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9 Rethinking the Role of NAIRU in Monetary Policy: Implications of Model Formulation and Uncertainty

Arturo Estrella and Frederic S. Mishkin

9.1 Introduction

Because the effects of monetary policy on the aggregate economy have long lags, monetary policy must necessarily be preemptive; that is, it must act well before inflation starts to rise.[1] This, of course, is easier said than done. In order to act preemptively, monetary policymakers must have signals that help them forecast future changes in inflation. One such signal that has received substantial attention both in the academic literature and in the press is the gap between unemployment and NAIRU, the nonaccelerating inflation rate of unemployment.[2] In other words, NAIRU is the unemployment rate at which inflation is expected to neither increase or decrease.

The NAIRU concept has come under quite serious attack in recent years. In the early to mid-1990s, the common view in the economics profession was that NAIRU in the United States was around 6 percent. However, when the un-

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1. If price stability has already been achieved, then inflation falling below its target is every bit as damaging as a rise in inflation above the target. Thus, in this situation, monetary policy must also be just as preemptive against declines in inflation below target levels.

2. See, e.g., Stiglitz (1997), Gordon (1997), Staiger, Stock, and Watson (1997a, 1997b), and Council of Economic Advisers (1997, 45–54). For a history of NAIRU, see Espinosa-Vega and Russell (1997). The NAIRU acronym would better be expressed as NIIRU (the nonincreasing inflation rate of unemployment) because it is the unemployment rate at which inflation is expected to neither increase or decrease.

employment rate began to fall below 6 percent in 1995 and remained well below that level thereafter without any increase in inflation—indeed inflation actually fell—concern arose that the NAIRU concept might be seriously flawed. In addition, recent academic research has shown that there is great uncertainty in the estimates of NAIRU (e.g., Staiger et al. 1997a, 1997b), suggesting that looking at the unemployment rate relative to NAIRU might not be a very helpful guide for monetary policy.

In this paper, we rethink the NAIRU concept and examine whether NAIRU might have a useful role in monetary policy making. We argue that the answer is yes. However, the positive answer depends critically on redefining NAIRU very carefully and distinguishing it from a long-run concept like the natural rate of unemployment, something that is not typically done in the literature. Furthermore, as we will see, the view that the NAIRU concept implies that the monetary authorities should try to move the economy toward the NAIRU, thus to some extent treating it as a target, is both incorrect and misguided.

The first step in our analysis, in section 9.2, is to think about defining NAIRU in the context of setting monetary policy instruments. We adopt a definition that focuses on NAIRU as a reference point for monetary policy and show that our definition of NAIRU is a short-run concept and is not the same as the natural rate of unemployment. Understanding that short-run NAIRU and the natural rate of unemployment differ is important, not only for the theoretical analysis to follow, but also because it suggests that short-run NAIRU is likely to be highly variable, in contrast to the natural rate of unemployment. One immediate implication is that thinking of NAIRU as a level at which the unemployment rate should settle is not very useful for policy purposes.

Our approach to the construction of short-run NAIRU is fairly general. Although we define this concept in the context of a particular model of inflation that is adapted from the current literature, the same approach can be applied to any predictive model of inflation in which unemployment plays an important role.

Once we have defined short-run NAIRU, we then go on to examine how it might be used in policy making. We do this in several steps. First, we look in section 9.3 at the certainty-equivalent case, when only inflation enters the policymakers' objective function and then when unemployment (or equivalently, output) as well as inflation are part of policymakers' objectives. Although the certainty-equivalent case is useful as a starting point for the analysis, we cannot stop here because several sources of uncertainty have important implications for how monetary policy should be conducted. In addition to uncertainty about estimates of the actual value of NAIRU, there is uncertainty about the estimated parameters of the model, especially the parameters that measure the effect of the NAIRU gap on inflation and the impact of monetary policy instruments on the NAIRU gap. We examine in section 9.4 what effect these sources of uncertainty have on how short-run NAIRU might be used in monetary policy making, again under the pure price stability objective and then

when unemployment as well as inflation enter the policymakers' objective function.

Our theoretical analysis shows that uncertainty about the level of short-run NAIRU does not necessarily imply that monetary policy should react less to the NAIRU gap. However, uncertainty about the effect of the NAIRU gap on inflation does require an adjustment to the reference point for monetary tightening in terms of the level of unemployment and to the weight applied to the gap between actual and target inflation. Furthermore, as in Brainard (1967), uncertainty about the effect of the monetary policy instrument on the NAIRU gap reduces the magnitude of the policy response.

