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5 Concepts of Quality in Input and Output Price Measures: A Resolution of the User-Value Resource-Cost Debate

Jack E. Triplett

5.1 Introduction

The appropriate treatment of quality change is a very old issue in the analysis of productivity, the measurement of capital, and in many other areas of economic measurement.

Many economists have advocated a "user-value" criterion. Under this concept, a new computer which does more calculations would be taken as a higher quality machine (provided this aspect of performance is valuable to the computer user). Price indexes would be adjusted for the value to the user of the performance difference, regardless of what it cost to produce the new computer. Because the performance difference has been removed from the price measure, it shows up in quantity measures. Despite the wide acceptance of the user-value criterion (based on my own informal poll), to my knowledge no explicit theoretical justification has ever appeared.

The alternative "production-cost" criterion is associated with Edward Denison[1] and accepted in the national accounts. This concept requires that quality differences among various computer models be evaluated using data on the resource cost of building computers, regardless of their relative performance in use.

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One might suppose the conceptual issue to be of small practical importance. In equilibrium the two methods should yield similar numbers; and whether at equilibrium or not, most practical quality adjustment proposals make use of market price information (hedonic methods and traditional "linking" methods share this property). Prices reflect, obviously, both value and cost, rendering a distinction between them inoperative.

Yet counterexamples abound. Griliches (1964) and Jaszi (1964) discussed an example (birth-control pills) for which resource cost and user-value treatment of a technical change gave different measures. More recently, a controversy over the appropriate price index treatment of legally mandated smog control and safety devices again showed that the conceptual treatment of quality change has a perceptible impact on economic measurements, and that resolving the conceptual and theoretical issues has clear practical importance. Such examples provide the motivation for the present paper.

The approach followed combines theoretical specifications that have been developed for input price indexes (among which are closely related theories of the "true cost-of-living index" and the "true input cost index") and for output price indexes (sometimes known as "true output deflators") with previous work of the author (Triplett 1971*b*, 1973, 1976). The latter argues that "quality," in economics, can best be understood by shifting the analysis from goods space to characteristics space, along lines proposed by Lancaster (1971). The results show that the Denison–Bureau of Economic Analysis production-cost criterion is correct if what is wanted is a measure of the output of capital goods (as the numerator, e.g., in a productivity measure for a machinery-producing industry). However, the user-value criterion is correct if one wants to construct an input measure—for example, a measure of capital services for incorporation into a production function.

The plan of the paper provides separate treatments for input and output price indexes. The distinction between the two is made in the first section, along with some discussion that sets the stage. Section 5.3 sets out the input price index case, with output price indexes discussed in Section 5.4. Each of these two central sections is organized along parallel lines—a first subsection which sets out the basic theory of input and output price indexes (these sections could be skipped by readers who are familiar with the technical index number literature), followed by a second section which explains the concept of characteristics (done separately for input and output price indexes because the characteristics concepts differ according to their use). Subsections 5.3.3 and 5.4.3 contain the core of the paper—the statements of input and output price index theory in characteristics space and the demonstration that theory leads to two different treatments of quality change. Section 5.5 concerns two arguments the protagonists of alternative approaches have made against each other's positions; the characteristics-space price index theory developed in this paper shows that both are false, thus illustrating its utility for clearing up many of the murky disputes that have so long dominated the literature on quality change. The final Section 5.6 contains an overall perspective on the paper and its conclusions.

5.2 Some Preliminary Observations

The distinction between input and output price indexes has been present in the price index literature for some time, at least implicitly, but Fisher and Shell (1972) were the first to work out the relationship between them.[2] It will be useful to discuss the setting in which the distinction is important.

Consider a simple two-sector model. Suppose a production function for "gadgets," in which computers, gadget-making machinery, and labor are inputs, and another production function for computers, which uses as inputs computer-making machinery and labor:

$$G = G(C, K_G, L), \tag{1}$$

$$C = C(K_C, L). \tag{2}$$

An output price index for computers is a price index for the output of the computer industry—that is, a price index for the left-hand side of equation (2). One might want an output price index for use in calculating measures of output and productivity for the computer industry, or in the computation of the national accounts.

An output price index for the computer industry presupposes that there are different kinds of computers (otherwise, there would be no aggregation problems in computing the industry's output price, and therefore no need for index numbers). Clearly, the interesting case occurs at the level of aggregation where one must deal with aggregation over products *and* with quality variations within a single "product." The former is a standard index number problem; the latter is the concern of the present study.

When computers figure in an input price index, it is a price index for the inputs to the gadget industry—in other words, a price index for the right-hand side of equation (1) where computers are one of three components in the input-price index. One might also wish to calculate a price index for computers used in the gadget industry. For example, the computer price index might be wanted for purposes of estimating the production function for gadgets. This would be a "subindex" (the term originates with Pollak [1975]) of the full input-price index. Another subindex would involve measures of wages. The theory of subindexes is relevant for many problems that arise in treating quality change.

In principle there are all kinds of economic differences between input and output price measures. The example involves a capital good—as does much of the quality change literature. The production function use really concerns the flow of capital services; an output measure is, of course, a flow of capital *goods.*[3] However, the distinction between a durable good and the flow of services is not central to the quality measurement issues the present paper addresses. And either case—output of goods or input of services—will probably be measured either directly or indirectly by deflating a value aggregate by a price index.[4]

Thus, in the actual computation of either an output measure or an input measure of capital goods, the point at issue comes down to the choice of the appropriate price index to deflate value data. This is, in fact, the issue on which the quality measurement debate has focused.

One point deserves emphasis. One frequently encounters in the literature some functional notation for the "production" of, say, computer services—that is, a measure of computer services is written as the "output" of a process in which the stock of computers, or the characteristics of the stock of computers, is taken as the "input." Whatever value such notation has for some purposes, this is not the meaning of inputs and outputs as used in the present study.

As a matter of practical computation, input and output price indexes for a commodity, industry, or sector may involve numerous other distinctions. Even at relatively detailed levels, a price index is still an aggregation, and even if the prices were all measured in the same way, the weights for input and output price indexes for a similarly named commodity would make them different measurements. A price index for the output of steel mills has weights that differ from those of an input price index for steel used in the auto industry. This paper's concern for input and output price measures is, however, limited to theoretical implications and to the quality measurement question. We pursue no other issues (see Early 1978).

5.3 A Theory of Quality Adjustment for Input Price Indexes

An input price index has many applications. For a firm or industry using a "KLEM" production process (capital, labor, energy, and materials), an input price index measures the price change of these four productive factors. The Bureau of Labor Statistics (BLS) Employment Cost Index is a form of input price index, in this case an index for only the labor portion of total inputs (in Pollak's [1975] term, it is a "sub-index" of the full input price index). Another example is the Consumer Price Index (CPI), which is an input price index for consumption.

Very little has been published on input price indexes for production. However, the theory can be developed by analogy to the theory of the cost-of-living index, on which the literature is voluminous (see Pollak 1971; Fisher and Shell 1972; Samuelson and Swamy 1974).

Section 5.3.1 states the theory of production input price indexes, drawing on the cost-of-living index literature. The standard theory applies to goods (or services), on the implicit assumption that they are homogeneous and the quality problem does not exist. In Section 5.3.2 quality change is defined as variation in the quantities of characteristics embodied in heterogeneous goods. Section 5.3.3 combines the first two sections, extending the theory of input price indexes from goods space into characteristics space; this extension proves the result that quality change in an input index must be handled by a measure of user value.

Readers who are familiar with theoretical index number literature may wish to turn directly to Section 5.3.2.

5.3.1 Input Price Indexes for Goods

The consumption price index literature distinguishes a cost-of-living index (sometimes termed a "true cost-of-living index") from the conventional fixed-weight price index formulas normally used by statistical agencies (the Laspeyres or Paasche formulas). We follow this practice, and define an "input cost index"[5] as a measure which answers the question: What is the cost change, between two periods, of collections of inputs sufficient to produce some specified output level? In the following, we speak of this as the "theoretical" or the "economic" input price index, in distinction to fixed-weight formulas.

The rationale for the input cost index definition is analogous to the one for the cost-of-living index. We begin with a production function,

$$Z = f(X_1, \ldots, X_j, \ldots, X_n), \tag{3}$$

where the X's are identified as quantities of market-purchased inputs (i.e., as goods). We ignore possible complications by assuming there is only a single output, or if there are multiple outputs that they may be aggregated into a suitable scalar measure. The theory does not require profit maximization (or any particular market structure on the output side) but does assume that the firm minimizes production cost.

Let Z^* be some output level relevant to the comparison, and designate the reference period as time 0. There is a cost-minimizing set of inputs $(X^*_{10}, \ldots, X^*_{j0}, \ldots, X^*_{n0})$, or, more compactly, $[X^*_0]$, for the reference period's set of input prices $(P_{10}, \ldots, P_{j0}, \ldots, P_{n0})$, which we write alternatively as $[P_0]$. The cost, C^*_0, of acquiring the optimal set of inputs can be determined from the cost function that is dual to the production function, f, but can also simply be added up:

$$C^*_0 = \sum_{i=1}^{n} X^*_{i0} P_{i0}.$$

We now consider some comparison period, t, with input prices $(P_{1t}, \ldots, P_{jt}, \ldots, P_{nt})$. For the same output level, Z^*, the production problem may again be solved for the cost-minimizing set of inputs, $[X_t^*]$. Wherever substitution is possible among factors, $[X_0^*]$ is not the same as $[X_t^*]$, unless relative factor prices are the same in $[P_0]$ and $[P_t]$. The cost of the set of inputs which minimizes the cost of producing Z^* in period t is C_t^*, and the input cost index is:

(4) $$I = \frac{C_t^*}{C_0^*} = \frac{\Sigma X_{it}^* P_{it}}{\Sigma X_{i0}^* P_{i0}}.$$

In words, the index is the ratio of the minimum cost of acquiring inputs sufficient for producing output Z^* in the comparison period (t) to the minimum cost of producing the same output in the reference period (0).

The input cost index can be thought of as pricing two sets of inputs which lie on the same production isoquant, with each set corresponding to the cost-minimizing point for one of the two periods. The isoquant for which the index comparison is made (output level Z^*) may correspond to the actual output of the reference period (Z_0 in fig. 5.1), or of the comparison period (Z_t), or to some other output.[6]

Consider the input cost index with Z^* defined to be the actual output of the reference period (Z_0). Designate this form of input cost index as I_L, or as the "Laspeyres-perspective" input cost index (because it takes the perspective of the initial period as the basis for the comparison). For this input cost index, point A of figure 5.1 provides the denominator and point B the numerator of equation (4).

For actual computation, $[X_0^*]$, or point A's inputs, are obtainable from survey data, Census of Manufactures, and other sources, and C_0^*, the denominator of equation (4), is reference period actual costs. Determining $[X_t^*]$, and therefore C_t^*, however, requires knowledge of the production function—point B in figure 5.1 is not observed directly but must be estimated.

