

This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: Macroeconomic Linkage: Savings, Exchange Rates, and Capital Flows, NBER-EASE Volume 3

Volume Author/Editor: Takatoshi Ito and Anne Krueger, editors

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-38669-4

Volume URL: [http://www.nber.org/books/ito\\_94-1](http://www.nber.org/books/ito_94-1)

Conference Date: June 17-19, 1992

Publication Date: January 1994

Chapter Title: Tariffs, Quotas, and Inventory Adjustment

Chapter Author: Kazumi Asako, Yoshiyasu Ono

Chapter URL: <http://www.nber.org/chapters/c8536>

Chapter pages in book: (p. 273 - 293)

---

# 11 Tariffs, Quotas, and Inventory Adjustment

Kazumi Asako and Yoshiyasu Ono

## 11.1 Introduction

The purpose of this paper is twofold. First, we examine the short-run dynamic behavior of a firm that faces both domestic and export markets. We focus on the adjustment process of inventories. Inventories smooth production processes and thereby are productive, and adjustment costs are necessary for the firm to accumulate or decumulate inventories. We want to see how the export decision is related to the short-run dynamics of inventories. Second, by taking into account short-run inventory dynamics, we examine the differential consequences of tariffs and quotas for both exporting and importing countries.

In a static competitive setting, tariffs and quotas exercise equivalent effects on both international and domestic prices and on the welfare of both importing and exporting countries as long as they realize the same import level. However, quotas are more restrictive than tariffs in the sense that the amount of import is completely inflexible under quotas, whereas it is still variable under tariffs. Therefore—in the presence of oligopoly in the importing country, for example—oligopolistic reactions by import-competing firms are very different under tariffs and under quotas.<sup>1</sup> Even without market imperfections, we conjecture that tariffs and quotas would cause critically different effects on the adjustment process if we explicitly consider the dynamic inventory adjustment of firms. In this paper, by considering the inventory adjustment process of a firm, we are able to compare the welfare effects of tariffs and quotas.

Kazumi Asako is professor of economics at Yokohama National University. Yoshiyasu Ono is professor of economics, Institute of Social and Economic Research, Osaka University.

The authors are very much indebted to John Helliwell, Takatoshi Ito, and Kazuo Nishimura for their comments. Any errors, however, are the sole responsibility of the authors.

1. See Itoh and Ono (1982, 1984) for the critical difference between the two trade policies in an oligopolistic setting.

If an import restriction is imposed, the inventory of an exporting firm will gradually decrease and eventually reach a new stationary state. By this process, an exporting firm can control not only domestic supply and employment but also the amount of export if a tariff is imposed. If a quota is imposed, however, there is no room for an exporting firm to control export. It can control only domestic supply and labor employment. This implies that an exporting country cannot help adjusting inventories faster under quotas than under tariffs.

In fact, we show that if a tariff realizes the same stationary-state import level as a quota does, the import level in interim states is always higher under tariffs than under quotas. Since optimal tariff theory implies that a marginal tariff (or quota) always benefits the importing country, a marginal quota benefits the importing country more than a marginal tariff that realizes the same stationary-state import. Since world welfare does not change under a marginal trade restriction, this implies directly that the exporting country prefers tariffs to quotas. This simply restates the well-known property of optimal tariffs and quotas that more restrictive trade policies benefit the importing country more as long as they are marginal.

Recently, trade restrictions have tended to be used as a means of reducing trade deficits rather than as a means of providing long-term protection for an industry, even though trade imbalance as a whole cannot improve through trade restrictions on a particular industry. Then, since a tariff is less restrictive than an equivalent quota, in the above sense, this industry's trade deficits should be higher under a tariff than under the equivalent quota (as long as the marginal revenue of the importing country's demand function is positive). Therefore, if a tariff is imposed to reduce the present value of trade deficits in this industry by the same amount as it would be reduced by a quota, it should be more restrictive than a tariff under which the stationary-state import equals the quota. Then, it is not clear which policy after all more improves the welfare level of the importing country. In this paper, we find that the importing country's welfare is still higher under a quota than under a tariff, even if the two trade policies have the same reduction effect on the present value of an industry's trade deficits.<sup>2</sup>

The rest of the paper consists of three sections. In section 11.2 the basic model of the firm is presented. We focus on two countries that are open to the world. There is a competitive industry whose commodity is traded between the two countries. The representative firm of the industry in question produces output by utilizing labor and inventories of goods in process. Adjustment costs are needed to change inventories, so that adjustments in inventories take place only marginally and continuously. After analyzing the basic features of our model, we consider an autonomous shock to domestic demand. We see that an

2. There are very few works on the welfare analysis of tariffs and quotas in a dynamic setting. Kimbrough (1985) and Ono and Ikeda (1990) are exceptions. The former uses a simulation method, whereas the latter ignores investment in inventory and simply assumes that the output of firms is constant.

initial jump in export occurs when domestic demand autonomously decreases because inventories and thereby output cannot adjust downward immediately. However, in due course, inventories start decreasing. These observations suggest that, depending on the adjustment stage of each industry, the correlation between inventories and exports can either be positive or negative.

In section 11.3, we examine the differential consequences of tariffs and quotas. After analyzing both the long-run and short-run consequences of these trade restrictions for the exporting country, we compare the welfare levels of the importing country under alternative policies. After lengthy calculations, we reach a proposition which, briefly put, states that the importing country prefers quotas to tariffs in order to reduce the trade deficit of an industry. Section 11.4 concludes the paper with several remarks.

## 11.2 The Basic Model

We focus on two countries that are open to the world. There is a competitive industry whose commodity is traded only between the two countries. Therefore, all prices except that of this industry and the interest rate are given. Furthermore, by assuming that the input of numeraire goods is only labor and that its production technology is linear, the wage rate is also fixed in terms of numeraire goods. In this setting, we can apply a partial-equilibrium analysis to the dynamics of the industry in question.<sup>3</sup>

### 11.2.1 Optimization by the Representative Firm

The representative firm of the industry in question produces output by utilizing labor and inventories of goods in process. Inventories smooth production processes and thus are productive, but firms must incur adjustment costs to accumulate or decumulate inventories.<sup>4</sup>

The problem for the firm is to maximize the discounted sum of profits:

$$(1) \quad \int_0^\infty (pS + qX - wL)e^{-rt} dt,$$

subject to the constraint

$$(2) \quad F(L, Z) = S + X + \dot{Z} + \Phi(\dot{Z}, Z),$$

where  $S$  = domestic supply,  $X$  = exports,  $L$  = labor,  $Z$  = inventories,  $p$  = domestic price,  $q$  = export price,  $w$  = wage rate, and  $r$  = given world interest rate.

