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3 Pricing Excess-of-Loss Reinsurance Contracts against Catastrophic Loss

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With the recent rise in catastrophic disaster losses and the resulting effect on insurance-company solvency, the insurance industry is increasingly calling for some form of federal assistance in meeting disaster claims. Most of these requests, including the industry-sponsored Natural Disaster Partnership Protection Act of 1995, incorporate some form of all-hazard federal reinsurance program or backstop. A recent paper by Lewis and Murdock (1996) argues that the lack of federal regulatory authority in the insurance industry, the prevalence of moral hazard, adverse selection, and other opportunities for risk shifting, and the well-documented inability of the federal government to set adequate premiums to control for these costs make any traditional federal reinsurance program problematic.

Instead, Lewis and Murdock propose an alternative form of federal reinsurance that provides targeted protection for the insurance industry against catastrophic events but limits the federal government's exposure to additional losses.[1] Under this alternative, the federal government would sell per occurrence excess-of-loss contracts to private insurers and reinsurers, where both

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1. The operating assumption in this paper is that the establishment of a federal reinsurance program does not include the provision of a taxpayer subsidy, which is viewed as a political decision outside the realm of this analysis.

the coverage layer and the fixed payout of the contract are based on insurance-industry losses, not company losses. In financial terms, the federal government would be selling earthquake- and hurricane-catastrophe call options to the insurance industry to cover catastrophic losses in the range of $25–$50 billion. Lewis and Murdock argue that this type of contract would expand capacity and stability for the insurance industry while limiting the taxpayer's exposure to the insured event: namely, a catastrophic disaster.

The purpose of the present paper is to develop a methodology for pricing the catastrophic reinsurance contracts proposed in Lewis and Murdock (1996) and to present price estimates based on both historical catastrophe-loss experience and engineering simulations. After briefly discussing the need for a federal role and summarizing the key provisions of the proposed reinsurance contract, the paper proceeds by developing a statistical model of losses that would be covered under the contract. We then discuss how insurers could use the proposed contracts in hedging catastrophic risk. Finally, we provide estimates of catastrophe frequency, severity, and expected total losses based on two sources—the Property Claims Services (1994) database on insured catastrophe property losses covering the period 1949–94 and simulated catastrophe losses obtained from Risk Management Solutions—and use these estimates to illustrate prices of the reinsurance contracts.

3.1 The Proposed Reinsurance Contracts

3.1.1 The Need for a Federal Role

The proposed federal catastrophe-reinsurance contracts are needed because of dislocations in insurance and reinsurance markets resulting from growing catastrophic property losses. Combined, Hurricane Andrew in 1992 ($18.4 billion) and the Northridge Earthquake in 1994 ($12.5 billion) resulted in nearly $31 billion in insured industry losses and caused the failure of at least ten insurance companies (see Scism and Brannigan 1996). The magnitude of insurance-industry losses associated with these two recent disasters is unprecedented. The next largest insured catastrophe loss was the $4.2 billion in losses associated with Hurricane Hugo. Over the period 1988–94, insured industry losses exceeded $35 billion in 1992 dollars, more than the cumulative total over the previous twenty-one years.

More troubling is the fact that Hurricane Andrew and the Northridge Earthquake may not represent "outlier" events. Research on the frequency and magnitude of hurricanes and earthquakes, as measured by the Saffir-Simpson and Richter scales, respectively, indicates a strong potential for increased disaster activity over the next twenty years (Gray 1990). In addition, given the 69 percent increase in insured coastal property values in the United States since 1988 (to $3.15 trillion), the losses associated with hurricanes are likely to be more severe than historical experience. Thus, the probability of disasters with losses at least as large as those incurred as a result of Hurricane Andrew and the Northridge Earthquake over the coming years remains significant.

The realization of this increased risk exposure has sent reverberations through the insurance markets, especially in Florida and California. Reinsurance companies have raised rates rapidly, in many cases by as much as 150 percent. Following suit, primary insurers have submitted requests for large rate increases to state insurance commissioners. However, most of these rate increases have been pared down by states before being approved, allegedly creating a gap between the reinsurance premium for a given layer of coverage and the amount the insurer can recover from the buyer through the primary market premium.[2] Thus, the recent jump in expected disaster-claim severity and frequency and the resulting recognition of the inadequacy of insurance premiums have prompted the industry to look to the federal government for some form of assistance, typically through a reinsurance mechanism.

On the basis of the federal government's superior ability to diversify intertemporally, Lewis and Murdock (1996) contend that a targeted, risk-specific federal reinsurance program could expand the supply of reinsurance without imposing a large liability on the federal government. Typically, the cost-of-funds advantage of private reinsurers over a ceding insurer relates to an improved ability to diversify risk geographically and, for some levels of risk, intertemporally. However, the ability of a reinsurer to diversify catastrophic risk is limited by the reinsurer's access to capital and the costs associated with the risk of insolvency. The actuarially fair premium for a hundred-year disaster is meaningless if the hundred-year disaster occurs in year 2 and bankrupts the reinsurer. Thus, even if a differential exists between the reinsurer's targeted economic return and the ceding insurer's required return, the risk premium required by the reinsurer for high-risk lines may make reinsurance unaffordable for the primary insurer. This is especially true for very high-risk exposures where the uncertainty with respect to the loss is also high.

The federal government, on the other hand, carries a near zero default rate. Therefore, a federal reinsurer would not be subject to insolvency risk and the limitations that insolvency risk places on a private reinsurer's access to capital. As a result, the risk premiums required by a federal reinsurer for upper layers of catastrophic risk would be significantly below the premiums required by private reinsurers. If this cost-of-capital advantage exceeds any efficiency losses associated with the federalization of this form of reinsurance, the supply of reinsurance will expand, creating additional capacity in the primary-insurance market. In addition, since catastrophic reinsurance capacity is almost nonexistent for upper layers of loss, inefficiency costs will most likely be small

2. Testing the existence of state "rate suppression" or "rate stickiness" with respect to catastrophe loads is outside the scope of this paper. However, it should be noted that the existence of state "rate stickiness" can account only for the failure of insurance rates to adjust to higher levels in the postdisaster environment in California since rates in the state were not approved before 1990.

as long as the federal reinsurance program is adequately targeted to insuring catastrophic risk.

3.1.2 The Proposed Contract

The objective of catastrophic reinsurance is to provide per occurrence protection for losses (L) that exceed some trigger level (T) based on the level of losses that the insurer can absorb. By design, once an event exceeds this trigger, the reinsurance pays (some fixed proportion of) disaster losses (L) usually up to some predetermined cap C on the reinsurer's exposure. Therefore, if losses are less than the trigger, the contract pays nothing. If losses fall in the range between the trigger and the cap, the contract pays out $L - T$. If losses exceed the cap, the contract pays out the difference between the cap and the trigger, or $C - T$. Using this basic structure, one can specify the payout (P) of the reinsurance as follows:[3]

$$
\begin{aligned}
P &= \text{Max}\,[0, \text{Min}\,(L - T, C - T)] \\
&= \text{Max}\,[0, L - T] - \text{Max}\,[0, L - C].
\end{aligned}
\tag{1}
$$

As the expression following the second equals sign in (1) reveals, the reinsurance contract is simply the difference between two call options with different strike prices, that is, a call-option spread, written on the loss exposure of the underlying event. This specification corresponds directly with conventional per occurrence reinsurance and with the structure of the catastrophe call options being traded on the Chicago Board of Trade (CBOT) (see Cummins and Geman 1995) and thus provides a financial framework for structuring the federal reinsurance role.

Under the proposed reinsurance program, the federal government would directly write and sell contingent claims against the upper (capped) layers of catastrophic disaster losses on a per occurrence basis. These contingent claims, hereafter referred to as *excess-of-loss* (XOL) contracts, would be available for qualified insurance companies, pools, and reinsurers and would cover industry losses from a disaster in the \$25–\$50 billion layer of coverage—a layer currently unavailable in the private market.

Like private catastrophe covers, these XOL contracts would provide coverage for a single event, not an aggregation of losses over a fixed period. However, like the CBOT options, the reinsurance trigger and cap would be based on insured industry losses to minimize the moral hazard and adverse-selection problems associated with writing company-specific reinsurance.[4] The payout

3. This specification oversimplifies somewhat the actual contract payoff. For a more detailed discussion, see Lewis and Murdock (1996).

4. For a review of the problems of adverse selection and moral hazard in insurance markets, see Dionne and Doherty (1992). For recent discussions involving catastrophe reinsurance and futures markets, see D'Arcy and France (1992), Cummins and Geman (1994), and Lewis and Murdock (1996).

function on these XOL contracts would also be a function of industry losses and would be fixed at the time the contract is issued. Thus, the expected payout of the contract would be reexpressed as follows:

$$P = \mathrm{Max}\,[0,\ \delta(L - T)] - \mathrm{Max}\,[0,\ \delta(L - C)], \tag{2}$$

where $\delta(\cdot)$ represents the payout function of the contract, which depends on the difference between the level of total industry losses and the contract trigger or strike price.

As mentioned above, the XOL contracts offered by the federal government would be analogous to writing a call option for the insurance industry that pays off when industry disaster losses exceed $25 billion, along with a "short" call option such that industry losses in excess of $50 billion are retained by the insurance industry (i.e., buyers of the contract would be "long" in the call option with strike price T and short in a call with strike price C).

Thus, the payout to insurers of the first (long) call-option component of the XOL contract rises as a fixed proportion of industry losses in excess of $25 billion once the $25 billion threshold is reached. However, once industry losses exceed $50 billion, the second (short) component of the XOL contract provides an equal offset to any additional industry losses above $50 billion. As a result, the federal government's exposure is limited to covering losses in the $25–$50 billion range. On the basis of this payout structure, insurance and reinsurance companies could decide on the optimal number of contracts to purchase in order to hedge their catastrophe-loss exposure.

Other aspects of the XOL contracts include the following:

a) As mentioned above, the contracts would cover insured property losses from hurricanes, earthquakes, and volcanic activity. Qualified lines of insurance for earthquake damage would include property losses in earthquake-shake policies (written separately or as part of a homeowner's policy), commercial multiperil, and commercial inland marine coverage associated with earthquakes. For hurricane damage, losses covered by homeowner's, wind (written separately or as part of a homeowner's policy), commercial multiperil, fire, allied, farmowner's, and commercial inland marine policies would be covered. For reporting purposes, estimates reported by the state insurance commissioner's office in each affected state would be used as an index of loss, with validation accomplished through year-end tax filings.

b) The XOL contracts would be sold annually with a maturity of one year. However, each contract would include a renewal provision that allows the holder of an exercised contract to purchase an additional contract to cover losses to the end of the original contract year at a cost of the original premium prorated to the remaining term on the exercised contract. An alternative form of the contracts, also under discussion, would cover the insurer for multiple events over a period of one year. This form would be equivalent to including an automatic renewability feature in the contract.

c) The trigger level of the contracts would be set at a level above the layers of reinsurance being provided in the private markets. On the basis of evidence provided by private reinsurers and the levels of coverage available in the CBOT market, the trigger (T) is set initially at $25 billion in industry losses. This trigger level of coverage would be adjusted or indexed annually to the rate of property-value inflation.

d) The fact that the actual distribution of catastrophic losses is unknown and must be estimated from imperfect data exposes the federal government to parameter-estimation risk as well as the risk of the underlying process. To place a cap on the government's exposure, the upper limit of the reinsurance contract is initially set at $50 billion. Again, the level of the cap can be adjusted annually to reflect property-value inflation.

e) The payout function of the XOL contract stipulates how much each individual contract will pay in the event the contract trigger is reached. As proposed, $\delta(\cdot)$ simply represents a scalar function relating industry losses to the desired denomination of each contract. That is, $\delta(\cdot)$ is set so that each contract pays out $1 million for every $1 billion by which industry losses exceeded the trigger:

$$\delta(\cdot) = \frac{1}{1000}(L - T). \tag{3}$$

This simple payout function, which by construction includes a contract payout cap equal to $25 million, would provide a total capacity of one thousand contracts being sold annually. A more complicated specification for $\delta(\cdot)$ is, of course, possible, but we will assume that the contracts will be based on the simple linear structure given by (3).

f) Only insured losses paid during the eighteen-month period immediately following the disaster and reported to the federal government within twenty-one months of the event date will be covered to limit the tail on the contract payout and to prevent the accumulation of losses over a series of events. While helping protect against fraud and abuse, this provision has the additional advantage of encouraging insurance companies to expedite the processing of claims in the wake of a disaster—a social good from a policy standpoint.

g) Private reinsurance firms, primary insurance companies, and state, regional, and national pools would be eligible to purchase and exercise federal XOL contracts, as long as they are licensed to write property-casualty insurance or reinsurance in a state in the United States and are actively providing insurance/reinsurance for property located in the United States.

To accelerate the development of a private reinsurance market to "crowd out" the federal government in the provision of catastrophe-reinsurance coverage layers, these XOL contracts could be priced using a private market cost-of-capital adjustment. That is, the reservation price established for the XOL contracts could be based, not on the federal government's cost of borrowing, but on a private market discount rate or hurdle rate established by the federal government. For additional details regarding the contracts, see Lewis and Murdock (1996).

3.2 Pricing Methodology

Pricing the contracts involves two steps: (1) estimating the loss distribution and the expected value of loss and (2) incorporating a risk premium and expense loading in the contract price. These steps are discussed below, following a general discussion of the pricing rationale for the contracts.

3.2.1 Pricing Rationale

Ideally, an insurance risk pool would be able to diversify risk across time as well as across exposure units in the pool at a given point in time. Most discussions of risk diversification through pooling consider only the latter dimension (for a review, see Cummins [1991]). Adding the time dimension can significantly reduce the standard deviation of losses from a pool of risks, reducing the residual risk faced by pool participants. As a simple example, consider a pool consisting of N independent, identically distributed exposure units with expected loss μ and variance of loss σ^2. The mean of the pool loss is then $N\mu$, and the variance is $N\sigma^2$, yielding a coefficient of variation (a standard measure of the "insurer's risk") of $\sigma/(\mu\sqrt{N})$. If the pool can also diversify across time, the coefficient of variation becomes $\sigma/(\mu\sqrt{TN})$, where $T =$ the number of time periods. Therefore, time diversification has the potential to reduce the insurer's risk significantly.

In a theoretical world, under the assumptions of perfect information, no transactions costs or contract-enforcement costs, and no probability of bankruptcy, time diversification would merely involve the pool's borrowing at the risk-free rate of interest when losses exceed the expected value of loss and lending (or repaying loans against the pool) when losses are less than their expected value. In principle, mutual insurers operate much like the theoretical risk pool, accumulating retained earnings when losses are less than expected and drawing down equity or borrowing to pay losses that are greater than expected. A stock insurer operates similarly except that the firm can raise funds by issuing equity as well as through borrowing and retaining earnings. Thus, at least in theory, both types of insurers diversify risk across time.

Because the assumptions underlying the pure risk pool hold only as approximations in the real world, however, insurance markets do not fully achieve time diversification. Time diversification fails most acutely in the case of very large losses, such as those resulting from catastrophes. Because capital is costly, insurers cannot maintain a sufficient equity cushion to guarantee the pool against bankruptcy. The possibility of bankruptcy, along with information imperfections in insurance and capital markets, is primarily responsible for the failure of time diversification for large losses.

The existence of these market imperfections implies that the cost of both debt and equity capital is likely to rise significantly following a large loss or an unusual accumulation of small losses. Prospective capital providers are concerned about the long-term viability of insurers that have suffered major loss shocks, and they are also worried that the insurers' reserves are not adequate to fund the losses from the catastrophe. The cost of capital reflects these information asymmetries, and, therefore, capital costs more following a loss shock than it would if the insurance and capital markets were frictionless and complete information were available. Thus, the insurer may not survive long enough to deliver a fair return on equity or repay loans needed to fund loss payments. The cost of capital would reflect these market imperfections, and capital would cost more than if the insurance and capital markets were frictionless and complete information was available. In the extreme, capital may not be available at any price to some insurers following a major loss shock.

