

Disaster Risk and Asset Returns: An International Perspective¹

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Abstract

Recent studies have shown that disaster risk can generate an equity premium similar to the data. Moreover, time variation in the risk of disasters can help explain the excess volatility of equity returns over that of government bill rates. However, these studies have ignored the cross-country asset pricing implications of the disaster risk model. This paper shows that standard disaster risk model assumptions lead to counterfactual international asset pricing implications. Given consumption pricing moments, disaster risk cannot explain the range of equity premia and government bill rates nor the high degree of equity return correlation. Moreover, the independence of disasters presumed in some studies generates counterfactually low cross-country correlations in equity markets. Alternatively, if disasters are all shared, the model generates correlations that are excessively high. We show that common and idiosyncratic components of disaster risk are needed to explain the pattern in consumption and equity co-movements.

The risk of disasters has long been proposed as an explanation for a variety of financial market anomalies. Key among these anomalies is the high equity premium in the face of relatively smooth consumption. As originally presented by Reitz (1988) and advanced by Barro (2006, 2009), a low probability of a large decline in output can sufficiently increase the variability in marginal utility to deliver the equity premium seen in U.S. data. Moreover, as Wachter (2013) shows, time varying disaster risk can help explain the excess volatility of equity returns over that of consumption.

Since disasters are rare in the US time series, this literature uses international data to measure both the frequency and size of these events. The typical approach assumes that every country faces the same disaster risk distribution, parameterized from a set of observed disasters across all countries.¹ However, this assumption carries important implications for the magnitude and co-movements in international asset returns. For example, if all countries face a similar disaster risk, this risk should affect the correlation of asset returns across countries.

In this paper, we consider the international asset pricing implications of disaster risk using consumption and asset price data for seven OECD countries. As in the literature, we begin by evaluating each country in isolation and assuming that they face the same disaster risk process. Within the constant disaster intensity framework as in Barro (2006), we ask whether variations in the impact of disasters can explain the cross-section of asset return moments across countries. To examine these implications, Simulated Method of Moments

¹Nakamura et al (2013) estimate endogenous differences in timing, magnitude, and length of disasters while maintaining the assumption that the frequency and size distribution is time invariant and the same across countries. They allow for correlation in the timing of disasters similar to our model below. However, they only use this information to match the US asset pricing levels and do not consider the international asset pricing implications. We discuss their approach relative to ours below.

(SMM) is used to derive model parameters that can fit the asset pricing moments. The evidence shows that the model fit varies widely across countries. Moreover, varying the size of the disaster as well as the probability of government default is required to explain differences. Nevertheless, the ranges generated by the model cannot explain the international variation in the data. We then allow for time-varying probabilities of disasters as in Wachter (2013). Time-varying disasters improves the fit for the range of return variability across countries but only modestly for their levels.

Given our best efforts to fit the model to individual country asset returns, we next turn to the implications for asset return correlations across countries. A standard empirical finding is that international consumption correlations are lower than equity return correlations.² We therefore evaluate the correlation relationships implied by the model. Under the typical assumption that disaster events occur independently, equity return correlations either match those of consumption correlations when disaster risk is constant or are much lower than consumption when disaster risk is time-varying.³ By contrast, when disaster risk is common, equity return correlations are near one, and are hence too high.⁴

To address the inconsistencies posed by these two extreme cases, we next posit that disaster risk depends upon a mixture of country-specific and common world disasters. We provide a framework that identifies the importance of each component using international asset return correlations. Our evidence shows that a high degree of common disaster risk is

²See, for example, the discussion in Lewis and Liu (2015) and Tesar (1995).

³For example, this implicit assumption is made in Barro (2006,2009). In a sample of 35 countries, the frequency of disasters is the average number of times that the decline in output in the 20th century was greater than 15% across all countries and years. Wachter (2013) follows a similar approach for declines in consumption, inferring the frequency and size of declines from a common cross-sectional country distribution.

⁴Correlation in disaster events also implies that the unconditional probability of disasters is lower than measured by the proportion of years that countries experience disasters, as often assumed. Nakamura et al (2013) demonstrate this point as well.

required to explain the pattern that asset return correlations are greater than consumption growth correlations.

A number of other papers have also addressed the impact of disasters on the macroeconomy and on asset markets and, as such, are related to our work here. Gabaix (2009,2012) considers disaster risk with variable severity of disasters arising from the resilience of an asset's recover rate through a "linearity generating" process. Martin (2008) solves for the welfare cost of business cycles due to disasters, but does so with CRRA utility, making it difficult to match to asset return data. Gourio (2008,2012) evaluates the impact of disasters in a real business cycle model allowing for recoveries after a disaster. Similarly, Nakamura et al (2013) allow for differing probabilities of entering disasters across countries. However, none of these papers consider the international asset pricing implications of disaster risk.

By contrast, Farhi and Gabaix (2016) examine the co-movements of returns and exchange rates with disasters. Given their emphasis on international markets, their paper is the most related to ours. Nevertheless, there are important differences. Farhi and Gabaix (2016) assume that markets are complete and focus upon exchange rate behavior. By contrast, our identification strategy allows for incomplete markets and we highlight the implications for asset market co-movements. As such, we view the contribution in our paper as complementary, but distinct from theirs.

The plan of the paper is as follows. In Section 1, we follow the literature and evaluate the model fit for countries in isolation. Section 2 describes the implications for correlations in consumption and asset returns across countries. Concluding remarks are in Section 3.

1 Individual Country Disaster Risk

We begin by evaluating the individual country disaster risk model that has been studied in the literature. Reitz (1988) first proposed the risk of rare, but severe, disasters as a potential resolution of the Mehra and Prescott (1985) equity premium puzzle in U.S. data. However, the infrequency of these events necessarily made this possibility difficult to quantify. For this reason, Barro (2006) proposed using data on disasters across a large sample of countries as independent observations to discipline both the size and frequency of disasters in the U.S. Clearly, under this assumption, the occurrence of disasters in non-U.S. countries would also impact the asset return behavior of those countries as well.

We therefore begin by asking how well the disaster risk model, commonly used to explain US asset returns, can be applied to those other countries. For this purpose, we develop a disaster risk framework, following Wachter (2013), which incorporates country specific parameters to target differences in asset returns.⁵ The framework also nests the Barro (2009) model when the probability of disaster is time-invariant, a special case we consider as well. We then evaluate the model fit for a group of seven OECD countries. In Section 2, we begin to develop the international implications of the model.

1.1 Preferences and Consumption

The world is comprised of J representative consumer-investors each living in a country, indexed by j . These consumers have identical preferences over a common homogenous good.⁶

⁵Wachter (2013) shows that time-variation in disaster risk is necessary for the model to more closely match the volatility of asset prices.

⁶In examining a common good framework across countries, this approach follows a standard assumption in the risk-sharing literature such as Obstfeld (1994) and Kalemli-Ozcan, Sorensen and Yosha (2003). However, the common consumption good could be viewed as a composite of multiple heterogeneous goods as described,

Consumption in that good for country j at time t is defined as C_t^j . As Barro (2009) argues, recursive preferences are needed to avoid the counterfactual implication that high price-dividend ratios predict high excess returns. We therefore assume that preferences are recursive over time as in Epstein and Zin (1989) and Weil (1990), and as formulated in continuous time by Duffie and Epstein (1992). For tractable, exact form solutions to our asset pricing moments below, we consider the case of unitary intertemporal elasticity of substitution in consumption.⁷ Thus, under these assumptions, utility at time t for representative consumer j , defined by V_t^j , is given by:

$$V_t^j = E_t \int_t^\infty f(C_s^j, V_s^j) ds \quad (1)$$

where

$$f(C_t^j, V_t^j) = \beta(1 - \gamma)V_t^j \left[\log C_t^j - \frac{1}{1 - \gamma} \log((1 - \gamma)V_t^j) \right] \quad (2)$$

and where $\beta > 0$ is the rate of time preference and $\gamma > 0$ is the coefficient of relative risk aversion.

We begin by considering countries individually as in the disaster risk literature. According to this literature, consumption in the data is presumed to be the endogenous outcome of a fuller production process.⁸ To provide a general relationship that will be analyzed more carefully below, we allow for differing effects of disaster risk across countries using a version

for example, in Adler and Dumas (1983).

⁷This assumption allows us to adapt the closed-form solutions from Wachter (2013) to individual country asset returns, in the case where disaster intensity are time-varying. Nevertheless, when the disaster probability is constant, we can allow the intertemporal elasticity of substitution to differ from one.

⁸In doing so, we follow the Euler equation approach formulated in Hansen and Singleton (1983). As articulated by Cochrane (1991) and Campbell (1993), among others, the relationship between consumption and asset returns may be viewed as an equilibrium arising from a richer, unspecified production economy.

of the consumption process in Wachter (2013). That is, the consumption process for country j is given by:

$$dC_t^j = \mu C_{t-}^j dt + \sigma^j C_{t-}^j dB_t^j + (e^{\omega^j Z_t} - 1) C_{t-}^j dN_t^j, \quad \forall = 1, \dots, J \quad (3)$$

where C_{t-} denotes $\lim_{s \uparrow t} C_s$ and C_t is $\lim_{s \downarrow t} C_s$. Further, dB_t^j is a standard Brownian motion, Z_t is a random variable with a time invariant distribution ν that is common across countries and N_t^j is a Poisson process with time-varying intensity parameter, λ_t^j , given by:

$$d\lambda_t^j = \kappa (\bar{\lambda} - \lambda_t^j) dt + \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j \quad (4)$$

where $dB_{\lambda,t}^j$ is also a standard Brownian motion. All processes, $\{dB_t^j, dB_{\lambda,t}^j, dN_t^j\}$, and realizations of Z_t are independent of each other. To capture the effect of disasters, $Z_t < 0$ and $\omega^j > 0$ so that realizations of dN_t^j lower the consumption level. In our quantitative application below, we follow Wachter (2013) in parameterizing the distribution of Z_t with the empirical distribution of disasters using the data from Barro and Ursua (2008).

Our specification of consumption processes in equations (3) and (4) assumes that some parameters are common, while others are country-specific. Country mean growth rates, μ , are set to be equal across countries for plausibility since our quantitative analysis will focus upon developed economies. For expositional simplicity, we also assume common intensity parameters, $\{\bar{\lambda}, \kappa, \sigma_\lambda\}$, although we relax this assumption below. To analyze the possibility for differing disaster effects across countries we specify the country-specific Poisson processes generating disasters as dN_t^j . In addition, we allow country specific parameters for consump-

tion volatility in normal times (σ^j) and a differing impact of disasters through ω^j . Clearly, a country with higher ω will experience a larger impact of disasters on consumption. Below we also consider country differences in asset return parameters measuring leverage (ϕ^j) and government bond default rate (q^j) to be detailed later.