3. If each household's utility is given by  $U = y + \sum u_i(x_i)$ , where we take  $y$  as numeraire, then the demand for commodity  $x_i$  depends only on its own price  $p_i$ . Therefore, by assuming this utility function, we can directly apply a partial-equilibrium analysis of each industry to this general equilibrium model.

4. Uzawa (1986, chap. 2) formulates a model in which inventories enter into the production function. Although we follow his formulation straightforwardly here, the essential feature of the present paper would not change if only inventories yield benefits to the firm.

The adjustment cost function and the production function are assumed homogeneous of degree one, so that  $\Phi(\dot{Z}, Z) = \phi(a)Z$  and  $F(L, Z) = f(\ell)Z$ , where  $a = \dot{Z}/Z$ , and  $\ell = L/Z$ . We assume the following conditions for the adjustment cost function:

$$(3) \quad \phi(0) = 0, \phi'(0) = 0, \text{ and } \phi(a) > 0, a\phi'(a) > 0, \phi''(a) > 0, \\ \text{for any } a \neq 0,$$

and the Inada conditions for the production function:

$$(4) \quad f(0) = 0, f'(0) = \infty, \text{ and } f'(\ell) > 0, f''(\ell) < 0, \text{ for any } \ell > 0.$$

Defining  $H$  to be the current-value Hamiltonian

$$(5) \quad H = pS + q[f(\ell)Z - S - aZ - \phi(a)Z] - w\ell Z + \theta aZ,$$

the first-order conditions, assuming the interior solution, are (2) and

$$(6) \quad H_s = 0 \text{ or } p = q,$$

$$(7) \quad H_\ell = 0 \text{ or } qf'(\ell) = w,$$

$$(8) \quad H_a = 0 \text{ or } q[1 + \phi'(a)] = \theta,$$

$$(9) \quad \dot{\theta} = r\theta + q\Phi_z(a) - qF_z(\ell),$$

where  $\Phi_z(a) = \phi(a) - a\phi'(a)$  and  $F_z(\ell) = f(\ell) - \ell f'(\ell)$ . The transversality condition must also follow:

$$(10) \quad \lim_{t \rightarrow \infty} \theta Z e^{-rt} = 0.$$

Condition (6) is nothing but the “law of one price,” which states that the same goods must be priced equally across domestic and foreign markets. Equations (7) and (8) solve  $\ell$  and  $a$ , respectively, as functions of price variables with the following derivatives:

$$(11) \quad \ell = \ell(q), \quad \ell_q = -f'/qf'' > 0,$$

$$(12) \quad a = a(\theta, q), \quad a_\theta = 1/q\phi'' > 0, \quad a_q = -\theta/q^2\phi'' < 0.$$

Equation (9) describes the dynamics of the imputed price of inventory.

### 11.2.2 Industry Equilibrium

We now move from the optimization problem of the representative firm to the determination of the industry equilibrium. To begin with, we postulate that domestic consumption demand, other than that for inventories or lost as adjustment costs, is given by a simple demand function  $D(p)$  with the derivative  $D_p = D'(p) < 0$ . Similarly, the import demand of the foreign country (or the world as a whole) is given by  $M(q)$  with  $M_q = M'(q) < 0$ . For simplicity we assume that there are no import-competing firms. Then the equalities between demand and supply mean that

$$(13) \quad D(p) = S,$$

$$(14) \quad M(q) = X,$$

where  $p = q$  from (6).

Second, the firm continuously adjusts investments in inventory by looking at the difference between  $\theta$  and  $q$  to satisfy condition (8). Investment in inventory then alters the accumulated inventory at the next instant.

Third, the equilibrium condition for the goods market as a whole is given by

$$(15) \quad D(q) + M(q) + [a + \phi(a)]Z = f(\ell)Z.$$

Equation (15) solves  $q$ , for exogenously given  $w$ , as a function of  $Z$  and  $\theta$ :

$$(16) \quad q = q(Z, \theta),$$

with

$$(17) \quad q_z = \frac{\partial q}{\partial Z} = \frac{f - (a + \phi)}{B} < 0,$$

$$(18) \quad q_\theta = \frac{\partial q}{\partial \theta} = -\frac{(1 + \phi')a_\theta Z}{B} > 0,$$

$$(19) \quad B = D_q + M_q + (1 + \phi')a_q Z - f' \ell_q Z < 0.$$

Fourth, the imputed price of inventory changes according to (9) with the given world interest rate.

### 11.2.3 Stationary State and Saddle-Point Path

The long-run stationary-state equilibrium is attained when  $\dot{Z} = 0$  or  $a = 0$  and when the imputed price of inventory remains constant over time,  $\dot{\theta} = 0$ . In this long-run stationary state (denoted hereafter with an asterisk), we have from (3) and (8)

$$(20) \quad \theta^* = q^*,$$

and from (3), (9), and (20)

$$(21) \quad F_z(\ell^*) = f(\ell^*) - \ell^* f'(\ell^*) = r.$$

Condition (21), which states the equality between the marginal productivity of inventory and the given world interest rate, determines  $\ell^*$  as a function of  $r$  alone. Because  $\ell^*$  in turn is a function of  $q$  given by (11),  $q^*$  is uniquely determined. The long-run inventory stock  $Z^*$ , and thereby aggregate supply  $f(\ell^*)Z^*$ , is determined by (15).

Away from the long-run stationary state, the dynamics of the perfect-foresight economy are regulated by two differential equations:

$$(22) \quad \dot{Z} = J(Z, \theta) = a(\theta, q)Z,$$

$$(23) \quad \dot{\theta} = K(Z, \theta) = r\theta + q\Phi_z(a(\theta, q)) - qF_z(\ell(q)).$$

From (22), the  $\dot{Z} = 0$  locus is a downward-sloping curve on the  $(Z, \theta)$  plane, because  $J_z = -q_z Z/q\phi'' > 0$  and  $J_\theta = (1 - q_\theta)Z/q\phi'' > 0$ . Above and to the right of this curve  $\dot{Z} > 0$ , while below and to the left of it  $\dot{Z} < 0$ . On the other hand, the  $\dot{\theta} = 0$  locus is either a downward-sloping curve or an upward-sloping curve because  $K_\theta = r - fq_\theta$  is not definite in sign. However, we have  $K_z = -fq_z > 0$  implying that  $\dot{\theta} > 0$  to the right of the  $\dot{\theta} = 0$  curve, while  $\dot{\theta} < 0$  to the left of it.