The federal government is better able than the private-insurance market to diversify large losses across time efficiently. The principal reason is that the riskless borrowing and lending assumption required for time diversification that does not apply to private insurers applies to the federal government, allowing it to borrow at the risk-free rate to fund losses arising from a catastrophe and then to repay the loans out of subsequent premium payments in periods when no severe catastrophes occur. Implicitly, premiums paid into a catastrophe-reinsurance program in excess of accumulated losses go to offset federal debt arising from other programs, so the pool is in effect "lending" at the risk-free rate during these periods. The ability of the federal government to time diversify would ensure the availability of reinsurance at a cost of capital that does not include a margin for information asymmetries and other market imperfections. Even with a risk premium to encourage private market crowding out of the federal contracts, the cost of capital would be lower than if the reinsurance were provided privately.

The discussion of time diversification suggests the following basic principle for pricing the federal XOL contracts: The contracts should be self-supporting in expected value; that is, the expected costs to the government of operating the program should be zero, where the expectation is defined as including the expected value of losses and other program costs across time. In principle, the price should also reflect financing costs. However, the net financing costs of the program are expected to be zero if the premiums are retained by the government. This is the case because the costs of borrowing to pay catastrophe losses are offset in expectation by the proceeds gained by "lending" the premium payments to the federal government during periods when no losses occur.

As mentioned above, the proposal calls for adding a risk premium to the contracts so that their prices will approximate the price that would be charged in the private marketplace in the absence of severe information imperfections (i.e., a normal risk charge or cost of capital). The risk can be viewed as a way to ensure that there would not be any unintended consequences in the private insurance and reinsurance markets due to the proposed program. This risk charge can also be viewed as compensating the government for other unforeseen costs that could arise under the program. Thus, the final price will be the expected value of loss plus administrative expenses and a risk loading.

3.2.2 The Loss Distribution and Its Moments

We develop the pricing model under two alternative assumptions regarding the design of the reinsurance contracts: (1) that the contracts cover only one loss, with the option to purchase an additional contract covering one loss for the balance of the year based on a price equal to the original price times the proportion of the year remaining after the first loss, and (2) that the contracts cover a theoretically unlimited number of multiple losses during a period of one year.

(Renewable) Contracts Covering a Single Loss Event

Using generalizations of standard actuarial formulas, the loss distribution under the proposed contract can be written as follows:

$$
\begin{aligned}
F(L) &= \sum_{N=0}^{\infty} p(N)q(L > T|N)S(L|L > T) \\
&= S(L|L > T)\sum_{N=0}^{\infty} p(N)q(L > T|N),
\end{aligned}
\tag{4}
$$

where $F(L)$ = the distribution function of catastrophic losses; $p(N)$ = the probability that N catastrophes occur during the contract period; $q(L > T|N)$ = the probability that one catastrophe exceeds the trigger level of losses, conditional on the occurrence of N catastrophes; and $S(L|L > T)$ = the distribution function of the severity of catastrophic loss, conditional on losses from a catastrophe exceeding the trigger. Thus, payment under the contract requires the occurrence of some number N of catastrophes (an event with probability $p(N)$) such that one loss exceeds the trigger level (T) (an event with probability $q(L > T|N)$). Both $p(N)$ and $q(L > T|N)$ are discrete probability distributions. The severity of the loss (loss amount), given that the loss exceeds the trigger, is assumed to follow the continuous probability distribution function $S(L|L > T)$. Even though the contract terminates following the first catastrophe where losses exceed the trigger, the exposure to a catastrophe of this magnitude increases with the number of catastrophes that occur.

The summation on the right-hand side of the second line of equation (4) gives the unconditional (on N) probability of a loss above the trigger point. We call this probability ($p*$). To obtain an expression for the unconditional probability ($p*$), we first derive $q(L > T|N)$. For any one catastrophe, let $\Pr(L \leq T) = P_<$ and $\Pr(L > T) = P_>$, and observe that

$$
\begin{aligned}
q(L > T|N) &= P_> + P_< P_> + P_<^2 P_> + \cdots + P_<^{N-1} P_> \\
&= P_> \frac{1 - P_<^N}{1 - P_<} = 1 - P_<^N .
\end{aligned} \tag{5}
$$

Intuitively, catastrophes are assumed to arrive sequentially throughout the year.[5] On the arrival of the first catastrophe, the reinsurance contract pays off if the losses from this catastrophe exceed T, an event that occurs with probability $P_>$. If the first catastrophe does not exceed the trigger level (an event with probability $P_<$), then the contract pays off if the second catastrophic loss exceeds T, with the result that the probability that the second catastrophe triggers the contract is $P_< P_>$, and so on.

The unconditional probability of a loss exceeding the trigger ($p*$) is then obtained as the expected value of $q(L > T|N)$ over N. The result is

$$
p* = E_p[q(L > T|N)] = [1 - E_p(P_<^N)] = \{1 - M_p[\ln(P_<)]\}, \tag{6}
$$

where $M_p[\ln(P_<)]$ = the moment-generating function of the probability distribution $p(N)$. Thus, if claim arrivals are Poisson distributed,

$$
p* = [1 - e^{\lambda(e^{\ln(P_<)} - 1)}] = [1 - e^{\lambda(P_< - 1)}] = [1 - e^{-\lambda P_>}], \tag{7}
$$

where λ = the parameter of the Poisson distribution. And, if claims arrive according to a negative binomial distribution, then

$$
p* = \left\{1 - \left[\frac{\rho}{1 - (1 - \rho)e^{\ln(P_<)}}\right]^{\alpha}\right\} = \left\{1 - \left[\frac{\rho}{1 - (1 - \rho)P_<}\right]^{\alpha}\right\}, \tag{8}
$$

where α and ρ are the parameters of the negative binomial distribution, which is

$$
p(k) = \binom{\alpha + k - 1}{k} \rho^{\alpha}(1 - \rho)^k \tag{9}
$$

for $k = 0, 1, 2. \dots$

The moments of $F(L)$ can be derived from the moment-generating function

$$
\begin{aligned}
M_F(t) &= (1 - p*) + p* \int_L^{\infty} e^{tL} s(L|L > T)dL \\
&= (1 - p*) + p* M_{S|L>T}(t),
\end{aligned} \tag{10}
$$

where $M_F(t)$ = the moment-generating function of $F(L)$, and $M_{S|L>T}(t)$ = the moment-generating function of the distribution $S(L|L > T)$. The mean and variance of $F(L)$ are, respectively,

5. Equation (5) is the distribution function of the geometric distribution. As shown by the rightmost expression in (5), the probability of an event that exceeds the threshold is one minus the probability that none of the N events exceeds the threshold.

(11a) $$E(L) = p^*\mu_1,$$

(11b) $$\mathrm{Var}(L) = p^*\mu_2 - p^{*2}\mu_1^2 = p^*(\mu_2 - \mu_1^2) + \mu_1^2 p^*(1 - p^*),$$

where μ_i = the ith moment about the origin of the distribution $S(L|L > T)$.

We next consider the severity distribution $S(L|L > T)$. We derive the expected severity under the assumption that the catastrophic loss distribution is shifted from the origin to point $0 < d \leq T$. This allows for the possibility of defining catastrophes as being events of some minimal size d, with the result that the support of the distribution is the interval $[d, \infty]$ rather than the usual support interval for loss severity of $[0, \infty]$.[6] Thus, L is distributed as $S(L - d)$, $L \geq d$; and the expected severity for a call option on L with strike price T is then given by

(12) $$\begin{aligned} E(L - T|L > T) = \mu_{1T} &= \int_T^\infty (L - T)dS(L - d|L > T) \\ &= \int_T^\infty [1 - S(L - d|L > T)]dL, \end{aligned}$$

where μ_{1T} = the expected severity of loss under a call with trigger T. The second moment about the origin for the call-option severity is

(13) $$\begin{aligned} E[(L - T)^2|L > T] = \mu_{2T} &= \int_T^\infty (L - T)^2 dS(L - d|L > T) \\ &= 2\int_T^\infty (L - T)[1 - S(L - d|L > T)]dL, \end{aligned}$$

where μ_{2T} = the second moment about the origin of the severity of loss under a call with trigger T.[7]

The corresponding moments of the severity of loss for the call spread can then be written conveniently as

(14) $$\begin{aligned} \mu_{1CT} &= \int_T^C (L - T)dS(L - d|L > T) \\ &\quad + (C - T)[1 - S(C - d|L > T)] \\ &= \int_T^C [1 - S(L - d|L > T)]dL, \end{aligned}$$

(15) $$\begin{aligned} \mu_{2CT} &= \int_T^C (L - T)^2 dS(L - d|L > T) \\ &\quad + (C - T)^2[1 - S(C - d|L > T)] \\ &= 2\int_T^C (L - T)[1 - S(L - d|L > T)dL, \end{aligned}$$

6. Shifting the distribution enables us to deal with data such as that on catastrophes collected by Property Claims Services (PCS), an insurance-industry statistical agent. PCS defines catastrophes as losses from catastrophic perils that cause insured property damage of $5 million or more. This database is analyzed in detail later in the paper.

7. The right-most expressions in (12) and (13) are obtained by integrating by parts.

where μ_{1CT} = the first moment about the origin of the severity of loss for a call spread with trigger T and cap C, and μ_{2CT} = the second moment about the origin of the severity of loss for a call spread with trigger T and cap C. The mean and variance of the call spread are then given by respectively,

(16a) $$E_F(L;\ T,\ C) = p^* \mu_{1TC},$$

(16b) $$\begin{aligned} \mathrm{Var}_F(L;\ T,\ C) &= p^* \mu_{2TC} - p^{*2} \mu_{1TC}^2 \\ &= p^*(\mu_{2TC} - \mu_{1TC}^2) + \mu_{1TC}^2 p^*(1 - p^*), \end{aligned}$$

where p^* is given by expression (6), and μ_{1CT} and μ_{2CT} are from (14) and (15).

Contracts Covering Multiple Losses

If the contracts cover multiple losses during a specified period, the frequency distribution becomes

(17) $$p_k(k;\ L > T) = \sum_{N=k}^{\infty} p(N) p_k(k;\ L > T|N),$$

where $p_k(k; L > T)$ = the probability that the XOL contracts are triggered k times during the coverage period, unconditional on the total number of catastrophes; and $p_k(k; L > T|N)$ = the probability that the XOL contracts are triggered k times during the coverage period, conditional on the occurrence of N total catastrophes. The distribution $p_k(k; L > T|N)$ is a binomial distribution with parameters $P_>$ and N. If $p(N)$ is Poisson with parameter λ, then it can be shown that $p_k(k; L > T) = p_k(k)$ is Poisson with parameter $\lambda P_>$. Similarly, if $p(N)$ is negative binomial with parameters ρ and α (see eq. [9]), $p_k(k)$ is also negative binomial with parameters α and $\beta = (\rho' P_>)/(1 - P_< \rho')$. The mean and variance of the call spread are then given by

(18a) $$E_F(L;\ T,\ C) = E_k(k)\mu_{1TC},$$

(18b) $$\mathrm{Var}_F(L;\ T,\ C) = E_k(k)(\mu_{2TC} - \mu_{1TC}^2) - \mathrm{Var}_k(k)\mu_{1TC}^2,$$

where $E_k(k)$ = the expected value of frequency based on the distribution $p_k(k; L > T)$, and $\mathrm{Var}_k(k)$ = the variance of frequency based on the distribution $p_k(k; L > T)$.

3.2.3 Risk and Expense Loadings

There are two primary approaches to incorporating risk loadings into prices of insurance and reinsurance contracts—the actuarial approach and the financial approach. An extensive literature exists on actuarial pricing principles (e.g., Goovaerts, de Vylder, and Haezendonck 1984; Buhlmann 1984; Wang 1995). The actuarial pricing principles usually imply that prices should have some desirable mathematical properties such as value additivity or that firms behave as if they were risk averse so that prices can be derived using utility functions. Although the lack of theoretical foundation for most additive risk-loading formulas and the assumption of firm utility functions would seem to rule out the actuarial approach, these approaches may provide some useful information in solving the pricing problem, under the interpretation that they provide a way to incorporate judgmental risk premiums in option prices on nonhedgeable stochastic processes. However, in this role, they should be considered subordinated to financial pricing approaches.

Financial pricing models are the most appropriate way to price the catastrophe-reinsurance contracts. Financial pricing models incorporate risk loadings that are based on an asset-pricing model or, minimally, avoid the creation of arbitrage opportunities. The classic paper on the pricing of options on jump processes is Merton (1976). More recent extensions are Naik and Lee (1990), Heston (1993), Aase (1993), and Chang (1995). The principal problem in applying option-pricing methodologies to options on catastrophes is that these methodologies are based on arbitrage arguments that do not apply in general to jump processes. The problem is one of market incompleteness when jumps in asset prices are possible. Market incompleteness implies that jump risk cannot be hedged, and therefore arbitrage arguments generally do not apply.

Because of the market-incompleteness problem, some additional assumptions are needed to price options on jump processes. Merton (1976) circumvents the problem by assuming that assets are priced according to the capital asset–pricing model (CAPM) and that jump risk is nonsystematic, that is, not correlated with the market portfolio of securities. If the risk of catastrophes is unsystematic, catastrophe risk can be diversified away by investors, and thus the return on the catastrophe reinsurance option is equivalent to the risk-free rate. Merton derives the formula for option prices on jump processes under these assumptions, with the magnitude of jump risk assumed to follow a lognormal distribution.[8]

If the assumption that jump risk is nonsystematic is not viewed as satisfactory—for example, because market prices respond to large catastrophes or because federal borrowing to fund the reinsurance contracts increases market interest rates—then other assumptions can be used to price the options. One approach is to assume that jumps can assume only a finite number of constant magnitudes and that a sufficient number of traded securities exist that are correlated with the jumps to permit the formation of portfolios to hedge the jump risk (see, e.g., Cummins and Geman 1995). This is equivalent to breaking up the severity of loss distribution applicable to catastrophes into a finite sequence of mass points. Gerber (1982) shows how this can be done while preserving

8. Chang (1995) shows that Merton's assumption of diversifiable jump risk is consistent with no arbitrage only when the aggregate consumption flow is not subject to jumps. If that assumption does not hold, Merton's formula underprices hedging assets and overprices cyclic assets. Cyclic assets are defined as assets subject to jumps that are negatively correlated with jumps in aggregate consumption, while hedging assets are defined as assets subject to jumps that are positively correlated with aggregate consumption jumps.

the moments of the severity distribution. If only the first few moments (such as the mean and the variance) of severity are of interest, this approach could prove to be effective. However, the moments do not uniquely characterize most probability distributions, including the lognormal (Johnson and Kotz 1970). Thus, if the first two or three moments are not sufficient for pricing the contracts, this approach may not be satisfactory.

An alternative approach that does not require constant jump sizes is to make an assumption about investor preferences. In a recent paper, Chang (1995) derives pricing formulas for traded and nontraded options under the following assumptions: (1) aggregate consumption follows a jump-diffusion process, and (2) preferences can be incorporated using the assumption that there exists a representative investor whose utility function is of the constant relative risk aversion type. Chang presents an option-pricing model that is "distribution free" in the sense that it places no restrictions on the probability distributions of the magnitudes of jumps in the value of the underlying asset or the "market portfolio," which in this case is aggregate consumption. Chang (1995) gives formulas for the option price in the case where jump sizes in aggregate consumption and in the strike price are jointly lognormal. However, it would also be possible to calculate option prices using other multivariate distributions that sometimes provide better models of catastrophic losses, such as the multivariate Burr 12 distribution (see Johnson and Kotz 1972). The approach could be implemented through numerical integration, based on Chang's pricing formulas.