1.2 Asset Returns

Using these processes, we examine the asset pricing relationships that would hold in equilibrium for each individual country. As noted earlier, we follow the literature in using the Euler equation of each country's consumption process to price asset returns. Under preferences given by equations (1) and (2) along with the consumption process in equations (3) and (4), the risk free rate of country j is:⁹

$$r_t^j = \beta + \mu - \gamma(\sigma^j)^2 + \lambda_t^j E \left[e^{-\gamma\omega^j Z_t} (e^{\omega^j Z_t} - 1) \right] \quad (5)$$

where the expectation is taken over the time invariant distribution ν of Z . As equation (5) shows, the only source of variation in the risk-free rate arises from time variation in disaster risk. Moreover, since $e^{\omega^j Z_t} < 1$, a higher probability of disasters, λ_t^j , reduces the risk free rate as it induces more precautionary savings.

Crises are often associated with a decline in the value of government securities, either through partial default or inflation. Typically, the literature following Barro (2006), assumes that a disaster is associated with a partial default in government debt. We define the

⁹The Appendix shows that our introduction of country-versus-common parameters directly extend the asset pricing solutions in derived in Wachter (2013), Appendices A and B. By contrast, Section 2 provides novel cross-country asset pricing moment implications.

probability of this government default for country j as q^j . Then, the instantaneous expected return on the government bill rate is:

$$r_t^{b,j} = r_t^j + \lambda_t^j q^j E \left[(e^{-\gamma \omega^j Z_t} - 1)(1 - e^{\omega^j Z_t}) \right] \quad (6)$$

Clearly the premium on government bills is increasing in probability of default, q^j . Moreover, the volatility depends upon the variation in the probability of disasters, λ_t^j . Importantly, note that in the absence of time-varying disasters, the government bill rate is constant so that its variance is zero.

We treat equity as a claim on levered consumption following Abel (1999). Specifically, dividends for equity in country j are defined as: $D_t^j = (C_t^j)^{\phi^j}$ where $\phi^j > 1$ is the leverage parameter on consumption. Using this definition along with the consumption process in equation (3), Ito's Lemma implies that the process of dividends for equity from country j is given by:

$$dD_t^j = \mu_D^j D_{t-}^j dt + \phi^j \sigma^j D_{t-}^j dB_t^j + (e^{\phi^j \omega^j Z_t} - 1) D_{t-}^j dN_t^j, \quad (7)$$

where $\mu_D^j = \phi^j \mu + \frac{1}{2} \phi^j (\phi^j - 1) (\sigma^j)^2$. Defining F_t^j as the price of a claim to all future dividends and π_t^j as the state price density both for country j , then this equity price can be written:

$$F_t^j = E_t \left[\int_t^\infty \frac{\pi_s^j}{\pi_t^j} D_s^j ds \right]. \quad (8)$$

Before deriving the equity price process, it is useful to examine the evolution of the state price density. Solving for the state price densities using the preferences and consumption

processes implies:

$$\frac{d\pi_t^j}{\pi_{t-}^j} = \mu_{\pi,t}^j dt - \gamma \sigma^j dB_t^j + b^j \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j + (e^{-\gamma \omega^j Z_t} - 1) dN_t^j, \quad (9)$$

where b is a positive constant that depends upon parameters of the time-varying disaster process, κ and σ_λ , the expected size of the disaster for country j , $\omega^j Z$, and preference parameters, β and γ .¹⁰

We can use this process to understand some of the basic relationships we find in our quantitative results below. First, note that the state price in equation (9) evolves with innovations to the exogenous variables in an intuitive way. In particular, π_t^j decreases in “good times”; that is, with increases in the Brownian on normal times consumption dB_t^j according to risk aversion, γ . By contrast, the state price increases in “bad times”; that is, with innovations to the Brownian on disaster probabilities, $dB_{\lambda,t}^j$, according to the current level of the disaster probability $\sqrt{\lambda_t^j}$ and the expected size of the disaster implied through the parameter b . Finally, since $Z_t < 0$, disaster events generated by dN_t^j increase the state price. Note that, in the absence of time-varying probabilities, the instantaneous variance of the state-price density during normal times would be driven by the variation in normal times consumption alone. Therefore, if disaster probabilities were constant (i.e, $\sigma_\lambda = 0$), then the instantaneous volatility of the state-price in the absence of disaster events would simply be $\gamma \sigma^j$.

Using this intuition, we can now evaluate the behavior of the stock price over time. Using

¹⁰Specifically, $b^j = \left(\frac{\kappa + \beta}{\sigma_\lambda^2} \right) - \sqrt{\left(\frac{\kappa + \beta}{\sigma_\lambda^2} \right)^2 - 2 \frac{E_\nu(e^{(1-\gamma)\omega^j Z_t} - 1)}{\sigma_\lambda^2}}$. In practice, the square-root imposes a restriction on the relationship between the expected size of disaster and the variation of the disaster probabilities, as described in Wachter (2013).

the evolution of the state price density, the diffusion for the stock price in equation (8) can be written as:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi^j \sigma^j dB_t^j + g\sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j + (e^{\phi^j \omega^j Z_t} - 1) dN_t^j, \quad (10)$$

where $\mu_{F,t}^j$ is the instantaneous mean and $g < 0$.¹¹

The evolution of the stock price follows the essential features of the state price density. In particular, the stock price increases with innovations in the Brownian on normal times consumption, dB_t^j , now augmented by the leverage parameter, ϕ^j . Moreover, the stock price decreases with innovations to the Brownian driving innovations to the probability of disasters, $dB_{\lambda,t}^j$, as well as disasters themselves. Also, note that in the absence of time-varying disaster probabilities, the stock price volatility in normal times would simply be that of the levered volatility of normal times consumption, $\phi^j \sigma^j$. Below, we analyze in detail the effect of each component on the mean and variance of equities. Overall, these relationships can then be used to generate the asset pricing moments, as we turn to next.

1.3 Data Across Countries

Since disasters occur infrequently in the U.S., Barro (2006) argued that international evidence is required in order to provide a larger data sample. We therefore calibrate our results to the long time series sample of consumption and asset return moments across countries reported in Barro and Ursua (2008). For the 21 OECD countries in the sample, this data set

¹¹Specifically, $g = G'(\lambda_t)/G(\lambda_t)$ where G is the price-dividend ratio. This price-dividend ratio also depends upon the state price diffusion in equation (9). Ensuring that the solution of G is not imaginary restricts the relationship between not just Z and the parameters of the time-varying densities as before, but also the leverage parameter ϕ .

provides consumption beginning in the range of 1800 to 1913. Availability of asset returns begin later, particularly for stock returns. For this reason, we focus upon seven countries with asset pricing data beginning earlier: Australia, Canada, France, Germany, Japan, the United Kingdom, and the United States. Among these countries, the United Kingdom's stock return data started the earliest in 1791 and Canada started the latest in 1934. In all cases, the bond return data began either earlier or at the same time as the stock return data.

Table 1 Panel A reports the means and standard deviations for the government bill rate, the equity return, and consumption for these seven countries.¹² As the table shows, the mean consumption growth rates of the countries are similar across countries, ranging between 1.47% and 2.48%. By contrast, the asset pricing estimates vary widely across countries, despite the long time series. For example, the highest mean stock return is for Australia at 10.27% while the lowest is France at 5.4%. Also, the standard deviations of equities are more similar for Australia, Canada, France, the U.K., and the U.S., but are higher at around 30% for Germany and Japan. A similar pattern may be seen in the standard deviation of the bill rate with substantially higher volatility in Germany and Japan. A range of mean bill rates is also apparent and are even negative for Germany and France.

One reason for these differences may be that disasters affect countries heterogeneously in the sample. Indeed, according to the disaster risk literature, the consumption and asset pricing moments may be affected by infrequent, but large, declines in consumption and stock prices. As such, it may be informative to condition the moments on years when disasters are absent. Therefore, following Wachter (2013), we also examine post war data as a subset

¹²For Canada, the bill rate is unavailable from Barro and Ursua (2008). We therefore use the bond rate for this country.

of our full sample, representing a data sample that excludes large disasters.

Again using the Barro and Ursua (2008) data, we re-compute consumption growth and asset pricing moments in Panel A for the period after 1949. Table 1, Panel B shows the range of annual data moments across the seven countries for this postwar sample. While the range of mean equity returns remains large, the standard deviations across countries are more similar, ranging from about 18% to 25% per annum. Not surprisingly, the standard deviations of consumption growth and government bill rates are uniformly lower in the postwar period (without disasters) than in the full sample. Both government bill rates and equity returns are also lower in the postwar data, as compared to the full sample, for all countries except Australia.

Next, we ask how well the model can be parameterized to fit these data moments.

1.4 Matching the moments: Constant Disaster Probability

Given the solutions for the asset pricing returns and the consumption processes in Section (1.1) and data moments in Section (1.2), we now ask how well the model fits each individual country's consumption and asset return data. We begin by considering the constant disaster risk model as in Barro (2006, 2009). Wachter (2013) shows that this model is nested in the framework described above with static instantaneous jump intensity. For this purpose, we assume the baseline parameters from Barro (2006) of relative risk aversion $\gamma = 4$, rate of time preference $\beta = 0.03$, normal time consumption growth rate $\mu = 0.025$, and $\lambda_t^j = \bar{\lambda} = 1.7\%$ for all country j .¹³ Moreover, a disaster is classified as a year when GDP dropped by 15% or greater. We also generate the distribution of Z_t from the sizes of disasters reported in

¹³Note that this probability level is more conservative than the 3.55% assumed by Wachter (2013).

that paper.

In Table 2, we evaluate the fit of the model for both the Unconditional Model Moments including disasters using the Barro-Ursua data in Panel A and the Conditional Model Moments during “normal times” using the postwar data in Panel B. For each version, we use Simulated Method of Moments (SMM) to provide the best fit of our model parameters for each country’s data moments. In order to fit these parameters, we target the following seven data moments across countries: (a) the mean government bill rate, (b) the standard deviation of the government bill rate, (c) the mean equity premium, (d) the standard deviation of the equity return, (e) the Sharpe ratio, and (f) the consumption growth standard deviation. Using these target moments, we estimate the following model parameters: (a) the volatility of consumption in normal times, σ^j ; (b) probability of government bond default, q^j ; (c) dividend-consumption leverage parameters, ϕ^j ; and (d) the proportion of disaster state consumption decline, ω^j , relative to the standard model.

