Insurers as Asset Managers and Systemic Risk *

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Abstract

Financial intermediaries often provide guarantees that resemble out-of-the-money put options, exposing them to tail risk. Using the U.S. life insurance industry as a laboratory, we present a model in which variable annuity (VA) guarantees and associated hedging operate within the regulatory capital framework to create incentives for insurers to overweight illiquid bonds ("reach-for-yield"). We then calibrate the model to insurer-level data, and show that the VA-writing insurers' collective allocation to illiquid bonds exacerbates system-wide fire sales in the event of negative asset shocks, plausibly erasing up to 20-70% of insurers' equity capital.

Keywords: Systemic risk; Financial stability; Inter-connectedness; Insurance companies.

JEL Classification: G11; G12; G14; G18; G22.

1 Introduction

This paper proposes a new mechanism through which financial institutions' off-balance sheet commitments induce reaching for yield behavior, asset interconnectedness, and ultimately systemic risk. While the literature has traditionally focused on financial institutions' funding risk, attention has recently shifted, particularly for non-bank financial institutions, towards asset interconnectedness: financial institutions holding similar risky assets in search of higher returns. Acharya and Yorulmazer (2007, 2008) show that 'too-many-to-fail' guarantees lead banks to herd in their lending behavior by investing in similar industries or exposing their balance sheet to a common risk factor. Greenwood, Landier and Thesmar (2015) show that fire sales, induced by regulatory microprudential requirements, can create contagion that spreads across banks holding the same assets.[1]

We contribute to the literature by showing that financial institutions invest in similar illiquid assets simply as a result of their shared business model. This endogenous *ex ante* reaching for yield can then generate *ex post* fire-sale externalities along the lines hypothesized by Greenwood et al. (2015). To study our proposed mechanism, we exploit the transformation of the largest U.S. life insurers that have significantly expanded the supply of variable annuities (VAs) with various embedded equity-linked investment guarantees (Koijen and Yogo (2017a,b)).[2] First, while the importance of financial guarantees is not unique to the insurance industry, U.S. insurance data offer a remarkable level of measurement detail with respect to portfolio holdings, policy generation, and regulatory constraints (see Ellul, Jotikasthira, and Lundblad (2011), for example). Second, given both their size and the nature of their return commitments, VAs are attracting attention from policymakers as a potential source of systemic risk.[3] As the U.S. retirement landscape has moved

[1] Domanski, Sushko and Shin (2015) examine portfolio adjustments by long-term investors to declining long-term interest rates and show that they can prompt further downward pressure on interest rates.

[2] A variable annuity is a life insurance policy generally sold to individuals approaching retirement. The policy consists of accumulation and payout phases, and combines insurance for life, longevity, and investment risks. We provide an overview of the different types of VAs in Appendix A.

[3] U.S. insurers have been implicated as a primary source of the market instability exhibited in February of 2018. (See, for example, the Financial Times February 22, 2018.) While insurers have increasingly turned to target volatility funds to help manage the risks associated with VA guarantees over the preceding several years, a significant component

away from employer-sponsored defined benefit plans, the VA business has grown to fill part of this gap. Figure 1 presents a history of the life insurance business over the last two decades, as its major product lines have evolved from traditional life insurance to asset management products, namely VAs.[4]

The emerging concerns about the potential risks to which insurers with VA guarantees are exposing themselves, and the financial system, in the event of a negative market-wide event can be captured by the systemic risk measure proposed by Acharya et al. (2017) and Brownlees and Engle (2016).[5] Figure 2 plots systemic risk (SRISK) for the largest banks and the largest insurers that underwrite VAs. In the post crisis period, the evolution of systemic risk for the two groups are strikingly different: while the banks' systemic risk measure spiked during the 2008-2009 financial crisis, it then decreased over time. However, the same measure for insurers selling VAs with guarantees increased during the crisis, and has not decreased in the same manner.

The importance of understanding how financial guarantees may create systemic risk is not restricted to the insurance sector. While the exact mechanism we highlight may be somewhat different, our model and these data help to shed light on the incentives and consequences of other guarantees that are pervasive throughout the financial system. First, defined benefit pension funds also provide various guarantees and share a degree of under-fundedness, both of which provide incentives for these funds to reach for yield. Second, the guarantees that banks provide to outside investors in securitization deals, used to minimize capital requirements, are also similar to those embedded in VAs. While data limitations for pension funds and banks make it impossible to comprehensively analyze the impact of guarantees in those other important sectors, our analysis makes an important contribution to a broader literature on the link between the incentives associated with guarantees and systemic risk.

Focusing on VAs, a critical feature of the embedded guarantees is that they promise minimum

of market-wide selling pressure was attributed to the unintended consequence of their collective strategy overlap on realized market volatility.

[4]Annuity premia and deposits earned by the U.S. life insurance industry increased from $286 billion in 2010 to $353 billion in 2014, making them one of the fastest growing areas of policy generation, accounting for almost 35% of U.S. life insurers' liabilities in 2015.

[5]See detailed description and data on NYU Stern's Systemic Risk website, http://vlab.stern.nyu.edu/welcome/risk.

returns to policy holders to be honored by the insurers. Given the put option-like nature of these investment products, two related problems manifest during a period characterized by equity market stress. First, individual insurer financial distress can arise as the moneyness of its guarantees explodes. Second, distress is now correlated across insurers as guarantees go in-the-money at the same time for all insurers with VAs.[6] It is precisely this type of shared risk that has raised significant concerns about financial stability across the insurance sector and, more broadly, across other parts of an interconnected financial system. This concern becomes very salient when considering the evidence in Billio et al. (2012), who show that core financial sectors have become highly linked over the past decade.

Insurers typically hedge guarantees dynamically, as markets for long-term put options are not well developed and such hedging is costly/difficult for insurers. In this paper, we show, both theoretically and through a calibration using U.S. insurance data, that insurers' attempts to dynamically hedge minimum-return guarantees induce reaching for yield by investing in similar (illiquid) assets, thereby increasing the likelihood of collective fire sales. The model considers a profit-maximizing insurer that sells VAs with guarantees linked to the stock market. The insurer optimally allocates its portfolio between (a) equity, (b) an illiquid, risky bond, and (c) a liquid, safe bond. In line with the existing literature, we assume that the liquid bond and stock can be sold at their expected values, but the riskier bond will be sold at a discount due to its illiquid nature. That said, the illiquid bond offers sufficient *ex-ante* risk compensation, even taking into consideration the fire sale price impact that may occur following a shock. The insurer is subject to risk-sensitive regulatory capital requirements.

Following practice, we assume that the insurer will hedge a proportion of the guarantees. However, consistent with the data, we assume that an insurer does not hedge via put options; from the National Association of Insurance Commissioners Schedule DB (derivatives) data, we observe that insurers hedge only about 5% of VAs with put options. We instead focus on delta

[6]The risks associated with these guaranteed investment products are very different from those that insurers faced when most of their products were traditional life insurance policies. Mortality risks, for example, are idiosyncratic and can be easily diversified away by issuing a large number of similar policies. This is not, however, the case with VAs linked to the equity market.

hedging, essentially selling short equities and investing the proceeds in both the safe and risky bonds. Depending on how much hedging is employed, the insurer will have a lower risk exposure which, in turn, implies a commensurately lower overall portfolio expected return, prompting the question as to how the insurer can still generate sufficient returns to meet its investment objectives. Any additional unit of risk that it wants to take will have to come from within the bond asset class, leading naturally to an overweight in illiquid bonds.[7]

The portfolio overweight on the riskier bonds chosen by the insurers with VAs will become problematic during a market downturn. In such states, the guarantees become in-the-money, regulatory reserves spike, and the insurers need to shore up their capital positions. While issuing equity is a possibility, it is precisely in these moments that such an avenue becomes impractical. This calls for an alternative action: selling of the illiquid bonds, and possibly of the other assets on the balance sheet, in a regulatory-induced fire sale. In the model, this will cause contagion to other insurance companies (and, outside of the model, likely to other parts of the financial system, including banks, because of the financial sector interconnectedness).

To calibrate the model, we use National Association of Insurance Commissioners (NAIC) data, obtained through SNL Financial, on guaranteed VAs' account values, gross reserves, reinsurance credits, portfolio holdings, and derivatives positions. To begin, we establish some important facts about the portfolio allocation of insurers with and without VAs. We find that only few life insurers underwrite VAs, and the ones that do tend to be very large. Further, we confirm that insurers that underwrite VAs hedge their exposures by selling common stocks and buying bonds, tilting their portfolios towards higher yielding illiquid bonds. That is, those insurers that underwrite VAs have significantly smaller allocations to liquid bonds and stocks and significantly larger allocations to illiquid bonds than insurers with no VAs.

To examine this link more formally, we infer an insurer's guarantee-induced stock market exposure, or delta, the main driver of delta hedging in our model. We calibrate the model by separately regressing an insurer's portfolio allocation to each asset group on the delta. We find

[7] A close empirical analogue is the reaching for yield behavior documented by Becker and Ivashina (2015) and Ellul, Jotikasthira, Lundblad, and Wang (2014).

that insurers delta hedge a large, but incomplete fraction (approximately 69%) of their guarantee exposures, and that insurers with VA guarantees reach for yield with larger allocations to illiquid bonds. From here, we take the implied model parameters from these regressions to determine three portfolios for each insurer: (i) an actual portfolio which reflects both delta (but incomplete) hedging and the associated reaching for yield behavior, (ii) a hypothetical portfolio that reflects only delta (but incomplete) hedging without reaching for yield behavior, and finally, (iii) a hypothetical portfolio which would obtain if the insurer had no guaranteed VAs at all. The difference between portfolios (i) and (ii) uncovers the effect of reaching for yield behavior, while the difference between portfolios (ii) and (iii) comes from the guarantee exposure that is incompletely hedged.

In the final step, we perform a quantitative exercise to assess the asset pricing impact of various market shocks that are attributable to the written VAs. To do so, we follow a framework similar in spirit to that of Greenwood et al. (2015). We focus on three types of shocks, all of which have magnitudes comparable to what we observed during the 2008-9 financial crisis: first, a shock to the equity market; second, a shock to the value of the guarantee; and finally, a shock to the illiquid bond (prices of illiquid bonds decline in a similar manner to those of mortgage-backed securities during the crisis). In each scenario, we examine in detail the differences in price impact across the different actual and hypothetical portfolios mentioned above to gauge the relative importance of the drivers of systemic risk. For example, we demonstrate that shocks to the equity market of 10-40% would result in insurers selling $114-$458 billion of illiquid bonds, with the corresponding system-wide fire sale costs representing up to 21% of insurers' total capital and surplus. If the stock market shock is also correlated with the shock to illiquid bonds, as we observed during the financial crisis, for example, the fire-sale costs will exponentially increase due to the externality. A 40% shock to the equity market plus an 8% shock to illiquid bonds may generate fire-sale costs that erase up to 89% of insurers' capital and surplus, with about 80% of the costs (equal to 70% of capital and surplus) directly attributable to the VAs. Further, we find that the largest culprit, by far, of *ex post* systemic risk is *ex ante* reaching for yield behavior, not the net VA exposures *per se*.

This paper makes a contribution to the systemic risk literature along several dimensions. First,

the existing literature has identified correlated investments as a potential source of systemic risk. Wagner (2010) and Allen et al. (2012) have theoretically shown that elevated asset similarity across institutions is one channel through which systemic risk may arise. Greenwood et al. (2015) show empirically how regulatory-induced fire sales can translate correlated investments into systemic risk. In this paper, we theoretically propose and empirically investigate one mechanism, different from regulation, that gives rise to asset inter-connectedness. While capital regulation is a part of our analysis, asset similarity across insurers instead arises from the business decision to provide investment products. While there is an emerging literature investigating the similarity across portfolios of financial institutions (e.g., Getmansky et al. (2016)), we are among the first to link this elevated portfolio correlation to the transformation of the insurance sector and the emergence of VAs with guarantees. To the best of our knowledge, our paper is the first to explicitly link VAs to interconnectedness and systemic risk.

Second, another strand of the literature addresses the issue of liquidity provision in times of market stress, proposing that some intermediaries, by virtue of their long horizons and balance sheet structure, can take on that vital role. Indeed, the spillover effects of fire sales will be attenuated, or perhaps even halted, if enough buying capital comes in, thus highlighting the importance of the stability of such intermediaries. Chodorow-Reich et al. (2016) propose that insurance companies behave like asset insulators during normal conditions, whereas they do not during market meltdowns. Aside from providing a micro-founded explanation for this result, our paper also shows that life insurers may not only stop insulating assets in a market meltdown but rather may themselves become a source of significant selling activity. More importantly, by virtue of asset inter-connectedness, they are now more likely to contribute to contagion through regulatory-induced fire sales. This has far reaching implications for the stability of the entire financial system.

Third, our analysis contributes to an understanding of the impact of financial guarantees on financial stability. Similar to that unearthed in our paper, defined benefit pensions face a tension between the types of assets in which they need to invest to generate returns commensurate with their objectives and the hedging that becomes necessary as a consequence. Safe assets are natural

choices for pension funds but they have the drawback of lower returns; this feature, in turn, leads pension funds to reach for yield by investing in riskier assets. In the case of a negative shock to those riskier assets, pensions funds' underfundedness widens and the subsequent corrective actions taken by funds can be a conduit for asset prices to affect the wider market.[8] Similarly, the guarantees embedded in banks' securitization deals, introduced to minimize capital requirements, have consequences that are not dissimilar from the ones we identify in our paper (see, for example, Acharya, Schnabl, and Suarez (2010)). Thus, while the mechanism uncovered in this paper for the insurance industry may be somewhat different, the analysis can still help deepen our understanding of how guarantees, pervasive throughout the financial system, can generate fire sales and systemic risk.

The rest of the paper is organized as follows. Section 2 provides the salient institutional features of the VA market and how the associated guarantees impact insurers' regulatory capital. Section 3 introduces the first part of the model to demonstrate how policy generation in the VA space leads to a riskier portfolio allocation and engenders inter-connectedness. Section 4 introduces the insurance-level data and calibrates the model's predictions. Section 5 provides several simulations exercises, based on the model's fire sale mechanism, to quantify the impact of various negative shocks. Section 6 discusses the implications of our results for other common forms of financial commitments, and Section 7 concludes.