We decided not to attempt to parameterize an option-pricing model for two primary reasons: (1) the option-model adjustment in the expected value price obtained from our pricing model is likely to be a second-order effect, and (2) the data available to parameterize the option-pricing model are likely to be inadequate to yield reliable parameter estimates. The problem is that the value of the underlying asset (property subject to insured catastrophe loss) is not available except at the time of the decennial U.S. Census. Thus, the calculation of essential quantities such as the instantaneous volatility parameter would have to rely on data that may be unreliable proxies for the actual value of insured property. Since option values are very sensitive to the key parameter estimates, this could introduce potentially serious error into the premium estimates.

Relying on the argument that the risk of loss from hurricanes and earthquakes is likely to be largely unsystematic, we propose using the expected-loss values based on our formulas as the basis for the price of the XOL options. This is essentially equivalent to using Merton's approach except that we substitute the loss estimates derived below for the lognormal distributions on which the Merton jump-option-pricing formula is based. As in our prices, no explicit market-risk premium is included in Merton's option-pricing formula. However, his formula does recognize the time value of money by discounting the anticipated payout under the option at the risk-free rate of interest. Our formula could also incorporate a discount factor for the time value of money. The appropriate period would be the expected time of the payment under the renewable option. However, under the multiple-claim option, technically it would be appropriate to discount each expected payment for its specific expected time of arrival. This could be done under the Poisson distribution using the duality between the Poisson process and the exponential distribution of the time of arrival between events. However, this, too, is an adjustment of second order in importance and probably not worth the extra effort.

Consequently, the final step in pricing is to incorporate expenses into the price of the contracts using the usual actuarial formula,

$$G(S,\ \tau) = \frac{F(S,\ \tau)}{(1 - e)(1 + r)^t}, \tag{19}$$

where $G(L; T, C, d)$ = the expected loss loaded for expenses and discounted, e = the expense ratio (ratio of expenses to the gross premium), r = the risk-free rate (e.g., the ninety-day Treasury bill rate), and t = expected time of arrival of the first event that triggers the contracts. The price based on (19) should be viewed as the federal government's *reservation price,* that is, the minimum price at which the contracts should be sold. If a higher price results when the contracts are auctioned, they should be sold at the auction price. The contracts should not be issued if the reservation price is not realized because that is likely to expose the government to an expected loss from issuing the contracts.

3.3 Hedging Catastrophe Risk with Federal XOL Reinsurance

This section illustrates how the proposed federal excess-of-loss (XOL) reinsurance contracts could be used by an insurer to hedge its exposure to the risk of hurricanes, earthquakes, and volcanic activity. It is assumed that the insurer's objective is to protect its equity capital and achieve other business objectives by optimally reducing the variance of its loss ratio.[9] This objective is consistent with the literature on insurance futures and options (e.g., Buhlmann 1995; Niehaus and Mann 1992). More general discussions of the rationale for managing firm risk are provided in Froot, Scharfstein, and Stein (1994), Mayers and Smith (1982), and Shapiro and Titman (1985). Insurers may find it advantageous to manage their net income risk in order to minimize taxes (Cummins and Grace 1994), protect franchise values, reduce regulatory costs, and avoid being penalized in the insurance market for changes in insolvency risk.

To model the insurer's loss ratio and hedging strategy, let L_{CA} = catastrophe losses of insurer A, L_{NA} = noncatastrophe losses of insurer A, L_{CI} = catastrophe

9. Because premiums can be treated as nonstochastic, it is not necessary to work with loss ratios. Loss ratios are used here because they provide a familiar and convenient framework for evaluating hedging strategies in insurance.

losses of the insurers included in the catastrophe loss index, P_A = premiums of company A in lines affected by catastrophes, and P_I = premiums for insurers reporting data to the catastrophe loss index. Insurer A's loss ratio is then defined as

$$R_A = \frac{L_{NA}}{P_A} + \frac{L_{CA}}{P_A}. \tag{20}$$

Assuming that the company buys some number N_A of XOL contracts, its loss ratio will be

$$R_A = \frac{L_{NA}}{P_A} + \frac{L_{CA}}{P_A} - \frac{N_A}{P_A}\left\{\frac{\text{Max}[L_{CI} - T,\ 0]}{1,000} - \frac{\text{Max}[L_{CI} - C,\ 0]}{1,000}\right\}. \tag{21}$$

It is assumed that L_{NA} is independent of L_{CA} and L_{CI} but that L_{CA} and L_{CI} are not independent.

As a first example, we assume that the company's objective is to cap its loss ratio due to catastrophes at the industrywide loss ratio represented by the trigger point of the XOL contract. Assume that industrywide premiums from policies covering perils included in the XOL contracts equal \$100 billion.[10] Then the federal XOL contracts can be viewed as providing a twenty-five/fifty loss ratio call spread, that is, as providing protection for loss ratios due to catastrophes ranging from 25 to 50 percent of premiums. The number of contracts that the insurer would purchase to implement this strategy is given by

$$N_A = \left(\frac{C - T}{P_I}\right)\frac{P_A}{S}, \tag{22}$$

where S = contract size = \$25 billion/1,000 = \$25 million.

A numerical example based on this hedging strategy is provided in Table 3.1. We assume that company A's share of the market for coverages affected by catastrophes is 1.2 percent and that its premium volume from policies covering perils included in the XOL pool is therefore \$1.2 billion.[11] We focus first on case A of table 3.1. This case assumes a catastrophic loss of \$40 billion, giving an industry loss ratio from catastrophic losses of 40 percent. It is also assumed that company A's catastrophic losses are perfectly correlated with the industry's catastrophic losses, with the result that company A's catastrophe-loss ratio is also 40 percent. Given company A's premium volume, this implies that company A suffers catastrophe losses of \$480 million. Equation (19) implies that the insurer purchases twelve XOL reinsurance contracts. The payoff per

10. Actual industry premiums for fire, allied lines, inland marine, farmowner's, homeowner's, commercial multiple peril, and auto physical damage, the coverages included in the federal XOL contracts, totaled \$94.5 billion in 1994 (A. M. Best Co. 1995).

11. The fifteenth largest property-liability insurer in the United States has a market share of 1.3 percent, and the twentieth largest has a market share of 1 percent (A. M. Best Co. 1995).

Table 3.1 Hedging Example Using Federal XOL Reinsurance

	Case A	Case B	Case C	Case D
Industry data				
Industry premiums ($)	100,000,000	100,000,000	100,000,000	100,000,000
Company A data				
Company A's market share (%)	1.20	1.20	1.20	1.20
Company A's premiums ($)	1,200,000	1,200,000	1,200,000	1,200.000
Federal XOL contracts				
Trigger expressed as loss ratio (%)	25.00	25.00	25.00	25.00
Cap expressed as loss ratio (%)	50.00	50.00	50.00	50.00
Contract size ($)	25,000	25,000	25,000	25,000
Hedging strategy				
Number of contracts	12.00	12.00	12.00	12.00
Evaluating the hedge				
Catastrophe size ($)	40,000,000	40,000,000	40,000,000	55,000,000
Industry catastrophe-loss ratio (%)	40.00	40.00	40.00	55.00
Company A's catastrophe losses ($)	480,000	504,000	444,000	660,000
Return per contract ($)	15,000	15,000	15,000	25,000
Company A's gain on cat contracts ($)	180,000	180,000	180,000	300,000
Company A's catastrophe-loss ratio:				
Without XOL reinsurance (%)	40.00	42.00	37.00	55.00
With XOL reinsurance (%)	25.00	27.00	22.00	30.00

Note: Case A = company A's and industry loss ratios perfectly correlated, loss less than $50 billion. Case B = company A's loss ratio greater than industry ratio, loss less than $50 billion. Case C = company A's loss ratio less than industry ratio, loss less than $50 billion. Case D = company A's and industry loss ratios perfectly correlated, loss greater than $50 billion. All dollar figures reported in thousands.

contract for a $40 billion loss is $15 million, so company A's gain from the reinsurance contracts is $15 million × 12, or $180 million. Company A's net loss from the catastrophe is $300 million ($480 million − $180 million), for a loss ratio of 25 percent. Thus, by purchasing the reinsurance contracts, company A has been able to cap its catastrophe-loss ratio at 25 percent.

Cases B and C of table 3.1 illustrate the effects of hedging when insurer A's losses are not perfectly correlated with industrywide losses. In case B, insurer A's loss ratio exceeds the industry loss ratio, and, in case C, insurer A's ratio is less than the industry ratio. In these cases, the hedge is not successful in holding the catastrophe-loss ratio to 25 percent, but the loss ratio is still substantially less than if no XOL contracts had been purchased. Case D shows the effects of a catastrophic loss ($55 billion) that exceeds the cap on the XOL contracts ($50 billion). Assuming that insurer A's losses are perfectly correlated with industry losses, insurer A's loss ratio under the XOL hedge is 30 percent, that is, the 55 percent unhedged-loss ratio minus the layer of XOL reinsurance coverage (25 percent).

The hedging strategy illustrated in table 3.1 is not necessarily optimal. To derive an optimal strategy, we consider the variance of the loss ratio. To simplify the notation, we disregard the upper limit in the XOL contracts and assume that the insurer can buy a call option with a strike price of T = $25 billion. The loss-ratio variance is given by

$$\text{Var}(R_A) = \sigma_A^2 = \frac{1}{P_A^2}(\sigma_{NA}^2 + \sigma_{CA}^2 + N_A^2\sigma_{CIT}^2 - 2N_A\sigma_{CA,\,CIT}), \tag{23}$$

where σ_A^2 = the variance of insurer A's loss ratio, σ_{NA}^2 = the variance of insurer A's noncatastrophe losses, σ_{CA}^2 = the variance of insurer A's catastrophic losses, σ_{CIT}^2 = the variance of losses included in the XOL reinsurance-contract pool, and $\sigma_{CA,CIT}$ = the covariance of insurer A's catastrophe losses with the losses included in the XOL pool. To find the number of contracts that minimizes the loss-ratio variance, we differentiate equation (20) with respect to N_A and set the resulting expression equal to zero, obtaining

$$N_A = \frac{\sigma_{CA,\,CIT}}{\sigma_{CIT}} = \frac{\rho_{CA,\,CIT}\,\sigma_{CA}}{\sigma_{CIT}}, \tag{24}$$

where $\rho_{CA,CIT}$ = the correlation coefficient of L_{CA} and Max[L_{CI} − T, 0]/1,000. Thus, to estimate the optimal number of contracts, the insurer would have to estimate the variance of its catastrophe losses, the variance of the losses in the XOL pool, and the correlation coefficient between its losses and the pool losses. The optimal number of contracts is increasing in the insurer's variance and the correlation coefficient and decreasing in the variance of the pool's losses.

3.4 Empirical Estimates of Catastrophic Losses and XOL Premiums

Two principal methods exist that could be used to develop empirical estimates of catastrophic losses: (1) fitting probability distributions to historical catastrophe-loss-experience data and (2) engineering simulation analysis. Both methods are utilized in this paper. Our historical catastrophe-loss-experience data are the insured catastrophic property losses reported to Property Claims Services (PCS), an insurance-industry statistical agent, for the period 1949–94. The engineering simulation analysis is based on catastrophe-loss simulations provided to us by Risk Management Solutions (RMS), a private firm that conducts research on the economic effects of catastrophes for insurance companies and other interested parties. The RMS analysis utilizes engineering and statistical techniques to simulate the probability and severity of catastrophes. This information is then merged with the firm's extensive database on insured-property exposures to estimate insured losses. Conducting the estimates on the basis of two sources of data provides a reasonableness check on the results and should provide the government and industry with a higher degree of confidence in the results than if only one source of data were used.

3.4.1 Loss-Severity Models

In modeling loss severity, it is important to fit a probability distribution to the observed data as well as evaluating the observed data directly. By fitting a probability distribution to the data, it is possible to model loss expectations in the tail of the loss distribution for ranges of losses larger than those contained in the data set. This is especially important when the sample size is small and/or very large events are possible but have low probabilities of occurrence.

On the basis of prior experience with modeling severity-of-loss distributions, we utilize four probability distributions as possible models for loss severity, the Pareto, the lognormal, the Burr 12, and the generalized beta of type 2 (GB2) (see Cummins et al. 1990; and Cummins and McDonald 1991). The density functions for these distributions are given below:

(25) Lognormal: $$s(L) = \frac{1}{(L - d)\sigma\sqrt{2\pi}} e^{-\left[\frac{\ln(L - d) - \mu}{\sigma}\right]^2}, \quad L > d,$$

(26) Pareto: $$s(L) = \alpha d^{\alpha} L^{-(1 + \alpha)}, \quad L > d,$$

(27) Burr 12: $$s(L) = \frac{|a| q (L - d)^{a-1}}{b^a \left[1 + \left(\frac{L - d}{b}\right)^a\right]^{q+1}}, \quad L > d,$$

(28) GB2: $$s(L) = \frac{|a| (L - d)^{ap-1}}{b^{ap} B(p, q) \left[1 + \left(\frac{L - d}{b}\right)^a\right]^{p+q}}, \quad L > d,$$

where $B(p, q)$ is the beta function. Because catastrophic losses are often defined as losses that exceed some monetary threshold, the probability distributions have been shifted so that they are defined for losses in excess of some threshold $d > 0$.

3.4.2 Loss Estimates Based on PCS Data

PCS defines a property catastrophe as a single event that gives rise to insured property damages of at least $5 million (the limit was $1 million prior to 1983). PCS obtains loss estimates by state for each catastrophe from individual insurers. The catastrophes reported include hurricanes, tornadoes, windstorms, hail, fires and explosions, riots, brush fires, and floods.[12]

It is important to be cautious in using historical loss data because systemic changes may have occurred such that prior catastrophes may not be representative of those that will occur in the future. Systemic changes may involve both frequency and severity of loss. For example, climatological changes may have occurred that increase either the frequency or the severity of various catastrophic perils. Economic and demographic changes can also affect catastrophe losses.

Fortunately, it is possible to adjust for most of the important systemic changes involving the frequency and severity of catastrophes. The two major factors affecting the severity of catastrophes are price-level changes (i.e., changes in construction costs and other factors that affect the prices of property exposed to loss) and changes in the amount of property exposed to loss. The latter adjustment is particularly important because several of the states with the highest exposure to catastrophe risk (such as California, Florida, and Texas) have been among the fastest-growing states over the past several decades.

We use two alternative approaches to adjust for changes in price and exposure levels. The first approach is to adjust for price-level changes affecting property values by using the U.S. Department of Commerce census fixed-weighted construction-cost index (taken from various years of the *Statistical Abstract of the United States*) to restate all catastrophe-loss values in 1994 dollars. To adjust for changes in the exposure base, we use data on population by state obtained from the U.S. Bureau of the Census. This approach assumes that the amount of property exposed to loss is highly correlated with population. Each catastrophe is adjusted to 1994 price and housing-value levels using the following formula:

$$L_{ijt}^{94} = L_{ijt} \frac{c_{94}}{c_t} \frac{v_{j,94}}{v_{jt}}, \tag{29}$$

where L_{ijt}^{94} = loss from catastrophe i in state j in year t, restated in 1994 dollars and exposure levels; L_{ijt} = loss from catastrophe i in state j in year t in year-t dollars; c_{94} = construction-cost index for 1994; c_t = construction-cost index for year t; $v_{j,94}$ = population of state j in 1994; and v_{jt} = population of state j in year t. In the discussion to follow, we refer to loss data based on equation (29) as *population-adjusted (PA) losses.*

As a second approach to adjusting for changes in price levels and the amount of property exposed to loss, we use data on the value of owner-occupied housing obtained from the U.S. Census of Housing, series HC80-1-A. This series provides the value of urban and rural owner-occupied buildings in each state

12. The catastrophe-insurance call spreads traded on the Chicago Board of Trade (CBOT) are also based on the PCS loss data.

at ten-year intervals based on the U.S. Census.[13] The series implicitly incorporates both changes in the price of housing and changes in the physical stock of property. Thus, no price indices are needed when adjusting catastrophes using the housing-value data. We refer to the data adjusted for changes in the value of owner-occupied housing as *value-adjusted (VA) losses.*

We consider the VA losses to be the primary data series for the estimation of XOL reinsurance premiums. Accordingly, most of the summary tables and graphs given below are based only on the VA series. However, the premium and loss-layer estimates are reported on the basis of both the PA and the VA series.

