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INTERSECTORAL SHIFTS AND AGGREGATE PRODUCTIVITY CHANGE

BY MICHAEL GROSSMAN AND VICTOR R. FUCHS*

This paper attempts to clarify the relationship between aggregate productivity and sector differentials and to provide quantitative estimates of possible effects. After reviewing the U.S. experience since 1929, the authors analyze the effects of shifts in sector employment shares on aggregate productivity. Computer simulations are used to identify the quantitative importance of these effects for secular trends and cyclical fluctuations. Given reasonable parameter values, the shift from industrial to service employment can have a major impact on aggregate productivity change for short-run fluctuations but not in the long-run.

INTRODUCTION

Economic growth nearly always has been associated with a rise in the service sector's share of total employment. Recently this shift has become the subject of renewed interest because of widespread concern about productivity. It has been suggested that the growth of the "low productivity" (service) sector imperils aggregate productivity and many observers ascribed the slow growth of output per manhour in 1969 and 1970 to that cause.

The purpose of this paper is to clarify the relationship between aggregate productivity and sector differentials and to provide some quantitative estimates of possible effects. The paper is divided into three sections. The first discusses some important conceptual distinctions. It also provides a brief review of U.S. experience since 1929 in order to indicate the order of magnitude of shifts in sector shares of output and employment and differential rates of change in sector productivity. The second presents a set of models to analyze the effects of shifts in sector shares on aggregate productivity. The third contains computer simulations that indicate the quantitative importance of these effects under various assumptions about sector differentials.

I. CONCEPTUAL AND EMPIRICAL BACKGROUND

(1) *Labor Productivity versus Total Factor Productivity*

It has long been recognized that changes in output per man or output per manhour provide only a partial (and sometimes misleading) indication of changes in efficiency.[1] To the extent that an increase in simple labor productivity is due to greater inputs of other factors of production (physical capital, human capital, intermediate inputs) there is no change in technological efficiency and no *a priori*

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[1] See, for instance, Kendrick (1961, pp. 6–8).

reason to view such an increase as a desirable social goal. This distinction is of particular importance when one makes sector comparisons. Since 1929, the industry-service differential in the growth of output per man has been about 1.3 percent per annum. Fuchs (1968, pp. 50–75) estimated that the differential in the rate of growth of output per total factor input was only about one-half as large. In the trend models that follow, we avoid the possible confusion introduced by changes in factor proportions by limiting inputs to a single homogeneous factor called "manhours." Thus, changes in output per manhour and in output per total factor input are identical by definition.

(2) *Shifts in Output Shares versus Employment Shares*

When one speaks of the shift to a "service economy," it is important to distinguish between shares of output and of employment. From the latter perspective, the U.S. has indeed become a "service economy." The service sector share of nonfarm employment (measured by persons engaged) has grown from 50.4 percent in 1929 to 60.2 percent in 1970.[2] From the point of view of output, however, there has been much less change. The service sector share of nonfarm output in constant (1958) dollars was 52.9 percent in 1929 and 50.7 percent in 1970. In the light of this historical record, the simulations to be presented mostly assume constancy of output shares; however, the effects of moderate shifts in sector shares of output are also shown.

(3) *Differences in Levels and Differences in Rates*

Sectors may differ with respect to levels of productivity (if output is valued at other than current year prices) and with respect to rates of change in productivity. Shifts in the relative importance of different sectors may, therefore, affect aggregate productivity because of the "level effect," the "rate effect," and the "interaction" between levels and rates. Two of the purposes of the simulations are to delineate clearly these separate effects and to indicate their probable relative importance.

It is worth noting that the importance of the "pure" level effect depends upon the weighting scheme used to calculate an index of aggregate real output. If current value weights are used (as in the Divisia indexes), then in the absence of a sector differential in rates of growth of productivity, shifts in sector shares of output and input cannot have any effect on aggregate productivity.[3] If, however, constant dollar output shares are used, then shifts in sector shares of output and input can affect aggregate productivity even in the absence of any subsequent differentials in sector productivity growth.

(4) *Secular versus Cyclical Implications*

Most of the discussion of the effects of sectoral shifts on aggregate productivity has been concerned with the secular or trend implications. It should be noted,

[2] The service sector includes trade; finance, insurance, and real estate; households and institutions; professional, personal, business, and repair services; and general government. Its shares of nonfarm employment and output were calculated from data in U.S. Department of Commerce, Office of Business Economics (various years).

[3] For a thorough discussion of Divisia indexes, see Richter (1966).

however, that there are implications for the cyclical behavior of aggregate output per manhour as well. These arise because of the tendency for output per manhour to fluctuate over the cycle more in the service sector than in industry (Fuchs 1968, pp. 173–177). At the same time, output and employment fluctuate more in industry than in services.

Output per manhour is more cyclically volatile in services because many service workers are more skilled than goods workers and because wages are more flexible in the service sector. The greater stability of service sector employment means that for equal cyclical declines in output, productivity will fall more in the service sector. Specifically, variations in short-run determinants of productivity such as "size of transaction" and "length of the production run"[4] will be greater in this sector. Comparisons of average cyclical changes (net of trend) of output per manhour in retail trade and manufacturing for the years 1947 through 1965 reveal a service industry differential of about two or three percent per annum (Fuchs, 1968, Chapter 7 and Appendix J).

