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5 Imperfect Knowledge, Inflation Expectations, and Monetary Policy

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5.1 Introduction

Rational expectations provide an elegant and powerful framework that has come to dominate thinking about the dynamic structure of the economy and econometric policy evaluation over the past thirty years. This success has spurred further examination of the strong information assumptions implicit in many of its applications. Thomas Sargent (1993) concludes that "rational expectations models impute much *more* knowledge to the agents within the model . . . than is possessed by an econometrician, who faces estimation and inference problems that the agents in the model have somehow solved" (3, emphasis in original).[1] Researchers have

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1. Missing from such models, as Benjamin Friedman (1979) points out, "is a clear outline of the way in which economic agents derive the knowledge which they then use to formulate expectations." To be sure, this does not constitute a criticism of the traditional use of the concept of "rationality" as reflecting the optimal use of information in the formation of expectations, taking into account an agent's objectives and resource constraints. The difficulty is that in Muth's (1961) original formulation, rational expectations are not optimizing in that sense. Thus, the issue is not that the rational-expectations concept reflects too much rationality but

proposed refinements to rational expectations that respect the principle that agents use information efficiently in forming expectations, but nonetheless recognize the limits to and costs of information processing and cognitive constraints that influence the expectations-formation process (Sargent 1999; Evans and Honkapohja 2001; Sims 2003).

In this study, we allow for a form of imperfect knowledge in which economic agents rely on an adaptive learning technology to form expectations. This form of learning represents a relatively modest deviation from rational expectations that nests the latter as a limiting case. We show that the resulting process of perpetual learning introduces an additional layer of interaction between monetary policy and economic outcomes that has important implications for macroeconomic dynamics and for monetary policy design. As we illustrate, monetary policies that would be efficient under rational expectations can perform poorly when knowledge is imperfect. In particular, with imperfect knowledge, policies that fail to maintain tight control over inflation are prone to episodes in which the public's expectations of inflation become uncoupled from the policy objective. The presence of this imperfection makes stabilization policy more difficult than would appear under rational expectations and highlights the value of effectively communicating a central bank's inflation objective and of continued vigilance against inflation in anchoring inflation expectations and fostering macroeconomic stability.

In this paper, we investigate the macroeconomic implications of a process of "perpetual learning." Our work builds on the extensive literature relating rational expectations to learning and the adaptive formation of expectations (Bray 1982; Bray and Savin 1984; Marcet and Sargent 1989; Woodford 1990; Bullard and Mitra 2002). A key finding in this literature is that under certain conditions an economy with learning converges to the rational-expectations equilibrium (Townsend 1978; Bray 1982, 1983; Blume and Easley 1982). However, until agents have accumulated sufficient knowledge about the economy, economic outcomes during the transition depend on the adaptive learning process (Lucas 1986). Moreover, in a changing economic environment, agents are constantly learning, and their beliefs converge not to a fixed rational-expectations equilibrium but to an ergodic distribution around it (Sargent 1999; Evans and Honkapohja 2001).[2]

rather that it imposes too little rationality in the expectations formation process. For example, as Sims (2003) has pointed out, optimal information processing subject to a finite cognitive capacity may result in fundamentally different processes for the formation of expectations from those implied by rational expectations. To acknowledge this terminological tension, Simon (1978) suggested that a less misleading term for Muth's concept would be "model consistent" expectations (2).

2. Our work also draws on some other strands of the literature related to learning, estimation, and policy design. One such strand has examined the formation of inflation expectations when the policymaker's objective may be unknown or uncertain—for example, during a transition following a shift in policy regime (Taylor 1975; Bomfim et al. 1997; Erceg and Levin 2003; Koz-

As a laboratory for our experiment, we employ a simple linear model of the U.S. economy with characteristics similar to more elaborate models frequently used to study optimal monetary policy. We assume that economic agents know the correct structure of the economy and form expectations accordingly. But, rather than endowing them with complete knowledge of the parameters of these functions—as would be required by imposing the rational-expectations assumption—we posit that economic agents rely on finite memory least squares estimation to update these parameter estimates. This setting conveniently nests rational expectations as the limiting case corresponding to infinite memory least squares estimation and allows varying degrees of imperfection in expectations formation to be characterized by variation in a single model parameter.

We find that even marginal deviations from rational expectations in the direction of imperfect knowledge can have economically important effects on the stochastic behavior of our economy and policy evaluation. An interesting feature of the model is that the interaction of learning and control creates rich nonlinear dynamics that can potentially explain both the shifting parameter structure of linear reduced-form characterizations of the economy and the appearance of shifting policy objectives or inflation targets. For example, sequences of policy errors or inflationary shocks, such as were experienced during the 1970s, could give rise to stagflationary episodes that do not arise under rational expectations with perfect knowledge.

Indeed, the critical role of the formation of inflation expectations for understanding the successes and failures of monetary policy is a dimension of policy that has often been cited by policymakers over the past two decades but that has received much less attention in formal econometric policy evaluations. An important example is the contrast between the stubborn persistence of inflation expectations during the 1970s, when policy placed relatively greater attention on countercyclical concerns, and the much-improved stability in both inflation and inflation expectations following the renewed emphasis on price stability in 1979. In explaining the rationale for this shift in emphasis in 1979, Federal Reserve Chairman Volcker highlighted the importance of learning in shaping the inflation expectations formation process.[3]

icki and Tinsley 2001; Tetlow and von zur Muehlen 2001). Another strand has considered how policymaker uncertainty about the structure of the economy influences policy choices and economic dynamics (Balvers and Cosimano 1994; Wieland 1998; Sargent 1999; and others). Finally, our work relates to explorations of alternative approaches for modeling aggregate inflation expectations, such as Ball (2000), Carroll (2003), and Mankiw and Reis (2002).

3. Indeed, we would argue that the shift in emphasis toward greater focus on inflation was itself influenced by the recognition of the importance of facilitating the formation of stable inflation expectations—which had been insufficiently appreciated earlier during the 1970s. See Orphanides (2004) for a more detailed description of the policy discussion at the time and the nature of the improvement in monetary policy since 1979. See also Christiano and Gust (2000) and Sargent (1999) for alternative explanations of the rise in inflation during the 1960s and 1970s.

> It is not necessary to recite all the details of the long series of events that have culminated in the serious inflationary environment that we are now experiencing. An entire generation of young adults has grown up since the mid-1960's knowing only inflation, indeed an inflation that has seemed to accelerate inexorably. In the circumstances, it is hardly surprising that many citizens have begun to wonder whether it is realistic to anticipate a return to general price stability, and have begun to change their behavior accordingly. Inflation feeds in part on itself, so part of the job of returning to a more stable and more productive economy must be to break the grip of inflationary expectations. (Volcker 1979, 888)

This historical episode is a clear example of inflation expectations becoming uncoupled from the intended policy objective and illustrates the point that the design of monetary policy must account for the influence of policy on expectations.

We find that policies designed to be efficient under rational expectations can perform very poorly when knowledge is imperfect. This deterioration in performance is particularly severe when policymakers put a high weight on stabilizing real economic activity relative to price stability. Our analysis yields two conclusions for the conduct of monetary policy when knowledge is imperfect. First, policies that emphasize tight inflation control can facilitate learning and provide better guidance for the formation of inflation expectations. Second, effective communication of an explicit numerical inflation target can help focus inflation expectations and thereby reduce the costs associated with imperfect knowledge. Policies that combine vigilance against inflation with an explicit numerical inflation target mitigate the negative influence of imperfect knowledge on economic stabilization and yield superior macroeconomic performance. Thus, our findings provide analytical support for monetary policy frameworks that emphasize the primacy of price stability as an operational policy objective—for example, the inflation-targeting approach discussed by Bernanke and Mishkin (1997) and adopted by several central banks over the past decade or so.

5.2 The Model Economy

We consider a stylized model that gives rise to a nontrivial inflation-output variability trade-off and in which a simple one-parameter policy rule represents optimal monetary policy under rational expectations.[4] In this section, we describe the model specification for inflation and output and the central bank's optimization problem; in the next two sections, we take up the formation of expectations by private agents.

4. Since its introduction by Taylor (1979), the practice of analyzing monetary policy rules using such an inflation-output variability trade-off has been adopted in a large number of academic and policy studies.

Inflation is determined by a modified Lucas supply function that allows for some intrinsic inflation persistence,

$$(1) \qquad \pi_{t+1} = \phi\pi^e_{t+1} + (1 - \phi)\pi_t + \alpha y_{t+1} + e_{t+1}, \qquad e \sim \text{i.i.d.}(0, \sigma^2_e),$$

where π denotes the inflation rate, π^e is the private agents' expected inflation rate based on time t information, i.i.d. indicates "independently and identically distributed," y is the output gap, $\phi \in (0, 1)$, $\alpha > 0$, and e is a serially uncorrelated innovation. As discussed by Clark, Goodhart, and Huang (1999) and Lengwiler and Orphanides (2002), this specification incorporates an important role for inflation expectations for determining inflation outcomes while also allowing for some inflation persistence that is necessary for the model to yield a nontrivial inflation-output gap variability trade-off.[5]

We assume that the policymaker can set policy during period t so as to determine the intended level of the output gap for period $t + 1$, x_t, subject to a control error, u_{t+1},

$$(2) \qquad y_{t+1} = x_t + u_{t+1} \qquad u \sim \text{i.i.d.}(0, \sigma^2_u).$$

This is equivalent to assuming that the intended output gap for period t + 1 is determined by the real rate gap set during period t, $x_t = -\xi(r_t - r^*)$, where r is the short-term real interest rate and r^* is the equilibrium real rate.[6] As will become clear, with this assumption the model has the property that under perfect knowledge both the optimal policy rule and the optimal inflation-forecast rule can be written in terms of a single state variable, the lagged inflation rate. This facilitates our analysis. Inflation expectations are fundamentally anchored by monetary policy, while output expectations are anchored by views of aggregate supply that are presumably less influenced by monetary policy. For this reason, we focus on the interaction between monetary policy and inflation expectations.

The central bank's objective is to design a policy rule that minimizes the loss, denoted by $\mathcal{L}$, equal to the weighted average of the asymptotic variances of the output gap and of deviations of inflation from the target rate,

$$(3) \qquad \mathcal{L} = (1 - \omega)\text{Var}(y) + \omega\text{Var}(\pi - \pi^*),$$

where Var(z) denotes the unconditional variance of variable z, and $\omega \in (0, 1]$ is the relative weight on inflation stabilization. This completes the description of the structure of the model economy, with the exception of the expectations formation process that we examine in detail below.

5. We have also examined the "New Keynesian" variant of the Phillips curve studied by Galí and Gertler (1999) and others, which also allows for some intrinsic inflation inertia. As we report in section 5.6, our main findings are not sensitive to this alternative.

6. Note, however, that this abstracts from the important complications associated with the real-time measurement of the output gap and the equilibrium real interest rate for formulating the policy rule. See Orphanides (2003a), Laubach and Williams (2003), and Orphanides and Williams (2002) for analyses of these issues.

5.3 The Perfect-Knowledge Benchmark

We begin by considering the "textbook" case of rational expectations with perfect knowledge in which private agents know both the structure of the economy and the central bank's policy. In this case, expectations are rational in that they are consistent with the true data-generating process of the economy (the model). In the next section, we use the resulting equilibrium solution as a "perfect-knowledge" benchmark against which we compare outcomes under imperfect knowledge, in which case agents do not know the structural parameters of the model but instead must form expectations based on estimated forecasting models.

Under the assumption of perfect knowledge, both the evolution of the economy and optimal monetary policy can be expressed in terms of two variables, the current inflation rate and its target level. These variables determine the formation of expectations and the policy choice, which, together with serially uncorrelated shocks, determine output and inflation in period $t + 1$. Specifically, we can write the monetary policy rule in terms of the inflation gap,

$$x_t = -\theta(\pi_t - \pi^*), \tag{4}$$

where $\theta > 0$ measures the responsiveness of the intended output gap to the inflation gap.

Given this monetary policy rule, inflation expectations are

$$\pi^e_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + \frac{1-\phi-\alpha\theta}{1-\phi}\pi_t. \tag{5}$$

Inflation expectations depend on the current level of inflation, the inflation target, and the parameter θ measuring the central bank's responsiveness to the inflation gap. Substituting this expression for expected inflation into equation (1) yields the rational-expectations solution for inflation for a given monetary policy,

$$\pi_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + \left(1 - \frac{\alpha\theta}{1-\phi}\right)\pi_t + e_{t+1} + \alpha u_{t+1}. \tag{6}$$

One noteworthy feature of this solution is that the first-order autocorrelation of the inflation rate, given by $1 - ([\alpha\theta]/[1 - \phi])$, is decreasing in θ and is invariant to the value of π^*. Note that the rational-expectations solution can also be written in terms of the "inflation expectations gap"—the difference between inflation expectations for period $t + 1$ from the inflation target, $\pi^e_{t+1} - \pi^*$,

$$\pi^e_{t+1} - \pi^* = \frac{1-\phi-\alpha\theta}{1-\phi}(\pi_t - \pi^*). \tag{7}$$

Equations (4) and (5) close the perfect-knowledge benchmark model.