The insured VA catastrophe losses from 1949 through 1994 are shown in figure 3.1. The largest losses were attributable to Hurricane Andrew, which caused $18.4 billion in insured losses in 1992, and the Northridge Earthquake, which accounted for $12.5 billion in insured losses in 1994. Value adjustment has a substantial effect on some of the earlier catastrophes. For example, after adjusting for property values, a windstorm loss in 1950 that affected eleven northeastern and Middle Atlantic states is the third most severe catastrophe. This loss ranks *much* lower in terms of the unadjusted data.

Summary statistics on VA catastrophe losses by cause of loss are shown in table 3.2. On the basis of the loss experience since 1949, earthquakes and hurricanes have been the most serious type of catastrophe, with by far the highest mean and standard deviation of loss. Earthquakes and hurricanes also have among the highest coefficients of variation and skewnesses.

In estimating potential catastrophe losses in the $25–$50 billion range, it is clear that one should focus on catastrophes that are sufficiently severe to cause damage in this layer. Relatively minor catastrophes, such as hailstorms in the Midwest, for example, have a negligible probability of ever generating a loss of $25 billion or more (the largest such loss to date was $443 million). Although windstorms other than hurricanes clearly have caused very large losses, most of the 864 windstorms in the sample were relatively minor storms, such as tornadoes, that likely did not have the potential to cause losses in the loss layer covered by the proposed reinsurance contracts.

Accordingly, we focus the remainder of the analysis on the catastrophic losses most likely to be representative of those that would generate covered losses under the reinsurance contracts—hurricanes and earthquakes. The number of events in these two categories from 1949 to 1994 was seventy-one, fifty-seven hurricanes and fourteen earthquakes (including the Northridge Earthquake). These hurricane and earthquake losses are graphed in figure 3.2. Four of the seventy-one hurricanes and earthquakes did not exceed the PCS defini-

13. Values for years in between the census years were based on the average growth rate in property values over each ten-year period. Comparable data on the value of rental properties and commercial and industrial buildings were not available. However, this should not cause a problem as long as the values of these types of properties are highly correlated with values of owner-occupied dwellings.

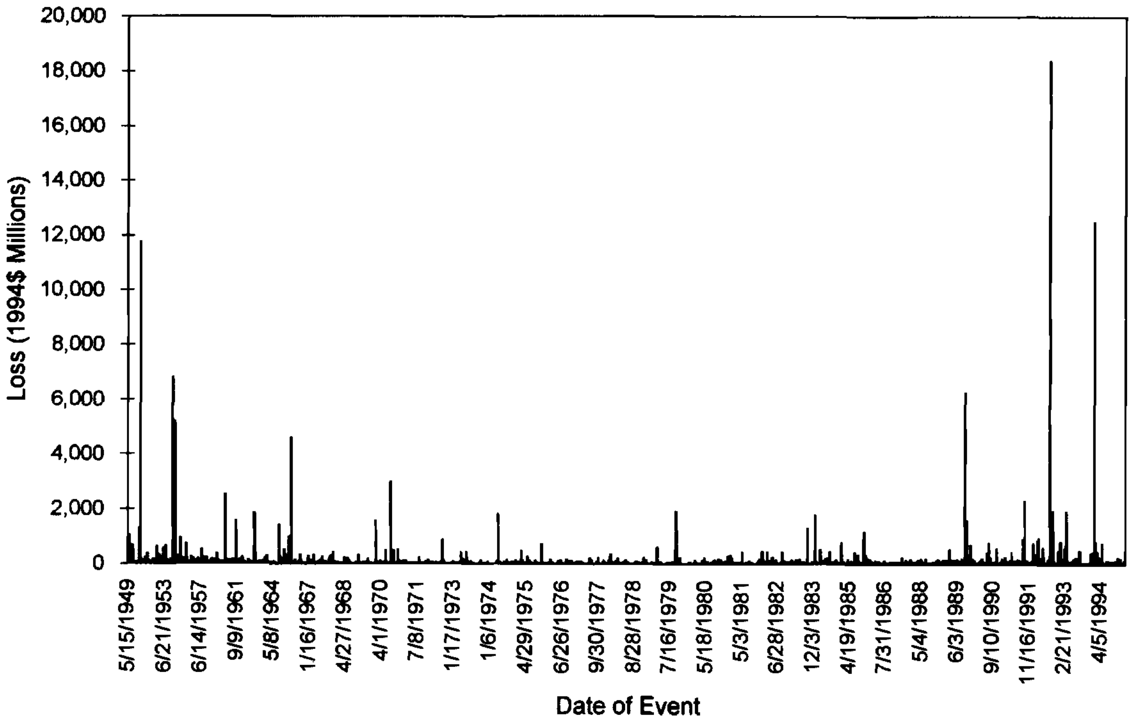

Fig. 3.1 U.S. property catastrophes, 1949–94
Source: Property Claims Services, Rahway, N.J.

Table 3.2 **Summary Statistics, U.S. Property Catastrophes, 1949–94**

Type of Catastrophe	*N*	Mean	SD	Coeff. of Var.	Skewness	Minimum	Maximum
Earthquake	14	1,079,919,991	3,313,558,418	3.07	3.64	11,852,852	12,500,000,000
Brush fire	27	228,427,389	434,833,689	1.90	4.44	3,769,473	2,296,609,302
Flood	14	73,110,934	117,528,364	1.61	2.20	7,022,724	356,502,769
Hail	53	82,098,301	90,209,509	1.10	2.11	7,992,680	443,331,807
Hurricanes	57	1,222,680,792	2,763,012,070	2.26	4.76	5,278,321	18,391,014,407
Ice	1	20,625,310	0			20,625,310	20,625,310
Snow	11	102,884,340	194,752,240	1.89	3.07	7,167,945	677,636,717
Tornado	21	74,586,127	116,138,156	1.56	3.67	3,246,349	546,706,772
Tropical storm	8	73,889,334	58,915,748	.80	1.81	19,991,072	204,946,131
Volcanic eruption	1	69,870,633	0			69,870,633	69,870,633
Wind	864	95,987,693	429,832,971	4.48	23.50	2,827,037	11,746,275,284
All other	66	108,959,698	191,921,889	1.76	3.25	3,777,433	983,118,263
Total	1,137	166,981,831	849,081,766	5.08	14.85	2,827,037	18,391,014,407

Source: Property Claims Services, Rahway, N.J.

Note: Losses were adjusted to 1994 exposure and price levels using U.S. Census of Housing's series HC80-1-A.

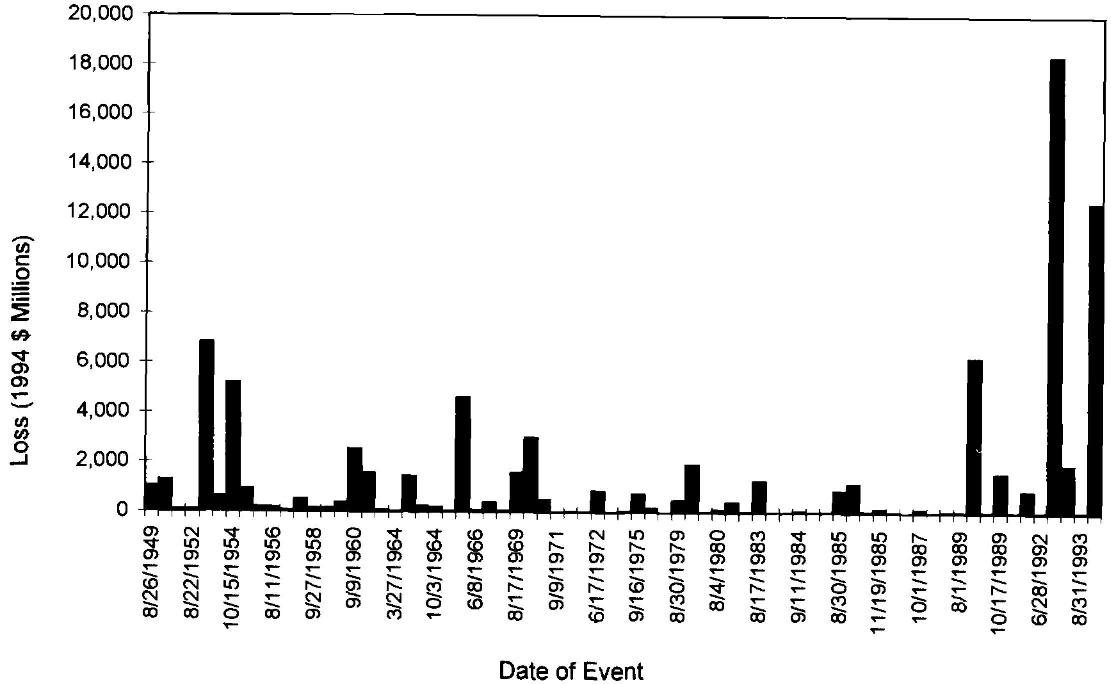

Fig. 3.2 U.S. hurricanes and earthquakes, 1949–94
Source: Property Claims Services, Rahway, N.J.

tion of a catastrophe ($12.04 million in 1994 dollars) when inflated to 1994. Consequently, these four events were dropped from the sample for the purposes of estimating severity distributions and XOL contract premiums. The final sample thus consists of sixty-seven events.

The next step is to estimate severity of loss distributions for the hurricane and earthquake data. As our value of d, the threshold that must be exceeded in order for a loss to be defined as a catastrophe, we use $12.04 million, which is the PCS lower-bound definition of a catastrophe ($5 million) inflated to 1994 price and exposure levels using the value of owner-occupied housing (i.e., the VA adjustment). Based on the PA adjustment, d = $6.85 million.[14]

We use maximum-likelihood-estimation techniques to estimate the parameters of the various probability distributions that we employed in this study. The parameters and log-likelihood function values are shown in table 3.3, along with the RMS parameters, which are discussed later.[15] Parameter estimates are shown for both PA and VA losses. The estimated probability distributions based on the VA losses are graphed in figure 3.3. Also shown in the graph is the empirical distribution function, calculated as $i/(n + 1)$, where $i = 1, 2, \ldots,$ n, and n = the number of observations. Both the lognormal and Burr 12 distributions provide excellent fits to the observed data. The GB2 (not shown) also fits well and is about the same as the Burr 12. However, the Pareto distribution tends to overestimate the amount of probability in the tail of the distribution. This is shown more clearly in figure 3.4, which graphs the tails of the estimated distribution functions, where the tail is defined somewhat arbitrarily as the largest third of the observations. From figure 3.4, it is clear that both the lognormal and the Burr 12 provide an adequate fit to the tail of the loss distribution. The tail of the Pareto is too heavy to represent the observed data. However, it is important to keep in mind the possibility of sampling error in a sample of this size, particularly if our adjustments for exposure are not sufficiently precise. Thus, we believe that the results based on the Pareto should also be considered when setting the premiums for the XOL contracts. In this sense, the Pareto can be viewed as providing a conservative upper bound for the premiums. However, on the basis of goodness of fit, we recommend basing the premiums on the lognormal, the Burr 12, or the GB2.

Analysis of the PA losses reveals that the Burr 12 and GB2 provide the best model for this data series. The lognormal underestimates the tail of the PA loss

14. Prior to 1983, PCS defined a catastrophe as any single event that generated insured losses greater than $1 million. Therefore, we also investigated an alternative threshold for catastrophic losses of $1 million inflated to 1994 dollars from the first year in the analysis, 1949. Using the housing-value index, this would have set the lower-bound definition of a catastrophe at $57.7 million. Reworking the analysis using this definition of a catastrophic event did not substantially change the results. Thus, they are not reported here.

15. The log-likelihood function values for the PCS and RMS samples are not directly comparable because the sample sizes differ—the PCS sample has sixty-seven events, and the RMS samples each have one thousand events.

Table 3.3 **Parameter Estimates Summary**

Distribution and Parameter	PCS Housing Value (VA)	PCS Population (PA)	RMS Nation	RMS California	RMS Florida	RMS Southeastern United States
Lognormal:						
μ	5.396	4.586	4.403	3.703	5.654	5.337
σ	2.064	2.168	2.195	2.059	2.259	2.211
$-\log(L)$	471.667	426.964	6,108.236	5,344.295	7,388.235	7,049.331
Pareto:						
α	.328	.343	.431	.564	.296	.323
d	12.040	6.850	12.040	12.040	12.040	12.040
$-\log(L)$	430.041	470.040	6,653.261	5,834.966	8,087.477	7,712.696
Burr 12:						
a	.659	.804	.910	1.099	.689	.760
b	874.302	95.780	44.600	18.006	690.888	308.502
q	1.991	.999	.737	.619	1.507	1.215
$-\log(L)$	502.537	461.542	6,609.184	5,813.172	7,889.142	7,539.407
GB2:						
a	.150	.078	.405	1.842	.179	.491
b	291,488,438.71	.001	23.515	21.847	1,349,162.97	469.254
p	10.970	121.909	3.816	.498	7.988	1.886
q	88.975	50.199	2.491	.335	34.485	2.590
$-\log(L)$	501.438	460.476	6,604.769	5,811.365	7,882.308	7,537.281
Frequency	2.20	2.20	6.67	3.60	.83	1.35

