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Chapter Author: J. Phillip Cooper, Stanley Fischer

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STABILIZATION POLICY AND LAGS: SUMMARY AND EXTENSION*

BY J. PHILLIP COOPER AND STANLEY FISCHER

This paper examines the effects of both the length and variability of lags on the effectiveness of countercyclical stabilization policy. The authors conclude that while the latter are an argument in favor of less vigorous use of stabilization policy, the former are not. The longer are lags, the more vigorously should stabilization policy be used. They also find that in their models, the constant growth rate value is never optimal and that the careful use of feedback controls is bound to be stabilizing.

INTRODUCTION

The major aim of this paper is to study the effects of both the length and variability of lags on the effectiveness of countercyclical stabilization policy. The chief tool of analysis is a simple difference equation, in which the value of a target variable (y) is determined as a function of its lagged value and concurrent and lagged values of a policy variable (x) as well as an additive stochastic term (u); the policy variable (x_t) is taken to be determined by a closed-loop feedback control rule responding to the lagged value of the target variable (y_{t-1})—proportional control—and the change in the value of the target variable ($y_{t-1} - y_{t-2}$)—derivative control.

The effectiveness of stabilization policy is evaluated by the value of the asymptotic variance of the target variable under the rules; various parameters of the difference equation determine both the mean length and the variability of the lags in the effect of policy. We are thus able to examine the results of changes in the length and variability of lags on the effectiveness of policy as policy is adjusted optimally (with respect to minimization of asymptotic variance) in response to these parameter changes. Our interest is not, however, confined to optimal policies and we also investigate other properties of the system, such as its stability and sensitivity to nonoptimal choice of control rules, as lags vary.

In Section 1 below we very briefly summarize results obtained in our "CC" (constant coefficients) model in which all lag parameters are constant. The notion of variable lags and our representation of the notion through the randomizing of lag coefficients are discussed in Section 2, when our "RC model" is introduced.[1] The effects of the variability of lags—as measured by the variance of the lag coefficients—on the outcome of policy rules is examined in Section 3. There is discussion in Sections 2 and 3 of the merits of a completely inactive policy which avoids any attempts at "fine tuning"—such policies have been recommended to the monetary authorities by Friedman [2] and others.

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[1] Although the CC model is a special case of the RC model, it is convenient to treat them separately so that the effects of the length of lags can be discussed apart from the effects of their variability. In addition, there are certain results which we obtain analytically for the CC model but numerically for the RC model.

This paper is a much-abbreviated summary of the paper presented at the NSF-NBER Conference on Control Theory and Economic Systems which is forthcoming in the *Journal of Political Economy.* Aside from the fact that many results are summarized but not fully developed here, the other major change between the Conference paper and this one is that we here use a different stochastic process for the behavior of the random lag coefficients. The process, described here as the "random β" case, is one which we now regard as a fairer representation of the notion of variable lags in the context of discussion of the relative merits of active and inactive countercyclical policies than the "random λ" case presented in the Conference paper. Our reasons for this view are discussed later.

1. The CC Model

A. Model Description

The model with constant coefficients is a standard first-order autoregressive scheme.

$$(1) \qquad y_t = \beta y_{t-1} + \sum_{i=0}^{\infty} \alpha_i x_{t-i} + u_t.$$

The restriction to a first-order autoregressive process is made for simplicity.

The variable y_t represents deviations of some economic variable from its target level in each period and will be referred to as "output"; x_t is to be understood as the deviations of some relevant instrument or policy variable (say, the rate of change of the money supply) from that path which would, in the absence of disturbances, keep the system on target at all times. Equation (1) may represent the reduced form of some structural model in which there is only one controllable exogenous variable. The value of u_t is not known at the time the current value of the policy variable, x_t, is chosen; information available at the time x_t is chosen consists of past levels of output and of the policy variable itself. The random variable u_t has mean zero, is serially uncorrelated, and without loss of generality has variance unity. It is assumed that $|\beta| < 1$ so that the system is stable in the absence of an active stabilization policy (i.e. if $x_t = 0$ for all t); generally we assume β positive.

