Chapter Title: Productivity Growth and Changes in the Terms of Trade in Japan and the United States

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Catherine Morrison and W. Erwin Diewert

7.1   Introduction

The productivity change in a closed economy going from year $t - 1$ to year t is usually defined as an index of outputs divided by an index of primary inputs. Under certain assumptions, a productivity improvement in an economy can be identified with an outward shift in the economy's production possibilities set.

In a small open economy, the domestic production possibilities set is augmented by the possibility of exchanging exports for imports at constant world prices. Over time, this augmented production possibilities set can shift outwards for at least two different reasons: (i) improvements in efficiency or productivity (as in a closed economy) and (ii) improvements in the economy's terms of trade; that is, the prices of imported goods fall relative to the prices of exported goods. A third source of outward shift in an open economy can also be distinguished: namely, the economy can increase its merchandise trade deficit. This will allow domestic consumption and investment (which we shall call domestic sales below) to increase in the short run. However, this third source of outward shift will generally be temporary in nature (unless the trade deficit is financed by gifts or increased foreign aid) since the deficit in the current period will have to be repaid in future periods.

A framework for measuring these three types of outward shift in the context of production theory was developed recently by Diewert and Morrison (1986). We shall use this framework in the present paper to measure Japa-

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nese and U.S. productivity and changes in the terms of trade for the years 1968–82.

In order to give the reader an intuitive, nonalgebraic explanation of the three types of outward shift mentioned above, we devote two sections of the paper to geometric exposition. In section 7.2 below, we illustrate the shifts using the traditional general equilibrium approach to (static) trade theory.[1] The problem with empirically implementing this traditional approach is that the informational requirements are very high: detailed price and quantity information on the allocations of producers, consumers, and governments is required.

Thus, in section 7.3 below, we illustrate the production theory approach that requires only producer information. In this approach, pioneered by Kohli (1978), all merchandise imports are channeled through the domestic production sector before being transformed into domestic goods. Also, exported goods are not regarded as domestic goods (but they can be highly substitutable with domestic goods).

In sections 7.4–7.7, we present the algebra of the production theory approach. The empirically oriented reader can skim over these sections and proceed to the empirical results.

In section 7.4, we define the sales function, which gives the maximum value of domestic sales a small, open, competitive economy can achieve, given the period t domestic technology and given domestic prices, export prices, import prices, domestic primary inputs, and the maximum merchandise trade deficit that the economy is allowed to run. In section 7.5, we use the sales function in order to define various theoretical productivity and terms-of-trade indexes.

In order to be able to evaluate these theoretical indexes using observable data, it is necessary to make some further assumptions. Thus, in section 7.6, we assume that the sales function in each period has a translog representation, and this assumption enables us to evaluate exactly the various theoretical indexes.

In section 7.7, we use a somewhat different approach in order to evaluate our theoretical indexes: a first- and second-order approximation approach. This approach and the previous translog approach were developed by Diewert and Morrison (1986).

In section 7.8, we turn to a description of the Japanese data, and we use this data in section 7.9 to calculate Japanese indexes of productivity, terms of trade, and "welfare" change. A similar program using U.S. data is followed in sections 7.10 and 7.11. Finally, section 7.12 offers some comparisons between the recent U.S. and Japanese productivity experience.

7.2   The Geometry of the Traditional General Equilibrium Approach

Consider an economy that produces two finally demanded goods, y_1 and y_2, using M primary inputs, $v_1, v_2, \ldots, v_M$, during two periods. For simplicity,

we shall hold the utilization of primary inputs constant during the two periods. The general equilibrium of the economy during the two periods is represented in figure 7.1.

The domestic production possibilities set for the economy in period 1 is the region enclosed by OT_1T_1'. The international trading line that the economy faces is P_1P_1' (which has slope $-p_1^1/p_2^1$, where p_i^1 is the internationally fixed price for good i in period 1, $i = 1,2$). Note that this price line is just tangent to the frontier of the domestic production possibilities set T_1T_1'. The trade-augmented production possibilities set is the region bounded by OP_1P_1'.

For simplicity, assume that there is only one consumer in the economy. The highest indifference curve that is just tangent to the trade-augmented production possibilities set is U_1U_1' and the point of tangency occurs at C in figure 7.1. Thus, in period 1, the economy will export AB units of y_2 in exchange for BC units of the imported good, y_1. We are assuming that there is no merchandise trade surplus or deficit in period 1.

We turn now to an analysis of the equilibrium in period 2. We assume that the economy's domestic production possibilities set shifts outward in period 2 (due to technical progress) to the frontier T_2T_2'. A measure of productivity gain in this economy might be $OT_2/OT_1 > 1$ (measured in terms of good 1) or $OT_2'/OT_1' > 1$ (measured in terms of good 2).

Instead of having balanced trade in period 2, let us assume that the economy is able to run a balance of trade deficit of size T_2T_3 in period 2; that is, we assume that these imports do not have to be paid for by exports in period 2. The effect of this assumption is to shift the domestic production possibilities frontier T_2T_2' to the right by T_2T_3 units, which results in a period-2 effective frontier T_3T_2'.

If the international prices of y_1 and y_2 remained unchanged in period 2, the economy's period-2 trade-augmented technology set would be OP_2P_2' and the
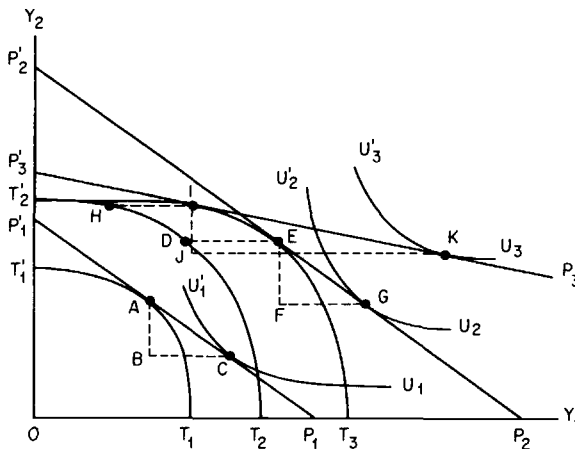

Fig. 7.1   The general equilibrium approach

highest indifference curve tangent to this set is $U_2 U_2'$ with the point of tangency at G. The point of domestic producer equilibrium would be D, the balance of trade deficit effect would be DE, and EF units of y_2 would be exported in exchange for FG units of imports.

However, the international prices are unlikely to remain constant. Suppose that the new period-2 international prices are p_i^2 for $i = 1,2$. The price line $P_3 P_3'$ has slope $-p_i^2/p_2^2$ and is tangent to the deficit-augmented domestic technology set $OT_3 T_2'$. The highest indifference curve tangent to $P_3 P_3'$ is $U_3 U_3'$, and the point of tangency is at K. The terms-of-trade effect is some measure of the distance between the indifference curves $U_2 U_2'$ and $U_3 U_3'$. In this case, the price of exports has increased relative to imports (i.e., $p_1^2/p_2^2 < p_1^1/p_2^1$), and so the terms-of-trade effect is positive and analogous to a domestic technology productivity improvement. Thus the final point of producer equilibrium in period 2 is at H, the distance HI represents the balance of trade deficit effect, and IJ units of exports are exchanged for JK units of imports.

We turn now to an alternative paradigm based on producer theory that will allow us to define counterparts to the above productivity, deficit, and terms-of-trade effects.

7.3 The Geometry of the Production Theory Approach

As in the previous section, we shall, for simplicity, hold the economy's primary inputs constant during the two periods. There are three additional goods in the economy: (i) a domestic consumption good, y_d, (ii) an exported good, y_x, and (iii) an imported good, y_m.

The frontier of the period-1 production possibilities set can be represented by a surface in y_d, y_x, and y_m space. We can represent this surface in a two-dimensional diagram by a family of domestic isoproduct curves. Thus, in figure 7.2, the curve $T_1 T_1'$ represents combinations of exports produced and imports utilized that are consistent with the production of a fixed amount of the domestic good, say $y_d = 95$. The curves $T_2 T_2'$, $T_3 T_3'$, and $T_4 T_4'$ represent com-
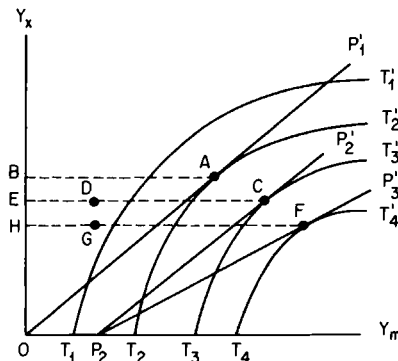

Fig. 7.2 The production theory approach

binations of exports and imports that are consistent with *higher* levels of domestic production in period 1, say y_d = 100, 105, and 110, respectively. Thus, for a fixed amount of imports, the economy's exports can increase only at the cost of diminishing domestic production.

In period 1, the rest of the world offers our small economy the trading line OP_1', which has slope p_m^1/p_x^1, where $p_m^1 > 0$ is the world price for a unit of the imported good and $p_x^1 > 0$ is the world price for a unit of the exported good. The highest domestic isoproduct curve tangent to the trading line OP_1' is T_2T_2' and the point of tangency is at A. Thus, in period 1, the economy imports AB units of y_m in exchange for OB units of y_x. Note that for simplicity we are assuming balanced trade in period 1.