5.3.1 Optimal Monetary Policy under Perfect Knowledge

For the economy with perfect knowledge, the optimal monetary policy, θ^P, can be obtained in closed form and is given by[7]

$$(8) \quad \theta^P = \frac{\omega}{2(1-\omega)}\left(-\frac{\alpha}{1-\phi} + \sqrt{\left(\frac{\alpha}{1-\phi}\right)^2 + \frac{4(1-\omega)}{\omega}}\right) \text{ for } 0 < \omega < 1.$$

In the limit, when ω equals unity (that is, when the policymaker is not at all concerned with output stability), the policymaker sets the real interest rate so that inflation is expected to return to its target in the next period. The optimal policy in the case $\omega = 1$ is given by $\theta^P = (1-\phi)/\alpha$, and the irreducible variance of inflation, owing to unpredictable output and inflation innovations, equals $\sigma_e^2 + \alpha^2\sigma_u^2$. More generally, the optimal value of θ depends positively on the ratio $(1-\phi)/\alpha$, and the parameters α and ϕ enter only in terms of this ratio. In particular, the optimal policy response is larger the greater the degree of intrinsic inertia in inflation, measured by $1-\phi$.

The greater the central bank's weight on inflation stabilization, the greater is the responsiveness to the inflation gap and the smaller the first-order autocorrelation in inflation. Differentiating equation (8) shows that the policy responsiveness to the inflation gap is increasing in ω, the weight the central bank places on inflation stabilization. As a result, the autocorrelation of inflation is decreasing in ω, with a limiting value approaching unity when ω approaches zero, and zero when ω equals 1. That is, if the central bank cares only about output stabilization, the inflation rate becomes a random walk, while if the central bank cares only about inflation stabilization, the inflation rate displays no serial correlation. And, as noted, this model yields a nontrivial monotonic trade-off between the variability of inflation and the output gap for all values of $\omega \in (0, 1]$. These results are illustrated in figure 5.1. Panel A of the figure shows the variability trade-off described by optimal policies for values of ω between zero and 1. Panel B plots the optimal values of θ against ω.

5.4 Imperfect Knowledge

As the perfect-knowledge solution shows, private inflation forecasts depend on knowledge of the structural model parameters and of policymaker

7. The optimal policy can be described in terms of the Euler equation that relates the intended output gap to the inflation rate and the intended output gap expected in the next period:

$$x_t = E_{t-1}\left(x_{t+1} - \frac{\omega}{1-\omega}\frac{\alpha}{1-\phi}\pi_{t+1}\right).$$

Under the assumption of serially uncorrelated shocks, the solution simplifies to the expression given in the text.

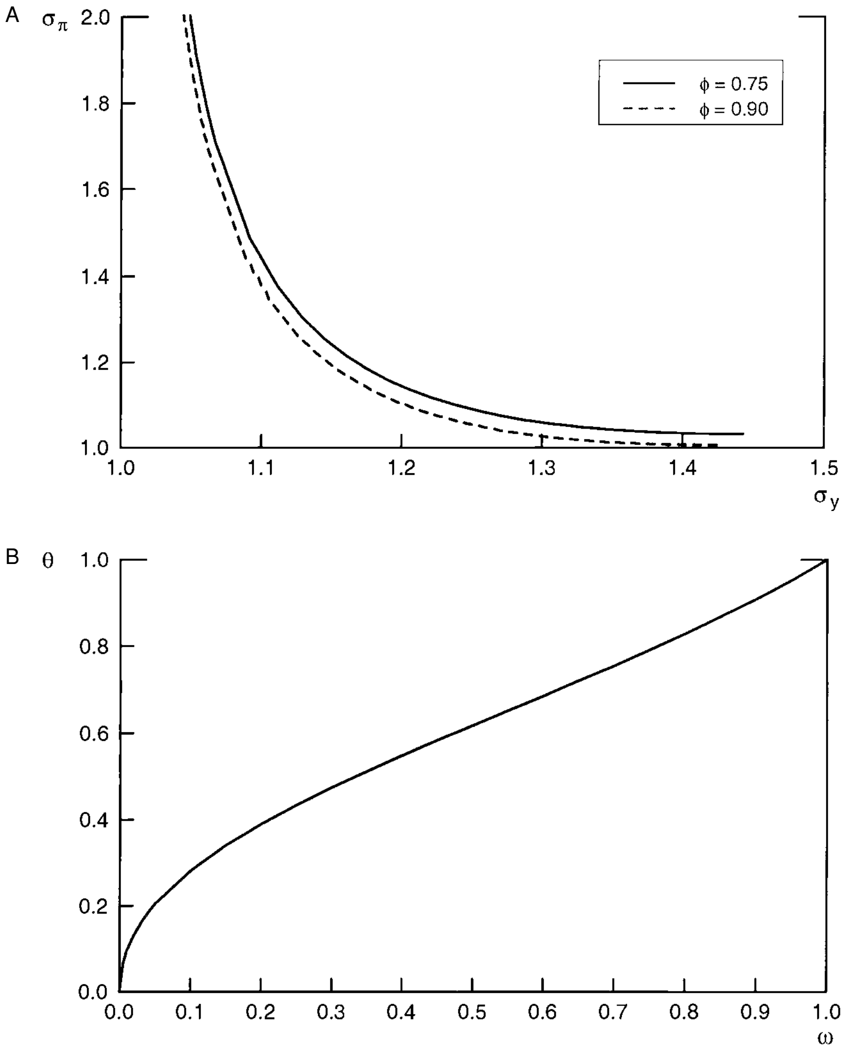

Fig. 5.1 ***A,* Efficient policy frontier with perfect knowledge; *B,* Optimal policy response to inflation**

Notes: The top panel shows the efficient policy frontier corresponding to optimal policies for different values of the relative preference for inflation stabilization ω, for the two specified parameterizations of α and ϕ. The bottom panel shows the optimal response to inflation corresponding to the alternative weights ω which are identical for the two parameterizations.

preferences. In addition, these parameters influence the expectations formation function nonlinearly. We now relax the assumption that private agents have perfect knowledge of all structural parameters and policymaker preferences. Instead, we posit that agents must somehow infer the information necessary for forming expectations by observing historical data, in essence acting like econometricians who know the correct specification of the economy but are uncertain about the parameters of the model.

In particular, we assume that private agents update the coefficients of their model for forecasting inflation using least squares learning with finite memory. We focus on least squares learning because of its desirable convergence properties, straightforward implementation, and close correspondence to what real-world forecasters actually do.[8] Estimation with finite memory reflects agents' concern for changes in the structural parameters of the economy. To focus our attention on the role of imperfections in the expectations formation process itself, however, we deliberately abstract from the introduction of the actual uncertainty in the structure of the economy which would justify such concerns in equilibrium. Further, we do not model the policymaker's knowledge or learning but instead focus on the implications of policy based on simple time-invariant rules of the form given in equation (4) that do not require explicit treatment of the policymaker's learning problem.[9]

We model perpetual learning by assuming that agents use a constant gain in their recursive least squares formula that places greater weight on more recent observations, as in Sargent (1999) and Evans and Honkapohja (2001). This algorithm is equivalent to applying weighted least squares where the weights decline geometrically with the distance in time between the observation being weighted and the most recent observation. This approach is closely related to the use of fixed sample lengths or rolling-window regressions to estimate a forecasting model (Friedman 1979). In terms of the mean "age" of the data used, a rolling-regression window of

8. This method of adaptive learning is closely related to optimal filtering, where the structural parameters are assumed to follow random walks. Of course, if private agents know the complete structure of the model—including the laws of motion for inflation, output, the unobserved states, and the distributions of the innovations to these processes—then they could compute efficient inflation forecasts that could outperform those based on recursive least squares. However, uncertainty regarding the precise structure of the time variation in the model parameters is likely to reduce the real efficiency gains from a method optimized to a particular model specification relative to a simple method such as least squares learning. Further, once we begin to ponder how economic agents could realistically model and account for such uncertainty precisely, we quickly recognize the significance of respecting (or the absurdity of ignoring) the cognitive and computational limits of economic agents.

9. We also abstract from two other elements that may further complicate policy design: the possibilities that policymakers may rely on a misspecified model or a misspecified information set for computing agents' expectations; see Levin, Wieland, and Williams (2003) and Orphanides (2003a), respectively, for a discussion of these two issues.

length l is equivalent to a constant gain κ of $2/l$. The advantage of the constant gain least squares algorithm over rolling regressions is that the evolution of the former system is fully described by a small set of variables, while the latter requires one to keep track of a large number of variables.

5.4.1 Least Squares Learning with Finite Memory

Under perfect knowledge, the predictable component of next period's inflation rate is a linear function of the inflation target and the current inflation rate, where the coefficients on the two variables are functions of the policy parameter θ and the other structural parameters of the model, as shown in equation (5). In addition, the optimal value of θ is itself a nonlinear function of the central bank's weight on inflation stabilization and the other model structural parameters. Given this simple structure, the least squares regression of inflation on a constant and lagged inflation,

$$\pi_i = c_{0,t} + c_{1,t}\pi_{i-1} + v_i, \tag{9}$$

yields consistent estimates of the coefficients describing the law of motion for inflation (Marcet and Sargent 1988; Evans and Honkapohja 2001). Agents then use these results to form their inflation expectations.[10]

To fix notation, let $\mathbf{X}_i$ and $\mathbf{c}_i$ be the 2×1 vectors $\mathbf{X}_i = (1, \pi_{i-1})'$ and $\mathbf{c}_i = (c_{0,i}, c_{1,i})'$. Using data through period t, the least squares regression parameters for equation (9) can be written in recursive form:

$$c_t = c_{t-1} + \kappa_t R_t^{-1} X_t(\pi_t - X_t' c_{t-1}), \tag{10}$$

$$R_t = R_{t-1} + \kappa_t(X_t X_t' - R_{t-1}), \tag{11}$$

where κ_t is the gain. With least squares learning with infinite memory, $\kappa_t = 1/t$, so as t increases, κ_t converges to zero. As a result, as the data accumulate this mechanism converges to the correct expectations functions and the economy converges to the perfect-knowledge benchmark solution. As noted above, to formalize perpetual learning—as would be required in the presence of structural change—we replace the decreasing gain in the infinite-memory recursion with a small constant gain, $\kappa > 0$.[11]

With imperfect knowledge, expectations are based on the perceived law of motion of the inflation process, governed by the perpetual-learning algorithm described above. The model under imperfect knowledge consists

10. Note that here we assume that agents employ a reduced form of the expectations formation function that is correctly specified under rational expectations with perfect knowledge. However, agents may be uncertain of the correct form and estimate a more general specification: for example, a linear regression with additional lags of inflation, which nests equation (9). In section 5.6, we also discuss results from such an example.

11. In terms of forecasting performance, the "optimal" choice of κ depends on the relative variances of the transitory and permanent shocks, as in the relationship between the Kalman gain and the signal-to-noise ratio in the case of the Kalman filter. Here, we do not explicitly attempt to calibrate κ in this way but instead examine the effects for a range of values of κ.

of the structural equation for inflation (1), the output-gap equation (2), the monetary policy rule (4), and the one-step-ahead forecast for inflation, given by

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t, \tag{12}$$

where $c_{0,t}$ and $c_{1,t}$ are updated according to equations (10) and (11).

We emphasize that in the limit of perfect knowledge (that is, as $\kappa \to 0$), the expectations function above converges to rational expectations and the stochastic coefficients for the intercept and slope collapse to

$$c_0^P = \frac{\alpha\theta\pi^*}{1-\phi},$$

$$c_1^P = \frac{1-\phi-\alpha\theta}{1-\phi}.$$

Thus, this modeling approach accommodates the Lucas critique in the sense that expectations formation is endogenous and adjusts to changes in policy or structure (as reflected here by changes in the parameters θ, π^*, α, and ϕ). In essence, our model is one of "noisy rational expectations." As we show below, although expectations are imperfectly rational, in that agents need to estimate the reduced-form equations they employ to form expectations, they are nearly rational, in that the forecasts are close to being efficient.

5.5 Perpetual Learning in Action

We use model simulations to illustrate how learning affects the dynamics of inflation expectations, inflation, and output in the model economy. First, we examine the behavior of the estimated coefficients of the inflation-forecast equation and evaluate the performance of inflation forecasts. We then consider the dynamic response of the economy to shocks similar to those experienced during the 1970s in the United States. Specifically, we compare the outcomes under perfect knowledge and imperfect knowledge with least squares learning that correspond to three alternative monetary policy rules to illustrate the additional layer of dynamic interaction introduced by the imperfections in the formation of inflation expectations.

In calibrating the model for the simulations, each period corresponds to about half a year. We consider values of κ of 0.025, 0.05, and 0.075, which roughly correspond to using forty, twenty, or thirteen years of data, respectively, in the context of rolling regressions. We consider two values for ϕ, the parameter that measures the influence of inflation expectations on inflation. As a baseline case, we set ϕ to 0.75, which implies a significant role for intrinsic inflation inertia, consistent with the contracting models of Buiter and Jewitt (1981), Fuhrer and Moore (1995), and Brayton et al.