The time form of the lag coefficients for the effects of policy, that is the α_i of (1), is assumed proportional to a density function belonging to the Pascal family [5].

$$(2) \qquad \alpha_i = \alpha \binom{r + i - 1}{i} (1 - \lambda)^r \lambda^i \qquad \begin{array}{l} r = 1, 2, 3, 4, \ldots \\ 0 < \lambda < 1. \end{array}$$

The parameters r and λ determine the structure of the coefficients α_i; we concentrate on the cases $r = 1$ and $r = 2$, particularly in the RC model below, but results holding for all members of the Pascal family are given in this section.

To standardize the long-run multiplier for monetary policy at unity, we set α in (2) equal to $(1 - \beta)$. It may be confirmed that then the ultimate effect on the level of y obtained by increasing x by one unit and holding x at its higher level forever, is to increase y by one unit, independent of the values of λ and β. We are thus assured that the "bang per buck" of policy stays constant for any permanently held values of λ and β.

Two comments: first, the α_i—which we call the "*direct*" (or "*policy*") lag coefficients—do not give the total effects on y_t of a unit input of x at time $t - i$, for changes in x at $t - i$ change output at time t through the autoregressive parameter β as well as directly. The level of output as a function of past levels of the policy variable and the random variable is

(3) $$y_t = \sum_{i=0}^{\infty} \hat{\alpha}_i x_{t-i} + \sum_{i=0}^{\infty} \beta^i u_{t-i}$$

where

(4) $$\hat{\alpha}_i = \sum_{j=0}^{i} \beta^j \alpha_{i-j}.$$

This is a convolution of the previous lag coefficients and we refer to the $\hat{\alpha}_i$ as the "*final form*" *lag distribution*. For $\beta = \lambda$, the final form lag coefficients are simply Pascal of order one higher than the order of the distribution for the α_i themselves.

Second, we use a particular structure for the α_i and a particular autoregressive structure in order to study the effects of the length of lags on stabilization policy; the mean final form lag of the effect of x on y is $\beta/(1 - \beta) + (r\lambda)/(1 - \lambda)$. The length of lag is thus an increasing function of r, λ and β.

The mean final form lag is the sum of the lag due to the autoregressive structure (the "*system*" lag) and that due to the policy lag. Thus, by distinguishing β from λ, we can discuss separately the effects of lags which are inherent in the economy (β) from those due to policy (λ and r). A long system lag (large β) automatically implies a long final form lag though policy may work slowly even if β is small.

B. The Constant Growth Rate Rule (CGRR)

We describe the policy $x_t = 0$ for all t as CGRR, i.e. a policy where no attempt is made to respond to deviations of y from trend. The asymptotic variance of output under a constant growth rate rule is

(5) $$\sigma_y^2 = \lim_{t \to \infty} E[y_t^2] = \frac{1}{1 - \beta^2}.$$

The minimal attainable variance of y_t is unity, obtained under any policy which succeeds in making $y_t = u_t$ for all t. Thus if $\beta = 0$, the optimal policy is CGRR. If β is not zero, there is room for improvement by use of some policy other than CGRR—the potential improvement increasing with $|\beta|$. It is useful to interpret β as a measure of the instability of the system in the absence of stabilization policy in much of what follows, the instability increasing as the system lag increases.

We shall refer to any policy which produces $y_t = u_t$ as *perfect control*. All other policies are *imperfect control*.

C. Policy Rules

The policy rules used are of the form

(6) $$x_t = \Gamma(B) \cdot By_t$$

where B is the backshift operator, and $\Gamma(B)$ is a polynomial in B of order n. For instance, one such rule with $n = 1$ is

$$x_t = \gamma_1 y_{t-1} + \gamma_2 y_{t-2} = g_1 y_{t-1} + g_2(y_{t-1} - y_{t-2}) \tag{7}$$

where g_1 is a proportional control and g_2 a derivative control.