For this reason, one normally approximates the theoretical input cost index I_L by some traditional fixed-weight formula, such as the Laspeyres index (dropping the i subscripts):

(5) $$L = \frac{\Sigma\, P_t\, X_0^*}{\Sigma\, P_0\, X_0^*}.$$

The only difference between L and I_L (i.e., between eqq. [4] and [5]) lies in the quantities in the numerator—($[X_t^*]$ in I_L, as opposed to $[X_0^*]$ in the L index). Note that neither index I_L nor L uses the *actual* inputs purchased in period t (in figure 5.1, period t's actual inputs are those corresponding to point C). The reason, of course, is that the index computation requires Z^* to be held constant in order to obtain a price measure; for various reasons, actual output and therefore actual inputs may change.

It is well known that the Laspeyres fixed-weight index provides an upper bound on the Laspeyres-perspective input cost index (the theoretical input cost index based on the reference period Z_0). This also implies that the Laspeyres index is an upward-biased measure of the true index, with the extent of the bias depending on the amount of substitution that takes place in response to changes in relative factor prices.[7] The mathematical proof of this proposition is identical to that provided by Pollak (1971) for the consumption case and need not be repeated here. However, for heuristic reasons, it is worth relating the result to the diagram of figure 5.1.

We have already noted that the Laspeyres-perspective input cost index of equation (4) compares the cost of point A's inputs at $[P_0]$ prices with point B's inputs at $[P_t]$ prices. Moreover, the actual cost (in prices $[P_0]$) of point A's inputs provides the denominator of both the input cost index I_L, and of L, its fixed-weight approximation.

Using the Laspeyres formula to obtain a measure of the change in the cost of inputs involves pricing point A at the new set of relative prices $[P_t]$, giving $\Sigma\, P_t X_0$, the numerator of L. The dashed line passing through point A shows combinations of inputs that could be acquired for the same cost as $\Sigma\, P_t X_0$ (i.e., the same cost in period t as input bundle A evaluated at period t's prices). However, Z_0 could be produced more cheaply in period t by substituting from point A to point B, an input substitution which would realize a cost saving shown by the distance between the parallel lines passing through A and through B. This cost saving is equal to the "substitution bias" in the fixed-weight Laspeyres index.

An alternative fixed-weight price index frequently employed is Paasche's formula, which is based on reference-period output level $(Z^* = Z_t)$ and input quantities:

$$(6) \qquad P = \frac{\Sigma\, P_t X_t^*}{\Sigma\, P_0 X_t^*}.$$

The national accounts imply this price index formula.

Corresponding to the Paasche fixed-weight index is an input cost index which uses the isoquant and input quantities of period t (the comparison period) rather than of period 0. We designate this input cost index as I_P, or the "Paasche-perspective input cost index." It is computed by comparing points C and D from figure 5.1. That is, I_P can be computed from equation (4) by specifying that Z^*—and therefore $[X_t^*]$ and $[X_0^*]$—be defined in terms of the isoquant which actually obtains in period t.

Thus, there are two economic indexes of input costs. They have orientations analogous to the differing orientations of Laspeyres and Paasche fixed-weight indexes: The I_L input cost index (in common with the Laspeyres formula) computes the change between periods 0 and t

from the perspective of the actual situation in period 0. The I_P index, on the other hand, derives from the perspective of the actual situation in period t, which likewise forms the basis for the weights for the Paasche fixed-weight formula.[8]

Except for special cases, these two input cost indexes will have different values, since they look at the change from different perspectives. This is the "index number problem" in its purest form.

A standard index number result is that the fixed-weight Paasche index *understates* the "Paasche-perspective" input cost index. In other words, in the Paasche index case the substitution bias in index P causes it to understate the true cost change, as measured by I_P. Index P provides an approximation to—and lower bound on—the true index, I_P. This relation is analogous to the upper-bound property of the L-index, relative to the I_L input cost index.

The relationships among the two true, or economic, indexes and their two fixed-weight bounds invalidate the widespread notion that the economic index must lie between values computed from Paasche and Laspeyres formulas. Liviatan and Patinkin (1961) give a particularly clear statement of the correct relation. The same relationships imply as well that the full difference between the fixed-weight Paasche and Laspeyres indexes cannot be taken as a measure of substitution bias or biases in fixed-weight indexes—some of the difference between L and P may be accounted for by the difference between I_L and I_P.

Comment. The approach to input cost indexes followed in this section amounts to a direct analogy to the cost-of-living index concept. Hence, the economic theory on which it is erected consists solely of the firm's cost-minimizing behavior, and ignores the fact that changes in input prices may cause the competitive firm to alter its output level as well as its factor proportions. Whether output effects should or should not be ignored in constructing an economic measurement depends, it seems to me, solely on the nature of the questions the economic measurement is supposed to answer. It does not necessarily depend, or certainly does not solely depend, on the nature of the maximization problem that the firm faces. These issues are too complex to be explored here; but comment is required because alternative approaches have been followed (see n. 18, and "Comment," p. 292). Note also that the theory is static and therefore abstracts from technical change. Fisher and Shell (1972) explore index number theory in the presence of taste and technical change.

Summary and Conclusions to Section 5.3.1

The section has discussed two input cost indexes and two associated fixed-weight price index formulas. It is conventional to think of these as the (two) theoretical indexes on the one hand and two practical indexes on the other hand. The content of the conventional theory of index numbers distinguishes a crucial property of these two sets of index numbers: it emphasizes that the input cost indexes take account of, as the fixed-weight indexes cannot, the effect of factor substitution in response to relative price change.

The purposes of the present paper, however, require emphasis on another property of input cost indexes: for either input cost index, I_L or I_P, output must be held constant in order to obtain the measure of price change. This is very important for the remainder of this paper. In the following, we refer to this property as the constant-output *criterion* for an input price measure.

The notion that the theoretical input price measure requires a constant-output criterion suggests an alternative interpretation of the role of fixed weights in conventional Paasche and Laspeyres index numbers. Textbooks usually explain that the quantity weights are held constant in a fixed-weight price index in order to decompose a value aggregate into price and quantity terms. The economic reason for fixed weights is quite different. Holding the quantities of inputs fixed is one way of holding output fixed, and it is output constancy that is required for an input price measurement. The bias introduced into fixed-weight indexes by substitution stems from the fact that they only hold output approximately fixed; the deviation from the constant-output criterion that both Paasche and Laspeyres indexes permit is the substitution bias of the fixed-weight index.

Thus, *constant output is the guiding rule for constructing any input price measure*, whether one of the true economic indexes, or one of the approximations. The concept of the index "criterion" plays a major role in this paper. We return to it in Section 5.3.3.

5.3.2 Modeling Quality Change: Input Characteristics

Conventional index number theory implicitly assumes that quality variation does not exist—it applies to a world of homogeneous goods. I incorporate quality change into the theory of index numbers by making use of the following proposition: when we use the term "quality" in economics, we are really making a kind of shorthand reference to the *quantities* in a vector of "characteristics." Under this way of looking at it, "quality change" is intrinsically quantifiable. It can be measured or evaluated in terms of quantities of characteristics—units that resemble in essential ways the goods whose quantities enter into conventional economic measurement.

Over the past decade or so, the term "characteristics" has been used in economics in a variety of ways. Moreover, in the present paper, we distinguish "input characteristics" from "output characteristics." For these reasons, some definitions and assumptions are required:

a) "Characteristics" are properties or attributes of goods. A house, for example, has characteristics such as floor space, number of bathrooms, or whatever is relevant. A machine's characteristics may include lifting or hauling capacities, cutting speeds, core size, and so forth. Labor services may also be described in terms of characteristics: standard human capital analysis distinguishes education and experience as productive characteristics of labor, and strength, dexterity, and other elements are frequently cited in the labor literature.

b) As the examples imply, characteristics are defined to be a lower level of aggregation than goods. That is, goods are aggregates of characteristics, not the other way around.

c) We assume that quantities of characteristics are the true inputs into the production function, and not quantities of goods or any other quantity obtained by reaggregating characteristics in some manner. This is primarily a simplifying assumption, intended to rule out a number of alternatives that have appeared in the literature but which are not especially relevant for present purposes and would greatly complicate the exposition of the basic theory. For example, much of the empirical literature is written as if labor, once disaggregated into human capital elements, must be reaggregated into some "labor aggregate," with the labor-aggregator function entering the production function. In this paper, we assume that labor characteristics enter the production function directly, without the necessity for any intervening "aggregator function." Another example concerns services of capital goods. For some purposes, investigators have combined a truck, let us say, with its associated labor and fuel inputs to produce a measure of "trucking services." That is, they have assumed a "production function for trucking services" (with trucks, labor, and fuel as inputs), and used the output of this secondary "production function" as the input into the primary production function. Parallels with the household production literature on the consumption side are obvious.

In the present paper, we dispense with all intervening aggregator functions. The basic reason is that these subproduction aggregator notions, even where relevant, unduly obscure and confuse the exposition, without adding much of consequence (though other reasons could also be cited).[9]

d) *Input* characteristics are defined by the following process. For a multifactor production function having at least one factor which is nonhomogeneous, consider substituting one example of this nonhomogeneous factor for another example of the same factor. That is, if the factor is labor, substitute Ms. B for Mr. A; if it is a machine, substitute the XYZ Co's model 200 for the ABC Co's model 55.

The substitute differs in quality from the original if there is a change in output when the substitution occurs, or if the substitution affects other factor usage in such a way that there would be a change in the value of the dual cost function if the substitute and the original were available at the same price. Another way of putting it is to say that quality variation in an input exists if substitution of different varieties or examples of this input creates variations in output or cost that are not explained by the factors included in the production or cost function. *A quantity is an input characteristic if it reduces that unexplained variation.* Years of education is a labor input characteristic if its use in the production or cost function accounts for all or part of the unexplained variation associated with the substitution of Ms. B for Mr. A; cutting speed is a machine input characteristic if its use in the production or cost function accounts for all or part of the unexplained variation associated with the substitution of the XYZ 200 for the ABC 55.

This definition of an input characteristic amounts to saying that something is an input only if it makes a contribution to production. A formal definition is warranted largely because the quality change literature has been filled with various taxonomies, and in order to make more clear the distinction between input characteristics and output characteristics (see Section 5.4.2).

e) We assume that only the quantities of characteristics matter, and not how they are embodied in goods. This means that a two-ton truck is assumed to provide equivalent hauling capacity to two one-ton trucks. This has sometimes been referred to as "linear" characteristics; "additive characteristics" is a more suitable term. The assumption is a very restrictive one.

The assumption is made primarily for the sake of simplicity. Obviously, even if two tons of hauling capacity makes an equivalent contribution to output whether in one vehicle or two, the number of vehicles used may imply variation in the labor input (one driver instead of two). But this is a complication to the basic characteristics theory rather than (as has sometimes been thought) an objection. Elsewhere (Triplett 1973) I have referred to it as the "package-size" problem, as it arises also in the consumption case. A recent paper by Trajtenberg (1979) has reopened this issue.

f) To avoid misinterpretation, a few disavowals must be listed. First, nothing in the present paper implies any linearity assumptions on the production function or on any other function or relationship. One could write a lengthy treatise attempting to straighten out what critics of different parts of the characteristics literature have put under the "linearity" rubric. This is not the place to deal with these matters; I have discussed some of them elsewhere (1980).