The phase diagram of figure 11.1 presupposes a downward-sloping  $\dot{\theta} = 0$  curve. For this to be the case, we need to assume a sufficiently small  $q_\theta$ . If  $q_\theta$  is large enough to approach unity, the  $\dot{\theta} = 0$  curve becomes upward-sloping. Nevertheless, for any  $0 < q_\theta < 1$ , the long-run stationary state exhibits saddle-point stability, because the characteristic equation

$$(24) \quad \eta^2 - (J_z + K_\theta)\eta + (J_z K_\theta - J_\theta K_z) = 0$$

has two roots opposite in sign, which is so because we obtain

$$(25) \quad J_z K_\theta - J_\theta K_z = \ell f' q_z Z/q\phi'' < 0.$$

Thus, for any historically given inventory stock, there is an optimal path, depicted by arrows, pointing to the stationary state, along which inventory eventually reaches the stationary state. Only when the economy is on this optimal saddle-point path is the transversality condition (10) satisfied. Note that, insofar as saddle-point stability exists, the optimal path is definitely downward-sloping whether the slope of the  $\dot{\theta} = 0$  curve per se is positive or negative.

#### 11.2.4 Export-Drive Hypothesis

When an autonomous shock occurs which shifts domestic demand downward, inventories become redundant. Then, because the imputed price of in-

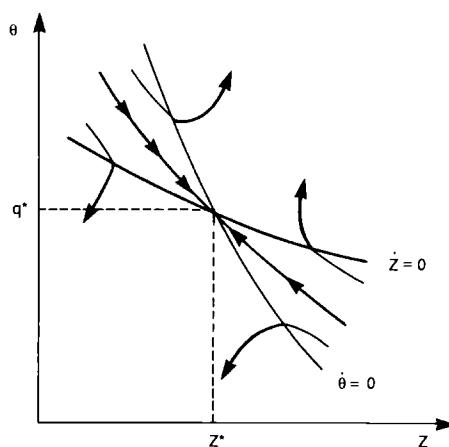


Fig. 11.1 Saddle-point path

ventories jumps down, the export price also jumps down, which in turn brings about an initial upward jump in exports. Throughout this process, inventories cannot decrease immediately, since changing inventories is costly. However, in due course, inventories start decreasing. Then, after the initial increase in exports, both inventories and exports start decreasing as we obtain from (14):

$$(26) \quad \frac{dX}{dZ} = M_q \left( q_z + q_\theta \left. \frac{d\theta}{dZ} \right|_{\text{opt}} \right) > 0,$$

since we have (17) and (18) and we know that  $\theta$  and  $Z$  are inversely related along the optimal path.

Exports keep decreasing to reach the former stationary-state level as the new export price is exactly the same as the former level. This fact is immediate, since we know that  $\ell^*$ , which is a function of  $q^*$ , is determined uniquely from (21). Thus, the dynamic effects of a decrease in domestic demand on inventory stock and exports are summarized without any rigorous proof as follows:<sup>5</sup>

*Proposition 1:* An autonomous decrease in domestic demand causes a gradual reduction in inventories. Exports first increase but later decrease to the former stationary-state level.

In Japan, it is usually pointed out that when the Japanese economy slows down more exports are driven, a presumption known as the export-drive hypothesis. This hypothesis has often been tested by checking whether the correlation between inventory stock and exports is positive. However, from the dynamic optimal behavior presented above, this simple relation may not necessarily hold even when export drive per se is present. In fact, from proposition 1, we find that if domestic demand gradually declines, exports increase because inventories cannot adjust instantaneously, and inventory stock stays too high for a while. In this process, the firm keeps reducing inventories to adjust them to a new stationary state. Thus, there should be a negative correlation between inventories and exports. When domestic demand stops declining, the firm continues to decrease inventories and to reduce exports as well. Thus, in the latter stage, inventories and exports will be correlated positively. Thus, depending on the adjustment stage of each industry, the correlation between inventories and exports can either be positive or negative.<sup>6</sup>

5. A proof of proposition 1 can be established by a phase diagram analysis. The optimal saddle-point path shifts to the left and downward when domestic demand autonomously decreases. This shift is qualitatively the same as the one initiated by the introduction of an import tariff or an import quota by the foreign country. See figure 11.2 of the next section.

6. Asako et al. (1993) examine the determinants of export-output ratios of Japanese manufacturing industries. They find that, whereas there are industries, such as ceramics, metal, and transportation machine manufacturing, for which the correlation between inventory stock and exports is positive, there are also industries, such as foods and textile manufacturing, for which the opposite of this relationship is the case. The implication of proposition 1 is totally consistent with these empirical observations.

### 11.2.5 Empirical Examination

In this subsection, we conduct an empirical examination utilizing data on Japanese manufacturing firms. The firm's export-output ratio is regressed on two main explanatory variables: inventory-output ratio (INV) and liquidity asset-output ratio (LIQ). The higher the latter ratio, the smaller the expected export-output ratio, because a firm is confronted with less export-drive pressure by the same amount of decrease in autonomous demand. A Tobit model is employed each year from 1964 to 1990 because there are firms each year for which the export-output ratio takes the value zero. The number of firms varies from 869 to 986 depending on the year. All data are taken from the Nikkei NEEDS Company data file.

The estimation results are summarized in table 11.1. The number of samples (SMPL) as well as the number of firms with positive export-output ratios (POS) are indicated in the same table. The SIGMA variable indicates the standard error of the regression of the Tobit model. The inventory-output ratio is