II. Models of Aggregate Productivity Change

The purpose of this section is to develop formal models of the effects of intersectoral shifts on aggregate productivity change. As part of the development of these models, a basic formula for an index number of aggregate productivity is derived. The analysis in this section pertains to trend phenomena, but with only minor modifications, it provides the framework for both the secular and the cyclical simulations that are performed in the next section. Because agriculture is now such a small part of the total economy (3.2 percent of current dollar GNP and 4.3 percent of employment in 1970), future changes in this sector are not likely to have important implications for aggregate productivity in the decades ahead. Therefore, in the formal models that are presented, there are two sectors, industry or goods and services, and one homogeneous labor input.[5] The following notation and definitions are adopted:

QG_t = quantity of goods output in physical units in year t
QS_t = quantity of service output
pg_b = price of goods output in the base period
ps_b = price of service output
$XG_t = pg_bQG_t$ = goods output in constant (base period) dollars
$XS_t = ps_bQS_t$ = service output in constant dollars
$X_t = XG_t + XS_t$ = total output in constant dollars
$x_t = XG_t/X_t$ = goods sector's share of output in constant dollars
HG_t = manhours employed in the goods sector
HS_t = manhours employed in the service sector
$H_t = HG_t + HS_t$ = total manhours employed
$h_t = HG_t/H_t$ = goods sector's share of manhours employed

[4] For a discussion of these concepts, see Alchian (1959).

[5] In the next section, we comment briefly on the effects of shifts in real output and employment away from agriculture on aggregate productivity change. For the purpose of this discussion, the goods sector is equated to agriculture, and the service sector is equated to the rest of the economy. With this one exception, our analysis of the effects of intersectoral shifts is based on shifts between industry and services.

$AG_t = XG_t/HG_t$ = output per manhour in the goods sector in constant dollars

$AS_t = XS_t/HS_t$ = output per manhour in the service sector in constant dollars

$A_t = X_t/H_t$ = aggregate output per manhour in constant dollars

rg = constant annually compounded rate of increase in AG_t

rs = constant annually compounded rate of increase in AS_t

$k_t = AS_t/AG_t$ = output per manhour in the service sector relative to output per manhour in the goods sector in year t

$Z_{t,i} = Z_t/Z_i$ = index number of Z (any variable) in year t relative to Z in year i, where $t > i$

r_t = annually compounded rate of increase associated with the index of aggregate output per manhour $A_{t,i}$

An index of aggregate output per manhour in year t (the terminal year) relative to aggregate output per manhour in year i (the initial year) is

$$A_{t,i} = h_tAG_t + (1 - h_t)AS_t/[h_iAG_i + (1 - h_i)As_i]. \tag{1}$$

This equation can be rewritten as[6]

$$\begin{aligned} A_{t,i} = {} & x_iAG_{t,i} + (1 - x_i)AS_{t,i} + (h_t - h_i)(1 - k_i)/[h_i + (1 - h_i)k_i] \\ & + (h_t - h_i)[(AG_{t,i} - 1) - k_i(AS_{t,i} - 1)]/[h_i + (1 - h_i)k_i]. \end{aligned} \tag{2}$$

Equation (2) gives the basic formula for analyzing the effects of intersectoral shifts on aggregate productivity change. It decomposes the productivity index into

[6] By definition,

$$\begin{aligned} A_t - A_i = {} & (AG_t - AG_i)h_i + (AS_t - AS_i)(1 - h_i) \\ & + (h_t - h_i)(AG_i - AS_i) + (h_t - h_i)[(AG_t - AG_i) \\ & - (AS_t - AS_i)]. \end{aligned}$$

Therefore,

$$\begin{aligned} A_{t,i} = {} & (h_iAG_t)(1/A_i) + (1 - h_i)(AS_t)(1/A_i) \\ & + (h_t - h_i)(AG_i - AS_i)(1/A_i) \\ & + (h_t - h_i)[(AG_t - AG_i) - (AS_t - AS_i)](1/A_i). \end{aligned}$$

Multiplication of the first term on the right-hand side of the last equation by AG_i/AG_i yields

$$(h_iAG_i/A_i)AG_{t,i} = x_iAG_{t,i}.$$

Similarly, multiplication of the second term by AS_i/AS_i yields

$$(1 - h_i)(AS_i/A_i)AS_{t,i} = (1 - x_i)AS_{t,i}.$$

Multiplication of the third term by AG_i/AG_i yields

$$(h_t - h_i)(1 - k_i)/[h_i + (1 - h_i)k_i].$$

Multiplication of the fourth term by AG_i/AG_i yields

$$\begin{aligned} & (h_t - h_i)(AG_{t,i} - 1)/[h_i + (1 - h_i)k_i] \\ & - (h_t - h_i)[AG_t(1/AG_i) - k_i]/[h_i + (1 - h_i)k_i]. \end{aligned}$$

The second part of the last expression can be rewritten as

$$-(h_t - h_i)k_i(AS_{t,i} - 1)/[h_i + (1 - h_i)k_i].$$

three parts, which are termed the "rate," "level," and "interaction" components. The rationale for this terminology will become apparent by considering a few implications of the basic formula.