2 VAs and Guarantees

From the perspective of an insurer, a VA policy is a combination of business lines related to asset management and life insurance. An insurer allocates policyholder savings to a separate account and acts as a delegated asset manager of policyholder's funds. Absent any guarantees, the separate account is a pass-through account in which a policyholder bears all investment risk. The life

[8] Klinger and Sundaresan (2018) provide evidence that swap spreads tend to be negative when pension plans are underfunded. Their explanation is that underfunded pension funds prefer to hedge their duration risk with swaps rather than buying Treasuries because swaps require only modest investment to cover margins, whereas buying a government bond requires outright investment. Rauh (2006) documents that firms' defined benefit pension plans contributions can reduce real investment.

insurance component of the VA is the option held by the policyholder to convert the funds to an annuity at retirement (see, Kartasheva (2014), for example).

The presence of guarantees, where we focus on those linked to the equity market, turns VAs into put option-like instruments. Given this characteristic, regulations were introduced to safeguard annuity investors by forcing insurers set aside reserves. In addition, insurers are required to hold additional capital to absorb extreme losses that might arise from the guarantees.[9] To determine the reserve and required risk-based capital (RBC), insurance regulators supply various scenarios for the joint path of several asset classes, namely treasury bonds, corporate bonds, and equity prices. Insurers then gauge any possible equity deficiency by simulating the values of their VA business under each supplied scenario (keeping the highest present value of equity deficiency in each path). The reserve is computed as the conditional mean over the upper 30 percentile of the equity deficiencies. It should be kept in mind, though, that there is no presumption that this value will be identical to the market value. The required capital is calculated as the conditional mean over the upper 10 percent of equity deficiencies minus the reserve.

VAs with embedded guarantees also introduce new features to an insurer's overall business model. Like any delegated portfolio manager, the size of assets under management becomes a significant driver of profitability. To attract funds in a highly competitive market, insurers are incentivized to offer generous guarantees that may later prove costly in certain states of the world. These guarantees also prompt more complex hedging strategies and induce more aggressive investments to offset the hedging costs. Making this issue of more immediate concern, the ultra-low yield monetary policy environment has placed additional pressure on insurers to search for yield among a limited set of available securities.

For traditional insurance risks, insurers' reserves are set to match (a profit margin adjusted) expected periodic payment to policyholders. Based upon standard asset-liability matching, reserves are usually invested in fixed-income securities. As an insurer may face insufficient reserves if it

[9] Specifically, Actuarial Guideline 43 (or Actuarial Guideline 39, prior to December 2009) determines the reserve value of variable annuities, and, since December 2005, the C-Phase II regulatory standard determines the contribution of variable annuities to required capital (Junus and Motiwalla (2009)).

underestimates, say, the average longevity risk of its clients, this risk would be largely idiosyncratic to the insurer. In sharp contrast, the size of the reserves associated with guarantees, now among the largest liabilities on insurers' balance sheets, fluctuates with stock market performance and interest rates. To demonstrate this feature, we plot in Figure 3 the evolution of reserves for the VA guarantees in relation to various insurance characteristics that directly measure different aspects of financial health: the evolution of insurers' gross reserves to capital (Panel A), their stock returns (Panel B), and their return on equity (Panel C) for the period from 2004 to 2013. Each year, life insurers with VAs are divided into two groups by the ratio of gross reserve to capital. The "high" ("low") group includes life insurers with a ratio of gross reserve to capital larger than (less than or equal to) the median. For comparison, the annual averages for life insurers without guaranteed VAs are also plotted.

As expected, reserve additions (and, as a consequence, capital reductions) spike when two conditions emerge: declining stock markets and interest rates, which are the conditions that feature in most recessions. Panel A also highlights the devastating effects on the reserves associated with the rapid deterioration of equity markets, such as the 2008-2009 financial crisis and the 2011-2012 price decline due to the Euro crisis. This is central for the understanding of how the VA business can create contagion during such a shock. As insurers' RBC ratios rapidly deteriorate, they will be under pressure to improve their financial health and this can be obtained through two ways: issuing new equity or reducing risk from their balance sheet by engaging in fire sales, akin to a de-leveraging process. Given that issuing equity may be very difficult during stress periods, engaging in regulatory-induced fire sales become more likely. The aggressive investments in risky and illiquid securities, also induced by the VAs, may further exacerbate the fire-sale externality.

While not necessarily implying causality at this stage between VAs and systemic risk, it is important to highlight the role that the growing non-traditional life insurance business may play in the event of a sustained market-wide stress event. The impact that such non-diversifiable risk poses to life insurers is borne out when one considers the experiences of several prominent insurers.[10]

[10] AIG, Hartford Financial Services Group, and Lincoln National were among those that aggressively wrote investment-oriented life policies that had minimum guarantees attached to them. Besides the well-known case of AIG,

3 A Model of Guarantees, Hedging, and Portfolio Choice

In this section, we develop a simple model that facilitates an understanding of the links between, on the one hand, an insurer's guarantee activities and any associated hedging and, on the other, its portfolio choice. We consider an insurer whose risk-taking is constrained by its (regulatory) capital position. The consequence is that whenever the insurer's risk in one part of the portfolio is, say, reduced (for example because of hedging), it has the capacity (and the will) to increase risk in other parts of its portfolio. The central focus of the analysis is on understanding how the insurer's portfolio allocations will be affected by (effective) guarantee-exposures, which will then be estimated using U.S. life insurance data in Section 4. In the analysis, we will take both the guarantees as well as any associated hedging as given. We can think of these characteristics as being determined endogenously by the business model of the insurer.

3.1 Setup

Consider one insurer and three dates $(0, 1, 2)$. The insurer has total funds A, of which E is capital and $A - E$ are liabilities. The insurer can use the funds to invest in three assets: stocks, an illiquid, risky bond, and a liquid, safe bond. The expected (date-2) returns on stocks and the illiquid bond are r_S and r_I ($r_S > r_I$), respectively; the return on the liquid bond is normalized to zero: $r_L = 0$.

At date 0, the insurer decides upon the fraction of funds to invest in stocks, the illiquid, and the liquid bond. We denote the respective portfolio weights as α_S, α_I, and α_L ($\alpha_i \in [0, 1]$). The portfolio choice has to obey two constraints. First, there is a constraint arising from the guarantee and any associated hedging. We consider an insurer who has sold (guaranteed) variable annuities where the stock market value underlies the associated guarantee. Specifically, the insurer has written guarantees of size $g > 0$, expressed as a fraction of its balance sheet (so the total guarantee is gA). We denote the (absolute value) of the option delta of a unit of the guarantee with $|\delta|$. Thus, when

Hartford Financial was also bailed out by the Troubled Asset Relief Program in 2009, and the reason was precisely the significant losses they faced arising from the VA business unit. Hartford eventually sold its VA business in 2013.

the stock market falls by one unit, the total value of the guarantee written increases by $|\delta|\, g$. We refer to $|\delta|\, g$ as the *effective* guarantee exposure. The option delta will reflect the characteristics of the guarantee written, for example, the degree to which it is out-of-the-money. We assume that the insurer hedges a proportion h ($h \in [0, 1]$) of the guarantee using delta hedging.[11] For this, the insurer sells short $h \cdot |\delta|\, g$ units of the stock market at date 0 and invests the proceeds into the remaining part of its portfolio. The guarantee thus amounts to the restriction that at least $h \cdot |\delta|\, g$ has be invested in the two bonds:

$$\alpha_L + \alpha_I \geq h \cdot |\delta|\, g. \tag{1}$$

Note that the guarantee does not impose a similar constraint on the stock market position, as we allow for short-selling of stocks.

Second, the insurer is subject to risk-sensitive capital requirements. We assume that the regulatory risk-weights on the three assets, denoted γ_i ($i \in \{S, I, L\}$), are proportional to their (date-2) returns (reflecting the fact that assets that are priced to have higher returns posses commensurately higher risk levels):

$$\frac{\gamma_S}{r_S} = \frac{\gamma_I}{r_I} \text{ and } \gamma_L = 0. \tag{2}$$

Note that the regulatory risk-weights do not reflect the potential fire-sale discount; that is, regulation does not (fully) capture illiquidity risk. This is the imperfection that will allow for the creation of systemic risk.

The insurer's total risk-weighted assets are then $(\alpha_S \gamma_S + \alpha_I \gamma_I)A$. Given a required capital-adequacy-ratio of ρ, the insurer's portfolio thus has to fulfill the following risk-weighted capital constraint:

$$\frac{E}{(\alpha_S \gamma_S + \alpha_I \gamma_I)A} \geq \rho. \tag{3}$$

The inequality presumes that the guarantee itself do not affect regulatory constraints (in Section 6 we

[11] As discussed previously, an alternative (although expensive) means of hedging is to buy outright puts. Such hedging has the effect of simply neutralizing (a part of) the guarantee. That is, the effective guarantee exposure when a proportion h_D is directly hedged becomes $(1 - h_D)\, |\delta|\, g$. In our estimations we will replace the effective guarantee exposure with this expression and hence also take into account direct hedging.

will discuss how the regulatory treatment of guarantees will affect our analysis). Note though that hedging – even though not directly apparent in equation (3) – does affect the regulatory constraint. For example, an insurer who invests the proceeds from hedging exclusively into liquid bonds will see its risk-weighted assets decline by $\triangle\alpha_S\gamma_S = h \cdot |\delta|\, g\gamma_S$. Note also that an alternative interpretation of equation (3), aside from a regulatory constraint, is that of the insurer's target ratio of risk.

At date 1 with probability π ($\in (0,1)$), we assume that a shock arrives which forces the insurer to sell a part of its portfolio. In Section 5, we model this explicitly as being the result of a capital shortfall. For now, we assume that the shock forces the insurer to sell a proportion s ($\in (0,1)$) of its portfolio. While we assume that the liquid bond and stocks can be sold at their expected value, we also assume that this is not the case for the illiquid bond (see Ellul, Jotikasthira and Lundblad (2011) for an analysis of why bonds held by insurance companies are subject to illiquidity concerns). Specifically, the illiquid bond can only be sold at a discount $c > 0$ to its date-0 value. We can think of c as the bond's fire-sale discount, which is determined by the total amount of selling in the economy and hence will be taken as given by an individual insurer.

Finally, we assume that all assets pay out their respective returns at date 2.

3.2 Portfolio Choice

At date 0, the insurer chooses portfolio weights to maximize its expected profits. When the shock does not arrive at date 1, the insurer will earn the respective return r_i (in expectation) on each asset. However, when the shock does arrive, the insurer's overall portfolio return will be reduced by $\alpha_S r_S + \alpha_I(r_I + sc)$. The insurer's expected date-2 return is hence given by $\alpha_S(r_S - \pi(r_S)) + \alpha_I(r_I - \pi(r_I + sc))$.[12]

We make the following three assumptions:

Assumptions: (i) $r_I > \frac{\pi}{1-\pi}sc$, (ii) $\gamma_I > \frac{E}{A\rho}$, and (iii) $h \cdot |\delta|\, g > 1 - \gamma_S\frac{E}{A\rho}$.

[12]We assume that the insurers remain solvent. Therefore, we can ignore the pay-outs to guarantee holders and holders of other liabilities. These payouts are a given to the insurer and hence will not affect the optimization problem. Similarly, the analysis ignores any influence of the traditional insurance activities – an issue to which we will return in Section 6.

The first assumption ensures that the illiquid bond, taking into account the probability that it has to be liquidated at date 1, has a positive expected return to the insurer.[13] The second assumption states that the capital constraint is sufficiently tight so that the insurer cannot invest everything into the illiquid bond (and hence also not stocks). The third assumption says that effective guarantee exposure (after hedging) has to be sufficiently large. If guarantees are very small, the bond holdings required to hedge the guarantee will be small as well, and will not affect the optimization problem (that is, the constraint (1) will not bind).

The insurer's maximization problem can be written as

$$\max_{\alpha_S,\alpha_L,\alpha_I} \alpha_S(r_S - \pi(r_S)) + \alpha_I(r_I - \pi(r_I + sc)), \text{ subject to} \tag{4}$$
$$\alpha_L + \alpha_I \geq h \cdot |\delta|\, g,\, \alpha_S\gamma_S + \alpha_I\gamma_I \leq \frac{E}{A\rho},\, \alpha_S + \alpha_L + \alpha_I = 1,\, \alpha_L, \alpha_I \geq 0.$$

The insurer's resulting optimal portfolio is given by

$$\alpha_S^* = 1 - h \cdot |\delta|\, g, \tag{5}$$
$$\alpha_I^* = (\frac{E}{A\rho} - (1 - h \cdot |\delta|\, g)\gamma_S)\frac{1}{\gamma_I}, \tag{6}$$
$$\alpha_L^* = h \cdot |\delta|\, g - (\frac{E}{A\rho} - (1 - h \cdot |\delta|\, g)\gamma_S)\frac{1}{\gamma_I}. \tag{7}$$

Proof. To build intuition, let us first consider the insurer's problem when there is no guarantee constraint (that is, we drop condition (1)). The insurer's objective is to maximize expected returns, subject to the condition that it has sufficient regulatory capital. The question thus becomes one of which asset gives the highest return per unit of risk-weight. Since risk-weights are proportional to date-2 returns, in the absence of fire-sales, the insurer is indifferent between investing in stocks and the illiquid bonds. However, due to the fire-sale cost, it strictly prefers stocks. The insurer will thus not hold any illiquid bonds ($\alpha_I = 0$), and invest up to the regulatory constraint (3) in stocks. From solving equation (3) for $\alpha_I = 0$, we obtain $\alpha_S = \frac{1}{\gamma_S}\frac{E}{A\rho}$. The remaining funds will be invested into

[13] This has the consequence that expectations of fire-sales at date 1 do not (directly) affect portfolio allocations at date 0. See Wagner (2011) for an analysis of optimal portfolio strategies in the presence of fire-sale risk.

the liquid bond. The holdings of the liquid bond can hence be obtained from the budget constraint ($\alpha_S + \alpha_L = 1$): $\alpha_L = 1 - \frac{1}{\gamma_S}\frac{E}{A\rho}$. Note that $\alpha_I + \alpha_L = 1 - \frac{1}{\gamma_S}\frac{E}{A\rho}$ fulfills the guarantee constraint (1) only when $h \cdot |\delta|\, g \leq 1 - \frac{1}{\gamma_S}\frac{E}{A\rho}$, that is, precisely when assumption (iii) is not fulfilled. It follows that insurers are constrained by the guarantee whenever assumption (iii) holds.