Second, nothing in the paper has anything directly to do with the form of hedonic functions or with their interpretation. In particular, the distinction between input characteristics and output characteristics has nothing whatever to do with the function Ohta-Griliches (1976) posited to exist between what they described as "engineering characteristics," such as "memory size", on one hand, and "performance characteristics," such as "computational capacity," on the other. Any engineering relation between "engineering" and "performance" characteristics, if it exists, is not one that is relevant for the theory of price indexes. The distinction arose from alternative empirical specifications for the variables in hedonic functions. This paper is not concerned with the empiricist tradition of hedonic studies.

Neither does the paper relate to the common technique of forming "hedonic price indexes" from the ratio of two period's regressions, as is so often done. One *might* use hedonic functions to help identify the characteristics that are wanted for input and output price indexes and, perhaps, to measure their costs as well (I say "perhaps" because of the problems of applying and using hedonic results, problems that will not be explored here [see Triplett 1971*a*; and Pollak 1979]). But how this has been done in the past, or how it should be done, is outside the scope of this paper.

5.3.3 Quality Change and Input Price Indexes for Characteristics

This section combines results of Sections 5.3.1 and 5.3.2 in order to examine the treatment of quality change in input price indexes. The method involves translating the "goods-space" index number theory of Section 5.3.1 into the notion of "characteristics space" developed in Section 5.3.2.[10]

Four results from Section 5.3.1 are required for this section:

1. An input cost index prices a collection of inputs in two periods that represent minimum-cost points on the same production isoquant.

2. For any comparison, a number of different input cost indexes can be computed, depending on which isoquant is used for the comparison; indexes based on isoquants appropriate to reference and comparison periods were designated, respectively, as "Laspeyres-perspective" (I_L) and "Paasche-perspective" (I_P) input cost indexes.

3. The usual fixed-weight Laspeyres and Paasche index formulas may be viewed as approximations to the true economic indexes, I_L and I_P. The Laspeyres index provides an upper bound to I_L, whereas the Paasche index is a lower bound to I_P.

4. Whether the economic index or the fixed-weight index is used, computing an input cost index requires that output be held constant over the span of the comparison; this condition was referred to as the "constant-output criterion" for the input cost index.

Carrying through the input cost index number analysis in characteristics space requires that characteristics—not the units in which market transactions are carried out—enter the production function. For certain advantages in the exposition (and no loss in generality), assume that only one input (the jth input) in the production function of equation (3) is nonhomogeneous and that this input has m characteristics. For a capital good (properly speaking, the services of a capital good; we use the terms synonymously), these characteristics may be thought of as cutting speeds, storage capacities, and so forth, as described in Section 5.3.2.

Substituting the m characteristics $(\chi_{j1}, \ldots, \chi_{jm})$ embodied in the nonhomogeneous good X_j for the good itself, we can rewrite (3) as

$$Z = g(X_1, \ldots, \chi_{j1}, \chi_{j2}, \ldots, \chi_{jm}, \ldots, X_n), \tag{3a}$$

so that the production function contains not n but $n + m - 1$ inputs (including the m characteristics of input j). Suppose input j, in equation (3), is a truck. In (3a), the input that enters the production function is no longer thought of as a quantity of trucks; rather, if characteristic $j2$, say, is a capacity measure, hauling capacity is the input. The firm chooses the number of units of truck capacity (as well as other truck characteristics, such as fuel economy, etc., and other labor, material, and capital inputs) that are optimal for its production process and output level.[11]

The Input Cost Index in Characteristics Space

The characteristics input cost index starts from the production function, g, from equation (3a). The index theory proceeds along similar lines to the formulation for the goods-space input cost index developed in Section 5.3.1.

We adopt the convention that a homogeneous input is itself a characteristic whose cost is equal to the market price, so that all $n + m - 1$ inputs in equation (3a) can be thought of as characteristics. With this convention, let $[\chi_0^*]$ designate the minimum-cost set of inputs, at time 0, sufficient to produce output level Z. The cost of acquiring this set of inputs can be written as:

$$C_0^* = C(\chi_0^*). \tag{7}$$

The "characteristics production cost" function, C, is interpreted as the minimum cost of producing output level Z^*, where the inputs to the production function are input characteristics rather than conventional goods.

The characteristics space cost function may be considerably more complicated than is the case of the analogous relation in the goods index section (the denominator of eq. [4]). In goods space, the input cost for any desired level of output can be determined from knowledge of the production function, f, and the prices of input goods (i.e., from the cost function that in conventional production theory is dual to the production function, f). The input characteristics case is more complicated because in general the costs of acquiring characteristics are not simple parameters (as are prices in the goods-space case), but are themselves determined by functional relations involving the market prices of input goods and the quantities of characteristics they contain. Because the purpose of this paper requires only the most general formulation of the notion of a characteristics input cost index, we will not extend the discussion to explore the complexities that arise in a more exact formulation (on this, see Pollak 1979).

The characteristics input cost index is defined analogously to the goods input cost index of Section 5.3.1: it is the ratio of the minimum-cost combination of inputs (characteristics) sufficient to produce output level Z^* in the reference and comparison period price regimes. Thus, it is the ratio of two values of equation (7)—the denominator embodies the cost regime of period 0, the numerator reflects period t's costs. This can be thought of as a modification of equation (4) to incorporate the characteristics production cost function of equation (7):

$$I = \frac{C_t^*}{C_0^*} = \frac{C(\chi_t^*)}{C(\chi_0^*)} \quad . \tag{4a}$$

The notation emphasizes that the cost function, C, is to be evaluated for a constant output level, Z^*, and for price or cost regimes corresponding to periods 0 and t.

In terms of index theory itself, the characteristics input-cost index has form, derivation, and properties similar to the conventional input-cost index for goods. In effect, the basic theory is identical—we merely adjust our thinking to let characteristics such as carrying capacities, cutting speeds, and so forth, play the roles conventionally assigned to "goods."

In particular, differences in characteristics quantities $[\chi_0^*]$ and $[\chi_t^*]$ arise from the same source as analogous effects in the goods index: changes in relative costs of acquiring inputs (characteristics) may lead the firm to substitute among inputs (characteristics). For example, changes in wages and in the price of fuel may lead to change in the optimal quantity of truck-carrying capacity in production function (3a). The characteristics input cost index would allow for such shifts, pricing a set of minimum-cost inputs that would produce constant output for the new structure of relative characteristics costs.

As with the goods input cost index, the characteristics input cost index can be based on the isoquant prevailing in the initial or reference period, the one for the comparison period, or some other one. If Z^* equals the reference period output level, Z_0, the characteristics input cost index is designated the "Laspeyres-perspective" index. The comparison period output level (Z_t) gives a "Paasche-perspective" index. In the goods index section, these alternative input cost indexes were referred to as the I_L and I_P indexes.

Quality Variation in the Characteristics Input Cost Index

In the ordinary meaning of the term, two varieties of a nonhomogeneous good may be thought to differ in quality whenever they are not exactly identical. The task of this section is to show that the only differences which matter in an input cost index are differences in input characteristics—that is, that only the user-value implications of goods need to be accounted for in the input cost index.

Suppose the I_L version of the input-cost index is to be computed and the nonhomogeneous input in equation (3a) is a machine. The denominator of the I_L index is the cost of the reference period's vector of inputs $[x_0^*]$. This vector implies a particular machine or combination of machines.

The theoretical input cost index of equation (4a) permits shifts in characteristics quantities in response to changing relative costs of acquiring characteristics. Accordingly, if the cost-minimizing characteristics in the numerator of equation (4a) differ from those in the denominator, this implies either a shift to some other machine, a shift in the mix of machines employed, or both.

This relation can be illustrated by supposing that the nonhomogeneous input of equation (3a) has two characteristics and that the relations between the two can be portrayed in figure 5.1. That is, the axes of figure 5.1 are interpreted as referring to two characteristics, χ_a and χ_b, instead of to two goods, and the solid and dashed isocost lines represent the relative costs of acquiring the two characteristics.[12] Under this interpretation, point A designates the bundle of characteristics embodied in machine A, and point B the characteristics of machine B. As figure 5.1 shows, a shift in relative characteristics costs (in the figure drawn as straight lines) induces a switch from one machine to the other.

An input cost index compares, as Section 5.3.1 shows, precisely such points as A and B. Thus, the characteristics input cost index need not be computed on identical varieties of nonhomogenous goods. Generally, an index which includes nonhomogeneous goods will encompass changes from one variety to another. In the everyday meaning of the term, this is a "quality change," and indeed machines A and B embody differing quantities of input characteristics. But in the context of the input cost index, the two machines are of equivalent quality. They both produce the same output level, and if introduced into equation (3a) they both have the same implications for the cost of the *other* inputs included in the production function. Thus, for the theory of input cost indexes, we say that machine A and machine B are of equivalent quality.[13]

Notice, however, that machines A and B do not necessarily sell for the same price in either period 0 or period t, and in both periods the using firm clearly chooses one over the other (a different one in each period). This is worth additional comment, in view of the fact that most of the quality change literature takes price equivalence in some period, or some "equally likely to be chosen" condition, as a definition of equal quality.

An intuitive rationale for the production-function definition of the present paper can be obtained by following through the analogy between the theory of the firm's choice of inputs when those inputs are assumed to be homogeneous goods and its choice of inputs when they are characteristics. Whether the theory relates to goods or to characteristics, there are a great many different bundles of inputs that are equally satisfactory in terms of their output productivity—all those bundles on a given isoquant. In both cases, only one of these equivalent bundles is actually chosen—the cost-minimizing one. In the characteristics case, this says that there may be many different machines that are equivalent in the production function; but the firm chooses the one (or combination) that provides the minimum-cost bundle of input characteristics.

In the conventional view, equal quality is inferred from identical selling prices for two goods. This amounts to a quality definition that takes all machines that lie on the same isocost line (in characteristics space) to be of equal quality. The characteristics-space analysis followed here shows that definition to be inconsistent with the nature of the input cost index. Equal quality is determined from equivalence of alternative machines in the production function that is the basis for the input cost index and not from equality in prices.[14]

Thus, substitution effects in the characteristics input cost index lead to changes in the varieties of nonhomogenous goods included in the index. This takes place because characteristics are obtained packaged in bundles; in order to change characteristics proportions, the purchaser must choose a different variety of good, or a different mix of varieties.

Quality "Adjustments" in the Input Cost Index

The preceding section has shown that a new machine whose characteristics imply that the firm has remained on the same production isoquant is treated in the input cost index as equivalent in quality to the old one. If, on the other hand, a new machine has characteristics inconsistent with remaining on the same production isoquant, some quality adjustment is called for in the measurement.

Suppose in period t a machine corresponding to point C of figure 5.1 is observed, and machine A (whose characteristics were included in $[\chi_0^*]$) is unavailable for inclusion in the index. Machine A's "unavailability" may simply acknowledge that it is not actually in use in the particular using firm or industry for which the input price index is being constructed, or that for some reason the agency or investigator compiling the index does not have access to data on machine A in time period t. It does not necessarily mean that it has disappeared from the face of the earth.[15]

We describe the quality difference between machines A and C by the vector of differences in their characteristics, or $\widetilde{\Delta}\chi$.[16] The question to be answered takes the form: How should $\widetilde{\Delta}\chi$ be dealt with in the input cost index of equation (4a)?

