

Aggregate Recruiting Intensity*

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June 26, 2016

Abstract

We develop a model of firm dynamics with random search in the labor market where hiring firms exert recruiting effort by spending resources to fill vacancies faster. Consistent with micro evidence, fast-growing firms invest more in recruiting activities and achieve higher job-filling rates. In equilibrium, individual recruiting decisions of hiring firms aggregate into an index of economy-wide recruiting intensity. We use the model to study how recruiting intensity responds to aggregate shocks, and whether it can account for the dynamics of aggregate matching efficiency around the Great Recession. Productivity and financial shocks can lead to sizable pro-cyclical fluctuations in matching efficiency through recruiting effort. Quantitatively, the main mechanism is that firms attain their employment targets by adjusting their recruitment effort as labor market tightness varies. Instead, fluctuations in new-firm entry have a negligible effect on aggregate recruiting intensity, despite their contribution to aggregate job creations.

Keywords: Aggregate Matching Efficiency, Firm Dynamics, Macroeconomic Shocks, Recruiting Intensity, Unemployment, Vacancies.

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*We thank Ricardo Lagos, Mark Gertler, Bob Hall, and Rob Shimer for helpful conversations at the early stages of this research, and our discussants Russell Cooper, William Hawkins, Nicolas Petrosky-Nadeau and Jeremy Lise for many useful suggestions.

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1 Introduction

A large literature documents cyclical changes in the rate at which the US macroeconomy matches job seekers and vacancies. Aggregate matching efficiency, measured as the residual of an aggregate matching function that generates hires H_t from inputs of unemployed workers U_t and vacancies V_t , often represents this crucial role of the labor market. Figure 1 shows that the Great Recession provides a particularly stark episode, featuring a decline in aggregate matching efficiency of around 60 percent.¹ Our reading of the data is that this decline contributed to a depressed vacancy-yield and to persistently higher unemployment following the crisis. Hence, identifying the deep determinants aggregate matching efficiency is necessary for a full understanding of the labor market dynamics during that period.

The literature has offered a number of explanations for the decline in aggregate matching efficiency over the recession. These have emphasized a shift in the composition of the pool of unemployed workers (Hall and Schulhofer-Wohl, 2013), a rise in occupational mismatch (Şahin, Song, Topa, and Violante, 2014), and a decline in worker search effort (Mukoyama, Patterson, and Şahin, 2013).

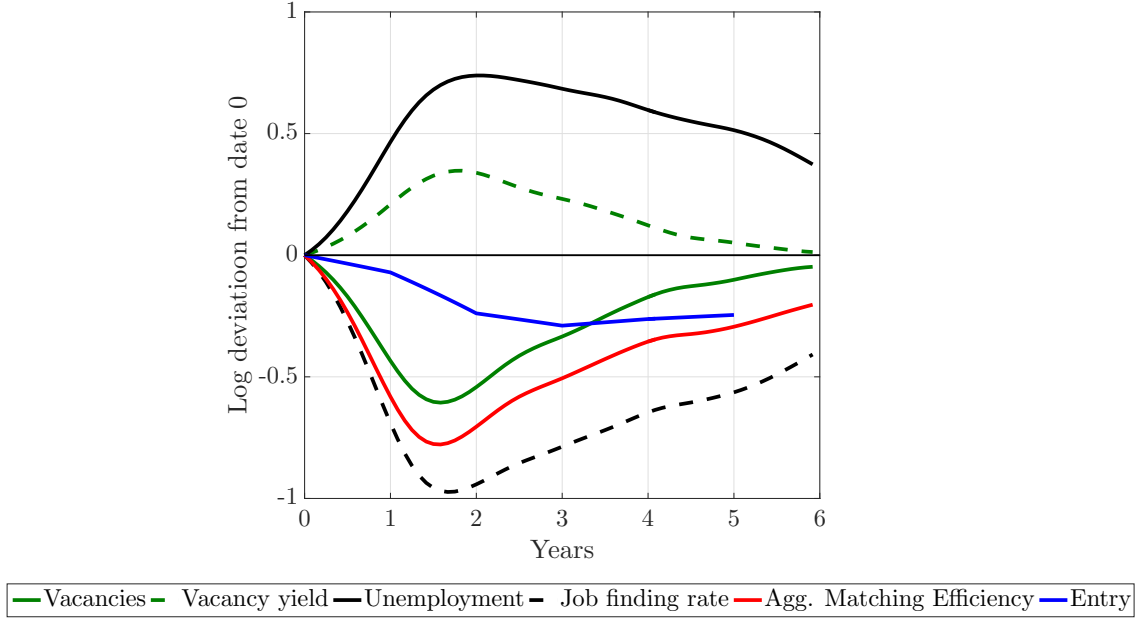
An alternative view is that fluctuations in the effort with which firms try to fill their open vacancies affect aggregate matching efficiency. When aggregated over firms, we call this factor *aggregate recruiting intensity*. The goal of this paper is to investigate whether this is an important source of the dynamics of aggregate matching efficiency, and to study the economic forces that shape how it responds to macroeconomic shocks.

Why study aggregate recruiting intensity? Our main motivation is the empirical analysis of recruitment intensity at the firm level in Davis, Faberman, and Haltiwanger (2013) (henceforth DFH)—the first paper to rigorously use JOLTS micro-data to examine what factors are correlated with vacancy-yields at the firm-level. The robust finding of DFH is that firms that grow faster fill their vacancies at a faster rate.² These results imply an obvious logic for how macroe-

¹In this figure, aggregate matching efficiency is the residual ε_t of $H_t = e^{\varepsilon_t} U_t^{1-\alpha} V_t^\alpha$, where we set $\alpha = 0.50$ and hires are measured as hires from unemployment. At this point this figure is only illustrative, and we are aware this may not be the most precise measure of match efficiency due to a number of factors, including: (i) vacancies are matched also to workers that are employed, (ii) some workers who are out of the labor force also search for jobs, (iii) many hires occur from unemployed workers on temporary layoffs and, as such, do not require frictional search on either side (Fujita and Moscarini, 2013).

²The numerous exercises in DFH show that this is a robust finding, not in any way spurious. For example: by definition, a firm that luckily fills a large amount of its vacancies will have both a higher vacancy yield and a

Figure 1: Dynamics of labor market variables in Great Recession (2008:01 - 2014:01)



Notes (i) Vacancies V_t , and hires H_t (used to compute vacancy-yield H_t/V_t) taken from monthly JOLTS data. (ii) Unemployment U_t is from the BLS. (iii) The job finding rate is H_t/U_t . (iv) Aggregate matching efficiency is equal to $H_t / (V_t^\alpha U_t^{1-\alpha})$ with $\alpha = 0.5$. (v) Given the availability of JOLTS data the first five series are measured from January 2001 to January 2014, expressed in logs and then HP-filtered. We plot level differences of these series from January 2008. (vi) (Firm) entry is taken from Census *Business Dynamics Statistics* and computed annually as the number of firms aged less than or equal to one year old at the time of survey and is available from 1977 to 2007. To this we fit and remove a linear trend. We plot log differences of this series from 2007.

conomic shocks affect aggregate recruiting intensity—and, thus, aggregate matching efficiency. If an aggregate negative shock depresses firm growth rates, aggregate recruiting intensity declines because hiring firms use lower recruitment effort to fill their posted vacancies.

This mechanism seems potentially consistent with the findings of several recent papers that document strong cross-sectional and time variation in firm growth rates. Haltiwanger, Jarmin, and Miranda (2010) find that firm growth depends more on firm age than size. Siemer (2013) and Chodorow-Reich (2014) find that young firm growth is particularly sensitive to financial shocks. Moreover, Bloom (2009) finds that dispersion in firm growth rates is strongly counter-cyclical, a fact attributed to counter-cyclical uncertainty shocks. Thus, in principle the shocks most-regularly studied by macroeconomists—i.e., productivity, financial, and uncertainty shocks—may impact aggregate recruiting intensity by shifting the firm growth rate distribution. For example, as Figure 1 shows, the rate at which firms entered the economy fell

higher growth rate. Or one may think that the result is driven by fixed heterogeneity in vacancy-filling rates across sectors and that this is correlated with growth rates and vacancy yields. The authors show that these, as well as several other explanations, do not drive their main result.

dramatically during the financial crisis. What is the impact of fewer young, fast-growing firms on aggregate recruiting intensity?

Our approach in this paper is to develop a model of firm dynamics in frictional labor markets which we use to examine the effect of these different macroeconomic shocks on aggregate recruiting intensity. Importantly the model is consistent with the facts that are salient to a discussion of the interaction between macroeconomic shocks, firm level growth rates and recruiting activities: (i) it matches the [DFH](#) finding that increases in firm hiring rates are realized chiefly through increases in vacancy yields rather than higher vacancy rates; (ii) it allows for credit constraints that slow the expansion of young firms; and (iii) is set in general equilibrium, since the composition and recruiting behavior of hiring firms depend on labor market tightness, which fluctuates strongly in the data ([Shimer, 2005](#)).

Our model is a version of the canonical Diamond-Mortensen-Pissarides random matching framework with decreasing returns in production and non-convex hiring costs ([Cooper, Haltiwanger, and Willis, 2007](#); [Elsby and Michaels, 2013](#); [Acemoglu and Hawkins, 2014](#)). The model simultaneously features a realistic firm life-cycle, as its classic competitive setting counterparts ([Jovanovic, 1982](#); [Hopenhayn, 1992](#)), and a frictional labor market with slack on both demand and supply sides. We augment this environment in three dimensions.

First, we allow for endogenous entry and exit of firms. This is a key element for understanding the effects of macroeconomic shocks on the growth rates of hiring firms, since it is well documented that young firms account for a disproportionately large fraction of job creation, grow faster than old firms ([Haltiwanger, Jarmin, and Miranda, 2010](#)), and are more sensitive to financial conditions ([Siemer, 2013](#)).

Second, we introduce a recruiting intensity decision at the firm level: besides the maximum number of open positions that they are willing to fill in each period, hiring firms choose the amount of resources that they devote to recruitment activities. This recruiting intensity generates heterogeneous job filling rates across firms. In turn, the sum of all individual firms' recruitment efforts, weighted by their vacancy share, aggregates to the economy's measured matching efficiency.

Third, we introduce financial frictions: firms face a nonnegativity constraint on dividends and a constraint on borrowing that restricts leverage to a multiple of collateralizable firm assets

as in [Evans and Jovanovic \(1989\)](#).³

We parameterize our model to match a rich set of aggregate labor market statistics and firm-level cross-sectional moments. Our analyses of the steady state properties of the model show that it produces firm life-cycles broadly consistent with those in the data: in particular, the “up-or-out” dynamics of young firms that [Haltiwanger \(2011b\)](#) shows to be important for the distribution of growth rates, and for the leverage of young firms. Moreover, the model’s age and size distributions of establishments and of employment are in line with the empirical ones.

The success in matching these facts suggests that our model is well-suited for a quantitative study of the effect of different macroeconomic shocks on aggregate recruitment intensity. Thus, we proceed to analyse the effect of productivity and financial shocks on aggregate recruitment intensity. We find that both shocks generate fluctuations in aggregate recruitment intensity that account for approximately 50 percent of the deviation in aggregate matching efficiency observed in the Great Recession. Moreover, financial shocks generate patterns of firm entry and leverage that are consistent with those observed during the 2008 recession. Productivity shocks do not.

Surprisingly, we find that the effects of the decline in firm entry (and, thus, of the shifts in the distribution of firm growth rates) on aggregate recruiting intensity is quantitatively small. Two counteracting forces dampen the drop in average growth rates after negative shocks: 1) hiring firms are selected, thus relatively more productive than in steady-state; and 2) the decline in market tightness allows productive firms to grow faster than in steady-state. Instead, aggregate recruitment intensity declines mainly because the number of available job seekers per vacancy increases, making it easier for firms to attain their recruitment targets without spending on recruitment costs.

To the best of our knowledge, [Kaas and Kircher \(2015\)](#) is the only other paper that focuses on heterogeneous job filling rates across firms. They consider a directed search environment in which different firms post different wages that attract jobseekers at differential rates, whereas we study how firms’ costly recruiting activities determine differential job filling rates.⁴ Moreover, while they consider the role of aggregate productivity shocks—as we do, as well—we

³Other papers that consider financial frictions in a model with search frictions in labor markets include [Wasmer and Weil \(2004\)](#) and [Petrosky-Nadeau and Wasmer \(2013\)](#).

⁴In Section 3, we use a hiring cost function that, once firms choose their optimal recruitment effort, is identical to the labor adjustment cost function in [Kaas and Kircher \(2015\)](#). We have reached this conclusion independently.

further analyse financial shocks, showing that the dynamics of macroeconomic variable during the Great Recessions are consistent with financial, rather than productivity, shocks. Finally, while in both models aggregate recruitment intensity drops after an aggregate negative shock, the reasons fundamentally differ. Kass and Kircher argue that the drop depends on recruitment intensity being a concave function of firms' hiring policies, whose dispersion across firms increases after a negative shock. Our rich decomposition of the contributions to aggregate recruitment intensity allows us to claim that the main reason for the drop is that the increase in the number of available job seekers per vacancy allows firms to reduce their recruitment effort. In Section 5 we closely discuss these differences.

The rest of the paper is organized as follows. Section 2 more precisely relates the above discussion of firm level recruiting intensity to aggregate match efficiency. Section 3 outlines the model economy and the stationary equilibrium. Section 4 describes the parameterization of the model, and highlights some cross-sectional features of the economy. Section 5 describes the dynamic response of the economy to various shocks and outlines the main results of the paper. Section 6 proposes a novel index of aggregate recruitment intensity based on our model, and illustrates its behavior over time. Section 7 concludes.

2 Recruiting Intensity and Aggregate Matching Efficiency

Starting from hiring decisions at the firm level we briefly describe how we can aggregate to an economy-wide matching function with an efficiency factor that has the interpretation of average recruitment intensity. This derivation follows DFH.

Any given hiring firm i chooses v_{it} , the maximum number of open positions, ready to be staffed, and costly to create, as well as e_{it} , an indicator of recruiting intensity. Let $v_{it}^* = e_{it}v_{it}$ be the number of *effective* vacancies in firm i . Integrating over all firms we obtain:

$$V_t^* = \int e_{it}v_{it}di, \tag{1}$$

the aggregate number of effective vacancies. Under our maintained assumption of a CRS Cobb-

Douglas matching function, aggregate hires equal:

$$H_t = (V_t^*)^\alpha U_t^{1-\alpha} = \Phi_t V_t^\alpha U_t^{1-\alpha}, \text{ with } \Phi_t = \left(\frac{V_t^*}{V_t} \right)^\alpha = \left[\int e_{it} \left(\frac{v_{it}}{V_t} \right) di \right]^\alpha, \quad (2)$$

which corresponds to DFH's generalized matching function. Therefore, measured aggregate matching efficiency Φ_t is an average of firm-level recruitment intensity weighted by individual vacancy shares, raised to the power of α , the economy-wide elasticity of hires to vacancies. Finally, consistency requires that each firm i faces hiring frictions, implying that

$$h_{it} = q(\theta_t^*) e_{it} v_{it}, \quad (3)$$

where $\theta_t^* = V_t^*/U_t$ is effective market tightness.⁵ Thus, $q(\theta_t^*) = H_t/V_t^* = (\theta_t^*)^{\alpha-1}$ is the aggregate job filling rate per effective vacancy, constant across all firms at date t .