Let us now solve for the optimal portfolio when the guarantee constraint binds (assumption (iii) holds). Since the insurer strictly prefers stocks to bonds, it will only invest into bonds up to the guarantee constraint ($\alpha_L + \alpha_I = h \cdot |\delta|\, g$) and invest the remaining funds into stocks: $\alpha_S^* = 1 - h \cdot |\delta|\, g$. The division between illiquid and liquid bonds in the bond portfolio (of size $h \cdot |\delta|\, g$) will be determined by the regulatory constraint: the insurer will invest into illiquid bonds until its capital adequacy ratio becomes binding: $(1 - h \cdot |\delta|\, g)\gamma_S + \alpha_I \gamma_I = \frac{E}{A\rho}$. Rearranging gives $\alpha_I^* = (\frac{E}{A\rho} - (1 - h \cdot |\delta|\, g)\gamma_S)\frac{1}{\gamma_I}$. The rest is held in liquid bonds: $\alpha_L^* = h \cdot |\delta|\, g - (\frac{E}{A\rho} - (1 - h \cdot |\delta|\, g)\gamma_S)\frac{1}{\gamma_I}$.

From (6) we can derive the following result:

Result 1 *Larger guarantee amounts will increase holdings of the illiquid bond ($\frac{\partial \alpha_I^*}{\partial g} > 0$).*

The intuition is the following. A higher amount of guarantees means that, because of delta hedging, an insurer will necessarily have lower stock market exposure (α_S is lower) and will hold more bonds ($\alpha_I + \alpha_L$ is higher). The insurer's regulatory risk thus declines, and it has room to pursue returns. As it cannot scale down on its overall bond holdings (because of the hedging constraint), it has to take the risk *within* the bond portfolio, that is, invest more in higher yielding illiquid bonds.

In the next section, we will calculate effective guarantee exposures $|\delta|\, g$ using U.S. life insurers' balance sheet data. We then use two of the model's implications on equilibrium portfolio holdings, (5) and (6), to estimate the hedging ratio, h, and the relative risk-weight of stocks, $\frac{\gamma_S}{\gamma_I}$, from insurers' actual holdings of stocks and illiquid bonds. The next result highlights the relationship between portfolio holdings and effective guarantees:

Result 2 *The sensitivities of stock and illiquid bond holdings to effective guarantees are given by*

$$\frac{\partial \alpha_S^*}{\partial\, |\delta|\, g} = -h \text{ and } \frac{\partial \alpha_I^*}{\partial\, |\delta|\, g} = h\frac{\gamma_S}{\gamma_I}. \tag{8}$$

The sensitivity of an insurer's stock market position to guarantee activity is simply the (negative of the) hedging ratio $-h$ because the associated hedging requires one-to-one short-selling in the stock market. The sensitivity of an insurer's holdings of illiquid bonds to guarantee activity is given by $h\frac{\gamma_S}{\gamma_I}$. This is because each unit of stocks sold relaxes the capital constraint by $\frac{\gamma_S}{\gamma_I}$ (since $\gamma_S > \gamma_I$, the effect is always larger than one). Since the insurer sells h units in total, it can increase its holdings of illiquid bonds by $h\frac{\gamma_S}{\gamma_I}$.[14]

4 Data and Model Calibration

To calibrate the model, we use the National Association of Insurance Commissioners (NAIC) data, obtained through SNL Financial, on VAs' account values, gross reserves, and reinsurance credits (from the General Interrogatories), portfolio holdings (from Schedules A, B, BA, and D), and derivatives positions (from Schedule DB). The data frequency is annual and the unit of observation is firm-year, where each firm refers to an independent life insurer or all life insurers in the same group (e.g., AIG). The sample period is from 2004 to 2013, although we have the data on VAs spanning 2003-2015.

Figure 4 plots the aggregated VAs' account value, summed across all insurers, and its associated gross reserve over time. The collective account value significantly increased from about \$840 billion in 2003 to almost \$1.5 trillion in 2007, due in part to the rise in the stock market. Over the same period, the associated collective gross reserve (part of insurers' liabilities) remained relatively low, potentially because the guarantees were deep out-of-the-money. In 2008, as the stock market collapsed, the aggregate account value dipped and the gross reserve spiked from about \$10 billion to almost \$60 billion. Since then, the account value recovered and eventually surpassed the previous peak in 2007. Despite the recovery, the gross reserve remained relatively high and volatile, reaching a new high at over \$70 billion during the 2011-12 European sovereign debt crisis.

Only a handful of life insurers underwrite VAs with guarantees, and even a smaller number do

[14] Our model implies that the insurer offsets reduced risk-taking in the equity market with higher risk in the bond universe. However, it should be noted that hedging still provides diversification benefits as it reduces overall exposure to the stock market (coming through both direct holdings of stocks and the guarantee).

so in a significant amount. In comparison to the average life insurer, these insurers tend to be very large and sophisticated. To create a relevant sample, we therefore start with the sample of insurers that underwrite VAs and add to it other insurers with average assets greater than or equal to the fifth percentile of the average assets of VA-underwriting insurers. Our sample includes a total of 176 unique insurers, of which 82 underwrite VAs at some point. Our sample covers about 33% by number and over 98% by assets of the life insurance industry during our sample period. To observe firm characteristics and asset allocations that are associated with VAs, we divide the insurers each year into three groups: [1] those with a greater than median VA exposure, as measured by the ratio of gross reserve to capital, [2] those with a lower than median VA exposure, and [3] those with no VA exposures. It is important to note that the VA exposures of insurers in groups [1] and [2] are vastly different; the average ratio of gross reserve to capital is 0.241 for the former and just 0.003 for the latter.

Table 1 presents summary statistics on several relevant firm characteristics as well as portfolio asset allocations across the three groups of insurers. The statistics are pooled across firm-year observations in each group. Panel A shows that insurers with high VA exposures are generally larger than others both in terms of assets (in the general account or on balance sheet) as well as capital and surplus. Their average assets and capital are $54,452 million and $4,959 million, respectively, larger than the corresponding averages for insurers with low VA exposures by $22,353 million (significant at 10%) and $1,363 million (not significant). Insurers with no VAs are the smallest, even with the size filter that we impose. Insurers with high VA exposures also have slightly lower RBC ratios, returns on equity, and stock returns than the other two groups; the differences, however, are not statistically or economically significant.

In Panel B of Table 1, we present the portfolio asset allocations across the three groups of insurers. To simplify our analysis and map the data to the model, we divide assets into four broad groups: (i) liquid bonds (L in the model), (ii) illiquid bonds (I in the model), (iii) common stock exposures (S in the model), and (iv) other assets (not in the model). The composition of each group is listed underneath the group's heading; for example, the liquid bonds comprise cash, synthetic

cash (from selling futures), bonds in NAIC classes 1 (rated A- and above) and 2 (rated BBB- to BBB+), and agency asset/mortgage backed securities (ABS) in NAIC classes 1 and 2. Both groups of insurers that underwrite VAs have significantly lower liquid bond allocations (about 8%) than insurers with no VAs. The differences are driven by cash and agency ABS in NAIC class 1 but are partially offset by synthetic cash from selling stock futures.

Insurers with high VA exposures have a significantly higher allocation to illiquid bonds than do both insurers with low or no VA exposures. The differences are in the same direction for all types of assets that we classify as illiquid bonds, although the magnitudes are most economically significant among private-label ABS, mortgages, and loans. For common stock exposures, we consider both cash stocks and stock futures, and find that insurers with high VA exposures, relative to the other two groups, both have smaller allocations to cash stocks and sell disproportionately more stock futures. Together, the summary statistics for the asset allocations are generally consistent with our model's predictions. Insurers that underwrite VAs hedge their guarantee exposures by selling common stocks and buying bonds, and tilt their portfolio towards illiquid bonds to utilize the reduction in risk-based capital and maximize their portfolio expected return. Insurers' allocations to other assets are relatively small and uninteresting.

To formally isolate the effects of VAs on insurers' asset allocations and evaluate their contribution to systemic risk, we calibrate the parameters of our model. We focus on two parameters, the delta hedge ratio (h) and the ratio of RBC requirements for common stocks and illiquid bonds (γ_S/γ_I), which together determine the tilt towards illiquid bonds. As shown in equations (5) and (8), h is the negative of the sensitivity of the common stock allocation to an insurer's guarantee-induced stock market exposure, or equivalently the delta normalized by total assets ($|\delta|\, g$). Equations (6) and (8) indicate that $h \cdot (\gamma_S/\gamma_I)$ is the sensitivity of the illiquid bond allocation to the normalized delta. We therefore calibrate our model by simply regressing asset allocations on the normalized delta.

We start by uncovering the normalized delta for each insurer. The insurer-level data on guaranteed VAs only tell us the account value and the gross reserve. Considering the gross reserve as a proxy for the value of the guarantee (which is admittedly imprecise), we can calculate the delta as

the change in gross reserve per a one unit change in the account value, given the existing VAs. Since we cannot separate the account value and gross reserve into the components associated with the existing VAs and new business, we rely on the law of motion for the account value and gross reserve under a set of simplifying assumptions. Ignoring non-linearities, the law of motion states that

$$\frac{reserve_t}{value_t} = \frac{reserve_{t-1} + \delta_{t-1} \cdot value_{t-1} \cdot ret_{stock,t-1,t} + newreserve_t}{value_{t-1} \cdot (1 + ret_{t-1,t}) + newvalue_t}$$

We infer $newvalue_t$ as the $value_t - value_{t-1} \cdot (1 + ret_{t-1,t})$, using a combination of stock returns (CRSP value-weighted index including dividends) and three-month interest rates as an estimate of $ret_{t-1,t}$. Based on the aggregate statistic from SNL Financial, we assume that 77% of the account value is associated with VAs with common stocks as underlying assets and the remaining is associated with money-market VAs.

We then calculate $newreserve_t$ by assuming that the guarantee remains at the same level as before, and therefore $newreserve_t = newvalue_t \cdot (\frac{reserve_t}{value_t})$. Finally, we assume that the reserve on the money-market VAs remains constant, such that the change in reserve on existing VAs is driven primarily by the stock market return, $ret_{stock,t-1,t}$, implying $\delta_{t-1} \cdot value_{t-1} = (reserve_t - reserve_{t-1} - newreserve_t)/ret_{stock,t-1,t}$. Finally, the normalized delta, $\delta_{t-1} \cdot \frac{value_{t-1}}{assets_{t-1}}$, corresponds to $|\delta|g$ in the model.

In practice, hedging complicates our calculation of the normalized delta since some hedge coverage can be used as a credit, under the regulation, to offset the gross reserve. We divide derivatives earmarked for hedging the VAs into two groups. The first group includes mostly (long) put options, which we assume are sufficiently 'effective' to be used as credit to offset the gross reserve. We refer to the use of these derivatives as direct, comprehensive hedging, which insurers report as either 'effective' or 'other hedging'. We assume that 70% (50%) of effective (other) hedging offsets the calculation of gross reserve [15] so that the change in gross reserve for existing VAs is induced by the unhedged assets plus 30% of assets covered by effective comprehensive

[15] According to Actuarial Guideline 43, up to 70% of derivatives used to hedge VA guarantees can be used to offset the gross reserve.

hedging plus 50% of assets covered by other comprehensive hedging. Because comprehensive hedging is, in reality, sufficiently effective regardless of the regulatory treatment, our calculated delta is biased upward as a measure of the sensitivity of the guarantees to the stock market. We therefore further adjust the normalized delta above by a factor equal to the ratio of unhedged assets to the sum of unhedged assets and uncredited comprehensive hedges.

The second group of derivatives includes linear derivatives, mostly stock market futures, which we assume are part of the delta hedge program highlighted in our model. We do not allow these derivatives to offset the gross reserve and layer them directly as synthetic cash and common stocks on top of insurers' asset allocations. To hedge the guarantee, insurers sell stock market futures, resulting in positive synthetic cash and negative synthetic common stocks, as observed in Table 1.

Having obtained the inputs, we then perform our main calibration by regressing each insurer's allocation to each asset group, liquid bonds, illiquid bonds, common stocks, and others, on the normalized delta. Table 2 reports the results. We include year fixed effects and control for insurers' capitalizations by also including the RBC ratio. In Panel A, we obtain the coefficient estimates by OLS with robust standard errors clustered by insurer. In Panel B, we use seemingly unrelated regressions (SUR) and impose a natural cross-equation constraint that the sum of coefficients on the normalized delta and the sum of coefficients on the RBC ratio equal zero (i.e., the sum of asset allocations is fixed at one). We calculate standard errors by boot-strapping, using 500 repetitions. The results in both panels are almost the same, and the cross-equation constraints in Panel B are not rejected. Nevertheless, we will use the SUR estimates in our subsequent analysis.

Many insurers in our sample have no or very low exposures to VAs, and therefore zero or very low normalized deltas. For this reason, the sample standard deviation of normalized delta is just 0.050. The coefficient estimates show that a one standard deviation increase in normalized delta is associated with an increase in the illiquid bond allocation of about 0.090 (or 9 percentage points), where 0.056 comes from liquid bonds (i.e., decrease in the liquid bond allocation of 0.056) and 0.033 comes from common stock exposures. These re-allocations are both statistically and economically significant. The decrease in other assets is insignificant and negligible in magnitude.

The coefficients on the RBC ratio show that insurers with high RBC ratio tend to allocate more of their assets to liquid bonds and less to the three other asset groups. This implies that the RBC ratio may reflect insurer-specific risk aversion and/or risk bearing capacity, and each insurer may strive to maintain its RBC ratio target over time, consistent with our model's underlying assumption.

Table 3 reports statistics on hedge coverage and the capital requirement for illiquid bonds, calculated from the data and from the estimates in Table 2, Panel B. We focus only on insurers with high exposures to guaranteed VAs, as these insurers together account for over 90% of all underwritten VAs in our sample. Moreover, our model predicts that the portfolio tilt towards illiquid bonds only occurs among insurers with significant exposures to VAs (above a certain level). The first two rows of Panel A report the statistics for effective and other comprehensive hedge coverage. Through the lens of regulation, insurers' use of options for hedging the guarantee is not considered effective, and thus the effective comprehensive hedge coverage is essentially zero for all firms. However, insurers do use options extensively, covering on average about 0.052 (or 5.2%) of the guarantee. The median insurer does not use options, possibly to avoid paying the option premia.