Substituting (6), (5) and (2) in (1), and using the operator B, one obtains for the general Pascal distribution:

$$y_t = \frac{(1 - \lambda B)^r u_t}{(1 - \lambda B)^r(1 - \beta B) - (1 - \lambda)^r(1 - \beta)\Gamma(B)B} \tag{8}$$

which is an autoregressive moving-average model of order $(r + 1, r)$ when $r \geq n$. By setting $n = r$ and choosing the coefficients in $\Gamma(B)$ appropriately it is always possible to obtain

$$y_t = u_t \tag{9}$$

which minimizes asymptotic variance—and also, any criterion function including only variances of output in each period. Thus optimal policies in the CC model are straightforward to obtain.[2]

To have a better idea of the properties of such policies, we turn for simplicity to the case $r = 1$, although similar results apply also for other values of r. The optimal policy for $r = 1$ is to use rule (7) with

$$\begin{aligned} g_1 &= -\frac{\beta}{1 - \beta} \\ g_2 &= -\frac{\beta\lambda}{(1 - \beta)(1 - \lambda)}. \end{aligned} \tag{10}$$

There are a number of interesting features of the rule (10).

(i) In this model, with perfect control, the proportional control depends only on the autoregressive parameter, and, in a sense, offsets the autoregressive component of the model, while the derivative control deals also with the direct lagged effects of policy.

(ii) Perhaps most interesting, the strength of the controls is an increasing function of the average length of lag, but increases in the length of lag do *not* increase the variance of output.[3]

(iii) The use of negative feedback controls cannot lead to the minimum variance policy if $\beta < 0$, that is, if the model itself, in the absence of control, contains only negative feedback.

We are also interested in the behavior of the system when policy is nonoptimal. Accordingly, we solved analytically for the value of the asymptotic variance of the system as a function of β, λ and the parameters in the control rule, for values

[2] It is easy to show that for perfect control,

$$\Gamma(B) = -\frac{\beta}{1 - \beta}\frac{(1 - \lambda B)^r}{(1 - \lambda)^r}$$

[3] Obviously the second half of this sentence must be true if perfect control can be attained. A stronger result is obtained in Howrey [3].

of $r = 1$ and $r = 2$. This expression is used in defining the stability conditions —in terms of β, λ and the control parameters—for the system; it is also used to define a region containing pairs of values of g_1 and g_2 which improve upon CGRR. For the cases $r = 1$ and $r = 2$ the following additional results are obtained.

(iv) The longer are both policy and system lags, the more likely is the system to be stable for given values of the control parameters.

(v) Weak negative feedback controls are bound to be stabilizing relative to CGRR (for $\beta > 0$).

(vi) The longer are both system and policy lags, the more likely is any choice of control parameters to be stabilizing relative to CGRR.

(vii) In cases where insufficient control parameters are used, long lags reduce the potential gains from an active stabilization policy relative to CGRR; however, they increase the likelihood that any given policy will be stabilizing relative to CGRR.

2. Variable Lags and the RC Model

Equation (1), the basic difference equation, can be rewritten as

$$y_t = \beta y_{t-1} + w_t + u_t \tag{11}$$

where w_t represents the total direct effects of policy, past and present. For the Pascal distribution with $r = 1$, the case on which we concentrate in this section,

$$w_t = (1 - \beta)(1 - \lambda) \sum_{i=0}^{\infty} \lambda^i x_{t-i} = (1 - \beta)(1 - \lambda)x_t + \lambda w_{t-1}. \tag{12}$$

In our random coefficients model, we continue to use (11) and (12) but modify them by making β a random variable. This has the effect of making both the autoregressive component and the effects of policy random. Specifically, we write β_t instead of β in both (11) and (12), and assume

$$\beta_t = \beta + \varepsilon_t \tag{13}$$

where ε_t has mean zero, variance σ^2, is serially uncorrelated and has zero covariance with all u_t. Substituting β_t for β in (11) and (12), the final form lag coefficients, $\hat{\alpha}_{it}$, which give the total effects on the level of output in period t of a unit change in x at time $t - i$, are

$$\hat{\alpha}_{it} = (1 - \beta_{t-i})(1 - \lambda) \sum_{j=0}^{i} \lambda^j \prod_{m=1}^{i-j} \beta_{t-m+1}. \tag{14}$$

In Figure 1 we present final form lag distributions generated for the RC model with $\lambda = 0.8$, β (now the mean of the distribution of β_t) $= 0.5$, and $r = 1$. The β_t used in Figure 1 were drawn from a beta distribution—for which the domain is $[0, 1]$— with the variance σ^2 stated on the diagram. The three cases shown were chosen from a set of ten distributions generated and represent the range of examples produced.