First, suppose that output per manhour in each sector remains constant between years i and t, so that $AG_{t,i}$ and $AS_{t,i}$ equal unity. In addition, suppose that output per manhour in the service sector in year i is less than output per manhour in the goods sector, so that k_i is less than unity. If the service sector's share of manhours increased between the two years, because of a shift in consumer's tastes in favor of service output, then $h_t - h_i$ would be negative. In this case, the aggregate productivity index would be less than unity, even though productivity within sectors remains constant. This decline in aggregate productivity can be traced to the shift in output and employment to the sector with a lower initial level of productivity.

Next, suppose that employment shares remain constant between years i and t. Then $h_t - h_i$ and the last two terms in equation (3) would equal zero. In this case, the aggregate productivity index would reduce to a weighted average of the sector productivity indexes, where the weights are initial year constant dollar output shares. Since annual rates of change in productivity are assumed to be constant in the model, the aggregate productivity index would be

$$A_{t,i} = x_i(1 + rg)^{t-i} + (1 - x_i)(1 + rs)^{t-i},$$

which immediately implies a value for r_t, the annually compounded rate of increase in aggregate productivity between year i and year t. This value would be essentially a weighted average of the rates of increase in productivity within sectors. If output and productivity rose faster in the goods sector than in the service sector, x_i would rise over time and so would the annual rate of increase in aggregate productivity.[7]

In the most general case, productivity grows at a differential rate between sectors and output and employment shares change over time. Therefore, the aggregate productivity index given by equations (1), (2), or (3) and its rate of change over time are influenced by the rate effect, the level effect, and the interaction between rates and levels.

It should be realized that equations (1) and (2) coincide with an index of aggregate productivity defined as the product of an aggregate output index in year t relative to year i and an aggregate input index in year i relative to year t:[8]

[7] If the income elasticities of demand for goods and services and the elasticity of substitution in consumption between goods and services all equaled unity, then a differential rate of growth in productivity between sectors would be consistent with no change in employment shares.

[8] By definition,

$$A_{t,i} = (X_t/H_t)(H_i/X_i),$$

or

$$A_{t,i} = (X_t/X_i)(H_i/H_t).$$

Since

$$(XG_t + XS_t)/X_i = x_iXG_{t,i} + (1 - x_i)XS_{t,i}$$

and since

$$(HG_i + HS_i)/H_t = h_tHG_{i,t} + (1 - h_t)HS_{i,t},$$

equation (5) immediately follows.

(3) $$A_{t,i} = X_{t,i}H_{i,t}$$

or

(4) $$A_{t,i} = [x_i XG_{t,i} + (1 - x_i)XS_{t,i}][h_t HG_{i,t} + (1 - h_t)HS_{i,t}].$$

In constructing the aggregate output index, the output indexes of goods and services are weighted by initial year constant dollar output shares. These weights are fixed in all indexes of aggregate output that have the same initial year in the denominator. In constructing the aggregate input index, on the other hand, the relevant weights are the terminal year employment shares. These weights obviously are not fixed in indexes with the same initial year but different terminal years.

It should also be realized that productivity can differ between sectors in year i only if the base period prices used to compute constant dollar output differ from the prices in year i. If prices in year i are used, k_i would equal unity, and the level component of the index number $A_{t,i}$ would equal zero.[9] Note, however, that indexes of the form $A_{t,j}$ $(j \neq i)$ constructed with year i prices as the base period prices might still be influenced by level effects.

It is revealing to consider the implications of equation (2) when output shares remain constant over time. In this case, the equation would reduce to[10]

(5) $$A_{t,i} = h_t AG_{t,i} + (1 - h_t)AS_{t,i}$$

According to equation (5), the index of aggregate output per manhour would become a weighted average of the indexes of output per manhour in each sector, where the weights are terminal year employment shares. Since there is a single homogeneous input in the model, these employment shares are equivalent to current (terminal) dollar expenditure or output shares.[11] Similarly, the rate of increase in aggregate productivity essentially would become a weighted average of the individual sector rates, where the weights are current dollar expenditure shares.

Equation (5) is particularly relevant to the historical experience of the U.S. economy because, as Section I indicated, real output shares were very stable between 1929 and 1970.[12] The equation shows that the effects of intersectoral

[9] Given constant returns to scale in production and perfect competition in product and factor markets,

$$pg_i QG_i/HG_i = ps_i QS_i/HS_i = w,$$

where w is the wage rate. Hence k_i would equal unity. In this case, it can be shown that the aggregate productivity index would coincide with the one suggested by Siegel (1952).