It is clear that the change in characteristics quantities implies a shift in output (in fig. 5.1, from Z_0 to Z_t). Thus, introduction of machine C in the production function g violates the constant-output criterion on which the input cost index is constructed. Beyond this, $\widetilde{\Delta}\chi$ may have implications in g that cannot be depicted in the two dimensions of figure 5.1—for example, a shift to trucks with greater load-carrying capability may imply a reduction of direct labor and increased use of fuel. Adjustments in other factor usage may indirectly violate the constant-output criterion.

The input cost index therefore must be adjusted for the quality change in order to maintain its constant-output criterion. Maintaining the con-

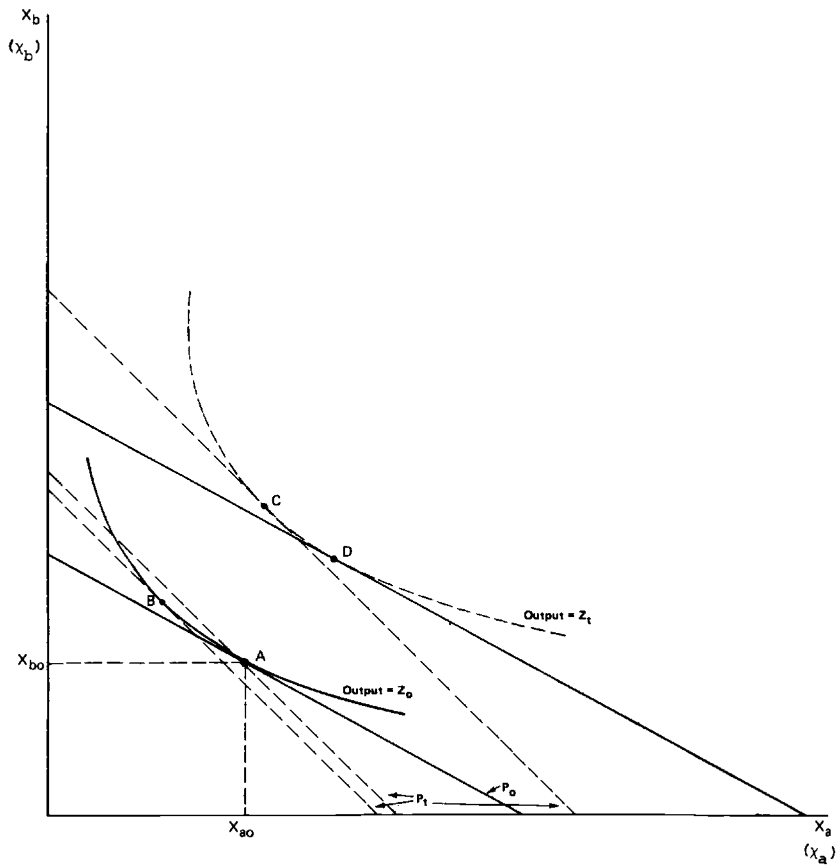

Fig. 5.1 Production isoquants in goods and characteristics spaces (basis for computing input price indexes and quality adjustments).

stant-output criterion implies that the appropriate adjustment is one exactly *equivalent to the output implications of* $\widetilde{\Delta}\chi$. That is, the appropriate compensation for the difference between machines A and C is equal to the difference in output between isoquants Z_0 and Z_t. The correct quality adjusted index is one that can be interpreted as the change in cost of a collection of inputs just sufficient to produce output level Z_0.

We have thus reached the conclusion that in a measure of input prices the adjustment for quality change must be output oriented. Adjustments are to be made only for changes in input characteristics that result in changed output (or reduced cost to the user), and the correct quality adjustment is exactly equal to the cost change or the value of the output change that they induce. In the literature, this is known as the user-value rule.

The conclusion that an input cost price measure requires evaluating quality change by an output standard will (experience presenting this paper has shown) strike many readers as paradoxical, at first glance. Yet, the ordinary goods-space input cost index is based on a constant-output criterion. An output-oriented quality adjustment is merely the characteristics-space application of the constant-output criterion inherent in any input cost index.

It may be helpful to apply the characteristics-space reasoning to the ordinary Laspeyres-formula index that will normally be computed as an approximation to the true input cost index. To construct the Laspeyres-price index in characteristics space, we redefine the index in terms of characteristics, rather than goods. In the conventional Laspeyres formula—equation (5)—the index weights are defined as quantities of goods used in the initial period's production. For the characteristics-space Laspeyres index, these weights are interpreted as quantities of characteristics, rather than quantities of goods. Thus, the quantity weights for the Laspeyres index for characteristics are quantities of the $n + m - 1$ input characteristics from equation (3a)—the actual characteristics quantities observed in the base period.

One could argue that because the characteristics Laspeyres index is computed for the purpose of approximating the theoretical characteristics input cost index, the theoretical treatment of quality change in the latter should be adopted for the fixed-weight index. An alternative argument that reaches the same conclusion may appeal to traditionalists.

Consider the interpretation that views the Laspeyres index merely as a formula for holding quantity weights constant in order to factor out price change from quantity change in a value aggregate. The characteristics Laspeyres index uses quantities of input characteristics as weights. We have interpreted quality change as a change in the quantities of characteristics. Thus, when quality change occurs, the quantity weights in the Laspeyres formula would no longer be held constant.

Making a quality adjustment to the Laspeyres index, therefore, may be interpreted as restoring the index quantity weights to their reference-period values. It has sometimes been said that there is no rationale for making quality adjustments within the Laspeyres framework. However, in the disaggregation approach to quality change the act of seeking a quality adjustment for the Laspeyres index has exactly the same significance and interpretation as holding constant the weights in the index formula—so long as weights are defined appropriately as characteristics quantities.

So far, discussion of quality change has been conducted within the framework of the Laspeyres-perspective characteristics input cost index. A similar line of reasoning holds for the Paasche-perspective index. For the I_P index it is, of course, isoquant Z_t that provides the constant-output criterion; accordingly, the quality adjustment appropriate to the I_P index is one that adjusts the reference period observation to assure that the I_P input cost index measures two combinations of inputs lying on the Z_t isoquant. This argument is a straightforward translation of the reasoning that has gone before.

Note that the existence of I_L and I_P versions of the characteristics input cost index implies that there are two possible quality adjustments for an input cost index. Both are based on a user-value rule, but one (the I_P index) takes the current user value as relevant, the other (the I_L index) is based on the initial period's user valuation. Obviously those two quality adjustments might not give the same answer, but this is nothing more than the old index number problem in a new and intriguing form.

Summary

The appropriate quality adjustment for an input cost index is one based on user value. The input cost index is derived from the cost of bundles of inputs that lie on the same production isoquant. Taking contribution to output as the quality adjustment for the input cost index assures that the inputs in both numerator and denominator of the index do correspond to points on the same production isoquant.

5.4 A Theory of Quality Adjustment for Output Price Indexes

The format of this section parallels the one followed in the section on input price indexes. We first sketch briefly the content of the theory of output price indexes applied to goods, then consider the concept of output characteristics. Section 5.4.3 extends the price index theory from goods space into characteristics space, from which the results on quality change emerge. Readers familiar with the theory of "goods" output price indexes may wish to skip directly to Section 5.4.2.

5.4.1 Output Price Indexes for Goods

The theory of output price indexes has been discussed from an economy-wide perspective by Moorsteen (1961), Fisher and Shell (1972), Samuelson and Swamy (1974), and others. The approach requires motivation. One method is to ask what properties we wish an output quantity measure to have, and to design an output price measure that is consistent with the quantity measure.

The relationship between an economy's fixed stock of resources and its output possibilities is represented by the standard textbook production possibility frontier diagram. Suppose that in the initial period the economy is at point A, as shown in figure 5.2, that in the subsequent period the

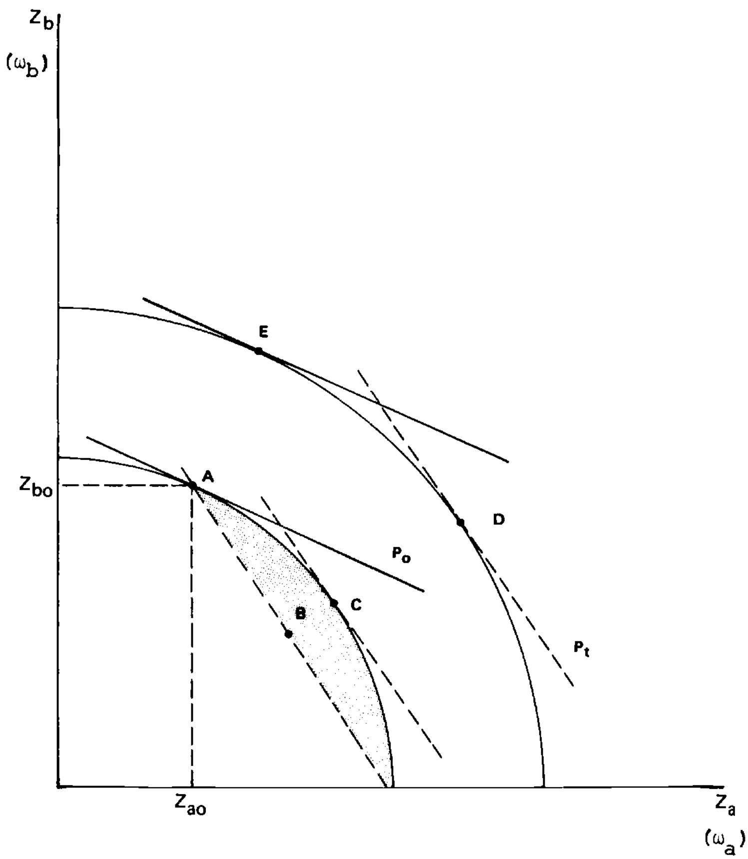

Fig. 5.2 Production possibility curves in goods and characteristics spaces (basis for computing output price indexes and quality adjustments).

economy grows, shifting to a higher production possibility frontier, and (because of changes in relative prices) moves to point D.

A natural method for measuring the output change from A to D is to allow for a movement along one production possibility curve, such as from A to C, and to take the distance between the curves at this point (CD) as a measure of the change in output. Of course, there are many such points: one could also move along the higher production possibility curve from D to E, and take the distance EA as the output measure. Unless the production possibility curves are radial displacements of each other, these two measures of output change may not agree. As we shall show, these alternative output measures correspond to the traditional Paasche-Laspeyres "index number problem."

We may wish to compute a price measure that is compatible with CD as the output quantity measure. Alternatively, we may wish to produce the output measure by deflation; if so, the correct output price index will produce CD as the measure of output.[17] Compatibility with CD as a measure of output requires the price index to treat two collections of outputs lying on the same production possibility curve (such as A and C) as equivalent. The theoretical output price index does just that: it compares the value of output collection A in period 0 prices with output collection C in period t's prices. This theoretical measure has usually been termed the "true output price deflator" in the price index number literature.

It is easy to show that the Laspeyres fixed-weight price index relates to the initial situation (point A) and that it *understates* the price change associated with remaining on a constant production possibility curve. Hence, when the Laspeyres index is used as a deflator it will produce an overestimate of the real output change CD. A geometric proof follows.