The formulation described above is for obvious reasons called the "random β" case. In the Conference paper we used the "random λ" case in which λ in (12)

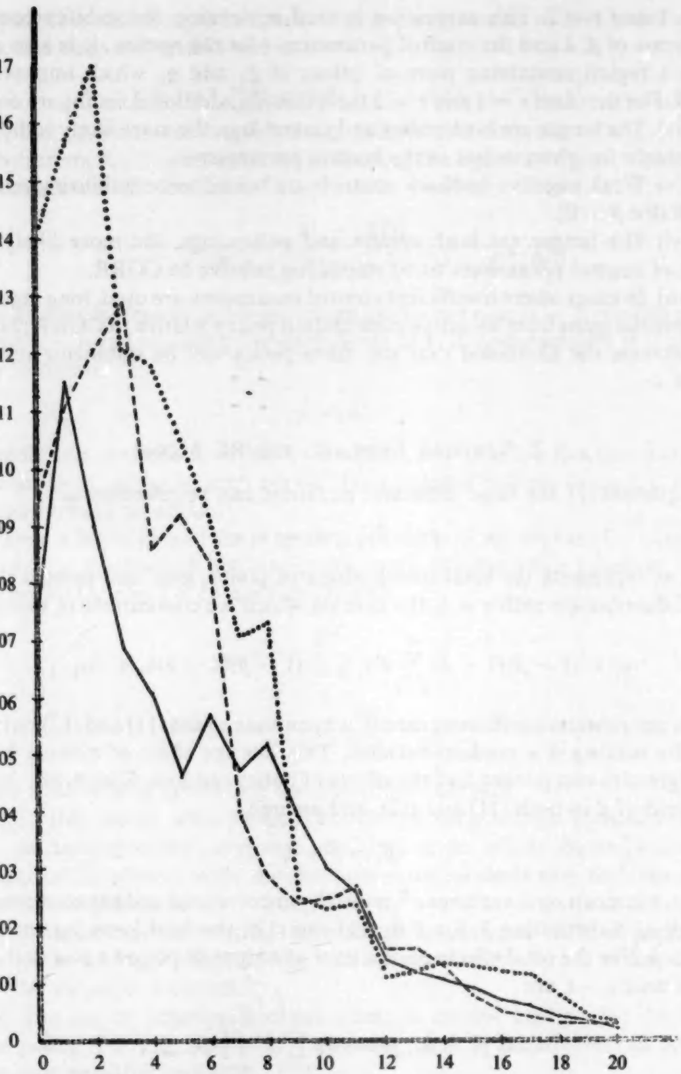

Figure 1 Three Realizations of Final Form Lag Distributions ($r = 1$, $\lambda = 0.8$, $\beta = 0.5$, $\sigma^2 = 0.0278$)

is a random variable. In the random λ case, randomness of the lag coefficients affects in the current period only the results of active countercyclical policy and not CGRR; in the random β case the results of *both* types of policy are affected. It can be shown that under CGRR

$$\sigma_y^2 = \frac{1}{1 - \beta^2 - \sigma^2}. \tag{15}$$

We believe it fairer to active policy to use the random β model since we do not believe that CGRR would lead to any less variability of behavioral parameters than other rules.

We believe that the lag formulations of (14), as shown in Figure 1, reflect the notion of "variable lags." The time pattern of the effects of any particular policy action is not likely to be the same as those of any other policy action; the consequences of any particular policy action are known with certainty neither in the period in which they are taken, nor in subsequent periods.

Our representation of variable lags treats these lags as stochastic, but variability of lags is possible in a deterministic model. For instance, the lags of monetary policy could vary systematically with the behavior of other exogenous variables in the economy, as they do in the FRB-MIT-Penn Econometric model. One might want to model variable lags by, say, having β_t be a function of a variable, the time path of which is specified in some suitable way. This is another possible route, but it is not one we have so far taken.

For the case $r = 2$, which we have also examined we have instead of (12)

$$w_t = (1 - \beta)(1 - \lambda)^2 x_t + 2\lambda w_{t-1} - \lambda^2 w_{t-2} \tag{16}$$

and then both the β in (11) and that in (16) become β_t, with β_t determined as in (13).