(1997).[12] In the alternative specification, we allow for a greater role for expectations and correspondingly give less weight to inflation inertia by setting $\phi = 0.9$, consistent with the findings of Galí and Gertler (1999) and others. To ease comparisons between the two values of ϕ, we set α so that the optimal policy under perfect knowledge is identical in the two cases. Specifically, for $\phi = 0.75$, we set $\alpha = 0.25$, and for $\phi = 0.9$, we set $\alpha = 0.1$. In all cases, we assume $\sigma_e = \sigma_u = 1$.

The three alternative policies we consider correspond to the values of θ, $\{0.1, 0.6, 1.0\}$, which represent the optimal policies under perfect knowledge for policymakers whose preferences reflect a relative weight on inflation, ω, of 0.01, 0.5, and 1, respectively. Hence, $\theta = 0.1$ corresponds to an "inflation dove" policymaker who is primarily concerned about output stabilization, $\theta = 0.6$ corresponds to a policymaker with "balanced preferences" who weighs inflation and output stabilization equally, and $\theta = 1$ corresponds to an "inflation hawk" policymaker who cares exclusively about inflation.

5.5.1 The Performance of Least Squares Inflation Forecasts

Even absent shocks to the structure of the economy, the process of least squares learning generates time variation in the formation of inflation expectations and thereby in the processes of inflation and output. The magnitude of this time variation is increasing in κ—which is equivalent to using shorter samples (and thus less information from the historical data) in rolling regressions. Table 5.1 reports summary statistics of the estimates of agents' inflation-forecasting models based on stochastic simulations of the model economy for the two calibrations we consider. As seen in the table, the unconditional standard deviations of the estimates increase with κ. This dependence of the variation in the estimates on the rate of learning is portrayed in figure 5.2, which shows the steady-state distributions of the estimates of c_0 and c_1 for the case of $\phi = 0.75$. For comparison, the vertical lines in each panel indicate the values of c_0 and c_1 in the corresponding perfect-knowledge benchmark.

The median values of the coefficient estimates are nearly identical to the values implied by the perfect-knowledge benchmark; however, the mean estimates of c_1 are biased downward slightly. Although not shown in the table, the mean and median values of c_0 are nearly zero, consistent with the assumed inflation target of zero. There is contemporaneous correlation between estimates of c_0, and c_1 is nearly zero. Each of these estimates, however, is highly serially correlated, with first-order autocorrelations just below unity. This serial correlation falls only slightly as κ increases.

Note that a more aggressive policy response to inflation reduces the vari-

12. Other researchers suggest an even smaller role for expectations relative to intrinsic inertia; see Fuhrer (1997), Roberts (2001), and Rudd and Whelan (2001).

Table 5.1 **Least squares learning, by value of κ**

	RE	$\phi = 0.75, \alpha = 0.25$			$\phi = 0.90, \alpha = 0.10$		
	0	0.025	0.050	0.075	0.025	0.050	0.075
$\theta = 0.1$							
Mean c_1	0.90	0.86	0.83	0.81	0.88	0.89	0.93
Median c_1	0.90	0.89	0.88	0.88	0.95	0.97	0.98
SD c_0	0.00	0.37	0.67	1.01	0.79	2.06	4.92
SD c_1	0.00	0.12	0.17	0.21	0.18	0.23	0.20
$\theta = 0.6$							
Mean c_1	0.40	0.37	0.34	0.32	0.37	0.35	0.33
Median c_1	0.40	0.38	0.37	0.36	0.40	0.41	0.42
SD c_0	0.00	0.25	0.38	0.50	0.40	0.66	0.91
SD c_1	0.00	0.20	0.28	0.33	0.31	0.42	0.50
$\theta = 1.0$							
Mean c_1	0.00	–0.02	–0.04	–0.05	–0.03	–0.03	–0.04
Median c_1	0.00	–0.02	–0.04	–0.05	–0.03	–0.04	–0.06
SD c_0	0.00	0.24	0.35	0.44	0.37	0.58	0.74
SD c_1	0.00	0.21	0.29	0.35	0.33	0.44	0.51

Notes: RE = rational expectations. SD = standard deviation.

ation in the estimated intercept, c_0, but increases the magnitude of fluctuations in the coefficient on the lagged inflation rate, c_1. In the case of $\theta = 1$, the distribution of estimates of c_1 is nearly symmetrical around zero. For $\theta = 0.1$ and 0.6, the distribution of estimates of c_1 is skewed to the left, reflecting the accumulation of mass around unity, but the absence of much mass above 1.1.

Finite-memory least squares forecasts perform very well in this model economy. As shown in table 5.2, the mean-squared error of agents' one-step-ahead inflation forecasts is only slightly above the theoretical minimum given in the first line of the table (labeled "Perfect knowledge").[13] Only when both inflation displays very little intrinsic inertia and the policymaker places very little weight on inflation stabilization does the performance of finite-memory least squares forecasts break down. Not surprisingly, given that we assume that the structure of the economy is fixed, agents' forecasting performance deteriorates somewhat as κ increases. Nonetheless, finite-memory least squares estimates perform better than those with infinite memory (based on the full sample), and the difference in performance is more pronounced the greater the role of inflation expectations in determining inflation. In an economy where inflation is determined by the forecasts of other agents who use finite-memory least squares, it is

13. This is consistent with earlier findings regarding least squares estimation. Anderson and Taylor (1976), for example, emphasize that least squares forecasts can be accurate even when consistent estimates of individual parameter estimates are much harder to obtain.

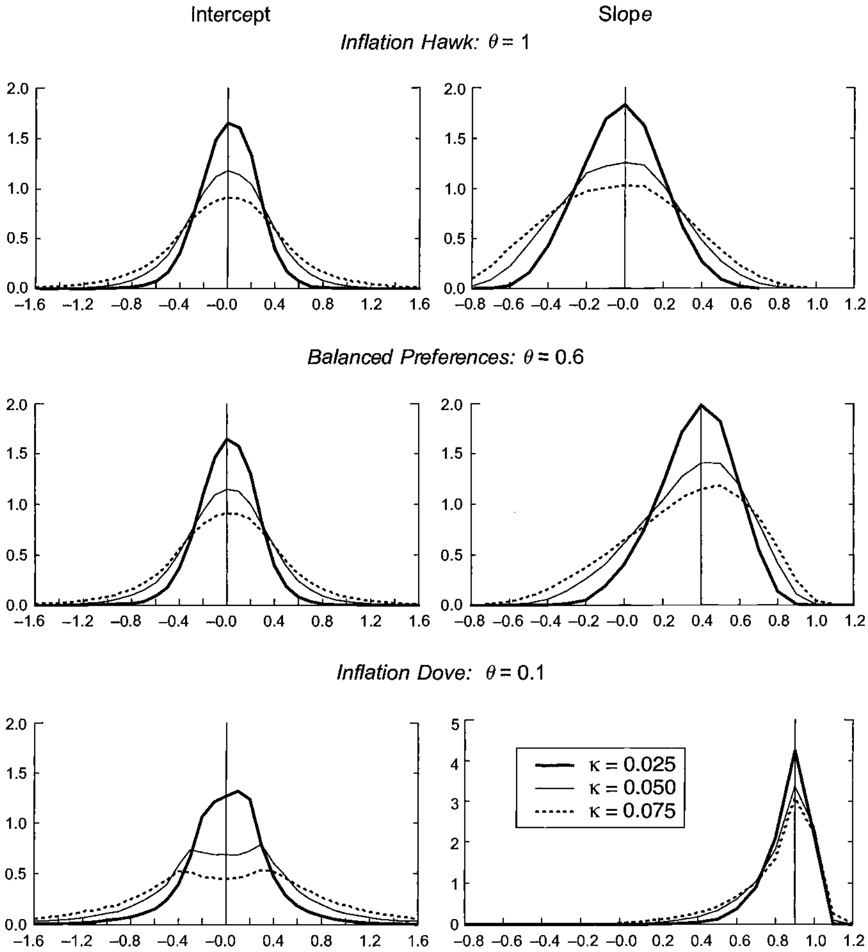

Fig. 5.2 Estimated expectations function parameters (ϕ = 0.75, α = 0.25)

Notes: The intercept and slope refer to the coefficients c_0 and c_1, respectively, in the agents' forecasting equation (9). The plots show the steady-state distributions of the estimates of c_0 and c_1 for different values of κ and θ.

better to follow suit rather than to use estimates that would have better forecast properties under perfect knowledge (Evans and Ramey 2001).

With imperfect knowledge, the private agents' ability to forecast inflation depends on the monetary policy in place, with forecast errors on average smaller when policy responds more aggressively to inflation. This effect is more pronounced the greater the role of inflation expectations in determining inflation. The marginal benefit from tighter inflation control on the ability of private agents to forecast accurately is greatest when the policymaker places relatively little weight on inflation stabilization. In this

Table 5.2 **Forecasting performance: Mean-squared error, by value of κ**

	$\phi = 0.75, \alpha = 0.25$			$\phi = 0.90, \alpha = 0.10$		
Forecast method	0.025	0.050	0.075	0.025	0.050	0.075
Perfect knowledge	1.03	1.03	1.03	1.01	1.01	1.01
$\theta = 0.1$						
LS (finite memory)	1.04	1.05	1.06	1.03	1.19	1.57
LS (infinite memory)	1.05	1.06	1.09	1.08	1.72	3.49
Long-lag Phillips curve	1.05	1.06	1.07	1.06	1.08	1.11
$\theta = 0.6$						
LS (finite memory)	1.04	1.04	1.05	1.01	1.01	1.02
LS (infinite memory)	1.06	1.09	1.12	1.10	1.20	1.31
Long-lag Phillips curve	1.05	1.07	1.08	1.07	1.11	1.17
$\theta = 1.0$						
LS (finite memory)	1.04	1.04	1.05	1.01	1.01	1.02
LS (infinite memory)	1.06	1.10	1.14	1.12	1.28	1.51
Long-lag Phillips curve	1.05	1.07	1.08	1.07	1.12	1.18

Note: LS = least squares.

case, inflation is highly serially correlated, and the estimates of c_1 are frequently in the vicinity of unity. Evidently, the ability to forecast inflation deteriorates when inflation is nearly a random walk. As seen by comparing the cases of θ of 0.6 and 1.0, the marginal benefit of tight inflation control disappears once the first-order autocorrelation of inflation is well below 1.

Finally, even though only one lag of inflation appears in the equations for inflation and inflation expectations, it is possible to improve on infinite-memory least squares forecasts by including additional lags of inflation in the estimated forecasting equation. This result is similar to that found in empirical studies of inflation, where relatively long lags of inflation help predict inflation (Staiger, Stock, and Watson 1997; Stock and Watson 1999; Brayton, Roberts, and Williams 1999). Evidently, in an economy where agents use adaptive learning, multiperiod lags of inflation are a reasonable proxy for inflation expectations. This result may also help explain the finding that survey-based inflation expectations do not appear to be "rational" using standard tests (Roberts 1997, 1998). With adaptive learning, inflation-forecast errors are correlated with data in the agents' information set; the standard test for forecast efficiency applies only to stable economic environments in which agents' estimates of the forecast model have converged to the true values.

5.5.2 Least Squares Learning and Inflation Persistence

The time variation in inflation expectations resulting from perpetual learning induces greater serial correlation in inflation. As shown in table 5.3, the first-order unconditional autocorrelation of inflation increases with κ. The first column shows the autocorrelations for inflation under per-

Table 5.3 Inflation persistence: First-order autocorrelation, by value of κ

		$\phi = 0.75, \alpha = 0.25$			$\phi = 0.90, \alpha = 0.10$		
θ	0	0.025	0.050	0.075	0.025	0.050	0.075
0.1	0.90	0.97	0.98	0.99	1.00	1.00	1.00
0.6	0.40	0.47	0.54	0.60	0.61	0.78	0.88
1.0	0.00	0.02	0.06	0.08	0.07	0.18	0.25

fect knowledge ($\kappa = 0$); note that these figures are identical across the two specifications of ϕ and α. In the case of the "inflation dove" policymaker ($\theta = 0.1$), the existence of learning raises the first-order autocorrelation from 0.9 to very nearly unity. For the policymaker with moderate preferences ($\theta = 0.6$), increasing κ from 0 to 0.075 causes the autocorrelation of inflation to rise from 0.40 to 0.60 when $\phi = 0.75$, or to 0.88 when $\phi = 0.9$.

Thus, in a model with a relatively small amount of intrinsic inflation persistence, the autocorrelation of inflation can be very high, even with a monetary policy that places significant weight on inflation stabilization. Even for the "inflation hawk" policymaker whose policy under perfect knowledge results in no serial persistence in inflation, the perpetual learning generates a significant amount of positive serial correlation in inflation. As we discuss below, the rise in inflation persistence associated with perpetual learning in turn affects the optimal design of monetary policy.