The economy depicted in figure 5.2 produces two outputs, Z_a and Z_b. For the price set $[P_0]$ (indicated by the slope of the line so labeled), the economy will produce at point A, with outputs Z_{b0} and Z_{a0}, which we label as the output set $[Z_0]$. The quantity $\Sigma P_{i0} Z_{i0}$ ($i = a, b$) is the value of output for the reference period, which is also the denominator of the Laspeyres-formula price index (eq. [5], from the section on input price indexes).

Let the slopes of the dashed lines in figure 5.2 represent relative prices prevailing in the new, or comparison, period (period t). The dashed line passing through point A (the reference period's output) corresponds to the quantity $\Sigma P_t Z_0$, that is, to the numerator of the Laspeyres-price index formula (eq. [5]). This quantity is interpreted as the reference period's outputs $[Z_0]$, valued at the comparison period's prices, $[P_t]$.

Any point on the dashed line passing through A (such as point B) represents an output combination whose value is equal to that of A. With resources sufficient to produce point A, however, the economy could attain a collection of outputs having higher value than those on the AB line. Any output in the shaded area of figure 5.2 would have greater value than the value of A's outputs. The output combination with highest value, at period t prices, is point C.

From the standpoint of resource use or opportunity cost point C is equivalent to point A, because they can both be produced from the same resource availability. The fact that the numerator of the Laspeyres index prices a collection of outputs with a value which lies below a portion of the production possibility frontier suggests a bias in the Laspeyres index. This bias is indicated by the distance between the line tangent to point C and the parallel line passing through point A (i.e., the distance BC). The output change BC is the "substitution" bias in the fixed-weight Laspeyres price index when it is used as an approximation to the theoretical, or "true," output price measure.

Note that the Laspeyres index is downward biased when used as an output price measure, though it is upward biased when used as an approximation to an input price index. The downward bias in the output deflator creates an exactly equal error of opposite sign in the output quantity measure. Using the Laspeyres price index as a deflator gives BD as the output measure, which is larger than CD (the correct measure) by BC, the substitution bias in the fixed-weight index.

Thus, an appropriate output-price index is one that compares the value of output bundles A, valued at prices $[P_0]$, and C, valued at prices $[P_t]$. This index is the economic or theoretical output-price index and is computed from equation (10), below. It is termed the "Laspeyres-perspective" output price index.

The implicit output price deflators emerging from the computation of the national accounts are Paasche-formula price indexes. The Paasche-price index can also be derived from figure 5.2.

Point D represents the actual output combination in period t. Parallel to the "Laspeyres-perspective" output price index is the "Paasche-perspective" index which compares the ratio of the values of outputs D and E (which lie on the same production possibility curve); deflation by this index gives EA as the true measure of output quantity change. By an analogous line of reasoning to that presented above for the Laspeyres case, it can be shown that the fixed-weight Paasche price index, which prices output collection D in both periods, gives an upward-biased measure of price change, compared to the value of the economic output price index based on points D and E of figure 5.2. Consequently, a measure of output produced through deflation by a Paasche price index will understate the true change in output. Note that the fixed-weight Paasche-formula index is upward biased because of substitution, when it is used as an output price index; when used as an input price index the substitution bias goes in the opposite direction (see Section 5.3.1).

A more formal statement of the ideas so far presented intuitively follows. Suppose there are n input quantities, $X_i (i = 1, \ldots, n)$, which are available in strictly fixed amounts ($X_i = X_i^*$). The production transformation relation between the n inputs and the economy's m outputs is given by

$$(8) \qquad T(Z_j, X_i^*) = 0$$

where $j = 1, \ldots, m$, and $i = 1, \ldots, n$.

We assume that inputs are not specialized, so that different combinations of outputs can be produced from them, and consider only those quantities of outputs that will completely exhaust the set of fixed available input quantities, $[X^*]$. The set of such output combinations (Z: $X = X^*$) defines the production possibility curve; for compactness, outputs lying on the production possibility curve will be designated as $[Z^*]$.

The revenue function

$$(9) \qquad R = R\,(Z^*, P)$$

indicates the revenue obtained from the set of outputs produced, given a set of output prices, $[P]$. The optimal set of outputs is determined by maximizing (9), with input quantities and output prices specified at levels appropriate to the comparison. For prices P_0, for input quantities $X^* = X_0$, and assuming a competitive structure, $Z^* = Z_0$ (optimal outputs at time 0 equal actual outputs). Thus, maximum revenue is $\Sigma\, Z_{j0}\, P_{j0}$, the actual value of output in the reference period. This is point A in figure 5.2.

We now pose the question, What output combination would have been forthcoming had prices been $[P_t]$, all else remaining as before—that is, with input quantities $X^* = X_0$, and unchanged transformation function T (which together imply an unshifted production possibility frontier). Maximizing the revenue function of equation (9) with reference-period inputs and prices $[P_t]$ yields a new set of revenue-maximizing quantities, which we designate $[Z_t^*]$. These, of course, are not the quantities actually produced in period t, as both resource availability and technology may have changed between the two periods (i.e., we are interested in determining point C on figure 5.2, not point D).

The output price index J can be found as the ratio of

$$(10) \qquad J = \frac{R_t}{R_0} = \frac{\Sigma Z_t^* P_t}{\Sigma Z_0^* P_0},$$

where it is understood that both halves of the ratio are computed using equations (8) and (9). When input quantities are fixed for both computations at the original resource endowment (i.e., $X^* = X_0$), the result is the

Laspeyres-perspective output price index, which compares points A and C in figure 5.2. This index is designated J_L.

There are (at least) two interesting output price indexes. The J_L index takes the perspective of the initial situation, answering the question: What is the (maximum) value ratio, between reference and comparison periods, of collections of outputs that can be produced using the reference period's resource endowment? This Laspeyres-perspective output price index, J_L, is analogous to the I_L input-cost index of Section 5.3.1.

A Paasche-perspective theoretical output price index can also be defined. We refer to this as J_P. The J_P index takes the comparison-period resource endowment as relevant, so that actual levels of inputs in time t are specified ($X^* = X_t$), and the production relations,(eq. [8]) are those prevailing at time t. It is therefore computed from equation (10) by letting Z^* in equation (9) be determined by the level of inputs available in time t, or X_t. For the J_P economic output price index, the ordinary fixed-weight Paasche price index is an approximation and *upper*bound (see Fisher and Shell 1972, Essay II).[18]

Comment. There is an interesting anomaly in the price index literature between the treatment of output price indexes, on the one hand, and input cost indexes and cost-of-living indexes, on the other. The theory of output indexes arose out of the need for international comparisons, and therefore its exposition has always proceeded from an economywide perspective rather than from that of an individual (multiproduct) firm. It is reasonable (though not strictly realistic) to specify that the economy has a fixed quantity of productive inputs; but a price index formulation based on the production possibility curve poses problems in applications where it seems inappropriate to assume fixed resource availability. An individual firm or a competitive industry faces, for example, not fixed amounts of inputs but rather fixed input prices. Fisher and Shell (1979) have worked on generalizing the traditional output price index theory to fit other situations.[19] This work is not developed far enough to be incorporated into this paper.

However, even if a more general theory can be worked out, it does not follow that the traditional approach is necessarily rendered obsolete. In some situations, it will still be meaningful to ask: How would the resources actually used in the reference period (or in the comparison period) have been allocated among the various outputs had relative output prices been different from those experienced at the time?

Summary and Conclusions to Section 5.4.1

This section has developed two theoretical output price indexes (J_L and J_P) and discussed the relation between these two indexes and the Laspeyres and Paasche (L and P) fixed-weight formulas. As in the input price index case, the economic indexes take account of substitution caused by changes in relative prices, as the fixed-weight indexes do not. However, the signs of substitution biases in L and P used as output price indexes are reversed, compared to the input price index case presented in Section 5.3.1.

All output price indexes hold constant the collection of productive inputs employed. We refer to this as the "constant-input" *criterion* for an output price measure. The theory of output price measures is thus symmetric with that of input price indexes, for the latter (as shown in Section 5.3.1) have a constant-output criterion.

When used as an output price index, the usual fixed-weight index (L or P) may be thought of as holding inputs constant through the device of holding base-period outputs constant. The superiority of the theoretical output price index arises from the fact that it holds inputs constant directly: it specifies a constant production possibility curve, and permits output combinations to shift along that curve rather than restricting the comparison to a single point on the production possibility curve. Again, this result is parallel to the input cost index, where the L and P indexes were thought of holding output constant by using fixed weights for inputs.

5.4.2 Modeling Quality Change: The Concept of Output Characteristics

As was true for the input cost index, I incorporate quality change into output price indexes by transforming the theory into characteristics space. The present section serves to define the concept of "output characteristics" required for the theoretical work in Section 5.4.3.

The term "characteristics," when used in economics, has become strongly identified with *input* characteristics. The well-known work of Lancaster (1971), for example, argued that disaggregation of consumer goods into characteristics gave a better explanation of consumer behavior because characteristics, rather than goods, were the true inputs into the utility function. We noted in Section 5.3.2 that a similar interpretation could be given to the human capital literature—years of education, of experience, and of training can be treated as labor characteristics on the specification that these, rather than the number of labor hours, are the true inputs into the production function.

It does not seem to have been recognized that a similar disaggregation can be applied to output. Any production function (such as eq. [3]) relates a set of inputs to output (usually written as a scalar quantity). If output is not a homogeneous good, it may be useful in understanding production to disaggregate output into characteristics, treating the production process as creating a set of jointly produced outputs (the characteristics). One relevant theoretical precedent for this is Dano (1966).

The distinction between input characteristics and output characteristics requires elaboration. A variable is an input characteristic if it acts as an input in a production or utility function. Put another way, one selects input characteristics empirically according to whether they provide something that is wanted or is valued in use.

An output characteristic is different. In this case what matters is the producing industry's production function, not that of the using industry. Referring to the two-sector model of equations (1) and (2), an input characteristic of computers is something that contributes to the production of gadgets; an output characteristic of computers, on the other hand, is something that requires productive activity in the computer industry.

One can overload the distinction between input and output characteristics. Normally, an output is not produced unless someone wants it, so in most cases an output characteristic will also be an input characteristic. But that does not invalidate the conceptual distinction between them; and there are exceptions—sometimes things get produced that are not wanted by users.

The idea of an output characteristic can be defined by a process symmetric to the one used for input characteristics in Section 5.3.2.

a) The production of a nonhomogeneous good is considered as the *joint output of* a set of characteristics. That is, instead of defining output as quantities of goods, such as trucks or boxes of soap, we break these goods down into outputs of characteristics, such as "load-carrying capacity," "number of ounces in the box," and so forth.

