Technology, Skill and the Wage Structure

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1. Introduction

This paper develops a flexible and tractable model with heterogeneous workers and technologies, many tasks/goods, and production complementarity between technology and human capital.

Motivation: in recent years inequality has increased, in the U.S. and elsewhere.

Empirical studies point to technical change that:

—involves some occupations, and not others;

—is skill-biased.

The goal is to develop a framework for understanding these changes.

1. Introduction

The main contribution here is to analyze the general equilibrium effects of technical change for a limited set of tasks on output levels, prices, and wages, throughout the economy.

Main model features:

—one-dimensional technology ladder; tasks differ by technology level;

—one-dimensional skill ladder;

—complementarity between technology and skill; and

—many tasks/goods, used in a CES aggregator with elasticity ρ, to produce a single final good.

1. Introduction: model features

Human capital is an asset that belongs to a single worker, who is the only one who can employ it in production. It is a "rival" input.

Technology is a nonrival input, used by all workers producing a particular good/task. This property also distinguishes it from physical capital.

Technology and human capital are inputs in a CES production function. The substitution elasticity is $\eta < 1$, so the function is log supermodular.

Labor markets are frictionless, so the low substitution elasticity means that the market (and efficient) allocation of labor across technologies displays positively assortative matching.

1. Introduction: preview of results

Preview of results, for $\rho > 1$:

Output rises and price falls for tasks enjoying the technical change.

For tasks/goods higher up the ladder:

- Employment at affected tasks expands to more skilled workers.
- Workers even higher up the skill ladder engage in task downgrading,
  - to fill the vacated jobs.
- Output falls and price rises for tasks higher up the ladder.
- Wages rise for workers at the upper end of the affected skill bin, and
  - for all workers higher up the ladder.

1. Introduction: preview of results (cont.)

For tasks/goods lower down the ladder there are two possibilities.

Employment at affected tasks may

(a) expand to less skilled workers, or

(b) contract, with some less-skilled workers leaving the affected tasks.

In case (a) workers lower down the skill ladder upgrade their tasks, to fill vacated jobs. Output falls and price rises for tasks lower down the ladder. All wages rise.

In case (b) workers lower down the ladder downgrade their tasks, as they are pushed out by more skilled workers. Output rises for tasks lower down the ladder. Some prices and wages may fall.

1. Introduction: Outline

2. Related literature
3. The model
4. Technical change
5. A multi-sector extension: revisit the rising tide question
6. Conclusions

2. Related Literature

A. Theoretical models of **skill-biased technical change**, such as Acemoglu (2002), Acemoglu and Autor (2010), Krusell, et al. (2000), Autor and Dorn (2013), Jovanovic (1998), Caselli (1999), Violante (2002), Sattinger (1975).

The model here is a special case of Costinot and Vogel (2010), with a low-elasticity CES production function. The specialization to CES allows a sharper characterization of the effects of changes.

B. Empirical work examining recent **trends in wage inequality**, such as Autor and Dorn (2013).

C. Models in the **search literature** with heterogeneous firms, such as Bagger and Lentz (2015) and Lise, Meghir, and Robin (2016).

In this literature there is only one good, and the "match surplus" for any worker-firm pair is exogenous. Absent search frictions, all workers would match with the "best" firm. Here, goods prices affect the surplus function, through downward sloping demand.

3. The model: final goods

Differentiated goods are distinguished by their technology $x_j > 0$, which determines their price p_j.

A single final good is produced competitively, with CRS, using differentiated goods as inputs.

Output of the final good is

$$y_F = \left(N \sum_{j=1}^{J} \gamma_j y_j^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)},$$

where $\rho > 1$ is the substitution elasticity, $\{\gamma_j\}_{j=1}^{J}$ are shares or weights for technologies $\{x_j\}_{j=1}^{J}$, and N is the number (mass) of tasks.

The price of the final good is normalized to unity, $p_F \equiv 1$.

3. The model: tasks/differentiated goods

Labor, differentiated by human capital level h, is the only input.