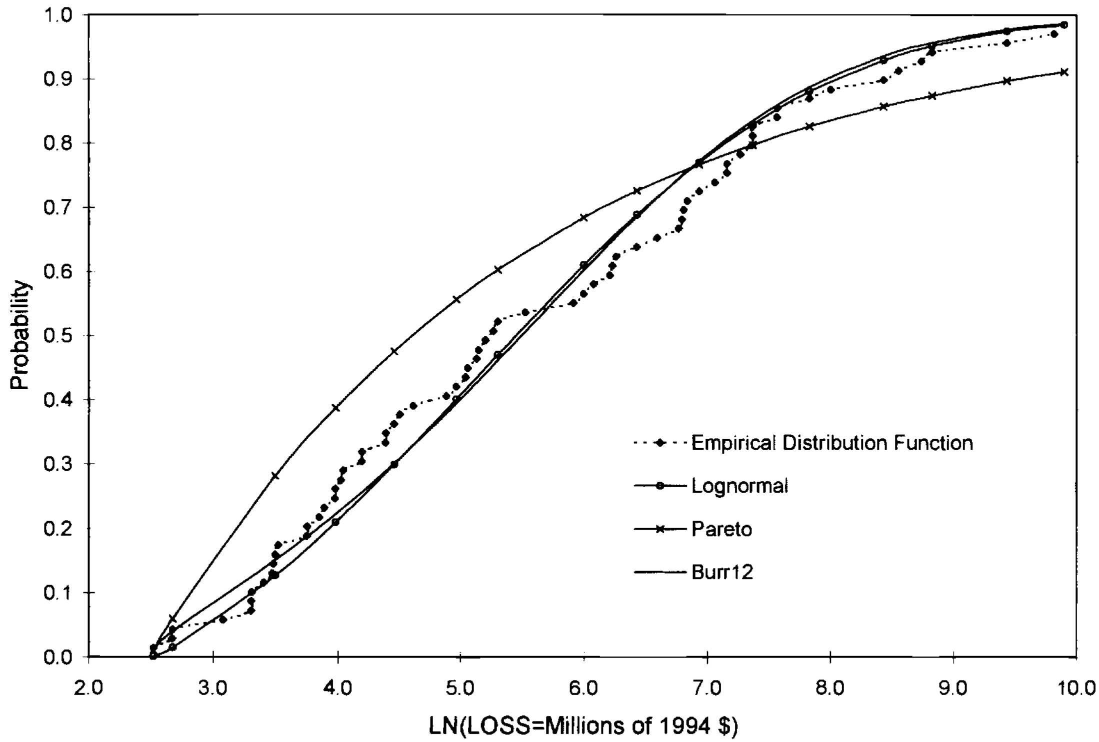

Fig. 3.3 Severity-of-loss distribution functions: earthquakes and hurricanes

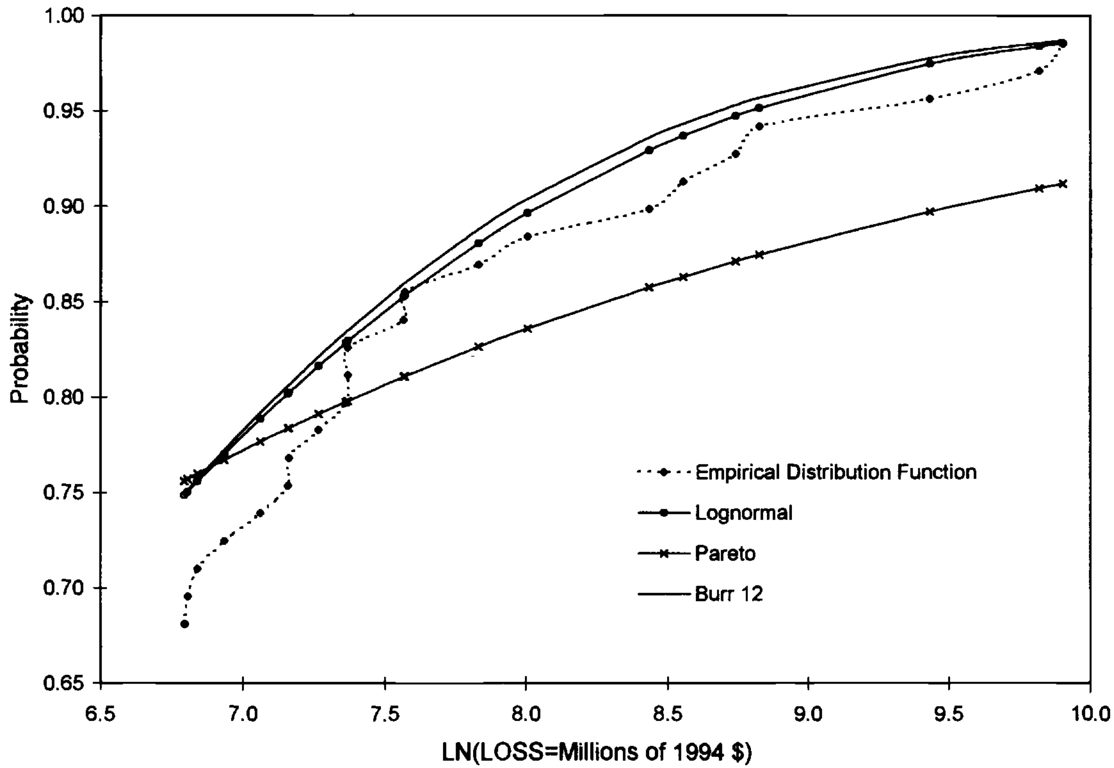

Fig. 3.4 Severity-of-loss distribution function tails: earthquakes and hurricanes

distribution, and the Pareto overestimates the tail, although not by as much as in the VA analysis.

The annual frequency of earthquakes and hurricanes is graphed in figure 3.5. The graph reveals a high degree of volatility in the frequency series. Frequency has been trending upward, as shown by the least-squares line and ten-year-moving-average lines in the figure. The variance of frequency also increased near the end of the period, as shown by the ten-year-moving-average variance line. On the basis of these apparent trends in the average number and variance of catastrophic events, we estimate the average number of events at 2.2 and use this estimate as the parameter of the Poisson frequency distribution in estimating p^*).[16]

Expected loss severities were calculated for various policy limits on the basis of the estimated severity distributions. For the VA loss data, these expected severities represent the expected value of loss for a policy that covers catastrophes in excess of $12.04 million up to the specified policy limit (e.g., $10 billion). For the PA data, the expected severities cover catastrophes in excess of $6.85 million up to the specified policy limits. Reinsurance-layer prices are obtained as differences between the expected policy limit severities. The results are presented in table 3.4 for both the PA and the VA data.

The Pareto distribution clearly gives the largest estimate of expected severity in the $25–$50 billion layer, $1.806 billion based on the VA data and $1.319 billion based on the PA data. For the lognormal, the expected severities in the $25–$50 billion layer are $170.2 based on the VA data and $81.0 million based on the PA data. The corresponding expected severities in the $25–$50 billion layer given by the Burr 12 and GB2, respectively, are $162.4 million and $112.0 billion, based on the VA data, and $211.0 million and $97.1 million, based on the PA data.

Table 3.4 also shows the overall expected loss for the four severity distributions, based on equations (11a) and (16a), and a Poisson frequency parameter of 2.2 events per year.[17] Based on the Pareto distribution, the expected loss is $3.636 billion for the VA data and $2.719 billion for the PA data. The lognormal gives expected losses of $370.0 million and $177.2 million based on the VA and PA series, respectively. The corresponding estimates based on the Burr 12 are $353.4 million (VA data) and $458.5 million (PA data), and those based on the GB2 are $244.2 million (VA data) and $212.1 million (PA data).

Overall, considering the goodness of fit to both data series, we recommend using either the Burr 12 or the GB2 model to calculate the XOL premiums. Taking the average of the Burr 12 estimates based on the VA and the PA data, one obtains an estimate of $405.9 million, or $405,900 for each $25 million

16. Our frequency estimate is based on a linear least-squares trend line fitted to the annual frequency observations and used to project trend to 1995.

17. The Poisson parameter is used in conjunction with eq. (7) to obtain an estimate of the loss probability p^*. The negative binomial results are very close to the Poisson results and hence are not shown.

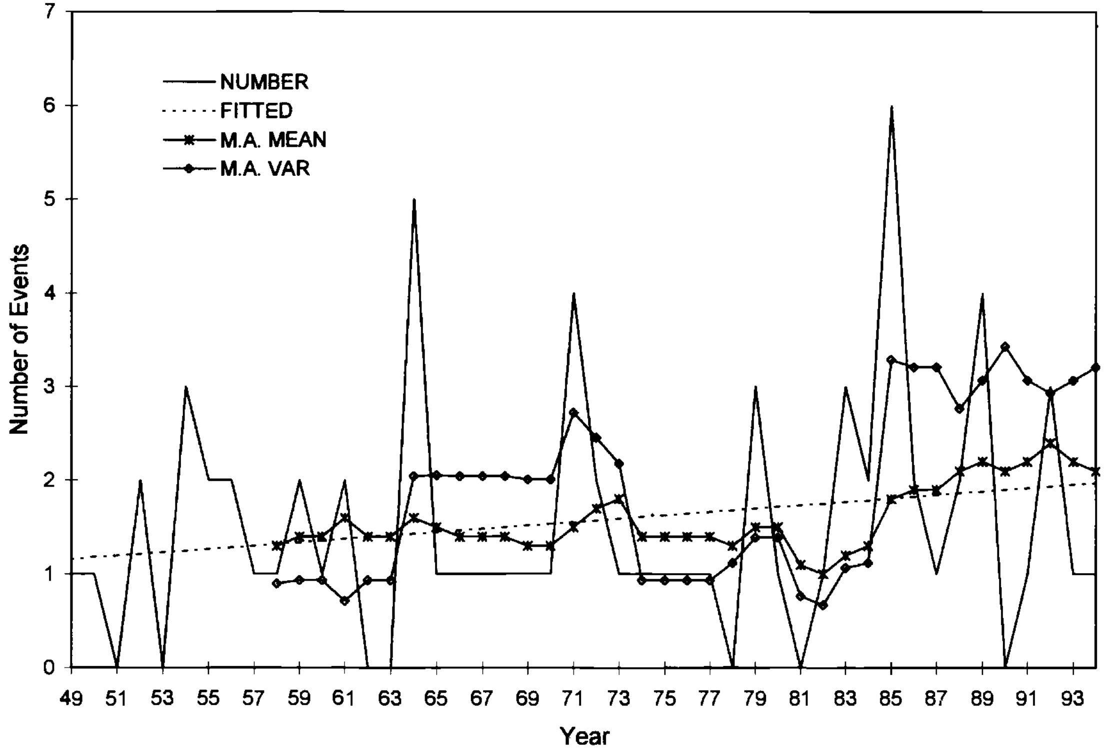

Fig. 3.5 Earthquake and hurricane frequency, 1949–94

Table 3.4 **Expected Loss Severities for Various Layers, Total Expected Loss for $25–$50 Billion Layer**

	Lognormal	Pareto	Burr 12	GB2
Losses inflated by housing values:				
$E(L)$	1,869.96M	Undefined	2,209.63M	1,511.32M
SD(L)	15,520.12M	Undefined	Undefined	55,723,965M
$E(L;$ \$12.04M, \$5B, \$12.04M)	864.75M	1,025.95M	841.38M	854.45M
$E(L;$ \$12.04M, \$10B, \$12.04M)	1,092.08M	1,637.97M	1,043.37M	1,059.38M
$E(L;$ \$12.04M, \$15B, \$12.04M)	1,220.31M	2,152.84M	1,156.47M	1,166.37M
$E(L;$ \$12.04M, \$20B, \$12.04M)	1,306.34M	2,613.28M	1,233.12M	1,233.67M
$E(L;$ \$12.04M, \$25B, \$12.04M)	1,369.34M	3,037.06M	1,290.16M	1,280.27M
$E(L;$ \$12.04M, \$30B, \$12.04M)	1,418.04M	3,433.75M	1,335.10M	1,314.54M
$E(L;$ \$12.04M, \$35B, \$12.04M)	1,457.10M	3,809.24M	1,371.88M	1,340.81M
$E(L;$ \$12.04M, \$40B, \$12.04M)	1,489.29M	4,167.49M	1,402.81M	1,361.57M
$E(L;$ \$12.04M, \$45B, \$12.04M)	1,516.37M	4,511.31M	1,429.37M	1,378.38M
$E(L;$ \$12.04M, \$50B, \$12.04M)	1,539.54M	4,842.81M	1,452.55M	1,392.24M
$E(L;$ \$25B, \$50B, \$12.04M)	170.20M	1,805.75M	162.39M	111.97M
Prob[$L >$ \$25|event occurs] = $P_>$ (%)	1.10	8.18	1.00	.79
Prob [$L >$ \$25] = p^* (Poisson param. = 2.2)	.0238	.1647	.0218	.0172
E ($L;$ \$25B, \$50B, \$12.04M| $L >$ \$25B)	15,518.11M	22,073.58M	16,194.72M	14,179.08M
Total $E(L)$: \$25–\$50B layer	369.97M	3,635.69M	353.35M	244.21M
Losses inflated by population:				
$E(L)$	1,036.65M	Undefined	Undefined	1,150.19M
SD(L)	10,733.19M	Undefined	Undefined	415,863,130M
E(L; \$6.85M, \$5B, \$6.85M)	551.51M	789.43M	541.34M	550.52M
$E(L;$ \$6.85M, \$10B, \$6.85M)	670.09M	1,247.59M	691.79M	675.25M
$E(L;$ \$6.85M, \$15B, \$6.85M)	734.59M	1,629.95M	790.83M	745.46M
$E(L;$ \$6.85M, \$20B, \$6.85M)	776.99M	1,970.14M	866.40M	792.87M
$E(L;$ \$6.85M, \$25B, \$6.85M)	807.62M	2,282.06M	928.20M	827.89M
$E(L;$ \$6.85M, \$30B, \$6.85M)	831.06M	2,573.14M	980.85M	855.23M
$E(L;$ \$6.85M, \$35B, \$6.85M)	849.71M	2,847.98M	1,026.92M	877.37M
$E(L;$ \$6.85M, \$40B, \$6.85M)	864.98M	3,109.64M	1,068.01M	895.79M
$E(L;$ \$6.85M, \$45B, \$6.85M)	877.76M	3,360.28M	1,105.19M	911.44M
$E(L;$ \$6.85M, \$50B, \$6.85M)	888.65M	3,601.54M	1,139.21M	924.95M
$E(L;$ \$25B, \$50B, \$6.85M)	81.03M	1,319.49M	211.01M	97.06M
Prob[$L >$ \$25|event occurs] = $P_>$ (%)	.53	6.01	1.13	.61
Prob[$L >$ \$25] = p^* (Poisson param. = 2.2)	.0116	.1239	.0246	.0134
$E(L;$ \$25B, \$50B, \$6.85M| $L >$ \$25B)	15,286.12M	21,950.14M	18,617.91M	15,839.40M
Total $E(L)$: \$25–\$50B layer	177.20M	2,719.11M	458.48M	212.06M

Note: $E(L; T, C, d)$ = expected value of loss severity (L) for a shifted distribution beginning at d for a reinsurance contract beginning at point of attachment T and having upper limit C. M = million; B = billion. The total $E(L)$ is based on the Poisson frequency distribution with mean of 2.2. Figures are dollar values unless otherwise specified.

contract. The corresponding estimate based on the GB2 is $228.1 million, or $228,100 for each $25 million contract. These premiums translate into *rates on line* for the $25–$50 billion layer of 1.62 percent and 0.91 percent, respectively.[18] The most conservative estimate of the premium is provided by the Pareto distribution, which gives a premium estimate of $3.177 billion or a 12.7 percent rate on line based on an average of the VA and PA results.

3.4.3 Loss Estimates Based on RMS Data

Risk Management Solutions supplied ten thousand simulated catastrophe losses for each of several geographic areas for use in this study. The geographic areas included the entire United States, the southeastern United States, California, and Florida. Preliminary analysis indicated that it is not necessary to work with all ten thousand observations when estimating the loss-severity distributions. Accordingly, we randomly selected subsamples of one thousand losses for each geographic-area definition to form the primary basis for the following discussion.[19] We use the full samples of ten thousand claims to provide empirical estimates of the premiums for comparison with the estimates based on the loss-severity distributions.

The RMS estimates also differ from the PCS estimates in the choice of the loss-frequency parameter. For the PCS data, loss frequency was estimated by fitting a trend line to a time series of observed annual catastrophe frequencies. However, RMS estimates frequency on the basis of engineering and meteorologic models that are used to predict the probabilities and severities of hurricanes and earthquakes. It is important to analyze the RMS frequency estimates because no catastrophe causing insured property damage in the $25–$50 billion range has been observed during the period covered by the PCS data (1949–present), but such events are possible and can be simulated using the RMS approach.

Summary statistics for the PCS sample and the national RMS sample are shown in table 3.5. The mean severities for the two samples are quite comparable, $1.284 billion for the PCS sample and $1.048 billion for the RMS sample. However, the coefficient-of-variation, skewness, and kurtosis estimates are considerably higher for the RMS sample, and the maximum loss in the RMS sample is about six times as large as the maximum in the PCS sample. The simulated RMS frequency of events larger than $12.04 million is also considerably higher than the corresponding PCS estimate (6.7 as opposed to 1.5). And the RMS estimate of frequency is about three times as large as our fre-

18. The rate on line is defined as the premium divided by the width of the layer, $25 billion in this case.

19. Estimation of the parameters of the loss distributions was very slow when all ten thousand observations were used. Preliminary analysis based on ten thousand and several random samples of one thousand revealed that the parameter estimates are very stable, i.e., not sensitive to the choice of sample. Accordingly, the remainder of the analysis was based on one-thousand-observation samples.

Table 3.5 **Summary Statistics: Actual Loss Experience and Simulated Loss Experience, Hurricanes and Earthquakes**

	Obs.	Mean ($)	SD ($)	Coeff. of Var.	Skewness	Kurtosis	Minimum ($)	Maximum ($)
Severity of losses reported by PCS, 1949–94, losses > $12.04 million	67	1,283,998.7	2,942,996.6	2.292	4.198	20.124	12,434.3	18,391,014.4
Severity of losses simulated by RMS:								
All losses	95,182	736,533.4	3,790,455.6	5.146	12.126	199.853	5,007.2	107,546,261.0
Losses > $12.04 million	66,138	1,047,983.0	4,493,486.8	4.288	10.193	141.061	12,058.2	107,546,261.0

	Obs.	Mean	SD	Coeff. of Var.	Skewness	Kurtosis	Minimum	Maximum
Frequency of losses reported by PCS, 1949–94	67	1.543	1.312	.85	1.477	2.539	0	6
Frequency of losses simulated by RMS:								
All losses	95,182	9.518	3.056	.321	.333	.152	0	23
Losses > $12.04 million	66,138	6.668	2.559	.384	.399	.195	0	19

quency estimate based on linear time trending (2.2 events per year). As discussed further below, the primary reason for the difference in frequencies is that RMS is simulating a larger number of earthquakes per year than have been observed historically.