**Table 11.1** Estimation Results

Year	Constant	INV	LIQ	SIGMA	SMPL	POS
1964	0.0055 (0.31)	0.2824 (4.55)	-0.0227 (3.31)	0.1686 (29.9)	869	509
1965	-0.0140 (0.74)	0.4335 (5.99)	-0.0170 (2.46)	0.1873 (30.8)	880	538
1966	-0.0088 (0.48)	0.4381 (6.40)	-0.0136 (2.03)	0.1770 (31.9)	882	572
1967	0.0026 (0.17)	0.4049 (6.59)	-0.0137 (2.28)	0.1573 (33.8)	885	623
1968	0.0038 (0.25)	0.3842 (6.72)	-0.0083 (1.29)	0.1557 (34.3)	890	640
1969	0.0176 (1.09)	0.3507 (5.89)	-0.0097 (1.47)	0.1595 (35.0)	898	663
1970	0.0218 (1.40)	0.3905 (6.55)	-0.0140 (2.18)	0.1547 (35.7)	912	689
1971	0.0254 (1.68)	0.4027 (6.94)	-0.0098 (1.72)	0.1634 (36.5)	913	710
1972	0.0116 (0.79)	0.2909 (4.86)	0.0048 (0.95)	0.1635 (36.6)	917	717
1973	0.0106 (0.71)	0.3095 (6.06)	-0.0014 (0.24)	0.1579 (36.8)	919	720
1974	0.0409 (2.71)	0.2781 (6.67)	-0.0053 (0.84)	0.1772 (36.9)	922	727
1975	-0.0039 (0.26)	0.3317 (6.96)	0.0126 (2.36)	0.1798 (37.0)	925	730
1976	0.0020 (0.14)	0.3711 (7.83)	0.0105 (1.82)	0.1851 (37.0)	931	731
1977	0.0026 (0.16)	0.3441 (5.40)	0.0161 (2.64)	0.1975 (37.2)	956	744
1978	-0.0023 (0.16)	0.3654 (6.17)	0.0150 (2.77)	0.1893 (37.3)	959	745
1979	-0.0220 (1.52)	0.4207 (6.52)	0.0226 (4.15)	0.1818 (37.5)	962	752
1980	0.0102 (0.78)	0.3251 (5.34)	0.0195 (3.89)	0.1905 (37.7)	967	763
1981	0.0137 (0.97)	0.3772 (5.77)	0.0153 (2.95)	0.1999 (37.9)	971	769
1982	0.0038 (0.28)	0.4358 (7.01)	0.0177 (3.41)	0.1913 (38.5)	977	787
1983	0.0167 (1.27)	0.3574 (5.87)	0.0175 (4.25)	0.1935 (38.6)	981	793
1984	0.0384 (2.98)	0.3814 (5.81)	0.0081 (2.57)	0.1967 (38.9)	984	804
1985	0.0341 (2.69)	0.3618 (5.59)	0.0111 (3.55)	0.1946 (38.9)	986	804
1986	0.0271 (2.35)	0.3526 (5.94)	0.0104 (4.19)	0.1852 (39.0)	986	806
1987	0.0212 (1.75)	0.3510 (5.40)	0.0092 (4.28)	0.1815 (39.0)	986	803
1988	0.0303 (2.62)	0.3212 (5.11)	0.0054 (2.87)	0.1770 (38.7)	986	796
1989	0.0350 (2.97)	0.2571 (4.11)	0.0065 (3.48)	0.1796 (38.7)	986	795
1990	0.0311 (2.48)	0.3296 (4.84)	0.0049 (2.45)	0.1808 (38.6)	986	793

*Note:* Figures in parentheses are absolute values of *t*-statistics.

estimated with positive sign every year at the 1 percent significance level. This indicates that the initial negative correlation between export and inventory pointed out in proposition 1 is, if it exists, not predominant in the present data set. Thus the larger the inventory-output ratio the higher the export-output ratio.

Meanwhile the LIQ variable takes negative signs only in the early years. From the mid-1970s the export-output ratio is higher, the larger the liquidity asset-output ratio. Since the positive correlation is significant during 1980s, we need a theory which explains this relationship. Although such a topic is interesting, it is beyond the scope of the present paper.

#### 11.2.6 Relative Adjustability of Inventory and Employment

In the argument above we assumed that employment can adjust flexibly whereas inventories adjust only gradually. Even if employment is not perfectly adjustable, proposition 1 holds as long as there is some factor of production, instead of employment, inputs of which can adjust flexibly. However, if there is no flexible factor and employment is not perfectly flexible, a firm cannot instantaneously adjust production. Therefore when domestic demand autonomously declines, exporting firms would first accumulate inventories and increase exports. Then they would gradually reduce inventories, employment, and export. In this way inventory and export may move in parallel. Thus the effect of an exogenous decrease in domestic demand on the movements of inventory and export would differ across countries and across industries, depending on whether employment can adjust flexibly or not.

Messmore (1992), for instance, presents empirical research on the flexibility of employment and inventories. She shows that flexibility of employment differs across countries. Especially in EC countries, such as Belgium, France, Sweden, and Norway, obstacles to the termination of employment contracts are serious, whereas in the United States they are unimportant. Japan and presumably East Asian countries are in between. Proposition 1 may fit the U.S. case more.

### 11.3 Tariffs and Quotas

In this section, we introduce tariffs and quotas as alternative means of restricting exports of the home country. Namely, the foreign country imposes either a tariff or a quota. A quota does not give any room for exporters to adjust the amount of exports even by reducing prices. Under a tariff, on the other hand, exporters can affect the amount of exports by changing prices. In a static setting, the effects of these two alternative restrictions are the same, as long as the same quantity of export is guaranteed. However, in a dynamic setting, exporters will change the export amount under a tariff in the stage of adjusting inventories since the marginal cost will differ over time. Under a quota the amount of exports is fixed regardless of changes in the marginal cost. There-

fore the effect of a tariff on the welfare of an importing country will differ from that of a quota.

Note that the following arguments still hold as long as there is some factor of production, the accumulation of which is costly. Therefore, even if we consider, instead of inventories, real capital accumulation, which is costly, we obtain the same results.

### 11.3.1 Tariffs

We begin with an import tariff set by the foreign country. The equilibrium condition for the export market of the home country is written, instead of (14), as

$$(27) \quad X = M(q(1 + \tau)),$$

where  $\tau$  stands for an import tariff. Substituting (27) into (15), we can rewrite the equilibrium condition of the industry as

$$(28) \quad D(q) + [a + \phi(a)]Z = f(\ell)Z - M(q(1 + \tau)).$$

From (28), in response to the marginal introduction of the tariff we have for  $\tau = 0$

$$(29) \quad Bdq = -(1 + \phi') a_b Z d\theta + \{f - (a + \phi)\}dZ - qM_q d\tau.$$

Therefore, besides (17) and (18), we get

$$(30) \quad q_\tau = \frac{\partial q}{\partial \tau} = -\frac{qM_q}{B} < 0.$$

We now discuss the effects of the marginal introduction of a tariff on the long-run stationary state. To begin with, we see from (21) that the labor-inventory ratio  $\ell^*$  depends solely on the interest rate, implying that  $\ell^*$  is unaffected by the tariff. Then, since the marginal productivity of labor is independent of the tariff, the price level  $q^*$  stays unaffected by the introduction of the tariff because the wage rate is kept intact in (7) or (11).

When  $q^*$  and thereby the imputed price of inventory  $\theta^*$  are unaffected by the tariff, we obtain from (29) that

$$(31) \quad \frac{dZ^*}{d\tau} = \frac{q^* M_q(q^*)}{f(\ell^*)} < 0.$$

Equation (31) means that the introduction of a tariff by the foreign country decreases the optimal long-run stationary-state inventory stock in the home country. The reduction of inventory should decrease the output supply and thereby exports. Domestic supply is invariant under tariffs as the price level is unaffected.

Next, we examine the short-run transitional impact of the introduction of a tariff. To begin, we observe that the free-trade long-run stationary state now

belongs to the region where both  $Z$  and  $\theta$  are increasing over time because we obtain from (22) and (23)

$$(32) \quad \frac{\partial \dot{Z}}{\partial \tau} = a_q q Z > 0,$$

and

$$(33) \quad \frac{\partial \dot{\theta}}{\partial \tau} = -f q_\tau > 0.$$

This implies that the saddle-point paths shift to the left and downward, as illustrated in figure 11.2.