Human capital has a continuous distribution, with density $g(h)$.

Task output depends on the size and quality of the workforce employed in its production, as well as its technology.

A task with technology x_j that employs workers with various skill levels, $\ell_j(h) \geq 0$ of skill h, has total output

$$y_j = \int \ell_j(h)\phi(h, x_j)dh,$$

where $\phi(h, x)$ is a CES function with elasticity $\eta < 1$.

[A CES function is log supermodular if and only if $\eta < 1$.]

3. The model: potential wages

Given prices $\{p_j\}$, let $w^p(h, x_j)$ denote the "potential" wage a worker with skill h would earn producing a task with technology x_j,

$$w^p(h, x_j) = p_j \phi(h, x_j), \qquad \text{all } h, \text{ all } j.$$

For fixed j, it is increasing in h.

As a function of x_j, there are two effects: on the intercept and the slope.

Since $\eta < 1$, efficiency requires positively assortative matching.

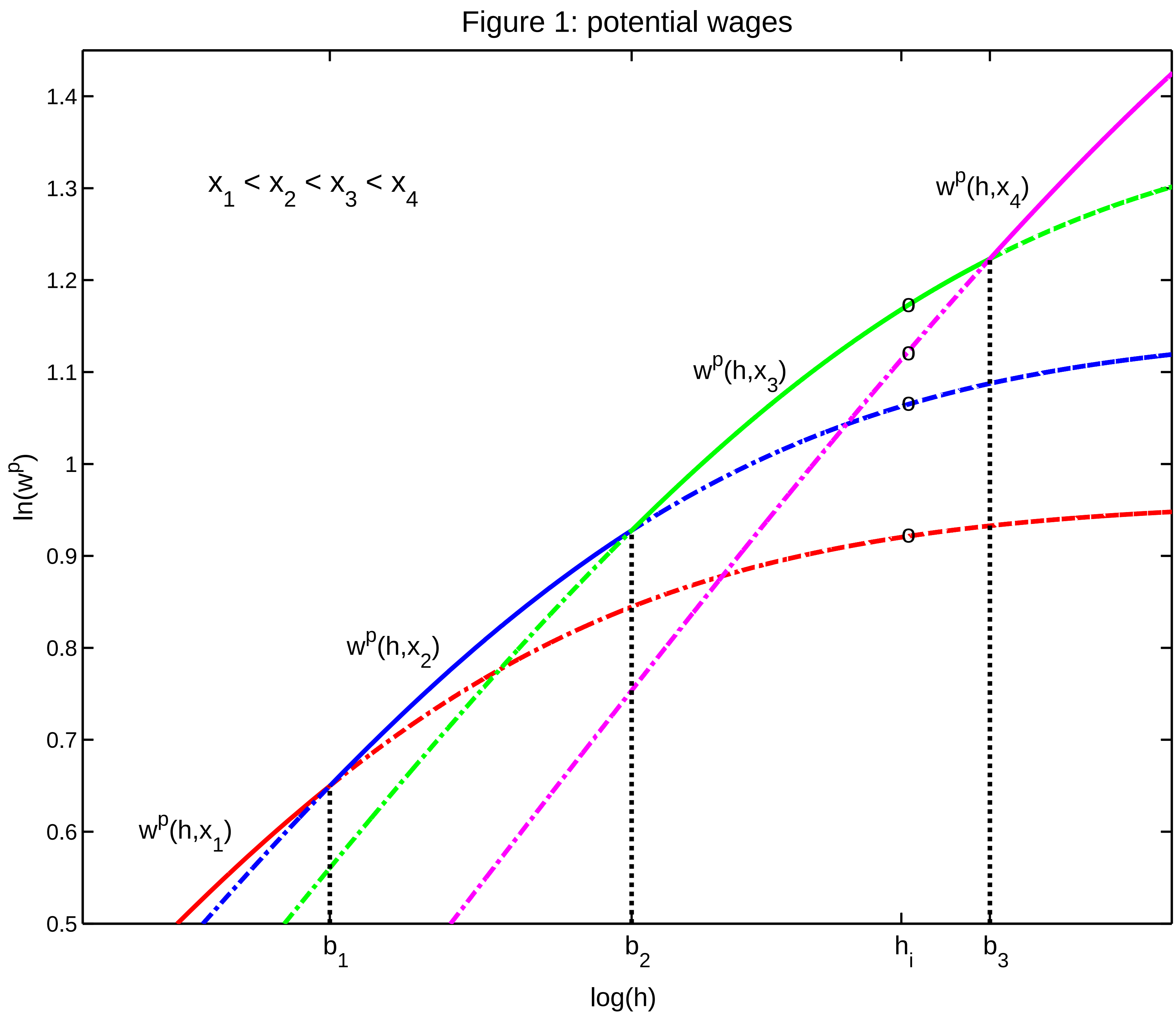


Figure 1: potential wages
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
ln(w^p)
x_1 < x_2 < x_3 < x_4
w^p(h,x_4)
w^p(h,x_3)
w^p(h,x_2)
w^p(h,x_1)
o
o
o
o
b_1
b_2
h_i
b_3
log(h)

3. The model: competitive equilibrium

Equilibrium is characterized by the allocation of labor across technologies:

cutoff levels $\{b_j\}_{j=1}^{J-1}$, with $b_0 = h_{\min}$, and $b_J = h_{\max}$.

An equilibrium exists, and it is unique and efficient.

Call the interval (b_{j-1}, b_j) **skill bin** j.

Workers with h in skill bin j produce goods with technology x_j,

Price is equal to unit cost for all tasks.

Goods with higher x_j have lower cost and price, $p_{j+1} < p_j$.

and they have higher output and revenue, $y_{j+1} > y_j$.

4. Technical change

Consider a technology improvement $dx_k = \varepsilon > 0$ for goods with technology x_k.

Questions:

1. What are the effects on outputs y_j, y_F, in the short run (SR), while labor is immobile, and in the long run (LR), when labor adjusts?