Insurers delta-hedge much of the remaining guarantee exposure. As discussed, our model interprets the negative of the coefficient on the normalized delta in the regression of common stock allocation as the delta hedge coverage, i.e., h = 0.655. We multiply h by the proportion of guarantee exposure that remains after comprehensive hedging to obtain the delta hedge coverage for each insurer. The average delta hedge coverage is 0.690, with the 90% confidence interval (calculated by bootstrapping) between 0.658 and 0.721. Thus, all together, insurers that underwrite most of guaranteed VAs hedge about three quarters of their guarantee exposure.

Our model also interprets the corresponding coefficient in the regression on the allocation to illiquid bond as the product of the delta hedge ratio and the relative capital requirements for common stocks vs. illiquid bonds (in excess of the capital requirement on liquid bonds). From this interpretation, and the fact that the capital requirement for common stocks is 0.30 (assuming beta of one), our coefficient of 1.809 implies that the capital requirement for illiquid bonds equal to 0.113, with the 90% confidence interval (calculated by bootstrapping) between 0.049 and

0.177. Our back-of-the-envelope calculation from each asset type that makes up the illiquid bond grouping suggests the capital requirement of about 0.060, which is slightly lower than our regression estimate but lies within the 90% confidence interval. Both the delta hedge coverage and the capital requirement for illiquid bonds implied by our estimation are reasonable and consistent with the data and anecdotes from conversations with practitioners. Below, we use our estimates to infer counter-factual portfolios.

Our model indicates that the portfolio we observe reflects the insurer's desire to both hedge a certain portion of its guarantee exposure as well as to maximize the expected return. The former leads to over-weighting bonds and under-weighting common stocks in general, while the latter tilts the bond allocation towards illiquid (and riskier) ones. The literature often refers to such tilt as "reaching for yield." To examine the contribution of VAs to systemic risk and to isolate the effect of reaching for yield, we create the following two counter-factual portfolios. Portfolio 1 captures the desired delta hedge coverage (same as reported above) but eliminates the reaching-for-yield tilt. We construct Portfolio 1 by keeping the sum of allocations to liquid and illiquid bonds the same as the actual portfolio, but re-allocate between them such that the ratio of liquid and illiquid bond allocations is the same as what we would observe should the insurer have no VAs. Reaching for yield thus explains the difference between the actual portfolio and Portfolio 1, as well as their differential systemic risk.

In contrast, Portfolio 2 explores the implication of completely eliminating the guarantee exposure. We construct it by unwinding the products of coefficients from Panel B of Table 2 and the normalized delta for each insurer, i.e., setting the normalized delta to zero. The guarantee exposure and its hedging account for the difference between Portfolios 1 and 2. Together, the guarantee exposure and the reaching for yield incentives that stem from delta hedging the stock market exposure comprise the main sources of systemic risk coming from VA underwriting. Hedging the guarantee exposure, in its pure form, however, helps reduce the risk.

Panel B of Table 3 reports allocations of Portfolios 1 and 2, and their bootstrapped standard errors. Compared to the actual portfolio, Portfolio 1, on average, allocates 0.109 less to illiquid

bonds and 0.109 more to liquid bonds. These are the effects of reaching for yield. Compared to the actual portfolio of insurers with no VAs, Portfolio 1 allocates 0.045 less to common stocks, reshuffling that amount to liquid and illiquid bonds, which helps hedge VA-induced stock market exposures. Note that Portfolio 1's allocations maintain the same levels of guarantee exposure and hedge coverage as the actual portfolio of insurers with high VA exposures.

By construction, Portfolio 2 has significantly less illiquid bonds and significantly more liquid bonds and common stocks than the actual portfolio. Without VAs, insurers no longer need to hedge their stock market exposures and therefore focus on maximizing the expected return, increasing stock and decreasing bond holdings. Portfolio 2 looks quite similar to the actual portfolio of insurers that do not underwrite VAs, confirming that our estimates in Table 2 that we use to construct Portfolio 2 fit the data well. This provides comfort that our calibration is realistic and internally consistent.

5 Fire Sales and Systemic Risk

Having calibrated our model to U.S. insurance data, we now analyze how a shock at date 1 can lead to fire sales. We expand the model introduced in Section 3 by assuming that there is a continuum of insurers, each of size A, with effective guarantee exposures, $|\delta|\, g$, and hedging ratios, h.

Consider a shock that reduces the total value of insurers' assets by ε (including any change that may come through the value of the guarantee). Following this, insurers will be in violation of their required (or target) capital adequacy ratios. We assume that insurers restore their capital by deleveraging.[16] As in Greenwood et. al (2015), we assume that they do so by selling assets proportionally to pay down debt.[17] Illiquid bonds potentially suffer from fire-sale discounts, as outlined in Section 3. We assume that when an amount S of illiquid bonds in the economy is sold, the bonds are traded at a discount of $c_0 S$ ($c_0 > 0$) to their date-0 value. The asset discount will

[16]Equivalently, one can also assume that insurers hold the size of their balance sheet constant and sell risk assets (stocks and illiquid bonds) in return for liquid bonds ("flight to quality").

[17]Proportional selling is the insurer's optimal liquidation as long as the fire-sale discount does not become too large and as long he does not change his hedging behaviour (equation (1)). This is because the insurer's date-1 portfolio problem is essentially the same as the one of date 0 (calculations available on request).

lead to an additional deterioration in insurers' capital positions, which in turn will require even more liquidations (and so on...). This is a fire-sale externality (e.g., Stein 2012): selling by some insurers will depress prices and hence worsen the capital positions of all insurers in the economy, thus forcing additional liquidations throughout the system.

The shock ε lowers insurers' capital by

$$\triangle E = -\varepsilon A - \alpha_I A \cdot c_0 S. \tag{9}$$

The first term ($-\varepsilon A$) is the direct reduction in equity arising from the negative asset shock. The second effect ($-\alpha_I A \cdot c_0 S$) is due to losses from the fire-sales discounts (these losses are permanent for the illiquid bonds that are sold but temporary for the ones remaining on balance sheet).[18] The reduction in insurers' assets is identical to the one for capital, except that assets are also reduced by asset sales s, expressed as a proportion of the balance sheet:

$$\triangle A = -\varepsilon A - \alpha_I A \cdot c_0 S - sA. \tag{10}$$

Following asset sales, each insurer will be again in fulfillment of its capital requirements. From equation (3), it hence follows that the equity multiplier, A/E, does not change between date 0 and date 1 (effectively, because the composition of assets does not change, the capital requirement becomes a leverage constraint). Capital and asset growth hence mirror each other:

$$\frac{\triangle E}{E} = \frac{\triangle A}{A}. \tag{11}$$

Using (9)-(11), we can then solve for an insurer's asset sales:

$$s = (\varepsilon + \alpha_I \cdot c_0 S)\frac{A - E}{E}. \tag{12}$$

[18] We assume that assets on the balance-sheet are marked-to-market, that is, the asset discount on illiquid bond also reduces date-1 capital through the illiquid bonds that are not sold. The regulatory accounting treatment of assets during crisis is nuanced in practice (see Ellul, Jotikasthira, Lundblad, and Wang (2014)); if assets on the balance sheet are not marked-to-market, fire-sale spirals will be less pronounced.

This expression illustrates the two channels through which a shock affects capital, and hence asset sales. The first term, $\varepsilon \frac{A-E}{E}$, is the direct effect of asset values, depending on the size of the shock and leverage. The effect of fire-sales on deleveraging is represented by the second term, $\alpha_I \cdot c_0 S \frac{A-E}{E}$. Aggregate liquidations of the illiquid bond reduce capital, thus causing more asset sales at a specific insurer than in the absence of fire sales. The equation also highlights the fire-sale externality: liquidations by some institutions ($S \uparrow$) will increase that at others ($s \uparrow$).

We now introduce superscript i to denote an individual insurer, and aggregate the amount of sales across insurers to obtain the economy-wide sales of the illiquid bond: $S = \sum_i s^i \alpha_I^i A^i$. Inserting s^i from equation (12) and solving for S give:

$$S = \frac{\varepsilon \sum_i \frac{A^i - E^i}{E^i} \alpha_I^i A^i}{1 - c_0 \cdot \sum_i \alpha_I^i \frac{A^i - E^i}{E^i} \alpha_I^i A^i}. \tag{13}$$

Recalling that α_I is increasing in $|\delta|\, g$ (equation (6)), we obtain

Result 3 *Total fire sales of the illiquid bond (S) are increasing in the effective guarantee exposure of each insurer ($|\delta|\, g$).*

The reason for this result is that more guarantees mean that insurers hold a larger proportion of their portfolio in illiquid bonds (because of hedging, α_I is higher, as shown in Section 3). This leads to more fire sales through two channels. The first channel is captured by the numerator; for a given shock, deleveraging requires proportional sales of all assets, including the illiquid bond. The second channel is the fire-sale externality, as captured by the denominator. A given amount of liquidation leads to more illiquid bond being sold, which depresses its price and leads to even more selling.

From equation (13), we can derive an expression for the total fire-sale cost. This can be viewed as a measure of *systemic risk* in our economy. Using that total costs are given by $C = S \cdot c_0 S$, we obtain:

$$C = c_0 \left(\frac{\varepsilon \sum_i \frac{A^i - E^i}{E^i} \alpha_I^i A^i}{1 - c_0 \cdot \sum_i \alpha_I^i \frac{A^i - E^i}{E^i} \alpha_I^i A^i} \right)^2. \tag{14}$$

As expected, higher guarantee exposures $|\delta|\, g$ (and thus higher α_I) lead to more systemic risk.[19]

[19] Systemic risk may lead to welfare losses as it can cause instability in the financial system, for example, due to a failure of insurers in fulfilling their commitments (potentially spreading to other connected financial institutions). In

In our simulations we will consider three types of shocks. First, a shock that hits the stock market. Specifically, we consider a reduction in the valuation of stocks by ε_S, which will directly reduce each insurer's equity by $\alpha_S \varepsilon_S A$ (omitting the superscript i for simplicity) and increase the value of the guarantee, leading to a further reduction in the insurer's equity (by increasing reserves) by $|\delta|\, g\varepsilon_S A$. The shock thus leads to a total reduction in the insurer's equity by $\alpha_S \varepsilon_S A + |\delta|\, g\varepsilon_S A$ and is thus identical to an asset shock of size $\varepsilon = (\alpha_S + |\delta|\, g)\varepsilon_S$. Second, a shock that proportionally reduces the value of illiquid bonds by ε_I. This shock will lower the value of insurer equity by $\alpha_I \varepsilon_I A$ and is hence identical to an asset shock of $\varepsilon = \alpha_I \varepsilon_I$. Finally, we consider a shock directly to the guarantee, increasing its value by ε_G percent (this may for example be the result of an increase in the volatility in the stock market or a persistent decline in interest rate). This is identical to an asset shock of $\varepsilon = g\varepsilon_G$. Note that we consider a shock to the value of the guarantee that does not come from the underlying stocks, and thus the δ plays no role here.

We now perform a simulation exercise to quantify the amount and costs of asset fire sales when insurers are hit by a market shock. We focus on insurers with high exposures to VAs, and parametrize equations (13) and (14) as follows. We use the price impact (c_0) of 18.6 basis points per \$10 billion of sale, which is the Net Stable Funding Ratio (NSFR) estimate for non-agency MBS by Duarte and Eisenbach (2016). Under the NSFR framework, assets are assumed to be slowly liquidated over one year. Therefore, our estimates of fire-sale amounts and costs may be on the conservative side, as insurers are more likely to take less than a year to adjust their portfolios and maintain their target RBC ratios.[20] We set the capital requirement for common stocks (α_S) at 0.30 and the total invested assets, capital and surplus, RBC ratio, and normalized delta of the guarantee at their averages during the period from 2010 to 2013 (after the crisis).

Clearly, sufficiently large shocks can induce fire sales by insurers, even if the insurers have no exposures to VAs. To distinguish between the fire sales that would have occurred anyway and those that are induced by the guarantee, we use the counter-factual portfolios in Panel B of Table 3 along

the absence of any instability issues, fire-sales are a pure zero-sum event, benefitting the buyers of illiquid bonds (e.g., focused distressed hedge funds) at the cost of the sellers.

[20] Greenwood et al. (2015) use the price impact of 10 basis points per \$10 billion of sale for all assets.

with the actual portfolios from Table 1. As discussed, Portfolio 2 is the portfolio that would have prevailed had the insurers not underwritten VAs, and thus the differences in fire-sale amount and cost between the actual portfolio and Portfolio 2 are attributable to the VAs, inclusive of hedging and reaching for yield. We further decompose the total VA-induced fire sales into three components. First, we layer VAs on top of Portfolio 2, and the differences between fire-sale amount and cost between Portfolio 2 with and without VAs are merely due to the gross VA exposure. Portfolio 1 lies between the actual portfolio and Portfolio 2, reflecting actual VA hedging but not reaching for yield. Thus, the differences between Portfolios 1 and 2, both without actual VA exposure, come from hedging. Finally, reaching for yield explains the differences between the actual portfolio and Portfolio 1.

We consider three types of shocks following our theoretical discussion above: a shock to the stock market, a shock to the illiquid bonds, and a shock to the guarantee. We also examine a categorical shock in which both stocks and illiquid bonds are proportionally affected to show that due to non-linearity, the total fire-sale costs of two asset shocks are greater than the sum of the costs associated with each shock. In each analysis, we assume that the price impacts are negligible for liquid bonds and common stocks, and focus on the fire sales of illiquid bonds.

Table 4 reports the results for negative stock market shocks, ranging in magnitude from 10% to 40%. Panel A considers the fire-sale externality, in which the amount of selling fully anticipates the aggregate price impact due to selling by other insurers. The results show that the stock market shocks of 10-40% would result in actual insurers selling \$114-\$458 billion of illiquid bonds, of which the majority (\$64-\$256 billion) is an amount attributable to the VAs (through the lens of our model). The corresponding fire-sale costs (value losses) are \$2-\$39 billion, representing 1-21% of insurers' total capital and surplus. Most of the fire sale costs, amounting to \$2-\$31 billion, are due to the VAs.

