THE SMOOTHING OF TIME SERIES
National Bureau of Economic Research, Inc.
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THE SMOOTHING OF TIME SERIES

By
FREDERICK R. MACAULAY

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NATIONAL BUREAU OF ECONOMIC RESEARCH
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F. R. M.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>17</td>
</tr>
<tr>
<td>I. The Smoothing of Economic Time Series. Curve Fitting and Graduation</td>
<td>31</td>
</tr>
<tr>
<td>II. Graduating by Simple Moving Averages and by the Mid Ordinates of Third-degree Parabolas Fitted by the Method of Least Squares</td>
<td>43</td>
</tr>
<tr>
<td>III. Third-degree Parabolic Graduation Formulas Having Smooth Weight Diagrams. Summation Formulas</td>
<td>51</td>
</tr>
<tr>
<td>IV. Fifth-degree Parabolic Summation Formulas for Graduation</td>
<td>63</td>
</tr>
<tr>
<td>V. Dr. E. C. Rhodes' Graduation Proposal</td>
<td>81</td>
</tr>
<tr>
<td>VI. The Whittaker-Henderson Method of Graduation</td>
<td>89</td>
</tr>
<tr>
<td>VII. Desirable Characteristics of Formulas for Graduating Monthly Time Series</td>
<td>100</td>
</tr>
<tr>
<td>VIII. The Extension of Summation Formulas to Cover the Range of the Data</td>
<td>113</td>
</tr>
</tbody>
</table>
APPENDICES

APPENDIX PAGEx

I. The Measurement of Probable Seasonal Fluctuations by Means of Operations on the Deviations of the Data from Graduated Curves. The Elimination of Seasonal Fluctuations before Graduation 121

II. A Computation Sheet Illustrating Graduation by Spencer's 15-term Third-degree Parabolic Formula 137

III. Three Sets of Twenty-five Term Third-degree Parabolic Graduation Weights 141

IV. The Weights Implied in Various Graduation Formulas 143

V. Seven Graduations of Dr. E. C. Rhodes' Test Mortality Data 149

VI. Paradigm for Graduating by Robert Henderson's Method 151

VII. The Results of Applying Nineteen Different Graduation Formulas to Equidistant Points on Indefinitely Extended Sine Series 157

VIII. Fourteen Different Graduations of Call Money Rates on the New York Stock Exchange for 97 Months—January 1886 to January 1894 inclusive 161
LIST OF CHARTS

I. Graduation Weight Diagrams 77
II. Whittaker-Henderson Graduation fitted to Dr. E. C. Rhodes' Mortality Data 84
III. Illustration of Robert Henderson's Method of Graduation 91
IV. Call Money Rates, Twelve-months Moving Average and 43-term Cyclical Graduation 108
V. A Criterion of Goodness of Fit. Twelve-months Moving Averages of the Three Graduations in Chart IV 109
VI. Call Money Rates and two Whittaker-Henderson Graduations 110
VII. Call Money Rates, Kenchington's 27-term Graduation and Spencer's 21-term Graduation 111
VIII. Call Money Rates Adjusted for a Constant Seasonal Fluctuation. 43-term Cyclical Graduation and Spencer's 15-term Graduation of the Adjusted Data 126
IX. Call Money Rates. 43-term Cyclical Graduation and Spencer's 15-term Graduation of Rates Adjusted for a Constant Seasonal 127
X. Call Money Rates, Spencer's 15-term Graduation of (1) the Unadjusted Rates (2) the Rates Adjusted for a Constant Seasonal Fluctuation and (3) the Rates Adjusted for a Changing Seasonal Fluctuation. The Changing Seasonal Fluctuations... 131
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—Sartor Resartus.