3 Model

Our starting point is an equilibrium random-matching model of the labor market in which firms are heterogeneous in productivity and size, and the hiring process occurs through an aggregate matching function. As discussed in the Introduction, we augment this model in three dimensions—all of which are essential to develop a framework that can address our question. First, our framework features endogenous firm entry and exit. Second, beyond the number of positions to open (vacancies), hiring firms optimally choose their recruiting intensity: by spending more on recruitment resources, they can increase the rate at which they meet job seekers. Third, once in existence, firms face two financial frictions: (i) they do not have access to equity markets, (ii) they can borrow, but debt must be collateralized.

In what follows, we present the economic environment in detail, outline the model timing, then describe the firm, bank, and household problems. Finally, we define a stationary equilibrium for the aggregate economy. Since our experiments will consist of perfect foresight transition dynamics, we do not make reference to aggregate state variables in agents' problems. We use a recursive formulation throughout.

⁵In the paper, we are faithful to the notation in this literature and denote *measured* labor market tightness V_t/U_t as θ_t .

3.1 Environment

Time is discrete and the horizon is infinite. Three types of agents populate the economy: firms, banks, and households.

Firms. There is an exogenous measure λ_0 of potential entrants each period, and an endogenous measure λ of incumbent firms. Firms are heterogeneous in their productivity $z \in Z$, stochastic and i.i.d. across all firms, and operate a decreasing-returns-to-scale (DRS) production technology $y(z, n', k)$ that uses inputs of labor $n' \in N$ and capital $k \in K$. The output of production is a homogeneous final good, whose competitive price is the numeraire of the economy.

All potential entrants receive an initial equity injection from households equal to a_0 . They then draw a value of z from the initial distribution $\Gamma_0(z)$ and, conditional on this draw, decide whether to enter and become an incumbent by paying the set-up cost χ_0 . Those that do not enter return the initial equity to the households. This is the only time when firms can obtain funds directly from households—throughout the rest of their lifecycle they must rely on the debt issuance.

Incumbents can exit exogenously or endogenously. With probability ζ , a destruction shock hits an incumbent firm, forcing it to exit. Surviving firms observe their new value of z , drawn from the conditional distribution $\Gamma(dz', z)$, and choose whether to exit or to continue production. Under either exogenous or endogenous exit the firm pays out its positive net-worth a to households. Those incumbents that decide to stay in the industry pay a per-period operating cost χ and then choose levels of inputs: labor and capital.

The labor decision involves either firing some existing employees or hiring new workers. Firing is frictionless, but hiring is not: a hiring firm chooses both vacancies v and recruitment effort e with associated hiring cost $\mathcal{C}(e, v, n)$, which also depends on initial employment. Given (e, v) , the individual hiring function (3) determines current period employment n' used in production. To simplify the wage setting, we assume firms make take-it-or-leave-it offers to workers, so the wage rate equals ω , the individual flow value from non-employment.

The capital decision involves borrowing capital from financial intermediaries (banks) in intraperiod loans. Due to imperfect contractual enforcement frictions, firms can appropriate a fraction $1/\varphi$ of the capital received by banks, with $\varphi > 1$. To pre-empt this behavior, a firm renting k units of capital is required to deposit k/φ units of their net worth with the bank. This

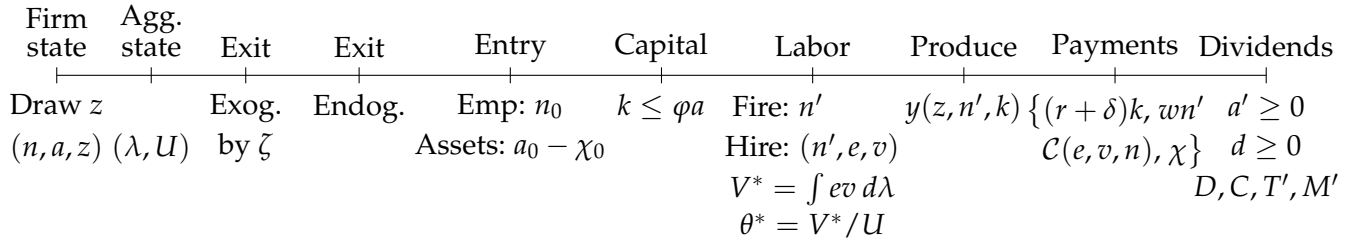


Figure 2: Timeline of the model

guarantees that, ex-post, the firm does not have an incentive to abscond with the capital. Thus, a firm with current net worth a faces a collateral constraint $k \leq \varphi a$.

Banks. The banking sector is perfectly competitive. Banks receive household deposits, freely transform this into capital, and rent capital to firms. The one-period contract with households pays a risk-free interest rate of r . Capital depreciates at rate δ in production, and so the price of capital charged by banks to firms is $(r + \delta)$.

Households. We envision a representative household with \bar{L} family members, U of which are unemployed. The representative household is risk-neutral with discount factor $\beta \in (0, 1)$. The household trades shares M of a mutual fund comprised of all firms in the economy and makes bank deposits T . It earns interest r on deposits, the total wage bill that firms pay to the employed family members, and D dividends per share held in the mutual fund. Moreover, unemployed workers produce ω units of the final good at home.

Before describing the firm's problem in detail, we outline the precise timing of the model, summarized in Figure 2. Within a period, the events unfold as follows: (i) realization of the productivity and firm destruction shocks, and exogenous exit of incumbents; (ii) exit decision by incumbent firms, and entry decision by potential entrants; (iii) borrowing decision by incumbents; (iv) hiring/firing decisions and labor market matching; (v) production and revenues from sales; (vi) payment of wage bill, costs of capital, hiring and operation expenses; dividend payment/saving decision by incumbent firms, and household consumption/saving decisions.

To be consistent with our transition dynamics experiments in Section 5, it is useful to note that we record aggregate state variables—the measures of incumbent firms λ and unemployment U —at the beginning of the period, between stages (i) and (ii). Moreover, even though the labor market opens after firms exit or fire, workers who separate in the current period can only

start searching in the next one.

3.2 Firm Problem

We first consider the entry and exit decisions, and then analyze the problem of incumbent firms.

Entry. A potential entrant who has drawn z from $\Gamma_0(z)$ solves the following problem

$$\max \left\{ a_0, \mathbb{V}^i(n_0, a_0 - \chi_0, z) \right\}, \quad (4)$$

where \mathbb{V}^i is the value of an incumbent firm, a function of (n, a, z) . The firm enters if the value to the risk-neutral shareholder of becoming an incumbent with one employee ($n_0 = 1$), initial net worth $a_0 - \chi_0$ —equal to the household equity injection a_0 minus the entry cost χ_0 —and productivity z exceeds the value of returning a_0 to the household. Let $i(z) \in \{0, 1\}$ denote the entry decision rule, which depends only on the initial productivity draw, since all the potential entrants share the same entry cost, the same initial employment and received the same ex-ante equity injection. As \mathbb{V}^i is increasing in z , there is an endogenous productivity cut-off z^* such that for all $z \geq z^*$ the firm chooses to enter. Thus, the measure of entrants is

$$\lambda_e = \lambda_0 \int_Z i(z) d\Gamma_0 = \lambda_0 [1 - \Gamma_0(z^*)]. \quad (5)$$

Exit. Firms exit exogenously with probability ζ . Conditional on survival the firm then chooses to continue or exit. An exiting firm pays out its net worth a to share-holders. The firm's expected value \mathbb{V} before the destruction shock equals

$$\mathbb{V}(n, a, z) = \zeta a + (1 - \zeta) \max \left\{ \mathbb{V}^i(n, a, z), a \right\}. \quad (6)$$

We denote by $x(n, a, z) \in \{0, 1\}$ the exit decision.

Hire or Fire. An incumbent firm i with employment, assets, and productivity equal to the triplet (n, a, z) chooses whether to hire or fire workers to solve

$$\mathbb{V}^i(n, a, z) = \max \left\{ \mathbb{V}^h(n, a, z), \mathbb{V}^f(n, a, z) \right\}, \quad (7)$$

where the two value functions associated with firing (f) and hiring (h) are described below.

The Firing Firm. A firm that has chosen to fire some of its workers (or not to adjust its workforce) solves

$$\begin{aligned} \mathbb{V}^f(n, a, z) &= \max_{n', k, d} d + \beta \int_Z \mathbb{V}(n', a', z') \Gamma(dz', z) \\ &s.t. \\ n' &\leq n, \\ d + a' &= y(n', k, z) + (1+r)a - \omega n' - (r + \delta)k - \chi, \\ k &\leq \varphi a, \\ d &\geq 0. \end{aligned} \tag{8}$$

Firms maximize shareholder value and, because of risk-neutrality, use β as their discount factor. Dividends d are given by revenues from production and interest on savings net of the wage bill, rental and operating costs, and change in net-worth $a' - a$. The last two equations in (8) reiterate that firms face a collateral constraint on the maximum amount of capital they can rent and a non-negativity constraint on dividends.

To help understand the budget constraint and preface how we take the model to the data, define firm debt by the identity $b \equiv k - a$, with the understanding that $b < 0$ denotes savings. Making this substitution reveals an alternative formulation of the model in which the firm owns its capital and faces a constraint on leverage. With state variable (k, b, n, z) the firm faces the following budget and collateral constraints

$$\begin{aligned} d + \underbrace{[k' - (1 - \delta)k]}_{\text{Investment}} &= \underbrace{[y(n', k, z) - \omega n' - \chi - rb]}_{\text{Operating Profit}} + \underbrace{[b' - b]}_{\Delta \text{ Borrowing}}, \\ b/k &\leq (\varphi - 1)/\varphi. \end{aligned}$$

This makes clear that the firm can fund equity payouts and investment in capital through either operating profits or expanding borrowing/reducing saving.

The Hiring Firm. The hiring firm additionally chooses the number of vacancies to post $v \in \mathbb{R}_+$ and recruitment effort $e \in \mathbb{R}_+$, understanding that, by a law of large numbers, its new hires

$n' - n$ equal the firm's job-filling rate qe of each of its vacancies times the number of vacancies v created: $n' - n = q(\theta^*)ev$. Note that the individual job-filling rate depends on the aggregate meeting rate q , which is determined in equilibrium and taken as given by the firm, as well as its recruiting effort e . The firm faces a variable cost function $\mathcal{C}(e, v, n)$, increasing and convex in e and v .

Note that a firm's continuation value depends on n' , not on the mix of recruiting intensity e and vacant positions v that generates it. As a result, one can split the problem of the hiring firm in two stages. First, the choice of n' , k and d . Second, given n' , the choice of the optimal combination of inputs (e, v) . The latter reduces to a static cost-minimization problem:

$$\begin{aligned} \mathcal{C}^*(n, n') &= \min_{e, v} \mathcal{C}(e, v, n) \\ \text{s.t.} \quad e &\geq 0, \quad v \geq 0, \quad n' - n = q(\theta^*)ev. \end{aligned} \tag{9}$$

yielding the lowest cost combination $e(n, n')$ and $v(n, n')$ that delivers $h = n' - n$ hires to a firm of size n , and implied cost function $\mathcal{C}^*(n, n')$.

The remaining choices of n' , k and d requires solving the dynamic problem

$$\begin{aligned} \mathbb{V}^h(n, a, z) &= \max_{n', k, d} d + \beta \int_Z \mathbb{V}(n', a', z') \Gamma(dz', z) \\ \text{s.t.} \quad n' &> n, \\ d + a' &= y(n', k, z) + (1 + r)a - \omega n' - (r + \delta)k - \chi - \mathcal{C}^*(n, n'), \\ k &\leq \varphi a, \\ d &\geq 0. \end{aligned} \tag{10}$$

The solution of this problem includes the decision rule $n'(n, a, z)$. Using this function in the solution of (9), we obtain decision rules $e(n, a, z)$ and $v(n, a, z)$ for recruitment effort and vacancies in terms of firm state variables.

Given the centrality of the hiring cost function $\mathcal{C}(e, v, n)$ to our analysis, we now discuss its

specification. In what follows, we choose the functional form

$$\mathcal{C}(e, v, n) = \left[\frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left(\frac{v}{n} \right)^{\gamma_2} \right] v, \quad (11)$$

with $\gamma_1 \geq 1$ and $\gamma_2 \geq 0$ being necessary conditions for convexity of the maximization problem (9).⁶ This cost function implies that the average cost of a vacancy, \mathcal{C}/v , has two separate components. The first is increasing and convex in recruiting intensity per vacancy e . The idea is that for any given open position, the firm can choose to spend resources on recruitment activities to make the position more visible or the firm more attractive as a potential employer, to assess more candidates per unit of time, or to better screen them, but all such activities are increasingly costly on a per-vacancy basis. The second component is increasing and convex in the vacancy rate, and captures the fact that expanding productive capacity is costly in *relative* terms: the implicit presumption is that, for example, creating 10 new positions involves a more expensive reorganization of production in a firm with 10 employees than in a firm with 1000 employees.