Finally, we note an important point: our basic assumption for the RC model is that the lags are "truly" stochastic—the distribution of the β_t is specified for all time.

3. Rules and the Variance of Output in the RC Model

For the case $r = 1$, using (11) and (12) and the policy rule (7), with $\beta_t = \beta + \varepsilon_t$, we obtain

$$\begin{aligned} y_t - [\beta_t + \lambda + (1 - \beta_t)(1 - \lambda)\gamma_1]y_{t-1} + [\lambda\beta_{t-1} - (1 - \beta_t)(1 - \lambda)\gamma_2]y_{t-2} \\ = u_t - \lambda u_{t-1} \end{aligned} \tag{17}$$

or

$$\begin{aligned} y_t - by_{t-1} + cy_{t-2} - (1 - (1 - \lambda)\gamma_1)\varepsilon_t y_{t-1} + \lambda\varepsilon_{t-1} y_{t-2} + (1 - \lambda)\gamma_2 \varepsilon_t y_{t-2} \\ = u_t - \lambda u_{t-1} \end{aligned} \tag{18}$$

where

$$b = \beta + \lambda + (1 - \beta)(1 - \lambda)\gamma_1$$

$$c = \lambda\beta - (1 - \beta)(1 - \lambda)\gamma_2.$$

The question of the stability conditions for equation (18) now arises. There are a number of concepts of stochastic stability,[4] and we shall use the finiteness of the asymptotic variance of output as our criterion; for $\sigma^2 = 0$ this gives the same stability conditions as those for the CC model. This is a convenient definition in view of the fact that we evaluate policies by this same criterion.

[4] See Kozin [4] for discussion of some of these concepts.

Deriving the asymptotic variance of y_t from (18), we obtain

$$(19)\quad \sigma_y^2 = \frac{(1+c)(1+\lambda^2) - 2\lambda b - \sigma^2[a_{22}(1-\lambda(b-\lambda)) - a_{12}\lambda]}{(1-c)[(1+c)^2 - b^2] - \sigma^2[(1+c)a_{11} - ba_{12} - b(1-c)a_{21} + (1-c^2)a_{22} - \sigma^2[a_{11}a_{22} - a_{21}a_{12}]]}$$

where

$$a_{11} = (1-(1-\lambda)\gamma_1)(1-(1-\lambda)\gamma_1 - \lambda b) + (1-\lambda)^2\gamma_2^2 + \lambda^2$$
$$a_{12} = (1-\lambda)\gamma_2[2(1-(1-\lambda)\gamma_1) - \lambda b], \qquad a_{21} = \lambda[1-(1-\lambda)\gamma_1],$$
$$a_{22} = \lambda(1-\lambda)\gamma_2.$$

It is possible—though very tedious—to show that

$$(20)\quad \left.\frac{\partial g_1^*}{\partial \sigma^2}\right|_{\sigma^2=0} \gtrless 0 \qquad \left.\frac{\partial g_2^*}{\partial \sigma^2}\right|_{\sigma^2=0} > 0 \qquad \left.\frac{\partial g_1^*/g_2^*}{\partial \sigma^2}\right|_{\sigma^2=0} > 0 \qquad \text{for } \beta > 0$$

where g_1^* and g_2^* are the optimal proportional and derivative controls. Thus, the presence of slight variability of the lag coefficients leads to weaker derivative and relatively stronger proportional controls than would be optimal in the absence of the variability. (The proportional control may actually increase absolutely.) Basically, feedback controls use the level of output and changes in output as guides to the behavior of the additive error term. When lag coefficients become variable, the level of output becomes a less safe guide to the behavior of the additive error—but the change in output is doubly less safe.[5] Thus, relatively more weight is thrown on the proportional control.