2. What are the LR effects on the skill allocation, prices, wages?

Do all wages increase? Does the rising tide lift all boats?

4. Technical change: final output

In the SR task output y_k increases from the direct effect of the technical change, and all other task outputs are unaltered.

Hence output of the final good increases only because y_k increases.

In the LR the labor allocation adjusts, but ....

4. Technical change: final output

PROPOSITION: To a first-order approximation, the reallocation of labor across goods has no additional effect on output of the final good, $\hat{y}_F^{LR} = \hat{y}_F^{SR}$.

The proof is basically the Envelop Theorem.

Since labor markets are competitive, the original (CE) allocation maximizes final output.

Hence for a small perturbation to technologies, reallocating labor has no first-order effect on final output.

But it does affect individual differentiated good outputs and prices, and it affects wages.

4. Technical change: labor allocation

Let $b_j^{(k)}(\varepsilon)$ be the thresholds for perturbation $dx_k = \varepsilon > 0$.

Differentiate the CE conditions to get

$$\underline{\beta}^{(k)} = M\underline{A}^{(k)},$$

where $\underline{A}^{(k)}$ is known, M is the inverse of a $(J-1)$ tridiagonal matrix, and

$$\beta_j^{(k)} = g(b_j)b_j'(\varepsilon), \qquad \text{all } j.$$

For any k, only two elements of $\underline{A}^{(k)}$ are non-zero, so only columns

$M_{\cdot k-1}$ and $M_{\cdot k}$ affect the solution. And because M is the inverse of a tridiagonal matrix, it has a lot of structure. These facts lead to ...

LEMMA 2: All thresholds at and above the k^{th} shift in the same direction, as do all those at and below the $(k-1)^{\text{th}}$.

4. Technical change: labor allocation

Lemma 2 leads to Propositions 4 - 7, which describe the labor, output, price, and wage changes.