Figure 11.3 depicts the transitional paths of inventory stock  $Z$ , imputed price of inventory  $\theta$ , export price  $q$ , domestic supply  $S$ , and exports  $X$ . The inventory stock gradually decreases to the new stationary-state level. Price variables  $\theta$  and  $q$  show qualitatively similar variations; i.e., these variables jump down initially and then keep rising until the common old equilibrium level  $\theta^* = q^*$  is recovered. The time profile of domestic supply is just the mirror image of that of export price; i.e., it initially jumps up and then decreases gradually until the old equilibrium level is reached. Exports of the home country jumps down initially and then keeps decreasing as the export price keeps rising.

### 11.3.2 Quotas

Next we consider the effect of an import quota set by the foreign country. Thus we impose the quantity constraint

$$(34) \quad X = \bar{X},$$

so that we have, in place of (28),

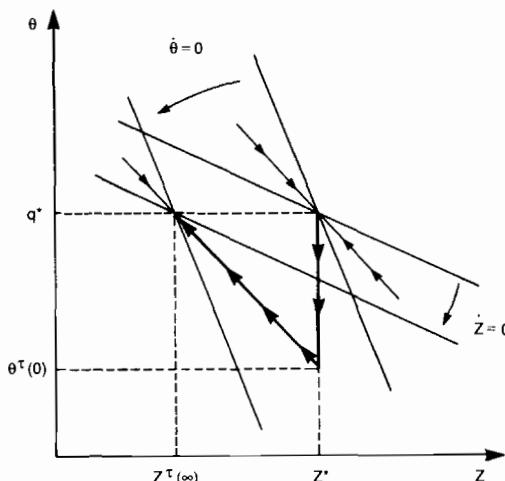


Fig. 11.2 Effect of a tariff

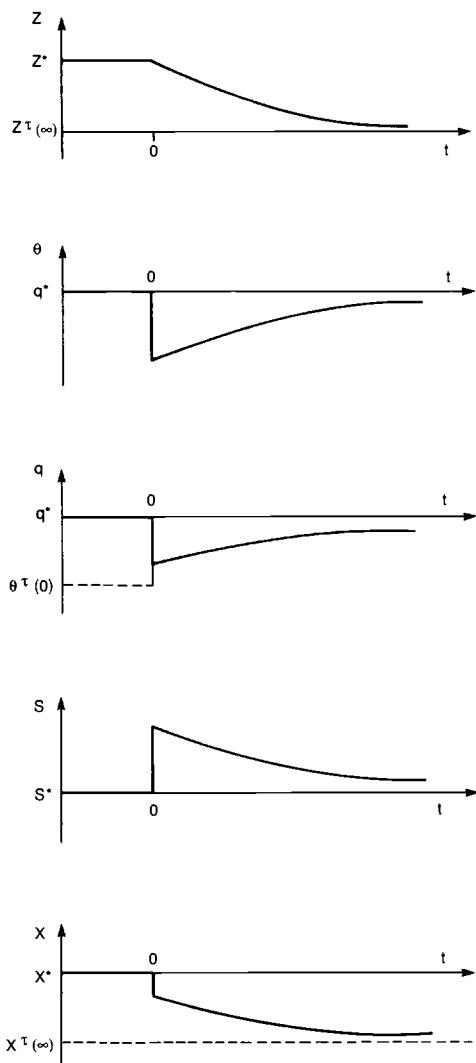


Fig. 11.3 Time profiles of variables under a tariff

$$(35) \quad D(q) + [a + \phi(a)]Z = f(\ell)Z - \bar{X}.$$

The long-run impacts of quotas are qualitatively similar to those of tariffs and, in fact, both policies exert quantitatively the same long-run impact if only

$$(36) \quad D(q^*(1 + \tau)) = \bar{X},$$

holds in the long-run stationary state.

To see the short-run transitional impacts, we first obtain from (35)

$$(37) \quad (B - M_q) dq = -a_0 Z^* d\theta + fdZ - d\bar{X},$$

where derivatives and parameters are evaluated in the stationary state. Comparing (29) and (37), we observe that the short-run dynamics of the transitional paths under quotas are to some extent very similar to those under tariffs, so that we do not depict any particular figure for this case.

However, once we take a closer look at the short-run dynamics, we know that the short-run transitional export price is lower than the long-run stationary level when inventories decumulate. This indicates that

$$(38) \quad M(q^*(1 + \tau)) > M(q^{**}(1 + \tau)) = \bar{X},$$

holds throughout the transitional path, where  $q^*$  denotes the export price under a tariff. Therefore, in the short run exports are greater under tariffs than under quotas. The opposite is the case when inventories accumulate.

### 11.3.3 Tariffs versus Quotas: Alternative Ways to Reduce Trade Deficits

In this subsection, we compare the welfare effect of a tariff with that of a quota when they decrease the trade deficit of an industry by the same amount.

Suppose that either a marginal tariff or a marginal quota is imposed in a competitive industry where the amount of trade is  $X^*$  and price is  $q^*$ . We derive from (7) and (21) that in a stationary state  $q^*$  is invariant even under trade restrictions. Therefore, we have

$$(39) \quad \Delta q^*(\infty) = q^*(\infty) - q^* = 0,$$

$$(40) \quad \Delta q^*(\infty) = q^*(\infty) - q^* = 0,$$

where  $q^*(t)$  and  $q^*(t)$ , respectively, represent the export price path under the quota and that under the tariff.

If a marginal tariff  $\tau$  ( $\equiv 0$ ) and a marginal quota  $\bar{X}$  ( $\equiv X^*$ ) have the same effect on the present value of trade deficit in this industry, we have

$$(41) \quad \int_0^\infty (X^* \Delta q^* + q^* \Delta \bar{X}) e^{-rt} dt = \int_0^\infty [X^* \Delta q^* + q^* (q^* \Delta \tau + \Delta q^*)] M_q e^{-rt} dt,$$

where using (39) and (40)  $\Delta q^*$  and  $\Delta q^*$  are given by

$$(42) \quad \Delta q^*(t) = [\Delta q^*(0) - \Delta q^*(\infty)] e^{\eta^x t} + \Delta q^*(\infty) = \Delta q^*(0) e^{\eta^x t},$$

$$(43) \quad \Delta q^*(t) = [\Delta q^*(0) - \Delta q^*(\infty)] e^{\eta^r t} + \Delta q^*(\infty) = \Delta q^*(0) e^{\eta^r t}.$$

Negative parameters  $\eta^x$  and  $\eta^r$ , respectively, are the solution of characteristic equation (24) under the quota and that under the tariff. Therefore, from (41) we obtain

$$(44) \quad [X^*(\Delta q^*(0)/\Delta \bar{X})/(r - \eta^x) + q^*/r] \Delta \bar{X} \\ = [(X^* + q^* M_q) (\Delta q^*(0)/\Delta \tau)/(r - \eta^r) + (q^*)^2 M_q/r] \Delta \tau.$$

Since we naturally assume that  $\Delta q^*(0)/\Delta \bar{X} > 0$  and  $\Delta q^*(0)/\Delta \tau < 0$  and that the marginal revenue of the foreign demand function is positive (i.e.,  $X^* + q^* M_q > 0$ ),<sup>7</sup> (44) implies that  $\Delta \bar{X}/\Delta \tau < 0$  when (41) is satisfied.