Table 2, Panel A reports the parameter estimates targeting the moments over the full Barro-Ursua sample, including years when disasters occur. Despite the range of data moments across countries, the estimates for the Unconditional Model Moments provide a relatively tight range of country-specific parameters. Indeed, for most of the parameters, the estimates correspond to those found in the literature for the U.S. For example, the probability of government default given that the country is in a disaster is in the range given by $q^j = [0.42, 0.45]$, close to the assumption of $q^j = 0.4$ in Barro (2006). Moreover, the fitted normal times consumption volatility estimates are around 2.3% and therefore near standard estimates. The range of the estimate for the leverage parameter ϕ^j are between 2.7 and 2.8, and thus near the Abel (1999) assumption of 3. Overall, these estimates are all relatively

in line with values required to fit asset pricing moments in the U.S., even though the target moments are international. By contrast, the range of estimates of ω are all lower than one, $\omega^j = [0.84, 0.88]$, indicating that the consumption loss in the event of a disaster is somewhat lower than that assumed in the standard U.S.-targeted model.

This tight range of fitted parameters creates difficulties in matching the wide range of asset pricing moments, however. As Table 2, Panel A reports, the annualized asset pricing moments from the model are much closer across countries than their data counterparts in Table 1, Panel A. For example, in the model, the mean and standard deviation of government bill rates are all relatively uniform and do not differ across countries by more than 1%. In the data, by contrast, the means vary by almost 4% and the standard deviation by over 10%. Similar discrepancies can be found in the equity returns. Even the more modest range of the consumption volatilities across countries cannot be generated by the model.

The bottom of the Panel A emphasizes the poor fit by reporting the Mean Squared Difference. This statistic represents the sum of the squared difference between targeted data moments and the corresponding model derived moments. As the numbers indicate, the model fits particularly poorly for Australia and Japan.

A potential problem with these results is that, depending upon the occurrence and severity of disasters, the data estimates may not accurately report the population data moments. For this reason, Panel B of Table 2 reports the results of an alternative SMM analysis targeting the model under normal times excluding disasters, given by the postwar data moments in Table 1, Panel B. Since the standard deviation of normal times consumption, σ_c , should match the consumption growth without disaster years, we set this parameter to 0.02 as the average of the postwar consumption growth volatility across countries. As before, to get an

aggregate measure of how the model fits across all the data moments, we report the Mean Squared Difference. The fit for Australia is improved as the Mean Squared Difference drops from 0.286 to 0.145. However, the Mean Squared Difference generally increases for the other countries.

The fitted parameters give a qualitatively similar pattern as before and, once again, are similar to the estimates found for the U.S. The values for the probability of government default q are now somewhat higher, between 0.506 and 0.533. Furthermore, the leverage parameter ϕ estimates range between 2.903 and 3.021, and thus are all close to 3. By contrast, the implied loss in consumption, captured by ω , is lower than standard disaster studies of the U.S. that assume $\omega = 1$. These estimates are virtually identical across countries at about 81% of the size generated by the distribution from the Barro data.

Given the similarities across parameters across countries, the model again shows little variation in implied moments across countries. The government bill rates are all about 3.9% across countries. Moreover, as noted earlier, during normal times, the model implies that the bill rate is constant so that the bill rate volatility is zero. The equity premium and the volatility of equity returns are fairly uniform around 5% and 6.5%, respectively. Thus, the model cannot explain the range of equity premia from 6.35% to 11.90% reported in Table 1, Panel B. Furthermore, in all cases, the model-implied volatility of equity is much lower than the 15% to 33% found in the data.

In light of these difficulties with matching moments across countries, we next study how variations in parameters affect the implied asset prices across countries. We begin by examining the impact of varying the severity of disasters across countries. For the purpose of this investigation, we focus upon the effects upon the Unconditional Model moments.

Under the first three columns labeled “Baseline,” Table 3 shows how asset pricing moments vary when the proportionate size of the disaster relative to the standard model ranges from $\omega = 0.8$ to 1, holding constant the other parameters. The return on the government bill rate decreases as the percent of consumption decline in the disaster state approaches the full impact of the world disaster consumption decline. In other words, as ω increases towards one, the return declines. As noted in Section 1, a more severe disaster increases savings, thereby reducing the risk-free rate and, as a consequence, the government bond rate as well. In the absence of time variation in probability risk, this increased savings has a large affect on the mean risk free rate but not necessarily the volatility of the risk free rate. By contrast for consumption growth, the increase in ω has a larger effect on volatility than on the mean.

Under the next three columns labeled “High ϕ ”, Table 3 shows the effects of a higher leverage parameter. Here we assume the leverage parameter ϕ to be 3.0 rather than 2.8, and then reexamine the results with varying ω . Comparing the results to the first three columns of Table 3 shows that a higher leverage ratio increases the equity premium. It also increases the standard deviation of the market return, albeit much more modestly. On the other hand, comparing the model moments in the Baseline Model to the High ϕ Model shows that these effects from higher leverage become muted when there is less sensitivity to disaster risk, or lower levels of ω . For example, comparing the Mean Equity Premium, the higher leverage ratio increases equity premium 24 bps from 7.88% to 8.12% when ω is one, but only 17 bps when ω is 0.8. The government bond rate is unaffected by any changes since, as noted earlier, it is independent of ϕ .

The last three columns of Table 3 labeled “Low q ” report the implications for the model moments when the probability of government bond default conditional on a disaster q declines

to 25% rather than 40% as in the Baseline Model. Comparing these results to those in the first three columns makes clear that a decrease in the probability of government default decreases the government bill rate. The intuition is clear. A lower probability of loss reduces the implied default risk premium as shown in equation (6). Moreover, this lower government bill rate correspondingly increases the equity premium. Reducing the likelihood of the default loss in disasters also reduces the volatility of the government bill rate.

Table 3 also reports the conditional model implied moments for each set of parameter values. For the conditional model moments, the increase in leverage ratio, ϕ , again produces a noticeable increase on the mean equity premium, and a slight increase on the equity return volatility. Similarly, the increase in q decreases the return and volatility of the government bill rate. Finally, given the low volatility of the equity return for the conditional moments, the model suggests implausibly large conditional Sharpe ratios that range between 1.11 and 1.46, for the cases when ω is one.

In summary, although the standard disaster risk model assumes identical disaster probability distributions across countries, the evidence shows that this assumption is unlikely to fit asset pricing moments across countries. Moreover, the model with constant probability of disaster fails to explain the volatility of asset returns. The volatility of equity returns are significantly lower than in the data, while the volatility of government bills during periods without crises is counterfactually zero. In the next section, we evaluate the effects of incorporating time-varying disaster risk on the cross-country variation in asset pricing moments, a step that also improves the match with the volatility of asset prices.

1.5 Matching the moments: Time-Varying Disaster Intensity

As the results in the constant disaster risk probability model show, the model generates a range of parameters that are too small to explain the range of international asset pricing moments. Moreover, the variability of asset returns is too small. Therefore, we now allow for time variation in the probability of disasters and ask whether the model can generate better fitting observed cross-country differences in asset returns.

The time-varying disaster intensity process in equation (4) introduces two new parameters, the volatility of the probability, σ_λ , and its mean reversion, κ . We therefore began by conducting SMM to target these two new parameters in addition to the four parameters formerly fitted. However, targeting these parameters generally meant that the SMM optimization would not converge because σ_λ and κ would tend to exceed conditions required for the distribution of λ to be well-defined.¹⁴ For this reason, in the results reported below, we instead constrain some of the parameters and calibrate others to obtain the best fit to the data.

We take as a base case for the time-varying probability model the parameters from Wachter (2013). In particular, we assume that time preference, $\beta = 0.12$, risk aversion, $\gamma = 3$ and as above, the intertemporal elasticity of substitution is one. Disasters are defined as declines in consumption of 10% or more. Moreover, the effects of disaster risk on consumption in the baseline model match the size and distribution of observed consumption declines documented in Barro and Ursua (2008). Thus, we set $\omega = 1$ and the average probability of

¹⁴Technically, the intensity process in equation (4) has a stationary Gamma distribution only for $\frac{1}{2}\sigma_\lambda^2 < \kappa\bar{\lambda}$. Thus, for given assumptions about the mean of disaster probabilities, $\bar{\lambda}$, the volatility of probabilities, σ_λ , cannot be too high and the degree of mean reversion, κ , cannot be too low. Otherwise, the distribution of probabilities will become degenerate.

disaster as $\bar{\lambda} = 3.55\%$. Given these assumptions, we then calibrate the model in two ways. First, we use the same disaster risk parameters (κ, σ_λ) as in Wachter (2013), and fit country specific leverage, government default, and consumption volatility (ϕ, q, σ) . Second, we hold fix parameters $\phi, q,$ and $\sigma,$ and fit κ and σ_λ to match data moments. Finally, we target the moments for a subset of countries chosen to be more likely to provide sufficient stability for well-defined probability distributions. For this reason, we focus upon three countries: the U.S., the U.K, and Australia.¹⁵

The first three columns of Table 4 show the results when we choose the leverage parameter, $\phi,$ probability of government default, $q,$ and volatility of normal times consumption, $\sigma,$ to match the target moments, while fixing the volatility of disaster intensity $\sigma_\lambda,$ its persistence parameter $\kappa,$ and ω to be one. In particular, normal times consumption volatility is calibrated to the standard deviation of non-disaster consumption growth. As the table shows, there is now a wider range of parameters, particularly for Australia with a leverage parameter and probability of government default now quite a bit lower at 1.6 and 0.3, respectively. The Mean Squared Difference is now substantially lower across all countries compared to those in the constant probability model in Table 2. Furthermore, the differences in parameters now generate a wider range in asset return moments across countries. Indeed, the mean of government bill rates becomes too low for Australia, although the model delivers the general pattern that the rate is lower there than in other countries. The model also generates equity volatility closer to the data. For the Conditional Model Moments, the government bill rate is now time-varying with positive variance.

¹⁵Of the other four countries, Germany and France have negative mean bill rates and Canada does not have government bill rates that are available from the Barro-Ursua data set. Moreover, Germany, France, and Japan underwent significant post-war reconstruction.

The last three columns of Table 4 report the estimates based upon reversing this process. Specifically, we now fix the leverage parameter ϕ and the probability of government default q , to those in Wachter (2013), and choose parameters in the distribution process for the intensity λ_t to best match the targeted Unconditional Model moments. Again, as earlier, we constrain ω to be one. As the table shows, the parameters vary in a narrow range. The degree of mean reversion, κ , varies between 0.085 for Australia to 0.145 for the U.K., while σ_λ varies only between 0.07 to 0.1. In practice, this narrow range is dictated by the condition that the distribution of λ be stationary. But the resulting effects on the government bill rates and equity premium are again that asset pricing moments cannot vary much across countries.