The national loss estimates based on the RMS empirical and fitted probability-of-loss distributions are shown in table 3.6. It is noteworthy that the estimated loss severities in the \$25–\$50 billion layer are quite comparable to our PCS estimates presented in table 3.4. The lognormal and Pareto PCS estimates in table 3.4 are actually larger than their RMS counterparts in table 3.5. The severity estimates based on the Burr 12 and GB2 distributions fitted to the RMS data (table 3.5) are somewhat larger than the corresponding estimates based on the PCS data (table 3.4). For example, the Burr 12 loss-severity estimate for the \$25–\$50 billion layer is \$279.2 million, whereas the PCS estimate is \$162.4 million (based on the VA adjustment).

The comparability of the fitted PCS and RMS loss estimates in the \$25–\$50 billion layer shows that using probability distributions to model the tails of loss distributions on the basis of relatively small samples can yield accurate estimates of expected values of large losses even if no losses in this range have been observed. This is also illustrated by figures 3.6 and 3.7, which show, respectively, the empirical distribution functions for the RMS and PCS data and the tails of the empirical distribution functions along with GB2 distributions fitted to the PCS and RMS data. Figure 3.6 shows that the tail of the PCS distribution is actually somewhat heavier than that of the RMS distribution for relatively small losses and is comparable for larger losses. Figure 3.7 shows that the GB2 distributions are also quite comparable for large losses, although the RMS distribution has a somewhat heavier tail.

Table 3.6 also shows the estimates of the total expected loss (i.e., the expected loss component of the XOL premiums) in the \$25–\$50 billion layer based on the RMS data. The expected loss estimates in table 3.6 are considerably larger than those based on the PCS data (table 3.4), primarily because of the difference between the RMS and the PCS loss-frequency estimates. The RMS-based estimates of the expected loss range from \$453.4 million (or a 1.8 percent rate on line) for the lognormal distribution to \$4.635 billion (or an 18.5 percent rate on line) for the Pareto. Again, however, the Pareto does not provide a very good fit to the data. The best fit is provided by the Burr 12 and GB2, and the premiums based on those models are \$1.758 billion (7 percent rate on line) and \$1.020 billion (4.1 percent rate on line), respectively.

Expressed per XOL contract, the expected-loss estimates from table 3.6 imply a price of \$1,758,000 per \$25 million national contract based on the Burr 12 and \$1,020,000 per contract based on the GB2. These are much larger than the \$405,900 per contract (Burr 12) and \$228,100 per contract (GB2) based on the PCS data. Whether the RMS or the PCS sample gives more reasonable estimates depends on the accuracy of the RMS prediction that 6.6 events will occur per year. A practical approach to resolving the uncertainty would be to

Table 3.6 **Expected RMS Loss Severities for Various Layers for the United States, Total Expected Loss for $25–$50 Billion Layer**

	Empirical	Lognormal	Pareto	Burr 12	GB2
Losses simulated by RMS					
$E(L)$	987.67M	922.63M	Undefined	132,471,003.8M	36,766.2M
SD(L)	4,433.98M	10,082.30M	Undefined	Undefined	Undefined
$E(L;$ $12.04M, $5B, $12.04M)	538.44M	492.68M	646.72M	498.66M	496.29M
$E(L;$ $12.04M, $10B, $12.04M)	669.39M	595.84M	963.60M	662.15M	628.83M
$E(L;$ $12.04M, $15B, $12.04M)	736.27M	651.67M	1,215.96M	776.88M	712.49M
$E(L;$ $12.04M, $20B, $12.04M)	780.90M	688.28M	1,433.83M	868.19M	774.28M
$E(L;$ $12.04M, $25B, $12.04M)	820.30M	714.68M	1,629.19M	945.26M	823.48M
$E(L;$ $12.04M, $30B, $12.04M)	850.34M	734.87M	1,808.30M	1,012.61M	864.44M
$E(L;$ $12.04M, $35B, $12.04M)	872.29M	750.92M	1,974.94M	1,072.81M	899.59M
$E(L;$ $12.04M, $40B, $12.04M)	882.29M	764.06M	2,131.60M	1,127.49M	930.39M
$E(L;$ $12.04M, $45B, $12.04M)	892.29M	775.05M	2,280.03M	1,177.78M	957.82M
$E(L;$ $12.04M, $50B, $12.04M)	902.29M	784.41M	2,421.50M	1,224.45M	982.56M
$E(L;$ $25B, $50B, $12.04M)	81.99M	69.73M	792.32M	279.19M	159.08M
Prob[$L >$ $25|event occurs] = $P_{>}$ (%)	.70	.46	3.73	1.43	.89
Prob [$L >$ $25] = p^* (Poisson param. = 6.7)	.0451	.0297	.2182	.0903	.0571
E ($L;$ $25B, $50B, $12.04M| $L >$ $25B)	11,713.10M	15,266.03M	21,246.44M	19,477.90M	17,847.16M
Total $E(L)$: $25–$50B layer	528.84M	453.36M	4,635.46M	1,758.16M	1,019.66M

Note: $E(L;\ T,\ C,\ d)$ = expected value of loss severity (L) for a shifted distribution beginning at d for a reinsurance contract beginning at point of attachment T and having upper limit C. M = million; B = billion. The total $E(L)$ is based on the Poisson frequency distribution with mean of 6.7. Figures are dollar values unless otherwise specified.

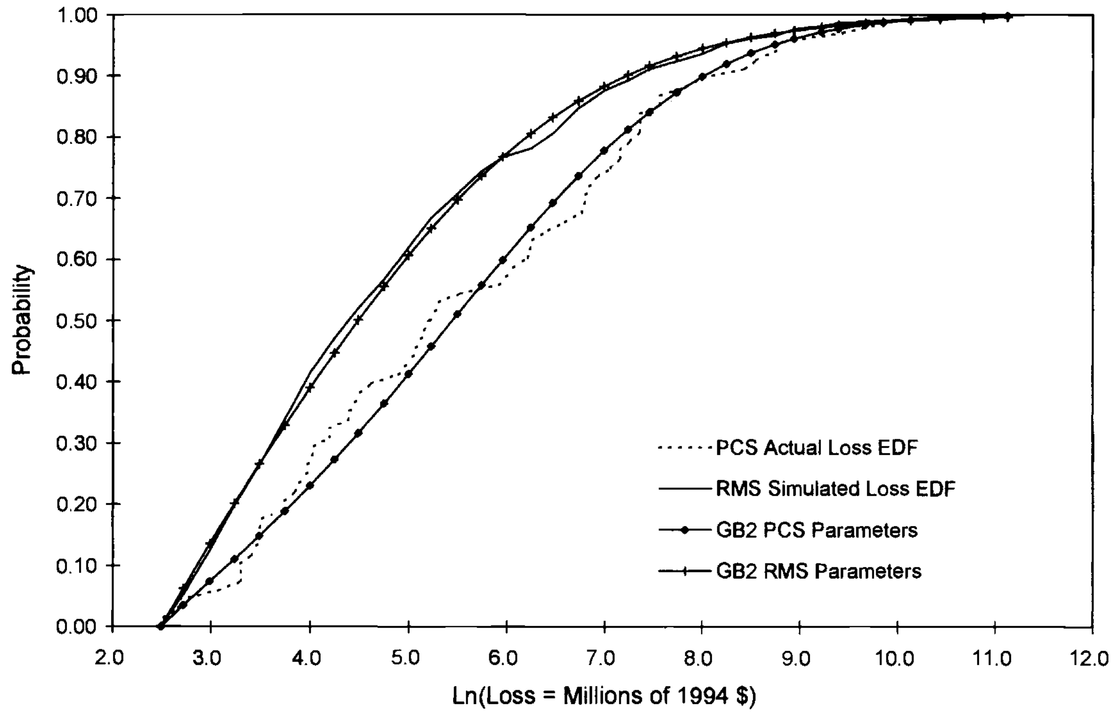

Fig. 3.6 Severity-of-loss distribution functions: GB2 hurricanes and earthquakes, PCS reported actual losses vs. RMS simulated losses

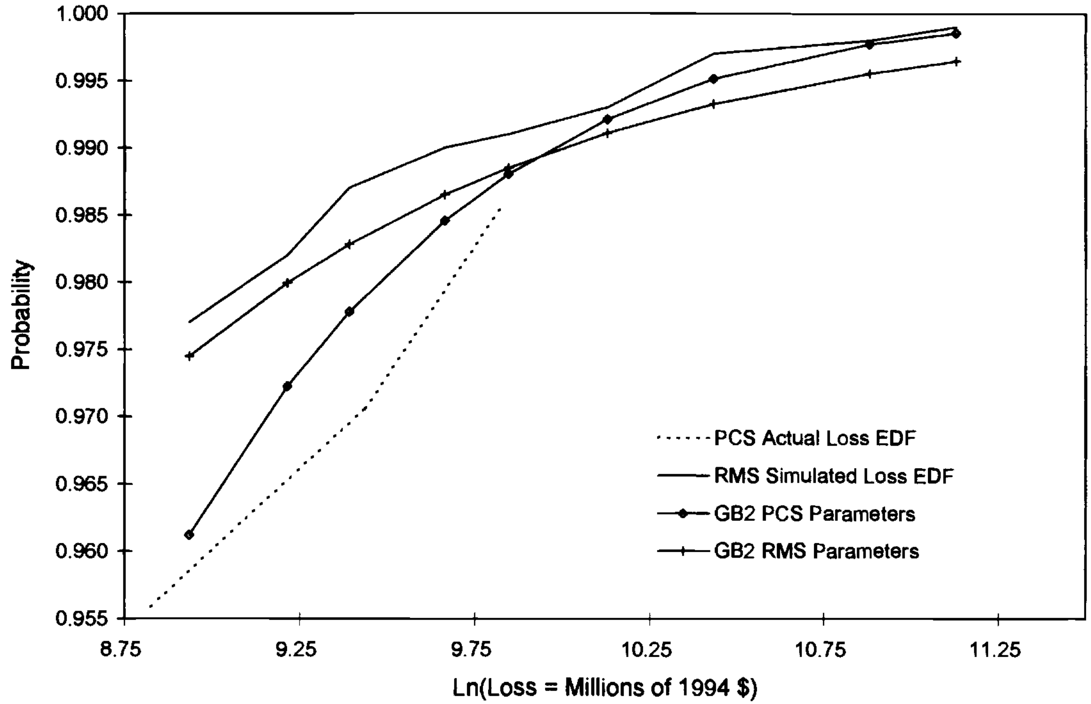

Fig. 3.7 Severity-of-loss distribution function tails: GB2 hurricanes and earthquakes, PCS reported actual losses vs. RMS simulated losses

set the reservation price as the average of the RMS and the PCS estimates. This would give a reservation price of $1.082 million (4.3 percent rate on line) based on the Burr 12 and $624,000 (2.5 percent rate on line) based on the GB2.

On the basis of the RMS data, we also estimated conditional and unconditional loss severities for California, Florida, and the southeastern region of the United States. Tables comparable to table 3.6 and based on these three samples are presented in appendix B. We also had enough PCS data on losses in the southeastern region of the United States to provide PCS estimates for this region. The results are summarized in table 3.7, which shows the conditional expected severity of losses in the $25–$50 billion layer, the probability of a loss in this layer based on the severity distribution (i.e., $P[L >$ $25 billion|an event occurs$] = P_>$]), and the unconditional expected severity in the layer (the product of the conditional severity and $P_>$). For purposes of comparison, the table also shows the comparable national statistics based on the PCS and RMS samples.

The RMS conditional loss severity is highest in California, $16.6 billion (table 3.7). The comparable conditional loss severities are $15.0 billion for Florida and $13.9 billion for the Southeast. However, the unconditional severities are higher in Florida and the Southeast than in California. The chance of breaching the $25 billion trigger is larger in Florida and the Southeast, but, given that a loss breaches the trigger, expected severity is higher in California. Graphic analysis (not shown) reveals that the GB2 is the best model for the RMS California severity data, while the Burr 12 and GB2 are the best models for Florida and the Southeast. Comparison of the PCS and RMS data for the Southeast reveals that the PCS data imply lower conditional severity, a lower probability of a loss exceeding the trigger, and lower unconditional severity than the RMS data, where these comparisons are based on the best-fitting GB2 distribution. This is likely due to the reduced PCS sample size and the absence of large events, such as the Northridge Earthquake, from the southeastern PCS sample.

The total expected-loss components of the PCS reservation-price estimates and their corresponding rates on line are summarized in table 3.8. The first panel of the table is based on historical loss-frequency estimates, while the second panel is based on the RMS frequency estimates. The differences between the results in the first two panels of the table can be attributed primarily to the loss-frequency estimates generated by RMS, which are higher than the historical averages nationally and those for California, Florida, and the Southeast. For example, the historical average number of events per year in California is 0.22, whereas the RMS estimate is 3.6. To see the effect that the higher-frequency estimates has on the reservation price, consider the best-fitting distributions—the Burr 12 and the GB2. Using the PCS VA (housing value) severity estimates and the historical frequencies for the national contracts, the estimated reservation rates on line are 1.41 percent for the Burr 12 and 0.98 for the GB2. However, using the corresponding RMS-provided frequency data

Table 3.7 **Summary: Expected Loss Severities and Expected Losses, \$25–\$50 Billion Layer**

	$E(L$; \$25B, \$50B, \$12.04M$\mid L >$ \$25B) (\$)				
Region	Empirical	Lognormal	Pareto	Burr12	GB2
PCS housing value (VA)	. . .	15,518.11M	22,073.58M	16,194.72M	14,179.08M
PCS population (PA)	. . .	15,286.12M	21,950.14M	18,617.91M	15,839.40M
RMS, United States	11,713.10M	15,266.03M	21,246.44M	19,477.90M	17,847.16M
RMS, California	16,601.38M	13,739.22M	20,229.10M	19,402.25M	19,844.63M
RMS, Florida	14,974.47M	16,839.55M	22,339.15M	17,514.89M	15,414.08M
PCS, Southeastern United States (VA)	. . .	16,677.02M	21,966.54M	15,853.33M	14,356.77M
RMS, Southeastern United States	13,867.94M	16,276.46M	22,111.64M	17,963.53M	16,776.73M
	Prob [$L >$ \$25B$\mid$Event Occurs] = $P_>$ (%)				
	Empirical	Lognormal	Pareto	Burr12	GB2
PCS housing value (VA)	. . .	1.10	8.18	1.00	.79
PCS population (PA)	. . .	.53	6.01	1.13	.61
RMS, United States	.70	.46	3.73	1.43	.89
RMS, California	.60	.09	1.34	.73	.92
RMS, Florida	1.70	2.39	10.44	2.13	1.70
PCS, Southeastern United States (VA)	. . .	1.57	7.41	.77	.59
RMS, Southeastern United States	1.20	1.52	8.47	1.66	1.33
	$E(L$; \$25B, \$50B, \$12.04M) (\$)				
	Empirical	Lognormal	Pareto	Burr12	GB2
PCS housing value (VA)	. . .	170.20M	1,805.75M	162.39M	111.97M
PCS population (PA)	. . .	81.03M	1,319.49M	211.01M	97.06M
RMS, United States	81.99M	69.73M	792.32M	279.19M	159.08M
RMS, California	99.61M	12.46M	271.42M	141.86M	182.84M
RMS, Florida	254.57M	402.06M	2,333.02M	372.95M	262.73M
PCS, Southeastern United States (VA)	. . .	261.57M	1,626.79M	122.47M	84.17M
RMS, Southeastern United States	166.42M	246.64M	1,873.72M	297.76M	223.63M

Note: M = million; B = billion.