There is another sense in which uncertainty about NAIRU may have an effect on policy. There may be uncertainty not just about the level of NAIRU or its effect but about the way it is modeled: the exact form of the model specification may be unknown. Errors in model selection may result in excess uncertainty regarding both inflation forecasts and the parameters of the model. Thus model selection has the potential to increase uncertainty about the effect of the NAIRU gap and to reduce the effectiveness of policy, and the magnitude of this problem may be more difficult to determine than that of simple parameter uncertainty. In section 9.5, we focus on the losses associated with leaving out key information from the model.

Although our theoretical framework shows the qualitative effects of uncertainty on how monetary policy should be conducted, it cannot tell us whether these effects are economically important. To examine this question, we estimate in section 9.6 a simple NAIRU gap model for the United States to obtain quantitative measures of uncertainty and to assess how these measures affect our view of the optimal reaction of monetary policy to movements in unemployment relative to short-run NAIRU. Using an analogous model based on monthly data, we then examine how in practice the short-run NAIRU concept could be used in the actual conduct of monetary policy. The estimated models provide us with measures of short-run NAIRU that indicate that it is highly variable, suggesting that trying to drive the unemployment rate toward NAIRU, whether it is a short-run or a long-run concept, would be an inappropriate way to think about how monetary policy should be conducted. In particular, we use our analysis to evaluate whether the setting of monetary policy instruments in the face of rapidly falling unemployment rates in recent years makes sense.

9.2 Defining Short-Run NAIRU: Why It Differs from the Natural Rate of Unemployment

The concept of the natural rate of unemployment was first developed by Friedman (1968) and Phelps (1968) to argue that there would be no long-run trade-off between unemployment and inflation. The natural rate of unemployment is defined as the level of unemployment to which the economy would

converge in the long run in the absence of structural changes to the labor market. An implication of this definition is that expansionary monetary policy that leads to higher inflation would not be able to produce lower unemployment on average. Indeed, as mentioned in Friedman (1968), higher inflation might even have the opposite effect of raising unemployment in the long run because it would interfere with efficient functioning of labor markets. The concept of a natural rate of unemployment leads to the following characterization of an expectations-augmented Phillips curve:

$$\pi_t = \pi_t^e + \beta(L)(u_t - \bar{u}_t) + \delta' z_t + \varepsilon_t,$$

where

π_t = inflation rate from $t - 1$ to t
π_t^e = inflation rate expected at $t - 1$
u_t = unemployment rate at time t
$\bar{u}_t$ = natural rate of unemployment at time t, which could be a constant but could shift with structural changes in the economy
z_t = a vector of variables such as supply shocks, which have zero ex ante expectation
ε_t = an unspecified disturbance term

In order to estimate this expectations-augmented Phillips curve, researchers typically assume that the expected inflation can be measured as a distributed lag on past inflation and other variables, and that the inflation rate is integrated of order one, so that $\Delta\pi_t$ is stationary. The resulting Phillips curve is then

$$\Delta\pi_t = \beta(L)(u_t - \bar{u}_t) + \gamma(L)\Delta\pi_{t-1} + \delta' z_t + \varepsilon_t. \quad (1)$$

The NAIRU concept was first developed in a paper by Modigliani and Papademos (1975) and is defined as the rate of unemployment at which there is no tendency for inflation to increase or decrease. In empirical work such as Staiger et al. (1997a, 1997b) and Gordon (1997), NAIRU is viewed as being equivalent to the natural rate of unemployment, $\bar{u}_t$, in equation (1) and is typically estimated by assuming that $\bar{u}_t$ is a constant, a random walk, or a linear transformation of some step function or spline.[3]

For policy purposes, equation (1) indicates that it is perfectly appropriate to think about the unemployment gap, $u_t - \bar{u}_t$, as one determinant of changes in the rate of inflation, recognizing that other factors, represented by the past history of inflation and the z_t variables, also affect the inflation process. However, current unemployment is frequently compared with the estimated value of NAIRU, and the resulting NAIRU gap is taken to be an indicator of inflationary pressure. Under a strong form of this view, if policymakers wish to drive inflation down, they need to raise the unemployment level above NAIRU,

3. See, e.g., Staiger et al. (1997a).

whereas if inflation is at its desired level, monetary policy needs to keep unemployment from falling below NAIRU.

Policy discussions, therefore, frequently focus on the difference between the current level of unemployment and NAIRU as estimated above, in other words, on the variable that enters the first term of equation (1) in a distributed lag. This implicit comparison has the advantage of simplicity: it focuses the discussion on a single indicator of inflationary pressure, the unemployment gap, that we know from the model should be zero in long-run equilibrium. However, this advantage is overwhelmed by a number of serious problems associated with this procedure.