5.5.3 The Economy Following Inflationary Shocks

Next, we consider the dynamic response of the model to a sequence of unanticipated shocks, similar in spirit to those that arose in the 1970s. The responses of inflation expectations and inflation do not depend on the "source" of the shocks—that is, on whether we assume the shocks are due to policy errors or to other disturbances.[14] The configuration of shocks we have in mind would not be expected to occur frequently, of course. It is, however, instructive in that it illustrates how in these infrequent episodes the evolution of inflation expectations with learning could dramatically deviate from the perfect-knowledge benchmark under some policies. Inflation expectations in these episodes can become uncoupled from the policymakers' objectives, resulting in a period of stagflation that cannot occur under the perfect-knowledge benchmark.

14. The policy error we have in mind is the systematic misperception of the economy's noninflationary potential supply following an unobserved shift in potential output growth or an increase in the natural rate of unemployment, as apparently experienced in the 1970s. (See, for example, Orphanides and Williams [2002] and Orphanides 2003b.) Because such changes can only be perceived with the passage of time, they yield errors that are recognized to be serially correlated only in retrospect. In our model, the effect of such errors on inflation dynamics is isomorphic to that of an exogenous serially correlated inflation shock.

Note that under least squares learning, the model responses depend nonlinearly on the initial values of the states c and R. In the following, we report the average response from 1,000 simulations, each of which starts from initial conditions drawn from the relevant steady-state distribution. The shock is 2 percentage points in period one, and it declines in magnitude from periods two through eight. In period nine and beyond there is no shock. For these experiments we assume the baseline values for ϕ and α, and set $\kappa = 0.05$.

With perfect knowledge, the series of inflationary shocks causes a temporary rise in inflation and a decline in the output gap, as shown by the dashed lines in figure 5.3. The speed at which inflation is brought back to target depends on the monetary policy response, with the more aggressive policy yielding a relatively sharp but short decline in output and a rapid return of inflation to target. With the inflation hawk or moderate policymaker, the peak increase in inflation is no more than 2.5 percentage points, and inflation returns to its target within ten periods. With the inflation dove policymaker, the modest policy response avoids the sharp decline in output, but inflation is allowed to rise to a level about 4.5 percentage points above target, and the return to target is more gradual, with inflation still remaining 1 percentage point above target after twenty periods.

Imperfect knowledge with learning amplifies and prolongs the response of inflation and output to the shocks, especially when the central bank places significant weight on output stabilization. The solid lines in the figure show the responses of inflation and output under imperfect knowledge for the three policy rules. The inflation hawk's aggressive response to inflation effectively keeps inflation from drifting away from target, and the responses of inflation and output differ only modestly from those under perfect knowledge. In the case of balanced preferences, the magnitude of the peak responses of inflation and the output gap is a bit larger than under perfect knowledge, but the persistence of these gaps is markedly higher. The outcomes under the inflation dove, however, are dramatically different. The inflation dove attempts to finesse a gradual reduction in inflation without incurring a large decline in output, but the timid response to rising inflation causes the perceived process for inflation to become uncoupled from the policymaker's objectives. Stagflation results, with the inflation rate stuck over 8 percentage points above target, while output remains well below potential.

The striking differences in the responses to the shocks under imperfect knowledge are a product of the interaction between learning, the policy rule, and inflation expectations. The lines in figure 5.4 show the responses of the public's estimates of the intercept and the slope parameter of the inflation-forecasting equation under imperfect knowledge. Under the inflation hawk policymaker, inflation expectations are well anchored to the policy objective. The serially correlated inflationary shocks cause some

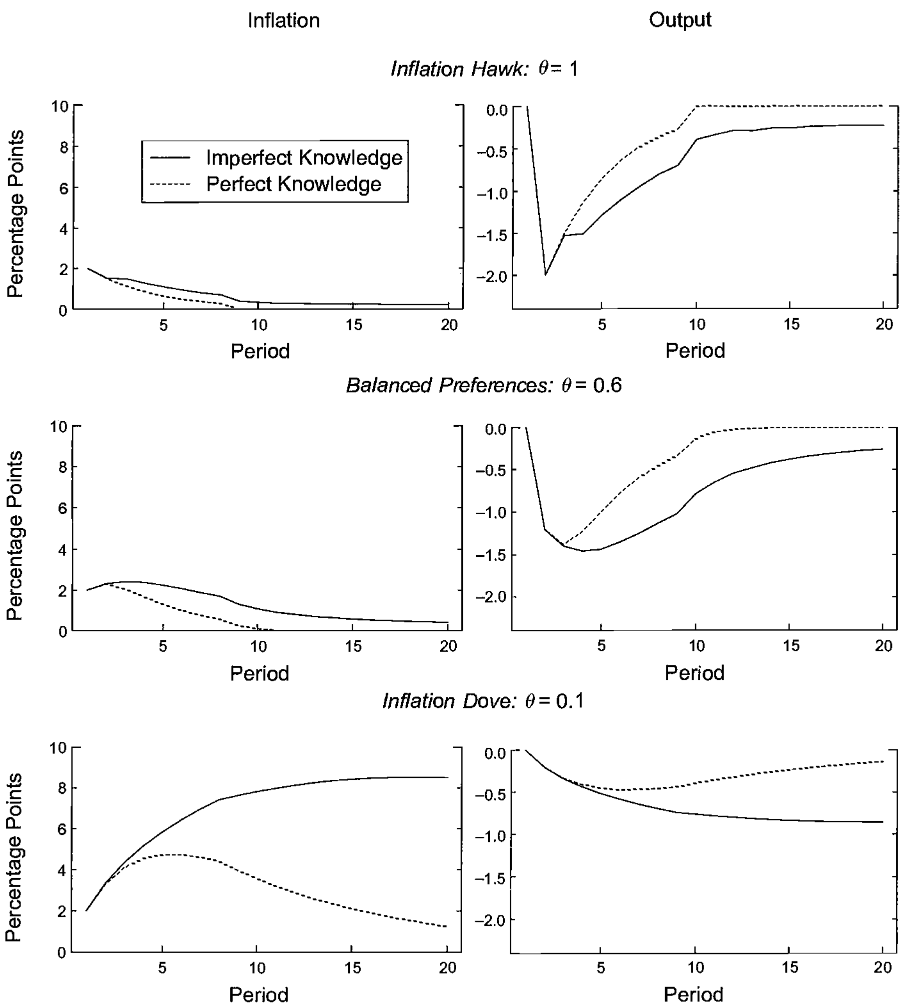

Fig. 5.3 Evolution of economy following inflation shocks (ϕ = 0.75, α = 0.25)

Notes: The plots show the mean responses of the inflation rate and the output gap to a series of inflationary shocks.

increase in both estimates, but the implied increase in the inflation target peaks at only 0.3 percentage point (not shown in the figure). Even for the moderate policymaker who accommodates some of the inflationary shock for a time, the perceived inflation target rises by just half of a percentage point. In contrast, under the inflation dove policymaker, the estimated persistence of inflation, already very high owing to the policymaker's desire to minimize output fluctuations while responding to inflation shocks, rises steadily, approaching unity. With inflation temporarily perceived to be a

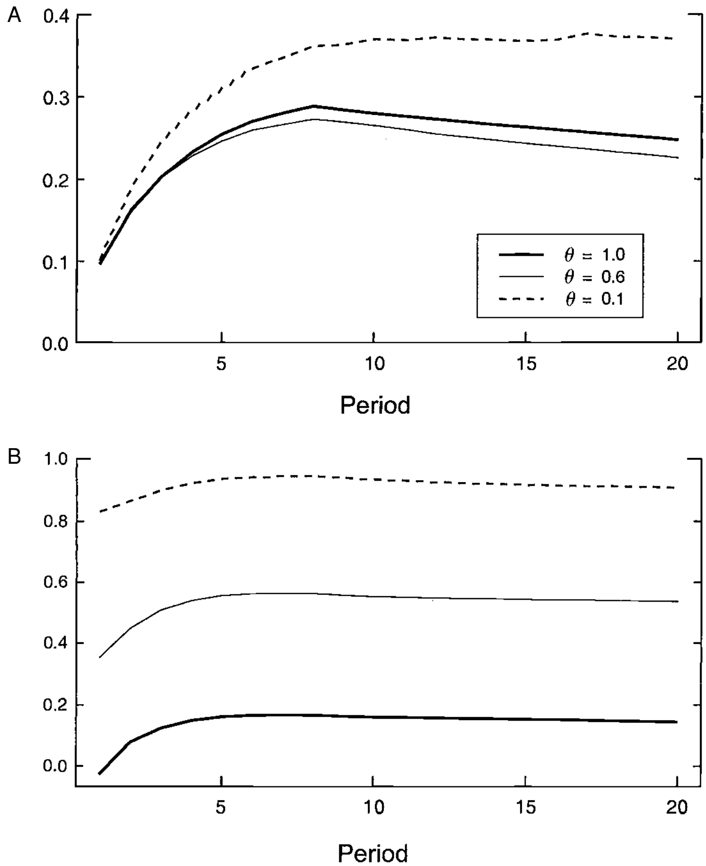

Fig. 5.4 ***A,* Estimated intercept following inflation shocks (ϕ = 0.75, α = 0.25); *B,* Estimated slope following inflation shocks**

Notes: The intercept and slope refer to the coefficients c_0 and c_1, respectively, in the agents' forecasting equation (9). The plots show the mean responses of the coefficients c_0 and c_1 to a series of inflationary shocks.

near-random walk with positive drift, agents expect inflation to continue to rise. The policymaker's attempts to constrain inflation are too weak to counteract this adverse-expectations process, and the public's perception of the inflation target rises by 5 percentage points. Despite the best of intents, the gradual disinflation prescription that would be optimal with perfect knowledge yields stagflation—the simultaneous occurrence of persistently high inflation and low output.

Interestingly, the inflation dove simulation appears to capture some key characteristics of the U.S. economy at the end of the 1970s, and it accords well with Chairman Volcker's assessment of the economic situation at the time:

> Moreover, inflationary expectations are now deeply embedded in public attitudes, as reflected in the practices and policies of individuals and economic institutions. After years of false starts in the effort against inflation, there is widespread skepticism about the prospects for success. Overcoming this legacy of doubt is a critical challenge that must be met in shaping—and in carrying out—all our policies.
>
> Changing both expectations and actual price performance will be difficult. But it is essential if our economic future is to be secure. (Volcker 1981, 293)

In contrast to this dismal experience, the model simulations suggest that the rise in inflation—and the corresponding costs of disinflation—would have been much smaller if policy had responded more aggressively to the inflationary developments of the 1970s. Although this was apparently not recognized at the time, Chairman Volcker's analysis suggests that the stagflationary experience of the 1970s played a role in the subsequent recognition of the value of continued vigilance against inflation in anchoring inflation expectations.

5.6 Imperfect Knowledge and Monetary Policy

5.6.1 Naive Application of the Rational-Expectations Policy

We now turn to the design of efficient monetary policy under imperfect knowledge. We start by considering the experiment in which the policymaker sets policy under the assumption that private agents have perfect knowledge when, in fact, they have only imperfect knowledge and base their expectations on the perpetual-learning mechanism described above. That is, policy follows equation (4) with the response parameter, θ, computed using equation (8).

Figure 5.5 compares the variability pseudo-frontier corresponding to this equilibrium to the frontier from the perfect-knowledge benchmark. Panel A shows the outcomes in terms of inflation and output-gap variability with the baseline parameterization, $\phi = 0.75$. Panel B shows the results of the same experiment with the more forward-looking specification for inflation, $\phi = 0.9$. In each case, we show the imperfect-knowledge equilibria corresponding to three different values of κ.

With imperfect knowledge, the perpetual-learning mechanism introduces random errors in expectations formation—that is, deviations of expectations from the values that would correspond to the same realization of inflation and the same policy rule. These errors are costly for stabilization and are responsible for the deterioration in performance shown in figure 5.5.