Because changes in the importing country's surplus under a quota and under a tariff are given as

$$(45) \quad \Delta W^*/\Delta \bar{X} = - \int_0^\infty X^*[\Delta q^*(t)/\Delta \bar{X}]e^{-rt}dt = - X^*(\Delta q^*(0)/\Delta \bar{X})/(r - \eta^*),$$

$$(46) \quad \Delta W^*/\Delta \tau = - \int_0^\infty X^*[\Delta q^*(t)/\Delta \tau]e^{-rt}dt = - X^*(\Delta q^*(0)/\Delta \tau)/(r - \eta^*),$$

we obtain from (44) that

$$\begin{aligned} & [X^*(\Delta q^*(0)/\Delta \bar{X})/(r - \eta^*) + q^*/r][\Delta W^*/\Delta \tau - (\Delta W^*/\Delta \bar{X})(\Delta \bar{X}/\Delta \tau)]X^* \\ &= -[X^*(\Delta q^*(0)/\Delta \bar{X})/(r - \eta^*) + q^*/r](\Delta q^*(0)/\Delta \tau)/(r - \eta^*) \\ (47) \quad & \quad + [(X^* + q^* M_q)(\Delta q^*(0)/\Delta \tau)/(r - \eta^*) \\ & \quad + (q^*)^2 M_q/r](\Delta q^*(0)/\Delta \bar{X})/(r - \eta^*) \\ &= (q^*/r)\{[(\Delta q^*(0)/\Delta \bar{X})/(r - \eta^*)][q^* + r(\Delta q^*(0)/\Delta \tau)/(r - \eta^*)]M_q \\ & \quad - (\Delta q^*(0)/\Delta \tau)/(r - \eta^*)\}. \end{aligned}$$

Therefore, we have

$$(48) \quad \text{sgn}[\Delta W^*/\Delta \tau - (\Delta W^*/\Delta \bar{X})(\Delta \bar{X}/\Delta \tau)] = \text{sgn}(\Omega),$$

where

$$(49) \quad \Omega = [(\Delta q^*(0)/\Delta \bar{X})/(r - \eta^*)][q^* + r(\Delta q^*(0)/\Delta \tau)/(r - \eta^*)]M_q \\ - (\Delta q^*(0)/\Delta \tau)/(r - \eta^*).$$

By calculating  $\Omega$ , we derive the following property:

*Proposition 2:* As long as a quota reduces the trade deficit of an industry by the same amount as a tariff does, it is more advantageous for the importing country to impose a quota.

*Proof:* See appendix.

Thus, the importing country would prefer quotas to tariffs to reduce the trade deficit of an industry.

## 11.4 Concluding Remarks

In this paper, we have analyzed the dynamic inventory behavior of competitive firms that face both domestic and export markets. We have been especially

7. This can be negative when the marginal cost of the exporting firm is very low. Then, tariffs or quotas even increase trade deficits. Here, we treat the case where trade restrictions reduce trade deficits.

interested in the dynamic interactions between inventory adjustment and exports. The analytical results can be summarized in two theoretical propositions.

Proposition 1 indicates that the correlation between inventories and exports can either be positive or negative, depending on the adjustment stage of each industry. Proposition 2 compares tariffs and quotas in the presence of inventory adjustment. It turned out that an importing country's welfare is higher under a quota than under a tariff when the two trade policies have the same reduction effect on the present value of trade deficits.

Our analytical framework can be utilized to analyze a variety of problems left unanswered in the present paper. These include the incorporation of inter-temporal optimization by consumers, consideration of endogenous growth, and extension to a truly standard general equilibrium framework.

## Appendix

### *Proof of Proposition 2*

We obtain  $\Delta q^*(0)/\Delta \bar{X}$  and  $\Delta q^*(0)/\Delta \tau$ , and derive  $\Omega$ . At  $t = 0$ , we obtain from the market equilibrium condition under a quota  $\bar{X}$ :

$$(A1) \quad D(q^*(0)) + \bar{X} = \{f(\ell(q^*(0))) - [a(0) + \phi(a(0))]Z(0)\},$$

so that

$$(A2) \quad D_q(dq^*(0)) + d\bar{X} = -\{[(f')^2/(q^*f'')]dq^* + da(0)\}Z^*,$$

since  $dZ(0) = 0$ ,  $Z(0) = Z^*$ , and  $\phi'(a(0)) = \phi(0) = 0$ . Deriving  $da$  from the first-order condition (8) as  $da = (d\theta - dq)/(q\phi'')$  and substituting the result into (A2), we obtain

$$(A3) \quad -(B^* - M_q)dq^*(0)/d\bar{X} = 1 + [Z^*/(q^*\phi'')]d\theta^*(0)/d\bar{X},$$

where  $B^*$  equals expression (19) evaluated in the stationary state. Since from the dynamic equation of  $\theta^*(t)$   $\theta^*(0)$  is written as

$$(A4) \quad \theta^*(0) = q^* + [K_z^*/(\eta^* - K_\theta^*)](Z(0) - Z^*(\infty)),$$

where  $K^*(z, \theta)$  represents (23) under a quota, we get

$$(A5) \quad d\theta^*(0)/d\bar{X} = -[K_z^*/(\eta^* - K_\theta^*)]dZ^*(\infty)/d\bar{X}.$$

In the new stationary state to be reached at  $t = \infty$ , the equilibrium condition of the industry becomes

$$(A6) \quad D(q^*(\infty)) + \bar{X} = f(\ell(q^*(\infty)))Z^*(\infty),$$

where  $q^*(\infty)$  is invariant regardless of trade policies and equals  $q^*(0) = q^*$ . This implies that we obtain

$$(A7) \quad dZ^*(\infty)/d\bar{X} = 1/f(\ell^*).$$

Therefore, we obtain from (A3), (A5), and (A7) that  $dq^*(0)/d\bar{X}$  satisfies

$$(A8) \quad \begin{aligned} & -(B^* - M_q)dq^*(0)/d\bar{X} \\ & = 1 - [K_z^*/(\eta^* - K_\theta^*)][Z^*/(q^*f\phi'')] \\ & = 1 - \{fZ^*/[(r - \eta^*)(B^* - M_q)q^*\phi'' + fZ^*]\}, \end{aligned}$$

from which we obtain

$$(A9) \quad -[dq^*(0)/d\bar{X}]/(r - \eta^*) = q^*\phi''/[(r - \eta^*)(B^* - M_q)q^*\phi'' + fZ^*].$$