[10] Substitute $HG_{i,t} = AG_{t,i}/XG_{t,i}$ and $HS_{i,t} = AS_{t,i}/XS_{t,i}$ into equation (4) to obtain

$$A_{t,i} = [x_i XG_{t,i} + (1 - x_i)XS_{t,i}][h_t AG_{t,i}(1/XG_{t,i}) + (1 - h_t)AS_{t,i}(1/XS_{t,i})].$$

If output grows at the same rate in each sector, then $XG_{t,i} = XS_{t,i} = X_{t,i}$ and

$$A_{t,i} = (X_{t,i})(1/X_{t,i})[h_t AG_{t,i} + (1 - h_t)AS_{t,i}].$$

[11] If pg_t and ps_t are the prices of goods and service output, respectively, in year t, then the current dollar goods sector expenditure share would be $pg_t QG_t/(pg_t QG_t + ps_t QS_t)$. By definition,

$$pg_t QG_t/(pg_t QG_t + ps_t QS_t) = wHG_t/wH_t = h_t.$$

[12] Suppose productivity rose at a more rapid rate in the goods sector than in the service sector. Then output shares would remain constant if the differential rate of growth in productivity multiplied by the elasticity of substitution in consumption between goods and services equaled the difference between the income elasticities of services and goods. Obviously services would have to be more income elastic than goods to fulfill this condition.

shifts on aggregate productivity change would depend only on the differential rate of growth in productivity between sectors and the change in employment shares. Indeed the difference between indexes in which the number of years between the terminal and initial years at various points in time was constant would be given by

$$A_{t+i,t} - A_{t,i} = (h_{t+i} - h_t)[(1 + rg)^i - (1 + rs)^i].$$

When output shares remain constant, a Divisia index of aggregate productivity would coincide with the index given by equation (5). It has been shown that the aggregate productivity index described in this section is based on an aggregate output index that is constructed with constant dollar output shares. Unlike this index, the Divisia productivity index is based on an aggregate output index that is constructed with current dollar output or expenditure shares.[13] If constant dollar output shares are fixed over time, the two indexes coincide because output indexes of goods and services are identical. Therefore, the index of total output is unaffected by the set of weights used to combine the sector output indexes.

The trend simulations in the next section do not specifically deal with Divisia productivity indexes because the actual index of aggregate output per manhour published by the Bureau of Labor Statistics employs constant dollar output weights.[14] This does not mean that we advocate the use of the BLS index rather than the Divisia index on theoretical grounds. Instead, the purpose of the simulations is to show the forces that affect an index of labor productivity that is widely used by many persons. Several of the simulations assume that output shares are constant. Therefore, the results obtained in these simulations are identical to the results that would be obtained with a Divisia index.

III. Computer Simulations

Trend Simulations

To quantify the effects of differential rates of growth in sector productivities and intersectoral shifts in output and employment on aggregate productivity change, a set of computer simulations has been performed for a 50 year period of time. In order to perform this analysis, values for the following parameters must be specified:

(1) the constant annually compounded rates of increase in output per manhou: in the goods and service sectors (rg and rs, respectively);

(2) the ratio of output per manhour in the service sector to output per manhour in the goods sector in the first year of the period (k_1); and

(3) The goods sector's share of output in constant dollars in year 1 (x_1). In addition, it is necessary to specify the behavior of the goods sector's share of output (x_t) over time. This information enables one to compute all the variables in years t

[13] For discussions and applications of Divisia productivity indexes, see Solow (1957); Richter (1966); and Jorgenson and Griliches (1967).

[14] For a description of this index, see U.S. Department of Labor, Bureau of Labor Statistics (1966, pp. 175–179).

and i in equation (2), the basic formula for the index of aggregate productivity:[15]

$$AG_{t,i} = (1 + rg)^{t-i}$$
$$AS_{t,i} = (1 + rs)^{t-i}$$
$$k_t = k_1[(1 + rs)/(1 + rg)]^{t-1}$$
$$h_t = x_t k_t/(1 - x_t + x_t k_t).$$

TABLE 1

VALUES OF rg, rs, k_1 AND x_1 IN THE TREND SIMULATIONS

rg (percent per annum)	rs (percent per annum)	k_1	x_1
2.0	1.0	0.80	0.4
3.0	1.0	1.00	0.5
		1.33	0.6
		2.00	

Table 1 shows the two sets of values of rg and rs, the four values of k_1, and the three values of x_1 that were chosen to give 24 simulations of aggregate productivity change. The two sets of values of rg and rs are both consistent with the U.S.