Our decomposition shows that the fire sales are largely associated with *ex ante* reaching for yield. Without the tilt of hedge coverage towards illiquid assets, the fire-sale amount would be 44% smaller, and the fire-sale costs would have been 69% smaller. These results imply that the exposure

to VAs alone, sufficiently hedged, is not the main culprit behind the generation of systemic risk; in contrast, the reaching for yield behavior is, contributing more to fire sales than the guarantee exposure itself. Insurers hedge about three quarters of their guarantee exposures, and as a result reduce their vulnerability to stock market shocks by 71% in terms of fire-sale amount and 77% in terms of fire-sale costs. However, they tilt their hedge towards illiquid bonds to take advantage of the reduction in RBC from under-weighting common stocks, and in the process, make themselves even more vulnerable to fire sales and illiquidity.

Panel B of Table 4 does not consider the fire-sale externality, shutting down the c_0 term in equations (13) and (14). The results show that for a given magnitude of shock, the amount of fire sales decreases by almost half, compared to the counterpart in Panel A. Further, the costs fall by about three quarters, suggesting that it is not the fire sales, *per se*, but rather the fire-sale externality that is the most important driver behind the systemic risk among financial institutions.

In Table 5, we examine the separate effects of negative shocks to illiquid bonds (Panel A), categorical shocks (Panel B), and positive shocks to the value of the guarantee (Panel C). The results show that the shocks to illiquid bonds of 2-8% (equivalent to 10-40% stock market shocks given the capital requirements of the two assets) would result in actual insurers selling $108-$431 billion of illiquid bonds, of which $55-$220 billion are attributable to the VAs. The associated fire-sale costs are $2-$26 billion, representing 1-19% of insurers' total capital and surplus. About three quarters of the fire-sale costs are due to the VAs, and virtually all of the effects are again a result of reaching for yield. Note that hedging, even without the reaching-for-yield tilt, increases rather than decreases insurers' vulnerability here because the insurers hedge only their stock market exposure and do so using a combination of liquid and illiquid bonds.

We construct a categorical shock simply as a shock to all assets with relative magnitudes proportional to their expected returns. For example, a shock of -10% would decrease the common stock price by 10% and the illiquid bond price by 2% (0.06/0.30*10%). In this sense, a categorical shock can be thought of as the sum of shocks to stocks and illiquid bonds. The results show that when risky assets move in tandem, the illiquidity costs increase exponentially as the price impact per

unit of sale increases in the sale amount. Although the fire-sale amounts for the 10-40% categorical shocks are approximately the sums of the corresponding amounts in the cases of independent shocks to stocks and illiquid bonds, the fire-sale costs are much larger than the sums of the two. The costs attributable to VAs range from \$8 billion for a 10% shock to over \$129 billion for a 40% shock, representing 4-70% of the insurers' capital and surplus. Since risky assets are often correlated in an economic downturn, though not perfectly as in the case we examine here, our results suggest that regulators and researchers take into account asset correlations in assessing financial institutions' capital adequacy.

Finally, the positive shocks to the value of guarantee of 20-80% would induce insurers to sell \$115-\$460 billion of illiquid assets. These effects are exclusively due to the VAs, by construction. The costs associated with these fire sales are \$2-\$39 billion, of which about 72% are attributed to reaching for yield. It is important to note that we hold the magnitudes of the shocks comparable across the scenarios in Tables 4 and 5, consistent with the capital requirements and the variation in gross reserve we observe in our sample. For categorical shocks, these magnitudes are stressful and similar to what we observe during the financial crisis, and for shocks to value of the guarantee, these magnitudes are consistent with the jump in gross reserve from 2010 to 2011. Our results highlight several channels through which VAs create vulnerabilities and systemic risk in the insurance sector.

6 Discussions

In this section, we discuss (i) how relaxing some of our assumptions will affect the results, and (ii) some broad implications of our results beyond the insurance industry. To begin, our analysis has taken a simplified view on the regulatory treatment of guarantees. First, insurers have to put aside reserves to account for, loosely speaking, expected losses from the guarantees. This will reduce insurers capital already at $t = 0$. However, insurers will also receive a fee for offering the guarantee, which will add to equity and offset the reserve (precisely neutralizing it when the guarantee fee is fairly priced). In addition, we should keep in mind that our analysis is *conditional*

on insurer's leverage and capital positions. So, effectively we compare insurers that have the same capital position but differ in their guarantee exposures, thus isolating any effect that comes through changes in equity.

Second, we only consider capital requirements for the asset portfolio. In practice, an insurer also needs to hold capital against business risk, including that arising from the written guarantees as well as other insurance activities. In Appendix B, we extend the model to take account of these capital requirements. Specifically, we consider an insurer who expands into VAs with guarantees while at the same time scaling down his traditional insurance activities to maintain the same regulatory risk level, as measured by the RBC ratio. We show that this leaves unchanged the impact of underwriting the guarantees on the holdings of stocks and bonds (equation (8) still holds) but changes the impact of VAs on fire-sales and systemic risk. If the insurer can only adjust its assets but not its business risk and liabilities, then upon a negative shock at $t = 1$, it will have to sell a larger proportion of its assets, compared to our baseline results, in order to maintain its target RBC ratio. Fire-sales, and systemic costs, hence increase. In addition, the move into VAs now changes the overall risk exposures, as the guarantees and the traditional insurance business are not equally affected by a common shock. For example, the liabilities arising from writing certain type of insurance may be fairly insensitive to changes in the valuation of stocks.[21] In this case, the effect of guarantees will be the same as in the baseline model. By contrast, if the value of the traditional business is also affected by the shock, the impact of guarantees on fire-sales may be intensified or mitigated.[22]

We make various other simplifying assumptions in our baseline analysis. Relaxing or changing these assumptions may increase or decrease our estimates of fire sales and systemic risk. First, we follow Greenwood et at. (2015) in assuming that upon a shock, insurers sell assets proportionally to stay within their target RBC ratio. As noted, this requires that $|\delta|\, g$ remains constant and the insurers maintain the same hedge ratio, in which case each insurer needs the same asset mix to

[21] As noted by Koijen and Yogo (2017a,b), the risks of traditional insurance are largely idiosyncratic; for example, the number of deaths among policy holders is unlikely to be meaningfully correlated with the stock market.

[22] Consider an increase in interest rates. This reduces the value of the insurer's asset portfolio. At the same time, it may also reduce the value of liabilities arising from the insurance business, thus mitigating the effect of the shock on the insurer's equity. This means that an insurer that moves from traditional insurance to VAs increases his interest rate sensitivity through two channels, resulting in higher fire-sales in the event of interest rate shocks.

hedge its VAs and maintain the RBC ratio. In reality, a decline in stock prices or a categorical shock is likely to increase the guarantee's moneyness and hence increase $|\delta|\, g$. If the insurer wants to maintain the same hedge ratio, it will need to go further short the stocks and long the bonds, both liquid and illiquid. This will counteract the fire sales of illiquid bond (but increase the sale of stocks, which may lead to a further decline in stock prices should we permit the price impact for the stocks in our model).[23]

We also assume that insurers mark to market all their assets such that a decline in asset prices, including the fire-sale feedback effect, is immediately reflected on their balance sheets. In practice, life insurers are required to mark to market their stock holdings but not their bond holdings; they can continue to hold bonds at modified historical costs unless the bonds are in NAIC category 6 (in or near default). The use of historical cost accounting (HCA), as opposed to marking to market (MTM), would reduce our estimates of VA-induced fire-sale costs, particularly those that occur through the externality. However, as noted by Ellul et al. (2015), the accounting of bonds suffering a large negative shock is not black or white; the insurer using historical cost accounting is still required to recognize Other-Than-Temporary-Impairment (OTTI) if the change in value of their bond holdings is deemed permanent. Various NAIC reports show that the insurers holding mortgage-backed securities recognized over $25 billion of OTTI in 2008-10. Hence, our estimates of fire sales, based on MTM, may not be that much higher than reality, even if insurers largely use HCA.

Often, large price declines are accompanied by rating downgrades and worsened liquidity. For example, Ellul et al. (2015) show that during the 2008-9 crisis, prices of mortgage and asset backed securities held by insurers declined across the board, with the median loss around 30%. At the same time, almost all of these securities were significantly downgraded, with many from AAA to speculative grades. Worsened credit ratings increase capital requirements, tighten the capital constraint, and therefore increase the amount of fire sales of risky assets, including both illiquid

[23] Another potential departure from proportional selling is that insurers often hold a tiny fraction of their portfolios in stocks and the amount of cash stocks that can be sold to maintain the RBC ratio may be limited. This implies that insurers may have to sell more illiquid bonds once they have depleted all of their cash stock holdings.

bonds and stocks. Further, worsened liquidity increases the price impact of fire sales, and hence the overall fire-sale costs.

Finally, our empirical estimates consider the fire-sale externality only among life insurers with greater than median exposures to VAs. Although these insurers are the largest, they still represent just about 50% of the life insurance industry in the U.S. In addition, property and casualty insurers and other financial institutions also hold stocks and illiquid bonds, and are regulated by similar capital adequacy requirements. For example, according to the 2017-Q3 Flow of Funds Report, life insurers together hold $2.7 trillion of corporate bonds and asset backed securities while P&C insurers and chartered depository institutions hold another $0.9 trillion. If we were to account for other related illiquid assets, such as loans and mortgages, the holdings of institutions outside our sample would become even more significant. Taking into account the propagation of the fire sale externality by these other institutions (one unit of fire sales generating even more fire sales), the overall systemic risk induced by VAs could be even larger.

Although we study the VA guarantees, our results have implications that extend beyond VAs and the insurance industry. Similar forms of guarantees exist in other contexts such as defined benefit pensions and asset backed securities' conduits. Defined benefit pension plans commit to paying out a certain amount of cash flows to retirees. These obligations are supported by the plans' assets, which are managed by the plan sponsors, often private and public employers. Like the insurers that underwrite VAs, the plan sponsors face the tension between hedging and return (or more broadly profits) as a perfect hedge would require that they invest in Treasuries and very safe assets but since these assets offer low returns, they need to contribute more and suffer lower profits. These sponsors therefore have incentives to reach for yield, by either investing in riskier assets (deviating from the hedge or tilting the hedge towards illiquid bonds) or by taking more risk in their normal business, e.g. increasing leverage etc. When the sponsors or the plans' assets get hit by a negative shock, underfundedness emerges and widens, forcing the sponsors to put in even more money and move towards safer assets. These can affect corporate investments, day-to-day business, and default risk of the sponsors, not necessarily inducing fire sales in the literal sense. However, if many sponsoring

firms engage in this reaching for yield behavior, they may collectively be a conduit in transmitting asset price shocks to the real economy.

The guarantees that sponsoring banks provide investors in a securitization deal are also similar to the guarantee in VAs (see Acharya, Schnabl, and Suarez (2013)). These guarantees are structured (through a recourse conduit) to minimize capital requirements, while helping to obtain lower yields in the securitized bonds. It is thus natural that the relieved capital is used to support other risky activities, resulting in the overall risk (including the off balance-sheet guarantees) being greater than intended by the capital regulation. While we do not know whether banks try to hedge these guarantees by shorting the securitized bond exposures and buying Treasuries or other fixed income assets, their ability to reduce the required capital on the guarantee permits these banks to take on more risk (in mostly illiquid assets anyway), working in the same way as VA hedging permitting insurers to overload illiquid bonds. As the securitized assets suffer a negative shock, the guarantee becomes binding and losses pile up, forcing the sponsoring banks to cut down risky assets and loans on their balance sheets and potentially engendering systemic fire sales.

7 Conclusion

We explore how systemic risk may arise from the inter-connectedness of the asset side of financial institutions' balance sheets. Specifically, we propose an innovative mechanism, namely, an incentive that arises from the financial institutions' business model rather than regulation per se, that can induce correlated investments across financial institutions. While capital regulation is an essential part of our analysis, asset similarity across insurers instead arises from insurers' business decisions to provide (and hedge) investment products. This endogenous *ex ante* reaching for yield can then generate *ex post* fire sale externalities that will propagate systemic risk. To study this, we employ the U.S. life insurance sector as a laboratory and focus on the recent transformation of the largest life insurers from institutions offering traditional insurance products to ones that offer investments products in the form of variable annuities.

To investigate how variable annuity guarantees induce correlated investments by insurance companies operating in this space, we present a theoretical model that captures the underlying economics and then calibrate the model by using insurer-level data. We propose a theoretical model that shows that, as a result of the fact that hedging the risks associated with these variable annuities lowers expected profitability, individual insurers have an incentive to aggressively reach for yield in the limited riskier asset space. We calibrate the model using insurance-level data and confirm that insurers operating in the VA space allocate their portfolios to riskier, illiquid bonds. Investing in similar illiquid assets emerges in equilibrium, increasing the likelihood of fire sales in the event of common shocks and in turn imposing externalities among insurers.

Our paper shows that the transformation of the life insurance industry has made these institutions less likely to behave as asset insulators, able to absorb temporary market dislocations. More importantly, by virtue of the asset inter-connectedness, they are now *more* likely to contribute to systemic risk through correlated regulatory-induced fire-sales. This has far reaching implications for the stability of the financial system.

Finally, it is worth reminding that the importance of the particular guarantees embedded in the VA market are not necessarily unique. The correlated risk-taking that these types of guarantees incentivized may not be restricted to the insurance sector. Dangerously underfunded private and U.S. state pension funds may too be induced to reach for yield. As various forms of guarantees pervade the global finance system, made perhaps only more problematic in the ultra-low interest rate environment after the finance crisis, regulators should perhaps be wary of the implications of asset side vulnerability to the broader financial sector associated with the mechanism that we identify.

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Appendix A: Types of Variable Annuity Guarantees

A variable annuity (VA) is a long-term investment product, primarily designed to facilitate retirement. The retirement system in the U.S. consists of three pillars. Pillar 1 is the Social Security retirement benefit that provides a government-sponsored minimum retirement income. Pillar 2 includes employer-sponsored defined benefit and defined contribution plans. Finally, Pillar 3 consists of tax-deferred private asset accumulation schemes, which can be used to supplement the other two sources of retirement income. The VA contract belongs to the third group.

VAs are contracts between an insurer and a policyholder that consist of accumulation and payout phases. During the accumulation phase, the policyholder contributes funds to the annuity account, and the balance is invested in stock and bond market instruments such as mutual funds, ETFs, etc. The policyholder has some discretion to allocate investments among different asset classes offered by an insurer. She can also terminate the contract and withdraw the account value of the policy, subject to any surrender costs. Once the contract reaches the payout phase, a policyholder may receive her purchase payments plus investment income and gains (if any) as a lump-sum (thereby terminating the contract), or may choose to receive them as a stream of regular payments (annuitize). If she decides to annuitize, the annuity payments can last a fixed number of years or until the end of life, depending on the contract terms.