As these results show, the means of asset returns are still similar to that of the US market, although there is a higher variation in standard deviations of returns. Furthermore, when the leverage parameter ϕ and the government default probability q are allowed to vary across countries, the model delivers a more plausible range of asset moments across countries than when the parameters determining the distribution of the disaster intensities differ. Therefore, we now ask whether variations in the impact of disasters can affect the moments.

The results are given in Table 5. For the base case model, reported in column 1, we first report the implications for Unconditional and Conditional Model Moments when the parameters are constrained to be equal to those of the model and, hence, $\omega = 1$. By contrast, the second and third columns report the results when $\omega = 0.85$ and $\omega = 0.95$, respectively.¹⁶

As the results show, reducing ω from 1 to 0.85 increases the bill rate since the disaster has

¹⁶We also considered the cases when $\omega > 1$, but these violated the condition that is required to give a non-imaginary solution of b in the state price density given in Equation (9). In other words, the restriction that $\left(\frac{\kappa+\beta}{\sigma_\lambda^2}\right)^2 > 2\frac{E_\nu(e^{(1-\gamma)\omega Z_t} - 1)}{\sigma_\lambda^2}$ was violated. See footnote 9.

less of an impact. It correspondingly reduces the size of the unconditional equity premium from 7.6% to 4.6%. At the same time, the unconditional equity volatility also decreases as the impact on consumption is lessened, declining from 20% to 16.6%.

In the following three columns, labeled “High ϕ ”, we consider the impact from increasing the leverage parameter ϕ to 3. The results are quite similar for the bill rate and volatility. However, the unconditional equity premium and volatility is higher as a direct implication of the higher leverage. Moreover, the range of potential sizes of declines is narrower since the model generates imaginary solutions for $\omega = 0.85$.

Finally, the last three columns, labeled “Low q ”, report the same analysis as the base case, but setting the probability of government default conditional on a disaster at $q = 0.25$, down from 0.40. The lower risk of default increases precautionary savings at the benchmark disaster size, $\omega = 1$, so that the unconditional mean bill rate is an implausibly low value of 0.6%. As the size of disaster declines to $\omega = 0.95$ and $\omega = 0.85$, the precautionary motive is offset somewhat and the bill rate increases and the equity premium correspondingly shrinks.

Similar patterns hold for the Conditional Model Moments. In general, the precautionary motive for holding bonds decreases with lower ω and q so that government bill rates increase as these variables are lowered. Generally, the equity premium moves inversely with these relationships.

Overall, the time-varying model does allow us to match the volatility of asset returns better as in Wachter (2013). This improvement, coupled with the fact that there is less variability across countries in the asset return variances, allows the model, even within a narrow range of parameters, to fit these moments better.

2 International Implications of Disaster Risk

The analysis above examined the country effects of disaster risk in isolation, as typically assumed in the disaster risk literature. In this section, we begin to consider the international implications of these same relationships. For this purpose, note first that the disaster risk framework specified above presumes some segmentation of markets. For example, the consumption processes in equation (3) are taken as given by the data and the state price densities are country-specific so that assets are potentially priced differently in each country. In following this approach, we are consistent with a large literature demonstrating that markets are not perfectly integrated.¹⁷ For example, Dumas et al (2003) demonstrate that the international equity markets do not fully incorporate the risk of their respective country outputs.

In this section we continue to treat the market given by the asset pricing relationships in Section 1, regarding consumption in the data as the result of some equilibrium decision from an unspecified production and asset market. Based upon these relationships from the data, we ask what the pattern of co-movements across countries would imply about the presence of disaster risk.

¹⁷Although Stulz (1981) and Adler and Dumas (1983) established the international capital asset pricing model under full market integration, studies such as Dumas and Solnik (1995) demonstrated the empirical problems with the model. Moreover, a large literature has studied the degree of segmentation in international markets. See, for example, Bekaert, Harvey, Lundblad, and Siegel (2011), Carrieri, Chaieb, and Errunza (2013), Carrieri, Errunza, and Hogan (2007), Christoffersen, Errunza, Jacobs, and Langlois (2012), and Pukthuanthong and Roll (2009), among others.

2.1 International Co-movements

An extensive literature has examined the co-movements of consumption and asset returns across countries. A general finding is that consumption correlations are low while those of asset returns, particularly equity, are high (see Lewis and Liu (2015) and Tesar (1995)). To evaluate the disaster risk model's ability to generate a similar pattern, we consider the co-movement of disasters.

2.1.1 Co-movements Assuming Independent Disasters

The disaster risk literature has typically treated the occurrence of a disaster as independent across countries.¹⁸ In this light, we begin by considering the pattern of consumption co-movements implied by equation (3), repeated here for convenience:

$$\frac{dC_t^j}{C_t^j} = \mu dt + \sigma^j dB_t^j + (e^{\omega^j Z_t} - 1) dN_t^j, \quad \forall = 1, \dots, J$$

If indeed, the Poisson process generating the disaster is independent across countries, then the instantaneous consumption correlations are just given by the correlation of the Brownians,

$$Corr\left(\frac{dC_t^i}{C_t^i}, \frac{dC_t^j}{C_t^j}\right) = Corr(dB_t^i, dB_t^j) \equiv \rho^{ij} \quad (11)$$

Similarly, asset price correlations are also only affected by the correlation of Brownians, implying that there is no effects on the instantaneous correlation across countries due to disaster risk. To see why, consider the state price processes across countries from equation

¹⁸Nakamura et al (2013) and Farhi and Gabaix (2016) provide exceptions, as noted above. Moreover, we relax this assumption below.

(9) under the assumptions that dN_t^j realizations and the Brownians on their intensities are independent across countries; i.e., that dB_t^j and dB_t^i are uncorrelated. In this case, the correlation of the state price processes are also $Corr\left(\frac{d\pi_t^i}{\pi_t^i}, \frac{d\pi_t^j}{\pi_t^j}\right) = \rho^{ij}$. Thus, the state price processes are only correlated across countries due to their normal times consumption correlations since agents view the impact of disasters on consumption as uncorrelated. As a result, the instantaneous correlation in stock prices across countries using the process for equities in equation (10) and the assumptions of independent disasters is also:

$$Corr\left(\frac{dF_t^i}{F_{t-}^i}, \frac{dF_t^j}{F_{t-}^j}\right) = \rho^{ij} \quad (12)$$

Furthermore, the independence of disasters risk implies that the government bill rates will be independent. Simple inspection of the government bill rate in equation (6) makes clear the reason. As noted earlier, these rates only vary due to the changes in the probability of disasters, according to λ_t^j . When λ_t^j and λ_t^i are uncorrelated for all i, j , then so will be all government bond rates. Therefore, the instantaneous correlation of government bill rates under independent disasters is:

$$Corr\left(\frac{dr_t^{b,i}}{r_t^{b,i}}, \frac{dr_t^{b,j}}{r_t^{b,j}}\right) = 0 \quad (13)$$

Note also that if disasters are independent, the correlation of consumption and asset pricing moments will be the same whether using a full sample including disasters or a sub-sample excluding those disasters.

2.1.2 Co-movements Assuming Common Disasters

By contrast to the assumption of independence, most of the disasters identified in the data by Barro (2006) and Barro and Ursua (2008) occur at roughly the same time for the OECD countries. For example, during the periods of the Great Depression and the World Wars, most of the countries were in disaster states, whether measured by declines in GDP of at least 15% as measured by Barro (2006), or by declines of consumption of at least 10% as classified by Wachter (2013). Thus, an alternative assumption may be that all countries share the same disaster risk process so that $dN_t^j = dN_t^w, \forall j$ where dN_t^w is a Poisson world disaster event shock that has an intensity λ_t^w . In this case, consumption for country j follows:

$$\frac{dC_t^j}{C_{t-}^j} = \mu dt + \sigma^j dB_t^j + (e^{\omega^j Z_t} - 1) dN_t^w, \quad \forall = 1, \dots, J \quad (14)$$

where dN_t^w is a Poisson jump process and the intensity process follows:

$$d\lambda_t^w = \kappa \left(\bar{\lambda}^w - \lambda_t^w \right) dt + \sigma_\lambda \sqrt{\lambda_t^w} dB_{\lambda,t}^w. \quad (15)$$

For expositional convenience, we assume that the mean reversion parameter κ and the volatility of the probability σ_λ are the same as defined earlier, and are common across countries. However, the impact of the disaster may affect countries differently through the size of ω^j .

When the disaster event is common, the correlation of consumption and asset returns will also depend upon the correlation of the disaster components. To build intuition, consider the effect on consumption correlations from these common disasters assuming for simplicity

that the intensity on the world disaster shock is constant so that $\lambda_t^w = \bar{\lambda}^w$. Defining the size of the decline in the consumption growth during disasters as $K^j \equiv (e^{\omega^j Z} - 1)$ and using properties of Poisson processes, the correlation of consumption across countries is given by:

$$Corr \left(\frac{dC_t^i}{C_t^i}, \frac{dC_t^j}{C_t^j} \right) = \frac{\sigma^i \sigma^j \rho^{ij} + K^i K^j \lambda^w}{\sqrt{(\sigma^i)^2 + (K^i)^2 \lambda^w} \sqrt{(\sigma^j)^2 + (K^j)^2 \lambda^w}} \quad (16)$$

Or, in the case where the effect of disasters is the same so that $\omega^i = \omega^j \forall j$, and $K^j = K^i = K$, the instantaneous correlation is:

$$Corr \left(\frac{dC_t^i}{C_t^i}, \frac{dC_t^j}{C_t^j} \right) = \frac{\sigma^i \sigma^j \rho^{ij} + (K)^2 \lambda^w}{\sqrt{(\sigma^i)^2 + K^2 \lambda^w} \sqrt{(\sigma^j)^2 + K^2 \lambda^w}} \quad (17)$$

Using the relevant assumptions about the parameters, it can be shown that the correlation of consumption growth in equation (17) is greater than the correlation of normal times consumption, ρ^{ij} . Thus, in this case of shared disaster risk, consumption is more correlated.