It is clear that, in the RC model, we do not obtain certainty equivalence results. This is a consequence of the fact that the current policy variable, x_t, affects current income subject to a multiplicative error.[6]

It can also be shown—once more at some length—that

$$(21)\quad \left.\frac{\partial \sigma_y^2}{\partial g_1}\right|_{\substack{g_1=0\\g_2=0}} \sim \beta \qquad \left.\frac{\partial \sigma^2}{\partial g_2}\right|_{\substack{g_1=0\\g_2=0}} \sim \beta \qquad \text{where } \sim \text{ means "of the same sign as."}$$

That is, the use of weak proportional or derivative controls is bound to be stabilizing relative to CGRR if β is positive—whatever the variability and length of the lags.

[5] This explanation requires the first autocovariance of income to be small, which it is at $\sigma^2 = 0$ *and* with optimal control. (In fact, there is zero autocovariance at this point.)

[6] See Brainard [1] for a fuller discussion of circumstances under which certainty equivalence is obtained. Our rules which give perfect control in the CC model are certainly equivalence rules. Also, if in (11), we had made the first β (that multiplying y_{t-1}) stochastic, and had otherwise had constant coefficients, we would have obtained the same rules for the RC model as for the CC—which illustrates the certainty equivalence principle.

It is, unfortunately, difficult to minimize (19) analytically with respect to γ_1 and γ_2 to study the behavior of the system. Accordingly, we have used (19) to compute optimal rules, stability regions and isovariance loci numerically for a number of combinations of λ, β and σ^2. In Figure 2 we present a typical diagram —for the case $\lambda = 0.8, \beta = 0.5, \sigma^2 = 0.0278$—produced in our numerical analysis; the large shaded area is the stability region in that values of g_1 and g_2 outside that area make the system completely unstable; the inner drawn locus is the CGRR isovariance locus—values of g_1 and g_2 within this region reduce variance

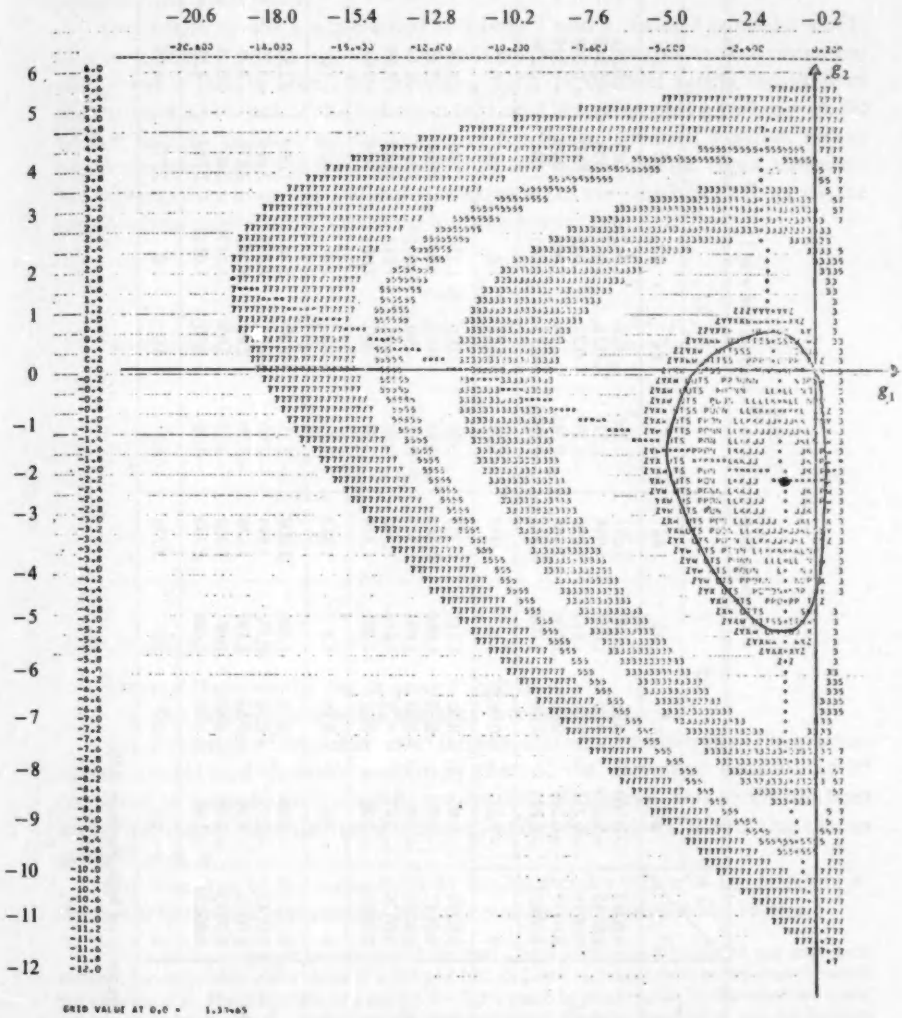

Figure 2 Stability Area and Isovariance Contours for $\beta = 0.5, \lambda = 0.8, r = 1$, and $\sigma^2 = 0.0278$