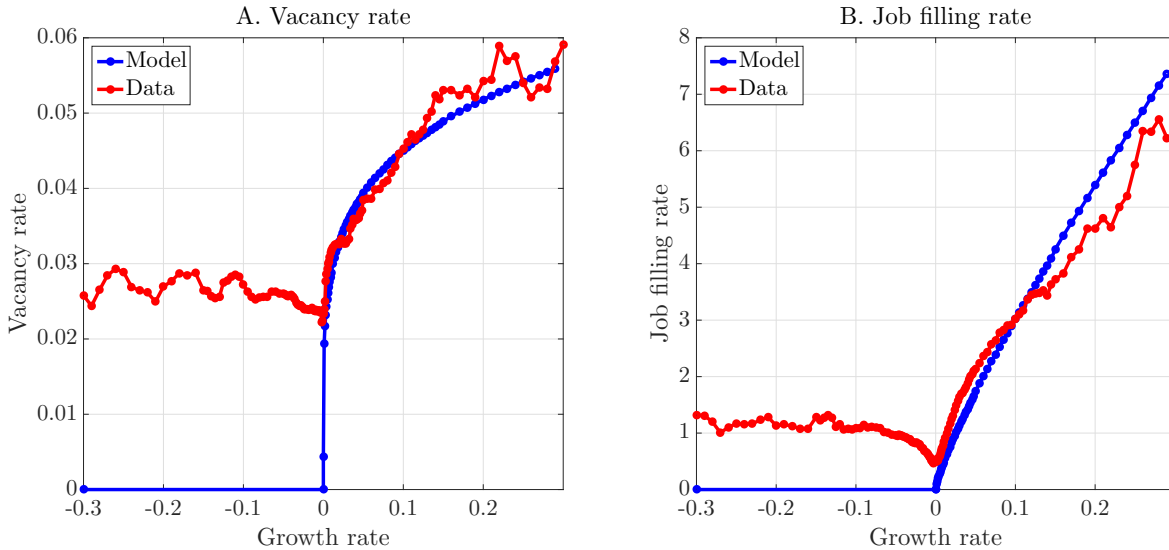
In Appendix A we derive several results for the static hiring problem of the firm (9) under this cost function and derive the exact expression for $\mathcal{C}^*(n, n')$ used in the dynamic problem (10). We show that, by combining first-order conditions, we obtain the optimal choice of e

$$e(n, n') = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_2}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}, \quad (12)$$

and, hence, the firm-level job filling rate $f(n, n') \equiv q(\theta^*) e(n, n')$. Equation (12) demonstrates that the model implies a log-linear relation between the job filling rate and employment growth at the firm level, with elasticity $\gamma_2/(\gamma_1 + \gamma_2)$. This is the key empirical finding of DFH, who estimated this elasticity to be 0.82. In fact, one could interpret our functional choice for \mathcal{C} in equation (11) as a “reverse-engineering” strategy in order to obtain, from first principles, the empirical cross-sectional relation between firm-level job-filling rate and firm-level hiring rate uncovered by DFH. Put differently, micro data sharply discipline the recruiting cost function of the model.

⁶In the limiting case $\gamma_1 = 1, \gamma_2 = 0$, the model collapses to the standard matching model without recruiting intensity margin: the optimal effort choice is 1 and the job filling rate is equal across all firms.

Figure 3: Cross-sectional relationships between monthly employment growth and the the vacancy rate and the job filling rate. Data from DFH online supplemental materials.



A second empirical finding of DFH is that the firm-level hiring technology $h = g(q, v)$ is constant-returns-to-scale in v , a feature we have also included in our individual hiring function. In our implementation, as shown in (12), these two findings imply a specific log-linear relationship between the job-filling rate and q .

Appendix A also shows that, once (11) includes the optimal choice of e , \mathcal{C} is equivalent to the hiring cost function assumed by Kaas and Kircher (2015).

Why does firm optimality imply that the job filling rate increases with the growth rate with elasticity $\gamma_2/(\gamma_1 + \gamma_2)$? For two reasons. First, recruiting intensity and the vacancy rate (v/n) are complements in the production of the hires per employee $(n' - n)/n$, the firm's growth rate—see the last equation in (9). Thus, a firm that wants to grow a lot will optimally create more positions and, at the same time, spend more in recruiting effort. Second, the stronger the convexity of \mathcal{C} in the vacancy rate (γ_2), relative to its degree of convexity in effort (γ_1), the more an expanding firm finds it optimal to substitute away from vacancies into recruiting intensity to realize its target growth rate. In the special case when $\gamma_2 = 0$, all the adjustment occurs through vacancies and recruiting effort is irresponsive to the growth rate and to macroeconomic conditions, as in the canonical model of Pissarides (2000).

Figure 3 plots the cross-sectional relationship between the vacancy rate and employment growth (panel A) and the job filling rate and employment growth (panel B) in the model and

in the DFH data, with the elasticity of the job filling rate to firm's growth $\gamma_2/(\gamma_1 + \gamma_2) = 0.82$.⁷ Since we assume that the individual hiring function is linear in vacancies, the elasticity of the vacancy rate to firm's growth equals $\gamma_1/(\gamma_1 + \gamma_2) = 0.18$.

3.3 Household Problem

The representative household solves

$$\begin{aligned} W(U, T, M) &= \max_{T', M', C > 0} C + \beta W(U', T', X') & (13) \\ &\text{s.t.} \\ C + \bar{Q}T' + PM' &= \omega\bar{L} + (D + P)M + T, \\ U' &= U - H(\theta^*) + F(\theta^*), \end{aligned}$$

where C denotes household consumption; T are bank deposits; M are shares of the mutual fund composed of all firms in the economy, with the aggregate number of shares normalized to one; \bar{L} denotes the number of household members; and U denotes the number of unemployed members. The share price is P and owning shares entitles the household to dividends D , the sum of all firm dividends.⁸ The total wage bill is the integral over all wage payments from firms, while workers that are idle this period and begin next period as unemployed job seekers produce ω units of the final good via home production. Unemployment evolves due to masses of hires $H(\theta^*)$ and separations of mass $F(\theta^*)$, which the household takes as given and we characterize later.

From the first-order conditions for deposits and share holdings, we obtain $\bar{Q} = \beta$ and $P = \beta(P + D)$ which imply a constant return of $r = \beta^{-1} - 1$ on both deposits and shares and, thus, the household is indifferent over portfolios. Since the household is risk neutral, it is also indifferent over the timing of consumption.

⁷In Figure 3, the model implies zero hires for firms with negative growth rates, whereas in the data time aggregation leads to positive vacancy rates and vacancy yields also for those firms.

⁸Households consider the initial equity injections into start-ups as negative dividends.

3.4 Stationary Equilibrium and Aggregation

Let Σ_N , Σ_A , and Σ_Z be the Borel sigma algebras over N and A , and Z . The state space for an incumbent firm is $S = N \times A \times Z$, and we denote with s one of its points (n, a, z) . Let Σ_S be the sigma algebra on the state space, with typical set $\mathcal{S} = \mathcal{N} \times \mathcal{A} \times \mathcal{Z}$, and (S, Σ_S) be the corresponding measurable space. Denote with $\lambda : \Sigma_S \rightarrow [0, \infty)$ the stationary distribution of incumbent firms at the beginning of the period, following the draw of firm level productivity, before the exogenous exit shock.

To simplify the exposition of the equilibrium, it is convenient to use $s \equiv (n, a, z)$ and $s_0 \equiv (n_0, a_0 - \chi_0, z)$ as the argument for incumbents' and entrants' decision rules.

A stationary recursive competitive equilibrium is a collection of firms' decision rules $\{i(z), x(s), n'(s), e(s), v(s), a'(s), d(s), k(s)\}$, value functions $\{\mathbb{V}, \mathbb{V}^i, \mathbb{V}^f, \mathbb{V}^h\}$, a measure of entrants λ_e , share price P and aggregate dividends D , wage ω , a distribution of firms λ , and a value for effective labor-market tightness θ^* such that: (i) the decision rules solve the firms problems (4)-(10), $\{\mathbb{V}, \mathbb{V}^i, \mathbb{V}^f, \mathbb{V}^h\}$ are the associated value functions, and λ_e is the mass of entrants implied by (5); (ii) the market for shares clears at $M = 1$ with share price

$$P = \int_S \mathbb{V}(s) d\lambda + \lambda_0 \int_Z i(z) \mathbb{V}^i(s_0) d\Gamma_0$$

and aggregate dividends

$$D = \zeta \int_S a d\lambda + (1 - \zeta) \int_S \{[1 - x(s)] d(s) + x(s) a\} d\lambda - \lambda_0 \int_Z i(z) a_0 d\Gamma_0;$$

(iii) the stationary distribution λ is the fixed point of the recursion:

$$\begin{aligned} \lambda(\mathcal{N} \times \mathcal{A} \times \mathcal{Z}) &= (1 - \zeta) \int_S [1 - x(s)] \mathbf{1}_{\{n'(s) \in \mathcal{N}\}} \mathbf{1}_{\{a'(s) \in \mathcal{A}\}} \Gamma(\mathcal{Z}, z) d\lambda \\ &\quad + \lambda_0 \int_Z i(z) \mathbf{1}_{\{n'(s_0) \in \mathcal{N}\}} \mathbf{1}_{\{a'(s_0) \in \mathcal{A}\}} \Gamma(\mathcal{Z}, z) d\Gamma_0, \end{aligned} \quad (14)$$

where the first term refers to existing incumbents and the second to new entrants; (iv) effective market tightness θ^* is determined by the balanced flow condition

$$\bar{L} - N(\theta^*) = \frac{F(\theta^*) - \lambda_e n_0}{p(\theta^*)}, \quad (15)$$

where $N(\theta^*)$ is aggregate employment

$$N(\theta^*) = (1 - \zeta) \int_S [1 - x(s)] n'(s) d\lambda + \lambda_0 \int_Z i(z) n'(s_0) d\Gamma_0, \quad (16)$$

$F(\theta^*)$ are aggregate separations

$$F(\theta^*) = \zeta \int_S n d\lambda + (1 - \zeta) \int_S x(s) n d\lambda + (1 - \zeta) \int_S [1 - x(s)] (n - n'(s))^- d\lambda, \quad (17)$$

which include all employment losses from firms exiting exogenously and endogenously, plus all the workers fired by shrinking firms, which we have denoted by $(n - n'(s))^-$.⁹ In the three equations above, the dependence of N and F on θ^* comes through the decision rules and the stationary distribution, even though, for notational ease, we have omitted θ^* as their explicit argument.

The left-hand side of (15) is the definition of unemployment—labor force minus employment—whereas the right-hand side is the steady-state Beveridge curve, i.e., the law of motion for unemployment in steady state:

$$U' = U - p(\theta^*) U + F(\theta^*) - \lambda_e(\theta^*) n_0. \quad (18)$$

Exactly as in [Elsby and Michaels \(2013\)](#), the two sides of (15) are independent equations determining the same variable—unemployment—and, combined, they yield equilibrium market tightness θ^* .¹⁰ Note that equations (15) and (18) account for the fact that every new firm enters with n_0 workers hired ‘outside’ the frictional labor market (e.g., the entrepreneurs).

Clearly, once θ^* is determined, so is U from either side of (15) and, therefore, V^* . Finally, we note that measured aggregate matching efficiency, in equilibrium, is $\Phi = (V^*/V)^\alpha$, where

⁹Entrant firms never fire, as they enter with the lowest value on the support for N , n_0 .

¹⁰Our computation showed that, typically, $N(\theta^*)$ is decreasing in its argument and the right-hand side of (15) is always positive and decreasing. Thus, the crossing point of left- and right-hand side is unique, when it exists. However, an equilibrium may not exist. For example, for very low hiring costs, $N(\theta^*)$ may be greater than \bar{L} . Conversely, for large enough operating or hiring costs, no firms will enter the economy. In this case, there is no equilibrium with market production (albeit there is always some home-production in the economy).

Table 1: Externally set parameter values

Parameter		Value	Target
Discount factor (monthly)	β	0.9967	Ann. risk-free rate = 4%
Mass of potential entrants	λ_0	0.02	Meas. of incumbents = 1
Size of labor force	\bar{L}	24.6	Average firm size = 23
Elasticity of matching function wrt V_t	α	0.5	JOLTS

measured and effective vacancies are respectively

$$\begin{aligned}
 V &= (1 - \zeta) \int_S [1 - x(s)] v(s) d\lambda + \lambda_0 \int_Z i(z) v(s_0) d\Gamma_0, \\
 V^* &= (1 - \zeta) \int_S [1 - x(s)] e(s) v(s) d\lambda + \lambda_0 \int_Z i(z) e(s_0) v(s_0) d\Gamma_0.
 \end{aligned}$$

4 Parameterization

4.1 Externally Calibrated

We begin from the subset of parameters that are calibrated externally. The model period is one month. We set β to replicate an annualized risk-free rate of 4%. Since the measure of potential entrants λ_0 scales λ —see equation (14)—we choose λ_0 to normalize the total measure of incumbent firms to one. We normalize the size of the labor force \bar{L} so that, given a measure one of firms, under our target unemployment rate of 7 percent, the average firm size will be 23 consistent with Business Dynamics Statistics (BDS) data over the period 2001-2007.¹¹ In line with empirical studies, we set α , the elasticity of aggregate hires to aggregate vacancies in the matching function, to 0.5. Table 1 summarizes these parameter values.

4.2 Internally Calibrated

Table 2 lists the remaining 19 parameters of the model that are set by minimizing the distance between an equal number of empirical moments and their equilibrium counterparts in

¹¹The unemployment rate is $u = \bar{L}/N(\theta^*) - 1$, and with a unit mass of firms the average firm size is simply $N(\theta^*)$. Hence given $u = 0.075$, \bar{L} determines average firm size.

Table 2: Parameter values estimated internally

Parameter		Value	Target	Model	Data
Flow of home production	ω	1.000	Monthly separ. rate	0.033	0.030
Scaling of match. funct.	$\bar{\Phi}$	0.208	Monthly job finding rate	0.411	0.400
Prod. weight on labor	ν	0.804	Labor share	0.627	0.640
Midpoint DRS in prod.	σ_M	0.800	Employment share n : 0-49	0.294	0.306
High-Low spread in DRS	$\Delta\sigma$	0.094	Employment share n : 500+	0.430	0.470
Mass - Low DRS	μ_L	0.826	Firm share n : 0-49	0.955	0.956
Mass - High DRS	μ_H	0.032	Firm share n : 500+	0.004	0.004
Persistence of z shocks	ρ_z	0.992	Std. dev ann emp growth	0.440	0.420
Std. dev. of z shocks	ϑ_z	0.052	Mean Q1 emp / Mean Q4 emp	75.161	76.000
Mean $z_0 \sim \text{Exp}(\bar{z}_0^{-1})$	\bar{z}_0	0.390	$\Delta \log z$: Young vs. Mature	-0.246	-0.353
Cost elasticity wrt e	γ_1	1.114	Elasticity of vac yield wrt g	0.814	0.820
Cost elasticity wrt v	γ_2	4.599	Ratio vac yield: <50/>50	1.136	1.440
Cost shifter wrt e	κ_1	0.101	Hiring cost / monthly wage	0.757	0.600
Cost shifter wrt v	κ_2	5.000	Vacancy share $n < 50$	0.287	0.340
Exogenous exit probability	ζ	0.006	Survive ≥ 5 years	0.497	0.500
Entry cost	χ_0	9.354	Annual entry rate	0.099	0.110
Operating cost	χ	0.035	Fraction of JD by exit	0.210	0.340
Initial wealth	a_0	10.000	One year old Debt to Output	1.394	1.250
Collateral constraint	φ	10.210	Aggregate debt-to-assets	0.276	0.330

the model.¹² Table 2 lists the targeted moments, their empirical values, and their simulated values from the model. Even though every targeted moment is determined simultaneously by all parameters, in what follows we discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power.

We set the flow of home production of the unemployed ω to replicate a monthly separation rate of 0.03. We choose the shift parameter of the matching function (a normalization of the value of Φ in steady state) in order to pin down a monthly job finding rate of 0.40. Together, these two moments yield a steady-state unemployment rate of 0.07.