Now consider the effects on asset prices. The correlation in the state prices in equation (9) will include a common disaster shock, dN_t^w , thereby increasing the correlation in state prices. Moreover, equity prices will share this same higher correlation due to the common disaster shock. To see this relationship, we rewrite the stock price in equation (10) with the common disaster risk under constant disaster risk:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi^j \sigma^j dB_t^j + (e^{\phi^j \omega^j Z_t} - 1) dN_t^w, \quad (18)$$

When disaster intensities are constant, instantaneous variations in equity returns are generated exclusively through the Brownian on normal times consumption dB_t and the

disaster event shock dN_t^w as in consumption, except that the effects are magnified by the leverage parameter ϕ . Thus, the correlations in equity prices are the same as those of consumption in equation (16) except that $K^j \equiv (e^{\phi^j \omega^j Z} - 1)$ and σ^j are replaced by $\sigma^j \phi^j$.

When disaster intensities are time-varying, the correlation of asset prices can be higher than consumption, however. For example, rewriting the stock price in equation (10) to include the common disaster event and common time-varying probabilities, the process becomes:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi \sigma^j dB_t^j + g \sigma_\lambda \sqrt{\lambda_t^w} dB_{\lambda,t}^w + (e^{\phi^j \omega^j Z_t} - 1) dN_t^w. \quad (19)$$

In this case, stock price changes have a higher correlation due to the perfect correlation in $dB_{\lambda,t}^w$, the changes to the probability of a common world disaster.

We next evaluate the implications for both common and independent disasters on the correlations across countries.

2.2 Matching the Moments: Correlations

In order to understand the degree of co-movement between returns, we focus upon two countries, the U.S. and the U.K. As a reference, these two countries in the post-war data have a correlation of 0.49 for consumption growth rates, 0.75 for equity returns, and 0.63 for government bill rates, see Lewis and Liu (2015).

Table 6, Panel A reports the correlations of consumption, equity returns, and government bills under the assumption that the disaster probability is constant and that $\lambda_t^j = \lambda_t = \bar{\lambda} = 3.5\%$ for $j = 1, 2$. The table reports both the “Unconditional” correlations over all

realizations as well as the “Conditional” correlations excluding the disaster events. The first column gives the results when the effects upon consumption due to disasters are assumed to be uncorrelated across countries as in equation (11). As described above, the correlation of equity during normal times, consistent with the “Conditional” results, is determined by normal times consumption. Over the whole sample including disasters, however, consumption and equity return correlations are driven down by independent realizations of dN_t^j . Alternatively, the second column of reports the results assuming a common world disaster as in equation (14). Since countries are affected by a common world disaster, the disasters across countries are not only perfectly correlated, but also have the same magnitude or realizations of Z_t . Once again, the consumption and equity return correlations are similar during normal times, as the “Conditional” results show. However, now the periods of disasters are shared across countries, both for the timing and size of disasters. As a result, the periods of disasters generate a strong common component during those periods, driving the consumption correlations very high. Nevertheless, the correlation of consumption is higher than that of equity returns, inconsistent with the data.

Panel B of Table 6 provides the correlation when the probability of disasters is time varying as in equation (16). In this case, an important component of the correlation of equity returns is determined by the co-movement of time-variation in disaster probabilities, λ_t . The first column shows that when these disaster probabilities are uncorrelated, even the normal times Conditional correlations in equities are lower than consumption at 0.0556. As in the static probability case, the independence of disasters renders the Unconditional correlations in both consumption and equity to be implausibly low at about 5%. Also, as described in the previous subsection, the correlation of the bill rates is driven entirely

by the correlation in the disaster probabilities. Thus, when the probabilities of disasters are independent across countries, the correlation of bill rates is also essentially zero since simulated model correlations are do not deviate zero by more than -1% .

By contrast, when disaster events and the changes to their probabilities affect consumption in all countries the same, the correlations in asset returns are much higher. For this case, reported in the second column of Table 6 Panel B, equity return correlations are significantly higher than consumption at 0.946. Moreover, this high correlation is maintained in the full sample at 0.958. At the same time, government bill rates carry a correlation of one both conditionally and unconditionally, since they are driven by the same common disaster probability, as explained above.

In summary, the investigation highlights problems with both versions of disaster risk. When disasters are independent, correlations of asset returns are too low, indeed lower than consumption correlations. On the other hand, when disasters are shared, correlations are too high and near one.

2.3 Country and World Disasters

As the results above demonstrate, standard assumptions about disaster risk across countries provides counterfactual implications for the normal times correlation in equity returns. In the absence of time-varying disaster intensities, the variation in equity returns is too low and the equity return correlation is driven entirely by the correlation in consumption. In the presence of time-variation in disaster intensities, however, the equity return correlation is too high if the disaster events are common and too low if disaster events are independent.

This observation suggests that a more plausible assumption is that some disasters are shared while some are country-specific. To allow for this possibility, we specify the consumption process as dependent upon two Poisson jump processes. In addition, we assume that conditional on being in a disaster state, each country draws independent consumption declines, defined by Z_t^j .¹⁹ In theory, there are two potential dimensions in which disaster risk can be correlated, the disaster event and the size of consumption decline conditional on disaster. To focus upon the role of disaster events, we assume that disasters are only correlated through the Poisson process that guides the timing of the disaster, and allow the size of disasters to be independent, defined by Z_t^j for each country j . Then the consumption process in this case is:

$$\frac{dC_t^j}{C_{t-}^j} = \mu dt + \sigma^j dB_t^j + (e^{\omega^j Z_t^j} - 1)(dN_t^j + dN_t^w) \quad (20)$$

where N_t^j has disaster intensity λ_t^j and N_t^w has disaster intensity λ_t^w . In other words, the probability of a disaster in each country can be generated by a world disaster shock, dN_t^w , or a country-specific shock, dN_t^j . In turn, each of these shocks are driven by their own time-varying probability processes as in equations (4) and (15).

As such, the correlation of consumption is a mixture of the independent and common jump processes. For example, in the special case when the probability of disasters is constant,

$\lambda_t^i + \lambda_t^w = \lambda^i + \lambda^w, \forall t$, we can define the consumption correlation as:

¹⁹Note that the assumption is only made to connect with the data on the prior section. In principle, we could consider the case where the world disaster is big and the country-specific disaster is smaller, or vice versa.

$$Corr \left(\frac{dC_t^i}{C_t^i}, \frac{dC_t^j}{C_t^j} \right) = \frac{\sigma^i \sigma^j \rho^{ij} + K^i K^j \lambda^w}{\sqrt{(\sigma^i)^2 + (K^i)^2 (\lambda^i + \lambda^w)} \sqrt{(\sigma^j)^2 + (K^j)^2 (\lambda^i + \lambda^w)}} \quad (21)$$

where $K^j \equiv (e^{\omega^j Z} - 1)$. As before, this relationship simplifies when the effect of disasters on consumption is the same across countries such that $\omega^i = \omega^j$ and $K^i = K, \forall i$ so that the correlation becomes:

$$Corr \left(\frac{dC_t^i}{C_t^i}, \frac{dC_t^j}{C_t^j} \right) = \frac{\sigma^i \sigma^j \rho^{ij} + K^2 \lambda^w}{\sqrt{(\sigma^i)^2 + K^2 (\lambda^i + \lambda^w)} \sqrt{(\sigma^j)^2 + K^2 (\lambda^i + \lambda^w)}} \quad (22)$$

In this case, the unconditional consumption correlations will increase due to the common world disaster, but decrease due to the uncorrelated country-specific disasters. The realizations of these common and country-specific disaster events therefore affect equity correlations as well. The probability of a disaster for country i given that country j is in a disaster will lie between the extreme cases of 0 and 1.

Note, however, in all of these versions, the correlation of consumption in “normal times” is still simply given by the correlation of the Brownians in Equation (12). The effects of disasters will only be seen in unconditional correlations. As long as there is a world disaster component, this effect will always increase the correlation in consumption as Equations (16) and (21) show. However, any time-variation in the probability of disasters will appear in the normal times equity returns.

Note that the probability of disasters is now given by the sum of the probabilities, $\lambda_t^i + \lambda_t^w$. In order to match the data, we impose the condition that the means of the two intensity processes equals that of the data; or $\tau^i \bar{\lambda}^w + (1 - \tau^i) \bar{\lambda}^i = \tilde{\lambda}^i$ where $\tilde{\lambda}^i$ is the weighted mean

of the joint Poisson process and $0 < \tau^i < 1$ is the share of disaster risk of country i that is due to country-specific disasters.

Identifying the share of country-specific versus world disaster probabilities poses a difficulty with empirically evaluating the differing disaster risks across countries. The disaster risk literature takes crisis events from other countries to try to identify the frequency and intensity in the US since there are not enough events in a given country. However, Lewis and Liu (2015) propose an identification approach that is useful in this context. Given the pattern of consumption correlation such as in equation (21) along with the implied correlation patterns of asset returns in the model, we can recover the relevant patterns for identification such as λ^i, λ^w . Using the framework above, we can vary the share of country-specific disaster risk through τ^i to match the observed correlation patterns.

Table 7, Panel B demonstrates the relationship implied by varying τ^i between 0 and 1. To be consistent with our prior analysis, we set $\tilde{\lambda}^i = 3.55\%$. We continue to maintain the assumption that the variance of the probability of country-specific and world disasters are the same, as for the case of the US and UK example. As the weight on the world disaster increases, the correlation of asset returns increases. During conditional “normal times” periods, the correlation of equity returns increases from 0.055 when disasters are uncorrelated to 0.946 when they are perfectly correlated. A similar pattern holds for the bill rate.

The table results suggest combinations of world and country-specific disasters that may match the pattern of correlations. For example, when $\tau = 0.8$, the correlation of normal times consumption is 0.495 as in the data, but also the correlation of equity returns is 0.75, as in the data between the U.S. and the U.K.

Overall, the evidence suggests a high degree of world disaster risk is needed to explain the degree of co-movement between asset returns.

3 Concluding Remarks

A growing literature examines the impact of disaster risk on the macroeconomy and asset returns. Indeed, the relevance of this risk has become more evident since the recent financial crises. A standard approach in this literature is to use international data to make inferences about the frequency and size of these disasters. Nevertheless, the focus of these studies has largely remained on the U.S. asset market in isolation.

In this paper, we evaluated international asset returns through the lens of a canonical disaster risk model as articulated in Barro (2006, 2009) and allowing for time-varying disasters as in Wachter (2013). Our analysis led to three main findings. First, while the disaster risk model does well in explaining U.S. asset returns, it is less successful in matching the range of asset return behavior observed internationally. Second, the degree to which the model can explain international asset return co-movements hinges largely on the importance of a common disaster risk across countries. Specifically, if the frequency and size of disasters is independent across countries, the correlation of asset returns is implausibly low. By contrast, if all disasters are common, these correlations are near one and, hence, unrealistically large. Third, these findings suggest that international correlations of asset returns and consumption can provide an identification of the importance of world versus country-specific disasters. Calibrating the model to the correlations between the U.S. and the U.K. implies that 80% of the disaster risk is common between the two countries. Overall, this paper

shows that the international dimensions of standard disaster risk models carry important implications for their saliency.