VAs expose a policyholder to market risk as the amount that she can eventually annuitize is uncertain and depends on future realized stock market performance. To help policyholders manage this risk, insurers offer a host of guarantees. The latter come in various forms, effectively providing an assurance for a policyholder that her savings and annuity payments are protected from adverse market conditions.

Guarantees are sold as additional contracts (often referred to as "riders" by insurers) in conjunction with a variable annuity contract. In particular, there is a separate schedule of fees paid by a policyholder for each guarantee. While there are many different types of guarantees associated with VAs, we focus on those that are linked to the equity market.

The guarantees can be divided in two broad categories, a guaranteed minimum death benefit (GMDB) and guaranteed living benefits (GLB). GLBs are riskier than GMBD to an insurer. A GMDB payment is contingent on a life event and is limited to the purchase amount. By contrast, GLBs can be exercised strategically by a policyholder, in particular when the purchased amount substantially exceeds the account value. This can be particularly problematic for insurers following poor capital market performance.

The table below summarizes the most common types of guarantees.

Guarantee	**Description**
Guaranteed minimum death benefit (GMDB)	The GMDB guarantees that if a policyholder dies, a beneficiary receives the greater of the account value, or all purchased payments minus all withdrawals. A policyholder may choose a stepped-up feature under which the guaranteed amount is based on a higher value than all purchases minus withdrawals. For example, a policyholder purchases a $150,000 variable annuity and selects a GMDB. Following poor capital market performance, the value of the account is $75,000 in 10 years. A policyholder dies in year 10 of the policy. A beneficiary receives $150,000.
Guaranteed lifetime withdrawal benefit (GLWB)	The GLWB provides a guarantee to withdraw a fixed percentage of the purchased payments regardless of the market performance. It does not require annuitization. For example, a policyholder purchases a $150,000 variable annuity and a GLWB of 4% annually. Following poor capital market performance, the value of the account is $75,000 in 10 years. A policyholder is in a good position though because she will receive $6,000 (4%*150,000) for life; the lifetime income is guaranteed and not limited to $150,000.

Guarantee	Description
Guaranteed minimum income benefit (GMIB)	GMIB provides a floor or a guaranteed minimum annuity payment regardless of investment performance. However, a policyholder also has the option to lock-in a higher payout when annuitizing if investments have performed well and the contract value exceeds what it would be with the minimum income benefit. For example, a policyholder purchases a $150,000 variable annuity and selects a GMIB that provides 4% annually. Following poor market performance, the variable annuity contract value is only $75,000 at the end of 10 years. But a policyholder has $222,036 to annuitize as a result of the GMIB.
Guaranteed minimum withdrawal benefit (GMWB)	The GMWB provides a guarantee to withdraw a fixed percentage of the total annuity premia each year regardless of market performance. The income payments are guaranteed until the total purchased amount is recovered. The GMWB does not require annuitization. For example, a policyholder purchases a $150,000 variable annuity and selects a GMWB that provides 4% annually. Following poor capital market performance, the variable annuity contract value is only $75,000 at the end of 10 years. A policyholder is in a good position though because she will receive $6,000 ($150,000 x 4%) per year until the $150,000 is recovered.
Guaranteed minimum accumulation benefit (GMAB)	The GMAB guarantees a minimum contract value-regardless of investment performance-after a set period of time. The minimum contract value is typically equal to or greater than the total purchased amount. For example, a policyholder purchases a $150,000 variable annuity and selects a GMAB. Following poor capital market performance, the variable annuity contract value is only $75,000 at the end of 10 years. A policyholder is in a good position though because the variable annuity contract value is still $150,000 at the end of 10 years.

Appendix B: Guarantees and Other Insurance Activities

We consider an insurer who has next to his financial portfolio (consisting of stocks and the two bonds) and guarantee exposures, also traditional insurance activities $t > 0$ (denoted in units of the size of the financial portfolio, A). We assume that when an insurer increases his guarantee-business, he does so in a manner that is neutral to overall (date-0) regulatory risk in his insurance activities (otherwise a risk change would be hard-wired into the analysis). Specifically we assume that the insurer keeps constant the total risk from insurance activities (both traditional and guarantee business):

$$(\gamma_t t + \gamma_g g)A = I, \tag{15}$$

where γ_t and γ_g are the risk-weight for the insurance business and the guarantee and I is total risk-taking in the insurance business. Rearranging above equation we obtain

$$t = \frac{I}{\gamma_t A} - \frac{\gamma_g}{\gamma_t} g, \tag{16}$$

showing that the traditional business declines when more guarantees are written. The new equation for the capital requirement is

$$\frac{E}{(\alpha_S \gamma_S + \alpha_I \gamma_I + \gamma_t t + \gamma_g g)A} \geq \rho. \tag{17}$$

The insurer's optimization problem at $t = 0$ is otherwise identical to the one described in Section 3.

The new expressions for the insurer's optimal date-0 (financial) portfolio are given by (derivation is analogous to equations (5)-(7) in the paper):

$$\alpha_S^* = 1 - h \cdot |\delta|\, g, \tag{18}$$

$$\alpha_I^* = (\frac{E}{A\rho} - \frac{I}{A} - (1 - h \cdot |\delta|\, g)\gamma_S)\frac{1}{\gamma_I}, \tag{19}$$

$$\alpha_L^* = h \cdot |\delta|\, g - (\frac{E}{A\rho} - \frac{I}{A} - (1 - h \cdot |\delta|\, g)\gamma_S)\frac{1}{\gamma_I}. \tag{20}$$

It follows that Result 1 and 2 still apply, in particular, the sensitivity of holdings of illiquid bonds to changes in the guarantee is unchanged. This means that we have the same search-for-yield effect as in the baseline model, resulting in higher holdings of illiquid bonds when guarantee exposure increases.

At date 1, as before, the insurer has to liquidate his (financial) portfolio when the shock hits. There are two differences to the baseline model. First, shocks now also affect equity through their impact on the value of the insurance business. Second, the insurer has to hold capital against the guarantee and the insurance activities.

Following the (partial) liquidation of his portfolio, the insurer will again be in his fulfillment of capital requirements. Hence, the capital constraint (17) will hold with equality at $t = 1$ and $t = 0$:

$$\frac{E}{(\alpha_S \gamma_S + \alpha_I \gamma_I)A + I} = \rho. \tag{21}$$

Total differentiating this condition with respect to time we obtain

$$\triangle E = \rho(\frac{E}{A\rho} - \frac{I}{A}) \triangle A. \tag{22}$$

Using the conditions for the change in equity and assets (equation (9) and (10) in paper) to substitute $\triangle E$ and $\triangle A$ and solving for asset sales s we obtain:

$$s = (\varepsilon A + \alpha_I A \cdot c_0 S)\frac{A - E - \rho I}{E - \rho I}.$$

This expression is identical to the one in the baseline model (equation (12)), except that the "equity multiplier" is now $\frac{A-E-\rho I}{E-\rho I}$ as we also have to take into account the insurance business.

We can make two observations. First, a given asset shock ε now has a larger effect on fire-sales. This can be appreciated by noting that $\left|\frac{\partial s}{\partial \varepsilon}\right|$ is increasing in I (as the equity multiplier has increased). The reason is that since the shock hits the overall balance sheet of the insurer, but the financial portfolio is only is part of it, a larger share of financial assets have to be liquidated to restore the

capital position following a shock. The baseline model thus understates the fire-sale problem.

Second, the impact of a shock on the insurer's assets A may differ from the baseline model. We can conceptually divide the shock ε into two parts, one that affects the traditional insurance business, ε_t, and one that effects the rest of the portfolio, ε_{-t} (which is the one we considered in the baseline model). How the introduction of the insurance business affects the overall impact of a shock (and hence fire-sales) will thus depend on the correlation of ε_t with ε_{-t}. For example, if the insurance business is unaffected by the shock ($\varepsilon_t = 0$), the asset shock will be the same as in the baseline model (but fire-sales will still be larger because of the effect described earlier). If the two shocks are positively correlated, there will be an offsetting effect when the insurer moves into the guarantee business (as this will reduce the traditional business, and hence exposure to ε_t). Similarly, the effect will become more pronounced when the two exposures are negatively correlated.

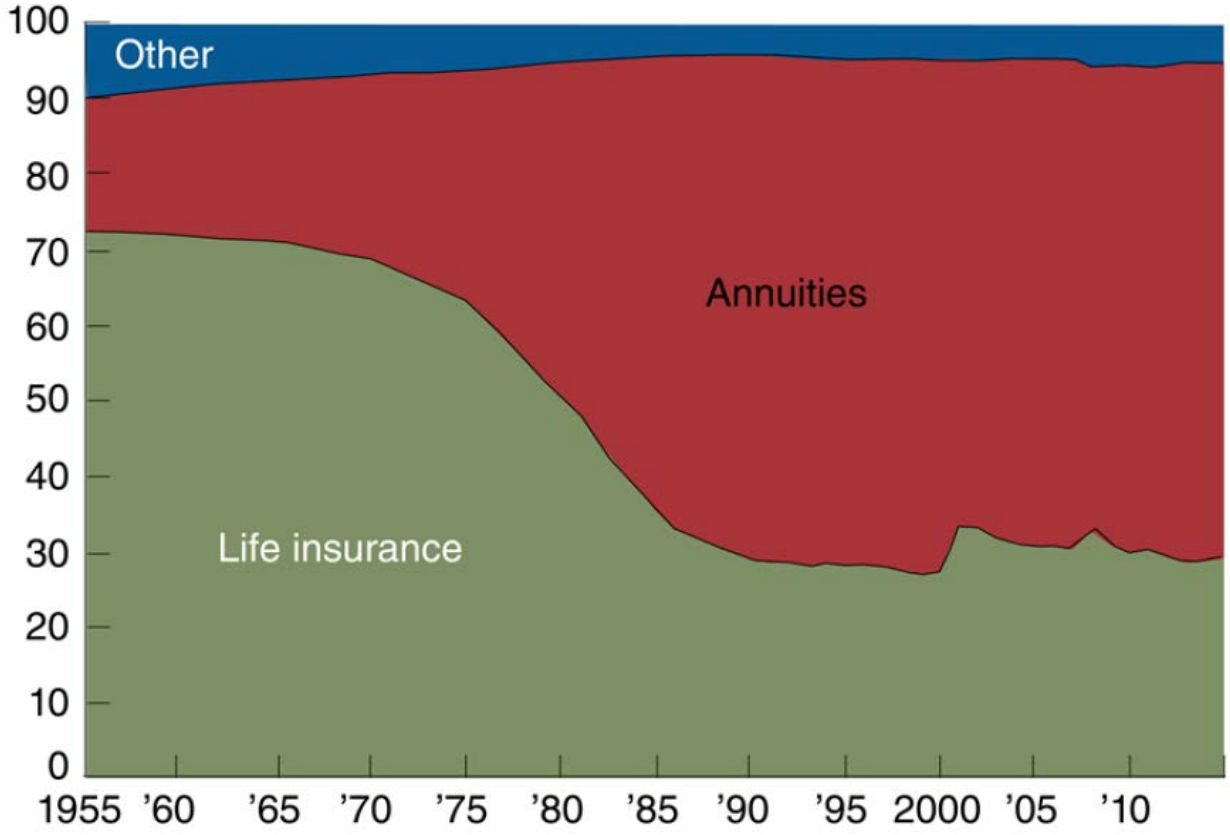


Figure 1: Life insurance product shares over the period 1955-2014.

Source: American Council of Life Insurers, 2015 Life Insurers Fact Book, and Alejandro H. Drexler , Zain Mohey-Deen, Richard J. Rosen.

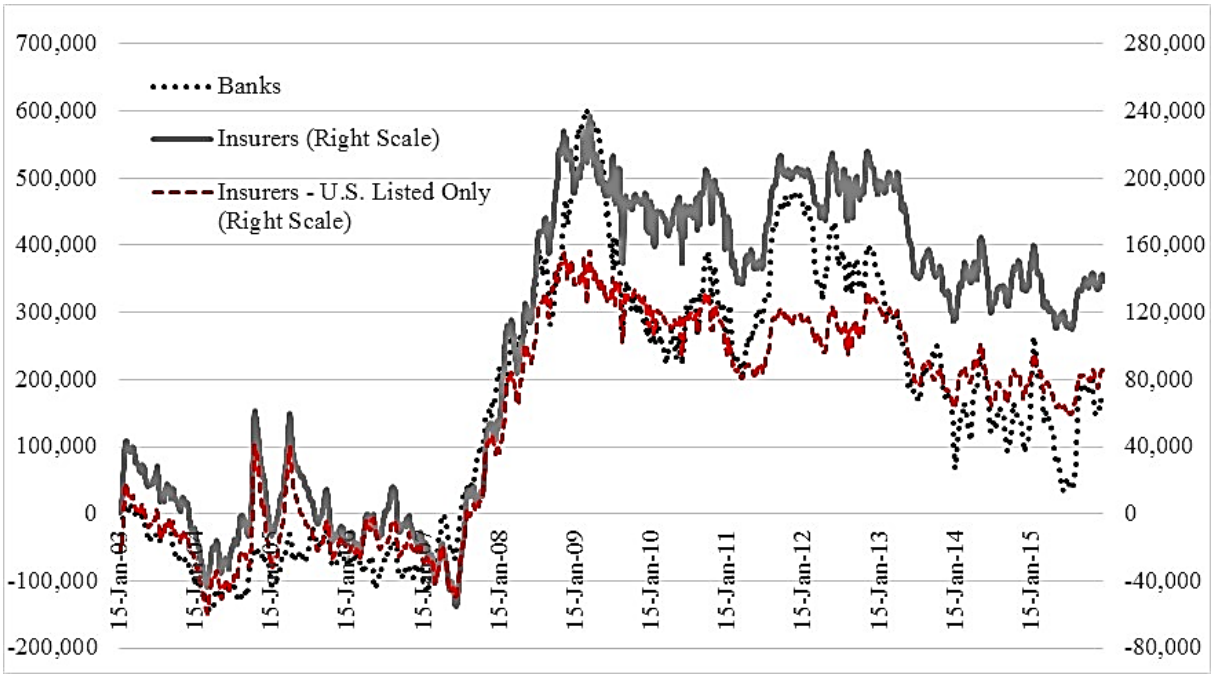


Figure 2: SRISK of Largest Banks and Insurers over Time. This figure plots the time series of (sum of) SRISK ($ million) for (i) ten largest publicly traded banks in the U.S. (dotted line), (ii) ten publicly traded insurers with the largest outstanding guaranteed variable annuities in the U.S. (solid line), and (iii) a subset of insurers in (ii) whose stocks are publicly traded in the U.S. (dashed line). The SRISK data are from NYU Stern Volatility Lab (vlab.stern.nyu.edu/welcome/risk) from January 1, 2003 until end of December 2015.