First, monetary policy does not generally focus only on long-run equilibrium, so the gap as defined above may be of limited usefulness. Second, even if equation (1) is viewed as a short-run forecasting equation, the dependent variable is contemporaneous monthly or quarterly inflation, which is quite unlikely to be the policy target in practice. Third, the current unemployment gap is only one of many explanatory variables in the equation, including several lags of the gap itself. Focusing on only one variable gives an incomplete picture. Fourth, the equation may not even represent the optimal forecast of inflation, since other potentially important variables may be omitted.

Finally, focusing on the unemployment gap may create the impression that the goal of policy is to drive unemployment toward NAIRU as a target level. As equation (1) illustrates, the current unemployment gap, $u_t - \overline{u}_t$, is only one of many explanatory variables in the Phillips curve equation. The presence of lags of $\Delta\pi$ in the equation suggests that inflation may decelerate because expected inflation is falling, even if the unemployment rate is below the natural rate of unemployment. Similarly, if there have been favorable supply shocks, inflation in the future may decelerate even though the unemployment rate is well below the natural rate. The presence of lags of the unemployment gap suggests complicated dynamics in which a current negative unemployment rate could also be associated with decelerating inflation. The presence of many other variables besides the current unemployment gap in the expectations-augmented Phillips curve equation therefore implies that the unemployment rate at which there is no tendency for inflation to rise or fall over the policy horizon can be quite different from the natural rate of unemployment, $\overline{u}_t$. In other words, it can be quite misleading to focus on NAIRU, as an estimate in equation (1) of the natural rate of unemployment, because it is not clear that the introduction of policy shocks designed to drive unemployment toward this characterization of NAIRU will do anything to control inflation either in the short run or in the long run.

Therefore, we propose an alternative way of thinking about NAIRU as a reference point for unemployment that reflects inflationary pressures over the short- or intermediate-run policy horizon. The key idea is that the reference point for unemployment at which inflation will neither increase nor decrease over the relevant policy horizon, which can be thought of as a short-run

NAIRU, embodies not only $\bar{u}_t$, the natural rate of unemployment, but also the other variables that help predict inflation. In other words, we would like to express the change in inflation over the relevant policy horizon as a function of $u_t - n_t$, where n_t is an appropriately constructed short-run NAIRU.

Thus suppose that the policy horizon for inflation is from j to $j + k$ months ahead and define

$$\Delta\pi_t^{(j,k)} = (1200/k)\log(p_{t+j+k}/p_{t+j}) - 100\log(p_t/p_{t-12})$$

as the difference between current annual inflation and inflation over the policy horizon, where p_t is the price level in month t. We then construct equation (2):

$$\Delta\pi_t^{(j,k)} = \alpha + \beta(L)u_t + \gamma(L)\Delta\pi_t + \delta' x_t + \varepsilon_t, \tag{2}$$

which is similar to equation (1), save for the dependent variable and the inclusion of a vector x that contains any predetermined variables that help predict inflation at the targeted horizon.[4]

In order to express the change in inflation as a function of the difference between unemployment and a short-run NAIRU, equation (2) can always be rewritten as

$$\Delta\pi_t^{(j,k)} = \beta_0(u_t - n_t) + \varepsilon_t \tag{3}$$

with

$$\begin{aligned} n_t &= \text{short-run NAIRU} \\ &= -[\alpha + (\beta(L) - \beta(0))u_t + \gamma(L)\Delta\pi_t + \delta' x_t]/\beta(0), \end{aligned} \tag{4}$$

where all the predictive power of the equation has been subsumed in the short-run NAIRU n_t. This short-run NAIRU is not an estimate of the long-run equilibrium natural rate, but a reference rate that represents the level of current unemployment that would correspond to a forecast of no inflation change over the policy horizon.[5] Another important point that immediately falls out of this equation is that since short-run NAIRU is related to past lags of unemployment, inflation, and any other variables that help forecast changes in inflation, short-run NAIRU may undergo substantial fluctuations even if the natural rate of unemployment is a constant.

Equation (3) has several important advantages over equation (1). In contrast

4. The variable x differs from z in the Gordon (1997) and Staiger et al. (1997a, 1997b) equations in that z represents primarily supply shocks that are contemporaneous with the dependent variable, whereas x is more general in that it includes any predetermined variables other than unemployment and inflation (and their lags) that help predict future inflation.

5. Eq. (4) is a generalization of the model of short-run NAIRU in Estrella (1997). After writing this paper, we discovered that Layard and Bean (1988) also have a similar definition of short-run NAIRU in the context of a one-period change in inflation.

to the conventional equation, the dependent variable in equation (3) is the change in inflation over the target horizon. Second, the current NAIRU gap, $u_t - n_t$, is the only explanatory variable in the equation and it subsumes all the predictive power of the equation. Third, the equation provides an optimal forecast of targeted inflation, given current information.