In period 2, the frontier of the production possibilities set will be a new surface in y_d, y_x, y_m space. Thus surface can be represented by a new family of domestic isoproduct curves in y_x, y_m space. For simplicity, we shall assume that this new family of curves coincides with the old period-1 family, except that, due to technical progress, the old curves represent higher levels of domestic output; for example, the curve T_1T_1' now represents, say, y_d = 100 instead of 105, while T_2T_2', T_3T_3', and T_4T_4' now represent domestic output levels of, say, 105, 110, 115. This relabeling (or shifting in the general case) of the curves T_iT_i' represents the productivity effect in our new paradigm.

Suppose that in period 2, the economy is allowed to run a merchandise trade deficit of size OP_2. If the world prices of exports and imports remain constant, the new period-2 trading line would be P_2P_2'; the highest isoproduct curve tangent to this line is T_3T_3', and the point of tangency is C. The economy would trade CD units of imports for OE units of exports and receive an additional $ED = OP_2$ units of imports by running a trade deficit. In this paradigm, the deficit effect is OP_2.

However, it is unlikely that the prices of exports and imports will remain constant. Thus, suppose that p_x^2 and p_m^2 are the prices of exports and imports in period 2 and the price line P_2P_3' has slope equal to $p_m^2/p_x^2 < p_m^1/p_x^1$. The highest domestic isoproduct curve tangent to P_2P_3' is T_4T_4', and the point of tangency occurs at F. Note that the economy's improved terms of trade have allowed it to move from C to F; that is, to a higher level of domestic output. This increase in domestic output is the terms-of-trade effect. Thus, in period 2, the economy will exchange FG units of the imported good for OH units of the exported good, and the balance of trade deficit effect is HG equal to OP_2.

We now turn to the derivation of general analytical techniques that will allow us to quantify the above three effects in the case where we have many domestic, import, export and primary goods and the utilization of primary factors of production is not held constant.

7.4   The Sales Function and Price and Quantity Effects

All of our theoretical indexes and effects may be defined in terms of the economy's period $t-1$ and period t sales functions. The period t *sales func-*

*tion S^t for the economy's period t technology set Γ^t (or production function) may be defined as follows:[2]

$$
(1) \qquad S^t(p_d, p_x, p_m, v, v_0)
$$

$$
\equiv \max_{y_d,\ y_x,\ y_m} \{ p_d \cdot y_d : (y_d, y_x, y_m, v) \quad \text{belongs to } \Gamma^t;
$$

$$
p_x y_x - p_m \cdot y_m + v_0 \geq 0 \}.
$$

The sales function S depends on six sets of variables: (i) t indexes the technology set Γ^t, which corresponds to the period t domestic technology set that the economy can utilize; (ii) $p_d \equiv (p_{d1}, p_{d2}, \ldots, p_{dN_d})$, a vector of positive prices of the N_d domestic goods in the economy; (iii) $p_x \equiv (p_{x1}, p_{x2}, \ldots, p_{xN_x})$, a vector of positive prices of the N_d export goods that the economy can produce, denominated in units of domestic currency; (iv) $p_m \equiv (p_{m1}, p_{m2}, \ldots, p_{mN_m})$, a vector of positive prices of the N_m imported goods that the economy utilizes, denominated in units of domestic currency; (v) $v \equiv (v_1, v_2, \ldots, v_M)$, a vector of M positive amounts of primary inputs that the economy is utilizing; and (vi) v_0 is the balance of trade deficit (denominated in domestic currency) that the economy is allowed to run (if v_0 is negative, then $-v_0 > 0$ is the surplus that the economy is accumulating).

The nonnegative vectors y_d, y_x, and y_m of dimension N_d, N_x, and N_M, respectively, are vectors of domestic production, exports, and imports respectively. The notation $p_d \cdot y_d$ stands for the inner product of the vectors p_d and y_d; that is, $p_d \cdot y_d \equiv \sum_{i=1}^{Nd} p_{di} y_{di}$.

The sales function $S^t (p_d, p_x, p_m, v, v_0)$ tells us how much domestic output the period t economy can produce (valued at the reference prices p_d) given that the vector of primary inputs v is available, exports may be sold at prices p_x, imports may be purchased at prices p_m, and the private production sector is allowed to utilize a balance of trade deficit of size v_0. The sales function is the producer theory counterpart to Woodland's (1980) indirect trade utility function.

Define the private production sector's period t (net) deficit (surplus if negative) on merchandise trade by

$$
(2) \qquad v_0^t = p_m^t \cdot y_m^t - p_x^t \cdot y_x^t = \text{value of imports} - \text{value of exports.}
$$

When we evaluate the sales function S^t at the observed period t arguments, using the assumption of competitive profit maximizing behavior and a constant-returns-to-scale assumption on the technology set Γ^t, we find that

$$
(3) \qquad S^t (p_d^t, p_x^t, p_m^t, v^t, v_0^t) = p_d^t \cdot y_d^t = w^t \cdot v^t + v_0^t.
$$

In addition to the above assumptions, we assume that S^t is differentiable with respect to its arguments when evaluated at p^t, v^t, v_0^t. Then, adapting the arguments in Diewert (1983, 1092–94), we find that the first-order partial derivatives of S^t are equal to the following observable vectors:

$$\nabla_{p_d} S^t(p_d^t, p_x^t, p_m^t, v^t, v_0^t) \; = \; y_d^t,$$

(4)
$$\nabla_{p_x} S^t(p_d^t, p_x^t, p_m^t, v^t, v_0^t) \; = \; y_x^t,$$

$$\nabla_{p_m} S^t(p_d^t, p_x^t, p_m^t, v^t, v_0^t) \; = \; -y_m^t,$$

$$\nabla_v S^t (p_d^t, p_x^t, p_m^t, v^t, v_0^t) \; = \; w^t,$$

and
$$\frac{\partial S^t}{\partial v_0}(p_d^t, p_x^t, p_m^t, v^t, v_0^t) = 1.$$

The notion $\nabla_v S^t(p_d^t, p_x^t, p_m^t, v^t, v_0^t) \equiv [\partial S^t/\partial v_1, \partial S^t/\partial v_2, \ldots, \partial S^t/\partial v_M]$ stands for the vector of first-order partial derivatives of S^t with respect to the M components of $v \equiv [v_1, v_2, \ldots, v_M]$. We note also that $w^t \equiv [w_1^t, w_2^t, \ldots, w_M^t]$ is the vector of wage rates and rental prices that the primary factors charge to producers in period t.

It is evident from (3) and (4) that the deficit v_0 plays a role that is similar to the role of a primary input: a bigger deficit (holding other things constant) will lead to a bigger equilibrium value of domestic sales.

A last bit of notation will be required, on occasion, in what follows. Define $N \equiv N_d + N_x + N_m$ as the total number of nonprimary input goods in the economy, and define the following N dimensional vectors of prices and quantities: $p \equiv (p_d, p_x, p_m) = (p_1, p_2, \ldots, p_N)$ and $y \equiv (y_d, y_x, -y_m) \equiv (y_1, y_2, \ldots, y_N)$.

We shall conclude this section by utilizing the sales function in order to define various price and quantity effects. These effects will be useful in subsequent sections.

For each nonprimary input good n, define the period t theoretical Paasche and Laspeyres price effects, P^t_{Pn} and P^t_{Ln} and their geometric mean by:

(5) $\quad P^t_{Pn} \equiv S^t(p^t, v^t, v_0^t)/$
$$S^t(p_1^t, \ldots, p_{n-1}^t, p_n^{t-1}, p_{n+1}^t, \ldots, p_N^t, v^t, v_0^t);$$

(6) $\quad P^t_{Ln} \equiv S^{t-1}(p_1^{t-1}, \ldots, p_{n-1}^{t-1}, p_n^t, p_{n+1}^{t-1}, \ldots, p_N^{t-1}, v^{t-1}, v_0^{t-1})/$
$$S^t(p^{t-1}, v^{t-1}, v_0^{t-1});$$

(7) $\quad P^t_n \equiv (P^t_{Ln} P^t_{Pn})^{1/2}; \quad n = 1, 2, \ldots, N.$

The indexes P^t_{Ln} and P^t_{Pn} provide answers to the following hypothetical global comparative-statistics-type question: What is the proportional change in domestic sales that can be attributed to the change in the nth output price going from period $t-1$ to t, p_n^{t-1}, to p_n^t, holding constant other prices and primary input availabilities, holding the technology constant at the period $t-1$ or period t level, and holding the economy's balance of trade deficit (or surplus) constant at the period $t-1$ level, v_0^{t-1}, or at the period t level v_0^t? We call P^t_{Pn} a Paasche-type index because the constant reference prices and quantities are current period or period t variables, while the reference variables being held constant in the Laspeyres-type index P^t_{Ln} are the base period or period $t-1$ variables.