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Table 1: Data Moments (Annual %)

Panel A: Full-Sample Data Moments (Barro/Ursua)							
	AUS	CAN	FRA	GER	JPN	UK	US
Mean Govt Bill r^b	1.26	3.92	-0.61	-1.53	0.43	1.79	1.99
Std Dev Govt Bill r^b	5.66	11.99	9.96	17.88	14.75	6.24	4.82
Mean Equity Premium	9.01	3.89	6.04	9.11	8.85	4.62	6.28
Mean Equity Return	10.27	7.81	5.43	7.58	9.28	6.41	8.27
Std Dev Equity Return	16.16	17.54	20.78	29.76	30.17	17.65	18.66
Sharpe Ratio	0.56	0.22	0.29	0.31	0.29	0.26	0.34
Mean Cons Growth	1.54	1.92	1.62	1.89	2.48	1.47	1.85
Std Dev Cons Growth	5.06	4.74	6.74	5.7	6.89	2.83	3.6
Panel B: Post-War Data Moments (Barro-Ursua)							
	AUS	CAN	FRA	GER	JPN	UK	US
Mean Govt Bill r^b	0.55	1.46	-0.84	0.11	-2.57	1.11	0.70
Std Dev Govt Bill r^b	5.14	3.81	9.86	12.05	15.67	3.67	3.19
Mean Equity Premium	8.49	6.35	8.15	10.63	11.90	7.77	8.16
Mean Equity Return	9.04	7.82	7.31	10.74	9.32	8.88	8.86
Std Dev Equity Return	20.09	15.82	25.42	33.66	33.73	22.51	17.44
Sharpe Ratio	0.42	0.40	0.32	0.32	0.35	0.35	0.47
Mean Cons Growth	2.45	2.22	3.34	3.56	5.47	2.38	2.40
Std Dev Cons Growth	3.19	2.24	3.70	3.36	6.19	2.21	2.07

Table 2: Constant Disaster Model Fit - SMM

Panel A: Target Unconditional Data Moments							
	AUS	CAN	FRA	GER	JPN	UK	US
Fitted Parameters							
ω	0.884	0.840	0.854	0.843	0.844	0.843	0.846
ϕ	2.664	2.795	2.63	2.805	2.755	2.749	2.741
q	0.421	0.442	0.431	0.429	0.447	0.445	0.436
σ	0.022	0.023	0.023	0.023	0.023	0.023	0.0224
Unconditional Model Moments (Annual %)							
Mean Govt Bill r^b	2.86	3.31	3.18	3.20	3.30	3.30	3.24
Std Dev Govt Bill r^b	2.35	2.29	2.11	2.42	2.22	2.08	2.27
Mean Equity Premium	5.36	4.74	4.85	4.8	4.68	4.76	4.73
Std Dev Equity Return	9.95	10.22	9.94	10.67	10.36	10.19	10.18
Sharpe Ratio	0.54	0.46	0.49	0.45	0.45	0.47	0.46
Std Dev Cons Growth	2.12	2.34	2.31	2.43	2.17	2.23	2.34
Mean Sq Diff	0.286	0.052	0.079	0.093	0.286	0.067	0.105
Panel B: Target Conditional Data Moments							
	AUS	CAN	FRA	GER	JPN	UK	US
Fitted Parameters							
ω	0.818	0.819	0.811	0.810	0.816	0.812	0.818
ϕ	2.903	2.925	3.021	2.988	2.952	2.994	2.957
q	0.510	0.506	0.524	0.517	0.520	0.533	0.514
σ	0.020	0.020	0.020	0.020	0.020	0.020	0.020
Conditional Model Moments (Annual %)							
Mean Govt Bill r^b	3.89	3.87	4.00	3.98	3.95	4.01	3.9
Std Dev Govt Bill r^b	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mean Equity Premium	4.91	4.94	4.97	4.89	4.94	4.85	5.03
Std Dev Equity Return	6.43	6.45	6.62	6.58	6.53	6.61	6.55
Sharpe Ratio	0.77	0.78	0.76	0.75	0.76	0.74	0.78
Std Dev Cons Growth	2.04	2.02	2.01	2.02	2.03	2.02	2.03
Mean Sq Diff	0.145	0.150	0.238	0.277	0.277	0.186	0.112

Table 3: Constant Disaster - Varying ω

ω	1.0	0.9	0.8	1.0	0.9	0.8	1.0	0.9	0.8
Parameters	Baseline			High ϕ			Low q		
μ	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
σ	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
ϕ	2.8	2.8	2.8	3.0	3.0	3.0	2.8	2.8	2.8
q	0.4	0.4	0.4	0.4	0.4	0.4	0.25	0.25	0.25
Unconditional Model Moments (Annual %)									
Mean Govt Bill r^b	1.44	2.78	3.69	1.44	2.78	3.69	0.59	2.18	3.26
Std Dev Govt Bill r^b	2.77	2.81	2.83	2.77	2.81	2.83	2.07	2.1	2.12
Mean Equity Premium	7.88	5.56	3.91	8.12	5.76	4.08	8.73	6.17	4.34
Std Dev Equity Return	10.74	10.27	9.79	11.27	10.79	10.31	10.74	10.27	9.79
Sharpe Ratio	0.60	0.27	0.02	0.59	0.28	0.04	0.76	0.39	0.11
Mean Div Growth	5.20	5.38	5.56	5.58	5.77	5.96	5.20	5.38	5.56
Std Dev Div Growth	16.76	15.28	13.82	17.96	16.37	14.81	16.76	15.28	13.82
Mean Cons Growth	1.86	1.92	1.99	1.86	1.92	1.99	1.86	1.92	1.99
Std Dev Cons Growth	5.99	5.46	4.94	5.99	5.46	4.94	5.99	5.46	4.94
Conditional Model Moments (Annual %)									
Mean Govt Bill r^b	1.66	3.00	3.91	1.66	3.00	3.91	0.72	2.30	3.39
Std Dev Govt Bill r^b	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mean Equity Premium	8.79	6.40	4.67	9.07	6.64	4.87	9.73	7.09	5.19
Std Dev Equity Return	6.19	6.13	6.08	6.65	6.58	6.53	6.19	6.13	6.08
Sharpe Ratio	1.15	0.55	0.13	1.11	0.55	0.15	1.46	0.78	0.30
Mean Div Growth	6.99	6.99	6.99	7.49	7.49	7.49	6.99	6.99	6.99
Std Dev Div Growth	5.60	5.60	5.60	6.00	6.00	6.00	5.60	5.60	5.60
Mean Cons Growth	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
Std Dev Cons Growth	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00

Table 4: Time Varying Disaster Model Fit - Calibration

	US	UK	AUS	US	UK	AUS
	Common Disaster			Country Disaster		
Parameters:						
κ	0.08	0.08	0.08	0.11	0.15	0.09
σ_λ	0.067	0.067	0.067	0.080	0.100	0.070
ϕ	2.5	2.6	1.6	2.6	2.6	2.6
q	0.7	0.8	0.3	0.4	0.4	0.4
σ	0.018	0.017	0.022	0.020	0.020	0.020
Unconditional Model Moments (Annual %)						
Mean Govt Bill r^b	1.99	2.3	0.69	1.03	1.03	0.97
Std Dev Govt Bill r^b	4.13	4.35	3.65	3.75	3.85	3.81
Mean Equity Premium	6.36	6.28	5.3	6.68	6.09	7.5
Std Dev Equity Return	18.98	19.77	11.74	18.45	16.98	19.86
Sharpe Ratio	0.36	0.34	0.46	0.37	0.37	0.39
Mean Cons Growth	1.63	1.63	1.63	1.6	1.6	1.61
Std Dev Cons Growth	6.29	6.28	6.41	6.34	6.29	6.42
Mean Sq Diff	0.001	0.009	0.013	0.002	0.014	0.031
Conditional Model Moments (Annual %)						
Mean Govt Bill r^b	2.55	2.93	0.99	1.39	1.4	1.35
Std Dev Govt Bill r^b	1.02	0.68	2.32	2.06	2.15	2.02
Mean Equity Premium	7.39	7.27	6.12	7.93	7.34	8.74
Std Dev Equity Return	16.75	17.52	9.63	16	14.26	17.63
Sharpe Ratio	0.44	0.41	0.60	0.49	0.50	0.49
Mean Cons Growth	2.52	2.52	2.52	2.50	2.50	2.50
Std Dev Cons Growth	1.75	1.70	2.14	1.75	1.70	2.14

Table 5: Time Varying Disaster - Varying ω

ω	1.0	0.95	0.85	1.0	0.95	0.85	1.0	0.95	0.85
Parameters									
	Baseline			High ϕ			Low q		
μ	0.0252	0.0252	0.0252	0.0252	0.0252	0.0252	0.0252	0.0252	0.0252
σ	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
ϕ	2.6	2.6	2.6	3.0	3.0	3.0	2.6	2.6	2.6
q	0.4	0.4	0.4	0.4	0.4	0.4	0.25	0.25	0.25
κ	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
σ_λ	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067	0.067
Panel A: Unconditional Moments (Annual %)									
Mean Govt Bill r^b	1.01	1.33	1.86	1.01	1.33	N/A	0.56	0.95	1.59
Std Dev Govt Bill r^b	3.79	3.55	3.11	3.79	3.55	N/A	3.55	3.26	2.76
Mean Equity Premium	7.62	6.34	4.61	8.56	7.18	N/A	8.07	6.73	4.88
Std Dev Equity Return	19.95	18.79	16.62	23.39	22.36	N/A	19.95	18.79	16.62
Sharpe Ratio	0.39	0.35	0.29	0.37	0.33	N/A	0.41	0.36	0.30
Mean Div Growth	4.24	4.36	4.59	4.89	5.03	N/A	4.24	4.36	4.59
Std Dev Div Growth	16.53	15.79	14.31	19.08	18.22	N/A	16.53	15.79	14.31
Mean Cons Growth	1.63	1.68	1.76	1.63	1.68	N/A	1.63	1.68	1.76
Std Dev Cons Growth	6.36	6.07	5.50	6.36	6.07	N/A	6.36	6.07	5.50
Panel B: Conditional Moments (Annual %)									
Mean Govt Bill r^b	1.38	1.69	2.18	1.38	1.69	N/A	0.82	1.20	1.81
Std Dev Govt Bill r^b	2.00	1.74	1.32	2.00	1.74	N/A	2.48	2.16	1.64
Mean Equity Premium	8.87	7.55	5.73	9.97	8.55	N/A	9.43	8.03	6.10
Std Dev Equity Return	17.73	16.57	14.41	21.19	20.20	N/A	17.73	16.57	14.41
Sharpe Ratio	0.49	0.49	0.49	0.49	0.49	N/A	0.49	0.49	0.49
Mean Div Growth	6.55	6.55	6.55	7.56	7.56	N/A	6.55	6.55	6.55
Std Dev Div Growth	5.16	5.16	5.16	5.96	5.96	N/A	5.16	5.16	5.16
Mean Cons Growth	2.52	2.52	2.52	2.52	2.52	N/A	2.52	2.52	2.52
Std Dev Cons Growth	1.99	1.99	1.99	1.99	1.99	N/A	1.99	1.99	1.99