This deterioration in performance is especially pronounced for the policymaker who places relatively low weight on inflation stabilization. As

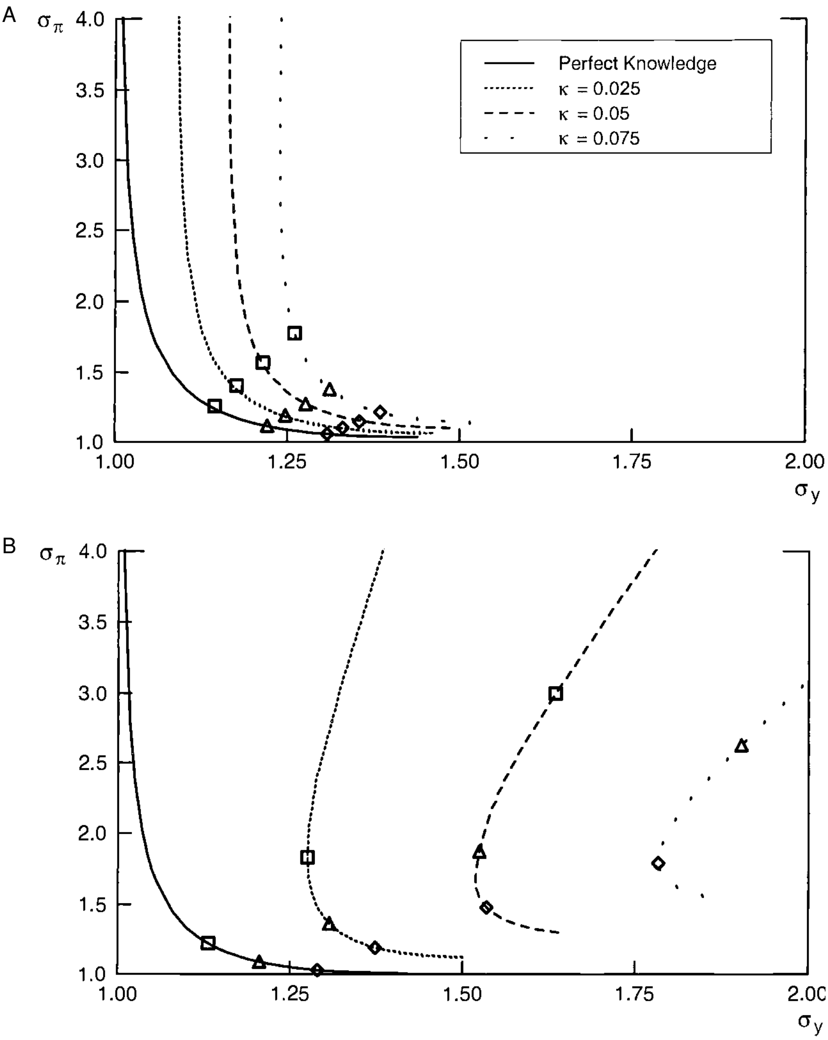

Fig. 5.5 ***A*, Outcomes with RE policy (ϕ = 0.75, α = 0.25); *B*, Outcomes with RE policy (ϕ = 0.9, α = 0.1)**

Notes: Each panel shows the efficient frontier with perfect knowledge and corresponding outcomes when the RE-optimal policies are adopted while, in fact, knowledge is imperfect. The square, triangle, and diamond correspond to preference weights $\omega = \{0.25, 0.5, 0.75\}$, respectively.

seen in the simulations of the inflationary shocks reported above, for such policies the time variation in the estimated autocorrelation of inflation in the vicinity of unity associated with learning can be especially costly. Furthermore, the deterioration in performance relative to the case of the perfect-knowledge benchmark is larger the greater is the role of expectations in determining inflation. With the higher value for ϕ, if a policymaker's preference for inflation stabilization is too low, the resulting outcomes under imperfect knowledge are strictly dominated by the outcomes corresponding to the naïve policy equilibrium for higher values of ω.

5.6.2 Efficient Simple Rule

Next we examine imperfect-knowledge equilibria when the policymaker is aware of the imperfection in expectations formation and adjusts policy accordingly. To allow for a straightforward comparison with the perfect-knowledge benchmark, we concentrate on the efficient choice of the responsiveness of policy to inflation, θ^S, in the simple linear rule

$$x_t = -\theta^S(\pi_t - \pi^*),$$

which has the same form as the optimal rule under the perfect-knowledge benchmark.[15]

The efficient policy response with imperfect knowledge is to be more vigilant against inflation deviations from the policymaker's target relative to the optimal response under perfect knowledge. Figure 5.6 shows the efficient choices for θ under imperfect knowledge for the two model parameterizations; the optimal policy under perfect knowledge—which is the same for the two parameterizations considered—is shown again for comparison. As before, we present results for three different values of κ: our baseline $\kappa = 0.05$ and also a smaller and a larger value. The increase in the efficient value of θ is especially pronounced when the policymaker places relatively little weight on inflation stabilization—that is, when inflation would exhibit high serial correlation under perfect knowledge. Under imperfect knowledge, it is efficient for a policymaker to bias the response to inflation upward relative to that implied by perfect knowledge. This effect is especially pronounced with the more forward-looking inflation process. Consider, for instance, the baseline case $\kappa = 0.05$. In the parameterization with $\phi = 0.9$, it is never efficient to set θ below 0.6, the value that one would choose under balanced preferences ($\omega = 0.5$) under perfect knowledge.

Accounting for imperfect knowledge can significantly improve stabilization performance relative to outcomes obtained when the policymaker naively adopts policies that are efficient under perfect knowledge. Figure 5.7 compares the loss to the policymaker with perfect and imperfect knowledge for different preferences ω. Panel A shows the outcomes for the

15. In Orphanides and Williams (2003), we explore policies that respond directly to private expectations of inflation, in addition to actual inflation. These rules are not fully optimal; with imperfect knowledge, the fully optimal policy would be a nonlinear function of all the states of the system, including the elements of c and R. However, implementation of such policies would assume the policymaker's full knowledge of the structure of the economy—an assumption we find untenable in practice.

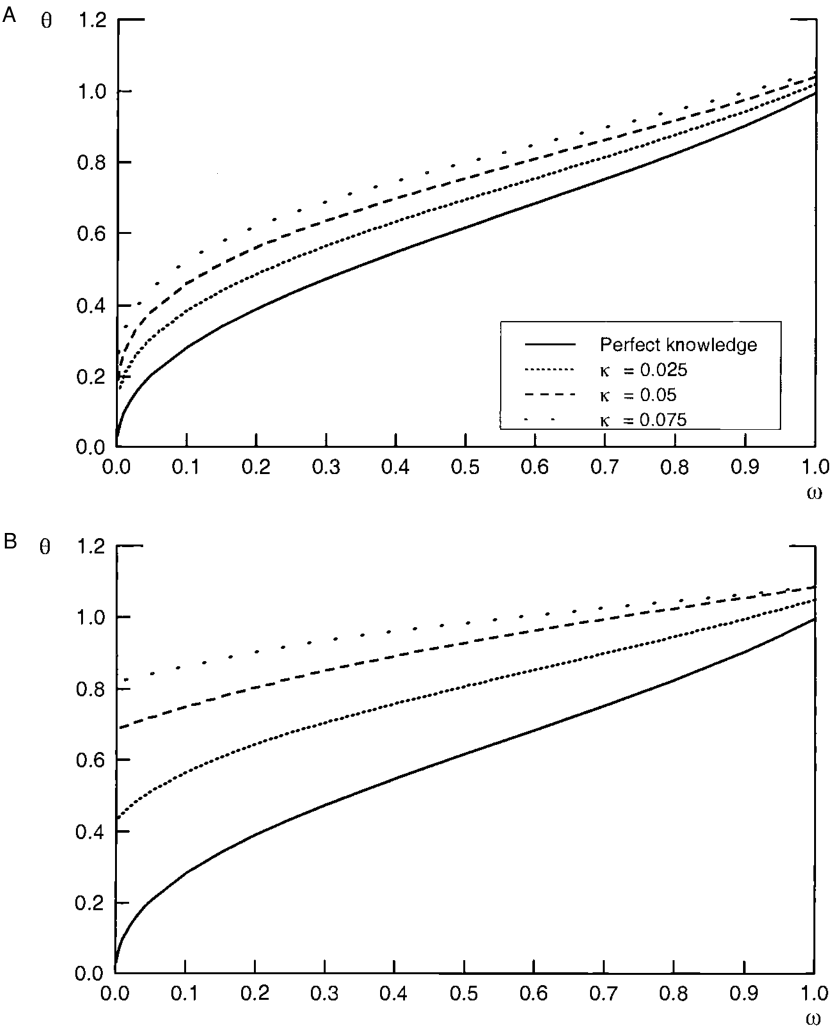

Fig. 5.6 ***A,* Efficient policy response to inflation (ϕ = 0.75, α = 0.25); *B,* Efficient policy response to inflation (ϕ = 0.9, α = 0.1)**

Notes: The solid line in each panel shows the optimal value of θ under perfect knowledge for alternative values of the relative preference for inflation stabilization ω. Remaining lines show the efficient one-parameter policy under imperfect knowledge.

baseline parameterization, $\phi = 0.75$, $\alpha = 0.25$; panel B reports the outcomes for the alternative parameterization of inflation, $\phi = 0.9$, $\alpha = 0.1$. In both panels, the results we show for imperfect knowledge correspond to our benchmark case, $\kappa = 0.05$. The payoff to reoptimizing θ is largest for policymakers who place a large weight on output stabilization, with the gain huge in the case of $\phi = 0.9$. In contrast, the benefits from reoptimiza-

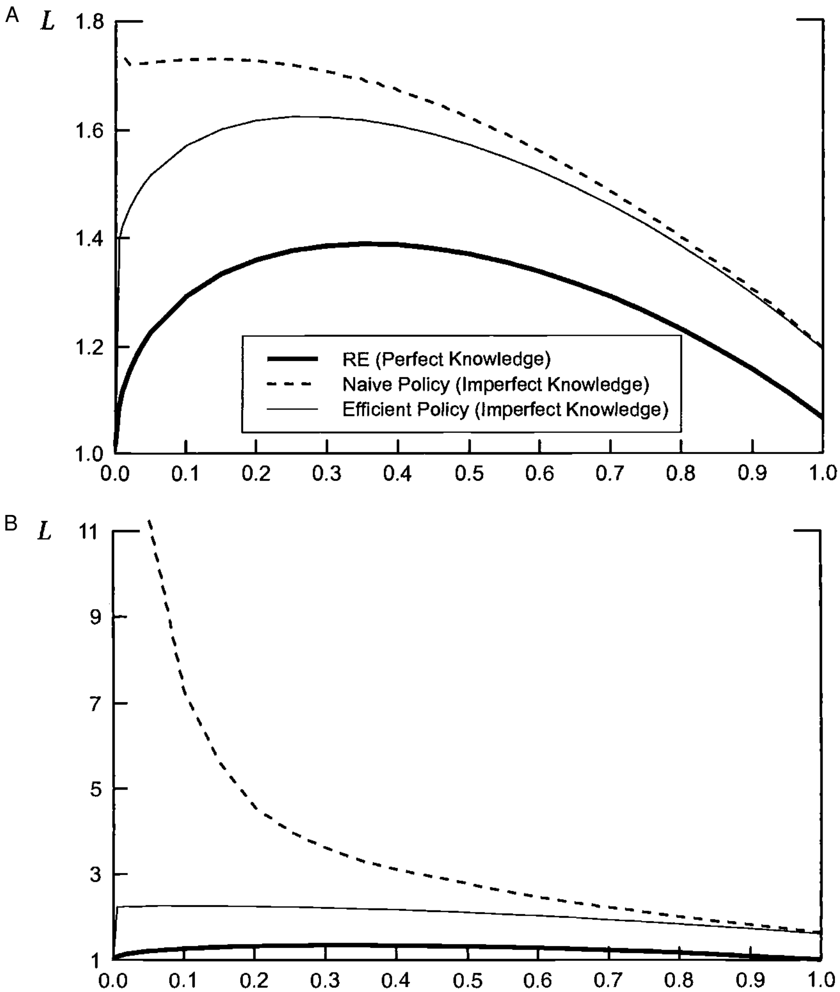

Fig. 5.7 ***A*, Policymaker loss (ϕ = 0.75, α = 0.25); *B*, Policymaker loss (ϕ = 0.9, α = 0.1)**

Notes: The two panels show the loss corresponding to alternative values of the relative preference for inflation stabilization ω for different assumptions regarding knowledge and different model parameterizations. The thick-solid line shows the case of perfect knowledge. The dashed line shows the outcomes assuming the policymaker chooses θ assuming perfect knowledge when knowledge is in fact imperfect. The thin-solid line shows the outcomes for the efficient one-parameter policy under imperfect knowledge.

tion are trivial for policymakers who are primarily concerned with inflation stabilization regardless of ϕ.

The key finding that the public's imperfect knowledge raises the efficient policy response to inflation is not unique to the model considered here and carries over to models with alternative specifications. In particular, we find the same result when the equation for inflation is replaced with the "New Keynesian" variant studied by Galí and Gertler (1999; see also Gaspar and Smets 2002). Moreover, we find that qualitatively similar results obtain if agents include additional lags of inflation in their forecasting models.

5.6.3 Dissecting the Benefits of Vigilance

In order to gain insight into the interaction of imperfections in the formation of expectations and efficient policy, we consider a simple example where the parameters of the inflation-forecast model vary according to an exogenous stochastic process.

From equation (5), recall that expectations formation is driven by the stochastic coefficient expectations function:

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t. \tag{13}$$

For the present purposes, let $c_{0,t}$ and $c_{1,t}$ vary relative to their perfect-knowledge benchmark values; that is, $c_{0,t} = c_0^P + v_{0,t}$ and , $c_{1,t} = c_1^P + v_{1,t}$, where $v_{0,t}$ and $v_{1,t}$ are independent zero-mean normal distributions with variances σ_0^2 and σ_1^2.