We note, however, that our approach to short-run NAIRU is fairly general and is largely independent of the particular form of equation (3). The definition of short-run NAIRU in equation (4) simply collects all the systematic terms in equation (3), other that the current rate of unemployment. Hence, this technique is applicable to any forecasting equation for $\Delta\pi_t^{(j,k)}$, as long as the current unemployment rate u_t enters significantly in the equation.[6]

The analysis of this paper will focus on equations (2) and (3) and on our corresponding definition of short-run NAIRU. For the purposes of theoretical analysis, we use a simplified version of these equations with a limited lag structure. We return to the more general specification, however, when we consider empirical estimates using monthly data in section 9.6.

9.3 The Role of NAIRU in Policy Making: The Certainty-Equivalent Case

9.3.1 Objective Function with Inflation Only

For the theoretical analysis, we start with a simple joint model of unemployment and inflation that is isomorphic to the one employed by Svensson (1997) with an output gap. In addition to inflation π and an unemployment gap $\tilde{u}$, the model contains an exogenous variable x and a monetary policy control variable r. This model will be the basis for the next few sections of the paper. However, some specific assumptions will be adjusted in subsequent sections in order to address particular issues. Assume for the purposes of this section that the parameters of the model are known with certainty.

$$\pi_t = \pi_{t-1} - a_1\tilde{u}_{t-1} + a_3x_{t-1} + \varepsilon_t, \tag{5}$$

$$\tilde{u}_t = b_1\tilde{u}_{t-1} + b_2r_{t-1} + b_3x_{t-1} + \eta_t, \tag{6}$$

$$x_t = c_3x_{t-1} + \nu_t, \tag{7}$$

where $\tilde{u}_t = u_t - \bar{u}$ and r_t is the monetary policy variable. Equation (5) is a dynamic Phillips curve in which both unemployment and x are predictors of inflation one period ahead, say a year. Equation (6) is an IS curve, and equation (7) defines the dynamics of the exogenous variable x. The equilibrium level of

6. In eq. (2), we think of π as an I(1) process, which is consistent with current econometric evidence and practice. See, e.g., Stock (1991) and King and Watson (1994, sec. 4). Alternatively, one could think of π as an I(0) process and include a level of π in the x-vector in eq. (2).

all the variables is zero. Note, therefore, that the policy variable r might be more similar to a change in the interest rate rather than the level.

The reduced-form expression for inflation two periods ahead based on current values of the variables is

$$\begin{aligned}\pi_{t+2} &= \pi_t - a_1(1 + b_1)\tilde{u}_t - a_1 b_2 r_t \\ &\quad + [a_3(1 + c_3) - a_1 b_3]x_t + \xi_{t+2},\end{aligned} \tag{8}$$

where

$$\xi_{t+2} = -a_1\eta_{t+1} + a_3\nu_{t+1} + \varepsilon_{t+1} + \varepsilon_{t+2}.$$

Assume now that the policy objective is to minimize

$$E_t(\pi_{t+2} - \pi^*)^2 = (E_t\pi_{t+2} - \pi^*)^2 + V_t\pi_{t+2}.$$

Although this assumption seems simplistic, Svensson (1997) has shown that the solution obtained in this manner is equivalent to the dynamic solution of a model in which the target is a weighted sum of all future squared deviations of inflation from the target level. Note also that equation (8) is analogous to equation (2) above in that it corresponds to an optimal forecast of inflation acceleration over the policy horizon, which is given by

$$E_t\pi_{t+2} = \pi_t - a_1(1 + b_1)\tilde{u}_t - a_1 b_2 r_t + [a_3(1 + c_3) - a_1 b_3]x_t.$$

The conditional variance of inflation is

$$V_t\pi_{t+2} = \sigma_\xi^2.$$

Since the variance of inflation does not depend on the policy variable, the result is determined by certainty equivalence; that is, the optimal rule may be obtained by setting expected inflation equal to the target, π^*, and solving for the value of the policy variable. The optimal value of the policy variable is given by

$$\begin{aligned}r_t^* &= -\frac{1 + b_1}{b_2}\tilde{u}_t + \frac{a_3(1 + c_3) - a_1 b_3}{a_1 b_2}x_t + \frac{1}{a_1 b_2}(\pi_t - \pi^*) \\ &= -\frac{1 + b_1}{b_2}(\tilde{u}_t - n_t) + \frac{1}{a_1 b_2}(\pi_t - \pi^*),\end{aligned} \tag{9}$$

where the short-run NAIRU (defined as a deviation from $\bar{u}$) is

$$n_t = \frac{a_3(1 + c_3) - a_1 b_3}{a_1(1 + b_1)}x_t. \tag{10}$$

Equation (9) is a variant of the Taylor (1993) rule, which differs in that it is expressed in terms of unemployment rather than output. In addition, it allows

for the reference point for monetary tightening in terms of the level of unemployment to be a short-run NAIRU rather than a fixed natural rate. In effect what this variation on the Taylor rules does is bring in additional information that helps forecast inflation in deriving an optimal setting of the policy instruments.

