function derived from data for one particular enterprise necessarily reflects the prevailing conditions of production in it, since these conditions represent a selection from among numerous possible forms of organization. Whether the influence of rigidities is sufficiently great to cause the shape of the empirical functions to differ essentially from the theoretical functions can be ascertained only after continued research in the field. Third, a set of problems arises from the attempt to approximate the static conditions assumed in theory. Any firm selected for study operates in a changing environment to which it continually adjusts. Most important are the variations in the prices of factors of production, which, unlike the influence of rigidities, can be removed from the cost data by a process of 'deflation'.

In general, the divergence between the production process analyzed and the theoretical situation assumed to exist can be minimized by three methods: (1) careful selection of a sample, with due regard to both the firm and the time period chosen, in order to reduce the influence of dynamic elements; (2) rectification of the data to remove directly the influence of disturbing factors that could be measured adequately; (3) multiple correlation analysis of the relation between cost and other variables whose influence was not directly measurable, in order to examine and remove their effects.

The application of these three methods is discussed in Sections 3, 4, and 5. The statistical findings are presented in Section 6 and their validity appraised in Section 7. The concluding section attempts to interpret and qualify the findings, reconciling them with the results prescribed by the static theory of cost behavior.

3 Collection of Data

It is much easier to isolate the relation between cost and output if the observations in the statistical sample are as free as possible from the influence of other variables affecting cost. We were fortunate, in this regard, to secure the active cooperation of a leather transmission belting manufacturer, one of whose plants met our sampling requirements admirably. In addition, the nature of the accounting data of the firm facilitated statistical analysis, since costs were kept in considerable detail as totals of expense for four-week accounting periods. Complete records of output measured in several different ways were available, as well as supplementary information concerning operating conditions affecting costs. All these records were comparable for several years.

The assumption, discussed above, under which short-run cost curves are drawn is that changes in the rate at which the plant is utilized are not accompanied by changes in the scale of the plant, in the technical methods of production, or in prices of input factors sufficiently great to induce
substitution among the factors of production. These conditions seemed to be approximately fulfilled by the firm during the period studied. Changes in the level of cost and output were frequent and large, as can be seen from Chart 9, and were unaccompanied by changes in scale or in basic methods of operation. Moreover, the technical processes were such as to make it unlikely that the technical coefficients of production would be altered significantly by changes in the price of input factors. As a rule, short-run cost functions are easier to determine for typical establishments in declining industries. Capital investment during the analysis period is usually small, output being less than plant capacity. Since, logically, capital is retired only when there is a probability that sales will not cover variable cost and return on the liquidation value of the equipment, changes in scale are minimized. Moreover, it is reasonable to suppose that technical progress is less likely to become a complication of any significance.

Analysis of cost is simplified when a single physically homogeneous product is manufactured. Difficult problems arise when cost must be apportioned among several products because of joint processes or common use of production facilities. In the leather belt plant the assumption of product homogeneity was not altogether justified, for not only were several grades of belting made but there were also certain highly dissimilar minor products. Nevertheless, the differences in grades and sizes of belting did not appear to affect the cost functions significantly. Since the products other than leather belting constituted a relatively small share of total output and were made in separate departments, the cost attributable to them was isolated and omitted from the analysis.

A further difficulty in analyzing cost arises when there is a pronounced and irregular lag between the incurring of cost and the recording of the corresponding output. The concern here is not with the engineering problem of determining the length of the production cycle, but with the statistical problem of correcting for a lag arising from the practice of recording cost in the period in which it occurred, instead of at the time the output to which it contributed was completed. This lag-correctation may become complicated when it is necessary to readjust each element of expense. Corrections for this lag become progressively less reliable as the production cycle lengthens. In the belt shop, however, the production cycle was short — a few days only.

7 Changes in factor prices resulting from changes in the firm's own output are, of course, not excluded from the discussion of cost theory in general although they were omitted from the models outlined above. The firm did not appear to be in a monopolistic position, i.e., so influential in the factor markets that the prices paid for materials and labor were affected by the quantities of these factors purchased.

8 The effects of variation in the composition of the output were tested by introducing as independent variables in a multiple correlation analysis various aspects or dimensions of the product, such as width, thickness, and weight per square foot. This procedure is described in Section 5.
Another reason for selecting this particular plant was that the operating conditions causing cost variation were few and those exerting the greatest influence on cost were measurable.