Table 6: Model Implied Correlation

Panel A: Static Disaster		
	No Disaster Correlation	Perfect Disaster Correlation
Unconditional:		
Corr Cons Growth	0.034	0.951
Corr Equity Return	0.117	0.873
Corr Bill Rate	-0.004	1.0
Conditional:		
Corr Cons Growth	0.495	0.495
Corr Equity Return	0.495	0.495
Corr Bill Rate	N/A	N/A
Panel B: Time Varying Disaster		
	No Disaster Correlation	Perfect Disaster Correlation
Unconditional:		
Corr Cons Growth	0.046	0.950
Corr Equity Return	0.045	0.958
Corr Bill Rate	-0.005	1.000
Conditional:		
Corr Cons Growth	0.490	0.489
Corr Equity Return	0.056	0.946
Corr Bill Rate	-0.009	1.000

[†] Data Correlation from Lewis and Liu (2015) for U.S. and U.K. during the postwar period are as follows: Consumption Growth = 0.49, Equity Return = 0.75, Bill Rate = 0.63

Table 7: Model Implied Correlation

Panel A: Static Disaster						
	$\tau = 0.0$	$\tau = 0.2$	$\tau = 0.4$	$\tau = 0.6$	$\tau = 0.8$	$\tau = 1.0$
Unconditional:						
Corr Cons Growth	0.048	0.144	0.270	0.373	0.491	0.614
Corr Equity Return	0.122	0.242	0.367	0.495	0.634	0.766
Corr Bill Rate	0.000	0.053	0.114	0.200	0.279	0.304
Conditional:						
Corr Cons Growth	0.495	0.494	0.494	0.495	0.495	0.494
Corr Equity Return	0.494	0.493	0.493	0.494	0.494	0.494
Corr Bill Rate	N/A	N/A	N/A	N/A	N/A	N/A
Panel B: Time Varying Disaster						
	$\tau = 0.0$	$\tau = 0.2$	$\tau = 0.4$	$\tau = 0.6$	$\tau = 0.8$	$\tau = 1.0$
Unconditional:						
Corr Cons Growth	0.042	0.169	0.281	0.387	0.482	0.613
Corr Equity Return	0.037	0.236	0.396	0.565	0.722	0.920
Corr Bill Rate	0.002	0.117	0.230	0.357	0.434	0.561
Conditional:						
Corr Cons Growth	0.492	0.495	0.494	0.492	0.495	0.494
Corr Equity Return	0.055	0.257	0.416	0.586	0.748	0.946
Corr Bill Rate	0.007	0.225	0.416	0.612	0.793	1.000

[†] Data Correlation from Lewis and Liu (2015) for U.S. and U.K. during the postwar period are as follows: Consumption Growth = 0.49, Equity Return = 0.75, Bill Rate = 0.63

A Appendix: Solving the Value Function for Individual Countries

In the text, we use solutions for asset returns as a function of the consumption process for each country. This section describes the value function used to solve for these returns. Then, Appendix B uses this formulation to solve for asset returns.

Determining the value function for each individual country follows as a natural modification of the solution in Wachter (2013). For each country, the consumption process follows equation (3) in the text, repeated here for convenience:

$$dC_t^j = \mu^j C_{t-}^j dt + \sigma^j C_{t-}^j dB_t^j + (e^{\omega^j Z_t} - 1) C_{t-}^j dN_t^j$$

where B_t^j is a standard Brownian, $\omega^j Z_t$ is the reduction in country j consumption during a disaster, N_t^j is a poisson process with an intensity parameter λ_t^j , driven by another standard Brownian motion, $dB_{\lambda,t}^j$, with diffusion given by equation (4) also repeated here:

$$d\lambda_t^j = \kappa (\bar{\lambda} - \lambda_t^j) dt + \sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j$$

The representative agent in each country j has recursive Epstein-Zin-Weil preferences as formulated in continuous time by Duffie and Epstein (1992) and defined as V_t^j given by equations (1) and (2) in the text:

$$V_t^j = E_t \int_t^\infty f(C_s^j, V_s^j) ds$$

where, assuming that the intertemporal elasticity of substitution is one,

$$f(C_t^j, V_t^j) = \beta(1 - \gamma)V_t^j \left[\log C_t^j - \frac{1}{1 - \gamma} \log((1 - \gamma)V_t^j) \right]$$

and where $\beta > 0$ is the rate of time preference and $\gamma > 0$ is the coefficient of relative risk aversion. In some of our results below, we also consider the case where the intertemporal elasticity of substitution, defined as ψ , is not equal to one. In this case, the felicity function is given by:

$$f(C^j, V^j) = \left(\frac{\beta}{1 - \frac{1}{\psi}} \right) \frac{C^{j, 1 - \frac{1}{\psi}} - ((1 - \gamma)V^j)^{\frac{1}{\theta}}}{((1 - \gamma)V^j)^{\frac{1}{\theta} - 1}} \quad (23)$$

where $\theta \equiv \frac{1 - \gamma}{1 - \frac{1}{\psi}}$.

To solve for the value function assuming unitary intertemporal elasticity, we specify the Hamilton-Jacobi-Bellman equation for an investor who allocated wealth between a risk free rate, r_t^j , and a risky asset that pays out in consumption each period.²⁰ For this purpose, note that when the intertemporal elasticity of consumption is one, the price-dividend ratio for this consumption claim is a constant, see Weil (1990). Defining the price of a claim on consumption for the representative agent in country j as S_t^j and this price-dividend ratio for country j as $l^j \equiv (S_t^j/C_t^j)$, the consumption process equation (3) implies:

$$dS_t^j = \mu S_{t-}^j dt + \sigma^j S_{t-}^j dB_t^j + (e^{\omega^j Z_t} - 1) S_{t-}^j dN_t^j \quad (24)$$

²⁰Note that this definition implies multiple "risk-free" rates, defined relative to the consumption process of each country. In Section 3, we will solve for the single world risk-free rate based upon integrated international economy.

Defining α_t as the fraction of wealth W_t^j that the representative agent allocates to the risky asset S_t , and $(1 - \alpha_t)$ fraction of their wealth to the risk free asset, then the wealth process for country j follows:

$$dW_t^j = (\alpha_t^j W_{t-}^j (\mu - r_t^j + (l^j)^{-1}) + W_{t-}^j r_t^j - C_t^j) dt + \alpha_t^j W_{t-}^j \sigma^j dB_t^j + \alpha_t^j W_{t-}^j (e^{\omega^j Z_t} - 1) dN_t^j \quad (25)$$

Then, defining the value function as $J(W, \lambda)$, at the optimum, the instantaneous expected change in the value function, plus flow utility must equal zero. That is, at the optimum, the Hamilton-Jacobian-Bellman equation must equal zero. Solving for this HJB as a direct application of Ito's lemma with jumps (see Duffie 2010). Therefore, this optimum must satisfy:

$$\left. \begin{array}{l} \sup \\ \alpha_t^j, C_t^j \end{array} \left\{ \begin{array}{l} J_W (\alpha_t^j W_t^j (\mu - r_t^j + (l^j)^{-1}) + W_t^j r_t^j - C_t^j) + J_{\lambda\kappa} (\bar{\lambda} - \lambda_t^j) + \frac{1}{2} J_{WW} (\alpha_t^j W_t^j \sigma^j)^2 + \frac{1}{2} J_{\lambda\lambda} \sigma_\lambda^2 \lambda_t^j \\ + \lambda_t^j E_\nu [J(W_t(1 + \alpha_t^j (e^{\omega^j Z} - 1)), \lambda_t^j) - J(W_t^j, \lambda_t^j)] + f(C_t^j, J) \end{array} \right\} = 0 \right. \quad (26)$$

where J_i and J_{ij} are the first and second derivatives of J with respect to i and to i and j , respectively and E_ν is the expectation taken over the time invariant distribution of Z given by ν .

Using the fact that in equilibrium, the representative agent holds all the claims on the consumption asset, $\alpha_t = 1$ and C_t^j is given by the consumption process in equation (3), we solve for this value function by guess-and-verify. Therefore, we follow the conjecture for the value function form in Wachter (2013) given by:

$$J(W_t^j) = I^j(\lambda_t^j)^{1-\gamma} \frac{(W_t^j)^{1-\gamma}}{1-\gamma} \quad (27)$$

subsuming the superscripts and subscripts for expositional transparency. Using this form of the value function and the envelope condition such that $J_W = f_C(C, V)$, it follows that: $l^j = l = \beta^{-1}$. Thus, all countries have the same price-dividend ratio, even though they are potentially priced in distinct markets.

Further conjecturing the form:

$$I^j(\lambda_t^j) = e^{a^j + b^j \lambda_t^j} \quad (28)$$

then following the same steps as Wachter (2013), it can be shown that:

$$b^j = \left(\frac{\kappa + \beta}{\sigma_\lambda^2} \right) - \sqrt{\left(\frac{\kappa + \beta}{\sigma_\lambda^2} \right)^2 - 2 \frac{E_\nu(e^{(1-\gamma)\omega^j Z_t} - 1)}{\sigma_\lambda^2}} \quad (29)$$

and that

$$a^j = \frac{1-\gamma}{\beta} \left(\mu - \frac{1}{2} \gamma (\sigma^j)^2 \right) + (1-\gamma) \log(\beta) + b^j \frac{\kappa \bar{\lambda}}{\beta}. \quad (30)$$

Note that the effects on the value function from time-variation in the disaster intensities, λ_t , as captured by b^j in equation (29) only differ across countries according to how much consumption declines when a disaster occurs, as measured by ω^j . However, the constant effect as captured by a^j in equation (30) differs also across countries according to the volatility of normal times consumption, σ^j , reflecting cross-country heterogeneity across countries from the standard certainty equivalent consumption measure. These differences will be important

in considering welfare gains in a fully diversified economy.