PROPOSITION 4-7: For any k, an increase in technology x_k implies, $\beta_k^{(k)} > 0$, so skill bin k expands at the upper end, and

—output rises and price falls for tasks of type k;

—output falls and price rises for tasks of type $j > k$;

—wages rise for all workers in skill bins $j > k$, and for workers at the upper end of bin k; and

—proportionate output, price and wage effects are damped for technologies and skill bins more distant from k.

4. Technical change: labor allocation

PROPOSITION 4-7 (CONT): the sign of $\beta_{k-1}^{(k)}$ is ambiguous.

If skill bin k is not too wide and ρ is not too close to unity. Then

$\beta_{k-1}^{(k)} < 0$, so skill bin k also expands at the lower end, and

—output falls and price rises for tasks of type $j < k$;

—wages rise for all workers in all skill bins;

—proportionate changes are damped, moving away from k.

But it is possible that $\beta_{k-1}^{(k)} > 0$, so skill bin k contracts at the lower end. In this case

—output rises for tasks $j < k$, and the proportionate change is damped, moving away from k;

—price changes are ordered, and some (near k) may fall; and

—in each skill bin, the wage change reflects the price change.

4. Technical change: low elasticity

These results are for a high elasticity of substitution across tasks, $\rho > 1$.

In this case an improvement in x_k means labor should be moved **toward** tasks using that technology.

If $\rho < 1$, an improvement in x_k means labor should be moved **away** from tasks using that technology, to increase output of complementary tasks. Hence some results are "mirrored."

4. Technical change: homogeneous technologies

Suppose all goods have the same technology level, x^{HT}, chosen so that final output y_F is unchanged. Then total wages are also unchanged, and all differentiated goods have the same price. (Keep $p_F = 1$.)

For each skill bin, there is a direct (own-productivity) effect, and a price effect. The latter tends to offset the former.

If the DFs for technology and skill "match" in a certain sense, then the change in the wage function is quadratic,

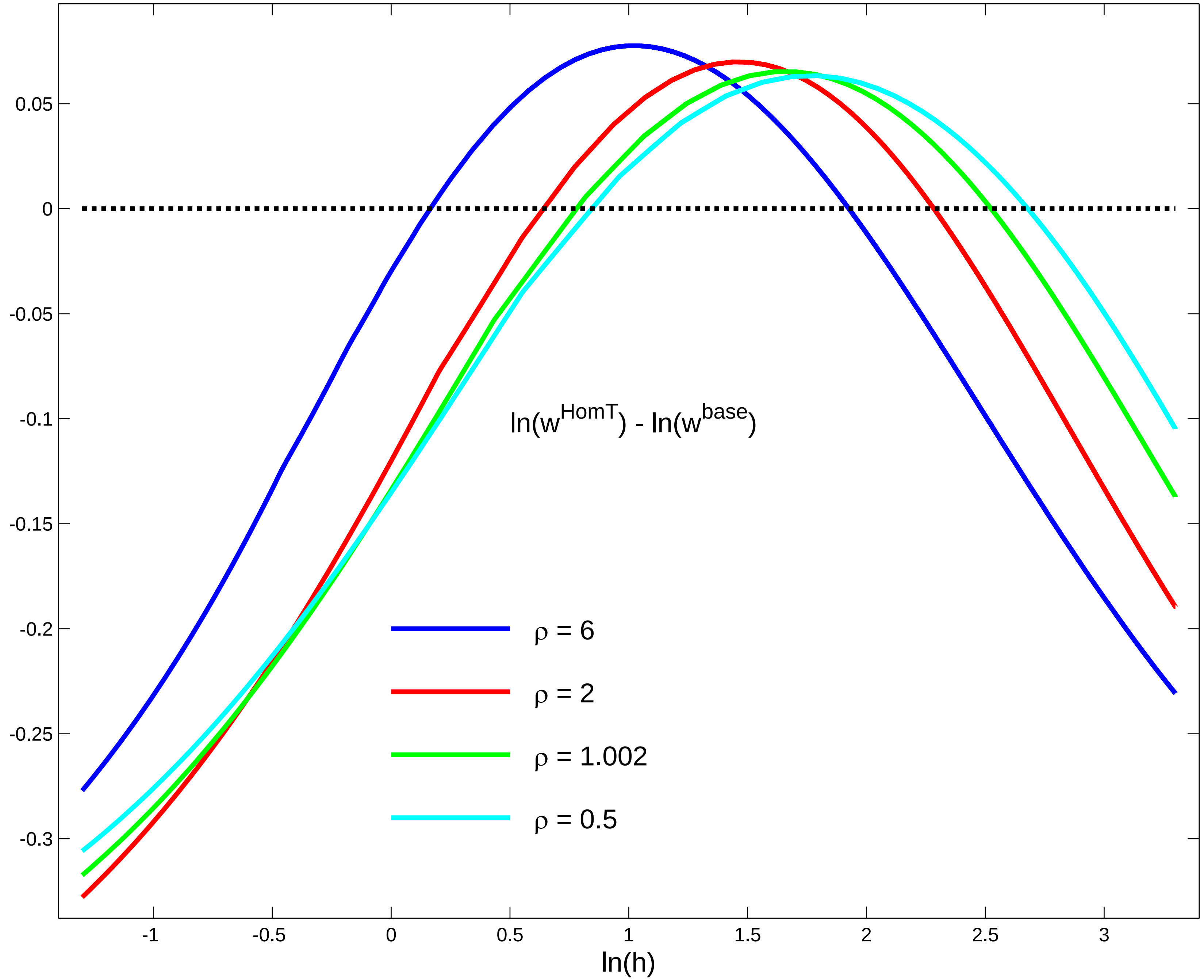
$$\Delta \ln w(h_j) \approx \chi_0 - \frac{1}{2}\chi_1 \Delta_{xj}^2,$$