We turn now to the input side and for each primary input m, we define the period t theoretical Paasche and Laspeyres quantity effects, Q_{Pm}^t and Q_{Lm}^t, and their geometric mean as follows:

$$(8) \qquad Q_{Pm}^t \equiv S^t(p^t, v^t, v_0^t)/S^t(p^t, v_1^t, \ldots, v_{m-1}^t, v_m^{t-1}, v_{m+1}^t, \ldots, v_M^t, v_0^t);$$

$$(9) \qquad Q_{Lm}^t \equiv S^{t-1}(p^{t-1}, v_1^{t-1}, \ldots v_{m-1}^{t-1}, v_m^t, v_{m+1}^{t-1}, \ldots, v_M^{t-1}, v_0^{t-1})/$$
$$S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1});$$

$$(10) \qquad Q_m^t \equiv (Q_{Lm}^t Q_{Pm}^t)^{1/2}; \quad m = 1, 2, \ldots, M.$$

The indexes Q_{Lm}^t and Q_{Pm}^t provide answers to the following hypothetical questions: What is the proportional change in domestic sales that can be attributed to the change in the mth primary input going from period $t-1$ to period t, v_m^{t-1} to v_m^t, holding constant output prices and other primary input availabilities and also holding the technology and the balance of trade deficit constant?

Finally, we define the Paasche and Laspeyres theoretical deficit effects, Q_{PO}^t and Q_{LO}^t, and their geometric mean as follows:

$$(11) \qquad Q_{PO}^t \equiv S^t(p^t, v^t, v_0^t)/S^t(p^t, v^t, v_0^{t-1});$$

$$(12) \qquad Q_{LO}^t \equiv S^{t-1}(p^{t-1}, v^{t-1}, v_0^t)/S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1});$$

$$(13) \qquad Q_0^t \equiv (Q_{LO}^t Q_{PO}^t)^{1/2}.$$

The indexes Q_{LO}^t and Q_{PO}^t provide answers to the following hypothetical question: What is the proportional change in private domestic sales that can be attributed to a change in the private sector's balance of trade deficit from v_0^{t-1} to v_0^t holding constant output, export, and import prices, and holding constant the technology set and primary input availabilities?

The indexes or effects defined above by (5)–(13) have been called theoretical effects because, in general, they cannot be evaluated using only observable price and quantity data. However, in section 7.6 below, we shall show that the above geometric mean effects can be evaluated if we assume that the technology can be represented by translog sales functions. In the following section, we shall define some additional theoretical indexes where we vary more than one variable at a time.

7.5   Theoretical Productivity, Terms of Trade, and Welfare Indexes

We define the period t theoretical Paasche and Laspeyres productivity indexes, R_p^t and R_L^t, and their geometric mean as follows:

$$(14) \qquad R_p^t \equiv S^t(p^t, v^t, v_0^t)/S^{t-1}(p^t, v^t, v_0^t);$$

$$(15) \qquad R_L^t \equiv S^t(p^{t-1}, v^{t-1}, v_0^{t-1})/S^{t-1};(p^{t-1}, v^{t-1}, v_0^{t-1});$$

$$(16) \qquad R^t \equiv (R_L^t R_P^t)^{1/2}.$$

The productivity index R^t_p calculates a hypothetical rate of increase in private domestic product going from the period $t-1$ technology to the period t technology but holding constant prices, primary inputs, and the trade deficit at their period t levels. The Laspeyres theoretical productivity index R^t_L undertakes the same type of computation except that output prices, primary input quantities, and the trade deficit are held constant at their period $t-1$ levels.

The period t theoretical Paasche and Laspeyres terms-of-trade adjustment indexes, A^t_p and A^t_L, and their geometric mean may be defined as follows:

(17) $A^t_p \equiv S^t(p^t_d, p^t_x, p^t_m, v^t, v^t_0)/S^t(p^t_d, p^{t-1}_x, p^{t-1}_m, v^t, v^t_0);$

(18) $A^t_L \equiv S^{t-1}(p^{t-1}_d, p^t_x, p^t_m, v^{t-1}, v^{t-1}_0)/S^{t-1}(p^{t-1}_d, p^{t-1}_x, p^{t-1}_m, v^{t-1}, v^{t-1}_0);$

(19) $A^t \equiv (A^t_L A^t_p)^{1/2}.$

The theoretical terms of trade adjustment index A^t_p calculates a hypothetical rate of increase in domestic product due to a change in export and import prices from the period $t-1$ values, p^{t-1}_x, p^{t-1}_m, to the period t values, p^t_x, p^t_m, holding constant the technology, domestic prices, primary inputs, and the trade deficit at their period t levels. The theoretical Laspeyres index A^t_L is similar, except that the constant variables are fixed at their period $t-1$ levels.

The combined effects of productivity improvements and changes in the terms of trade are exhibited in the following theoretical Paasche and Laspeyres "welfare" change indexes,[3] W^t_p and W^t_L, and their geometric mean:

(20) $W^t_p \equiv S^t(p^t_d, p^t_x, p^t_m, v^t, v^t_0)/S^{t-1}(p^t_d, p^{t-1}_x, p^{t-1}_m, v^t, v^t_0),$

(21) $W^t_L \equiv S^t(p^{t-1}_d, p^t_x, p^t_m, v^{t-1}, v^{t-1}_0)/S^{t-1}(p^{t-1}_d, p^{t-1}_x, p^{t-1}_m, v^{t-1}_0),$

(22) $W^t \equiv (W^t_L W^t_p)^{1/2}.$

Finally, the combined short-run effects of productivity improvements, changes in the terms of trade, and changes in the allowed merchandise trade deficit are contained in the following theoretical Paasche and Laspeyres "total welfare" change indexes, T^t_p and T^t_L, and their geometric mean:

(23) $T^t_p \equiv S^t(p^t_d, p^t_x, p^t_m, v^t, v^t_0)/S^{t-1}(p^t_d, p^{t-1}_x, p^{t-1}_{w6m}, v^t, v^{t-1}_0);$

(24) $T^t_L \equiv S^t(p^{t-1}_d, p^t_x, p^t_m, v^{t-1}, v^{t-1}_0)/S^{t-1}(p^{t-1}_d, p^{t-1}_x, p^{t-1}_m, v^{t-1}, v^{t-1}_0);$

(25) $T^t \equiv (T^t_L T^t_p)^{1/2}.$

In the following two sections, we show how the various theoretical indexes and effects defined in this section and the previous section can be evaluated using observable data.

7.6   Exact Indexes: The Translog Approach

Suppose that the sales function in period t, S^t, has the following translog functional form:

$$
\begin{aligned}
\ln S^t(p, v, v_0) \equiv \alpha_0^t &+ \sum_{n=1}^{N} \alpha_n^t \ln p_n + (1/2) \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_{ij} \ln p_i \ln p_j \\
&+ \sum_{m=0}^{M} \beta_m^t \ln v_m^t + (1/2) \sum_{i=0}^{M} \sum_{j=0}^{M} \beta_{ij} \ln v_i \ln v_j \\
&+ \sum_{n=1}^{N} \sum_{m=0}^{M} \gamma_{nm} \ln p_n \ln v_m
\end{aligned}
$$

(26)

where $\alpha_{ij} = \alpha_{ji}$, $\beta_{ij} = \beta_{ji}$ for all i and j, and the parameters satisfy various other restrictions that ensure that S^t (p_d, p_x, p_m, v, v_0) is (i) linearly homogeneous nondecreasing and concave in v_0, v for fixed $p \equiv (p_d, p_x, p_m)$, (ii) linearly homogeneous convex and nondecreasing in p_d for fixed p_x, p_m, v, v_0, and (iii) homogeneous of degree zero and quasi convex in p_x, p_m, and v_0 for fixed p_d and v. The definition (26) requires that all prices and quantities be positive. In particular, we require $v_0 > 0$. If $v_0 < 0$, then we replace v_0 in (26) by $-v_0$.

Note that the coefficients corresponding to the quadratic terms in (26) do not depend on time t but that the other coefficients (α_0^t, α_n^t, and β_m^t) are allowed to be different in each time period. Also, the quadratic nature of (26) means that the translog sales function can provide a second-order approximation to an arbitrary twice continuously differentiable sales function $S(p, v, v_0)$; that is, the translog sales function is a flexible functional form.