¹²Specifically, the vector of parameters Ψ is chosen to minimize the minimum-distance-estimator criterion function

$$f(\Psi) = (\mathbf{m}_{data} - \mathbf{m}_{model}(\Psi))' \mathbf{W} (\mathbf{m}_{data} - \mathbf{m}_{model}(\Psi))$$

where \mathbf{m}_{data} and $\mathbf{m}_{model}(\Psi)$ are the vectors of moments in the data and model, and $\mathbf{W} = \text{diag}(1/\mathbf{m}_{data}^2)$ is a diagonal weighting matrix.

We assume the revenue function $y(z, n', k) = z [(n')^\nu k^{1-\nu}]^\sigma$ and introduce a small degree of permanent heterogeneity in the parameter σ .¹³ Specifically we consider a three-point distribution with support $\{\sigma_L, \sigma_M, \sigma_H\}$ —symmetric about σ_M —leaving four parameters to choose: (i) the value of σ_M ; (ii) the spread $\Delta\sigma \equiv (\sigma_H - \sigma_L)$; and (iii)-(iv) the weights on the low and high DRS firms μ_L, μ_H . This allows the model to match the firm size distribution in the same spirit as the use of permanent heterogeneity in productivity in the quantitative applications of [Elsby and Michaels \(2013\)](#) and [Kaas and Kircher \(2015\)](#). As noted by [Sedlacek and Sterk \(2014\)](#) the assumption of heterogeneity in σ is useful in a model of entry and exit since it can generate old, small firms. The values of these four parameters allow the model to match the BDS statistics on employment and establishment shares of firms of size 0-49 and 500+.¹⁴

Firm productivity z follows an AR(1) process in logs: $\log z' = \rho_z \log z + \varepsilon$, with $\varepsilon \sim \mathcal{N}(-\vartheta_z^2/2, \vartheta_z)$. We calibrate ρ_z and ϑ_z to match two measures of firm dispersion, one in employment growth and one in employment levels: the standard deviation of annual employment growth for continuing establishments in the Longitudinal Business Database ([Elsby and Michaels, 2013](#)), and the ratio of the mean size of fourth to first quartile of the firm distribution ([Haltiwanger, 2011a](#)).¹⁵

The initial productivity distribution for entrants Γ_0 is Exponential, with mean \bar{z}_0 chosen to match the productivity gap between entrants and incumbents, specifically the differential in revenue productivity between firms older than 10 and younger than 1 year old ([Foster, Haltiwanger, and Syverson, 2016](#)).

We now turn to hiring costs. The cost function (11) has four parameters: the two elasticities (γ_1, γ_2) and the two cost shifters (κ_1, κ_2) . Recall, from the discussion surrounding equations (11) and (12), that the cross-sectional elasticity of job filling rates to employment growth rates, estimated to be 0.82 by [DFH](#), is a function of the ratio of these two elasticities.¹⁶ The second moment used to separately identify the two elasticities is the ratio of vacancy yields of small

¹³Since we specify the *revenue* function, we do not take a stand on whether z represents demand or productivity shocks, or whether σ represents DRS in production or the interaction of a weakly IRS production function with a downward sloping demand curve, which would be trivial to add to the model. Given this understanding we discuss the revenue function as if it were a production function: σ represents DRS and z is total factor productivity.

¹⁴In terms of the description of the model and stationary equilibrium, one should add σ to the firm's state vector s , but nothing substantial in the firm problem and the definition of equilibrium would change.

¹⁵In the numerical solution and simulation of the model, z remains a continuous state variable.

¹⁶We cannot map $\gamma_2 / (\gamma_1 + \gamma_2)$ directly into this value since in [DFH](#), and in the model's simulations for consistency, the growth rate is the Davis-Haltiwanger growth rate normalized in $[-2, 2]$.

(< 50 employees) to large (> 50 employees) firms from JOLTS data on hires and vacancies by firm size. Intuitively, when $\gamma_2 = 0$ this ratio is one as recruiting effort is constant across firms.

We use two targets to pin down the cost shift parameters. The first is the total hiring cost as a fraction of monthly wage per hire, a standard target for the single vacancy cost parameter that usually appears in vacancy posting models. We have two sources for this statistic: [Silva and Toledo \(2009\)](#) estimate it to be 0.4; [O'Leonard \(2011\)](#) reports a value around 0.8, based on a survey on firm recruitment practices. In the absence of better information, we target 0.6. The second target is the vacancy share of small ($n < 50$) firms from JOLTS: κ_2 determines the size of hiring costs for small (low n) firms and, thus, the amount of vacancies they create.

The parameters χ and ζ have large effects on firm exit. The operation cost χ mostly impacts exit rates of young firms; therefore, we target the five-year survival rate found in BDS data, which is approximately 50 percent. The parameter ζ contributes to the exit of large and old firms; hence we target the fraction of total job destruction due to exit. To pin down the set-up cost χ_0 , we target the annual entry-rate of 11 percent from the BDS.¹⁷

The remaining two parameters are the size of the initial equity injection a_0 and the collateral parameter φ . To inform their calibration, we target the debt-output ratio of one-year old firms computed from the Kaufman Survey ([Robb and Robinson, 2014](#)), and the aggregate debt to total assets ratio from the Flow of Funds.

4.3 Cross-Sectional Implications

We now explore the main cross-sectional implications of the calibrated model, at its steady-state equilibrium.

Table 3 reports some empirical moments that we did not target in the calibration and their model-generated counterparts. The fact that the ratio of dividend payments to profits in the model is close to its empirical value reinforces the view that our collateral constraint is neither too tight nor too loose. The model can also replicate well the distribution of employment by growth rate and by age, neither of which was explicitly targeted.

¹⁷When computing moments designed to be comparable to their counterparts in the BDS, we carefully time-aggregate the model to an annual frequency. For example, the entry-rate in the BDS is measured as the number of age zero firms in a given year divided by the total number of firms. Computing this statistic in the model requires aggregating monthly entry and exit over 12 months.

Table 3: Non-targetted moments

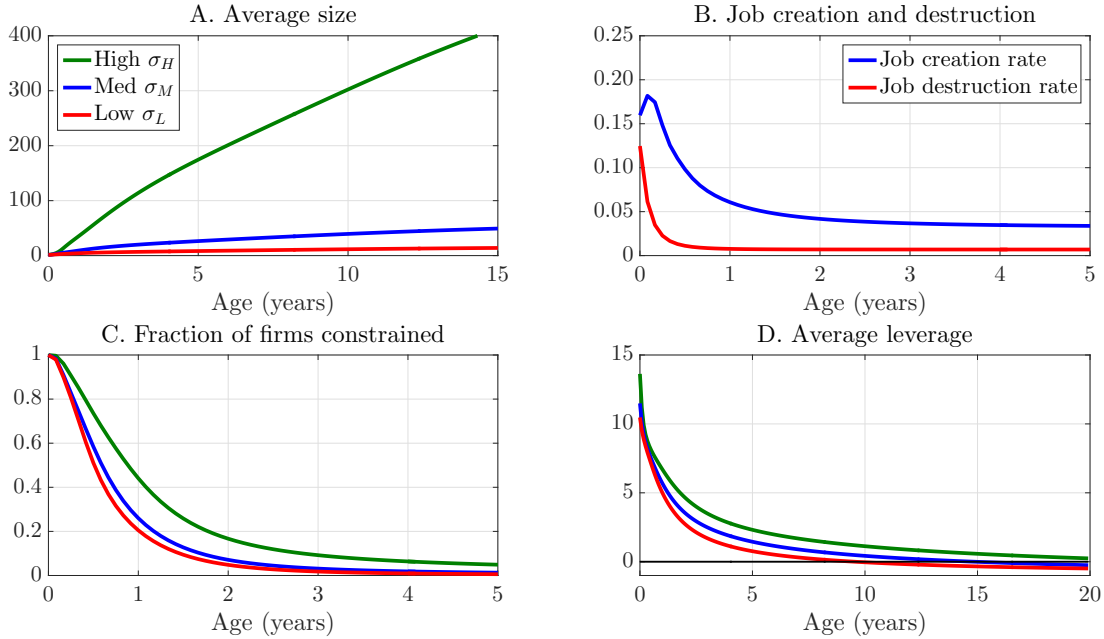
Moment	Model	Data	Source
Aggregate dividend / profits	0.411	0.400	NIPA
¹ Aggregate capital / output	1.100	1.370	Flow of funds
² Employment share: $growth \in [-2.00, -0.20)$	0.070	0.076	Kaas-Kircher
Employment share: $growth \in (-0.20, -0.20]$	0.828	0.848	Kaas-Kircher
Employment share: $growth \in (0.20, 2.00]$	0.102	0.076	Kaas-Kircher
Employment share: $Age \leq 1$	0.013	0.020	BDS
Employment share: $Age \in (1, 10)$	0.309	0.230	BDS
Employment share: $Age \geq 10$	0.678	0.750	BDS

In Figure 4 we plot the average firm size, job creation and destruction rates, fraction of constrained firms and leverage (debt/saving over total assets, b/a) for firms from birth through to maturity. Panel A shows that σ_H -firms, those with closer to constant returns in production, account for the upper tail in the size and growth-rate distributions. On average, though, firm size grows by much less over the life cycle, since these ‘gazelles’—as they are often referred to in the literature—are only a small fraction of the total. On average, the model and the data line up well: average size grows by a factor of 3 between ages 1-5 and 20-25 in the model and 3.4 in the data. Convex recruiting costs and collateral constraints slow down growth: most firms reach their optimal size around age 10, and σ_H -firms keep growing for much longer.

Panel B plots job creation and destruction rates by age. It is a stark representation of the ‘up-or-out’ dynamics of young firms documented in the literature (Haltiwanger, 2011b). Panel C depicts the fraction of constrained firms (defined as those with $k = \varphi a$ and $d = 0$) over the life cycle. In the model, financial constraints bind only for the first few years of a firm’s life, when net worth is insufficient to fund the optimal level of capital. Panel D illustrates that the debt-asset ratio declines with age and after age 10 the median firm is saving (i.e. negative debt). Firms have a precautionary saving motive due to the simultaneous presence of three elements: (i) a concave payoff function because of DRS; (ii) stochastic productivity; and (iii) the collateral constraint.

Panel A of Figure 5 shows that recruiting intensity and the vacancy rate are sharply decreasing with age. These features arise because our cost function implies that both optimal hiring

Figure 4: Average life cycle of firms in the model



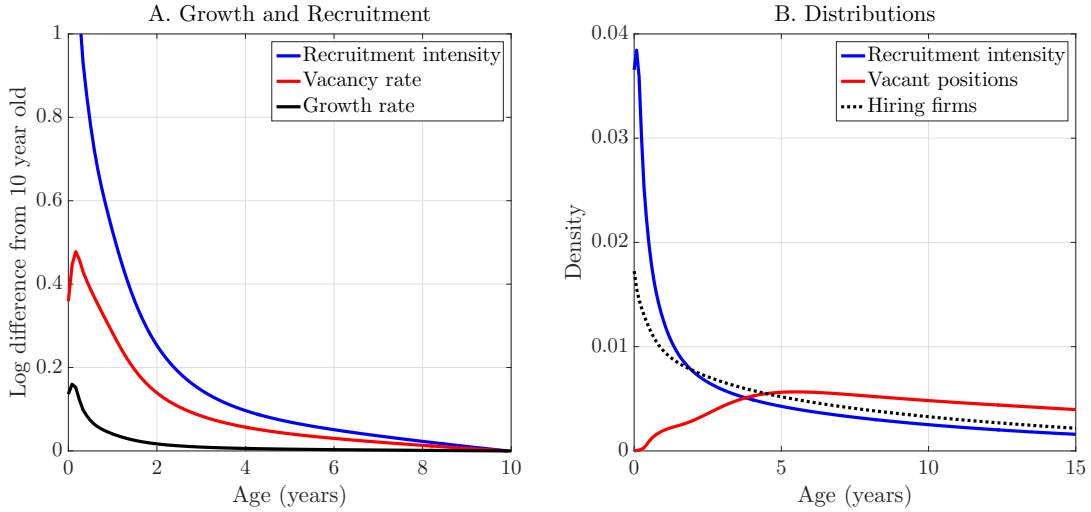
effort and optimal vacancy rates are increasing in the growth rate, and young firms are those with the highest desired growth rates. Moreover, the stronger convexity of \mathcal{C} in the vacancy rate (γ_2), relative to its degree of convexity in effort (γ_1) implies that a rapidly expanding firm prefers to substitute away from vacancies into recruiting intensity to realize its target growth rate. Thus, young firms find it optimal to limit the number of new positions, but recruit very aggressively for the ones that they open. As firms age, growth rates fall and this force weakens.

Panel B shows that, relative to the steady-state age distribution of hiring firms, the effort distribution is skewed towards young firms, whereas the vacancy distribution is skewed towards older firms. In the model the age-distribution of vacancies is almost uniform: young firms grow faster than old ones and, thus, post more vacancies per worker; however, they are smaller and, thus, they post fewer vacancies for a given growth rate. These two forces counteract each other and the ensuing vacancy distribution over ages is nearly flat.

Finally, Figure 5 highlights that the JOLTS notion of vacancy as ‘open position ready to be filled’ is a good metric of hiring effort for old firms, for whom recruiting intensity is nearly constant, whereas it is imperfect for young firms aged 0-5, whose average recruiting intensity and variance of recruitment effort are much higher than those of mature firms.¹⁸

¹⁸We will add an additional panel in Figure 5 to show that the variance of recruitment effort is decreasing in age.

Figure 5: Vacancy and Effort Distributions by Age



5 Aggregate Recruiting Intensity and Macroeconomic Shocks

Our main experiments examine the equilibrium of the economy along perfect foresight paths for shocks to aggregate productivity A and to the financial constraint parameter φ . Appendix C provides details on the solution of the model along these perfect foresight path.