TABLE 2

RATES OF CHANGE IN AGGREGATE PRODUCTIVITY[a] AND INDEXES OF AGGREGATE PRODUCTIVITY[b] DURING SELECTED DECADES

k_1	First Decade	Fifth Decade	First Decade	Fifth Decade	First Decade	Fifth Decade
	$x_1 = 0.4$		$x_1 = 0.5$		$x_1 = 0.6$	
	rg = 2 percent per annum, rs = 1 percent per annum					
0.80	1.9 (121.1)	1.4 (115.4)	1.4 (115.3)	1.3 (114.2)	0.9 (109.7)	1.3 (113.2)
1.00	1.6 (116.9)	1.5 (116.0)	1.5 (115.9)	1.4 (114.8)	1.4 (114.9)	1.3 (113.8)
1.33	1.1 (111.6)	1.6 (116.8)	1.6 (116.7)	1.5 (115.6)	2.0 (122.0)	1.4 (114.5)
2.00	0.5 (104.8)	1.7 (117.9)	1.7 (117.8)	1.6 (116.8)	2.8 (132.2)	1.5 (115.7)
	rg = 3 percent per annum, rs = 1 percent per annum					
0.80	2.4 (127.1)	1.7 (118.1)	1.8 (120.0)	1.5 (116.1)	1.3 (113.3)	1.4 (114.4)
1.00	2.1 (123.3)	1.8 (119.3)	1.9 (121.3)	1.6 (117.1)	1.8 (119.3)	1.4 (115.2)
1.33	1.7 (118.5)	1.9 (120.9)	2.1 (123.0)	1.7 (118.5)	2.5 (127.5)	1.5 (116.5)
2.00	1.2 (112.3)	2.1 (123.3)	2.3 (125.3)	1.9 (120.8)	3.4 (139.5)	1.7 (118.5)

[a] Percent per annum, annually compounded.
[b] In parentheses and multiplied by 100.

[15] The formula for h_t is obtained from the identity

$$HG_t/HS_t = (XG_t/XS_t)(AS_t/AG_t),$$

or

$$h_t/(1 - h_t) = [x_t/(1 - x_t)]k_t.$$

historical experience of a more rapid rate of growth in productivity in the goods sector than in the service sector. The functional relationship between x_t and t took the form

$$x_t = \hat{x} + b/t,$$

where b is a constant, $x_1 = \hat{x} + b$, and $\hat{x}$ is the asymptotic value of x_t as t approaches infinity. For x_1 equal to 0.5, b was assumed to be zero, so that x_t remains constant (and equal to 0.5) over time. This case closely corresponds with the actual trends in the U.S. For x_1 equal to 0.4, b was chosen to make x_{50} equal to 0.6. For x_1 equal to 0.6, b was chosen to make x_{50} equal to 0.4.

Table 2 presents annually compounded rates of growth in aggregate productivity during the first decade (year 1 through year 11) and the fifth decade (year 40 through year 50) of each simulation. The table also shows indexes of

TABLE 3

INDEXES OF AGGREGATE PRODUCTIVITY FOR SELECTED DECADES, RATE COMPONENT

(all numbers multiplied by 100)

k_1	First Decade	Fifth Decade	First Decade	Fifth Decade	First Decade	Fifth Decade
	$x_1 = 0.4$		$x_1 = 0.5$		$x_1 = 0.6$	
	rg = 2 percent per annum, rs = 1 percent per annum					
0.80	115.0	117.3	116.2	116.2	117.3	115.0
1.00	115.0	117.3	116.2	116.2	117.3	115.0
1.33	115.0	117.3	116.2	116.2	117.3	115.0
2.00	115.0	117.3	116.2	116.2	117.3	115.0
	rg = 3 percent per annum, rs = 1 percent per annum					
0.80	120.0	124.8	122.4	122.4	124.8	120.1
1.00	120.0	124.8	122.4	122.4	124.8	120.1
1.33	120.0	124.8	122.4	122.4	124.8	120.1
2.00	120.0	124.8	122.4	122.4	124.8	120.1

aggregate productivity for year 11 relative to year 1 and year 50 relative to year 40 in parentheses. Tables 3, 4, and 5 decompose these indexes into rate, level, and interaction components, respectively.

The 12 simulations that are based on a 1 percent per annum differential in rates of growth in sector productivity are the most consistent with the actual U.S. differential reported in Section I. According to these 12 simulations, the annual rate of growth in aggregate productivity over a ten year span could be as low as 0.5 percent per annum or as high as 2.8 percent. This range is due to alternative assumptions about relative levels of productivity in the initial period and shifts in the sector shares of output and employment. The very slow growth of 0.5 percent per annum would occur if the sector with a more rapid rate of growth in productivity also had a relatively low initial level of productivity and experienced an increase in its share of real output and employment. The very high rate of 2.8 percent per

TABLE 4

Indexes of Aggregate Productivity for Selected Decades, Level Component

(all numbers multiplied by 100)

k_1	First Decade	Fifth Decade	First Decade	Fifth Decade	First Decade	Fifth Decade
	$x_1 = 0.4$		$x_1 = 0.5$		$x_1 = 0.6$	
	rg = 2 percent per annum, rs = 1 percent per annum					
0.80	3.6	−1.4	−0.5	−1.4	−4.5	−1.3
1.00	0.0	−0.9	0.0	−0.9	0.0	−0.9
1.33	−4.5	−0.2	0.7	−0.2	6.2	−0.2
2.00	−10.3	0.7	1.7	0.8	15.0	0.8
	rg = 3 percent per annum, rs = 1 percent per annum					
0.80	3.1	−4.5	−1.1	−4.3	−5.0	−3.8
1.00	0.0	−3.6	0.0	−3.4	0.0	−3.1
1.33	−3.9	−2.2	1.4	−2.2	6.9	−2.1
2.00	−8.9	−0.3	3.4	−0.3	17.0	−0.3

annum would be realized if output and employment shares fell in the sector with a low initial level of productivity but a more rapidly growing rate of increase in productivity. The latter situation was experienced in the U.S. in agriculture, and the shift of real output away from agriculture undoubtedly made a significant contribution to the growth of aggregate productivity in the past.