Panel A: Ratio of Gross Reserve to Capital

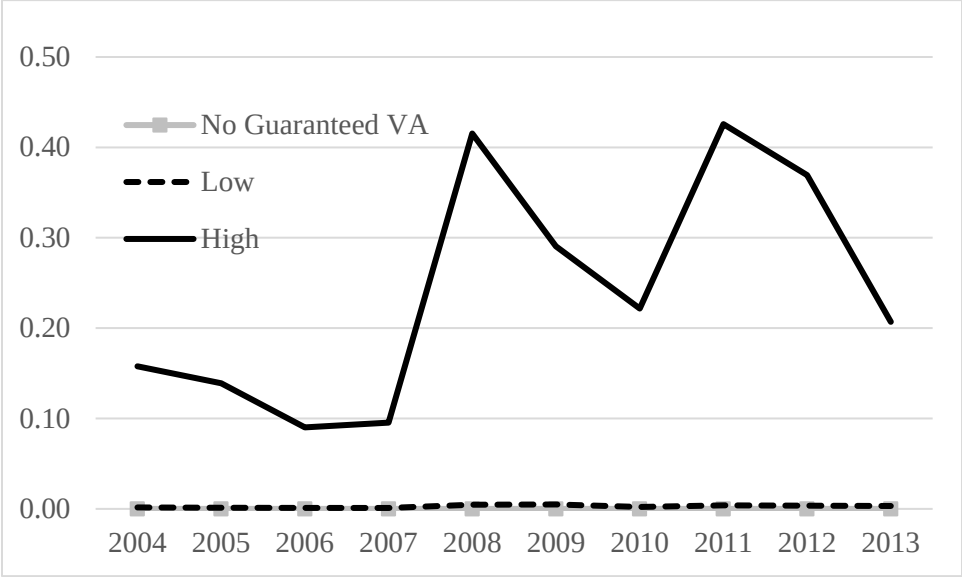
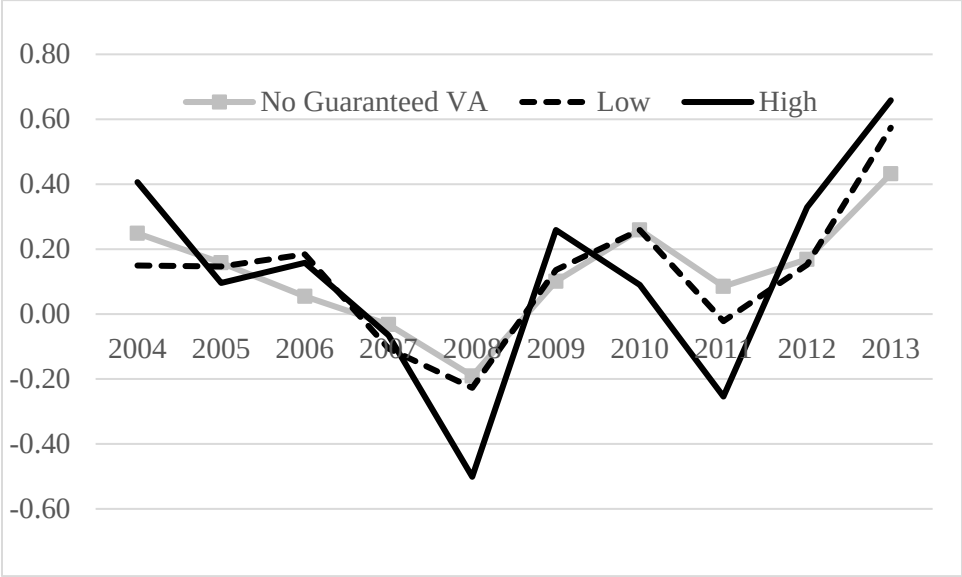


Figure 3: Exposure to Guaranteed Variable Annuities (VA) and Firm Performance. This figure plots the time series of the ratio of gross reserve to capital (Panel A), stock return (Panel B), and return on equity (Panel C) for the period from 2004 to 2013. Each year, life insurers with VA are divided into two groups by the ratio of gross reserve to capital. The "high" ("low") group includes life insurers with ratio of gross reserve to capital higher than (less than or equal to) the median. For each variable, the annual averages across life insurers in each group are plotted— the solid (dashed) line representing the high (low) group. For comparison, the annual averages for life insurers without VA are also plotted (solid line with square markers). Only life insurers that have averaged total assets greater than the fifth percentile of the sample of life insurers with VA are included.

Panel B: Stock Return



Panel C: Return on Equity

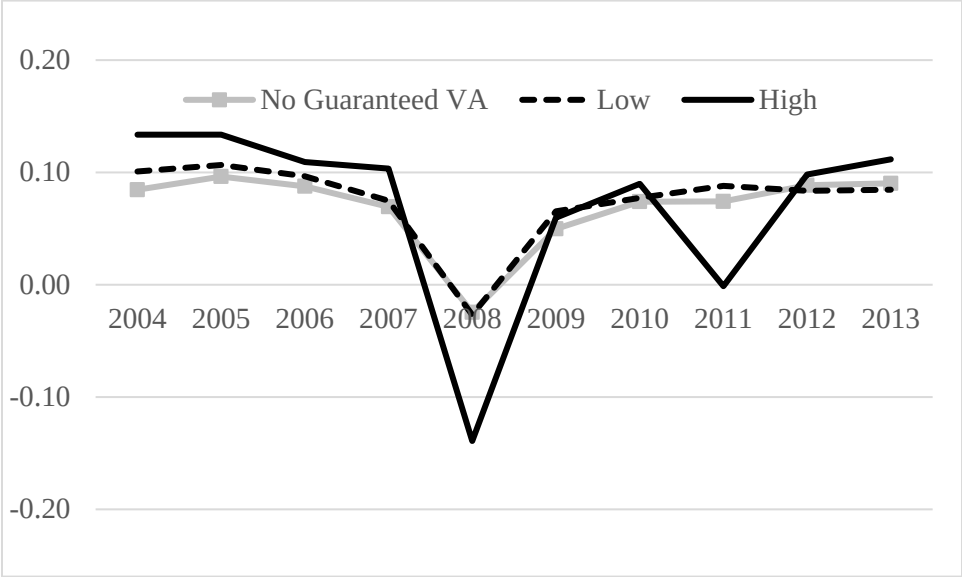


Figure 3, cont'd: Exposure to Guaranteed Variable Annuities (VA) and Firm Performance.

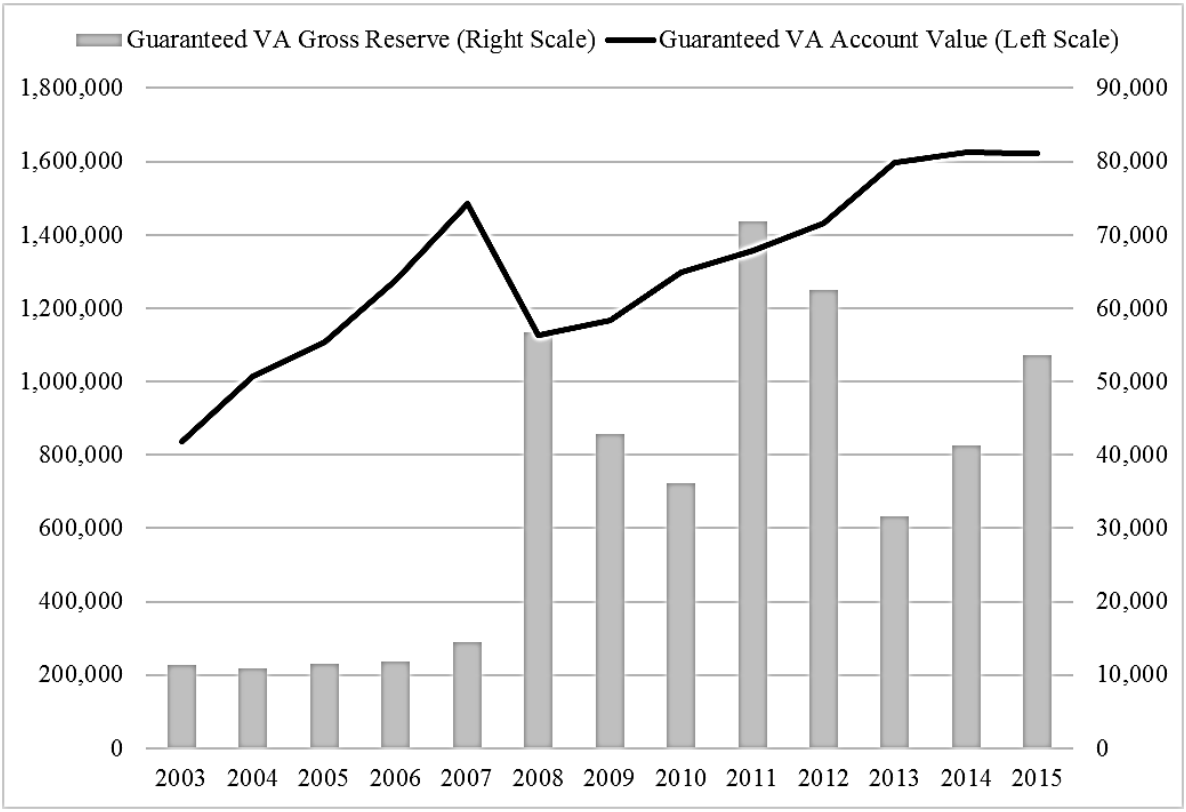


Figure 4: Guaranteed Variable Annuities (VA) Account Value and Gross Reserve over Time. This figure plots the time series of account value (line) and gross reserve (bar) ($ million) for guaranteed VA, as reported annually by life insurers to the NAIC in the general interrogatories form. The sample period is from 2003 to 2015. The account value and gross reserve are summed over individual life insurers with outstanding VA.

Table 1: Summary Statistics of Life Insurers' Characteristics and Asset Allocations

This table presents summary statistics of general firm characteristics (Panel A) and asset allocations (Panel B). The data are from the NAIC, obtained through SNL Financial. The sample period is from 2004 to 2013, and the observation frequencies are firm-year. The sample includes only (group-level) insurers whose average assets in the general account are greater than or equal to the fifth percentile of average assets of insurers that underwrite guaranteed variable annuities (VA). A total of 176 unique insurers are included, of which 82 have guaranteed VA at some point during the sample period. Each year, insurers are divided into three groups by the ratio of guaranteed VA gross reserve to capital. Insurers in groups 1 (2) have the ratios of guaranteed VA gross reserve to capital that are higher (at or lower) than the median. Insurers in group 3 have no guaranteed VA, and are used as a benchmark. For each insurer, assets include all invested assets reported at statutory accounting values, and capital and surplus are assets minus liabilities. Risk-based capital ratio (RBC ratio) is the (adjusted) statutory capital divided by the required risk-based capital. Return on equity is net income divided by common equity, as reported under GAAP. Stock return includes both percentage price change and dividend, and is available only for public insurer groups. Assets are divided into NAIC-defined categories, which are then grouped into liquid bonds, illiquid bonds, common stocks, and others. Asset allocation is the value of assets in each category or broad group divided the value of all invested assets. Tests of difference in mean are conducted using pooled panel regressions with standard errors clustered by calendar year-month. *, **, and *** refer to statistical significance at 10%, 5%, and 1% levels.

	[1] High			[2] Low			[3] No Guarantee			[1] - [2]	[1] - [3]
	Mean	Std. Dev.	Median	Mean	Std. Dev.	Median	Mean	Std. Dev.	Median	Mean	Mean
Panel A: Firm Characteristics											
Assets ($ Million)	54,452	66,070	32,894	32,099	50,509	11,027	5,404	11,198	1,702	22,353*	49,047***
Capital and surplus ($ Million)	4,959	5,827	3,048	3,596	5,721	1,225	712	1,208	244	1,363	4,247***
RBC ratio	9.395	4.945	8.760	10.335	4.605	9.142	10.944	11.248	8.691	-0.940	-1.549
Return on equity	0.065	0.167	0.087	0.074	0.082	0.078	0.069	0.171	0.070	-0.008	-0.003
Stock return	0.116	0.372	0.125	0.127	0.283	0.109	0.120	0.304	0.114	-0.011	-0.003