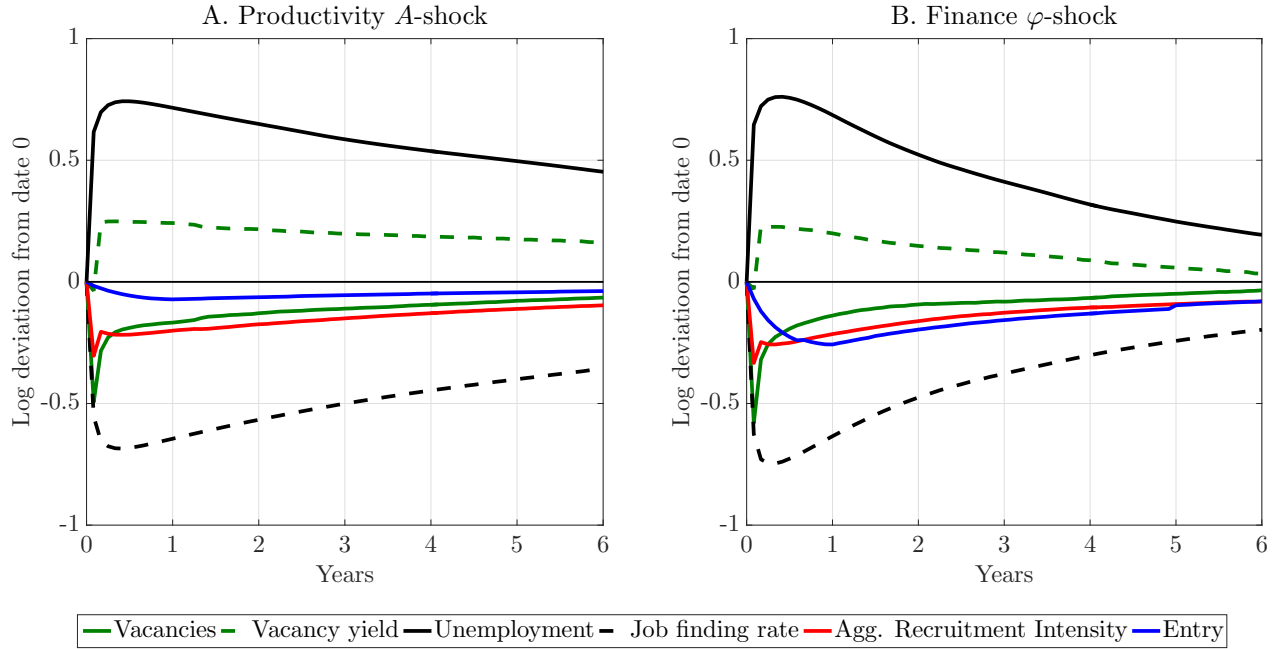
We frame these experiments in the context of the Great Recession. Specifically, we consider AR(1) shocks, choosing the size of the shock so that the model matches the maximum deviation of detrended output over 2008-2012 from its value in 2007, a value of -10 percent (Fernald, 2015). The auto-regressive parameters of each shock are chosen such that the shock has a half-life of six years. This results in a three-percent shock to A , and a 75-percent shock to φ .¹⁹

Figure 6 displays the dynamics of key labor market variables and annual firm entry rate, and Figure 7 displays the dynamics of additional macroeconomic variables, including output, which we match through our choice of the magnitude of the shocks.²⁰ Figure 6 shows that the model accounts extremely well for the joint behavior of labor market variables displayed in Figure 1. While the labor market responds similarly to both shocks, the differential responses of other variables clearly identify a financial shock in the 2008 recession. Specifically, young-

¹⁹These values imply that the auto-regressive parameters depend on the size of the shocks. Thus, the shock to φ is less persistent than the shock to A . In future versions of the paper, we will choose the auto-regressive parameter to match a fixed half-life of output rather than a fixed half-life of the shock.

²⁰Additionally, Figure B2 in Appendix B displays the dynamics of the fraction of firms that are financial constrained and aggregate leverage over the transition.

Figure 6: Dynamics of labor market variables



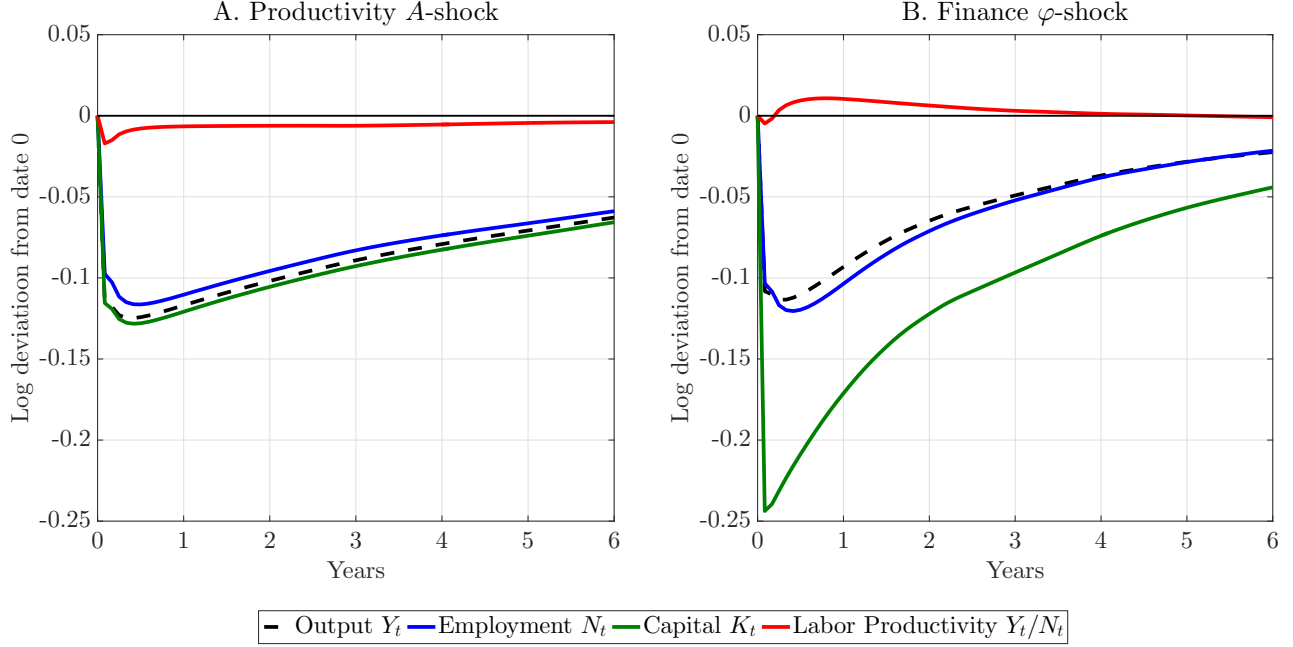
firm values decline sharply, since a large fraction of them are constrained (recall Figure 4), leading to a decline in entry (by 27 percent) that is remarkably close to its empirical value (29 percent).²¹ Constrained firms are also forced to deleverage, thereby reducing the capital stock by 25 percent, which approximates the 26 percent decline observed in the data between 2007 and 2009.²² These patterns indicate that the financial structure of our model, along with the financial shock, are well posed, commending the conclusions of the analysis that follows.

Figure 6 delivers two additional results regarding aggregate recruiting intensity. First, aggregate recruiting intensity declines by 33 percent after the financial shock, thereby accounting for approximately half of the decline observed in the data in Figure 1. Second, the patterns of aggregate recruiting intensity are startlingly similar after both shocks, depressing the aggregate vacancy yield and increasing unemployment. These similar patterns may seem puzzling, since Figure 6 shows that the entry rate of new firms—which account for a disproportionate share of job creation—remarkably differs under the two shocks.

²¹Entry in the data is measured as the number of firms reporting an age of zero divided by the total number of firms in the LBD. The survey is in March and so this measure excludes firms which enter and exit between surveys. In the model the measure is computed using a rolling twelve month window.

²²We measure capital in the data from the Flow of Funds, Table B.103 as the sum of inflation adjusted Real-estate, Equipment and Inventories (lines 3-5). If we remove inventories, capital drops by 28.5 percent.

Figure 7: Dynamics of standard macroeconomic variables



5.1 Decomposing Aggregate Recruiting Intensity

We now explain this apparent puzzle of why aggregate recruiting intensity responds in the same way to both shocks. Our explanation highlights the key role of the main elements of our general-equilibrium model with heterogeneous firms. Specifically, we show that new-firm entry has a quantitatively sizeable effect on aggregate recruiting intensity only (i) in partial equilibrium; and (ii) if we ignore the role of selection in hiring.

To guide our explanation, we return to our expression for aggregate recruiting intensity, using λ^h to denote the distribution of hiring firms:

$$\Phi_t = \left(\frac{V_t^*}{V_t} \right)^\alpha = \left[\int e_{i,t} \left(\frac{v_{i,t}}{V_t} \right) d\lambda_t^h \right]^\alpha. \quad (19)$$

Substituting the policy function for recruitment effort (12) into the above equation and taking log differences, we obtain:

$$\Delta \log \Phi_t = \underbrace{-\alpha \frac{\gamma_2}{\gamma_1 + \gamma_2} \Delta \log q(\theta_t^*)}_{\text{Substitution effect}} + \underbrace{\alpha \Delta \log \left[\int g_{i,t}^{\frac{\gamma_2}{\gamma_1 + \gamma_2}} \left(\frac{v_{i,t}}{V_t} \right) d\lambda_t^h \right]}_{\text{Composition effect}}. \quad (20)$$

We call the two effects in equation (20) the *substitution* and *composition* effect, respectively.

The Substitution Effect. The substitution effect is the change in aggregate recruiting intensity Φ_t due to firms changing effort in response to movements in $q(\theta_t^*)$, holding constant growth rates $g_{i,t}$, vacancies $v_{i,t}$ and the distribution of hiring firms λ_t^h .

In a recession, labor market tightness falls, as firms reduce vacancies and a spike in separations increases the pool of unemployed workers. The surge in unemployment raises the probability that any vacancy matches with an unemployed worker $q(\theta_t^*)$. Therefore, given the hiring technology $g_{i,t} = q(\theta_t^*)e_{i,t}v_{i,t}/n_{i,t}$, a growing firm with a target growth rate $g_{i,t}$ now reoptimizes its combination of recruiting inputs $e_{i,t}$ and $v_{i,t}$. Since costs are increasing in both effort and vacancies, the firm decreases both: a slack labor market makes it easier for employers to hire, so employers spend less to attract workers.

The policy functions for effort e and the vacancy rate vr highlight the relative strength of the substitution effect for these two margins in response to a change in $q(\theta_t^*)$:

$$e(g, \theta^*) = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_2}{\gamma_1 + \gamma_2}} g^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}, \quad (21)$$

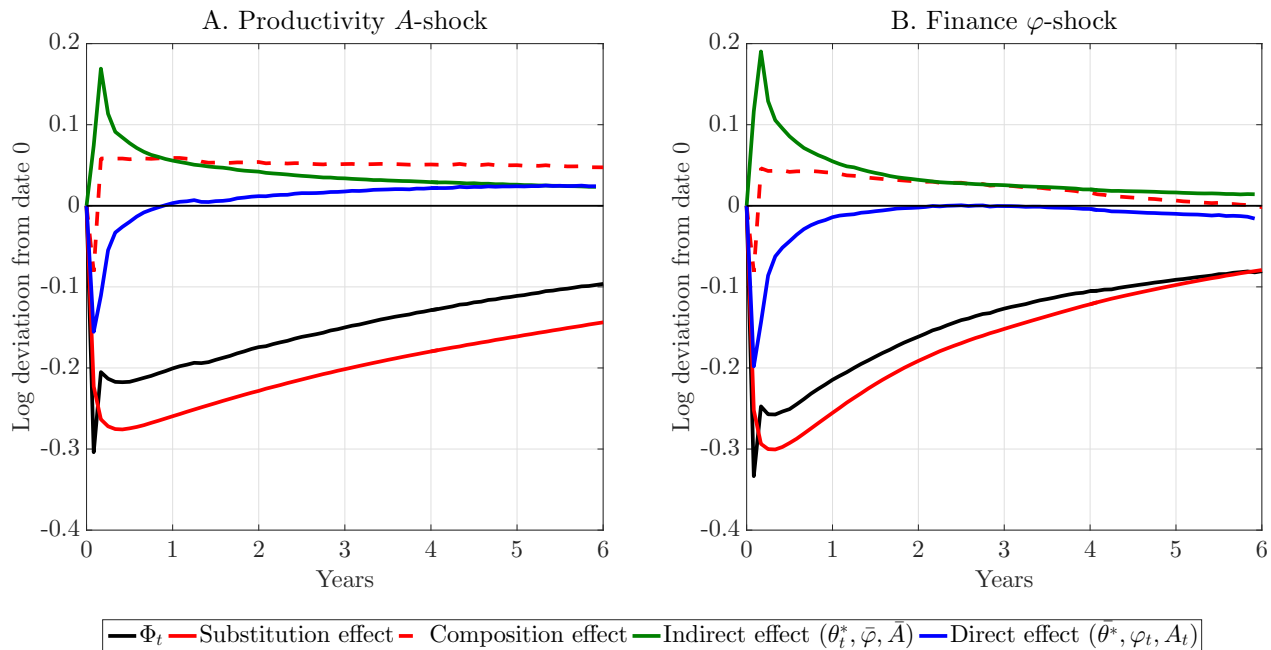
$$vr(g, \theta^*) = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}} (g)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}}. \quad (22)$$

Since $\gamma_2/(\gamma_1 + \gamma_2) = 0.82 > 0.5$ the firm reduces effort relatively more than it reduces its vacancy-rate. Therefore, in the aggregate, V_t^* declines more than V_t does (or equivalently, Φ_t falls), leading to a negative substitution effect.

The Composition Effect. The composition effect is defined residually, thereby including the effect of changes in growth rates $g_{i,t}$, vacancy policies $v_{i,t}$, and the distribution of hiring firms λ_t^h on aggregate recruiting intensity.

It is useful to split the composition effect into its two main components. The first is a *direct composition effect*: the response to the shock in a partial-equilibrium economy, while keeping θ_t^* at its steady state level, denoted $\bar{\theta}$ (in what follows, we use variables with a bar to denote their steady-state values, e.g., $\bar{\theta}$). The direct effect *reduces* aggregate recruiting intensity, since a negative aggregate shock lowers firm growth rates and reallocates hiring away from young, fast-growing firms. Moreover, this reduction should be larger after the financial shock than after

Figure 8: Decomposition of aggregate recruitment intensity



the productivity shock, since the former disproportionately hits young, fast-growing firms.

The second is the *indirect composition effect*: the response of the economy under the equilibrium path for θ_t due to the shock, while keeping $\varphi_t = \bar{\varphi}$ and $A_t = \bar{A}$ at their respective steady-state values. The indirect effect *increases* aggregate recruiting intensity, since firms grow *faster* when $q(\theta_t^*)$ rises, as they meet job seekers more easily. Therefore, this indirect effect partially offsets the decline in the growth rates of firms.²³

Figure 8 plots these components of aggregate recruitment intensity in response to the productivity (panel A) and financial shocks (panel B), revealing several interesting patterns. First, the substitution effect (red line) is quantitatively the largest one, accounting for virtually all the decline in aggregate recruitment intensity (black line). Second, the composition effect (red dashed line) is positive but small, as the direct (blue line) and the indirect (green line) components almost offset each other. Third, comparing the two plots confirms that the direct effect is somewhat larger after the financial shock than after the productivity shock, consistent with the

²³We should point out that the composition effect also varies over time because it is concave in $g_{i,t}$, since $\gamma_2/(\gamma_1 + \gamma_2) < 1$; hence, a mean-preserving spread in growth rates lowers Φ_t . However, our calibration implies that the quantitative magnitude of the change in Φ_t due to the increase in the dispersion of growth rates is negligible and, thus, we focus here on the two main components. Section 5.3 and Appendix D provide further details.

notion that a financial crisis has a larger adverse effect on firm entry.