where

$$\chi_0, \chi_1 > 0, \qquad \Delta_{xj} = \ln x^{HT} - \ln x_j.$$

Wage rises for skill near the mean, and fall away from the mean.

Figure 3: wage change from eliminating technology heterogeneity



5. A multi-sector extension

In numerical examples, it is hard to get wages to fall.

It easier in an extended with two tiers in production:

in each sector, tasks/goods are used to produce aggregates;

sector aggregates are used to produce the final good.

Each sector has its own set of varieties $\{y_{sj}\}$.

Key assumption: $\rho > \sigma$, so **varieties within a sector are more substitutable** than sector aggregates.

Demand for task y_{sj} is increasing in sector output Y_s and sector price P_s.

But Y_s, P_s are also linked through demand by final goods producers.

With $\rho > \sigma$, Y_s has a **stronger effect through price than directly**.

An increase in Y_s reduces price P_s so sharply that demand y_{sj} falls.

5. A multi-sector extension: Example 1

If the technology distributions are the same in all sectors,

$$\gamma_{sj} = \gamma_j, \qquad \text{all } s, j,$$

then the skill bins are exactly as in the one-sector model.

5. A multi-sector extension: Example 2

Two sectors, Low-tech and High-tech, $s = L, H$.

Final good technology has $\sigma = 1$.

Three technology levels, $x_1 < x_2 < x_3$, and skill levels, $h_1 < h_2 < h_3$.

In sector H, all tasks have technology x_3 and employ h_3.

In sector L, some tasks have technology x_2 and employ h_2,

and some have x_1 and employ h_1.

For a small increase in x_2, the labor allocation is unchanged.

Wages necessarily rise for h_2 and h_3, but fall for h_1, if

$$\theta_L + 1/\rho < 1.$$

6. Conclusions

Other questions the model can address:

—changes in the skill distribution (from immigration);

—changes in the demand structure (from int'l trade);

—role of labor market frictions in generating unemployment and creating job ladders.

Forthcoming soon: a paper featuring investment on both sides, shifting both 'ladders' at the same rate.