In principle, treating an output as the joint production of a set of characteristics amounts to assuming a production process not materially different from other and better known joint production examples (such as beef and hides)—except that we assume that characteristics can be produced in variable proportions, at least over some ranges. As in the beef-hide case, there may be limits on the proportions in which characteristics can be produced—there may be some characteristic which from the technological view must be present in every output bundle. So even though we treat characteristics as if they were separate outputs, there may well be technical reasons why they are bundled together in the first place.

b) As in the input case, output characteristics are a finer level of aggregation than are goods, and goods, of course, are the units in which market transactions take place.

c) To define an output characteristic, suppose output is computers. Suppose further that the substitution of a model 490 computer for a model 390 causes changes in input usage (in the production function of eq. [2]) that are not explained by the count of numbers of machines produced. *Something is an output characteristic if it accounts for, or partly accounts for, the unexplained variation in resource usage occasioned by changes in the varieties of nonhomogeneous goods produced.*

This definition of an output characteristic says, in effect, that an output is something that uses resources. Indeed, that is the reason why the theory of production is concerned with the transformation of inputs into outputs. An output is not an output because someone wants it; being useful or desired is the definition of an input (compare the definition of an output characteristic with the input characteristic definition in Sec. 5.3.2).

d) We assume that only the quantity of characteristics matters in the production function and not how the output of characteristics is packaged into the output of goods. This assumption implies there are no "packaging size" economies or diseconomies associated with building larger quantities of characteristics into a single variety of an output good (an equivalent assumption was made on the input side in Sec. 5.3.2). It also rules out production complementarities between the "quality" of output goods (the amounts of characteristics they contain) and the quality of the inputs. Making more comfortable and longer wearing shoes is assumed to require more leather and more shoemaking labor, not different kinds of materials or more highly skilled shoemakers. The assumption is wholly a simplifying one, though it does eliminate interesting, relevant, and realistic cases.

5.4.3 Characteristics Output Price Indexes and the Treatment of Quality Change

This section proceeds in parallel with Section 5.3.3 to extend the "goods space" or conventional output price index theory to incorporate the idea of output characteristics. I first summarize results from Section 5.4.1 that are required for this section.

1. The theoretical output price index is constructed from the values of collections of outputs in two periods that represent maximum revenue points on the same production possibility frontier.

2. For any comparison, a number of output price indexes can be computed, depending on which production possibility frontier is used for the comparison; indexes based on production possibility frontiers appropriate to reference and comparison periods are designated, respectively, as "Laspeyres-perspective" and "Paasche-perspective" output price indexes (J_L and J_P).

3. The usual fixed-weight Laspeyres and Paasche index formulas (L and P) may be viewed as approximations to the true economic indexes. The Laspeyres index provides a lower bound to J_L, whereas the Paasche index is an upper bound to J_P (i.e., $L \leq J_L$ and $P \geq J_P$).

4. The bounding conditions in (3) are a reversal of the bounds for input cost indexes (as noted in Sec. 5.3.1, $L \geq I_L$ and $P \leq I_P$).

5. Computing an output price index, whether the theoretical index or the fixed-weight approximation, requires that resource use be held constant over the comparison in order to eliminate from the price measure shifts in the production possibility curve—the only unambiguous measure

of output change; this condition is referred to as the constant-input "criterion" for the output price index.

6. The output index criterion stated in (5) is a symmetric reversal of the criterion for the input cost index; the input cost index required a constant-output criterion (Sec. 5.3.1).

It remains to use the concept of output characteristics from Section 5.4.2 to extend the basic price index theory from goods space to characteristics space.

Assuming, for simplicity, that only one of the economy's outputs (eqq. 8 and 9, above) is nonhomogeneous, we treat that output as the joint output of a set of output characteristics. We can, without loss of generality, specify that the first good is the nonhomogeneous one, with r characteristics. Adopting the convention that a homogeneous good is itself a characteristic, the production transformation equation (8) is rewritten as

$$T(\omega_{11}, \ldots, \omega_{1r}, \omega_2, \ldots, \omega_m; X_i) = 0, \tag{8a}$$

where $i = 1, \ldots, n$. This says that if the good "boxes of soap," for example, has output characteristics "size of box," a measure of "washing power per ounce," and some sort of packaging convenience element, then the economy's outputs are the quantities of these characteristics it produces and not the quantity "number of boxes of soap." Once these substitutions have been made, equation (8a) relates $m + r - 1$ attainable output characteristics (each ω) to the quantities of the available n inputs. Note that the definition of an "output characteristic" presented in Section 5.4.2 is implied by the construction of the production transformation function in characteristics space.[20]

A Characteristics-Space Index for Output Prices

The transformation of the goods output price index of Section 5.4.1 into an equivalent price measure in characteristics space is parallel to the development of the characteristics input cost index in the subsection "The Input-Cost Index in Characteristics Space" under Section 5.3.3.

Considering the output characteristics in the production transformation function (8a), a characteristics revenue function can be defined by modifying equation (9) to incorporate output characteristics. That is, the set of output characteristics is substituted for the nonhomogeneous output, Z_1. This function is written:

$$R = R(\omega^*, P). \tag{9a}$$

The nature of the characteristics concept makes equation (9a) a complex one, because it must depict the maximum revenue obtainable from various combinations of characteristics. A similar difficulty has already been encountered in the discussion of the characteristics input-cost index and need not, therefore, be discussed a second time.

Any theoretical output price index is formed out of ratios of maximum revenues obtainable, under two price regimes, for collections of outputs located on the same production possibility curve.[21] What we will term the "theoretical characteristics output price index"[22] is formed by taking ratios of equation (9a) under two output price regimes, or

$$\text{(10a)} \qquad J = \frac{R_t}{R_0} = \frac{R(\omega^*, P_t)}{R(\omega^*, P_0)} .$$

In words, the characteristics-space output price index amounts to pricing points from the same production possibility curve, with the two points corresponding to optimal output mixes under two different price regimes; the major difference from the output price index for goods is that the production possibility curve and the revenue function are both defined on characteristics of goods rather than on the goods themselves.

To understand the interpretation of the output price index in the characteristics context, it may intuitively be helpful to examine the conventional Laspeyres-index formula, where (output) quantities provide the weighting structure.

To construct the Laspeyres output price index in characteristics space, the index is redefined in terms of characteristics rather than goods. The index weights in the conventional Laspeyres price index formula—equation (5)—are then reinterpreted to be quantities of characteristics rather than quantities of goods. Reverting to our soap example, there is an output weight for base-period production of "number of ounces," for the measure of "washing power," and for the "packaging convenience" element.

Several points need to be emphasized. First, as in the characteristics input cost index, the characteristics output price index has a form, derivation, and properties similar to the output price index for goods.

Second, in common with the goods output price index, the characteristics output price index must be computed for a given production possibility curve.[23] This means that the characteristics output price index is computed by holding the input dosage constant, because changes in quantities of productive inputs (as well as changes in the technology) mean a shift in the production possibility curve.

Third, as with the goods index, there may be more than one characteristics output price index. In effect, figure 5.2 still applies. We wish to decompose the change between outputs A and D into a change in output and a change in price, and there are (at least) two perspectives—that of A and that of D. The argument leads to Laspeyres-perspective (J_L) and Paasche-perspective (J_P) indexes and is the same as the one presented in

Section 5.4.1 for the goods output price index. The resulting two indexes, J_L and J_P, have as counterparts the I_L and I_P characteristics input cost indexes of the subsection on "The Input Cost Index in Characteristics Space" under Section 5.3.3 (pp. 281–82 above).

Quality Variation and Quality Adjustments in Output Price Indexes

Much of the present paper involves a series of parallelisms between the output price index and input price index cases; crucial results are often reversed in the two cases. Both the parallelisms and the reversals are straightforward consequences of the relations between inputs and outputs in the theory of production. As the patterns have become familiar by now, the exposition of the characteristics output price index case can be shortened by noting that it amounts to a translation of the parallel discussion of the characteristics input cost index (in the second and third subsections under Sec. 5.3.3, pp. 283–87), but with the results transformed according to the framework of output price index theory.

As was true of the characteristics input cost index, the characteristics output price index may include a different machine in the numerator and denominator of the measure. The theoretical index always incorporates shifts (in this case in the output of characteristics) that occur in response to relative price changes. Therefore, if the relative revenue a seller receives from two characteristics changes, the producer has an incentive to change the proportions of those characteristics embodied in the machine it sells. In the ordinary view of things, this will take the form of a new "model"; if the "price" of speed has risen in relative terms and that of fuel economy has fallen, the new model may have more speed relative to its fuel economy. However, if the two machines correspond to revenue-maximizing sets of characteristics in comparison and reference periods, respectively, they are included in the numerator and denominator of the theoretical output price index. The reader is referred to the subsection "Quality Variation in the Characteristics Input Cost Index" under Section 5.3.3 for a comparable result for the input cost index case.

One could use figure 5.2 to illustrate the theoretical characteristics output price index by supposing that the axes of the figure correspond to characteristics ω (rather than to goods Z), with the production possibility curve and revenue lines similarly reinterpreted. Then points such as A and C are interpreted as two machines having different combinations of characteristics, but which can be produced with the same resources used in the reference period. Points such as A and D, on the other hand, represent machines using different quantities of resources. The reader is cautioned that this use of the figure is heuristic.

Thus, a constant-quality output price index can be thought of as an index based on two sets of characteristics (two machines) that lie on the same production possibility curve but correspond to maximum revenue points under two price regimes (the comparison and the reference periods).

If there is a change in output characteristics which, taken together, implies movement to a different production possibility curve (comparison of points such as C and D in fig. 5.2), some "quality adjustment" is called for in order to restore the constant-input criterion for an output price index. The adjustment required is equal to the value of the resources required to move the set of output characteristics included in the index back to the same production possibility curve. This is precisely the resource cost quality measurement rule that has been argued in the literature.

Note that, as in the input cost index case, there are two possible adjustments and that they correspond to the J_L and J_P forms of the output price index. One can adjust the new machine to correspond to a set of characteristics that could have been produced with the initial period's resource stocks and technology; this gives C-D in figure 5.2 as the appropriate quality adjustment, with the price index based on comparison of points A and C. Because this adjustment corresponds to the J_L form of the output price index in characteristics space, one could look at it as a "Laspeyres-perspective" quality adjustment. The appropriate quality adjustment for the J_P index, on the other hand, involves points E and A. This quality adjustment yields an output price index based on period t's resource utilization and technology, that is, a price index computed from comparison of points D and E.[24]

It may be useful, heuristically, to go through the quality adjustment process in terms of the familiar Laspeyres index. The traditional interpretation refers to this index as a formula for holding quantity weights constant in order to factor out price from quantity change in a value aggregate. Note the similarity of the following to a comparable line of reasoning in Section 5.3.3.

In the characteristics Laspeyres output price index, output characteristics quantities are the weights. Quality change is interpreted as a change in the quantity of characteristics embodied in a good, which means the quality change amounts to a change in the (output) weights in the characteristics Laspeyres index. Making a quality adjustment to the "goods" Laspeyres index can be interpreted as an adjustment which holds constant the weights in the "characteristics" Laspeyres output price index. Because in the output characteristics case, characteristics are resource using (on the argument pursued in Sec. 5.4.2), the adjustment to the Laspeyres output price index is, as in the case of the theoretical index, referenced by resource use.

Summary

In output price indexes (the fixed-weight forms as well as the theoretical ones based on production possibility curves), the quality adjustment required is equal to the resource usage of the characteristics that changed. Only with a resource-cost adjustment does the index price a set of outputs that can be produced with the resources available in the reference period (for the characteristics-space form of index J_L). Alternatively, only a resource-cost quality adjustment assures that the J_P form of the output price index prices a collection of outputs that could have been produced with comparison-period resources. This resource-cost adjustment is precisely the production-cost criterion for quality measurement advocated for price measures by such economists as Denison (1975) and Jaszi (1971).[25]

5.5 Using Characteristics-Space Theory to Resolve Quality Change Issues

Imbedding the analysis of quality change in the theory of index numbers proves a powerful tool for clearing up confusions found in the quality change literature. Two examples are addressed in this section.