We now discuss the magnitudes of these components in further detail. We focus on the financial shock, although we have verified that the intuition and quantitative results extend to the case of the productivity shock.

5.2 Why Is the Substitution Effect Large?

Figure 8 shows that the substitution effect is the most important factor in determining aggregate recruiting intensity. We now explain how this magnitude obtains, proceeding in three steps.

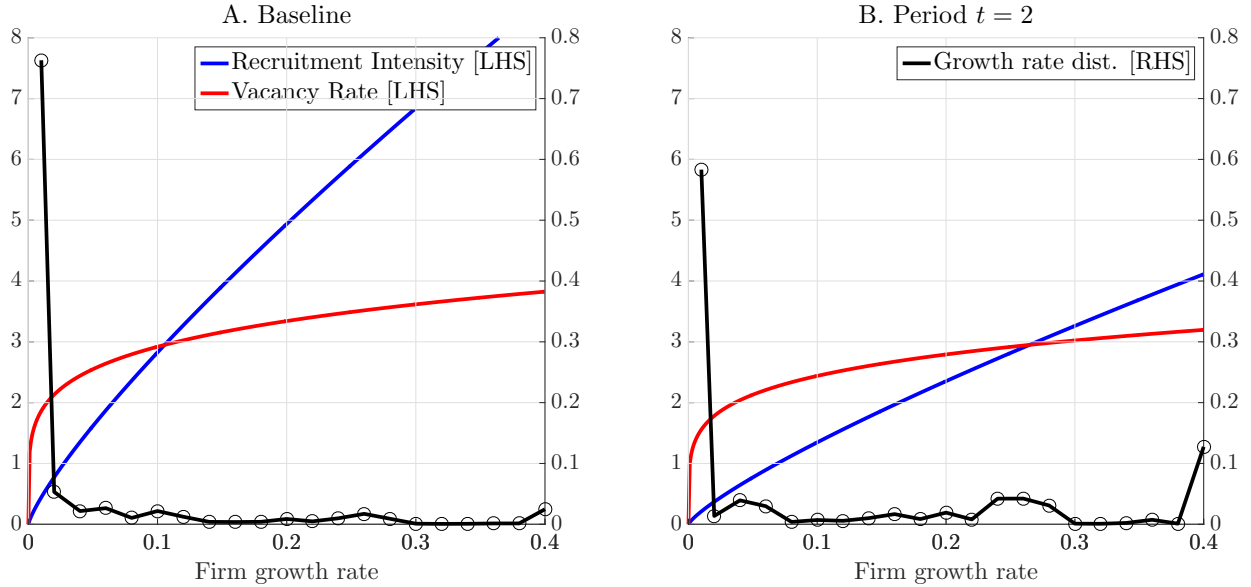
First, we note that the policy functions for effort and the for the vacancy rate—i.e., equations (21) and (22)—imply that the relative magnitudes of γ_1 and γ_2 determine the magnitudes of the elasticities of each policy with respect to g/q . DFH’s cross-sectional regressions of firms’ growth rates $g_{i,t}$ on the vacancy-filling rate (*including* time fixed effects) impose $\gamma_2/(\gamma_1 + \gamma_2) = 0.82$, which means that effort is more elastic than the vacancy rate.

Second, these policy functions highlight that firms’ efforts and vacancy rates respond symmetrically to a change in g keeping q fixed, and to a change in q keeping g fixed. These symmetric responses arise because q , e and v/n enter the growth ‘production’ technology symmetrically: $g = q(\theta^*)e(v/n)$.

Figure 9 puts together the two previous points: (i) the elasticity of effort is larger than that of the vacancy rate with respect to g ; (ii) the elasticity of effort with respect to g has the same magnitude, but opposite sign, as to that with respect to q . Specifically, we plot firms’ effort and vacancy-rate policies in steady-state (panel A) and two periods after the shock (panel B). Panel A shows that in steady state recruitment intensity increases much faster than the vacancy rate as firm growth rates increase, due to its higher elasticity. Since q increases substantially after the shock (Figure 6), the comparison between panel A and panel B shows that recruitment intensity declines substantially more than the vacancy rate does.

Third, our quantitative results displayed in Figure 6 and 7 imply that changes in aggregate q are substantially larger than changes in g across firms, which allows us conclude that the substitution effect is negative and large. The next Section leverages our general-equilibrium model to explain in more detail why the change in firm growth rates is small (and, thus, the composition effect is small).

Figure 9: Recruitment policies and distribution of growth rates of hiring firms



Note (i) Panel B plots distribution of growth rates and recruitment policies two periods after the shock. To be precise, period 0 the economy is in steady-state, in period 1 the path for the shock is realized, period 2 then follows. We choose period 2 rather than period 1 since due to the timing of the model workers that are fired in period 1 do not enter the labor market as unemployed workers until period 2. (ii) The growth rate distribution is computed over bins of width 0.02.

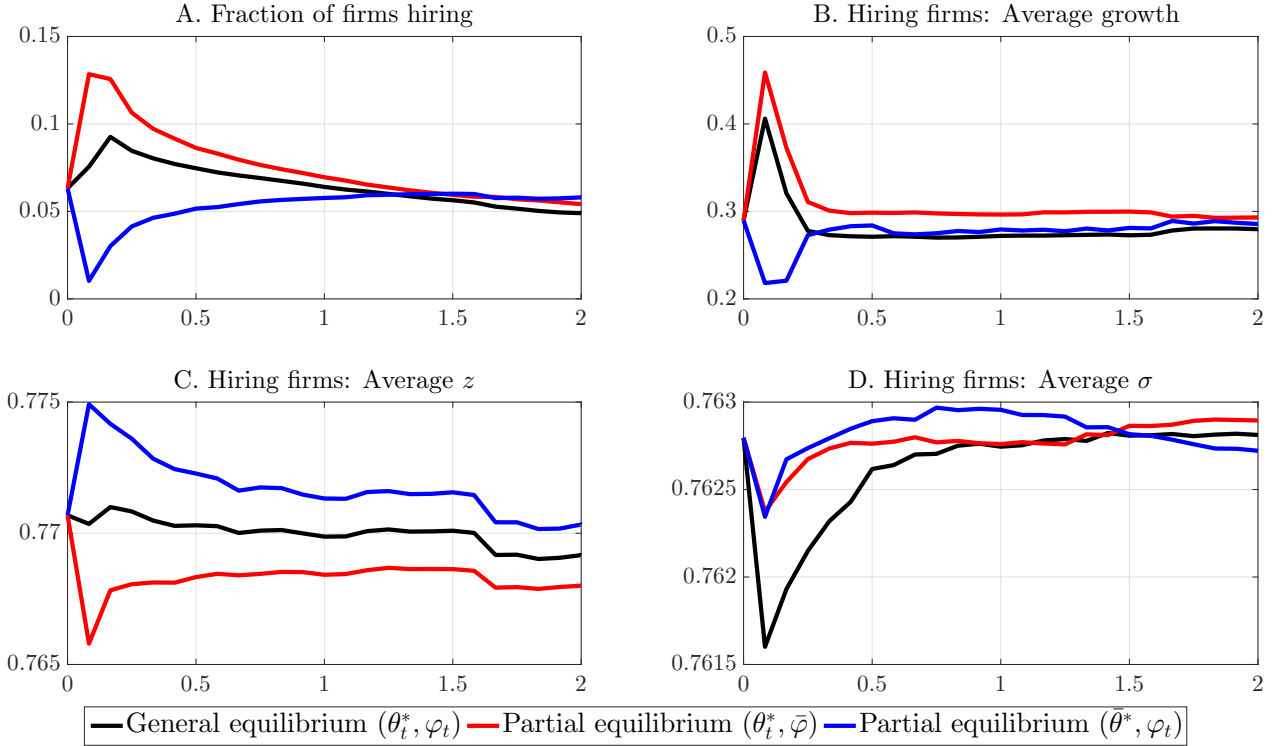
5.3 Why Is the Composition Effect Small?

An important implication of the plots displayed in Figure 9 is that the growth rate distribution of hiring firms changes only slightly after the shock. We have already emphasized that the direct and the indirect effects are off-setting forces: the former reduces firm growth rates, due to the negative shock; the latter increases firm growth, as per worker hiring costs fall. A further consideration is that the positive selection of hiring firms on productivity tempers both effects. Specifically, the decline in φ_t increases the average productivity of hiring firms, since some firms that hired in steady state no longer hire after the shock. Conversely, the increase in $q(\theta^*)$ decreases the average productivity of hiring firms, since some firms that did not hire in steady state find it easier to hire after the shock.

Figure 9 also displays the distribution of growth rates in steady-state (panel A) and two periods after the shock (panel B). The net effect is that the average growth rate of hiring firms increases.

Figure 10 illustrates the role of the changes in the productivity distribution of firms on the direct and the indirect composition effects. Specifically, we plot in panel A): the fraction of firms

Figure 10: Selection of hiring firms on productivity z and DRS σ



hiring; B) the average growth rates of hiring firms; C) average productivity, and; D) average σ (the DRS parameter). All panels display their respective values computed over three different paths for the transition dynamics: (i) the general equilibrium transition dynamics that generate Figure 6 (black); (ii) the partial equilibrium dynamics along the equilibrium path for θ_t^* , keeping $\varphi_t = \bar{\varphi}$ (red) corresponding to the indirect composition effect; and (iii) the partial equilibrium dynamics along the equilibrium path for φ_t , keeping $\theta_t = \bar{\theta}$ (blue), corresponding to the direct composition effect.

Panels C and D indicate that selection—i.e., changes in the productivity distribution of firms—has a quantitatively large impact on the direct effect: the blue lines show that the average productivity of hiring firms (panel C) and their average returns-to-scale (panel D) both increase substantially after the shock; in turn, these attenuate the decreases of the fraction of firms hiring (panel A) and of their average growth (panel D). Instead, selection seems to have a quantitatively smaller impact on the indirect effect: the red lines show that fraction of firms hiring (panel A) and their average growth (panel B) spike after the shock. Overall, Figure 10 shows that the positive selection of hiring firms in a recession is quantitatively substantial and

Table 4: Cross-sectional firm statistics for general and partial equilibrium transitions

	Unweighted				Vacancy-weighted			
	SS ($\bar{\theta}^*, \bar{\varphi}$)	GE (θ_t^*, φ_t)	PE ($\theta_t^*, \bar{\varphi}$)	PE ($\bar{\theta}^*, \varphi_t$)	SS ($\bar{\theta}^*, \bar{\varphi}$)	GE (θ_t^*, φ_t)	PE ($\theta_t^*, \bar{\varphi}$)	PE ($\bar{\theta}^*, \varphi_t$)
A. Hiring firms								
Ave g	0.290	0.406	0.459	0.387				
Std dev g	0.405	0.541	0.578	0.544				
Ave z	0.771	0.762	0.766	0.762				
Ave σ	0.763	0.762	0.762	0.762				
B. All firms								
Ave g	0.016	0.025	0.069	0.027				
Std dev g	0.126	0.183	0.244	0.165				
Ave z	0.888	0.882	0.882	0.882				
Ave σ	0.762	0.762	0.762	0.762				

Note (i) All non-steady-state statistics are computed in period 2 of transition dynamics as in Figure 9.

prevents the composition effect from having a large impact on aggregate recruiting intensity.²⁴

An additional source of fluctuations in Φ_t that enters into the composition effect arises because the composition effect is concave in $g_{i,t}$, since $\gamma_2/(\gamma_1 + \gamma_2) < 1$. A mean-preserving spread in growth rates therefore has a negative effect on Φ_t . This mechanism bears some similarities to that which generates fluctuations in aggregate recruitment intensity in [Kaas and Kircher \(2015\)](#). Returning to the experiment considered in Figure 9, we report in Table 4 the vacancy-weighted standard deviation of growth rates across hiring firms, as well as other related statistics. We are well positioned to assess this effect since (i) we match the standard deviation of growth rates in steady state, (ii) the financial shock generates a 45 percent increase in the standard deviation of growth rates, which compares well to the 39 percent increase estimated by [Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry \(2012\)](#). We find that this mechanism has a negligible contribution to the composition effect, since $\gamma_2/(\gamma_1 + \gamma_2) = .82$ is close to 1 and, thus, $\mathbb{E} \left[g^{\frac{\gamma_2}{\gamma_1 + \gamma_2}} \right] \approx \mathbb{E} [g]^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}$. See Appendix D for a more thorough discussion.

²⁴A more nuanced exercise to isolate the role of selection—which we plan to add to our analysis—would be to take the measure of hiring firms over the transition

$$\lambda_t^h = \mathbf{1}_{[h_{i,t} > 0]} \lambda$$

and use this to compute a counter-factual series for the composition effect under the steady-state policies

$$Comp_t = \left[\int \bar{e} \frac{\bar{v}}{V} d\lambda_t^h \right]^\alpha.$$

Finally, Table 4 illustrates that the moments of the distribution of growth rates are similar whether we compute them over the distribution of hiring firms directly, or weighted by vacancies, as the analytical expression of recruitment intensity in equation (19) requires. This similarity further highlights that firms attain their hiring targets mostly by modifying their recruitment effort rather than their posted vacancies.

6 A New Index of Aggregate Recruitment Intensity

Based on the results from our general-equilibrium model we now construct a rule-of-thumb index of aggregate recruiting intensity. The index is easy to compute from observable labor market aggregates. We further compare it with the one that DFH provide via their “generalized matching function.”