Suppose that S^{t-1} and S^t are translog sales functions defined by (26) with $v_0^{t-1} v_0^t > 0$ (so that the trade deficit has the same sign in periods $t-1$ and t).[4] Then Diewert and Morrison (1986, 671–74) showed that the following theoretical indexes (defined in the previous two sections) may be *exactly* computed using observable price and quantity data as follows: the price effects P_n^t defined by (7) for $n = 1, \ldots, N$ may be computed using

(27) $\ln P_n^t = (1/2)[(p_n^t y_n^t / p_d^t \cdot y_d^t) + (p_n^{t-1} y_n^{t-1} / p_d^{t-1} \cdot y_d^{t-1})] \ln (p_n^t / p_n^{t-1});$

the quantity effects Q_m^t defined by (10) for $m = 1, \ldots, M$ may be computed using

(28) $\ln Q_m^t = (1/2)[w_m^t v_m^t / p_d^t \cdot y_d^t) + (w_m^{t-1} v_m^{t-1} / p_d^{t-1} y_d^{t-1})] \ln (v_m^t / v_m^{t-1});$

the deficit effect Q_0^t defined by (13) may be computed using

(29) $\ln Q_0^t = (1/2)[(v_0^t / p_d^t \cdot y_d^t) + (v_0^{t-1} / p_d^{t-1}) \cdot y_d^{t-1})] \ln (v_0^t / v_0^{t-1});$

the productivity index R^t defined by (16) may be computed using

(30) $R^t = p_d^t \cdot y_d^t / p_d^{t-1} \cdot y_d^{t-1} \left[\prod_{n=1}^{N} P_n^t \right] \left[\prod_{m=0}^{M} Q_m^t \right],$

where the P_n^t are defined by (27) and the Q_m^t are defined by (28) and (29); the terms-of-trade adjustment index, A^t, defined by (19), may be computed using

$$\ln A^t = \sum_{i=1}^{N_x} (\tfrac{1}{2})[(p_{xi}^t y_{xi}^t / p_d^t \cdot y_d^t)$$

(31)
$$+ (p_{xi}^{t-1} y_{xi}^{t-1} / p_d^{t-1} \cdot y_d^{t-1})] \ln (p_{xi}^t / p_{xi}^{t-1}) - \sum_{j=1}^{N_m} (\tfrac{1}{2})[(p_{mj}^t y_{mj}^t / p_d^t \cdot y_d^t)$$

$$+ (p_{mj}^{t-1} y_{mj}^{t-1} / p_d^{t-1} \cdot y_d^{t-1})] \ln (p_{mj}^t / p_{mj}^{t-1}),$$

and the welfare change index, W^t, defined by (22), may be computed as follows:

(32)
$$W^t = R^t A^t,$$

where R^t and A^t are defined by (30) and (31). Thus the welfare change index decomposes nicely into the product of a productivity index times a terms-of-trade adjustment index.

Finally, under our translog assumptions, the theoretical total welfare change index, T^t, defined by (25), may be computed as follows:

(33)
$$T^t = W^t Q_0^t = R^t A^t Q_0^t,$$

where R^t, A^t, and Q_0^t are defined by (30), (31), and (29), respectively.

In subsequent sections of this paper, we shall evaluate the indexes defined in this section using Japanese and U.S. data. However, there is a problem with the exact translog approach outlined in this section: in order to theoretically justify our results, we must have the trade deficit retaining the same sign in the two periods under consideration. Since this assumption is not always satisfied (for either the Japanese or U.S. data), we need to utilize another approach to evaluate our theoretical indexes when the trade deficit changes sign. This alternative approach (due to Diewert and Morrison [1986, 674–77]) will be explained in the following section.

7.7   A Nonparametric First-Order Approximation Approach

Recall equations (4), which enable us to evaluate the first-order partial derivatives of the sales function S^t, evaluated at the period t prices and quantities p^t, v^t, v_0^t. If we replace t by $t-1$, then we may also use equations (4) to evaluate the first order partial derivatives $S^{t-1} (p^{t-1}, v^{t-1}, v_0^{t-1})$. We can use equations (4) to form first-order Taylor series approximations to the theoretical indexes, defined in sections 7.4 and 7.5, that can be evaluated numerically using observable data. Thus we define the following first-order approximations $\tilde{R}_P^t$ and $\tilde{R}_L^t$ to the theoretical productivity indexes R_P^t and R_L^t defined by (14) and (15) as follows:

(34)
$$\tilde{R}_P^t \equiv S^t(p^t, v^t, v_0^t) / [S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1})$$
$$+ \nabla_p S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1}) \cdot (p^t - p^{t-1})$$
$$+ \nabla_v S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1}) \cdot (v^t - v^{t-1})$$
$$+ \nabla_{v_0} S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1}) \cdot (v_0^t - v_0^{t-1})]$$

$$(35) \qquad = p_d^t \cdot y_d^t / [p_d^{t-1} \cdot y_d^{t-1} + y^{t-1} \cdot (p^t - p^{t-1})$$
$$+ w^{t-1} \cdot (v^t - v^{t-1}) + v_0^t - v_0^{t-1}],$$

where we have used (3) and (4) to derive (35) from (34). Similarly, we define the Laspeyres approximate productivity index $\tilde{R}^t_L$ by (36) and derive (37) using (3) and (4):

$$(36) \; \tilde{R}^t_L \equiv [S^t(p^t, v^t, v_0^t) + \nabla_p S^t(p^t, v^t, v_0^t) \cdot (p^{t-1} - p^t)$$
$$+ \nabla_v S^t(p^t, v^t, v_0^t) \cdot (v^t - v^{t-1})$$
$$+ \nabla_{v_0} S^t(p^t, v^t, v_0^t)(v_0^{t-1} - v_0^t)]/S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1})$$

$$(37) \qquad = [p_d^t \cdot y_d^t + y^t \cdot (p^{t-1} - p^t) + w^t \cdot (v^{t-1} - v^t) + v_0^{t-1} - v_0^t]/p_d^{t-1} \cdot y_d^{t-1}.$$

Now define the geometric mean of the above two approximate productivity indexes by

$$(38) \qquad\qquad\qquad \tilde{R}^t \equiv (\tilde{R}^t_L \tilde{R}^t_P)^{1/2} ,$$

where $\tilde{R}^t_P$ and $\tilde{R}^t_L$ are defined by (35) and (37), respectively.

The quadratic approximation lemma of Denny and Fuss (1983a, 1983b) leads us to believe that the index $\tilde{R}^t$, defined by (38), will approximate the true index R^t, defined by (16), to the second order.

Analogous first-order approximations to the theoretical deficit effects Q^t_{P0} and Q^t_{L0} defined above by (11) and (12) are given by (39) and (40) below, and their geometric average is defined by (41):

$$(39) \; \tilde{Q}^t_{P0} \equiv S^t(p^t, v^t, v_0^t)/\{S^t(p^t, v^t, v_0^t) + [\partial S^t(p^t, v^t, v_0^t)/\partial v_0](v_0^{t-1} - v_0^t)\}$$
$$= [1 - (v_0^t - v_0^{t-1})(p_d^t \cdot y_d^t)^{-1}]^{-1};$$

$$(40) \; \tilde{Q}^t_{L0} \equiv \{S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1}) + [\partial S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1})/\partial v_0](v_0^t - v_0^{t-1})\}$$
$$/S^{t-1}(p^{t-1}, v^{t-1}, v_0^{t-1})$$
$$= 1 + (v_0^t - v_0^{t-1})(p_d^{t-1} \cdot y_d^{t-1})^{-1};$$

$$(41) \; \tilde{Q}^t_0 \equiv (\tilde{Q}^t_{L0} \tilde{Q}^t_{P0})^{1/2}.$$

First-order approximations to the theoretical terms-of-trade sales adjustment indexes defined by (17) and (18) and their geometric mean are defined by:

$$(42) \; \tilde{A}^t_L \equiv [p_d^{t-1} \cdot y_d^{t-1} + y_x^{t-1} \cdot (p_x^{t-1} - p_x^{t-1}) - y_m^{t-1} \cdot (p_m^{t-1} - p_m^{t-1})]/p_d^{t-1} \cdot y_d^{t-1},$$

$$(43) \; \tilde{A}^t_P \equiv p_d^t \cdot y_d^t / [p_d^t \cdot y_d^t + y_x^t \cdot (p_x^{t-1} - p_x^t) - y_m^t \cdot (p_m^{t-1} - p_m^t)],$$

$$(44) \; \tilde{A}^t \equiv (\tilde{A}^t_L \tilde{A}^t_P)^{1/2}.$$

Similar first-order approximations to the theoretical welfare indexes defined by (20) and (21) are defined by (45) and (46) (these indexes incorporate changes in productivity and changes in the terms of trade but hold the balance-of-trade deficit constant):

(45) $\quad\tilde{W}_L^t \equiv [p_d^t \cdot y_d^t + y_d^t \cdot (p_d^{t-1} - p_d^t) + w^t \cdot (v^{t-1} - v^t)$
$\qquad\qquad + (v_0^{t-1} - v_0^t)]/p_d^{t-1} \cdot y_d^{t-1},$

(46) $\quad\tilde{W}_P^t \equiv p_d^t . y_d^t / [p_d^{t-1} \cdot y_d^{t-1} + y_d^{t-1} \cdot (p_d^t - p_d^{t-1})$
$\qquad\qquad + w^{t-1} \cdot (v^t - v^{t-1}) + (v_0^t - v_0^{t-1})],$

(47) $\quad\tilde{W}^t \equiv (\tilde{W}_L^t \tilde{W}_P^t)^{1/2}.$

Finally, first-order approximations to the theoretical total welfare change indexes defined by (23) and (24) are defined by (48) and (49) (these indexes are like the welfare indexes except that they also incorporate changes in the balance of trade deficit):

(48) $\quad\tilde{T}_L^t \equiv p_d^t \cdot y_d^t + y_d^t \cdot (p_d^{t-1} - p_d^t) + w^t \cdot (v^{t-1} - v^t)]/p_d^{t-1} \cdot y_d^{t-1},$

(49) $\quad\tilde{T}_p^t \equiv p_d^t . y_d^t / p_d^{t-1} \cdot y_d^{t-1} + y_d^{t-1} \cdot (p_d^t - p_d^{t-1}) + w^{t-1} \cdot (v^t - v^{t-1})],$

(50) $\quad\tilde{T}^t \equiv (\tilde{T}_L^t \tilde{T}_p^t)^{1/2}.$

Since our new geometric mean indexes $\tilde{R}^t$, $\tilde{Q}_0^t$, $\tilde{A}^t$, $\tilde{W}^t$, and $\tilde{T}^t$ do not depend on any functional form assumptions, we call them nonparametric indexes.