TABLE 1

OPTIMAL CONTROLS AND VARIANCES FOR $r = 1$*

$\beta =$	$\lambda =$ / $\sigma^2 =$	0.2				0.5				0.8			
		g_1^*	g_2^*	σ_*^2	σ_0^2	g_1^*	g_2^*	σ_*^2	σ_0^2	g_1^*	g_2^*	σ_*^2	σ_0^2
0.2	0.0267	−0.200	−0.05	1.042	1.071	−0.210	−0.18	1.042	1.071	−0.231	−0.655	1.045	1.071
	0.0145	−0.225	−0.05	1.023	1.058	−0.225	−0.21	1.023	1.058	−0.243	−0.795	1.025	1.058
	0.0076	−0.235	−0.06	1.012	1.050	−0.235	−0.23	1.012	1.050	−0.248	−0.880	1.013	1.050
	0.0035	−0.245	−0.06	1.006	1.045	−0.245	−0.24	1.006	1.045	−0.249	−0.950	1.006	1.045
	0.00032	−0.250	−0.06	1.001	1.042	−0.250	−0.25	1.001	1.042	−0.250	−1.000	1.001	1.042
	0	−0.25	−0.0625	1.0	1.042	−0.25	−0.25	1.0	1.042	−0.25	−1.0	1.0	1.042
0.5	0.0278	−0.80	−0.20	1.113	1.385	−0.86	−0.65	1.123	1.385	−1.09	−2.15	1.161	1.385
	0.0147	−0.90	−0.20	1.060	1.360	−0.93	−0.80	1.067	1.360	−1.15	−2.75	1.093	1.360
	0.0076	−0.93	−0.25	1.031	1.347	−0.96	−0.90	1.035	1.347	−1.13	−3.25	1.052	1.347
	0.0035	−0.97	−0.25	1.014	1.340	−0.98	−0.95	1.016	1.340	−1.09	−3.60	1.025	1.340
	0.00031	−1.00	−0.25	1.001	1.334	−1.00	−1.00	1.001	1.334	−1.01	−3.95	1.002	1.334
	0	−1.0	−0.25	1.0	1.333	−1.0	−1.0	1.0	1.333	−1.0	−4.0	1.0	1.333
0.8	0.0267	−2.05	−0.35	1.680	3.000	−2.15	−1.10	1.761	3.000	−2.55	−3.00	2.022	3.000
	0.0145	−2.75	−0.50	1.373	2.894	−2.90	−1.75	1.439	2.894	−3.75	−5.25	1.658	2.894
	0.0076	−3.30	−0.65	1.197	2.838	−3.50	−2.40	1.243	2.838	−4.75	−7.95	1.400	2.838
	0.0035	−3.65	−0.80	1.091	2.805	−3.85	−3.05	1.117	2.805	−5.15	−10.80	1.213	2.805
	0.00032	−3.95	−1.00	1.008	2.780	−4.00	−3.90	1.011	2.780	−4.35	−15.15	1.025	2.780
	0	−4.0	−1.0	1.0	2.778	−4.0	−4.0	1.0	2.778	−4.0	−16.0	1.0	2.778

* Precision on these numbers differs.

below that obtained under CGRR; plus signs (+) and asterisks (*) trace the loci on which the first-order conditions for g_1 and g_2, respectively, are satisfied; the optimal values of g_1 and g_2 are, of course, at the intersection of these two loci.