As a further extension that may be used in our next version, we consider the case of preferences without unitary intertemporal elasticity in consumption. For this case, we assume for now that the intensity of disasters is constant. Subsuming the dependence upon country j and time t for expositional clarity, recall that $J(W) = I^{1-\gamma} \frac{W^{1-\gamma}}{1-\gamma}$, which implies that:

$$J_W = W^{-\gamma} I^{1-\gamma} = (lC)^{-\gamma} I^{1-\gamma} \quad (31)$$

Next, solving for the derivative of felicity with respect to consumption, we have:

$$f_C(C, V) = \frac{\beta C^{-\frac{1}{\psi}}}{((1-\gamma)V)^{\frac{1}{\theta}-1}} = \beta C^{-\gamma} (lI)^{\frac{1}{\psi}-\gamma} \quad (32)$$

where the second equality follows from substitution the below relationship for the value function:

$$V = J(W) = I^{1-\gamma} \frac{(lC)^{1-\gamma}}{1-\gamma} \quad (33)$$

Substituting this form for the value function into equation (32) and using the fact that along the optimum, $J_W = f_C(C, V)$, this relationship implies that the price-dividend ratio becomes:

$$l = \beta^{-\psi} I^{\psi-1} \quad (34)$$

Note, since we defined $\frac{S}{C} = l$ and in equilibrium $W = S$, then we have that $\frac{W}{C} = l$. Using

this equality, and some algebra substituting the above definition for $J(W)$, implies:

$$f(C(W), J(W)) = \frac{\beta}{1 - \frac{1}{\psi}} \frac{(lW)^{1 - \frac{1}{\psi}} - ((1 - \gamma)J(W))^{\frac{1}{\theta}}}{((1 - \gamma)J(W))^{\frac{1}{\theta} - 1}} = \frac{\beta}{1 - \frac{1}{\psi}} W^{1 - \gamma} \frac{l^{\frac{1}{\psi} - 1} - I^{1 - \frac{1}{\psi}}}{I^{\gamma - \frac{1}{\psi}}} \quad (35)$$

Substituting the above into the first two parts of the JMB Equation and equating $\alpha = 1$, yields:

$$W^{1 - \gamma} j^{1 - \gamma} (\mu - r + l^{-1}) + W^{1 - \gamma} j^{1 - \gamma} r - W^{1 - \gamma} j^{1 - \gamma} (l^{-1}) + \frac{1}{2} (-\gamma) W^{1 - \gamma} I^{1 - \gamma} (\sigma^2) = W^{1 - \gamma} I^{1 - \gamma} (\mu - \frac{1}{2} \gamma \sigma^2) \quad (36)$$

$$\begin{aligned} \lambda E_v [J(W_t(1 + (e^{\omega Z} - 1))) - J(W_t)] &= \lambda(1 - \gamma)^{-1} W^{1 - \gamma} I^{1 - \gamma} E_v [(1 + (e^{\omega Z} - 1))^{1 - \gamma} - 1] \\ &= \lambda(1 - \gamma)^{-1} W^{1 - \gamma} I^{1 - \gamma} E_v [(e^{\omega(1 - \gamma)Z} - 1)] \end{aligned} \quad (37)$$

Putting all 3 pieces together, and dividing by $W^{1 - \gamma}$, and substituting into the JMB implies:

$$j^{1 - \gamma} (\mu - \frac{1}{2} \gamma \sigma^2) + \lambda(1 - \gamma)^{-1} I^{1 - \gamma} E_v [(e^{\omega(1 - \gamma)Z} - 1)] + \frac{\beta}{1 - \frac{1}{\psi}} \left[\frac{l^{\frac{1}{\psi} - 1} - I^{1 - \frac{1}{\psi}}}{I^{\gamma - \frac{1}{\psi}}} \right] = 0 \quad (38)$$

Substituting equation (34) into the above Bellman Equation, and solving for I and recalling the country specific components, subscripted by j , implies that the price-dividend ratio is given by:

$$l^j = [-\mu + \frac{1}{2} \gamma (\sigma^j)^2] - \lambda(1 - \gamma)^{-1} E_v [(e^{\omega^j(1 - \gamma)Z} - 1)] + \frac{\beta}{1 - \frac{1}{\psi}} (1 - \frac{1}{\psi})^{-1} \quad (39)$$

In this case, the price-dividend ratios will differ across countries.

B Appendix: Equilibrium Consumption Asset Price and Risk Free Rate for Individual Countries

In this appendix, we describe briefly the equilibrium solutions for the price of the consumption asset and the risk free rate under unit elasticity. Using the definition of $J(W) = j^{1-\gamma} \frac{W^{1-\gamma}}{1-\gamma}$ in the above Hamiltonian-Jacobi-Bellman equation in (26) and taking the derivative with respect to α , we get the following first order condition:

$$\begin{aligned} I_t^{1-\gamma} (W_t^{1-\gamma}) ((\mu - r_t^j + l^{-1}) + I_t^{1-\gamma} (-\gamma) (W_t^{1-\gamma}) (\alpha_t) (\sigma^j)^2) \\ + I_t^{1-\gamma} W_t^{1-\gamma} \lambda_t E_v [(1 + \alpha_t (e^{\omega^j Z} - 1))^{-\gamma} * (e^{\omega^j Z} - 1)] = 0 \end{aligned} \quad (40)$$

In equilibrium, α must equal 1, since the price of the consumption equity claim is the wealth of the economy. Using this relationship and rearranging implies:

$$(\mu - r_t^j + l^{-1}) + (-\gamma) (\sigma^j)^2 = -\lambda_t E_v [(1 + (e^{\omega^j Z} - 1))^{-\gamma} * (e^{\omega^j Z} - 1)] \quad (41)$$

Therefore, the risk free rate for country j , r_t^j , can be written as:

$$r_t^j = \mu + l^{-1} - \gamma (\sigma^j)^2 + \lambda_t^j E_v [(e^{\omega^j Z})^{-\gamma} (e^{\omega^j Z} - 1)]. \quad (42)$$

Rearranging and noting that $l^{-1} = \beta$ verifies the risk-free rate equation (5) given in the

paper:

$$r_t^j = \beta + \mu - \gamma(\sigma^j)^2 + \lambda_t^j E \left[e^{-\gamma\omega^j Z_t} (e^{\omega^j Z_t} - 1) \right]$$

Let r^C denote the instantaneous expected return on the consumption claim, defined as drift in price plus the dividend plus the expectation of the jump, all as proportion of the price:

$$r_t^C = \mu + l^{-1} + \lambda_t E_v[(e^{\omega^j Z} - 1)] \quad (43)$$

Therefore, the instantaneous premium on an asset paying out consumption, conditional on no disaster is:

$$r_t^C - r_t = \gamma(\sigma^j)^2 - \lambda E_v[((e^{\omega^j Z})^{-\gamma} + 1) * (e^{\omega^j Z} - 1)] \quad (44)$$

Equilibrium Dividend Asset Price: Let D_t be the dividend process, which is a levered claim on consumption, $D_t = C_t^\phi$, where ϕ is the leverage parameter. Note that $\frac{dD}{dC} = \phi C^{\phi-1}$ and $\frac{d^2 D}{dC^2} = \phi(\phi - 1)C^{\phi-2}$. Again, by Ito's Lemma, and dropping j superscripts for clarity:

$$\begin{aligned} dD_t &= (\mu C_{t-} \frac{dD}{dC} + \frac{1}{2}(\sigma^2 C_{t-}^2) \frac{d^2 D}{dC^2}) dt + \sigma C_{t-} \frac{dD}{dC} dB_t + [(C_{t-} + (e^{\omega Z} - 1)C_{t-})^\phi - C_{t-}^\phi] dN_t \\ &= \phi(\mu + \frac{1}{2}\sigma^2(\phi - 1))D_{t-} dt + \sigma\phi D_{t-} dB_t + [(e^{\omega Z})^\phi - 1]D_{t-} dN_t \end{aligned} \quad (45)$$

From Duffie and Skiadas (1994), we have the following relationship between the state price

density, π_t , and the value function

$$\pi_t = \exp\left(\int_0^t f_v(C_s, V_s) ds\right) f_c(C_t, V_t) \quad (46)$$

With the state price density specified above, we can price any asset claim. Let $F(D_t)$ denote a claim on the dividends:

$$F(D_t) = E_t\left(\int_t^\infty D_s \frac{\pi_s}{\pi_t} ds\right) \quad (47)$$

By no-arbitrage conditions F must satisfy the follow (see Appendix A.2 of Wachter 2013), where $F_t = F(D_t)$:

$$\pi_t(DF_t) + F_t(D\pi_t) + D_t\pi_t + (\delta\pi_t)(\delta F_t) + (\lambda\pi_t)(\delta F_t) + \lambda\mathcal{J}(\pi_t F_t) = 0 \quad (48)$$

To solve for this process, we guess-and-verify a constant ratio between price and dividend. In other words, we conjecture that $F(D_t) = l_D D_t$, where clearly l_D is defined as the ratio. Then using the same arguments as above for the consumption claim, we can solve l_D through the relationship:

$$\phi\left(\mu + \frac{1}{2}\sigma^2(\phi - 1)\right) + l_D^{-1} - r = \phi\gamma\sigma^2 + \lambda E_\nu[e^{-\gamma\omega_{iz}} - 1] - \lambda E_\nu[e^{(\phi-\gamma)\omega_{iz}} - 1] \quad (49)$$

Using this relationship and the evolution of the state price density, the instantaneous return on the equity claim as the drift in price, plus dividends, plus the expectation of the

jump price can be written:

$$r^e = \phi\left(\mu + \frac{1}{2}\sigma^2(\phi - 1)\right) + l_D^{-1} + \lambda E_\nu[e^{\phi_i \omega_i z} - 1] \quad (50)$$

It follows from the risk free rate equation (42), that the instantaneous equity premium for the dividend-paying asset is:

$$r^e - r = \phi\gamma\sigma^2 + \lambda E_\nu[e^{-\gamma\omega_i z}(1 - e^{\phi_i \omega_i z}) + e^{\phi_i \omega_i z} - 1] \quad (51)$$

Similarly, using the solution for the equity price in equation (47) verifies the equity price diffusion given in equation (10) in the text:

$$\frac{dF_t^j}{F_{t-}^j} = \mu_{F,t}^j dt + \phi\sigma^j dB_t^j + g\sigma_\lambda \sqrt{\lambda_t^j} dB_{\lambda,t}^j + (e^{\phi^j \omega^j Z_t} - 1) dN_t^j,$$

We use this price process in combination with the dividend payout process to generate equity returns in the quantitative analysis.