Even in this relatively simple setting, short-run NAIRU n_t is not a constant but is instead a function of the exogenous variable x. If lags of inflation, unemployment, and the policy variable appear in equations (5) and (6), their role in the policy rule—and therefore in the definition of short-run NAIRU—would be like that of x in the model. Of course, if the only variable that helps predict inflation over the policy horizon, other than the unemployment rate, is a constant, then NAIRU will be constant as in a more standard formulation. Note also that, like $\tilde{u}$, the short-run NAIRU of our theoretical model is measured in relation to $\bar{u}$. In empirical applications, we would want to focus on the equivalent of $n_t + \bar{u}$ as a measure of short-run NAIRU.

Equation (9) also helps to clarify the proper use of NAIRU for policy purposes. The policy objective is not to drive unemployment to NAIRU, which is a temporary and variable reference point, but to use the NAIRU unemployment gap as one indicator of the direction to move the policy variable, by an amount dictated by the coefficients of the model. Also, the NAIRU gap indicator is not to be interpreted in isolation but must be weighed against the effect on the optimal setting of the policy variable suggested by the other indicator that is also included in the reaction function, the gap between actual and target inflation.

It is also important to recognize that our equation (9) variant of the Taylor rule is completely consistent with the result of Svensson (1997). Setting the policy instrument according to equation (9) is equivalent to setting expected inflation over the policy horizon equal to the inflation target π^*, which is the Svensson (1997) optimality condition if only inflation is in the objective function.

We can also draw some conclusions about the sign of the coefficient of x in the definition of NAIRU, based on whether x represents a supply or a demand effect. For example, if x is a supply effect such as an oil price shock, then a_3 and b_3 would have the same sign. Since the other parameters in equation (10) were chosen to have positive values, the two terms in the coefficient would be offsetting and the net effect of x on short-run NAIRU would be indeterminate. In contrast, if x represents a demand effect, then a_3 and b_3 would have opposite signs and the two terms would be reinforcing. The sign of the effect is positive if the demand variable x increases inflation and vice versa. In other words, a demand shock that raises inflation would lead to a higher value of short-run NAIRU, which implies more tightening given the same value of unemployment.

Supply and demand shocks also have differential effects on the overall implication about the optimal setting of the policy variable. The cumulation of

supply effects would tend to drive both unemployment and inflation in the same direction, producing offsetting effects in equation (9). Cumulated demand effects, however, would drive inflation and unemployment in different directions, providing an unambiguous policy reaction. Therefore, demand effects that raise inflation should provoke a policy tightening.

9.3.2 Output as Well as Inflation in the Objective Function

Even when inflation is the only concern of policymakers, as in subsection 9.3.1, the optimal policy assigns a significant role to the level of unemployment or to the unemployment gap, as seen in equation (9). In this section, we explore how policy should be conducted when policymakers include both inflation and output in their objectives. We do this by including a second term in the objective function, which now becomes

$$E_t(\pi_{t+2} - \pi^*)^2 + \lambda E_t \tilde{u}_{t+1}^2.$$

The economic significance of this change is that the policy objective assigns some weight to reducing the variability of unemployment around zero, which is the equilibrium level.[7]

The optimal value of the policy variable in this case is

$$r_t^{(\lambda)} = \frac{1}{(a_1^2 + \lambda)b_2}\{-[(1 + b_1)(a_1^2 + \lambda) - \lambda]\tilde{u}_t + [a_1 a_3(1 + c_3) - (a_1^2 + \lambda)b_3]x_t + a_1(\pi_t - \pi^*)\}.$$

The modification of the objective function to reflect an unemployment target changes the weights on u, x, and $\pi_t - \pi^*$ in the optimal policy rule but does not affect its general form. Specifically, the weight on $\tilde{u}_t$ relative to the weight on $\pi_t - \pi^*$ rises with λ. In the extreme, if the weight on unemployment becomes infinitely large (λ approaches infinity), the optimal rule simplifies to

$$r_t^{(\infty)} = -\frac{b_1}{b_2}\tilde{u}_t - \frac{b_3}{b_2}x_t,$$

in which the inflation gap has disappeared and only an unemployment gap remains. This result may also be obtained by certainty equivalence, setting expected unemployment equal to its equilibrium level and solving for the value of the policy variable.

7. Once again, this is a relatively simple objective function designed to highlight the key points of this paper. A more complex dynamic solution of a similar model may be found in Svensson (1997), which exhibits properties that are qualitatively analogous to those of the simpler model of this paper.