Our new nonparametric indexes do not have the nice multiplicative properties that the translog indexes defined in the previous section had: recall (32), $W^t = R^t A^t$, and (33), $T^t = R^t A^t Q_0^t = W^t Q_0^t$. However, in our empirical work, we shall find that our new indexes had the above multiplicative properties to a high degree of approximation. We turn now to the empirical implementation of the indexes defined in this section and the previous section.

7.8 The Japanese Data

The Japanese data used for this study were developed from the *Economic Statistics Annual* (Bank of Japan 1986) from the Research and Statistics Department of the Bank of Japan. The data required are the prices and quantities of output (value added), labor, capital, exports, and imports for each calendar year. The capital and labor series were generated from data on gross fixed capital formation, operating surplus, consumption of fixed capital, compensation of employees, and number of employees. Value added was then computed as the sum of the values of capital and labor. The export and import data were generated from more detailed value and "quantum" data for six different types of exports and seven imports. This will allow us to assess the impact of the energy price shock in the early 1970s.

More specifically, the data on capital was constructed by using a benchmark capital level (for 1966), supplied by John Helliwell and his associates at the University of British Columbia and based on Organization for Economic Cooperation and Development (OECD) data, and then using the investment data from the Bank of Japan series on gross fixed capital formation, along with a

12.5% rate of depreciation, to construct the capital quantity series. The total value of capital ($w_K \cdot v_K = V_K$) was assumed to be the sum of the operating surplus plus the consumption of fixed capital. The price of capital was then computed as V_K/v_K. Bank of Japan series were also available for total compensation of employees ($w_L \cdot v_L = V_L$) and the number of employees (v_L), which were used to compute a price of labor as $w_L = V_L/v_L$.[5]

The export and import data, as mentioned above, included the value of six exports and eight imports plus totals. The export data encompassed separate information on food, textiles, chemicals, nonmetallic minerals, metal and metal products, and machinery and equipment. The import data included food, textiles, metals, mineral fuels, other raw materials, chemicals, and machinery and equipment. The prices of each component were computed by dividing each value by the "quantum" indicator, which is described by the Bank of Japan as the total value divided by the unit value. The resulting prices were used to calculate aggregate prices for exports and imports by using a translog aggregation procedure. The resulting total values for exports and imports did not exactly coincide with the full totals due to a small miscellaneous component that was not provided with a quantum index. The quantities (or quantum values) were therefore regenerated by using the aggregated prices (p_x and p_m) along with the full total values of exports and imports (V_x and V_m) to compute the constant dollar quantity indexes y_x and y_m.[6]

Finally, value added ($p \cdot y$) was calculated as $V_y = V_L + V_K$, and the corresponding price (p) was assumed to be equal to the implicit gross domestic product (GDP) deflator provided by the Bank of Japan. The value of domestic sales could then be calculated as $V_s = V_y - V_x + V_m$ and the price calculated implicitly as a translog index of the prices of these components of absorption. These data may be found in table 7.1.

Looking at the data in table 7.1, a number of trends emerge. For example, the price of labor increased dramatically, while the number of employees stayed relatively constant. Compensation per employee increased by at least a factor of seven during this time period, while the number of employees increased by only 20%. By contrast, the data indicate that the capital rental price increased by approximately two times and the stock level by close to three times.[7] During the same time span, output increased substantially; value added in constant dollars increased by a factor of almost three. The corresponding price of output also increased to approximately 275% of its value in the beginning of the sample.

The pattern of prices of traded goods is particularly interesting. The unit price of exported goods from Japan only doubled during this time period. The price of imported goods, however, provides a strong contrast to this. Although the price of some imported goods was actually falling slightly in the early 1970s, from 1972 to 1982—in response to dramatic increases in costs of raw materials and especially fuel—the price of imported goods increased by a factor of four. Since these price trends are so different and international trade is fairly substantial in Japan, explicit consideration of terms of trade adjust-

Table 7.1   **Japanese Price and Quantity Data**

Year	K	P_K	L	P_L	Y	P_Y	X	P_X	M	P_M	S	P_S
1967	29,024.2	.78710	41,305.5	.45343	54,194.4	.76405	3,940.108	.96745	3,836.849	.95994	57,191.9	.72517
1968	30,654.1	.87577	41,994.0	.52581	61,100.4	.80350	4,954.146	.96083	4,355.298	.96187	62,222.5	.77682
1969	33,070.7	.96136	42,313.0	.60483	68,797.4	.84201	5,882.085	.99749	4,992.975	.96506	68,738.2	.82042
1970	36,316.2	1.03890	42,766.3	.73013	75,335.9	.90373	6,821.044	1.04710	5,981.378	1.01768	76,251.3	.89196
1971	40,369.7	.95311	42,993.0	.86284	78,818.1	.95054	8,266.639	1.03638	5,989.071	1.07381	77,216.8	.95025
1972	44,320.0	1.00000	43,035.0	1.00000	85,815.0	1.00000	8,928.027	1.00000	6,597.156	1.00000	85,221.0	1.00000
1973	48,708.2	1.08639	44,151.6	1.22489	93,380.8	1.11940	9,512.114	1.09361	7,953.468	1.18050	95,460.4	1.12054
1974	53,883.1	1.08656	43,966.9	1.55597	92,531.9	1.34997	11,192.440	1.50515	7,869.874	2.15937	95,156.1	1.34471
1975	57,530.7	1.04612	43,849.4	1.81640	95,026.3	1.45533	11,278.783	1.55822	7,040.110	2.43634	97,603.6	1.43201
1976	60,671.8	1.10302	44,252.4	2.04039	100,089.4	1.54842	13,820.367	1.52077	7,706.426	2.47283	103,938.5	1.49972
1977	63,747.6	1.12564	44,848.4	2.24906	104,862.2	1.63615	15,047.310	1.50203	7,920.279	2.40759	109,137.5	1.55385
1978	66,957.2	1.21046	45,402.5	2.39676	110,281.2	1.71132	15,200.701	1.39419	8,242.751	2.00481	117,282.7	1.58858
1979	70,777.9	1.22444	45,998.6	2.55838	115,269.0	1.75614	15,027.924	1.57770	9,095.688	2.61892	126,517.7	1.63097
1980	74,882.9	1.25878	46,477.1	2.75886	120,847.1	1.80579	17,555.877	1.72185	8,724.714	3.55662	132,832.1	1.69111
1981	78,665.4	1.23931	46,854.9	2.94883	125,788.3	1.85284	19,466.441	1.80288	8,275.875	3.79426	135,839.3	1.71974
1982	82,516.0	1.25230	47,333.5	3.12253	129,723.4	1.88582	18,882.223	1.96821	8,153.281	4.03833	143,399.4	1.73282

Note: K is the quantity of capital services, L is the quantity of labor, Y is the real value added, X is the quantity of exports, M is the quantity of imports, and S is the quantity of domestic sales. P_K, P_L, P_Y, P_X, P_M and P_S are the corresponding price indices.

ments should have a relatively large impact on indexes for Japan. In addition, the balance of payments, $V_m - V_x$, is increasingly negative over this period; the value of exports becomes larger over time. We turn now to an evaluation of the indexes defined in sections 7.6 and 7.7.

7.9   Japanese Indexes of Productivity and Welfare Change

We need to reconcile the notation used in the previous section with the notation we used earlier in the theoretical sections. We are now assuming that N_d, N_x, and N_m (the number of domestic goods, exported goods, and imported goods respectively) all equal one, so that $N = N_d + N_x + N_m = 3$. The price vector $p^t \equiv (p_d^t, p_x^t, p_m^t) \equiv (p_1^t, p_2^t, p_3^t) = (P_S^t, P_X^t, P_M^t)$ where P_S^t is the price of sales in period t, P_X^t is the price of exports in period t, and P_M^t is the price of imports in period t. These three price series may be found in table 7.1. The quantity vectors y_d^t, y_x^t, and y_m^t, which occur in sections 7.6 and 7.7, are actually scalars in our present application and are equal to the quantity series S^t, X^t, and M^t which are listed in table 7.1. The period t quantity vector y^t, which occurs in sections 7.6 and 7.7 is defined to be the following three dimensional vector: $y^t \equiv (y_1^t, y_2^t, y_3^t) \equiv (S^t, X^t, -M^t)$. Note that the first two components of y^t are positive, while the third component is negative. The period t balance of trade deficit is defined as $v_0^t \equiv P_M^t M^t - P_X^t X^t$. Finally, the primary input vector $v^t \equiv (v_1^t, v_2^t)$ is defined to be (L^t, K^t) and the corresponding period t price vector $w^t \equiv (w_1^t, w_2^t)$ is defined to be (P_L^t, P_K^t), where L^t, K^t, P_L^t, P_K^t are listed in table 7.1.