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	[1] High			[2] Low			[3] No Guarantee			[1] - [2]	[1] - [3]
	Mean	Std. Dev.	Median	Mean	Std. Dev.	Median	Mean	Std. Dev.	Median	Mean	Mean
Panel B: Asset Allocation											
Liquid bonds	0.653	0.115	0.634	0.657	0.135	0.655	0.737	0.138	0.753	-0.004	-0.084***
Cash	0.035	0.036	0.025	0.024	0.025	0.018	0.050	0.081	0.028	0.010***	-0.015**
Synthetic cash	0.032	0.048	0.003	0.006	0.027	0.000	0.000	0.000	0.000	0.025***	0.032***
Bonds in NAIC 1	0.297	0.127	0.274	0.282	0.204	0.279	0.350	0.261	0.378	0.015	-0.053*
Bonds in NAIC 2	0.208	0.064	0.212	0.228	0.111	0.207	0.205	0.139	0.193	-0.020	0.003
Agency ABS in NAIC 1	0.081	0.066	0.066	0.116	0.086	0.106	0.131	0.129	0.104	-0.035**	-0.050***
Agency ABS in NAIC 2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Illiquid bonds	0.326	0.113	0.347	0.288	0.120	0.289	0.195	0.126	0.178	0.038*	0.131***
Long-term assets	0.024	0.021	0.020	0.021	0.022	0.012	0.012	0.018	0.004	0.003	0.013***
Bonds in NAIC 3-6	0.034	0.018	0.032	0.032	0.020	0.032	0.028	0.032	0.019	0.002	0.006
Agency ABS in NAIC 3-6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Private ABS in NAIC 1	0.108	0.060	0.106	0.104	0.083	0.096	0.078	0.090	0.045	0.004	0.031***
Private ABS in NAIC 2	0.011	0.011	0.009	0.008	0.012	0.004	0.007	0.012	0.002	0.002	0.004***
Private ABS in NAIC 3-6	0.008	0.008	0.006	0.005	0.006	0.003	0.004	0.008	0.001	0.003***	0.004***
Mortgages	0.087	0.062	0.097	0.077	0.059	0.087	0.041	0.065	0.005	0.010	0.046***
Loans	0.045	0.047	0.030	0.036	0.031	0.024	0.025	0.031	0.014	0.009	0.021**
Derivatives for income gen.	0.008	0.013	0.003	0.005	0.010	0.000	0.001	0.003	0.000	0.004*	0.008***
Common stock exposures	0.000	0.051	0.010	0.041	0.058	0.026	0.046	0.063	0.021	-0.040***	-0.045***
Common stocks	0.032	0.033	0.022	0.047	0.051	0.027	0.046	0.063	0.021	-0.015*	-0.013*
Synthetic common stocks	-0.032	0.048	-0.003	-0.006	0.027	0.000	0.000	0.000	0.000	-0.025***	-0.032***
Other assets	0.017	0.019	0.010	0.013	0.016	0.010	0.021	0.027	0.012	0.003	-0.004
Preferred stocks	0.008	0.014	0.003	0.008	0.015	0.003	0.013	0.024	0.003	0.000	-0.005**
Real estates	0.006	0.013	0.002	0.004	0.006	0.002	0.006	0.011	0.002	0.002	0.000
Securities lending/borrowing	0.003	0.006	0.001	0.002	0.004	0.000	0.002	0.006	0.000	0.001*	0.001

Table 2: Guaranteed Variable Annuities (VA) and Asset Allocations

This table reports estimates of equation-by-equation OLS regressions (Panel A) and seemingly unrelated regressions (SUR, Panel B) of asset allocations on VAs' delta with respect to the stock market. The sample period is 2004-2013, excluding 2008-2009. Both the asset allocations and the delta are normalized by total invested assets. All models include risk-based capital ratio (RBC ratio) and year fixed effects. Columns (1) - (4) are for liquid bonds, illiquid bonds, common stock exposures, and other assets, as defined in Table 1. In Panel A, standard errors in parentheses are clustered by insurer. In Panel B, the coefficients of both Delta/Assets and RBC ratio are constrained to sum to zero to keep the sum of asset allocations to 100%, and standard errors in parentheses are calculated by bootstrapping, using 500 repetitions. *, **, and *** refer to statistical significance at 10%, 5%, and 1% levels.

Panel A: Equation by Equation OLS

	Asset Allocations			
	Liquid Bonds (1)	Illiquid Bonds (2)	Common Stocks (3)	Others (4)
Delta/Assets	-1.194***	1.857***	-0.667***	-0.034
	(0.349)	(0.340)	(0.221)	(0.045)
RBC ratio	0.003***	-0.002***	-0.000	-0.000**
	(0.001)	(0.001)	(0.000)	(0.000)
Year fixed effects	YES	YES	YES	YES
Observations	1,071	1,071	1,071	1,071
R-squared	0.038	0.043	0.018	0.057

Panel A: Seemingly Unrelated Regressions

	Asset Allocation			
	Liquid Bonds (1)	Illiquid Bonds (2)	Common Stocks (3)	Others (4)
Delta/Assets	-1.119***	1.809***	-0.655***	-0.035
	(0.357)	(0.311)	(0.216)	(0.042)
RBC ratio	0.003***	-0.002***	-0.000	-0.000**
	(0.001)	(0.001)	(0.000)	(0.000)
Year fixed effects	YES	YES	YES	YES
Cross-equation restrictions	Sums of coefficients of Delta/Assets and RBC ratio across equations (1) - (4) are zero.			
Observations	1,071	1,071	1,071	1,071
R-squared	0.038	0.043	0.018	0.057

Table 3: Calibration Results and Counterfactual Portfolios

This table reports summary statistics on actual and calibrated hedge coverage and risk-based capital (RBC) requirement for illiquid bonds (Panel A) and counterfactual asset allocations (Panel B) for the sample of insurers with guaranteed variable annuities (VA) greater than the median. In Panel A, total hedge coverage is divided into comprehensive (effective plus others) and delta hedging. Comprehensive hedging includes hedging using options with negative delta, e.g. buying put options. Effective and other hedging are categorized based on hedging effectiveness and separately reported by insurers. Only derivatives that have common stocks underlying, and are earmarked for hedging VAs are included. Delta hedging is estimated using the coefficient of Delta/Assets in column (3) of Table 2 Panel B, scaled by the fraction of exposure that remains after taking into account comprehensive hedging. RBC requirements for illiquid bonds are calculated in two ways-- (a) from the data as the weighted average of RBC requirements for all asset categories that comprise illiquid bonds, and (b) from the model estimates in Table 2 Panel B as negative of the coefficient of Delta/Assets in column (3) divided by the coefficient of Delta/Assets in column (2) and then multiplied by the RBC requirement of 0.30 for common stocks. In Panel B, the allocations for Portfolio 1 are calculated by keeping the estimated delta hedge coverage the same as actual but without tilting the allocation towards illiquid bonds, i.e. keeping the mix between liquid and illiquid bonds as if the insurers had not underwritten the VAs. The allocations for Portfolio 2 are calculated by adjusting the actual allocations by negative of the products of the coefficients in Table 2 Panel B and Delta/Assets. The pooled averages of Portfolios 1's and 2's allocations are reported, along with their differences from the actual asset allocations as well as the allocations of insurers with no VA exposures. Bootstrapped standard errors, calculated using 500 repetitions, are in parentheses. *, **, and *** refer to statistical significance at 10%, 5%, and 1% levels.

Panel A: Hedge Coverages and Implied Capital Constraint

	Data			Estimation		
	Mean	Std. Dev.	Median	Mean	PCT5	PCT95
Comprehensive hedging - effective	0.000	0.000	0.000	-	-	-
Comprehensive hedging - others	0.052	0.121	0.000	-	-	-
Delta hedging	-	-	-	0.690	0.658	0.721
RBC requirement for illiquid bonds	0.060	0.020	0.058	0.113	0.049	0.177

Table 3, cont'd: Calibration Results and Counterfactual Portfolios

Panel B: Counterfactual Portfolios

	Portfolio 1: Same Level of Guaranteed VA and Hedge Ratio; No Reaching for Yields			Portfolio 2: No Guaranteed VA		
	Mean	Mean - Actual	Mean - Actual of No VA Insurers	Mean	Mean - Actual	Mean - Actual of No VA Insurers
Liquid bonds	0.762***	0.109***	0.025*	0.718***	0.065***	-0.019
	(0.014)	(0.015)	(0.015)	(0.021)	(0.022)	(0.021)
Illiquic bonds	0.217***	-0.109***	0.021	0.221***	-0.105***	0.025
	(0.036)	(0.037)	(0.037)	(0.018)	(0.019)	(0.019)
Common stock exposures	0.000	0.000	-0.045***	0.039***	0.038***	-0.007***
	(0.001)	(0.003)	(0.002)	(0.001)	(0.003)	(0.002)
Other assets	0.017***	0.000	-0.004	0.019***	0.002	-0.002
	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)

Table 4: Fire-Sale Amount and Costs under Stock Market Shock

This table presents estimates of fire-sale amount and fire-sale costs incurred by insurers with VA exposures greater than the median, given a negative shock to stock market. Panel A considers the fire-sale externality in which the amount of selling fully anticipates the aggregate price impact due to selling by other insurers. Panel B does not consider the fire-sale externality. The price impact (c_0) of 18.6 basis points per $10 billion of sale is assumed, following the NSFR estimate for non-agency MBS by Duarte and Eisenbach (2016). The allocations of actual portfolio, Portfolio 1, and Portfolio 2 are as reported in Tables 1 and 3. The risk-based capital (RBC) requirements for common stocks and illiquid bonds are 0.30 and 0.06, respectively. The total invested assets, capital and surplus, RBC ratio, and (percentage) delta exposure of the guarantee are the averages from 2010 to 2013 (after the crisis). The decomposition attributes the difference between the effects on actual portfolio and Portfolio 1, both with actual VA exposures, to "reaching for yield" and the difference between the effects on Portfolios 1 and 2, both with actual VA exposures, to "hedging." The fire-sale effects coming from the "gross VA exposures" are calculated from Portfolio 2 with and without VA exposures.

Panel A: Fire Sales with Full Externality

	Fire-Sale Amount ($ Million)				Decomposition of Fire-Sale Amount ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
10%	114,387	63,792	96,153	50,343	50,595	-32,361	45,810
20%	228,775	127,584	192,306	100,685	101,191	-64,722	91,620
30%	343,162	191,376	288,459	151,028	151,786	-97,083	137,431
40%	457,549	255,168	384,611	201,370	202,382	-129,444	183,241

	Fire-Sale Costs ($ Million)				Decomposition of Fire-Sale Costs ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
10%	2,434	757	1,720	471	1,677	-963	1,248
20%	9,735	3,028	6,879	1,886	6,707	-3,851	4,993
30%	21,903	6,812	15,477	4,243	15,091	-8,665	11,234
40%	38,939	12,111	27,514	7,542	26,829	-15,404	19,972

Table 4, cont'd: Fire-Sale Amount and Costs under Stock Market Shock

Panel B: Fire Sales without Full Externality

	Fire-Sale Amount ($ Million)				Decomposition of Fire-Sale Amount ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
10%	57,120	41,069	64,448	33,743	16,051	-23,379	30,705
20%	114,240	82,138	128,895	67,485	32,101	-46,757	61,410
30%	171,359	123,207	193,343	101,228	48,152	-70,136	92,114
40%	228,479	164,276	257,790	134,971	64,203	-93,514	122,819

	Fire-Sale Costs ($ Million)				Decomposition of Fire-Sale Costs ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
10%	607	314	773	212	293	-459	561
20%	2,427	1,255	3,090	847	1,173	-1,835	2,243
30%	5,462	2,823	6,953	1,906	2,638	-4,129	5,047
40%	9,710	5,020	12,361	3,388	4,690	-7,341	8,972

Table 5: Fire-Sale Amount and Costs under Other Asset and Liability Shocks

This table presents estimates of fire-sale amount and fire-sale costs incurred by insurers with VA exposures greater than the median, given a negative shock to illiquid bonds (Panel A), a negative categorical shock (Panel B), or a positive shock to value of the guarantee (Panel C). Both panels fully consider the fire-sale externality. The price impact (c_0) of 18.6 basis points per $10 billion of sale is assumed, following the NSFR estimate for non-agency MBS by Duarte and Eisenbach (2016). The allocations of actual portfolio, Portfolio 1, and Portfolio 2 are as reported in Tables 1 and 3. The risk-based capital (RBC) requirements for common stocks and illiquid bonds are 0.30 and 0.06, respectively. The total invested assets, capital and surplus, RBC ratio, and (percentage) delta exposure of the guarantee are the averages from 2010 to 2013 (after the crisis). In Panel C, the value of guarantee is assumed to equal the gross reserve, and the shock is assumed to operate through channels other than a decline in value of the underlying stocks. The decomposition attributes the difference between the effects on actual portfolio and Portfolio 1, both with actual VA exposures, to "reaching for yield" and the difference between the effects on Portfolios 1 and 2, both with actual VA exposures, to "hedging." The fire-sale effects coming from the "gross VA exposures" are calculated from Portfolio 2 with and without VA exposures.

Panel A: Negative Shock to Illiquid Bonds

	Fire-Sale Amount ($ Million)				Decomposition of Fire-Sale Amount ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
2%	107,805	59,493	52,898	52,898	48,312	6,595	0
4%	215,610	118,986	105,797	105,797	96,624	13,189	0
6%	323,415	178,479	158,695	158,695	144,936	19,784	0
8%	431,220	237,972	211,594	211,594	193,248	26,378	0
	Fire-Sale Costs ($ Million)				Decomposition of Fire-Sale Costs ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
2%	2,162	658	520	520	1,503	138	0
4%	8,647	2,633	2,082	2,082	6,013	551	0
6%	19,455	5,925	4,684	4,684	13,530	1,241	0
8%	34,587	10,533	8,328	8,328	24,054	2,206	0

Table 5, cont'd: Fire-Sale Amount and Costs under Other Asset and Liability Shocks

Panel B: Negative Categorical Shock

	Fire-Sale Amount ($ Million)				Decomposition of Fire-Sale Amount ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
10%	235,653	130,617	155,472	109,662	105,035	-24,855	45,810
20%	471,306	261,235	310,945	219,324	210,071	-49,710	91,620
30%	706,959	391,852	466,417	328,987	315,106	-74,565	137,431
40%	942,612	522,470	621,890	438,649	420,142	-99,420	183,241
	Fire-Sale Costs ($ Million)				Decomposition of Fire-Sale Costs ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
10%	10,329	3,173	4,496	2,237	7,156	-1,323	2,259
20%	41,316	12,693	17,984	8,947	28,623	-5,290	9,037
30%	92,961	28,560	40,463	20,131	64,401	-11,903	20,332
40%	165,264	50,773	71,935	35,789	114,491	-21,162	36,146

Table 5, cont'd: Fire-Sale Amount and Costs under Other Asset and Liability Shocks

Panel C: Positive Shock to Value of Guarantee

	Fire-Sale Amount ($ Million)				Decomposition of Fire-Sale Amount ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
20%	114,964	60,588	58,195	0	54,376	2,392	58,195
40%	229,927	121,175	116,391	0	108,752	4,784	116,391
60%	344,891	181,763	174,586	0	163,128	7,177	174,586
80%	459,854	242,351	232,782	0	217,504	9,569	232,782
	Fire-Sale Costs ($ Million)				Decomposition of Fire-Sale Costs ($ Million)		
Magnitude of Shock	Actual Portfolio + VAs	Portfolio 1 + Actual VAs	Portfolio 2 + Actual VAs	Portfolio 2 + No VAs	Reaching for Yield	Hedging Guarantee Exposure	Gross Guarantee Exposure
20%	2,458	683	630	0	1,776	53	630
40%	9,833	2,731	2,520	0	7,102	211	2,520
60%	22,125	6,145	5,669	0	15,980	476	5,669
80%	39,333	10,925	10,079	0	28,408	846	10,079