9.4 NAIRU and Policy Making: Implications of Parameter Uncertainty

9.4.1 Objective Function with Inflation Only

Uncertainty about the Natural Rate of Unemployment

We begin to examine the consequences of uncertainty in the model of section 9.3 by looking at the effects of uncertainty regarding the natural rate of unemployment or, equivalently, long-run NAIRU. We start with this particular question for two reasons. First, it seems that in the policy discussion on the use of NAIRU, it is this question that is most frequently in the mind of the policymaker, although it is not always precisely formulated. Second, the examination of this narrower issue provides helpful intuition for the more general results that follow in the rest of this section.

Thus consider a more focused version of the model of section 9.3 in which traditional long-run NAIRU is the appropriate reference point for monetary policy in terms of the unemployment rate:

$$\begin{aligned} \pi_t &= \pi_{t-1} - a_1(u_{t-1} - \bar{u}) + \varepsilon_t \\ &= \pi_{t-1} - a_1 u_{t-1} + a_0 + \varepsilon_t, \end{aligned} \tag{5a}$$

$$u_t - \bar{u} = b_1(u_{t-1} - \bar{u}) + b_2 r_{t-1} + \eta_t, \tag{6a}$$

where $a_0 = a_1\bar{u}$ and, as in section 9.3, $\bar{u}$ is the natural rate and r_t is the monetary policy variable. We write these equations explicitly in terms of $\bar{u}$ in order to focus on uncertainty with regard to this parameter. For the same reason, we assume that the parameters b_1 and b_2 in equation (6a) are known.

The second expression for equation (5a), under the natural stochastic assumptions, may be estimated using least squares. It is straightforward then to calculate the asymptotic distribution of the parameter estimates, which are consistent. In particular, we can derive that $TV(\hat{a}_1, \hat{a}_0)$, the asymptotic variance of the vector of estimates $(\hat{a}_1, \hat{a}_0)$ multiplied by the number of observations T, is

$$\begin{pmatrix} \sigma_\varepsilon^2 \\ \sigma_u^2 \end{pmatrix} \begin{bmatrix} 1 & \bar{u} \\ \bar{u} & \bar{u}^2 + \sigma_u^2 \end{bmatrix},$$

where $\bar{u}$ and σ_u^2 are the unconditional asymptotic mean and variance of u_t and $\sigma^2\varepsilon$ is the variance of ε_t. Now, if J is the Jacobian of the transformation $(a_1, a_0) \mapsto (a_1, \bar{u}) = (a_1, a_0/a_1)$, then asymptotically $TV(\hat{a}_1, \hat{\bar{u}}) = TJ V(\hat{a}_1, \hat{a}_0)J'$, which equals

$$\begin{bmatrix} \sigma_\varepsilon^2/\sigma_u^2 & 0 \\ 0 & \sigma_\varepsilon^2/a_1^2 \end{bmatrix},$$

where we have made use of the fact that the unconditional mean of equation (5a) is $\overline{\Delta\pi} = 0$.

The foregoing derivations may now be incorporated into the optimization problem of section 9.3, again with the objective function $E_t(\pi_{t+2} - \pi^*)^2$, but now

$$E_t\pi_{t+2} = \pi_t - a_1(1 + b_1)(u_t - \bar{u}) - a_1 b_2 r_t$$

and

$$V_t\pi_{t+2} = \sigma^2_{a_1}[(1 + b_1)(-u_t + \bar{u}) - b_2 r_t]^2 + a_1^2(1 + b_1)^2\sigma^2_{\bar{u}} + \sigma^2_{a_1}\sigma^2_{\bar{u}}(1 + b_1)^2 + \sigma^2_{\xi}.$$

In the expression for the variance, the terms that include $\sigma^2_{\bar{u}}$ do not depend on the policy variable. Since the estimators of $\bar{u}$ and a_1 are orthogonal, the optimal rule will not depend on the uncertainty with regard to $\bar{u}$, as shown in the expression

$$r_t^* = -\frac{1 + b_1}{b_2}(u_t - \bar{u}) + \frac{1}{1 + \tau_1^{-2}} \cdot \frac{\pi_t - \pi^*}{a_1 b_2},$$

where $\tau_1 = a_1/\sigma_{a_1}$.