The three *price effects*, P_1^t, P_2^t, and P_3^t, defined by (27), the two *quantity effects*, Q_1^t and Q_2^t, defined by (28), and the *deficit effect* Q_0^t defined by (29) are evaluated using the Japanese data listed in table 7.1 and are listed in table 7.2. Recall that P_i^t is a measure of the proportional increase in the value of domestic sales due to the change in the ith price from its actual period $t-1$ value to its period t value, holding constant the trade deficit, the technology, and other prices and quantities. Similarly, Q_j^t is a measure of the proportional increase in the value of domestic sales due to the change in the jth primary input from its period $t-1$ value to its period t value, holding constant the trade deficit, the technology, other primary input utilization, and the prices of domestic output, exports, and imports. Finally, Q_0^t is a measure of the proportional increase in the value of domestic sales due to the change in the country's balance of trade deficit holding constant the technology, the prices of domestic goods, exports and imports, and the utilization of primary inputs.

Information on single determinants of production trends is evident from the individual comparative statics indexes in table 7.2. For example, Q_1^t in table 7.2 shows the impact on the change in domestic product from increasing the use of labor. This index indicates that increases in the labor input have contributed to a greater product in all but two years—1974 and 1975, when labor growth was negative—but the effect is negligible. By contrast, the contribu-

Table 7.2   **Translog Price and Quantity Effects for Japan**

Year	P_1^t	P_2^t	P_3^t	Q_1^t	Q_2^t	Q_0^t
1967	1.077	1.003	1.000	1.008	1.019	1.017
1968	1.071	.999	.999	1.007	1.030	.987
1969	1.056	1.003	.999	1.003	1.043	.992
1970	1.087	1.005	.995	1.004	1.053	1.000
1971	1.065	.998	.995	1.002	1.058	.981
1972	1.052	.996	1.005	1.000	1.049	1.000
1973	1.120	1.009	.986	1.013	1.049	1.055
1974	1.200	1.037	.933	.997	1.049	
1975	1.064	1.004	.984	.998	1.029	
1976	1.047	.996	.998	1.005	1.023	.986
1977	1.036	.998	1.003	1.007	1.021	.989
1978	1.022	.990	1.018	1.007	1.021	.997
1979	1.026	1.014	.973	1.007	1.024	
1980	1.036	1.010	.961	1.005	1.023	1.000
1981	1.016	1.006	.991	1.004	1.020	
1982	1.007	1.013	.991	1.006	1.020	.997
Mean	1.061	1.005	.989	1.005	1.034	1.000

Note: P_1^t is the domestic sales price effect, P_2^t is the price of exports effect, P_3^t is the price of imports effect, Q_1^t is the quantity of labor effect, Q_2^t is the quantity of capital effect, and Q_0^t is the deficit effect.

tion of increases in the capital stock represented by Q_2^t is quite high; in 1970–74 in particular, around 5% of product growth can be attributed to an increase in capital.

The impact of changes in prices in domestic product can also be determined from table 7.2. For example, P_1^t indicates the year t increase in the value of domestic product attributable purely to domestic sales price increases. This index increased by a positive but decreasing proportion from about 7.8% in 1967 to .8% in 1982.

Looking at the price effects P_2^t, the changes in the price of exports caused increased total value of product for most years. However, in some years— 1968, 1971, 1972, 1976, 1977, and 1978—changes in the price of exports contributed to a very small decrease in product value.

The impacts of import price changes on domestic sales, the P_3^t, are particularly interesting. The substantial increase in import prices during the two energy crises leads to decreases in output for many years. This is particularly true for 1974, where the increase in the import price alone would have caused a 7% decrease in sales if not attenuated by changes in other determinants of the sales level. Note, however, that later—in 1977 and 1978—a slight increase in sales could be attributed to import price changes; the aggregate price of imported goods actually declined in this period due partly to a drop in total fuels imported.

The translog productivity indexes, R^t, defined by (30), the translog terms of trade adjustment indexes, A^t, defined by (31), the translog welfare change indexes, W^t, defined by (32), and the translog total welfare change indexes, T^t, defined by (33), are shown in table 7.3 using the Japanese data in table 7.1. The corresponding nonparametric indexes, $\tilde{R}^t$, defined by (38), $\tilde{A}^t$, defined by (44), $\tilde{W}^t$, defined by (47), and $\tilde{T}^t$, defined by (50), are also listed in table 7.3.

Note that the translog indexes R^t, W^t, and T^t are not defined for years when the merchandise trade deficit changes sign. The nonparametric indexes $\tilde{R}^t$, $\tilde{A}^t$, $\tilde{W}^t$, and $\tilde{T}^t$ are always well defined. Note that in years when the translog and nonparametric indexes are both defined, they approximate each other rather closely.

The productivity indexes R^t and $\tilde{R}^t$ show a substantial decrease in productivity growth in the 1970–71 period and an even stronger impact in 1973–74, when the rates of growth actually became negative. The post-1975 years were characterized by very healthy productivity growth rates, although not quite as high as in the earlier years of the sample, particularly for 1981, which exhibited growth of only .2%. The largest percentage growth in the post–energy crisis years was the "snapback" in 1976, when growth jumped back up to 4.9%; this is closely followed by a 4.7% increase in 1980.

The terms-of-trade adjustment indexes, A^t and $\tilde{A}^t$, which show the effects on domestic sales of combined changes in export and import prices, are rather close to one for most years. However, in three years, the terms-of-trade adjustment factor was significantly below one, which indicates an increase in import prices relative to export prices. These three years corresponded to the OPEC price shock years, and the combined effect of changes in export and import prices in these years was a *decrease* in growth of over 3% in 1974, 1.3% in 1979, and almost 3% in 1980. Thus, we are able to measure rather precisely the effects on growth of the adverse changes in Japan's terms of trade during these years.

Adjusting the productivity growth measures by these terms-of-trade indexes results in the welfare measures W^t and $\tilde{W}^t$, which are closely comparable and closely related to the productivity indexes since the A^t are close to 1.0. The impacts of the energy "crisis" are, of course, more evident in these "welfare" indexes; welfare growth in 1971 and 1974 was negative: about -3% and -5.7% respectively.

Finally, for the sales and first-order approximation approaches, the combined indexes incorporating productivity, terms-of-trade changes and the impact of the deficit are represented by the translog index T^t and the nonparametric index $\tilde{T}^t$, defined by (33) and (50), respectively. These indexes are nearly identical for those years where the translog index is defined. Years where the merchandise trade deficit grew significantly, thus causing $\tilde{T}^t$ to exceed $\tilde{W}^t$ by more than about 1%, were 1967, 1973, and 1979. Years where the trade deficit declined significantly were 1968, 1971, 1977, and 1981.

Table 7.3   **Japanese Productivity, Terms-of-Trade Adjustment, and Welfare Change Indexes**

Year	R^t	$\tilde{R}^t$	A^t	$\tilde{A}^t$	W^t	$\tilde{W}^t$	T^t	$\tilde{T}^t$
1967	1.059	1.064	1.004	1.004	1.064	1.068	1.082	1.082
1968	1.062	1.060	.999	.999	1.061	1.059	1.047	1.047
1969	1.059	1.059	1.003	1.003	1.063	1.063	1.055	1.055
1970	1.046	1.046	1.000	1.000	1.046	1.046	1.047	1.047
1971	.977	.976	.994	.994	.971	.971	.954	.953
1972	1.047	1.047	1.001	1.001	1.049	1.049	1.050	1.050
1973	1.003	1.035	.995	.995	.998	1.030	1.054	1.053
1974		.971	.968	.970		.943		.951
1975		1.016	.989	.988		1.005		.997
1976	1.054	1.049	.995	.995	1.049	1.044	1.035	1.035
1977	1.029	1.029	1.001	1.001	1.031	1.030	1.020	1.020
1978	1.038	1.038	1.009	1.009	1.047	1.047	1.044	1.044
1979		1.029	1.986	1.987		1.016		1.045
1980	1.047	1.047	.972	.972	1.018	1.018	1.019	1.019
1981		1.017	.997	.997		1.015		.997
1982	1.026	1.026	1.004	1.004	1.031	1.031	1.028	1.028
Mean	1.037	1.032	.995	.995	1.036	1.027	1.036	1.026

Note: The productivity indexes, R^t and $\tilde{R}^t$, are defined by (30) and (38), the terms of trade adjustment indexes, A^t and $\tilde{A}^t$, are defined by (31) and (44), the welfare change indexes, W^t and $\tilde{W}^t$, are defined by (32) and (47), and the total welfare change indexes, which incorporate changes in productivity, in the terms of trade and in the trade deficit, T^t and $\tilde{T}^t$, are defined by (33) and (50).