To do so, we return to the aggregate matching function and obtain $\log(H_t/V_t) = \log \Phi_t + \log q_t$, which links the aggregate vacancy-yield, recruitment intensity, and matching rate. Totally differentiating this expression for (i) the aggregate hiring rate H_t/N_t , and (ii) the matching rate q_t , we obtain:

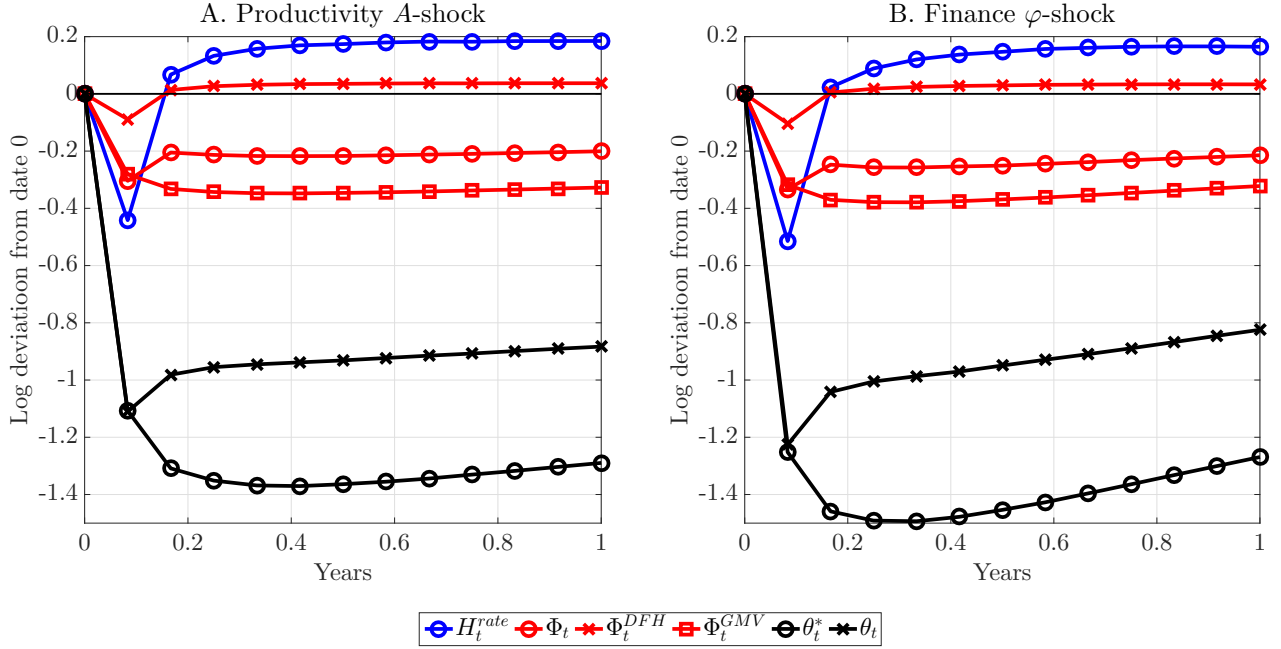
$$\frac{d \log H_t/V_t}{d \log H_t/N_t} + 1 = \frac{d \log \Phi_t}{d \log(H_t/N_t)} + \frac{d \log \Phi_t}{d \log q_t} + \frac{d \log q_t}{d \log(H_t/N_t)} + 1. \quad (23)$$

The working hypothesis of DFH is that (i) the second and third terms on the right-hand side are zero; and (ii) the term on the left-hand side—the macro-elasticity of the vacancy-yield to the hiring-rate—is the same as the estimated micro-elasticity $\zeta = 0.82$. These assumptions deliver the DFH measure of aggregate recruitment intensity: $d \log \Phi_t^{DFH} = \zeta d \log(H_t/N_t)$. Using data on H_t/N_t , DFH find that it accounts for 20 percent of the decline in job finding rates over the 2007-08 recession.²⁵

The transition dynamics of our model indicate that the general-equilibrium effect of changes in q_t on the composition of recruiting activities (the second term in the RHS of eq. 23) accounts for most movements in aggregate recruitment intensity. Thus, we propose an alternative back-of-the-envelope measure of aggregate recruitment intensity that focuses on it. Specifically, as

²⁵Under a standard matching function $d \log f_t = \alpha d \log(V_t/U_t)$. Under the augmented matching function $d \log f_t = \alpha d \log(V_t/U_t) + \alpha d \log \Phi_t^{DFH}$. With data on f_t and their constructed series Φ_t^{DFH} DFH’s result is computed by dividing $d \log f_t$ by $\alpha d \log \Phi_t^{DFH}$.

Figure 11: Dynamics of aggregate recruitment intensity



per equation (20) we set $\frac{d \log \Phi_t}{d \log q_t} = -\alpha \frac{\gamma_2}{\gamma_1 + \gamma_2} = -\alpha \tilde{\zeta}$. We still need to map observable aggregates into $q_t = \bar{\Phi} \theta_t^{*(1-\alpha)}$. We do not directly observe effective market tightness θ_t^* , but the quantitative evaluation of our model prompts us to infer θ_t^* from observed θ_t , with an elasticity of $\varepsilon_{\theta_t^*, \theta_t} = 1.45$.²⁶ Therefore, our expression for changes in aggregate recruitment intensity equals:

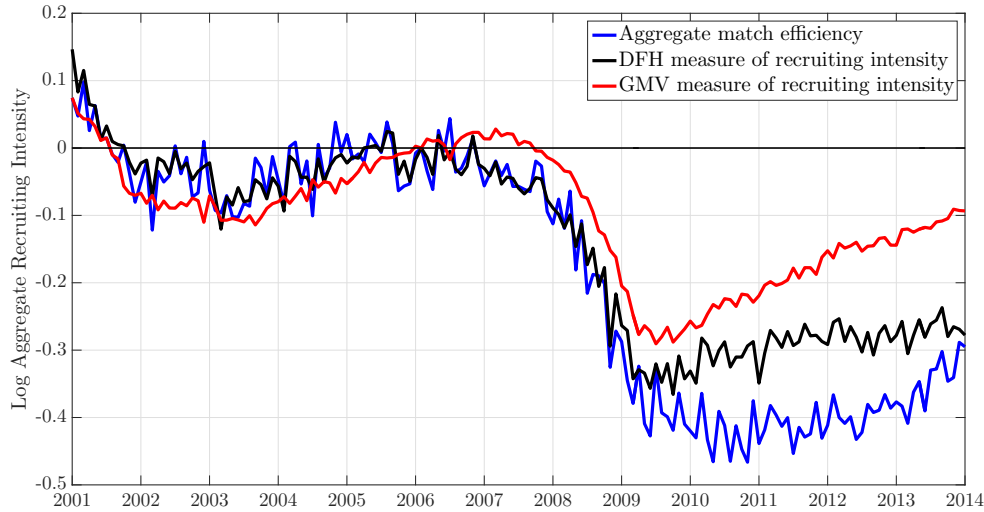
$$d \log \Phi_t^{GMV} = \underbrace{\alpha \tilde{\zeta} \times (1 - \alpha) \times \varepsilon_{\theta_t^*, \theta_t}}_{\approx 0.30} d \log \theta_t. \quad (24)$$

Figure 11 plots the equilibrium path of aggregate recruitment intensity (red circled line), our Φ_t^{GMV} index (red squared line), as well as the DFH index Φ_t^{DFH} . Our Φ_t^{GMV} index approximates remarkably well the path of aggregate recruitment intensity, more precisely than Φ_t^{DFH} does.

Figure 12 replicates Figure X of DFH, adding our index Φ_t^{GMV} , as well as aggregate matching efficiency. We should point out that the *causal* interpretations of the two indices differ. Our index Φ_t^{GMV} relies on the role of market-tightness, as our general-equilibrium model highlights that this is the primary driver of aggregate recruitment intensity. The DFH index Φ_t^{DFH} instead relies on the aggregate hiring rate. Nonetheless, the two indices line up closely, due to the

²⁶This elasticity lies between the elasticities estimated from the transition dynamics paths for θ_t^* and θ_t following the productivity shock (equal to 1.41) and financial shock (equal to 1.51).

Figure 12: Index of implied aggregate recruiting intensity (Jan 2007 = 0)



strong correlation between market-tightness and hiring rates in the time-series.

7 Conclusions

We have developed a rich model to study aggregate recruitment intensity and its role in explaining the fluctuations in aggregate matching efficiency. The model allows for numerous different mechanisms—of both partial- and general-equilibrium nature—to affect recruitment intensity at the firm level. Specifically, it allows for financial frictions, as well as firm age and size, to affect the growth rates of firms, which are the key determinants of firm-level recruiting intensity according to the empirical findings of [Davis, Faberman, and Haltiwanger \(2013\)](#). Our calibration of the unknown parameters of the model ties all of these features to the data.

The calibrated model generates sizeable movements in aggregate recruiting intensity. Moreover, our calibrated model indicates that, during the Great Recession, aggregate recruitment intensity declined mainly because the number of available job seekers per vacancy increased—i.e., market tightness declined—making it easier for firms to achieve their recruitment targets without having to spend on recruitment costs. The decline of new-firm entry played a small role for aggregate recruitment intensity, even though our model matches the large contribution of young firms to the aggregate number of vacancies and hires observed in the data.

As emphasized by [Faberman \(2016\)](#), making progress in understanding how firms' hiring

decisions respond to macroeconomic conditions is important since job creation policies that fail to recognize the determinants of employer's recruitment effort may fall short in achieving their goal.

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APPENDIX

This Appendix is organized as follows. Section [A](#) contains the derivations of the cost hiring function that we introduced in Section [4](#). Section [B](#) provides additional figures referenced in the main text. Section [C](#) details the algorithms for the computation of the stationary equilibrium and the transitional dynamics.

A The hiring cost function

In this section we show that, once we postulate the hiring cost function

$$\mathcal{C}(n, e, v) = \left[\frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left(\frac{v}{n} \right)^{\gamma_2} \right] v \quad (\text{A1})$$

then, through firm's optimization we obtain a log-linear cross-sectional relationship between the job-filling rate and the employment growth rate that is consistent with the empirical findings in [DFH](#). Next, we show that our cost function boils down to the one that [Kaas and Kircher \(2015\)](#) choose. Finally, by substituting the firm FOCs into [\(A1\)](#), we derive a formulation of the cost only in terms of (n, n') that we use in the intertemporal problem [\(10\)](#) in the main text.

As we explained in Section [\(3.1\)](#), the firm solves a static cost minimization problem: given a choice of n' , it determines the lowest cost combination of (e, v) that can deliver n' . The hiring firm's cost minimization problem is

$$\begin{aligned} \mathcal{C}(n, n') &= \min_{e, v} \left[\frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left(\frac{v}{n} \right)^{\gamma_2} \right] v \\ \text{s.t.} \quad &: \quad n' - n \leq q(\theta^*) e v \\ &e \in [0, 1], \quad v \geq 0 \end{aligned} \quad (\text{A2})$$

Convexity of the cost function [\(A1\)](#) in (e, v) requires $\gamma_1 \geq 1$ and $\gamma_2 \geq 0$. When $\gamma_1 = \gamma_2 = 0$, we have the standard model where every firm sets $e = 1$ and the cost of vacancy creation is linear. After setting up the Lagrangian, and ignoring for now the corner solution $e = 1$, one can easily derive the two FOCs with respect to e and v that, combined together, yield a relationship

between the optimal choice of e and the optimal choice of the vacancy rate v/n :

$$e = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1}} \left(\frac{v}{n} \right)^{\frac{\gamma_2}{\gamma_1}}. \quad (\text{A3})$$

Note that, if $\gamma_2 = 0$, as in [Pissarides \(2000\)](#), recruiting intensity is equal to a constant for all firms and it is independent of aggregate labor market conditions—both counterfactual implications. The following changes in parameters (*ceteris paribus*) result in a substitution away from vacancies and towards effort: $\uparrow \kappa_2, \downarrow \kappa_1, \uparrow \gamma_2$, and $\downarrow \gamma_1$. The effect of the cost shifter is obvious. A higher curvature on the vacancy rate in the cost function ($\uparrow \gamma_2$) makes the marginal cost of creating vacancies rising faster than the marginal cost of recruiting effort; since the gain in terms of additional hires from a marginal unit of effort or vacancies is unaffected by γ_2 , it is optimal for the firm to use relatively more effort.

Now, substituting the law of motion for employment at the firm level into [\(A3\)](#), we obtain the optimal recruitment effort choice, expressed only as a function of the firm-level variables (n, n') :

$$e(n, n') = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_2}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}} \quad (\text{A4})$$

which, in turn implies, for the job filling rate,

$$f(n, n') = q(\theta^*) e(n, n') = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}. \quad (\text{A5})$$

This equation demonstrates that the model implies a log-linear relation between the job filling rate and employment growth at the firm level, with elasticity $\gamma_2 / (\gamma_1 + \gamma_2) < 1$ as in the data. Moreover, firm-level job filling rates are countercyclical, through their dependence on $q(\cdot)$.

Finally, substituting [\(A5\)](#) into the firm-level law of motion for employment yields an expression for the vacancy rate

$$\frac{v}{n} = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}}. \quad (\text{A6})$$

Now, note that, by substituting the optimal choice for recruitment effort [\(A3\)](#) into [\(A1\)](#), we

obtain the following formulation for the cost function

$$\mathcal{C}(n, v) = \left[\kappa_2 \frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)} \left(\frac{v}{n} \right)^{\gamma_2} \right] v, \quad (\text{A7})$$

which is one of the specifications that [Kaas and Kircher \(2015\)](#) invoke.

Finally, if we use (A6) into (A7), we obtain a version of the cost function only as a function of (n, n') that we can use directly in the dynamic problem (10):

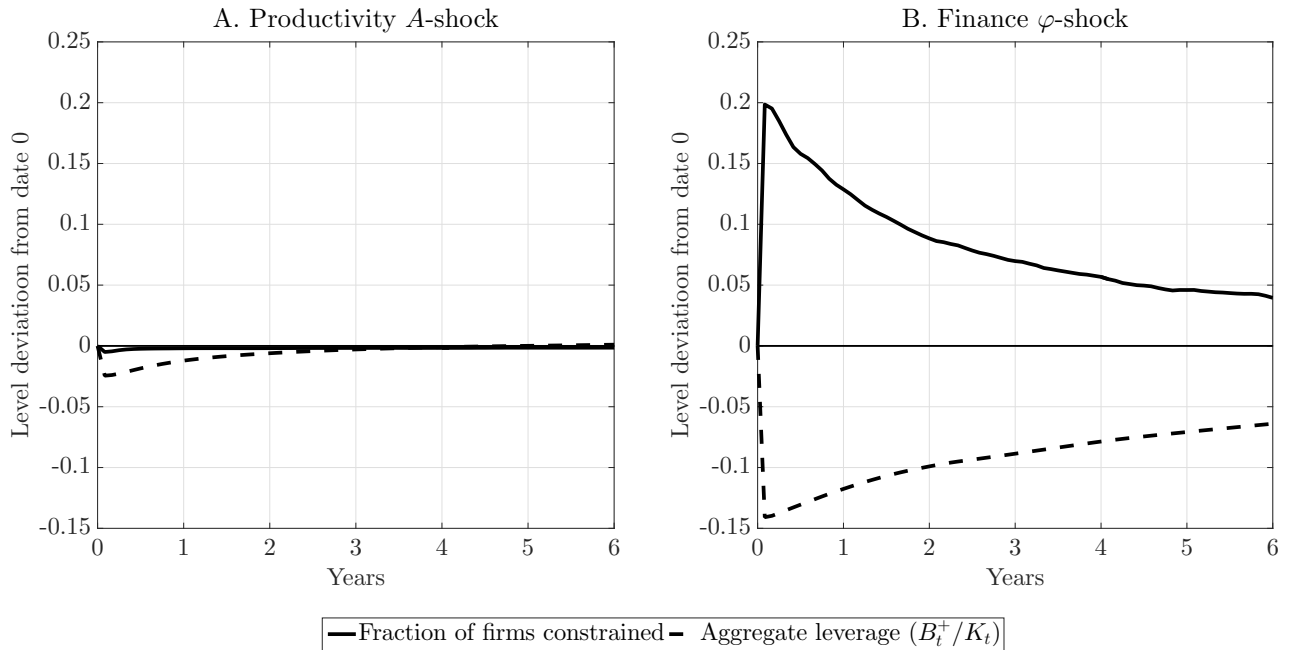
$$\mathcal{C}^*(n, n') = \kappa_2 \left[\frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)} \right] \left\{ \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}} \right\}^{1 + \gamma_2} n.$$

B Additional figures

Figure B1: Vacancy and Effort Distributions by Size



Figure B2: Dynamics of financial variables



C Computational details

C.1 Value and policy functions

We use collocation methods to solve the firm's value function problem (4)-(7). Let $s = (n, a, z)$ be the firm's idiosyncratic state, abstracting from heterogeneity in σ since this is fixed. We approximate the *expected* value function $\mathbb{V}^e(n', a', z)$ which gives the firm's expected value conditional on current decisions for assets and employment

$$\mathbb{V}^e(n', a', z) = \int_{\mathcal{Z}} \mathbb{V}(n', a', z') d\Gamma(z, z'),$$

where the integrand is the value given in (6).