We produced such diagrams for values of λ and β of 0.2, 0.5, and 0.8, and six values of σ^2 for each of the nine combinations of λ and β. In each case we took β_t as belonging to the beta distribution, and computed the variance of β_t for a number of integer-valued parameters of that distribution.[7] It is perhaps worth emphasizing that the results presented below are *not* simulation results—we use the analytical expression (19) for the asymptotic variance of output to compute optimal rules numerically.

Our major results are presented in Tables 1 and 2. Table 1 contains results for $r = 1$, Table 2 results for $r = 2$. The four entries in a row for each combination of λ, β and σ^2 are, in order, the optimal g_1 (g_1^*), the optimal g_2 (g_2^*), the value of the variance of output at the optimum (σ_*^2), and the value of the variance under CGRR (σ_0^2). In addition to the results of the tables, we shall mention results based on examination of the diagrams such as Figure 2 for the cases presented. We consider now in turn the effects of changes in (A) the variability of lags, (B) the length of the policy lag, (C) the length of the system lag.

TABLE 2

OPTIMAL CONTROLS AND VARIANCES FOR $r = 2$, $\lambda = 0.8$, $\beta = 0.5$

		$\lambda = 0.8$			
		g_1^*	g_2^*	σ_*^2	σ_0^2
$\beta = 0.5$	$\sigma^2 = 0.0278$	−0.5	−2.75	1.32	1.39
	0.0147	−0.6	−4.5	1.26	1.36
	0.0076	−0.7	−6.75	1.21	1.35
	0.0035	−0.6	−9.0	1.17	1.34
	0.00031	−0.4	−13.5	1.12	1.34
	0	−0.36	−14.4	1.11	1.33

A. Variability of Lags

Most of these results are in accord with intuition.

(i) The minimal attainable variance increases with σ^2.

(ii) For small σ^2, increases in σ^2 increase the relative strength of the proportional control and decrease absolutely that of the derivative control; as σ^2 continues to increase both controls are reduced absolutely. This result has been explained above. Note that the controls γ_1 and γ_2 (equation (7)) both decrease in strength with σ^2.

(iii) The area of the outer stability region shrinks with σ^2—the larger is σ^2 the more likely is any particular pair of controls to destabilize the system.

[7] We used those integer parameters of the beta distribution which produced the maximum variance for each value of the mean ($\beta = 0.2$ and 0.8), and then increased these parameters to reduce the variance of β_t. The maximum variance for $\beta = 0.5$ is much larger than that for the other two cases; this larger variance, 0.0833, results from the degeneration of the beta distribution into the uniform distribution. We do not present this case in Table 1 or Table 2.

(iv) Similarly, the area contained within the CGRR locus shrinks with σ^2—the more variable the lags, the less likely is any arbitrary pair of controls to outperform CGRR. But recall that—from (21)—the use of weak controls is bound to be stabilizing.

B. The Length of the Policy Lag, λ and r

(i) For given σ^2 and β^2, the minimal attainable variance increases with λ and r; thus the longer the lag of policy, for given β and σ^2, the smaller the potential gains from the use of active stabilization policy relative to CGRR.

(ii) The optimal controls increase with λ; thus the longer are policy lags the more vigorous should policy be—although the potential gains from attaining the optimal policy fall with λ. For $r = 2$, proportional control is weaker than for $r = 1$ while derivative control is stronger.

(iii) The area contained within the outer stability region increases with λ and r, for given β and σ^2.

(iv) The area contained within the CGRR locus increases with λ and r, for given β and σ^2.

C. The Length of the System Lag

(i) The minimal variance attainable increases with β. But note that, surprisingly, the ratio of the minimal attainable variance to the variance under CGRR decreases with β: the *longer* the system lag, the *greater* the potential gain from the use of active stabilization policy.

(ii) Each optimal control is an increasing function of the length of lag.

(iii) The areas contained within the stability region and the CGRR locus increase with β—the longer the system lag, the less likely is any given policy to be destabilizing, either absolutely or relative to CGRR.

In brief, our conclusions are that while variability of lags is an argument in favor of the less vigorous use of stabilization policy, the length of lags is not. Indeed, generally the longer are both policy and system lags, the more vigorously should stabilization policy be used. Further, we nowhere found CGRR optimal and we have shown that, in our models, the careful use of negative feedback controls is bound to be stabilizing.

University of Chicago

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