7.10   The U.S. Data

The data required to calculate the indexes include price and quantity information on national output, capital and labor inputs, exports, and imports. We have developed the output, import, and export data for 1967–82 from the National Income and Product Accounts (U.S. Department of Commerce 1981, 1982, 1983) and have used real capital stock data constructed by the Bureau of Labor Statistics (U.S. Department of Labor 1983) and real labor data updated from Jorgenson and Fraumeni (1981), since these series closely approximate our theoretically ideal indexes.

More specifically, we have calculated the value of output $(P_Y^t Y^t)$ as the gross domestic business product including tenant-occupied housing output, property taxes, and federal subsidies to businesses, but excluding federal, state, and local indirect taxes and owner-occupied housing. The corresponding price index (P_Y^t), was computed by cumulating the Business Gross Domestic Product Chain Price index. Note that our output series for the United States is conceptually somewhat different from our value-added output series for Japan. The Y^t and P_Y^t series for the United States may be found in table 7.4.

The values of merchandise exports $(p_x^t \cdot y_x^t)$ and imports $(p_m^t \cdot y_m^t)$ were deter-

Table 7.4   **U.S. Prices and Quantities**

Year	K	P_K	L	P_L	Y	P_Y	X	P_X	M	P_M	S	P_S
1967	254.516	.92252	543.092	.72204	788.585	.79501	36.0953	.84947	35.6903	.80249	779.516	.80182
1968	266.369	.95383	556.789	.77277	838.986	.81567	38.9682	.86270	43.2146	.81339	830.649	.82588
1969	278.533	.94054	576.440	.82668	872.243	.84667	40.8635	.89104	45.5838	.83500	860.247	.86056
1970	290.697	.90078	570.485	.88226	863.168	.88646	45.2778	.93788	47.4815	.88906	846.679	.90359
1971	300.990	.95611	573.463	.93318	888.341	.92636	44.6520	.96959	51.5564	.94030	869.694	.95239
1972	311.907	1.00000	595.496	1.00000	938.255	.96712	49.3530	1.00000	58.6285	1.00000	916.821	1.00000
1973	326.255	1.09631	625.866	1.06135	1,021.939	1.00000	61.2051	1.16592	62.7543	1.17375	981.204	1.04900
1974	340.914	1.08115	630.035	1.15423	1,037.676	1.05601	65.9362	1.48958	60.7473	1.76037	996.297	1.10873
1975	350.272	1.21930	611.574	1.22917	1,019.453	1.15632	63.9734	1.66677	53.1780	1.94587	960.740	1.22404
1976	356.822	1.32665	635.394	1.32006	1,038.203	1.26385	66.5975	1.71765	65.1605	1.96222	994.593	1.33298
1977	366.179	1.48218	666.360	1.40568	1,086.373	1.33211	66.9110	1.78929	72.8799	2.13216	1,076.490	1.40762
1978	379.279	1.57942	705.067	1.53092	1,188.673	1.41203	74.1455	1.90020	78.9832	2.29565	1,146.755	1.49912
1979	393.627	1.64533	736.629	1.66222	1,229.876	1.52217	82.6177	2.16876	80.0676	2.70617	1,182.880	1.61455
1980	407.663	1.70086	743.179	1.78329	1,214.455	1.66221	91.4960	2.40338	73.4325	3.41670	1,160.537	1.76632
1981	419.203	1.93726	759.257	1.93094	1,257.408	1.81181	88.0600	2.63230	74.0626	3.62441	1,185.064	1.95356
1982	435.110	1.86782	738.415	2.06104	1,177.835	1.98212	79.7703	2.62128	71.8893	3.53622	1,116.955	2.12937

Note: K is the quantity of capital services, L is the quantity of labor, Y is real value added, X is the quantity of exports, M is the quantity of imports, and S is the quantity of domestic sales. P_K, P_L, P_Y, P_X, P_M, and P_S are the corresponding price indexes.

mined by adding the durable and nondurable export and import values, respectively, reported in the national accounts (U.S. Department of Commerce 1981, 1982, 1983). Tariff revenues were added to the value of imports. Corresponding prices (P_X^t and P_M^t) were calculated as translog indexes of the components of each measure, and quantities (X^t and M^t) were determined implicitly. For 1967–82, value and price data for nine different types of exports and 10 types of imports were available, which were used to compute chain price indexes.

Using the values of imports and exports, $P_M^t M^t = p_m^t \cdot y_m^t$ and $P_X^t X^t = p_x^t \cdot y_x^t$, tax-adjusted gross domestic private business sales to domestic purchasers, or sales, was calculated as $P_S^t S^t = P_Y^t Y^t - P_X^t X^t + P_M^t M^t$. The corresponding price (P_S^t) was determined by cumulating the gross domestic purchases chain price index from the national accounts, and the constant dollar quantity S^t was calculated by division.

For our labor quantity series, L^t, we used the series constructed by Jorgenson and Fraumeni (1981), which is conveniently shown in a table elsewhere (U.S. Department of Labor 1983, 77). Our total private labor compensation series, $P_L^t L^t$, was taken from the same publication. The price of labor, P_L^t, was determined by division.[8]

For our capital services quantity series, K^t, we used the private business sector (excluding government enterprises) constant dollar capital services input as displayed by the U.S. Department of Labor (1983, 77). In order to ensure that the value of privately produced outputs equals the value of privately utilized inputs, we determined the price of capital services, P_K^t, residually, that is, $P_K^t \equiv (P_Y^t Y^t - P_L^t L^t)/K^t$. All of these U.S. series are presented in table 7.4.

The patterns in the data for the United States vary considerably from those seen for Japan. For example, the price of labor did not increase nearly as substantially as it did in Japan, and the corresponding change in labor quantity is much higher. Total compensation to labor, therefore, increased similarly to Japan, but, for the United States, this was a result of increased levels of labor input whereas for Japan the price adjustment was more important. The capital trends are more similar; the U.S. price of capital increased slightly more than for Japan and the quantity increased a bit less, but the magnitudes are closely comparable. The output trend is analogous to that for capital; the volume of output increased more in Japan and price increased less than that for the United States. The import and export price and quantity trends also follow expected patterns. Import prices increased substantially in the United States, particularly after 1973, but the price increase is greater for Japan, and the increase in quantity of imports is similar for the two countries. By contrast, export price increases are more substantial for the United States, and the corresponding increase in exports is much lower than for Japan. We turn now to the evaluation of the indexes defined in sections 7.6 and 7.7 for the U.S.

7.11   U.S. Indexes of Productivity and Welfare Change

We make exactly the same notational conventions with the U.S. data as we did with the Japanese data at the beginning of section 7.9.

The three translog *price effects*, P_1^t, P_2^t, P_3^t, defined by (27), the two translog *quantity effects*, Q_1^t, Q_2^t, defined by (28), and the translog *deficit effect* Q_0^t defined by (29) are listed in table 7.5 using the U.S. data listed in table 7.4.

The U.S. labor effect, Q_1^t, in table 7.5 is different from Japan's, as would be expected from the differing labor trends; increases in the labor input in the United States have contributed to greater product except in the worst recession years, including 1970, 1975, 1982. Overall, the contribution is strongly positive (and more so than in Japan, a circumstance that can be seen by comparing the respective Q_1^t indexes).

Q_2^t shows the impact on domestic sales' growth of growth in the capital stock. A comparison of the U.S. Q_2^t in table 7.5 with the Japanese Q_2^t in table 7.2 shows that the average U.S. capital effect of 1.2% is much smaller than the corresponding Japanese average capital effect of 3.4%. The smaller U.S. effect reflects its smaller rate of growth of the capital stock.

The individual price effects are particularly interesting for the United States; although the export price effects, P_2^t, induced increased product value in the United States in every year except 1982, changes in the price of imports reflected in the price effects P_3^t caused decreased product value except in 1982. The overall impacts are, however, especially for the earlier years, very small

Table 7.5   Translog Price and Quantity Effects for the United States

Year	P_1^t	P_2^t	P_3^t	Q_1^t	Q_2^t	Q_0^t
1968	1.030	1.000	.999	1.015	1.017	
1969	1.042	1.001	.998	1.002	1.016	1.000
1970	1.050	1.002	.996	.993	1.014	
1971	1.054	1.001	.997	1.003	1.012	
1972	1.050	1.001	.996	1.024	1.012	1.004
1973	1.044	1.009	.989	1.032	1.015	.991
1974	1.062	1.019	.966	1.004	1.015	1.006
1975	1.104	1.010	.991	.980	1.009	
1976	1.089	1.002	.999	1.024	1.006	
1977	1.056	1.003	.992	1.030	1.009	1.016
1978	1.065	1.004	.992	1.035	1.012	1.002
1979	1.077	1.011	.982	1.028	1.012	.998
1980	1.094	1.010	.973	1.005	1.011	.996
1981	1.106	1.009	.993	1.013	1.009	1.002
1982	1.090	.999	1.002	.982	1.012	1.002
Mean	1.067	1.005	.991	1.011	1.012	1.002

Note: P_1^t is the domestic sales price effect, P_2^t is the price of exports effect, P_3^t is the price of imports effect, Q_1^t is the quantity of labor effect, Q_2^t is the quantity of capital effect and Q_0^t is the deficit effect.

in magnitude. By contrast, the increase in product value from domestic price increases, P_j^t, is positive and quite large throughout; it does not show the declining effect over time that is found for Japan.