Substituting expectations into the Phillips curve and rearranging terms results in the following reduced-form characterization of the dynamics of inflation in terms of the control variable x:

$$\pi_{t+1} = (1 + \phi v_{1,t})\pi_t + \frac{\alpha}{1-\phi}x_t + \alpha u_{t+1} + e_{t+1} + \phi u_{0,t}. \tag{14}$$

In this case, the optimal policy with stochastic coefficients has the same linear structure as the optimal policy with fixed coefficients and perfect knowledge, and the optimal policy response is monotonically increasing in the variance σ_1^2.[16]

Although informative, the simple case examined above ignores the important effect of the serial correlation in v_0 and v_1 that obtains under imperfect knowledge. The efficient choice of θ cannot be written in closed form in the case of serially correlated processes for v_0 and v_1, but a set of stochastic simulations is informative. Consider the efficient choice of θ for our benchmark economy with balanced preferences, $\omega = 0.5$. Under perfect

16. See Turnovsky (1977) and Craine (1979) for early applications of the well-known optimal control results for this case. For our model, specifically, the optimal response can be written as

$$\theta = \frac{\alpha(1-\phi)s}{(1-\phi)(1-\omega) + \alpha^2 s},$$

where s is the positive root of the quadratic equation $0 = \omega(1-\omega)(1-\phi)^2 + (\omega\alpha^2 + [1-\omega][1-\phi]^2\phi^2\sigma_1^2)s + (\phi^2\sigma_1^2 - 1)\alpha^2 s^2$.

While the optimal policy response to inflation deviations from target, θ, is independent of σ_0^2, the variance of the $v_{0,t}$ differentiation reveals that it is increasing in σ_1^2), the variance of $v_{1,t}$. As $\sigma_1^2 \to 0$, of course, this solution collapses to the optimal policy with perfect knowledge.

knowledge, the optimal choice of θ is approximately 0.6. Instead, simulations assuming an exogenous autoregressive process for either c_0 or c_1 with a variance and autocorrelation matching our economy with imperfect knowledge suggest an efficient choice of θ approximately equal to 0.7—regardless of whether the variation is due to c_0 or to c_1. For comparison, with the endogenous variation in the parameters in the economy with learning, the efficient choice of θ is 0.75.

As noted earlier, for a fixed policy choice of policy responsiveness in the policy rule, θ, the uncertainty in the process of expectations formation with imperfect knowledge raises the persistence of the inflation process relative to the perfect-knowledge case. This can be seen by comparing the thick-solid and dashed lines in the two panels of figure 5.8, which plot the persistence of inflation when policy follows the rational-expectations (RE) optimal rule and agents have perfect and imperfect knowledge (with $\kappa = 0.05$), respectively. This increase in inflation persistence complicates stabilization efforts as it raises, on average, the output costs associated with restoring price stability when inflation deviates from its target.

The key benefit of adopting greater vigilance against inflation deviations from the policymaker's target in the presence of imperfect knowledge comes from reducing this excess serial persistence of inflation. More aggressive policies reduce the persistence of inflation, thus facilitating its control. The resulting efficient choice of reduction in inflation persistence is reflected by the thin-solid lines in figure 5.8.

5.7 Learning with a Known Inflation Target

Throughout the preceding discussion and analysis, we have implicitly assumed that agents do not rely on explicit knowledge regarding the policymaker's objectives in forming expectations. Arguably, this assumption best describes situations where a central bank does not successfully communicate to the public an explicit numerical inflation target and, perhaps, a clear weighting of its price and economic stability objectives. Since the adoption and clear communication of an explicit numerical inflation target is one of the key characteristics of inflation-targeting regimes, it is of interest to explore the implications of this dimension of inflation targeting in our model. To do so, we consider the case where the policymaker explicitly communicates the ultimate inflation target to the public; that is, we assume that the public exactly knows the value of π^* and explicitly incorporates this information in forming inflation expectations. Of course, even in an explicit inflation-targeting regime, the public may remain somewhat uncertain regarding the policymaker's inflation target, π^*, so that this assumption of a perfectly known inflation target may not be obtainable in practice and may be seen as an illustrative limiting case.

The assumption of a known numerical inflation target simplifies the

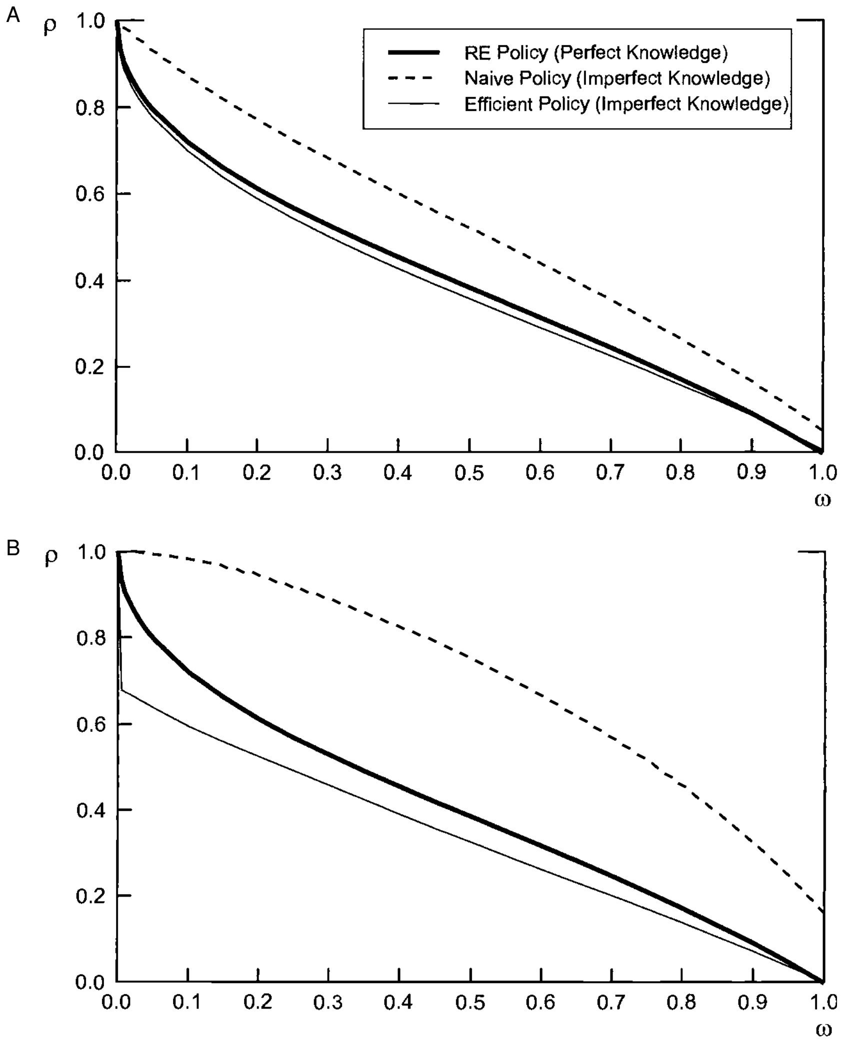

Fig. 5.8 ***A*, Inflation persistence ($\phi = 0.75$, $\alpha = 0.25$); *B*, Inflation persistence ($\phi = 0.9$, $\alpha = 0.1$)**

Notes: The figure shows the population first-order autocorrelation of inflation corresponding to policies based on alternative inflation stabilization weights ω. For each value of ω, the thick-solid line shows the inflation persistence in the benchmark case of rational expectations with perfect knowledge. The dashed line shows the corresponding persistence when policy follows the RE-optimal solution but knowledge is imperfect. The thin-solid line shows the persistence associated with the efficient one-parameter rule with imperfect knowledge.

public's inflation forecasting problem. From equations (7) and (8), the reduced-form equation for inflation under rational expectations is given by

$$(15) \qquad \pi_{t+1} - \pi^* = \left(1 - \frac{\alpha\theta}{1 - \phi}\right)(\pi_t - \pi^*) + e_{t+1} + \alpha u_{t+1}.$$

With a known inflation target, the inflation-forecasting model consistent with rational expectations is simply

$$(16) \qquad \pi_i - \pi^* = c_{1,t}(\pi_{i-1} - \pi^*) + v_i.$$

Note that in this forecasting equation only the slope parameter, c_1, is estimated; thus, in terms of the forecasting equation, the assumption of a known inflation target corresponds to a zero restriction on c_0 (when the forecasting regression is written in terms of deviations of inflation from its target). As in the case of an unknown inflation target, constant-gain versions of equations (10) and (11) can be used to model the evolution of the formation of inflation expectations in this case. The one-step-ahead forecast of inflation is given by

$$(17) \qquad \pi^e_{t+1} = \pi^* + c_{1,t}(\pi_t - \pi^*),$$

and again, in the limit of perfect knowledge (that is, as $\kappa \to 0$), the expectations function above converges to rational expectations with the slope coefficient $c_1^P = (1 - \phi - \alpha)/(1 - \phi)$. This formulation captures a key rationale for adopting an explicit inflation-targeting regime: to reduce the public's uncertainty and possible confusion about the central bank's precise inflation objective and thereby to anchor the public's inflation expectations to the central bank's objective.[17]

Eliminating uncertainty about the inflation target improves macroeconomic performance, in terms of both inflation and output stability. The thin-solid lines in panel A of figure 5.9 trace the RE-policy pseudo-frontiers in the case of a known inflation target. For comparison, the dashed lines show the pseudo-frontiers assuming that the inflation target is not known by the public (this repeats the curves shown in figure 5.5 for our benchmark case, $\kappa = 0.05$). Recall that the pseudo-frontier is obtained by evaluating the performance of the economy under imperfect knowledge for the set of policies for $\omega \in (0,1]$ given by equation (8) that would be optimal under perfect knowledge. As seen in the figure, economic outcomes are clearly more favorable when the inflation target is assumed to be perfectly known

17. The adoption of inflation targeting may affect the private formation of expectations in other ways than by tying down the ultimate inflation objective. For instance, Svensson (2002) argues that inflation-targeting central banks should also make explicit their preference weighting, ω, which in principle could further reduce the public's uncertainty about policy objectives. However, given the remaining uncertainty about model parameters (α and ϕ in our model), the uncertainty about the value of c_1 is not eliminated in this case. The extent to which this uncertainty may be reduced is left to further research.

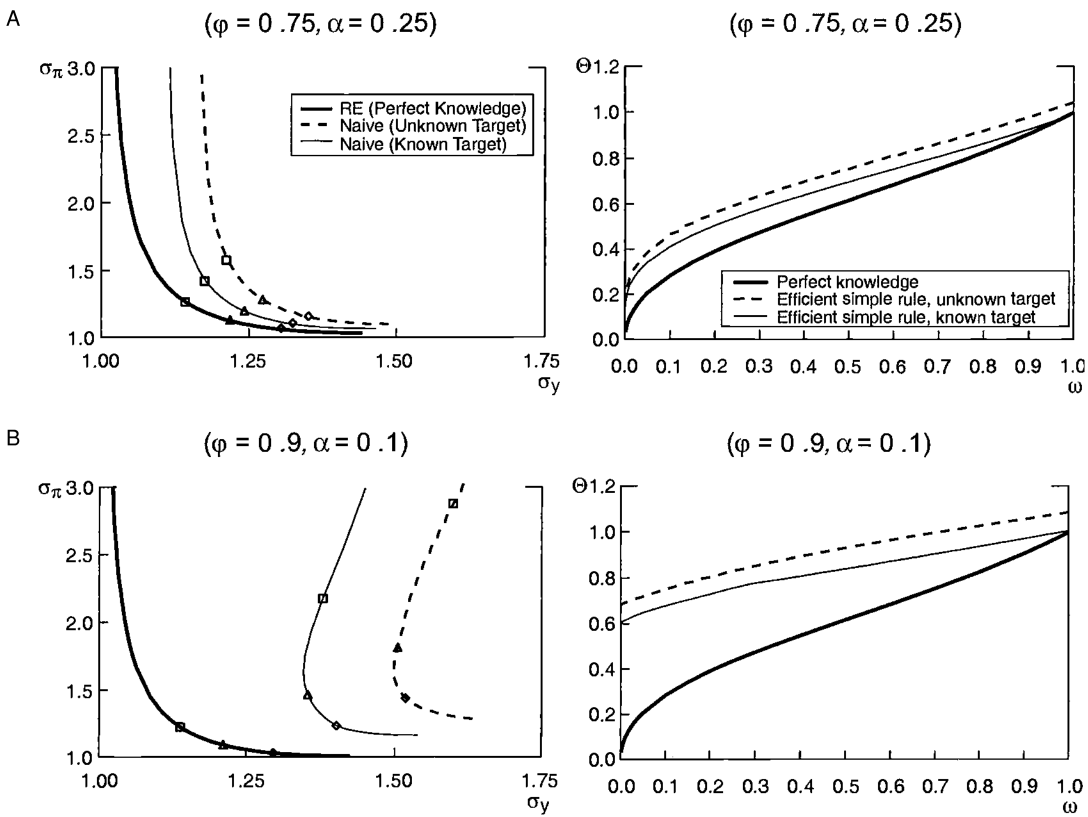

Fig. 5.9 Comparing policies with a known and unknown inflation target: *A*, Outcomes with RE policy; *B*, Efficient response to inflation

Notes: The thin-solid lines indicate economic outcomes (top panel) and efficient policy responses (bottom panel) with perpetual learning when the policy-maker's inflation target is assumed to be perfectly known. The thick-solid and dashed lines correspond, respectively, to the perfect-knowledge benchmark and the case of perpetual learning with an unknown inflation target. See also the notes to figures 5.5 and 5.6.

than otherwise. Still, the resulting pseudo-frontiers lie well to the northeast of those that would obtain under perfect knowledge. Evidently, imperfect knowledge of the dynamic process for inflation alone has large costs in terms of performance, especially when expectations are very important for determining inflation outcomes, represented by the case of $\phi = 0.9$.