5.5.1 "Costless" Quality Change

Adherents to the user-value view have often presumed that the resource cost method could not *in principle* deal with quality changes that increase performance but are cheaper to produce—the so-called costless quality change argument. This presumption, for example, underlies much of the discussion of output measures in the report of the Panel to Review Productivity Statistics (1979), and determines the report's conclusion that use of a resource-cost quality adjustment rule for output-price indexes is "not adequate for dealing with changes in quality that do not result from changes in cost" (the report cites improvements in computers as an example; see p. 8).

The discussion of costless quality change in the Panel's report (as elsewhere) was marred by confusion over what the resource-cost criterion really said and by considerable ambiguity about what was meant by the term "costless." We address the latter point first.

For simplicity (in order to permit the diagrammatic presentation of fig. 5.3), suppose computers have only one output characteristic (call it "computations") and are made from only one input (labor). Figure 5.3 graphs what is known in the production literature as a "factor requirements function," showing the minimum input requirements for, in this case, computers of different capacities. As figure 5.3 is drawn, in the initial period (requirements function F_1) minimum labor requirements per unit of computational capacity occur for computer size C_0; so we

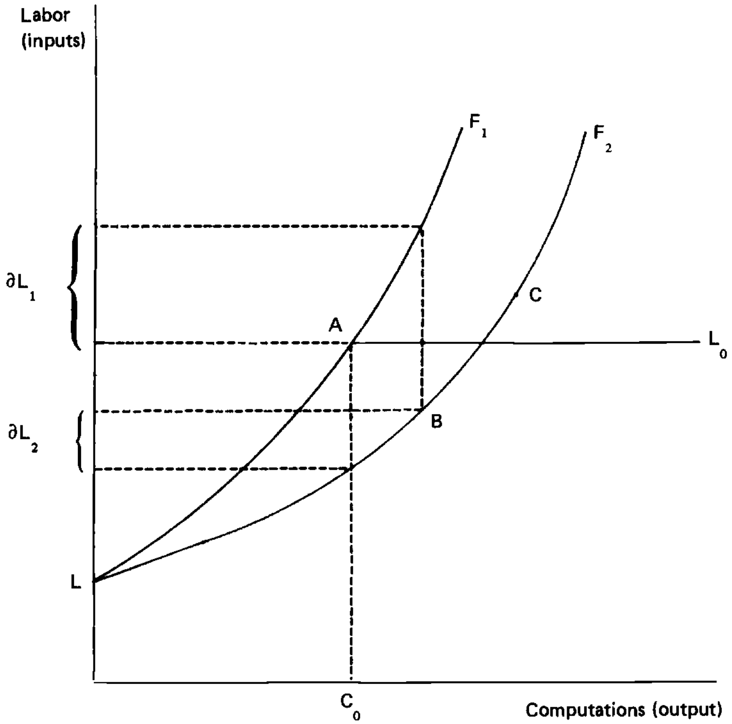

Fig. 5.3 Factor requirements function for the output characteristic "computations."

assume that in the initial period available computers cluster around size C_0.

If increments to computer capacity were really "costless" in the initial period, the factor requirements function would be horizontal beyond some point (such as the line LAL_0 in fig. 5.3). If this is what was meant by "costless" quality change, then the argument has a certain validity: a resource cost method would treat all computers lying on the horizontal segment of LAL_0 as equivalent and would indeed make no quality adjustments for expansions in computer capacity in this range.

However, it is doubtful that a horizontal factor-requirements schedule was the basis for the "costless" quality change discussion. Little of value is truly costless, and to my knowledge, functions such as LAL_0 have never been encountered empirically.

If not "costless" in this sense, how does one rationalize the frequent observation of technical improvements that cost less than what went before? For this empirical observation lies at the root of the "costless" discussion, and computers are a favorite example.

A better characterization of what its proponents must have meant by "costless" is to suppose a shift of the factor requirements function (corresponding to cost-reducing technical advances in producing computers), as in F_2 in figure 5.3. With minimum average labor requirements per computational unit now occurring at B, we may observe computer size B in the second period. Compared with machine A (produced only in the initial period), machine B in the second period requires fewer labor inputs to produce (thus it costs less), yet yields more computations. As a practical matter, if there are scale economies, or only a small number of computer varieties are marketed, something like this example may well show up in a comparison of different years' data. Without fully specifying it, parties to the costless quality change debate undoubtedly had comparison of points like A and B in mind.

There is no sense, however, in which the movement from A to B invalidates a resource-cost quality adjustment, despite frequent contrary assertions. In fact, there are *two* possible resource-cost adjustments. From the perspective of the initial period, function F_1 gives ∂L_1 as the real resource cost of the quality change. Alternatively, one could use F_2, giving ∂L_2 as the basis for making a quality adjustment.

Thus, the charge that a resource-cost criterion could not deal with "costless" quality change involved basic confusion between a shift in a schedule and a movement along it. The schedules involved are in characteristics space rather than goods space, and the insight which resolves the debate emerges from an explicit characteristics-space analysis. The resolution of the costless quality change issue is one example of the usefulness and power of the characteristics space analysis.

One or two additional points can be made about the computer example. Note that as drawn $\partial L_2 < \partial L_1$, which seems realistic: the incremental cost of "computations" is lower today than in the past. For this example, F_1 provides the appropriate adjustment for the "Laspeyres-perspective" output price index, J_L, of Section 5.4.3, F_2 is relevant for the sister J_P index. Thus, choice between the two is the classic index number problem.

The GNP deflators are Paasche-formula indexes (current period weights), making F_2 the relevant basis for adjustment. This implies a smaller quality adjustment (and a greater price increase) than if F_1 were used. Any empirical work using data from F_1 (or, alternatively, the average of F_1 and F_2) tends to overstate the quality correction to be applied to a Paasche-formula output price index. Empirical work on quality change in the deflators has ignored this distinction.

Although examples of "costless" quality change have nothing whatever to do with the feasibility or conceptual appropriateness of user-value and resource-cost alternatives for making quality adjustments, there is, to be sure, a practical problem facing index makers. Usually, neither requirements function is known. If any information is available at all, it may consist only of cost data for two points, such as A and B, or A and C, that lie on different functions. That is, there may be data on the cost of the old machine under the old technology and the cost of the new machine under the new technology, but no data at all on the cost of both under comparable technologies.

In this case, nothing can be done under the resource-cost criterion, for the necessary information is not available. Obviously, direct comparison of labor inputs for A and B adjusts the index in the wrong direction, whether by user-value or resource-cost criteria. And even though resource cost for A and C would go in the right direction, using this information as a quality adjustment also contains an error, for *either* quality adjustment criterion. However, this very serious practical difficulty has nothing whatever to do with the conceptual one of determining the theoretically appropriate basis for making the adjustment.

5.5.2 The Elimination of Productivity Change Objection

An objection frequently raised against the user-value criterion was cited in the report of the Panel to Review Productivity Statistics (1979, p. 13): "If capital is measured in terms of its output-producing capacity, the output capital ratio becomes an uninteresting statistic, since it will tend to show no change."

It is hard to see what is going on in this quotation if one remains in goods space.[26] However, moving into characteristics space makes it easy to show the Panel's statement to be wrong; it involves (as did the costless quality change matter) confusing a shift in a function with movements along it.

Consider the production function defined in equation (3a) from Section 5.3.3. In that case, inputs were defined as characteristics. Productivity change in a characteristics world is defined in the same way as it is in goods space—productivity change is a shift in the production function, in this case a shift in equation (3a). In the characteristics input cost index, quality change was defined by that same production function: changes in quantities of input characteristics embodied in input goods were counted as quality change. This means that a user-value measure of quality change represents a movement from one isoquant to another *along* the production function (3a).

Valuing quality change as movements along the using industry's production function (as the user-value criterion does) in no way precludes a production function shift—no more so than a production function shift is precluded by measuring inputs in any other way. Accordingly, productivity change (a production function shift) is not tautologically eliminated by employment of a user-value quality adjustment rule.

As with the "costless" issue discussed above, the element of validity in the "unchanged productivity" proposition arises when one has observations on but two points. If we only know inputs and outputs for two periods, *and do not know the production function*, then of course there is no way of partitioning the change between the contributions of increased characteristics (movement along a production function) and a shift in a production function. Attributing all the output change to input change would clearly eliminate measured productivity change. But that is not what the user-value criterion is about.

This section serves to show the power of characteristics-space analysis in resolving issues that have plagued the quality measurement literature for years. The technique will work profitably on other examples as well, but that can be done elsewhere.

5.6 Summary and Conclusions

The appropriate theoretical treatment of quality change has been an issue in economic measurement for years. There have long been two schools of thought.

The "user-value" approach looks at the output implications of quality change in some productive input; a machine is higher quality if it has higher productivity when used in producing something else. On the "resource-cost" view, the cost of making a machine is the proper basis for making quality adjustments, not the productivity of using machines to produce other goods.

The present paper resolves this old debate by noting (1) that there are two different uses of the data (input measures and output measures), (2) that it is well established in the index number literature that inputs and outputs imply different theoretical price index treatments, and (3) by showing that the difference in theoretical treatments carries over into the issue of adjusting for quality change.

Rather satisfying is the result that the two sides to the quality adjustment debate are both right, each for the purpose it (implicitly) had in mind—the "user-value" and "resource-cost" positions correspond to correct theoretical treatments of quality change for input-price indexes and output-price indexes, respectively. This "you are both right" resolution extends to prominent researchers on the subject as well as to positions taken by the major statistical agencies (the Bureau of Economic Analysis having taken, historically, the correct theoretical position for output measures, while the Bureau of Labor Statistics has historically taken different theoretical positions for input uses and for output uses of data—both, it turns out, correct for the uses specified).

The method of this paper is to extend the basic theories of input and output price indexes from goods space to characteristics space. That is, the usual theory (and the usual index computations) involve prices and quantities of goods; the extension redefines the variables of interest to be the costs and quantities of the *characteristics* of goods (and of labor services). The rationale for redefinition is the specification that "quality change" involves changes or rearrangements of the quantities of characteristics embodied in goods. Thus, rather than treating "quality change" as an ad hoc procedure outside the basic theory of index numbers (as has often been true in the past), the paper extends index number theory into the dimension in which empirical investigation has taken place.

The finding that there are two correct methods for making quality adjustments, not just one, fits in with other results for input and output price indexes. Much of the content of the theory of input cost indexes and output price indexes is parallel, except the results are, for the most part, essentially reversed. The "reversal" phenomenon is intuitively appealing, once it is understood that the perspectives of the two economic measurements are from opposite ends of the production process. The mirror-image view of production one gets from altering perspective from the input side to the output side is fundamental to the process of production itself.

The results of this paper preserve in characteristics space index number results that are well established for goods space. A "characteristics" *input* price index requires holding *output* constant, just as does a "goods" input price index; and a "characteristics" *output* price index, following in sequence the "goods" output price index, requires that the *resources* going into the productive process be held constant. These two conditions (constant output for input price indexes, constant inputs for output price indexes) were referred to in the body of the paper as index "criteria."