The translog productivity indexes, R^t, defined by (30), the translog terms of trade adjustment indexes, A^t, defined by (31), the translog welfare change indexes W^t, defined by (32), and the translog total welfare change indexes, T^t, defined by (33) are listed in table 7.6 using the U.S. data in table 7.4. The corresponding U.S. nonparametric indexes, $\bar{R}^t$, defined by (38), $\bar{A}^t$, defined by (44), $\bar{W}^t$, defined by (47), and $\bar{T}^t$, defined by (50) are also listed in table 7.6.

The translog productivity growth measure, R^t, and the nonparametric measure, $\bar{R}^t$, are represented in columns 2 and 3 of table 7.6. Note that these multifactor productivity indexes are quite similar for the years when the U.S. trade deficit did not change sign. There were large drops in productivity in 1970, 1975, 1979–80, and especially 1982. The year 1975 was a poor productivity year—there was a 2% decrease in productivity—which caused concern in the late 1970s about the observed "productivity slowdown." The late 1960s were also disappointing, but 1977 appeared very strong in terms of productivity growth. In addition, 1980 exhibited a 2% productivity decline, and 1982 was catastrophic with a 6% drop in productivity. These patterns suggest that productivity trends cannot be characterized by a unique productivity downturn in 1973, although there does appear to be a trend toward deterioration of productivity growth over time.

The U.S. terms-of-trade adjustment indexes, A^t and $\bar{A}^t$, are generally very close to 1.0, since internationally traded goods are such a small proportion of total output for the United States, even in the most recent years of the sample. However, in 1974 and 1980 (two energy shock years), increases in the prices of imported goods relative to exported goods were responsible for declines in real output of about 1½% in each year.

With the exception of these two years, the translog "welfare" index, W^t, (obtained by multiplying R^t and A^t together) and the nonparametric "welfare" index, $\bar{W}^t$, do not vary significantly from R^t; for a relatively closed economy like the United States, improvements in the terms of trade have a relatively small effect on economic welfare defined in this manner.

Since the U.S. merchandise trade deficits and surpluses were relatively small over the years 1967–82, the total welfare change indexes T^t and $\bar{T}^t$ do not differ much from the welfare change indexes W^t and $\bar{W}^t$. The exception to this is 1977, where the increase in the trade deficit relative to 1976 was large enough to account for an approximate 1.6% gain in the real domestic output.

7.12   Conclusion

Comparing the U.S. and Japanese productivity performance over the years 1967–82, the Japanese indexes $\bar{R}^t$ show only two years of decline throughout

Table 7.6   U.S. Productivity, Terms-of-Trade Adjustment, and Welfare Change
                   Indexes

Year	R^t	$\tilde{R}^t$	A^t	$\tilde{A}^t$	W^t	$\tilde{W}^t$	T^t	$\tilde{T}^t$
1968		1.025	1.000	1.000		1.025		1.031
1969	.996	.996	1.000	1.000	.996	.996	.996	.996
1970		.979	.999	.999		.978		.976
1971		1.005	.998	.998		1.004		1.011
1972	1.013	1.013	.998	.997	1.011	1.011	1.016	1.016
1973	1.029	1.029	.999	.998	1.029	1.027	1.020	1.020
1974	1.003	1.004	.985	.985	.989	.989	.995	.995
1975		.983	1.001	1.000		.984		.973
1976		.988	1.001	1.001		.990		1.003
1977	1.028	1.029	.995	.995	1.023	1.024	1.040	1.040
1978	1.015	1.015	.997	.997	1.012	1.012	1.015	1.015
1979	.997	.998	.994	.993	.992	.991	.990	.990
1980	.983	.984	.983	.982	.967	.967	.964	.963
1981	.992	.992	1.002	1.002	.995	.994	.997	.997
1982	.942	.942	1.002	1.002	.944	.944	.947	.947
Mean	1.000	.999	.997	.997	.996	.996	.998	.998

Note: The productivity indexes, R^t and $\tilde{R}^t$, are defined by (30) and (38), the terms of trade adjustment indexes, A^t and $\tilde{A}^t$, are defined by (31) and (44), the welfare change indexes, W^t and $\tilde{W}^t$, are defined by (32) and (47), and the total welfare change indexes, which incorporate changes in productivity, in the terms of trade and in the trade deficit, T^t and $\tilde{T}^t$, are defined by (33) and (50).

the sample period, 1971 and 1974, whereas the U.S. indexes show declines in productivity in many years, including 1969–70, 1975–76, and 1979–82. This is a large portion of a sample that includes only 15 data points. The growth in productivity over the entire sample period for Japan was large relative to the United States and showed a gradual decline from around 6% to 3% per year, although there is a lot of fluctuation around the trend. The worse years for Japan were worse than the worst years for the United States, but those years were very limited. Overall, both countries experienced a decreasing trend in yearly productivity growth over the sample period, but the U.S. decline was more pronounced, and the average level was substantially lower.

The terms-of-trade adjustment indexes also are interesting to compare. Although the A^t indexes are close to 1.0 for Japan, they are even closer to 1.0 for the United States. This is intuitively reasonable because the magnitude of trade relative to GNP is large in Japan as compared to the United States, and because the pattern of export prices as compared to import prices differs more for Japan than for the United States. This difference in price patterns at least partly results because Japan is more dependent on imported raw materials, expecially fuels, than is the United States. For example, the 1974 value of A^t for Japan, .969, is the lowest value over the sample period because of the

impact of energy price increases. This value indicates a decrease in potential product of about 3% in response only to the change in the relative prices of traded goods. This corresponds to a U.S. value of .986 in 1974, the second lowest value in the sample, indicating a smaller, 1.4% drop. On average, the Japanese terms-of-trade adjustment values tend to be slightly lower than for the United States and lower than unity; the means are .995 and .977, respectively. This indicates a lower level of welfare overall than is suggested by the pure productivity measures R^t, due to changes in the terms of trade.

Adjustment of the productivity measures by the A^t indexes to derive the W^t indexes has little effect on the comparative welfare found for Japan and the United States. The overall tendency is that the welfare indicators remain similar to the productivity indexes, although welfare growth is slightly lower than productivity growth, especially for the later years and for Japan.

To conclude, it should be recognized that productivity measures, although important, may obscure significant contributions to short-run welfare that are obtained by international trade. In this paper, we have outlined a method, following a more extensive treatment by Diewert and Morrison (1986), that can distinguish these additional "welfare" changes, resulting from changes in the terms of trade and the deficit, from productivity changes. To develop this approach we have used a production theory–based framework similar to that which provides a basis for much of the productivity literature.

This framework is used to construct productivity, terms-of-trade adjustment, and welfare indexes for the United States and Japan as combinations of individual comparative statics indexes representing the effects of output production, domestic output price, input use, the deficit, and export and import price changes on growth in domestic production or sales.

These indexes show that Japan's productivity from 1968 to 1982 has been significantly greater than that of the United States and, in fact, has been strongly positive in almost all years, whereas increases in productivity and welfare have been relatively low in the United States. An interesting implication of these numbers is that Japan's productivity growth appears not to have been declining as significantly as that of the United States; Japan experienced a minimal number of very poor productivity growth years around the first OPEC energy price shock and then snapped back relatively quickly, although not completely. In addition, adjusting for the relative terms-of-trade faced, and the deficit incurred, by the countries has a greater impact for Japan than for the United States.

These implications are obviously only a small subset of those which these indexes provide, but they highlight the richness of the information available from our procedures. Application of these procedures to later and more complete data for these and other countries should provide very useful indications of the effects of trade patterns on economic welfare.

Notes

1. For expositions of traditional trade theory, see Dixit and Norman (1980) and Woodland (1982, 165).

2. See Diewert and Morrison (1986, 669).

3. "Welfare" is perhaps best interpreted as potential welfare since we have not specified how the domestic product is to be distributed between various consumer groups.

4. We also require competitive profit-maximizing behavior on the part of producers and the international price vectors p_x^i and p_m^i must be expressed in terms of domestic currency.

5. Two other approximations were also tried for purposes of comparison. These included dividing the compensation of labor series by the "average month hours per worker" to generate a price of labor series and using "cash earnings per regular worker" to approximate a labor price. These two methods resulted in series that bounded the price of labor data used in the study.

6. It appeared important, particularly for mineral fuels, to decompose these indexes to allow for the individual impacts of the different categories to appear; the fuel component of imports exhibited a dramatic jump in value and price in the 1974 data which is important to capture explicitly.

7. This occurs even though the depreciation rate was assumed to be quite high—12.5%. This assumption was made as a result of evidence that replacement investment is a significantly higher portion of total investment relative to the U.S. experience.

8. The Bureau of Labor Statistics (BLS) labor quantity series is an unweighted man/hours series and hence is unsuitable for our purposes. We wish to thank Mike Harper at BLS and Barbara Fraumeni for their help in providing the updated data series.

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