Thus uncertainty about the natural rate, in and of itself, does not affect the solution to the policymaker's optimization problem, as defined in this section and in section 9.3. However, the uncertainty about the natural rate does increase the cost function because, as seen above, it increases the conditional variance of π_{t+2}. The uncertainty about the parameter a_1, the effect on inflation acceleration of the gap between unemployment and the natural rate, does figure in the optimal policy through the term $(1 + \tau_1^{-2})^{-1}$, which is a essentially a function of the t-statistic on a_1. Its effect, however, is not on the term containing the unemployment gap, but rather on the term containing the gap between current and target inflation. The greater the uncertainty about a_1, the lower τ_1 and therefore $(1 + \tau_1^{-2})^{-1}$, so the less weight the policymaker should place on the current inflation gap. This result is very robust, as it obtains in the models of subsequent sections, in which we introduce more complex specifications with fairly general parameter uncertainty.

General Parameter Uncertainty

Consider again the model defined by equations (5), (6), and (7) of subsection 9.3.1, but assume now that there is uncertainty at time t about all the coefficients of the model (a_1, a_3, b_1, b_2, b_3, c_3) and about the disturbance of the reduced form (ξ), but that the uncertainty in all of these variables is pairwise orthogonal. Although these uncertainty assumptions are not entirely general—on account of the assumed orthogonality—they are more extensive than those that the previous literature has examined.[8] The orthogonality assumptions are

8. Other papers that look at the effect of parameter uncertainty in a similar context are Svensson (1997), Clarida, Galí, and Gertler (forthcoming), and Wieland (1998).

easily relaxed for coefficients belonging to the same equation, but the inclusion of the corresponding covariances does not provide greater intuition and is therefore not pursued here. Thus, at time t, the expectation and variance of inflation at time $t + 2$ are given by

$$E_t\pi_{t+2} = \pi_t - a_1(1 + b_1)\tilde{u}_t - a_1b_2r_t + [a_3(1 + c_3) - a_1b_3]x_t$$

and

$$\begin{aligned} V_t\pi_{t+2} = {} & [a_1^2\sigma_{b_1}^2 + \sigma_{a_1}^2(1 + b_1)^2 + \sigma_{a_1}^2\sigma_{b_1}^2]\tilde{u}_t^2 + (a_1^2\sigma_{b_2}^2 + \sigma_{a_1}^2 b_2^2 + \sigma_{a_1}^2\sigma_{b_2}^2)r_t^2 \\ & + [a_3^2\sigma_{c_3}^2 + \sigma_{a_3}^2(1 + c_3)^2 + \sigma_{a_3}^2\sigma_{c_3}^2 + a_1^2\sigma_{b_3}^2 + \sigma_{a_1}^2 b_3^2 + \sigma_{a_1}^2\sigma_{b_3}^2]x_t^2 \\ & + 2\sigma_{a_1}^2[(1 + b_1)b_2\tilde{u}_t r_t + b_2b_3r_t x_t + (1 + b_1)b_3\tilde{u}_t x_t] + \sigma_\xi^2, \end{aligned}$$

where the values of the coefficients denote their expected values.[9]

As in subsection 9.3.1, the policy objective is to choose r_t so as to minimize the objective function

$$E_t(\pi_{t+2} - \pi^*)^2 = (E_t\pi_{t+2} - \pi^*)^2 + V_t\pi_{t+2}.$$

In this case, the optimal value of the policy variable is given by

$$\begin{aligned} r_t^* = \frac{1}{1 + \tau_2^{-2}}\Bigg[& -\frac{1 + b_1}{b_2}\tilde{u}_t - \frac{b_3}{b_2}x_t \\ & + \frac{1}{1 + \tau_1^{-2}}\left(\frac{\pi_t - \pi^*}{a_1b_2} + \frac{a_3(1 + c_3)}{a_1b_2}x_t\right)\Bigg], \end{aligned} \tag{11}$$

where $\tau_1 = a_1/\sigma_{a_1}$ and $\tau_2 = b_2/\sigma_{b_2}$. Equation (11) can be rewritten as

$$\begin{aligned} r_t^* = \frac{1}{1 + \tau_2^{-2}}\Bigg(& -\frac{1 + b_1}{b_2}(\tilde{u}_t - (n_t + \phi_t)) \\ & + \frac{1}{1 + \tau_1^{-2}} \cdot \frac{1}{a_1b_2}(\pi_t - \pi^*)\Bigg), \end{aligned} \tag{12}$$

where

$$\phi_t = -\frac{1}{1 + \tau_1^2} \cdot \frac{a_3(1 + c_3)}{a_1(1 + b_1)}x_t.$$

9. This convention economizes on notation and is correct by definition if the coefficient estimates are unbiased.

Comparison of equations (9) and (12) indicates that the presence of uncertainty introduces two multiplicative terms of the form $(1 + \tau_i^{-2})^{-1}$. These terms are essentially functions of the t-statistics corresponding to the parameters a_1 and b_2, respectively, which correspond to the one-period-ahead effects of unemployment on inflation and of the policy variable on unemployment. All other variance-related terms in the objective function drop out of the calculation. When there is no uncertainty about a_1 and b_2, the two multiplicative terms become one, reverting to the certainty-equivalent case of subsection 9.3.1.