TABLE 5

Indexes of Aggregate Productivity for Selected Decades, Interaction Component

(all numbers multiplied by 100)

k_1	First Decade	Fifth Decade	First Decade	Fifth Decade	First Decade	Fifth Decade
	$x_1 = 0.4$		$x_1 = 0.5$		$x_1 = 0.6$	
	rg = 2 percent per annum, rs = 1 percent per annum					
0.80	2.5	−0.5	−0.4	−0.5	−3.1	−0.5
1.00	1.8	−0.4	−0.3	−0.4	−2.4	−0.4
1.33	1.1	−0.3	−0.2	−0.3	−1.5	−0.3
2.00	0.1	−0.1	0.0	−0.2	−0.1	−0.2
	rg = 3 percent per annum, rs = 1 percent per annum					
0.80	4.0	−2.2	−1.4	−2.1	−6.5	−1.8
1.00	3.3	−2.0	−1.2	−1.9	−5.6	−1.7
1.33	2.4	−1.7	−0.9	−1.6	−4.2	−1.5
2.00	1.2	−1.2	−0.5	−1.2	−2.3	−1.2

The index numbers of aggregate productivity associated with the 2.8 and 0.5 percent per annum rates of growth can be decomposed as follows:

	$r = 2.8$	$r = 0.5$	*Difference*
Index	132.2	104.8	27.4
Rate	117.3	115.0	2.3
Level	15.0	−10.3	25.3
Interaction	−0.1	0.1	−0.2

Almost all of the 27.4 point difference in the two indexes is due to the difference in the level components. A similar conclusion emerges if one compares rates of growth in productivity for any given simulation. The greatest difference between such rates is 1.3 percent per annum. This difference is based on the 2.8 percent per annum rate of increase in the first decade and the 1.5 percent increase in the fifth decade of the simulation in which x_1 equals 0.6 and k_1 equals 2. The index numbers associated with these two rates can be decomposed as follows:

	$r = 2.8$	$r = 1.3$	*Difference*
Index	132.2	116.4	15.8
Rate	117.3	115.1	2.2
Level	15.0	1.4	13.6
Interaction	−0.1	−0.1	0.0

The level component obviously dominates the comparison of these two indexes.

If one assumes no shift in sector shares of output and a differential rate of growth in productivity of 1 percent per annum, then the range of possible rates of growth in aggregate productivity is reduced considerably. Moreover, the change over time in the course of aggregate productivity for any assumed initial relative level is very small—the rate of change in years 40 through 50 being only 0.1 percent per annum smaller than in years 1 through 11. It should be noted that the assumption of constant output shares coincides with the U.S. historical experience. This assumption also generates an aggregate productivity index that is equivalent to a Divisia index.

As demonstrated in Section II, if output shares are fixed, the aggregate productivity index takes the form

$$A_{t,i} = h_t AG_{t,i} + (1 - h_t) AS_{t,i}.$$

For a given value of k_1, differences in this index for different decades are due solely to the fall in the goods sector's employment share over time. If one assumes that k_1 equals unity,[16] then the goods sector's share of employment would have been 50 percent in year 1 and 38 percent in year 50. Therefore, a relatively large change in employment shares would produce a relatively small change in the rate of growth in aggregate productivity (from 1.5 percent per annum in the first decade to 1.4 percent per annum in the fifth decade.)

Not surprisingly, the average rate of change in aggregate productivity rises when productivity in the goods sector grows at 3 percent per annum instead of at 2 percent per annum. It is striking, however, that the *range* of possible outcomes remains fairly stable. If rg equals 3 percent per annum, the annual rates of growth

[16] The data in Section I imply that k equaled 1.10 in 1929.

in aggregate productivity over a 10 year span range from 1.2 percent to 3.4 percent. Tables 3–5 show that differences in the index numbers associated with these rates are almost completely "explained" by differences in their level components.

The simulations in which output shares remain constant and productivity in the goods sector rises by 3 percent per annum exhibit larger declines in the rate of change in aggregate productivity than those in which productivity rises by 2 percent in the goods sector. If k equals 1 and rg equals 3 percent, r falls by 0.3 percent between the first and fifth decades. In this simulation, the goods sector's share of total employment would fall from 50 percent in year 1 to 28 percent in year 50. These figures show that when the aggregate productivity index is simply a weighted average of the sector productivity indexes, substantial changes in weights are necessary to produce significant changes in the rate of growth of productivity. Such shifts would not occur unless the differential rate of growth in productivity between sectors were much larger than the U.S. historical experience indicates.

In summary, the trend simulations reveal that a given set of rates of growth in productivity within sectors is consistent with a fairly wide range of rates of change in aggregate productivity. By altering the assumptions about the initial levels of productivity and the behavior of output and employment shares over time, different rates of growth in aggregate productivity can be generated. Moreover, for any assumed initial levels of productivity, the course of aggregate productivity change can vary substantially over time. When large differences occur, they have been attributed almost entirely to a level effect rather than to rate or interaction effects. If, however, output shares are held constant, the variation in potential rates of growth in productivity is reduced considerably. In addition, for given initial productivity levels, productivity change will be fairly constant over time unless employment shares shift dramatically. If one abstracts from the decline in the agricultural sector, then the actual experience in the U.S. suggests that intersectoral shifts could not have had a major impact on aggregate productivity change.