The basic finding that, relative to the perfect-knowledge benchmark, policy should be more vigilant against inflation under imperfect knowledge also obtains in the case of a known inflation target. Panel B of figure 5.9 shows the optimal values of θ for the three cases we consider: perfect knowledge, imperfect knowledge with known π^*, and imperfect knowledge with unknown π^*. When π^* is known, the optimal choice of θ is slightly lower than when π^* is unknown. Even with a known inflation target, however, it remains optimal to be more vigilant against inflation relative to the perfect-knowledge case. An exception is the extreme case of $\omega = 1$ when the optimal value of θ is exactly unity, the same value that obtains under perfect knowledge.[18]

A striking result, seen most clearly in the case of $\phi = 0.9$, is that the optimal value of θ is relatively insensitive over a large range of values for the stabilization preference weight, ω, whether the inflation is known or unknown. By contrast, under perfect knowledge, the optimal value of θ is quite sensitive to ω. An implication of this finding is that with imperfect knowledge there is relatively little "cost" associated with policies designed as if inflation were the central bank's primary objective, even when policymakers place substantial value in reducing output variability in fact. By contrast, as shown above, the costs of optimizing policies that incorrectly place a large weight on output stability under the assumption of perfect knowledge can be quite large. This asymmetry suggests that the practice of concentrating attention primarily on price stability in the formulation of monetary policy may be seen as a robust strategy for achieving *both* a high degree of price stability *and* a high degree of economic stability.

5.8 Conclusion

We examine the effects of a relatively modest deviation from rational expectations resulting from perpetual learning on the part of economic agents with imperfect knowledge. The presence of imperfections in the formation of expectations makes the monetary policy problem considerably more difficult than would appear under rational expectations. Using a simple linear model, we show that although inflation expectations are nearly efficient, imperfect knowledge raises the persistence of inflation and

18. In this limiting case, estimates of c_1 are symmetrically distributed around zero. Hence, in terms of a simple rule of the form given by equation (4), there is no gain from overresponding, relative to the case of perfect knowledge, to actual inflation.

distorts the policymaker's trade-off between inflation and output stabilization. As a result, policies that appear efficient under rational expectations can result in economic outcomes significantly worse than would be expected by analysis based on the assumption of perfect knowledge. The costs of failing to account for the presence of imperfect knowledge are particularly pronounced for policymakers who place a relatively greater value on stabilizing output: a strategy emphasizing tight inflation control can yield superior economic performance, in terms of both inflation and output stability, than can policies that appear efficient under rational expectations. More generally, policies emphasizing tight inflation control reduce the persistence of inflation and the incidence of large deviations of expectations from the policy objective, thereby mitigating the influence of imperfect knowledge on the economy. In addition, tighter control of inflation makes the economy less prone to costly stagflationary episodes.

The adoption and effective communication of an explicit numerical inflation target also mitigate the influence of imperfect knowledge on the economy. Communication of an inflation target may greatly improve attainable macroeconomic outcomes and afford greater economic stability relative to the outcomes that are attainable when the public perceives the policymaker's ultimate inflation objective less clearly. These results highlight the potential value of communicating a central bank's inflation objective and of continued vigilance against inflation in anchoring inflation expectations and fostering macroeconomic stability.

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Comment George W. Evans

Introduction

This is a very nice paper. The main points are important, the structure is simple and clear, and I find the key arguments persuasive. In my comments I am going to begin by summarizing the heart of the Orphanides-Williams argument. Then I will locate their paper within the rapidly growing literature on learning and monetary policy. Finally I will return to their paper and offer a number of specific comments on natural extensions or alternative approaches.

Summary of the Argument

Orphanides and Williams (OW) work with a simple two-equation macro model. The first equation is an augmented Phillips curve with inertia:

$$\pi_{t+1} = \phi\pi^e_{t+1} + (1 - \phi)\pi_t + \alpha y_{t+1} + e_{t+1},$$

where π_{t+1} is the rate of inflation between period t and period $t + 1$, π^e_{t+1} is the rate of inflation over this period expected at time t, y_{t+1} is the level of the output gap in $t + 1$, and e_{t+1} is a white noise inflation shock. The second equation is an aggregate-demand relation that embodies a lagged policy effect,

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$$y_{t+1} = x_t + u_{t+1},$$

where x_t is set by monetary policy at t and u_{t+1} is white noise. Through monetary policy it is assumed that policymakers are able one period ahead to control aggregate output up to the unpredictable random disturbance u_{t+1}.

The combination of this aggregate-demand equation and the neoclassical (as opposed to neo-Keynesian) inflation equation yields a particularly tractable model for studying the effects of private agents' learning. In particular, the timing assumptions are carefully crafted to yield simplicity.

Policymakers choose the x_t process to minimize

$$(1 - \omega)Ey_t^2 + \omega E(\pi_t - \pi^*)^2.$$

This is a standard quadratic loss function. We can think of ω as reflecting policymakers' preferences, which may (or may not) be derived from the preferences of the representative agent.

Optimal Policy under Rational Expectations

Under rational expectations (RE), optimal policy takes the form of the feedback rule

$$x_t = -\theta^P(\pi_t - \pi^*),$$

where $\theta^P = \theta^P(\omega, \alpha/[1 - \phi])$. This leads to an efficiency frontier, described by a familiar trade-off between σ_π and σ_y, shown in their figure 5.1.

For this choice of feedback parameter, in the rational-expectations equilibrium (REE) inflation follows the process

$$\pi_t = c_0^P + c_1^P \pi_{t-1} + \text{noise}_t$$

$$E_t\pi_{t+1} = c_0^P + c_1^P\pi_t,$$

where c_0^P, c_1^P depend on $\theta^P\alpha/(1 - \phi)$. Here noise$_t$ is white noise. The superscript P refers to "perfect knowledge," which OW use as a synonym for RE.

Thus, under RE the problem is quite straightforward. How "aggressive" policy should be with respect to deviations of inflation from target depends in a natural way on the structural parameters ϕ, α and policymaker preferences as described by ω.

Least Squares Learning

Now we make the crucial step of backing away from RE. Instead of assuming that agents are endowed a priori with RE, we model the agents as forecasting in the same way that an econometrician might: by assuming a simple time series model for the variable of interest, and by estimating its parameters and using it to forecast. Specifically, suppose that private agents believe that inflation follows an AR(1) process, as it does in an REE, but that they do not know c_0^P, c_1^P. Instead they estimate the parameters of

$$\pi_t = c_0 + c_1\pi_{t-1} + v_t$$

by a least squares–type regression, and at time t forecast

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t.$$

Over time the estimates $c_{0,t}$, $c_{1,t}$ are updated as new data become available. We consider two cases for this updating.

Infinite Memory—"Decreasing Gain"

First we suppose that agents literally do least squares using all the data. We assume that policymakers do not explicitly take account of private agent learning and follow the feedback rule with $\theta = \theta^P$. Then, with "infinite memory" (no discounting of observations), one can show (e.g., Evans and Honkapohja 2001)

$$c_{0,t}, c_{1,t} \rightarrow c_0^P, c_1^P \text{ w.p.1},$$

so that asymptotically we get the optimal REE.

Technically the most convenient way to set up least squares learning by private agents is using the recursive least squares (RLS) algorithm.[1] In this algorithm the agents carry their parameter estimates (and an estimate of the second moment matrix of the regressors) into the next period. Updated estimates next period are then generated recursively using the most recent data point. Because each data point is counted equally by least squares, the "gain" κ_t (i.e., the effective weight placed on the last data point) is given by $\kappa_t = 1/t$ (i.e., by the inverse of the sample size). In the learning literature this is called the "decreasing gain" case, because $\kappa_t \rightarrow 0$ as $t \rightarrow \infty$.

I remark that convergence to the REE is not obvious. This is because the model is "self-referential": that is, the evolution of the data depends on expectations and hence on the estimated coefficients, and these in turn are updated using the data generated. Convergence to REE does take place because the equilibrium in this model satisfies the "E-stability" conditions that govern stability in such a system.

Finite Memory—"Constant Gain"

OW make a small but significant change to the standard least squares updating formula. Instead of assuming that all observations count equally, they discount or downweight past data. In terms of the RLS algorithm, this is accomplished technically by setting the gain, the weight on the most recent observation used to update estimates, to a small constant (i.e., setting $\kappa_t = \kappa$; e.g., 0.05).

Why would it be natural for agents to use a constant rather than de-

1. The technique of formulating learning as a recursive algorithm, and then applying stochastic approximation tools to analyze convergence, was introduced by Marcet and Sargent (1989).

creasing gain? The main rationale for this procedure is that it allows estimates to remain alert to structural shifts. As economists, and as econometricians, we tend to believe that structural changes occasionally occur, and we might therefore assume that private agents also recognize and allow for this. Although in principle one might attempt to model the process of structural change, this typically unduly strains the amount of knowledge we have about the economic structure. A reasonable alternative is to adjust parameter estimators to reflect the fact that recent observations convey more accurate information on the economy's law of motion than do past data, and "constant gain" estimators are one very natural way of accomplishing this downweighting of past data.[2]

Implications of Constant-Gain Least Squares

With constant-gain procedures, estimates no longer fully converge to the REE. The estimators $c_{0,t}$, $c_{1,t}$ converge instead to a stochastic process. Because of this, OW use the term "perpetual learning" to refer to the constant gain case.

If the gain parameter κ is very small, then estimators will be close to the REE values for most of the time with high probability, and output and inflation will be near their REE paths. Nonetheless, small plausible values like $\kappa = 0.05$ can lead to very different outcomes in the calibrations OW consider. In particular, they find the following:

1. The standard deviations of $c_{0,t}$ and $c_{1,t}$ are large even though forecast performance remains good.
2. There is a substantial increase in the persistence of inflation, compared to the REE.
3. Most strikingly, the policy frontier shifts out very substantially and in a nonmonotonic way (see their figure 5.5).

Policy Implications

Under perpetual learning if policymakers keep to the same class of rules

$$x_t = -\theta^S(\pi_t - \pi^*),$$

then they should choose a different θ. Here the notation θ^S is meant to indicate that we restrict policymakers to choose from the same "simple" class of policy rules. There are four main implications for policy in the context of constant-gain (perpetual) learning by private agents.

2. Two remarks are in order. First, an alternative rationale for constant gain is that it can be an equilibrium in learning rules, even if structural change is not present; see section 14.4 of Evans and Honkapohja (2001). Second, there are other ways of allowing for structural change: for example, through time-varying gain sequences or explicit models of structural variation.

1. Naive policy choice can be strictly inefficient. This is illustrated in the second panel of their figure 5.5. By "naive" policy is meant the policy that assumes RE (perfect knowledge) on the part of agents, when in fact the agents are following perpetual learning with $\kappa > 0$. In particular, there are cases in which increasing θ^S would decrease the standard deviations of *both* inflation and output.
2. In general, policy should be more hawkish; that is, under perpetual learning the monetary authorities should pick a larger θ^S than if agents had RE.
3. Following a sequence of unanticipated inflation shocks, inflation doves (i.e., policymakers with low θ reflecting a low ω) can do very poorly. This is illustrated in OW's figure 5.3.
4. If the inflation target π^* is known to private agents, so that they need estimate only the slope parameter c_1, then the policy frontier is more favorable than when it is not known. This is illustrated in the first panel of their figure 5.9.

I will return to a discussion of these and other specific results after discussing learning and monetary policy in a more general setting.

Learning in Monetary Policy

Recently, considerable research has begun to focus on the implications for monetary policy when explicit account is taken of the literature on adaptive/econometric learning in macroeconomics.[3]

I will give a selective overview of this recent research and locate OW within this context. Then I will return to a discussion of OW. There are four main issues I will use to group my general remarks: (a) the theoretical roles played by learning, (b) the question of who or what group of agents is learning, (c) the particular implications of constant-gain learning, and (d) some further (personal) thoughts on rationality.

Roles for Learning

There are three main types of result that can be delivered by incorporating learning into a monetary policy model.

Stability under Private Agent Learning

An REE need not necessarily be stable under private agent learning. It is logically possible that if agents follow least squares learning (with the

3. For example, two recent workshops or conferences have considered this topic, one at the Cleveland Federal Reserve Bank, in February 2001, on "Learning and Model Misspecification," and a second at the Atlanta Federal Reserve Bank, in March 2003, on "Monetary Policy and Learning."

usual decreasing gain) then the system fails to converge to an REE, even if their parameter estimates are initially close to the REE.

This theoretical possibility of instability turns out to be a genuine concern for monetary policy in New Keynesian or New Phillips curve models (as is the related but distinct issue of indeterminacy). Bullard and Mitra (2002) show that stability under private agent learning should not be taken for granted if policymakers follow Taylor-type rules. Depending on the specific formulation of the rule, instability can arise for certain choices of parameter settings. Evans and Honkapohja (2002, 2003a, 2003c) examine this issue in the context of optimal monetary policy. They show that stability under learning is a pervasive problem when the interest rate rule is formulated as a reaction to fundamental shocks, but it can be overcome when the rule reacts appropriately to private expectations. Recent work by Preston (2003) has considered this issue in the context of long-horizon agents.

