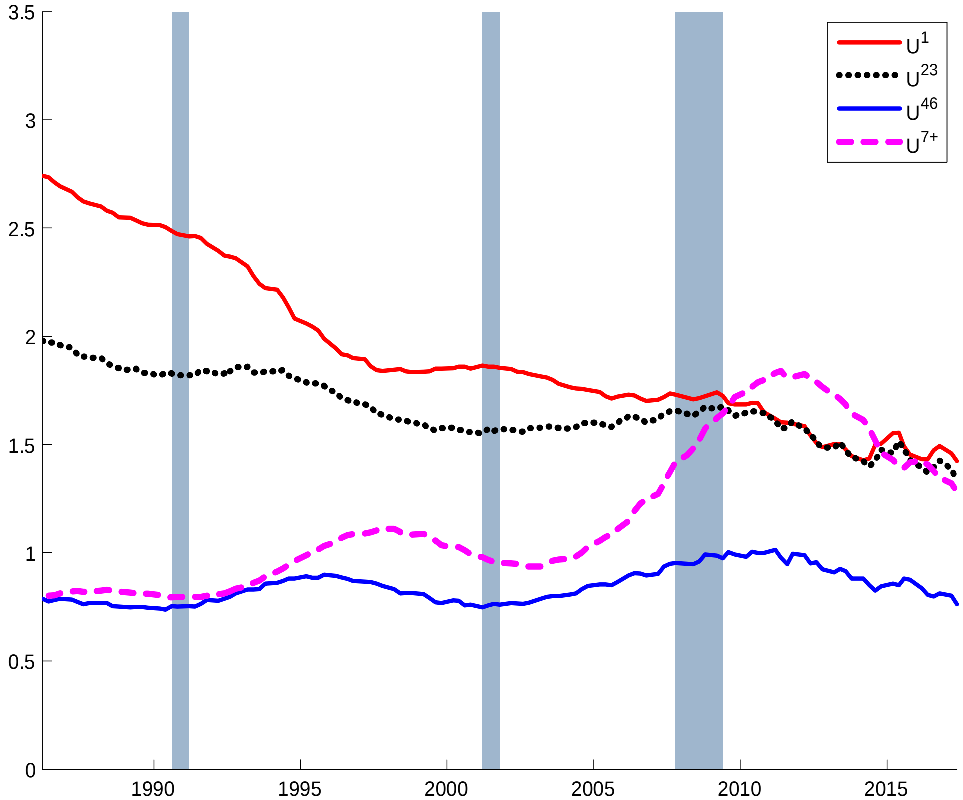


Figure 9. Duration components of structural unemployment rate
Notes to Figure 9. The shaded areas denote NBER recessions.