We next obtain  $\Delta q^*(0)/\Delta\tau$ . The market equilibrium condition under a tariff  $\tau$  is written as

$$(A10) \quad D(q^*(0)) + M((1 + \tau)q^*(0)) = \{f(\ell(q^*(0))) \\ - [a(0) + \phi(a(0))]\}Z(0),$$

so that we obtain

$$(A11) \quad -B^*dq^*(0)/d\tau = q^*M_q + [Z^*/(q^*\phi'')]d\theta^*(0)/d\tau.$$

Since from the dynamic equation of  $\theta^*(t)$   $\theta^*(0)$  is written as

$$(A12) \quad \theta^*(0) = q^* + [K_z^*/(\eta^* - K_\theta^*)](Z(0) - Z^*(\infty)),$$

where  $K^*(z, \theta)$  represents (23) under a tariff, we get

$$(A13) \quad d\theta^*(0)/d\tau = -[K_z^*/(\eta^* - K_\theta^*)]dZ^*(\infty)/d\tau.$$

Letting  $t \rightarrow \infty$ , the equilibrium condition of the industry becomes

$$(A14) \quad D(q^*(\infty)) + M((1 + \tau)q^*(\infty)) = f(\ell(q^*(\infty)))Z^*(\infty),$$

where  $q^*(\infty) = q^*$ , so that we obtain

$$(A15) \quad dZ^*(\infty)/d\tau = q^*M_q/f(\ell^*).$$

Therefore, we obtain from (A11), (A13), and (A15) that  $dq^*(0)/d\tau$  satisfies

$$(A16) \quad \begin{aligned} -B^*dq^*(0)/d\tau &= q^*M_q\{1 - [K_z^*/(\eta^* - K_\theta^*)][Z^*/(q^*f\phi'')]\} \\ &= q^*M_q\{1 - fZ^*/[(r - \eta^*)B^*q^*\phi'' + fZ^*]\}, \end{aligned}$$

from which we obtain

$$(A17) \quad -[dq^*(0)/d\tau]/(r - \eta^*) = (q^*)^2M_q\phi''/[(r - \eta^*)B^*q^*\phi'' + fZ^*].$$

Now, applying (A9) and (A17) to (49), we derive

$$(A18) \quad [(B^* - M_q)(r - \eta^x) + fZ^*/(q^*\phi'')] [B^*(r - \eta^r) + fZ^*/(q^*\phi'')] \Omega/(q^*M_q) = B^*\eta^r - (B^* - M_q)\eta^x,$$

which implies that

$$(A19) \quad \text{sgn}(\Omega) = \text{sgn}[(B^* - M_q)\eta^x - B^*\eta^r].$$

But,  $\eta^r$  and  $\eta^x$  are, respectively, negative roots of the following characteristic equations:

$$(A20) \quad (\eta^r)^2 - r\eta^r + [fZ^*(f - r)/(B^*q^*\phi'')] = 0, \text{ and}$$

$$(A21) \quad (\eta^x)^2 - r\eta^x + [fZ^*(f - r)]/[(B^* - M_q)q^*\phi''] = 0.$$

Then, because we know  $B^* < B^* - M_q < 0$  and because it is easy for us to derive  $\partial(\eta^r B^*)/\partial B^* < 0$ , by comparing (A20) and (A21) we get  $(B^* - M_q)\eta^x < B^*\eta^r$ . This implies that (A19) is negative, which in turn implies that

$$(A22) \quad \Delta W^r/\Delta \tau < (\Delta W^x/\Delta \bar{X})(\Delta \bar{X}/\Delta \tau).$$

Inequality (A22) proves proposition 2.

Q.E.D.

## References

- Asako, K., H. Izawa, A. Kawaguchi, and N. Tomita. 1993. Yusyutsu Hiritsu no Kettei Youin (Determinants of export ratio). Mimeograph.
- Itoh, M., and Y. Ono. 1982. Tariffs, quotas and market structure. *Quarterly Journal of Economics* 96 (May): 295–305.
- . 1984. Tariffs vs. quotas under duopoly of heterogeneous goods. *Journal of International Economics* 17 (November): 359–73.
- Kimbrough, Kent P. 1985. Tariffs, quotas and welfare in a monetary economy. *Journal of International Economics* 19: 257–277.
- Messmore, Lauren B. 1992. *The effect of labor turnover costs on inventory investment: A cross-country analysis*. Ph.D. thesis, Department of Economics, Harvard University.
- Ono, Yoshiyasu, and Shinsuke Ikeda. 1990. International welfare effects of saving controls and trade restrictions. ISER Discussion Paper no. 235, Osaka University, October.
- Uzawa, Hirofumi. 1986. *Keizai Dogaku no Riron (A theory of economic dynamics)*. Tokyo: University of Tokyo Press.

**Comment** Kazuo Nishimura

Asako and Ono present a dynamic trade model with inventory adjustment. Two main results are reported in this paper. I will discuss them in order.

The first result concerns the export-drive hypothesis, which says that a decrease in domestic demand leads to inventory accumulation, which drives a country to export more. Asako and Ono say that there is not necessarily a positive correlation between inventories and exports, which is contrary to the export-drive hypothesis. However I do not fully agree with their interpretation of proposition 1. The level of inventory  $Z$  is a state variable in the model. All other variables are functions of inventory. If there is exogenous shock (autonomous decline in demand), the domestic demand function  $D(p)$  shifts downward. This will change the values of the shadow price of inventory  $\theta$ , goods prices  $p = g$ , and the other variables  $a$  and  $l$  and also the amount of export associated with the given initial level of inventory. This is the only case when export increases. However inventory stays at the initial level in this case. Then, a decline in demand causes a change of stationary state, and inventory and export start to move toward a new stationary state. As inventory decreases, export will also decrease. Therefore, whenever inventory changes, we observe a positive correlation between inventory and export. In conclusion there is non-negative correlation between inventory and export and thus effect in the adjustment process contradicting the export-drive hypothesis.

The second result states that a quota is more advantageous than a tariff for the importing country, given the condition that the trade deficits under both policies are the same. Trade deficit here is restricted to mean the trade deficit of the imported good industry. Hence it is equivalent to the total value of the import. And by total value Asako and Ono mean the discounted present value of the import. The advantage of the importing country is measured by its welfare change. Welfare is measured by consumer surplus. Because the demand curve for the imported good by importing country is fixed, there is no substitute good for the imported good.