The conclusion that input price indexes require a user-value quality adjustment is a consequence of the requirement that input price indexes be based on a constant-output criterion. The result that output price indexes must use a resource-cost quality adjustment method flows from the fact that output price indexes are based on a constant-input criterion.

Having established that the theory requires different procedures for input and for output price measures, how much difference does the theory make for practical measurement? My answer is: For most cases, probably not a lot, but more for lack of data to implement the theoretical methods than for lack of relevance of the theory.

One would expect that in equilibrium the marginal cost of producing a quality change must approximate the incremental value of it to the user—otherwise a reallocation of resources would take place. Thus, real differences in the magnitude of the quality adjustments one gets from user-value and resource-cost adjustments presumably reflect shifts in functions, interference with competitive allocation, or wrong data. That does not mean, contrary to assertions made by some participants in the quality measurement debate, that working out the theoretical properties and appropriateness of the two quality adjustment systems is irrelevant.

There are two reasons for theoretical discussion of user-value and resource-cost adjustments. First, previous discussion of these issues has been so confused that it is worth trying to straighten it out.[27] Had it been realized that input measures and output measures call for different treatments of quality change, the past course of debate on this issue would have been far different. One might even have seen better decisions in statistical measurements. For this purpose, the index number theory contained in this paper is in no sense vitiated if empirically the numbers would come out about the same, provided quality change amounted to small movements around equilibrium points, curves were smooth, and we had all of the information necessary to make the theoretically correct adjustments in both input and output indexes.

Second, much of the data on quality change, taken at its face value, suggests that the world is not so neat as the theoretical model we have in our heads. It is of considerable importance to straighten out what the right model is in order to understand whether we are getting the wrong data, or whether the data that come to us are simply reflecting discontinuities, shifts, and other unfortunate attributes of the real world.

Ultimately, limitations on implementing the theory come from lack of data. Full implementation of the theoretical results requires, at minimum, estimation of production functions on characteristics for both supplying and using industries. What we have are, at most, fragmentary information about the outcomes. The practical reality is that in most actual situations there is not enough information to implement even one theoretically appropriate measure. We simply do not have the luxury of computing both measures and deciding on theoretical grounds which to choose. Nevertheless, it is still important to understand what should be done with the data if we were ever to get them.

Notes

1. The statement by Denison (1957) is still well worth reading and remains consistent with his later writings on the subject, incorporating as they do issues which have developed over the past two decades. See, e.g., his debate with Dale Jorgenson and Zvi Griliches (Denison 1972), and his article on pollution control and safety regulation expenditures (1979). The Bureau of Economic Analysis position is similar. See, for example, Jaszi's 1964 debate with Griliches.

2. They were also the first to deal with quality change in the context of input and output price indexes (see n. 25 below).

3. In a recent survey, Usher (1980) identified five uses for capital measures. Two of the five—"an argument in an investment function" and "use in the national accounts"—are those considered in this paper. An extension of the analysis in this paper to consideration of other uses for capital measures could lead to different concepts for quality measurement, so it should not be inferred that the alternatives considered here comprise all those that are

relevant in other contexts. See Christensen and Jorgenson (1973) for a development of measurement concepts of capital for different purposes.

4. This is not an innocuous step. It is well established in the economic theory of index numbers that deflation by the theoretically appropriate price index will not always produce the theoretically appropriate quantity index (see Pollak 1971).

5. The term "input cost index" has unfortunately also been used in a different way—an index of input prices computed *in lieu of* a measure of the price of output (wage rates and materials prices, e.g., as a proxy for a price index for new houses). That has nothing to do with the measure discussed in the present paper.

6. There could be many input cost indexes, for the isoquant one might like to use for comparisons is not limited to those corresponding to actual outputs in reference or comparison periods. E.g., one might compare input price change between 1979 and 1980 in terms of 1972 production levels (1972 being the last Census of Manufactures). This point is made for the consumption case in Pollak (1971).

7. For an empirical estimate of the size of the bias in a fixed-weight index relative to a true input price index (of consumption), see Braithwait (1980).

8. An analogous set of cost-of-living indexes exists for consumption measures; see Pollak (1971). The two alternative versions of the cost-of-living index correspond to the alternative decompositions of income and substitution effects in the standard theory of consumption.

9. In some forms, these aggregator functions are very restrictive assumptions, which, moreover, have mainly computational convenience (such as reducing the number of coefficients to be estimated in a multifactor production function) to recommend them. Others are simply not relevant for present purposes. I would interpret, e.g., a "production function for trucking services" as having something to do with subindexes (see Pollak 1975, or Blackorby and Russell 1978). One never needs to form subindexes in order to compute the aggregate index, and the latter is the primary concern of the present paper.

10. Klevmarken (1977) and Pollak (1979) have also discussed input price indexes defined in characteristics space. Both were concerned with the cost-of-living index (see also Diewert 1980). The approach in this section follows a similar methodology but was conceived independently. To my knowledge, the first suggestion that the analysis of quality change in index numbers would take the form of redefining the price index in characteristics space appears in Triplett (1971*b*).

11. The reader is reminded of simplifying assumption (*d*) in Section 5.2.2.

12. The example could be justified technically if it were possible to form a subindex on just the two characteristics, but the two-dimensional representation on figure 5.1 is intended to be heuristic. The full set of inputs in eq. (3a) yields a multidimensional production surface rather than an isoquant, but the argument is similar.

13. Production function equivalence, or equivalence with respect to other inputs in the cost function, was the quality definition introduced in Section 5.3.2. That definition was derived from and motivated by the characteristics input cost index analysis of the present section.

14. For small quality changes around point A, two machines whose characteristics lie on the same production isoquant will also have approximately the same selling price. Thus, the conventional view can be supported for small changes in the neighborhood of equilibrium.

15. Much of the quality literature is written as if the problem is to figure out what the price of a new (or old) machine would have been had it in fact been available. A change in the number and types of machines available would cause changes in relative prices of the varieties that were available, so one cannot use data on the varieties that were in fact available to infer anything about the price schedules that might exist under some other conditions. It seems better to avoid general equilibrium problems by assuming that the task of quality adjustment involves only the more limited problem of estimating the price of a variety that was offered on the market somewhere, but whose price was not collected by the

agency that was responsible for producing the indexes. I owe this distinction to Robert Gillingham.

16. In effect, this amounts to assuming that the new machine embodies simply a rearrangement or "repackaging" of characteristics previously available (the "repackaging" term stems from Fisher and Shell [1972]). If the new machine has a truly new characteristic, not available anywhere before, the method of analysis breaks down. If the characteristic is truly new, we are facing the intractable new product problem. A standard proposal for dealing with new products in the goods index literature is to use the demand reservation price (the lowest price at which none will be demanded) for periods in which the product does not exist. One could apply the same solution to truly new characteristics. Fortunately, it seems reasonable to assert that most new products do not involve characteristics that have never before existed. Burstein (1961) gives a persuasive argument that innovation in delivery and distribution methods is far more pervasive than provision of truly new characteristics (Burstein observed that television was just a new method for distributing ball games and vaudeville shows).

17. Note the caveat on forming quantity measures by deflation in n. 4.

18. Of course, these two are not the only possible relevant indexes since for some purposes some other basis for evaluation may be appropriate (a comparison of 1929 and 1969 prices using 1950 resources and production technology). See also n. 6.

19. An earlier attempt is an article by Archibald (1977) who follows a different course from the rest of the literature. Starting from the observation that the firm's goal is profit maximization, rather than optimization with respect to outputs and inputs themselves, Archibald constructs what he calls a "price index for profit"—defined as the ratio of the profit function under two price regimes. The usefulness of this "price index for profit" is not readily apparent, partly for reasons Archibald himself notes, as well as other considerations (it cannot be defined when profits are zero or negative, for instance). Nevertheless, Archibald derives the output price index as a "subindex" (Pollak 1975) of the price index for profit.

The notion of deriving a useful index (the output price index) as a component of a concept which itself may not be useful is not a very appealing procedure. Moreover, one can question the appropriateness of basing the theory on the firm's profit function. It is indisputable that the competitive firm maximizes profit and does not maximize revenue for its own sake. But the appropriate way to set up price index theory is determined by the questions one wants the index to address, and not necessarily by the nature of the optimization problem the economic unit is trying to solve. In any event, Archibald's output index section, standing on its own, largely duplicates Fisher and Shell (1972).

20. In Section 5.4.2 an "output characteristic" was defined as an attribute of a good that was *resource using* (as distinguished from an input characteristic, an attribute valued by the user). Eq. (8a) determines a production possibility curve in characteristics space; the information contained in a production possibility curve concerns output combinations that can be produced from a stock of inputs. Thus, if there existed a characteristic which was not resource using, increasing or decreasing its quantity would have no implications for the other outputs that can be produced. Accordingly, a characteristic which uses no resources has no role in eq. (8a), even if it were desired by users (color is perhaps an example).

21. But see the "Comment" on p. 292, above.

22. Terminology is made difficult by the extra complication of eq. (9a) over eq. (9): The revenue received from sale of a good is simply its price, but the values of characteristics are in general not fixed quantities (as we normally think of prices) but are themselves variables, which may depend on the quantities of characteristics embodied in output goods. Thus, it might have been more precise to speak of an "output value index" instead of an output price index (in parallel with the input cost index terminology of Section 5.3). This terminology was rejected as cumbersome, as it lacks precedent in either the theoretical or pragmatic index number literature. As already noted, Fisher and Shell (1972) use the terminology "true output deflator."

23. Again reference is made to the caveat expressed in the "Comment" on p. 292.

24. This distinction between reference-period and comparison-period resource-cost quality adjustments is elaborated upon in Sec. 5.5 below.

25. Fisher and Shell (1972) provide the only other theoretical discussion of quality change in the explicit context of input price indexes and output price indexes. Fisher-Shell's analytic method for incorporating quality change into price index theory is different from my own. They consider kinds of quality change that can be represented as a parameter shifting (as the case may require) a production, utility or transformation function defined on goods. Moving the analysis into characteristics space, as I do, may be considered a more explicit representation of one form of Fisher-Shell "parametrizable" quality change, a technique which is more powerful in the sense that it can be brought to bear on specific problems, yielding more explicit index number results than they were able to extract. The cost of so doing, it must be admitted, is the move into the intrinsically difficult characteristics-space notion, with all the problems that involves. Whichever method is advantageous for a particular case, the results of both agree. Speaking of quality change in output price indexes, Fisher-Shell write (1972, p. 106): "If more steel, labor, and other inputs are embodied in new cars than in old ones, then the production of a given number of cars represents a bigger output when new cars are involved than when old ones are. Moreover, this is true regardless of how consumers view the change." Thus, an output index requires a resource-cost quality adjustment rule.

26. Indeed, Fisher and Shell (1972), working in goods space, find quality and technical change indistinguishable.

27. One could note in this regard that the Panel to Review Productivity Statistics endorsed, though not without qualification, a user-value quality adjustment rule in its chapter on output, but a resource-cost criterion in its chapter on capital inputs—just the opposite from the theoretically correct output and input price index adjustments.

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