Table 3.8 **Reservation-Price Estimates of Federal XOL Contracts**

Region	Historical Frequency Estimates	Severity Distribution Assumption ($)			
		Lognormal	Pareto	Burr12	GB2
PCS housing value (VA)	2.2	369.97M (1.48)	3,635.69M (14.54)	353.35M (1.41)	244.21M (.98)
PCS population (PA)	2.2	177.20M (.71)	2,719.11M (10.88)	458.48M (1.83)	212.06M (.85)
RMS, United States	2.2	152.64M (.61)	1,673.51M (6.69)	604.63M (2.42)	346.57M (1.39)
RMS, California	.217	87.02M (.35)	500.57M (2.00)	80.74M (.32)	56.91M (.23)
RMS, Florida	.378	4.71M (.02)	102.34M (.41)	53.55M (.21)	68.99M (.28)
PCS, Southeastern United States (VA)	.844	219.31M (.88)	1,330.98M (5.32)	103.03M (.41)	70.86M (.28)
RMS, Southeastern United States	.844	206.84M (.83)	1,526.20M (6.10)	249.56M (1.00)	187.68M (.75)
	RMS Frequency Estimates				
PCS housing value (VA)	6.7	1,083.66M (4.33)	9,209.22M (36.84)	1,037.08M (4.15)	720.08M (2.88)
PCS population (PA)	6.7	525.46M (2.10)	7,188.55M (28.75)	1,341.85M (5.37)	627.69M (2.51)
RMS, United States	6.7	453.36M (1.81)	4,635.46M (18.54)	1,758.16M (7.03)	1,019.66M (4.08)
RMS, California	3.6	44.61M (.18)	950.54M (3.80)	502.23M (2.01)	645.12M (2.58)
RMS, Florida	.83	331.61M (1.33)	1,861.27m (7.45)	307.93M (1.23)	217.31M (.87)
PCS, Southeastern United States (VA)	1.35	349.41M (1.40)	2,089.95M (8.36)	164.48M (.66)	113.18M (.45)
RMS, Southeastern United States	1.35	330.30M (1.32)	2,395.22M (9.58)	398.38M (1.59)	299.86M (1.20)

Note: The rates online, given in percentages and shown in parentheses, are obtained by dividing the reservation prices by $25,000 million. M = million.

increases the estimates to 4.15 and 2.88 percent, respectively. A similar pattern can be observed for the other severity estimates reported in table 3.8 and across each of the different regions.

Whether the reservation price should be based on the historical data or on the RMS projections depends on the degree of credibility that should be assigned to the RMS projections. This is difficult to gauge in the absence of a full-scale engineering analysis or a few more years of historical experience. However, the difference between the two approaches provides a reasonable range that government officials could use when setting the reservation price. Also, as indicated above, these rates should be loaded for the expenses of administering the program and discounted to reflect the time lag between the premium payment and expected-loss-payment dates.

3.5 Conclusions

This paper analyzes a proposal for federal excess-of-loss (XOL) reinsurance contracts to assist insurers in hedging the risk of property catastrophes. Under the proposed reinsurance program, the federal government would directly write and sell per occurrence excess-of-loss reinsurance contracts protecting against catastrophe losses. These XOL contracts would be available for qualified insurance companies, pools, and reinsurers and would cover industry losses from a disaster in the $25–$50 billion layer of coverage—a layer currently unavailable in the private market.

The rationale for government provision of these contracts is that the capacity of the private insurance and reinsurance markets is presently inadequate to provide coverage for losses of this magnitude. The unavailability of capacity for large catastrophes has a number of serious effects on the viability of insurance markets and the ability of society to respond to a major disaster. The lack of capacity has led to shortages in the supply of insurance, with the resulting potential for higher federal disaster-relief expenditures as a result of a major catastrophe. The unavailability of high-limits reinsurance also increases the probability of insolvency for insurers participating in the property-insurance market, thus posing further risk to the stability of insurance markets.

Private market capacity for large losses is limited because the possibility of bankruptcy, along with information asymmetries in insurance and capital markets, constrains the ability of private insurers and reinsurers to diversify risk across time. Time diversification requires that insurers be able to raise debt and/or equity capital at reasonable rates following a large loss. However, the cost of capital to insurers tends to increase following a loss shock, and capital may be unavailable at any price for certain lines of coverage. Private insurance markets tend to function effectively in diversifying the risk of relatively small losses, but they are not very efficient in dealing with extremely large losses.

The federal government, on the other hand, has a superior ability to diversify risk across time through the exercise of federal borrowing power. While it is costly for private insurers to raise additional capital following a loss shock, federal debt is viewed as default-risk free, and thus the federal government would not find its cost of capital increasing significantly following a catastrophe. Thus, the federal government's superior financing and time-diversification capabilities would permit the federal XOL contract program to bypass the imperfections in the insurance, reinsurance, and capital markets that impede the private provision of disaster insurance.

The proposed XOL contracts would help solve the problems in insurance markets while potentially reducing the federal government's role in providing disaster-relief payments to propertyowners following a catastrophe. The contracts do not provide a subsidy to insurers but instead are designed to be self-supporting in expected value; that is, the contracts are to be priced so that the expected cost to the government is zero. If a loss occurred that exceeded the amount of premiums that had been paid into the program, the federal government would use its borrowing power to raise funds to pay the losses. During periods when the accumulated premiums paid into the program exceeded the losses that had been paid, the buyers of the contracts implicitly would be lending money to the Treasury, reducing the costs of government debt. The expected interest on these "loans" offsets the expected financing (borrowing) costs of the program as long as the contracts are priced at the expected value of loss plus program administrative expenses.

A risk premium could be added to the price of the contracts to provide an incentive for private market crowding out of the federal program. This could imply that the expected return to the government from the program would be positive rather than zero, but this would not necessarily be the case if the risk premium compensates the government for parameter-estimation risk or unforeseen program risk.

A methodology was developed for calculating premiums for the XOL contracts. The first step is to estimate the expected value of loss. This involves fitting severity distributions to catastrophe losses. We estimated loss-severity distributions on the basis of two samples—the historical data on hurricane and earthquake losses maintained by Property Claims Services (PCS) and a sample of simulated loss based on engineering analysis provided by Risk Management Solutions (RMS). Four severity-of-loss distributions were used, the lognormal, the Pareto, the Burr 12, and the generalized beta of type 2 (GB2) distributions. The Burr 12 and GB2 distributions generally provided the best fit to the data. Using our severity distributions, we estimated the expected-loss component of the government's reservation price for proposed XOL contracts covering the entire United States, California, Florida, and the Southeast. The reservation prices were computed using historical frequency data and using the frequency projections developed by RMS. The RMS frequency estimates are considerably higher than the historical averages. Thus, we suggest that the reservation price should be set using the range of PCS and RMS price projections based on the best-fitting Burr 12 and GB2 distributions as the expected-loss component of the reservation price as a guide for policymakers. The expected loss should be loaded for administrative expenses and discounted to obtain the final reservation price. The ultimate price for the contracts would be determined by auction, but the contracts should not be issued for less than the reservation price.

Future research is needed to explore the full implementation of the option model. In addition, the premium estimates could be improved by obtaining more comprehensive estimates of the value of property by state (e.g., to include rental, commercial, and industrial property) and physical measures of the severity of catastrophes. The incorporation of physical projections of the predicted frequency of major catastrophes would also improve the estimates.

Appendix A

Derivation of $p_k(k;\ L > T)$ When $p(N)$ Is Negative Binomial

Recall that the unconditional distribution of the number of catastrophes that breach the trigger T is

$$\text{(A1)}\qquad p_k(k;\ L > T) = p_k(k) = \sum_{N=k}^{\infty} p(N)p_k(k;\ L > T|N).$$

Assume that $p(N)$ is negative binomial, that is, that

$$\text{(A2)}\qquad p(N) = \frac{\Gamma(\alpha + N)}{\Gamma(\alpha)N!}\rho^{\alpha}(1 - \rho)^{N}.$$

Given N events and $P_> = S(L > T)$, $p_k(k; L > T|N)$ is binomial:

$$\text{(A3)}\qquad p_k(k;\ L > T|N) = \frac{N!}{(N - k)!k!}P_>^k(1 - P_>)^{N-k}.$$

Substituting (A2) and (A3) into (A1) and collecting terms yields

$$p_k(k;\ L > T) = p_k(k)$$
$$= \sum_{N=k}^{\infty} \frac{\Gamma(\alpha + N)}{(N - k)!k!\Gamma(\alpha)}P_>^k(1 - P_>)^{N-k}\rho^{\alpha}(1 - \rho)^{N}.$$

Changing the index of summation from N to $h = N - k$, setting $\rho' = (1 - \rho)$, and moving terms that do not involve h outside the summation sign, we obtain

$$\text{(A4)}\qquad p_k(k) = \frac{(\rho' P_>)^k\rho^{\alpha}\Gamma(\alpha + k)}{k!\Gamma(\alpha)(1 - P_<\rho')^{\alpha+k}}\sum_{h=0}^{\infty}\frac{\Gamma(\alpha + h + k)}{h!\Gamma(\alpha + k)}(P_<\rho')^h(1 - P_<\rho')^{\alpha+k}.$$

The expression to the right of the summation is a negative binomial distribution, so the summation equals one. Rearranging the expression to the left of the summation yields

$$p_k(k) = \frac{\Gamma(\alpha + k)}{k!\Gamma(\alpha)}\left(\frac{\rho' P_>}{1 - P_< \rho'}\right)^k \left(\frac{\rho}{1 - P_< \rho'}\right)^\alpha . \tag{A5}$$

This is a negative binomial distribution with parameters α and $\beta = \frac{\rho' P_>}{1 - P_< \rho'}$.

Appendix B

Table 3B.1 **Expected RMS Loss Severities for Various Layers for California, Total Expected Loss for $25–$50 Billion Layer**

	Empirical	Lognormal	Pareto	Burr 12	GB2
Losses simulated by RMS					
$E(L)$	682.41M	351.29M	Undefined	Undefined	Undefined
SD(L)	3,889.86M	2,796.35M	Undefined	Undefined	Undefined
$E(L;$ $12.04M, $5B, $12.04M)	322.04M	255.66M	367.81M	288.45M	299.39M
$E(L;$ $12.04M, $10B, $12.04M)	423.61M	285.60M	502.45M	373.19M	398.19M
$E(L;$ $12.04M, $15B, $12.04M)	492.21M	299.42M	602.25M	432.21M	469.34M
$E(L;$ $12.04M, $20B, $12.04M)	536.57M	307.57M	684.55M	478.98M	526.95M
$E(L;$ $12.04M, $25B, $12.04M)	570.97M	313.01M	755.88M	518.34M	576.22M
$E(L;$ $12.04M, $30B, $12.04M)	599.37M	316.90M	819.53M	552.66M	619.71M
$E(L;$ $12.04M, $35B, $12.04M)	619.95M	319.83M	877.43M	583.28M	658.93M
$E(L;$ $12.04M, $40B, $12.04M)	639.95M	322.12M	930.83M	611.05M	694.83M
$E(L;$ $12.04M, $45B, $12.04M)	655.58M	323.96M	980.58M	636.56M	728.05M
$E(L;$ $12.04M, $50B, $12.04M)	670.58M	325.46M	1,027.30M	660.20M	759.06M
$E(L;$ $25B, $50B, $12.04M)	99.61M	12.46M	271.42M	141.86M	182.84M
Prob[$L >$ $25|event occurs] = $P_>$ (%)	.60	.09	1.34	.73	.92
Prob [$L >$ $25] = p^* (Poisson param. = 3.6)	.0213	.0032	.0470	.0259	.0325
$E(L;$ $25B, $50B, $12.04M| $L >$ $25B)	16,601.38M	13,739.22M	20,229.10M	19,402.25M	19,844.63M
Total $E(L)$: $25–$50B layer	353.48M	44.61M	950.54M	502.23M	645.12M

Note: $E(L; T, C, d)$ = expected value of loss severity (L) for a shifted distribution beginning at d for a reinsurance contract beginning at point of attachment T and having upper limit C. M = million; B = billion. The total $E(L)$ is based on the Poisson frequency distribution with mean of 3.6. Figures are given in dollar values unless otherwise specified.

Table 3B.2 **Expected RMS Loss Severities for Various Layers for the Southeastern United States, Total Expected Loss for $25–$50 Billion Layer**

	Empirical	Lognormal	Pareto	Burr 12	GB2
Losses simulated by RMS					
$E(L)$	975.63M	2,408.49M	Undefined	Undefined	3,032.44M
$SD(L)$	4,433.98M	27,495.06M	Undefined	Undefined	Undefined
$E(L;$ \$12.04M, \$5B, \$12.04M$)$	900.14M	901.68M	1,047.91M	845.30M	861.95M
$E(L;$ \$12.04M, \$10B, \$12.04M$)$	1,156.90M	1,172.45M	1,678.27M	1,089.65M	1,096.86M
$E(L;$ \$12.04M, \$15B, \$12.04M$)$	1,302.95M	1,334.42M	2,209.87M	1,244.13M	1,236.16M
$E(L;$ \$12.04M, \$20B, \$12.04M$)$	1,390.10M	1,447.72M	2,686.02M	1,358.52M	1,334.13M
$E(L;$ \$12.04M, \$25B, \$12.04M$)$	1,452.58M	1,533.46M	3,124.78M	1,449.86M	1,409.07M
$E(L;$ \$12.04M, \$30B, \$12.04M$)$	1,502.66M	1,601.58M	3,535.86M	1,526.15M	1,469.39M
$E(L;$ \$12.04M, \$35B, \$12.04M$)$	1,538.50M	1,657.53M	3,925.29M	1,591.80M	1,519.61M
$E(L;$ \$12.04M, \$40B, \$12.04M$)$	1,568.99M	1,704.61M	4,297.07M	1,649.49M	1,562.49M
$E(L;$ \$12.04M, \$45B, \$12.04M$)$	1,593.99M	1,744.97M	4,654.10M	1,701.02M	1,599.78M
$E(L;$ \$12.04M, \$50B, \$12.04M$)$	1,618.99M	1,780.10M	4,998.50M	1,747.62M	1,632.70M
$E(L;$ \$25B, \$50B, \$12.04M$)$	166.42M	246.64M	1,873.72M	297.76M	223.63M
Prob[$L >$ \$25|event occurs] = $P_>$ (%)	1.20	1.52	8.47	1.66	1.33
Prob [$L >$ \$25] = p^* (Poisson param. = 1.35)	.0161	.0203	.1083	.0222	.0179
$E(L;$ \$25B, \$50B, \$12.04M| $L >$ \$25B$)$	13,867.94M	16,276.46M	22,111.64M	17,963.53M	16,776.73M
Total $E(L)$: \$25–\$50B layer	223.34M	330.30M	2,395.22M	398.38M	299.86M

Note: $E(L; T, C, d)$ = expected value of loss severity (L) for a shifted distribution beginning at d for a reinsurance contract beginning at point of attachment T and having upper limit C. M = million; B = billion. The total $E(L)$ is based on the Poisson frequency distribution with mean of 1.35. Figures are given in dollar values unless otherwise specified.