We set up a grid of collocation nodes $S = N \times A \times Z$ where $N = \{n_1, \dots, n_{N_n}\}$, with $N_n = N_a = N_z = 10$. The grid Z is constructed by first creating an equi-spaced grid in probabilities from 0.001 to 0.999, which is then inverted through the cdf of the stationary distribution implied by the process for z to obtain Z . The grids A and N are chosen to have a far higher density at lower values. The upper bound for employment, \bar{n} , is chosen so that the optimal size of the highest productivity firm $n^*(\bar{z}) < \bar{n}$. The upper bound for assets, \bar{a} , is chosen so that the optimal capital $k^*(\bar{z})$ can be financed, that is $k^*(\bar{z}) < \varphi\bar{a}$. Note that these are parameter dependent, therefore recomputed for each new vector of parameters considered in estimation.

We approximate $\mathbb{V}^e(s)$ on S using a linear spline with $N_s = N_n \times N_a \times N_z$ coefficients. Given a guess for the spline's coefficients we iterate towards a vector of coefficients that solve the system of N_s Bellman equations, which are linear in the N_s unknown coefficients. Each iteration proceeds as follows. Given the spline coefficients we use golden search to compute the optimal policies for all states $s \in S$, and the value function $\mathbb{V}(s)$. We then fit another spline to $\mathbb{V}(s)$ which facilitates integration of productivity shocks $\varepsilon \sim \mathcal{N}(0, \vartheta_z)$. To compute $\mathbb{V}^e(s)$ on S we approximate the integral by

$$\mathbb{V}^e(n, a, z) = \sum_{i=1}^{N_\varepsilon} w_i \mathbb{V}(n, a, \exp(\rho_z \log(z) + \varepsilon_i)).$$

Here $N_\varepsilon = 80$ and the values of ε_i are constructed by creating a grid of equi-spaced points between 0.001 to 0.999, then using the inverse cdf of the shocks (normal) to create a grid in ε .

The weights w_i are given by the probability mass of the normal distribution centered around each ε_i . Note that this differs from quadrature schemes where one is trying to minimize the number of evaluations of the integrand, usually with N_ε around four. Since $\mathbb{V}(s)$ is already given by an approximant at this step, and the integral is only computed once each iteration, this is not a concern and we compute the integral very precisely. We then fit an updated vector of coefficients to $\mathbb{V}^e(s)$ and continue.²⁷

C.2 Stationary distribution

To construct the stationary distribution we use the method of non-stochastic simulation from [Young \(2010\)](#), modified to accommodate a continuously distributed stochastic state. We create a new, fine grid of points S^f on which we approximate the stationary distribution using a histogram, setting $N_n^f = N_n^f = N_z^f = 100$. Given our approximation of the expected continuation value we solve for the policy functions $n'(s^f)$ and $a'(s^f)$ on the new grid and use these to create two transition matrices \mathbf{Q}_n and \mathbf{Q}_a which determine how mass shifts from points $s^f \in S^f$ to points in N^f and A^f , respectively. We construct \mathbf{Q}_x as follows for $x \in \{a, n\}$

$$\mathbf{Q}_x[i, j] = \left[\mathbf{1}_{x'(s_i^f) \in [X_{j-1}^f, X_j^f]} \frac{x'(s_i^f) - X_j^f}{X_j^f - X_{j-1}^f} + \mathbf{1}_{x'(s_i^f) \in [X_j^f, X_{j+1}^f]} \frac{X_{j+1}^f - x'(s_i^f)}{X_{j+1}^f - X_j^f} \right],$$

for $i = 1, \dots, N_s^f$ and $j = 1, \dots, N_x^f$.²⁸ This approach ensures that aggregates computed from the stationary distribution will be unbiased. For example if $x'(s) \in (X_j, X_{j+1})$, then masses w_j and w_{j+1} are allocated to X_j and X_{j+1} such that $w_j X_j + w_{j+1} X_{j+1} = x'(s)$. The transition matrix for the process for z is computed by $\mathbf{Q}_z = \sum_{i=1}^{N_\varepsilon} w_i \mathbf{Q}_z^i$, where \mathbf{Q}_z^i is computed as above under $z'(s^f) = \exp(\rho_z \log z + \varepsilon_i)$. Finally the overall incumbent transition matrix \mathbf{Q} is the tensor product $\mathbf{Q} = \mathbf{Q}_z \otimes \mathbf{Q}_a \otimes \mathbf{Q}_n$.

To compute the stationary distribution we still need the distribution of entrants. To allow for entry cut-offs to move smoothly we compute entrant policies on a dense grid of $N_z^0 =$

²⁷In practice, instead of this simple iterative approach to solve for the coefficients, we follow a Newton algorithm as in [Miranda and Fackler \(2002\)](#), which is two orders of magnitude faster. The Newton algorithm requires computing the Jacobian of the system of Bellman equations with respect to the coefficient vector. The insight of [Miranda and Fackler \(2002\)](#) is that this is simple to compute given the linearity of the system in the coefficients.

²⁸If exit is optimal on grid point s_i^f then we set row i of \mathbf{Q}_x to zero.

500 productivities. This is clearly important for us since it ensures that entry does not jump in the transition dynamics or across parameters in calibration. The grid Z^0 is constructed by taking an equally spaced grid in cumulative probabilities and inverting it through the cdf of potential entrant productivities (exponential). Let the corresponding vector of weights be given by \mathbf{P}_0 . Given the approximation of the continuation value \mathbb{V}^e we can solve the potential entrants policies $n'_0(s_0)$ and $a'_0(s_0)$, conditional on entry. We can then solve the firm's discrete entry decision. Finally we compute an equivalent transition matrix \mathbf{Q}_0 using these policies, where non-entry results in a row of zeros in \mathbf{Q}_0 .

The discretized stationary distribution \mathbf{L} on S^f is then found by the following approximation to the law of motion (14)

$$\mathbf{L} = (1 - \zeta)\mathbf{Q}'\mathbf{L} + \lambda_0\mathbf{Q}'\mathbf{P}_0,$$

which is a contraction on \mathbf{L} , solved by iterating on a guess for \mathbf{L} . The final stationary distribution is found by choosing λ_0 such that $\sum_{i=1}^{N_s^f} L_i = 1$.

C.3 Computation of moments

An aggregate X is computed by integrating λ over firm policies $x(s)$. Using the above approximation this is simply $X = \mathbf{L}'x(s)$.

For age based statistics, our moments in the data refer to firm ages in years. We therefore generate an 'age zero' measure of firms by allowing for 12 months of entry. We then iterate this distribution forward to compute age statistics such as average debt to output for age 1 firms, or the distribution of vacancies by age.

For statistics such as the average annual growth rate conditional on survival we need to simulate the model. In this case we draw 100,000 firms on S^f in proportion to \mathbb{L} and simulate these forwards solving (rather than interpolating) firm policies each period and evolving productivity with draws from the continuous distribution of innovations ε . To remove the effect of the starting grid, we simulate for 36 months and compute our statistics comparing firms across months 24 and 36.

C.4 Transition dynamics

Transition dynamics are solved for in the usual way. Consider the case of a shock to aggregate productivity A . A path for $\{A_t\}_{t=0}^T$ is chosen with $A_0 = A_T = \bar{A}$. Given a conjectured path for equilibrium market-tightness $\{\tilde{\theta}_t^*\}_{t=0}^T$ and the assumption that the date T continuation values of the firm are the same as in steady state, one can solve *backwards* for expected value functions \mathbf{V}_t^e at all dates $T-1, T-2, \dots, 1$. Setting the aggregate states $U_0 = \bar{U}$ and $\lambda_0 = \bar{\lambda}$, and using conjectured given paths for θ_t^* , the shocks and continuation values one can then solve *forwards* for a new market-clearing $\theta_t^{*'} that equates unemployment from labor demand U_t^{demand} and worker flows U_t^{flows} in every period using the labor demand and evolution of unemployment equations$

$$\begin{aligned} U_{t+1}^{flows} &= U_t - H(\theta_t^*) + F(\theta_t^*) - \lambda_{e,t} n_0 \\ U_{t+1}^{demand} &= \bar{L} - \int n'(s, \theta_t^*, A_t, \mathbf{V}_t^e) d\lambda_t \end{aligned}$$

Once we reach $t = T$ we set $\tilde{\theta}_t^* = \theta_t^*$ and iterate until the proposed and equilibrium paths for market tightness converge.

D Further discussion of dispersion results

In the main text we discussed a final source of fluctuations in Φ_t that here we discuss in more detail. This is the effect of changes in the dispersion of growth rates that would enter the composition effect. Due to the concavity of the composition effect in $g_{i,t}$ (since $\gamma_2/(\gamma_1 + \gamma_2) < 1$), a mean-preserving spread in growth rates of *hiring firms* would have a negative effect on Φ_t .

A version of this mechanism is responsible for the fluctuations in aggregate recruitment intensity in [Kaas and Kircher \(2015\)](#). In their model of competitive search, aggregate recruitment intensity is expressed can be expressed as an average of meeting rates in each market, where meeting rates are a concave function of market-tightness. In terms of our notation this would be given by $\Phi_t^{KK} = \int q(\theta_{m,t})(v_m/V)dm$ where m indexes markets. They find that productivity recessions are accompanied by an increase in the dispersion of market-tightness across markets, leading to a decline in Φ_t^{KK} . How important might this be for aggregate matching efficiency?

In our model, consistent with [DFH](#), dispersion in firm level meeting rates are driven by dispersion in firm level growth rates. [Table 4](#) shows that following a financial shock the standard deviation of firm growth increases by about 45 percent. Suppose that this increase in the variance were mean preserving. This would lead the average growth rate of *hiring firms* to *increase*. Is the Jensen's inequality effect large enough to offset this first order effect?

To answer this question we conduct a simple exercise. We posit a normal distribution of growth rates in steady-state with parameters $\bar{\mu}, \bar{\sigma}$.²⁹ Assuming a uniform distribution of vacancies, the composition effect in [equation \(20\)](#) is then

$$\Phi^C(\mu, \sigma) = \int_0^\infty g_i^{\frac{\gamma_2}{\gamma_1 + \gamma_2}} f(g_i) dg_i,$$

where f describes the normal density. We ask how changes to μ and σ affect Φ^C . To do this we consider an alternative measure $\tilde{\Phi}^C$ in which this curvature effect is zero

$$\tilde{\Phi}^C(\mu, \sigma) = \left[\int_0^\infty g_i f(g_i) dg_i \right]^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}.$$

In choosing values for μ and σ we turn to [Table 4](#), setting $\bar{\mu} = 0.016$ and $\bar{\sigma} = 0.126$. [Table 4](#)

²⁹We also conducted this experiment for a leptokurtic distribution of growth rates using a Pearson Type VII distribution and found our results to be unchanged.

Table D1: Effect of firm growth rate dispersion on the composition effect

	μ	σ	μ_h	Pct hiring	Φ^C	$\tilde{\Phi}^C$	$\Phi^C/\tilde{\Phi}^C$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. $\bar{\mu}, \bar{\sigma}$	0.016	0.126	0.107	55.1	1.000	1.000	0.861
2. $\bar{\mu}, \sigma_R$	0.016	0.183	0.152	55.1	1.299	1.307	0.856
3. σ_R and μ s.t. $\mu_h = \bar{\mu}_h$	-0.135	0.183	0.107	23.1	0.415	1.000	0.730

suggests a recession value of $\sigma_R = 0.183$. Recall that we found that this mapped well into the values found in Bloom et al. (2015) when studying the Great Recession. We compute Φ^C and $\tilde{\Phi}^C$ at (i) $(\bar{\mu}, \bar{\sigma})$, (ii) $(\bar{\mu}, \sigma_R)$, and (iii) under σ_R with μ chosen such that the average growth rate of hiring firms $\mu_h = 0.107$ as in steady-state.

Table D1 gives results. Column 4 shows the percent of firms hiring, columns 5 and 6 show ratios of $\Phi^C(\mu, \sigma)$ and $\tilde{\Phi}^C(\mu, \sigma)$ to their values at $(\bar{\mu}, \bar{\sigma})$. Column 7 gives the ratio $\Phi^C(\mu, \sigma)/\tilde{\Phi}^C(\mu, \sigma)$.

Column 7 shows that in steady state the curvature effect is sizeable, reducing the composition effect by 16 percent. Yet our real interest is the effect of cyclical changes in σ . When increasing only the standard deviation of growth rates (row 2) the average growth rate of hiring firms increases by 5 percent (column 3). This first order effect leads to a 29.9% increase in the composition effect (column 5). This is only negligibly offset by the curvature effect: in a model without the curvature effect the increase would have been 30.7% (column 6). To further isolate the curvature effect we decrease firm growth rates by a counterfactually large 13.5 ppt (row 3) so as to maintain the steady state average growth rate of hiring firms (column 3). Having set the first order effect to zero we find a 59% decline in the composition effect. This is large and around the same magnitude as the decline in aggregate match efficiency in the Great Recession. However this has required a decline in the annualized growth rate of more than 80 ppt (column 3), and the mass of hiring firms to be halved (column 4). Both are clearly counterfactual. As a benchmark, Bloom et al. (2016) find that the annual rates of sales growth declined by 21 ppt.

This experiment points to the additional ‘curvature’ effect being quantitatively small, and reiterates the first-order role of the selection effect generating *counter-cyclical* movements in aggregate recruiting intensity. The curvature effect is large only when the first order effect is set to zero, which requires counterfactually large declines in firm growth.