Selection Criterion

In some models the phenomenon of indeterminacy (i.e., multiple REE) arises. In this setting, learning can provide a natural way of choosing between equilibria. A particular question of interest is the following. It is known that when a steady state of a linear model is indeterminate there exist "sunspot" equilibria—that is, REE in which the solution is driven by extraneous noise. Such solutions, with economic fluctuations driven in a self-fulfilling way by extrinsic random variables, would usually be considered an unintended and undesirable by-product of economic policy. A particular question of interest, in cases of multiple equilibria, is whether the sunspot equilibria can be stable under learning.

It has been known for some time that it is possible in some cases for sunspot equilibria to be stable under learning. This was initially demonstrated by Woodford (1990) in the context of the overlapping-generations model of money. In general, whether a sunspot equilibrium is stable under learning depends on the model and the particular solution (see chap. 12 of Evans and Honkapohja 2001). There has been recent interest in whether stable sunspot solutions can arise in more realistic monetary models. In particular, Evans, Honkapohja, and Marimon (2003) look at when this can occur in cash-in-advance models, and Honkapohja and Mitra (forthcoming), Carlstrom and Fuerst (2004), and Evans and McGough (forthcoming) examine the issue for New Keynesian models.

Non-REE Learning Dynamics

Finally, we move to the possibility that the economy under learning generates solutions that in some way go beyond RE. Here it appears useful to group results into two broad categories. One possibility is that learning converges to a "restricted-perceptions equilibrium." This arises if agents are endowed with an econometric model that is misspecified asymptotically, as discussed in chapter 13 of Evans and Honkapohja (2001). For example, agents may omit some variables that help forecast the variables of interest, or their forecasting model may fail to capture nonlinearities that are present.

Somewhat more radically, learning may generate "persistent learning dynamics" (see chap. 14 of Evans and Honkapohja 2001) as a result of local instability of an REE under learning (as in Bullard 1994) or due to a learning rule that fails to fully converge to REE parameter values (as in constant-gain learning rules). The OW paper falls into this last class: private agents use a learning rule in which parameter estimates never quite converge to REE values. This "perpetual learning" then turns out to have major policy implications, even when the deviation from REE might be thought not too large.

Who Is Learning?

The earliest literature on learning focused on private agents (i.e. households and firms). In dynamic macroeconomic models private agents, in order to make optimal decisions, must make forecasts of relevant future variables. Clearly the expectations of households and firms do matter enormously for the actual evolution of the economy. The RE revolution made the crucial advance of defining and analyzing what it means for expectations to be consistent with the economic structure and optimizing agents. However, this has had the potential disadvantage of demoting private expectations as an independent force. Consequently it was natural that the initial focus of the learning literature was on private agent learning. The OW paper follows the primary strand of the literature in this respect.

However, policymakers also need to form expectations and make forecasts, and they too are not endowed with full knowledge of the economic structure or fully rational forecast functions. Some recent research has begun to tackle this issue. Most notably, Thomas Sargent's (1999) book on the disinflation in the 1990s emphasized learning by policymakers about a (misspecified) Phillips curve trade-off. Sargent's model incorporates a tantalizing combination of misspecification, learning, and optimal policy formulation.

Obviously it is possible to allow for separate learning by private agents and policymakers. In fact, Sargent (1999) actually allows for this in some cases, although much of his analysis, and that of Cho, Williams, and Sargent (2002), focuses on learning by policymakers with RE assumed for private agents. Simultaneous learning by policymakers is also analyzed in Honkapohja and Mitra (2002) and discussed in Evans and Honkapohja (2003c).

There is an additional asymmetry that should be noted. Both private agents and policymakers need to make forecasts of future aggregate variables, but in addition, implementation of optimal policy may require simultaneous estimation of structural parameters. This issue is considered in Evans and Honkapohja (2003a, 2003c).

Constant-Gain Learning

As already emphasized, the use of constant-gain (or "perpetual") learning plays a central role in OW. In general, constant-gain learning can lead to a number of phenomena. First, the work of Sargent (1999), Cho, Williams, and Sargent (2002), Williams (2002), and Bullard and Cho (2002) emphasizes the possibility of "escapes"—that is, occasional big deviations from a unique REE. This is a surprising finding: for significant periods of time learning dynamics can drive the economy away from the REE, but in a predictable direction.

When there are multiple REE, escapes can take a different form. The most widely examined case is the case of multiple distinct REE steady states. Here escapes take the form of periodic shifts between the different steady states as a result of large random shocks interacting with the learning dynamics. This phenomenon is seen in chapter 14 of Evans and Honkapohja (2001), the hyperinflation model of Marcet and Nicolini (2003c), the exchange rate model of Kasa (2004) and the liquidity trap model of Evans and Honkapohja (2003b).

Finally, it turns out that, even in a quite standard model with a unique REE and without the more exotic effects just described, constant-gain learning has significant implications for optimal policy. This is the important new finding that is demonstrated in the current paper by OW.

Some Further Thoughts on Rationality

In constructing economic models we have three kinds of agents: (a) private agents, (b) policymakers, and (c) economists (us). In the bad old days of adaptive expectations, private agents made systematic mistakes, but we the economists were very smart. We told policymakers what to do, so they were smart too.

The RE revolution changed all this. Now private agents became smart, and policymakers (and earlier economists) were mistaken, as shown by the Lucas critique. As theorists we were again smart (because we understood how private agents really formed expectations), but as econometricians we were not quite so smart. This is because as econometricians we had to estimate parameters that were known with certainty by the private agents and theorists.

The adaptive-learning viewpoint has the enormous advantage over these earlier approaches that it (potentially) achieves greater cognitive consistency between these three kinds of agents. In particular, private agents are modeled as behaving like econometricians—that is, like economists in our forecasting role. Of course, as theorists we still typically analyze models with a specified structure that is effectively known only to us, but at least it can be consistently treated as unknown to private agents, policymakers, and econometricians. Furthermore, the degree of smartness of each group is a matter of choice or judgement for us as theorists.

An important aspect of this "bounded rationality" approach is that many features of RE do carry over to the adaptive-learning approach. For example, the Lucas critique can apply under bounded rationality, as emphasized in Evans and Ramey (2003). The Lucas critique will often arise if agents attempt to forecast in an optimal way, even if they are not perfectly rational in the sense of "rational expectations."

Back to Orphanides and Williams

Returning now to the OW paper, let me make some specific critical comments and suggest some extensions.

1. *The inflation shocks experiment.* My first point concerns the inflation shocks scenario shown in OW's figure 5.3. OW examine a sequence of unanticipated positive inflation shocks starting with $e_1 = 2$ percent and declining to zero over nine (semiannual) periods. My main point is that this is more like a structural shift, and that the effects are the same as a decrease in potential output over four years. This raises several questions that would need to be explicitly addressed in a full treatment of this issue.

Suppose, for example, that e_{t+1} is partly predictable, as seems appropriate for a structural shift, and that the loss function is

$$L = E_0\left\{\sum_{t=0}^{\infty}[(1-\omega)(y_t - y_t^*)^2 + \omega(\pi_t - \pi^*)^2]\right\}.$$

Depending on the source of the shock, policymakers may want to lower their output target y_t^* (to $y_t^* = -\alpha^{-1}e_t$). Even if policymakers continue to set $y_t^* = 0$, policy should take into account expected $e_{t+1} > 0$.

This is perhaps a setup in which it would be particularly fruitful also to incorporate policymaker learning.

2. *Bias toward "hawkishness."* OW show that policymakers should be more hawkish. The intuition for this result is fairly intuitive. A more hawkish (high θ) policy helps to keep inflation expectations π^e_{t+1} "in line" (i.e., closer to RE values). This gives policy an additional role, besides stabilizing y and π, and this additional role means that under perpetual learning it is optimal for policymakers to be more hawkish than they would be, for given policymaker preferences, under RE.

This observation leads naturally to the question of how robust this result is. In particular, in New Keynesian models y^e_{t+1} also matters. The structure in such models is

$$y_t = -\varphi(i_t - \pi^e_{t+1}) + y^e_{t+1} + g_t$$
$$\pi_t = \lambda y_t + \beta\pi^e_{t+1} + \gamma\pi_{t-1} + u_t.$$

Will the presence of y^e_{t+1} in the "IS" curve (the first equation) make the direction of bias for the policymaker ambiguous? The answer is not clear a priori and would need to be explicitly analyzed.

3. *Choice of gain parameter* κ. The value of κ is taken as given and not explained. This is quite standard in the constant-gain learning literature. In one respect this is convenient, since it can then be treated as a parameter to be estimated empirically.

However, one can think about the issue further from a theoretical viewpoint. The most typical rationale for introducing constant gain, as indicated above, is that it is a way of allowing for structural shifts. The choice of κ can then be thought of as providing a balance between tracking and filtering: high values of κ allow the estimator to better track structural change, but with the disadvantage of yielding noisier estimators.

One possibility would then be to explicitly introduce structural shifts into the model and find the optimal value of κ. This type of exercise is done in chapter 14 of Evans and Honkapohja (2001) and in Evans and Ramey (2003). In OW this would add complexity and is unlikely to matter. However, the issue of the optimal choice of gain is likely to become important in future work.

4. *Smarter agents.* Using the bounded rationality approach one can always ask: should the agents be smarter? less smart? This is always a matter of judgment. There are several possible ways in which the private agents in OW could be "smarter." For example, private agents could be modeled as estimating an AR(p) instead of an AR(1). Indeed, one could consider the possibility that the agents choose the lag length p in the same way as an applied econometrician. Similarly, agents might consider forecasting based on a vector autoregression (VAR), perhaps using one of the standard statistics to choose the order of the VAR.

It seems likely that the qualitative results would be unaffected, but it would be of interest to know how the detailed results depend on such specification issues. It might appear unsatisfactory, compared to the lack of ambiguity in the RE approach, to be faced with questions about lag length and model specification. But this is really a strength of the adaptive-learning framework. Econometricians dealing with forecasting and estimation problems inevitably face precisely such issues in practice. It seems absurd to assume that private agents and policymakers have clear-cut answers to problems that in effect remain research issues for us as econometricians.

Conclusions

This is an important paper. Theoretically, Orphanides and Williams provide a new reason for studying adaptive learning, based on optimal policy when agents follow "perpetual learning" rules. From an applied viewpoint, the paper suggests another factor that can generate stagflation, and it provides policy recommendations that are intuitive and plausible. I hope (and confidently anticipate) that the authors (and others) will do more work along these lines.

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Discussion Summary

Lars Svensson remarked that the Orphanides-Williams model provides an important argument for announcing an inflation target: namely, that this simplifies private-sector learning, stabilizes inflation expectations, and thereby allows the central bank to respond less aggressively to inflation than it would have to do otherwise. This should have some bearing on the Federal Reserve's decision on an inflation target. He also suggested that, in addition to the simpler linear policies presented in the paper, the authors should also compute the optimal, nonlinear policies.

Ricardo Caballero commented on the arbitrariness of the particular model of learning used in the paper, and suggested that it would be more convincing to consider forms of learning in which the degree of learning depended upon the magnitude of observed shocks.

Olivier Blanchard pointed out that the aggressive policy responses to inflation are driven by the assumptions about the source of model uncertainty and might be overturned in a setting in which uncertainty about output was more important. In the current U.S. situation, for example, there was greater uncertainty about growth going forward than about inflation.

Frank Smets asked whether the form of adaptive learning used in the paper provided a rationale for price-level targeting.

Donald Kohn argued that output stabilization had an important role to play for agents' ability to learn about permanent income. Moreover, he suggested considering a situation in which both the central bank and private agents were learning, which would allow for private agents' and policymakers' inflation expectations to be different, a situation that had been important between 1994 and 2001.

John Berry stressed that how agents interpreted policy outcomes depended importantly on communication between policymakers and the public. The wage-price controls of 1972, for example, were judged by the press as a failure despite the fact that inflation was merely a few tenths of a percentage point above the announced target rate of 2.5 percent. Based on this experience, he asked whether there was an appropriate role for ambiguity in communications with the public.

Mark Gertler questioned whether the restrictions placed on agents' information sets played an important role in the results, and suggested allowing agents to use lags of both inflation and the output gap in their forecasting rules.

Gregory Mankiw pointed out that the particular value of the gain chosen by the authors was a suspicious free parameter, and he argued that it would be more convincing to derive the optimal gain from explicit modeling of the source of uncertainty, which would lead back to rational expectations.

Christopher Sims argued that the results under learning depended critically on the specific form of the equation that agents are learning about. In the present case, an important question was whether agents had to learn about the intercept or the slope of the Phillips curve, and at which rates they were updating their estimates of either of these parameters.

John Williams responded that their results remained robust even when agents used several lags of both inflation and the output gap in their inflation forecasts, due to the high degree of inflation persistence generated by the model. He argued that, while the precise value of the gain used in learning could be determined inside the model, a constant-gain formula for learning was both realistic and robust. Adding uncertainty about the future output gap was a topic of work in progress, but it should not overturn the results presented in their paper because of the important role played by the persistence of inflation in deriving inflation forecasts.