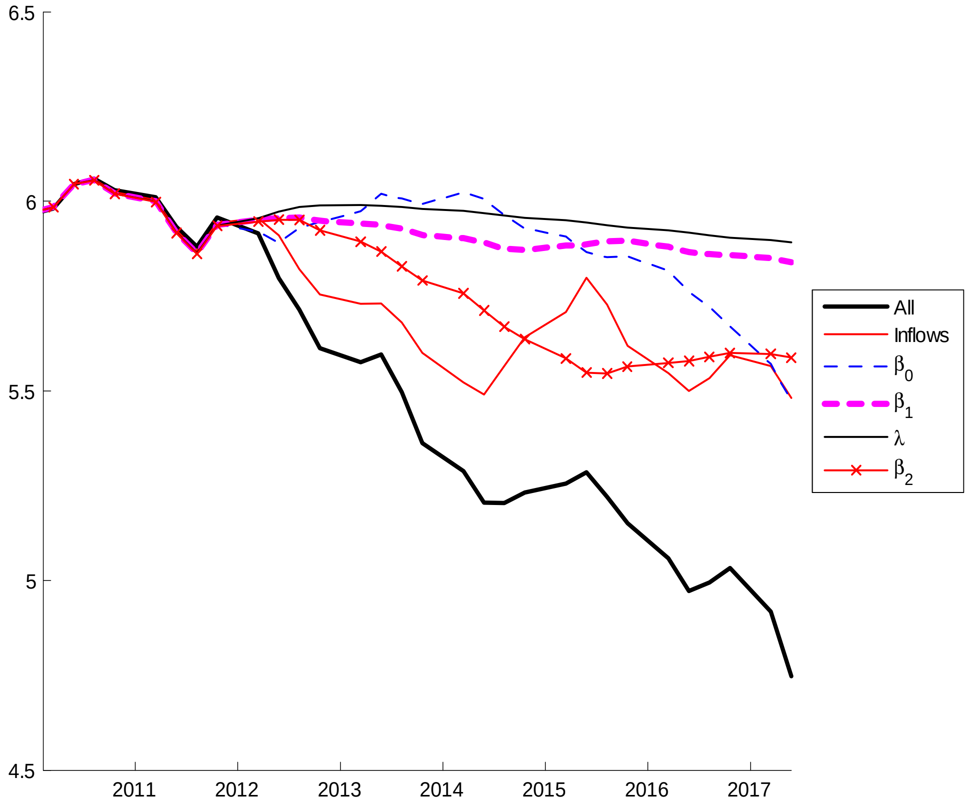


Figure 10. Historical decomposition of structural unemployment rate after the Great Recession

Notes to Figure 10. The shaded areas denote NBER recessions.

Appendix

Estimation algorithm

The system (15) and (4)-(8) can be written as

$$\begin{aligned} x_t &= Fx_{t-1} + v_t \\ y_t &= h(x_t) + r_t \end{aligned}$$

for $x_t = (\xi_t', \xi_{t-1}', ..., \xi_{t-47}')'$, $E(v_t v_t') = Q$, and $E(r_t r_t') = R$. The function $h(.)$ as well as elements of the variance matrices R and Q depend on the parameter vector

$\theta = (R_1, R_{2.3}, R_{4.6}, R_{7.12}, R_{13+}, \sigma_w, \sigma_{\beta_0}, \sigma_{\beta_1}, \sigma_{\beta_2}, \sigma_\lambda)'$. The extended Kalman filter (e.g., Hamilton, 1994b) can be viewed as an iterative algorithm to calculate a forecast $\hat{x}_{t+1|t}$ of the state vector conditioned on knowledge of θ and observation of $Y_t = (y_t', y_{t-1}', ..., y_1')'$ with $P_{t+1|t}$ the MSE of this forecast. With these we can approximate the distribution of y_t conditioned on Y_{t-1} as $N(h(\hat{x}_{t|t-1}), H_t' P_{t|t-1} H_t + R)$ for $H_t = \partial h(x_t)/\partial x_t' | x_t = \hat{x}_{t|t-1}$ from which the likelihood function associated with that θ can be calculated and maximized numerically. The forecast of the state vector can be updated using

$$\hat{x}_{t+1|t} = F\hat{x}_{t|t-1} + FK_t(y_t - h(\hat{x}_{t|t-1}))$$

$$K_t = P_{t|t-1} H_t (H_t' P_{t|t-1} H_t + R)^{-1}$$

$$P_{t+1|t} = F(P_{t|t-1} - K_t H_t' P_{t|t-1})F' + Q.$$

A similar recursion can be used to form an inference about x_t using the full sample of available data, $\hat{x}_{t|T} = E(x_t | y_T, ..., y_1)$ and these smoothed inferences are what are reported in any graphs in this paper; see our online appendix for further details.

Prior to the starting date June 1976 for our sample, BLS aggregates are available but not the micro data that we used to construct $U_t^{13.+}$. For the initial value for the extended

Kalman filter, we estimate $\hat{x}_{1|0}$ from pre-sample values for aggregates as described in the online appendix. By setting large diagonal elements of $P_{1|0}$, the particular value of $\hat{x}_{1|0}$ has little influence on any of the results.

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