The importance of the level effect with regard to shifts away from agriculture and the unimportance of this effect with regard to shifts from industry to services are demonstrated in a recent study by Nordhaus (1972). He finds that, for the years 1948 to 1971, reductions in agriculture's shares of output and employment played a substantial role in the growth of labor productivity. He also finds that, due to the decline in importance of the agricultural sector, the level effect associated with the shift out of this sector decreased in magnitude over the period. This explains the slower rate of growth in productivity during the latter part of the period. His simulations reveal almost no further reduction in the rate of growth in productivity for the years 1972 to 1980. These simulations, like ours, show that, when output shares remain constant, the level effect becomes unimportant.[17]

[17] It should be noted that Nordhaus concludes from his simulations: "As the movement toward low-productivity sectors continues, we should expect a further productivity deceleration (1972, p. 527)." This conclusion is *not* supported by the simulations. The predicted rates of productivity growth cyclically corrected are 2.2 percent per annum for the years 1965 to 1971, 2.2 percent for 1971 to 1976, and 2.1 percent for 1976 to 1980 (Nordhaus, 1972, Table 16).

Cyclical Simulations

Section I indicated that cyclical fluctuations in productivity in the service sector probably exceed cyclical fluctuations in the goods sector. To quantify the effect of this differential on aggregate productivity change during the course of a business cycle, computer simulations of business cycles have been performed. It is worth repeating that the sources of this differential are the greater educational level of service sector workers and the greater flexibility in wages in the service sector. The basic formula for aggregate productivity was developed for a single homogeneous input. It can, however, be applied to the cyclical analysis provided it is interpreted as an index of output per manhour rather than as a more refined measure of productivity such as output per total factor input.

The computer simulations are based on the fundamental proposition that the amplitude of fluctuation in an expansion net of trend equals the amplitude of fluctuation in the preceding contraction. Long-run values are specified for the goods sector's share of real output (x) and the ratio of output per manhour in service sector to output per manhour in the goods sector (k). These two variables determine the long-run share of employment in the goods sector (h), exactly as in the trend simulations. Long-run values are those that would be observed in the absence of a business cycle. A variable is assumed to equal its long-run value at the midpoint of an expansion or a contraction. Since output is more volatile in the goods sector than in the service sector during the course of a cycle, x behaves in a procyclical manner. In all cyclical simulations, the long-run value of x was equal to 0.5, but allowance was made for the procyclical behavior of this variable.

In addition to choosing values for k, values must be chosen for the rates of change in output per manhour in expansions and contractions in the goods and service sectors (rg and rs) and the duration of expansions and contractions. Table 6

TABLE 6

VALUES OF rg, rs, k, AND DURATION OF EXPANSIONS AND CONTRACTIONS IN THE CYCLICAL SIMULATIONS

rg (percent per annum)	rs (percent per annum)	Duration of Expansion[a] (in years)	k
2.0	4.0	2	0.67
		3	0.80
			1.00
			1.33

[a] Duration of contraction is always equal to one year.

shows the set of values of rg and rs, the two cycle lengths, and the four values of k that were employed to generate data for 8 hypothetical business cycles. The values of rg and rs are based on those reported by Fuchs (1968, Chapter 7) for cyclical fluctuations in output per manhour in manufacturing and retail trade over reference cycles. Expansions are assumed to last longer than contractions because this is a basic characteristic of business cycles in the U.S. The three year expansion is the most representative of the post World War II experience.

It is important to realize that a given value of *rg* or *rs* in Table 6 equals the annual rate of change in productivity in an expansion minus the rate of change in a contraction.[18] Since expansions last longer than contractions and since the simulations assume equal amplitudes in the two phases of the cycle, the absolute value of the annual rate of decrease in productivity during a contraction must *exceed* the annual rate of increase in an expansion. In particular, if e is the length of the expansion and c is the length of the contraction, then

$$rge = [c/(c + e)]rg$$

$$rgc = -[e/(c + e)]rg,$$

where rge and rgc are, respectively, the "pure" cyclical rates of change in goods productivity in expansions and contractions. These formulas and the use of continuous, rather than annual, compounding insure that amplitudes will be the same in expansions and contractions. Together with the long-run values of x and k and the behavior of x over the cycle, they provide the necessary information to carry out the cyclical simulations.[19]

Table 7 presents the annual rate of change in aggregate output per manhour in expansions and contractions for each simulation. It also shows the index of aggregate output per manhour at the peak of the cycle relative to the trough and its rate and level component. The interaction component is extremely small in all simulations and is not shown.