Since the model of this paper is formulated to explain the dynamics of the exporting country, proposition 2 should be a result on the welfare of the exporting country. The welfare implication for the importing country may be given as a corollary or in a remark after proposition 2. The import function  $M(q)$  is the only place in this model where the importing country plays a part. Therefore the welfare implication for the importing country given in proposition 2 should not be counted as a main result.

My comments above are only on the interpretation or statement of the results. Reading through the paper, I must say that Asako and Ono derive the comparative dynamics in a very clever way. I think that comparative *dynamics*

are much more difficult than comparative statics in most cases. Their contribution is certainly an important addition to the literature.

## Comment John F. Helliwell

This paper provides an imaginative use of costly inventory adjustment as a means of modeling differentially the effects of tariffs and quotas and of analyzing the movements of exports and inventories following an unexpected drop in domestic demand. The two main parts of the paper focus on separate issues and depend for their results on different features of the model setup, so they should be considered separately.

### Inventories and Exports

The authors' main conclusion in this section is that the correlation between inventories and exports can be either positive or negative, depending on the stage of an industry's response to an unexpected change in tariffs or quotas. In the first stage of response to an unexpected drop in domestic sales, inventories will be dropping gradually while exports will be rising. This, they argue, upsets the commonly held presumption, known as the export-drive hypothesis, whereby a positive relation is to be expected. The fact that their model upsets the export-drive hypothesis depends on very special features of their model setup, and in particular on assumptions that are contrary to those that underlie the export-drive hypothesis. I think that their assumption may be less plausible than an alternative setup that would be more compatible with the export-drive hypothesis. To focus more clearly on the issue, I shall first outline what I take to be the logic of the export-drive hypothesis and then spell out how that hypothesis assumes a quite different set of assumptions about inventory and factor adjustment costs than is employed by Asako and Ono.

The export-drive hypothesis starts with the notion that inventories accumulate in the wake of an unexpected drop in domestic sales, and producers then make a drive to expand export markets in order to help eliminate their excess inventories. Thus periods with high inventories tend to be associated with high exports as well, at least in those instances where the high inventories are due to an unexpected drop in domestic sales. Obviously, if the sales surprise took the form of an unexpected increase in export demand, then high exports would be associated with low inventories, following the same logic about the buffer stock use of inventories. Hence, the correlations that are being talked about in

John F. Helliwell is Mackenzie King Visiting Professor of Canadian Studies at Harvard University, professor of economics at the University of British Columbia, and a research associate of the National Bureau of Economic Research.

the export-drive hypothesis are specific to a particular shock and cannot be tested by simple correlations.

By contrast, when the Asako-Ono model is confronted with unexpectedly low domestic demand, inventories are gradually reduced, while exports are increased at first, before eventually returning to the former stationary-state level. Both models show an increase in exports, above the path that otherwise would have taken place, but some of the implications for inventory stocks are strikingly different. In the Asako-Ono setup and in the model implicitly underlying the export-drive hypothesis, long-run target inventories are reduced in response to any reduction in long-run expected sales. What distinguishes the two approaches is the nature of the short-run inventory adjustment. In the Asako-Ono setup, inventories drop right from the outset. Inventory stocks do not drop immediately to the new target level, since they are subject to adjustment costs that increase with the speed of adjustment. However, inventories cannot increase in response to an unexpected drop in sales since there are assumed to be no costs of adjusting other factor inputs. Hence output immediately falls by more than the unexpected drop in sales, by an extra amount equal to the partial adjustment of inventories toward their new target level.

In the model implicitly underlying the export-drive hypothesis, by contrast, there is room for inventories to act as an optimal buffer between unexpected sales changes and a stabilized production level because there are assumed to be costs entailed by changing the level of output and none to changing the level of inventories, beyond those that might flow from the fact that factor inputs, and hence output, are costly to adjust.

Thus the Asako-Ono conclusion about the movement of inventories in response to an unexpected drop in domestic sales differs from the export-drive hypothesis because of fundamentally different assumptions about adjustment costs. A judgment about the relative plausibility of the two results requires us to choose between the two conflicting sets of assumptions about adjustment costs.

Which is the more likely assumption about adjustment costs? I would be inclined to cast my vote for assuming that output is costly to adjust, which amounts to assuming that all measured factor inputs are costly to adjust, and that abnormal rates of factor utilization, while possible, are not costless. With these assumptions about adjustment costs, it is not necessary to assume any further adjustment costs in the level of inventories, and the door is opened for inventory stocks to play a buffering role between uncertain changes in sales and costly-to-adjust production levels.<sup>1</sup> Indeed, I am inclined to think that something of this sort probably underlies Asako and Ono's initial argument that inventories are productive because they smooth production processes. This would in turn seem to imply that there are cost advantages to a smooth produc-

1. Some evidence favoring such a buffer-stock role for inventories in the industrial countries is presented in Helliwell and Chung (1981).

tion process, and if inventories are to play such a role, it is presumably because they are available to be run up or run down to help buffer unexpected changes in demand or cost conditions.

### Tariffs and Quotas

On the tariffs versus quotas issue, the main point the authors emphasize is that costly adjustment alters the relative present values of the costs of using tariffs and quotas in favor of using quotas to achieve any given target reduction in the present value of trade deficits. I conjecture that this result is quite general and is likely to hold for a variety of specifications of costs of adjustment. This is likely because the result depends on the central distinction between tariffs and quotas, that the latter bite immediately on quantities and hence have a larger ratio of short-term to long-term trade balance effects in the presence of adjustment costs. To illustrate the generality of this result and its independence from the specific role of inventories, I think it would be possible to relabel inventories as plant and equipment and the analysis would carry through just as it does now, providing we continue to make the assumption that capital goods are an undifferentiated part of the homogeneous output of the representative firm.

Given the practical importance of the choice between tariffs and quotas and the likelihood that the result does not depend on the authors' specific assumptions about the role and costliness of inventory adjustment, it would be helpful to provide some indication of how it might be possible to assess the likely magnitude of the partial preference of quotas over tariffs. If the likely effect is small, then it can safely be treated as a secondary member of the larger set of efficiency issues surrounding the revenue, distribution, and political consequences of the use of tariffs and quotas. In any event, fine-tuning the choice between tariffs and quotas, as applied by one country acting on its own, ought to be subservient to the larger questions of the likelihood of retaliation of others and of the systemic effects of making industry-specific tariffs and quotas a feasible goal for firms that might better be spending their scarce entrepreneurial talents finding new products, improving production techniques, and developing new markets.

Although I regard the comparison of tariffs and quotas to be more convincing than the arguments about the short-run dynamic linkages between inventories and exports, the quality of the analysis is high throughout the paper, reflecting the sensible application of sharp tools to interesting issues.

### Reference

- Helliwell, J. F., and A. Chung. 1981. Aggregate output with variable rates of utilization of employed factors. *Journal of Econometrics* 19:597–625.