Table 3B.3 **Expected RMS Loss Severities for Various Layers for Florida, Total Expected Loss for \$25–\$50 Billion Layer**

	Empirical	Lognormal	Pareto	Burr 12	GB2
Losses simulated by RMS					
$E(L)$	987.67M	3,675.52M	Undefined	132,471,003.80M	2,557.73M
SD(L)	4,433.98M	46,828.08M	Undefined	Undefined	234,918,436.58M
$E(L;$ \$12.04M, \$5B, \$12.04M)	1,135.33M	1,099.92M	1,189.12M	1,066.08M	1,104.32M
$E(L;$ \$12.04M, \$10B, \$12.04M)	1,509.92M	1,480.25M	1,940.19M	1,397.98M	1,445.22M
$E(L;$ \$12.04M, \$15B, \$12.04M)	1,704.34M	1,718.89M	2,582.86M	1,604.34M	1,642.39M
$E(L;$ \$12.04M, \$20B, \$12.04M)	1,832.49M	1,891.23M	3,163.90M	1,754.58M	1,775.82M
$E(L;$ \$12.04M, \$25B, \$12.04M)	1,929.21M	2,024.88M	3,703.04M	1,872.77M	1,873.74M
$E(L;$ \$12.04M, \$30B, \$12.04M)	2,002.29M	2,133.18M	4,210.97M	1,970.17M	1,949.35M
$E(L;$ \$12.04M, \$35B, \$12.04M)	2,060.75M	2,223.63M	4,694.35M	2,052.98M	2,009.83M
$E(L;$ \$12.04M, \$40B, \$12.04M)	2,103.77M	2,300.85M	5,157.65M	2,124.98M	2,059.48M
$E(L;$ \$12.04M, \$45B, \$12.04M)	2,143.77M	2,367.91M	5,604.09M	2,188.66M	2,101.07M
$E(L;$ \$12.04M, \$50B, \$12.04M)	2,183.77M	2,426.94M	6,036.06M	2,245.72M	2,136.47M
$E(L;$ \$25B, \$50B, \$12.04M)	254.57M	402.06M	2,333.02M	372.95M	262.73M
Prob[$L >$ \$25|event occurs] = $P_>$ (%)	1.70	2.39	10.44	2.13	1.70
Prob [$L >$ \$25] = p^* (Poisson param. = .83)	0.0141	0.0197	0.0833	0.0176	0.0141
$E(L;$ \$25B, \$50B, \$12.04M| $L >$ \$25B)	14,974.47M	16,839.55M	22,339.15M	17,514.89M	15,414.08M
Total $E(L)$: \$25–\$50B Layer	210.56M	331.61M	1,861.27M	307.93M	217.31M

Note: $E(L;\ T,\ C,\ d)$ = expected value of loss severity (L) for a shifted distribution beginning at d for a reinsurance contract beginning at point of attachment T and having upper limit C. M = million; B = billion. The total $E(L)$ is based on the Poisson frequency distribution with mean of 0.83. Figures are given in dollar values unless otherwise specified.

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Comment Sanjiv Ranjan Das

Catastrophic losses ensue from large acts of God, such as earthquakes, hurricanes, etc. The insurance industry uses the term *cats* to describe these events and the contracts underwritten for these events. The paper by Cummins, Lewis, and Phillips is a paper on what I will denote *bigcats*. Bigcats are single events that result in losses exceeding $25 billion. The objective of this paper is to price one-year reinsurance contracts on bigcats. These are underwritings on single events on loss magnitudes that we have not as yet experienced. The motivation is simple: losses of this size will eventually occur, and, without a good mechanism to handle them, the reinsurance industry as well as consumers of insurance will suffer severe economic crises.

Cummins, Lewis, and Phillips offer the following in the paper: (i) a proposal that the government write these bigcat insurance covers and (ii) a mathematical exposition of what these contracts will cost. By examining the past distribution of large losses, they develop a methodology to price these contracts and then provide indicative prices. Using, for example, the Burr 12 distribution, they arrive at a severity-of-loss estimate of about $17 billion on average, and multiplying this by a 2 percent probability of occurrence results in an expected loss of about $350 million.

My review of the paper falls into four categories: (1) the need for these contracts; (2) an examination of the proposed mechanics of these contracts; (3) an examination of the pricing method; and (4) a proposal for binary contracts.

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Necessity of the Bigcat Contract

Is the bigcat contract much ado about nothing? Should the government expend costly resources on this proposal? It may be akin to the development of the superheavyweight category in world boxing tourneys when people were satisfied with the heavyweight version. A little probability work shows, however, that the proposal is not only justified but timely as well.

Consider some of the data in the paper. The expected number of cats per year is $\lambda = 2.2$ (Poisson arrival rate parameter λ). A simple back-of-the-envelope calculation from the data in table 3.3 of the paper will show that the probability of the loss exceeding $25 billion is 0.005. Thus, the probability of occurrence of a bigcat is on the order of 1 percent per year. On average, we will see one bigcat every one hundred years. This does not sound like a cause for concern. However, this is not the correct way to look at this question. What we need to examine is the probability of at least one occurrence in one hundred years, not the average number of occurrences:

N	Poisson Probability
0	.1108
1	.3679
2	.1839
3	.0613

This turns out to be 89 percent! Hence, it is clearly a matter requiring attention. Similar analysis for twenty-five years gives a probability of at least one bigcat as 23 percent, and for ten years the probability is 10 percent. It is clear that, if the numbers hold, we will see a bigcat in the near future.

The issue is, can we live with it? In assessing this matter, we must consider whether allowing a certain number of reinsurance firms to go down will have a detrimental effect on the insurance industry. On balance, it probably will. On another tack, discussion with industry representatives suggests that the pricing at lower levels of coverage seems to be affected by the difficulty of reinsuring bigcat risk. The paper argues in addition that the government is better placed to provide the time diversification needed to hedge these contracts and that the provision of this contract will provide better assessment of this risk as well as formalizing a system where the participants pay for the coverage rather than relying on endgame government bailouts.

Eventually, cross-sectional diversification will overtake time diversification as the means to manage this risk. Cross-sectional pooling of risk also offers a better market in that there will likely exist larger numbers of buyers and sellers of the risk (i.e., a two-way market), as opposed to the current proposal, which envisages one seller and many buyers. In sum, there is a clear need for this type of contract, as a starter to a full-fledged market in bigcat risk.

Contract Mechanics

The toughest issues to be dealt with in any proposal arise from the implementation mechanics. This proposal is no exception. There are several issues that need attention.

1. The first is delayed settlement: the contract is written on losses aggregated over eighteen months after the date of the event. Given this, buyers of the contract must wait to obtain the proceeds from the government. With what is almost certainly a badly eroded capital base, the reinsurance firms may not be able to wait that long. Of course, they may securitize their expected claim against the government, but this will mean taking a discount on the value of the claim, which may be quite large, given both the "distress" nature of the sale and the uncertainty of the true value of the claim.

2. The contract with one seller who sets a reservation price may actually distort the fair value of the insurance in this market.

3. The third issue is auction design: here several issues need addressing, especially in the light of the fact that both price and quantity risk are severe. The winner's curse would be large, making the setting of the reservation price critical, to ensure not only that the government achieves a fair reserve but also that the reservation price offers a good signal of value. The likelihood of one firm garnering a disproportionate amount of this cover appears high as well.

4. The fourth issue is the rollover version of the contract: the contract envisages a rollover option that allows the buyer to renew the contract for the rest of the year if within the year a bigcat occurs. The renewal is made for the remaining part of the year at a time-prorated value of the original contract. This rollover design has two flaws: (i) If the analogy of options is used, the time value of the contract is not equally distributed over the life of the option and tends to decay rapidly at times closer to maturity. If this is the case, the rollover would be overpriced. (ii) If the arrivals of cats are not independently and identically distributed but positively autocorrelated, as is surely the case with hurricane risk, then the rollover is underpriced. Hence, a more careful specification of the rollover contract is called for.

One possible suggestion may be to write contracts on the change in the price of insurance within a short period immediately after the cat. We observe that the price of insurance rises when a catastrophic event occurs and is correlated with the size of the event. This happens because the insurance industry needs to raise prices to cover higher costs of capital given capital erosion, and the rise in prices is partly demand driven since people seem to rush out and buy insurance when alarmed by a large catastrophe. This has the advantage of (i) being directly and immediately measurable, (ii) being traded, (iii) avoiding the delayed-settlement problem, all of which would make for a more liquid market. However, this assumes that prices react to cat size with a high degree of correlation (i.e., low-basis risk). It is not clear that this is so, and it calls for further empirical investigation.

Pricing Approaches

The paper makes a difficult evaluation problem appear easy. The approach examines the fit of several statistical distributions to the data on loss severity and finds that the Burr 12 and GB2 distributions provide a good fit. While an options approach may also be used to come up with the insurance value of the contracts, it is hard to justify it, given that risk-neutral valuation does not apply in the absence of an ability to replicate the option with an underlying security/asset.

Pricing these contracts requires three separate analyses: (1) a scheme to develop a distribution of loss severity; (2) estimation of the hazard or event rate; and (3) use of an appropriate cost-of-capital or discount rate to compute the present value of the losses. These three are clearly in descending order of complexity, and the paper rightly focuses on the most complex analysis, developing a satisfactory and pleasing methodology.

If one has to raise a red flag, it is about what is often called the "Star Trek" problem. Since we have never seen a bigcat as yet, we are employing statistical analysis to make forecasts about a zone where we have never been before! The degree of confidence in this exercise must perforce be weak.

Finally, one suggestion about the modeling: we do know that the standard deviation of losses tends to increase as we go into higher and higher layers of risk; that is, volatility of loss severity $\sigma(T)$ is increasing in the trigger level (T), or $\partial\sigma(T)/\partial T > 0$. Use of this fact may bring more structure to the modeling method and sharpen the confidence levels in the pricing results.

Binary Contracts

In some cases, loss occurrence is easier to forecast than loss severity. A good example of this is hurricanes, where advances in weather-forecasting technology have made the prediction of hurricane arrivals more facile. By writing binary contracts that pay off a fixed amount only on the occurrence of the bigcat, not on the severity, the writer of the contract will be able to price it more accurately. While this increases basis risk substantially, at the bigcat level, with triggers of $25 billion, the reinsurer is more concerned with credit risk than basis risk. There are several advantages of the binary contract, and, indeed, several participants in the market already trade such instruments. The benefits are the following: (1) The binary contract avoids the "Star Trek" problem. (2) The per contract risk can be quantified by the writer of the contract. (3) The market is forced to forecast severity but now has an available instrument to trade it. (4) It is easier to offer multiple maturities of contracts. (5) Implied hazard rates can be traded, just the way the market for equity options trades implied volatilities. (6) It will be easier to make a market, and demand and supply will therefore set price in a liquid two-way market. It avoids the paradigm of one seller and many buyers. (7) Pure jump-process option-pricing technology may be used. (8) Plenty of physical forecasting expertise now exists for these risks.

Conclusion

The paper makes an important contribution in highlighting the urgent need for bigcat contracts. While there are several mechanics and pricing issues that need sorting out, the authors should be pleased that their work will provide a first benchmark for contracting in this area and set a standard of quality for future work. With the establishment of indexes on cat risk, a new generation of pricing models such as this one will soon be spawned. It is truly an exciting time for modelers in this industry—this paper is the tip of the iceberg.

Comment James A. Tilley

One thing that is becoming clear is that it is easier to be a discussant the later in the conference one speaks because some of the good points one wants to make have already been very well expressed by others. It is too difficult for me to resist making comments about the role of the federal government and the proposed manner of federal government involvement in the catastrophic loss problem, but I will also live up to my assigned task of commenting on the pricing methodology proposed by the authors.

The first key question is, Does the federal government need to be involved at all—can the private sector handle the problem by itself? Many advocates of leaving it to the private sector base their view on the huge capital base of the insurance/reinsurance industry, the ability of the industry to diversify across risks other than property-catastrophe, the possibility of adopting more aggressive investment strategies that have high returns and high volatilities but little if any correlation with the insurance risks underwritten, and the hope that constraints on the state regulation of premium rates for personal lines can be eased or eliminated. Still, I have not seen any thorough, definitive analysis to suggest that the private sector alone can cope with the financial consequence of mega-disasters, either now or in the future.

The second key question is, If the federal government should participate, are the proposed \$25–\$50 billion excess-of-loss (XOL) contracts the best way? Several participants have made good points already. I will repeat them briefly and then add a few points not yet raised.

1. The fundamental issue is whether federal government involvement should be on a basis of *risk transfer* or *risk financing*. An associated issue is

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whether the involvement should be ex ante (risk transfer) or ex post (risk financing). Chris Milton of AIG has already argued in discussion here for the use of "transitory capital" in lieu of permanent capital. I think that that is consistent with a financing view; that is, the federal government should be ready to lend when required.

2. I agree completely with Aaron Stern's comment that, if the proposed federal XOL contracts are used, they should cover losses on an aggregate basis rather than a per occurrence basis. An aggregate basis is consistent with the notion of the federal government serving an industry "backstop" role, whereas an occurrence basis is not.

3. If the federal XOL contracts are utilized, how high should the attachment be? Should it be set at a twenty-five-, fifty-, or one-hundred-year return likelihood, or should the probability of attachment be even more remote? Again, this is a question of private- versus public-sector involvement and underscores the role of the federal government as providing a backstop to the industry.

4. Should there be more than one layer of federal XOL contracts so that even higher caps can be provided, thus enhancing the value of the federal government backup to the insurance industry?

5. Is the basis risk imposed on the purchasers of the call-spread contracts so great as to largely negate the hedging value of the contracts? To what level of geography would the contracts have to be refined to deal satisfactorily with basis risk?

6. One idea that may bear fruit is the *mandatory* purchase of the federal XOL contracts by the industry, assuming that the attachment point were high enough and the price low enough (e.g., the reservation price). Such mandatory purchase could mitigate basis risk greatly because the federal government would in essence be underwriting the entire industry. If the issue of individual insurer attachment points could be dealt with satisfactorily, each insurer could make recovery in proportion to underwritten net losses (UNL) in lieu of a predetermined market share of industry losses. This approach is equivalent to the use of federal XOL contracts in combination with intercompany swaps of industry-based recovery in return for UNL-based recovery.

Let me now turn to my explicit assignment of commenting on the authors' proposed pricing methodology for the federal XOL contracts. A good starting point is the fundamental actuarial formula:

$$\begin{aligned}\text{reinsurance premium} &= \text{“pure premium” to cover expected losses}\\ &\quad + \text{ expense loading } + \text{ risk loading.}\end{aligned}$$

The risk-loading component is usually developed from notions of required capital to support the risks underwritten via the contract and the reinsurer's cost of that capital. The concept *required capital* should account for the spread of risk in the reinsurer's entire portfolio, both cross-sectionally and over time.

The variance of the reinsurer's loss distribution and more particularly, the risk of ruin underpin the calculation of the risk loading.

The problem posed by catastrophic losses is that the ratio of the standard deviation to the mean of the loss distribution applying to an XOL contract increases dramatically as the attachment for the XOL cover increases, actually rendering risk transfer uneconomical at some point. That is, at such a point, there is no price acceptable to both reinsurer and cedent.

The authors point out that the federal government is in a unique position to take a nearly *infinite time horizon* and thus fully exploit the benefit on intertemporal spreading to reduce the risk loading to *zero*. However, the authors seem to advocate building a risk load into the XOL contract pricing in order to avoid "crowding out" the private sector. My question to the authors is, If the federal government backstop of the insurance industry triggers only at a very high level, and if the federal government has a critical competitive advantage due to long-run intertemporal spreading, why would the private sector even consider playing the game? The industry has more than enough to worry about without taking on the megacatastrophic problem as well.

As a final point, I would like to comment on the calibration of the pure-premium component of the pricing formula. The authors' work demonstrates that distribution assumptions matter a great deal, as one expected they would—RMS versus PCS, GB2 versus lognormal, etc., all make a difference. It is difficult to see how the federal government would be able to gauge, even over the very long run, whether the reservation price has been established properly. Moreover, would federal politicians be able to resist the "payback" mentality of the reinsurance industry following a megacatastrophe? Would the prices for federal XOL contracts then be jacked up? The notions of infinite-horizon intertemporal spreading and "here-and-now" political decision realities seem to conflict.

In summary, I think that the product structure and pricing concepts advanced in the authors' paper merit serious attention, but, as always for proposed solutions to vexing issues, early work often raises even more questions—that is its essential value, after all. The authors are to be congratulated for their contributions.

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