Selection of Sampling Period

The period January 1, 1935 to June 1, 1938 was chosen for study because it fulfilled the following conditions most satisfactorily:

1) The rate of output and other measurable cost determinants varied sufficiently to yield observations over a wide range and to afford fairly uniform coverage of this range.

2) Detailed and complete records of variations in cost and in the chief operating variables affecting cost were available.

3) The plant and equipment remained unchanged during the analysis period, permitting the observation of short-run adjustments uninfluenced by long-run changes.

4) Since technical methods of production were constant, the cost-output relation was not obscured by simultaneous changes in production methods.

5) The data were sufficiently recent to allow their use for cost forecasting and budgeting.

The four-week accounting period used by the company in its cost records and production plans was taken as the observation time unit for this study. During the three and a half years of observation, there were forty-five such accounting periods or 'months'. Since any theoretical analysis is properly in terms of rates of output, it is desirable in empirical studies of this kind to choose an observation unit in which the flow of output is uniform. In general the shorter the period the more likely is this condition to be fulfilled. In the plant studied there was evidence that fluctuations in the production flow within the four-week planning and recording period were small. Attempts to allow for the variation that did exist are discussed in Section 5.

Selection of cost elements

In selecting elements of cost for the analysis, we omitted expenses that were arbitrarily allocated to the leather belt shop and that bore no apparent relation to its operating conditions. For example, certain general overhead and head office and administrative expenses, which were distributed among the various plants operated by the firm, were excluded.

To ascertain whether the omission of overhead common to several plants led to an underestimate of the marginal cost of the leather belt shop, an exploratory graphic correlation analysis was made, using the common overhead as the dependent variable and the level of operations of the belt shop and the other plants as independent variables. The level
of operations was measured by output and also by labor input. Since this analysis showed no significant partial regression of overhead on the activity of the belt shop, the absence of marginal general overhead cost was indicated. However, in the measurement of combined cost, certain elements of overhead cost as well as all elements of direct cost were included.

Although combined cost was of central theoretical and practical significance, it was found desirable to study also the behavior of individual constituents of cost. Combined cost was first broken down into two components, ‘direct’ and ‘overhead’ cost. The terms ‘direct’ and ‘overhead’ cost refer to accounting classifications and should not be confused with the economic categories of fixed and variable cost. The distinction between direct and overhead cost is based upon the ease of identifying cost with the particular units of output that give rise to it, whereas the distinction between fixed and variable cost depends upon whether cost varies with changes in output. In the plant studied none of the overhead cost was completely fixed.

The components of cost were broken down still further into their elements. Since the forces affecting cost vary in their impact upon different elements, separate correction of each element was necessary to remove irrelevant variations caused by changes in the prices of input factors and lags in recording. Furthermore, separate analysis of the behavior of the elements provides a basis for setting cost standards and flexible budgets in sufficient detail to be managerially useful. This detailed knowledge also makes possible more specific and exact allowances for changes in factor prices or minor alterations in the technique of production. Individual analyses were carried out, first, for direct cost and its elements: direct labor, leather, and cement; and second, for overhead cost and its elements: indirect labor, supplies, repairs, depreciation, taxes, insurance, salaries, sundries, dies and rings, heat, light and power, and water.

4 Rectification of Data

Rectification of the data is designed to eliminate influences that tend to obscure the true relation between cost and output. Since the influences on cost (apart from output) affect the various elements of combined cost differently, composite correction is unlikely to be accurate. For this reason it was found desirable to use specialized rectification devices for the various elements of cost.\(^9\) There are two sources of distortion that necessitate rectification: (1) the time lag between the recording of cost and

\(^9\) This procedure is substantiated by the experience of Ehrke and Schneider in their statistical analysis of cost in a cement mill. Their correction for price changes in the factors was first undertaken by using the \textit{Großhandels Preissindex}. Finding this unsatisfactory they constructed a special index for the prices of labor, limestone, clay, coal and coke. See Kurt Ehrke, \textit{Die Ueberzeugung in der Zementindustrie} (Jena: Gustav Fischer, 1953) §2, Die Verbesserung der Daten, in the statistical part.