The main result of these simulations is that for given values of *rg*, *rs*, and the duration of the cycle, variations in the long-run level of relative productivity cause substantial variations in the rate of change in aggregate output per manhour.[20] Since real output shares are the same in simulations with identical values of *rg*, *rs*, and c and since interaction effects are extremely small, these differences are entirely due to differences in the level effect. It is striking that if k is as small as 0.67, then the rate of change in aggregate output per manhour *equals* the rate of

[18] Let me be the observed rate of change in a series in expansions and let mc be the observed rate of change in contractions. If the trend component of the series is independent of the stage of the cycle, then

$$me = r + re$$

$$mc = r + rc,$$

where r is the secular rate of growth, re is the "pure" cyclical rate of increase in expansions, and rc is the pure cyclical rate of decrease in contractions. Subtracting mc from me, one obtains

$$me - mc = re - rc,$$

where $-rc$ is positive. The cyclical simulations assume that $me - mc$ equals 2 percent per annum for output per manhour in the goods sector and equals 4 percent per annum for output per manhour in the service sector.

[19] Fuchs's evidence (1968, p. 265) suggests that the annual rate of change in x in expansions minus the annual rate of change in contractions is 5 percent (*not* 5 percentage points). This value was used in all simulations.

[20] We also performed simulations in which *rg* equaled 3 percent and *rs* equaled 6 percent. The results of these simulations (not shown) support the conclusion reached in the text with regard to the effects of variations in the long-run level of relative productivity.

TABLE 7

CYCLICAL RATES OF CHANGE IN AGGREGATE OUTPUT PER MANHOUR[a] AND INDEXES OF AGGREGATE OUTPUT PER MANHOUR AND COMPONENTS, PEAK RELATIVE TO TROUGH[b]

k	Duration of Expansion (in years) 2	3
rg = 2 percent per annum, rs = 4 percent per annum		
	Rates of Change in Expansions and Contractions[c]	
0.67	4.2	4.0
0.80	3.6	3.6
1.00	3.0	3.2
1.33	2.1	2.0
	Indexes of Aggregate Output per Manhour	
0.67	102.8	103.2
0.80	102.5	102.8
1.00	102.0	102.3
1.33	101.4	101.6
	Rate Components	
0.67	102.0	102.3
0.80	102.0	102.3
1.00	102.0	102.3
1.33	102.0	102.3
	Level Components	
0.67	0.8	0.9
0.80	0.5	0.5
1.00	0.0	0.0
1.33	−0.6	−0.6

[a] Percent per annum, continuous compounding.
[b] All index numbers multiplied by 100. Interaction component not shown.
[c] Rate of change in expansion minus rate of change in contraction.

change in output per manhour in the service sector. Put differently, the cyclical behavior of aggregate productivity is dominated completely by the behavior of its most cyclically volatile component. The reverse occurs if k is as large as 1.33.

The sizable level effects in the cyclical simulations can be attributed to the large shifts in output and employment shares during the course of the cycle. If, for example, k equals 0.67 and c equals 3 years, then the goods sector's share of real output would be 49 percent at the trough of the cycle and 52 percent at the peak. In the same simulation, the goods sector's share of employment would be 39 percent at the trough of the cycle and 41 percent at the peak. These shifts are extremely large relative to those that occur in the trend simulations in a three year time period.[21]

The value of k in the simulation just described is consistent with the value of this variable in 1970 indicated in Section I. Because of the more rapid secular

[21] In the trend simulation in which k_1 equals 1.0, rg equals 2 percent, rs equals 1 percent, and x is constant, the goods sector's share of employment falls from 50 percent to 38 percent over a 50 year period. In simulations in which the goods sector's share of output varies, it is only allowed to change from 60 percent to 40 percent (or vice versa) over a 50 year period.

rate of growth in productivity in the goods sector than in the service sector, k has fallen over time. This decline in k has probably increased the cyclical rate of change in aggregate output per manhour in recent business cycles[22] and may explain a substantial part of the slow growth in output per manhour during the most recent recession.

SUMMARY AND CONCLUSION

The purpose of this paper has been to examine and quantify the effects of sector differentials in productivity growth on aggregate productivity change. These differentials cause output and employment shares to shift and may have a substantial impact on aggregate productivity, especially if sectors also differ with regard to their initial level of productivity. A formal model of aggregate productivity change has been developed, and it has been shown that an index number of aggregate productivity can be decomposed into rate, level, and interaction components. Secular and cyclical computer simulations have been performed to quantify the relative importance of various effects. Given reasonable parameter values, the simulations reveal that shifts between industry and services can have a major impact on aggregate productivity change in the short-run but not in the long-run.

This finding should relieve those who have been worried about the slow rate of growth in output per manhour in the past few years. Our results show that the positive secular differential between the rates of growth in productivity in the goods and service sectors has probably caused the cyclical rate of change in aggregate output per manhour to rise. Output per manhour probably now falls at a more rapid rate in a recession (or rises at a slower rate) than it would have in past recessions. Provided the secular differential in productivity growth continues, output per manhour will fall at an even faster rate in future recessions. But if "what goes down must come up" is the rule of the business cycle, output per manhour will also rise at a more rapid rate in future expansions. Consequently, shifts between industry and services should continue to play the same relatively minor role in long-run productivity growth in the future that they have played in the past.

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[22] The term probably is used because cyclical fluctuations in output have declined over time. Taken by itself, this factor would reduce cyclical fluctuations in output per manhour.

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