One of the two uncertainty effects—the one related to b_2, the coefficient on the policy variable in equation (6)—takes a form that is predictable from the analysis by Brainard (1967). Specifically, as σ_{b_2} rises, the term $(1 + \tau_2^{-2})^{-1}$ falls so that uncertainty about the magnitude of the effect of the policy variable leads to a partial policy reaction—a reaction that is less than that in the certainty-equivalent case.

In contrast, uncertainty about a_1, the effect of unemployment on the change in inflation in equation (5), has an effect not on the scale of the policy reaction, but rather on the weight applied to $\pi_t - \pi^*$ and on the reference point in terms of unemployment at which that reaction occurs. Specifically, as σ_{a_1} rises, the term $(1 + \tau_1^{-2})^{-1}$ falls so that the weight on $\pi_t - \pi^*$ falls. A rise in σ_{a_1} causes the term $(1 + \tau_1^2)^{-1}$ and the absolute value of the adjustment term ϕ_t to rise. If x has a positive impact on inflation (i.e., $a_3 x_t$ is positive), then ϕ_t is negative and so the reference point for monetary tightening in terms of unemployment, $n_t + \phi_t$, falls.

The effect of uncertainty about a_1 on how the reference point responds to change in x is somewhat more complex. The net effect on the reference point $n_t + \phi_t$ depends on whether x is a supply or demand variable, as discussed in subsection 9.3.1. Consider the combined expression

$$n_t + \phi_t = \left(\frac{1}{1 + \tau_1^{-2}} \cdot \frac{a_3(1 + c_3)}{a_1(1 + b_1)} - \frac{b_3}{1 + b_1} \right) x_t .$$

If x is a supply variable, the direction of the effect of uncertainty on the magnitude of the reference point is unclear. It is clear, however, that as uncertainty about a_1 approaches infinity, the sign of the coefficient is the same as the sign of $-b_3$. If x is a demand variable, uncertainty reduces the absolute magnitude of the reference point unambiguously.

9.4.2 Output as Well as Inflation in the Objective Function

We now modify the results of the previous subsection by assuming that the policy objective function includes both inflation and unemployment. As in subsection 9.3.2, the objective function becomes

$$E_t(\pi_{t+2} - \pi^*)^2 + \lambda E_t \tilde{u}_{t+1}^2 .$$

The optimal value under parameter uncertainty is

$$r_t^{(\lambda)} = \frac{1}{1 + \tau_2^{-2}}\left[\left(-\frac{1 + b_1}{b_2} + \frac{\lambda}{a_1^2 + \sigma_{a_1}^2 + \lambda}\right)\tilde{u}_t + \left(-\frac{b_3}{b_2} + \frac{a_1 a_3(1 + c_3)}{a_1^2 + \sigma_{a_1}^2 + \lambda}\right)x_t + \frac{a_1(\pi_t - \pi^*)}{a_1^2 + \sigma_{a_1}^2 + \lambda}\right].$$

The effect of including a target for unemployment, as represented by λ, is analogous to the effect of uncertainty about a_1. In the above equation, these two terms occur additively in the same expression in the terms corresponding to the exogenous variable and the inflation gap. Only in the unemployment term does λ appear separately. Intuitively, the reason for this is that uncertainty about a_1 makes the relationship expressed in equation (5) less reliable, so policy becomes more concerned with affecting the "intermediate target" of equilibrium unemployment.

If the weight on unemployment becomes infinitely large, the optimal rule simplifies to

$$r_t^{(\infty)} = \frac{1}{1 + \tau_2^{-2}}\left(-\frac{b_1}{b_2}\tilde{u}_t - \frac{b_3}{b_2}x_t\right)$$

in which, as in the certainty-equivalent case, the inflation gap has disappeared and only an unemployment gap remains. Here the only effect of uncertainty is of the rescaling type, as identified by Brainard (1967).

9.5 NAIRU and Policy Making: The Implications of Model Selection

In this section, we discuss another type of uncertainty that affects the definition of short-run NAIRU, its computation, and the policy rule that results from inflation targeting. Specifically, we focus on uncertainty regarding the correct form of the basic model and the associated problem of model selection. Whereas in section 9.4 we assumed that the form of the model was known but that the parameters were estimated with uncertainty, we now suppose that the policymaker ignores some key information variable in the optimization problem.[10]

In general, if inflation two periods ahead is the policy target, and if a variable helps predict inflation at that horizon, it is inefficient not to include the information in the model. For example, the models of sections 9.3 and 9.4 define the policy rule in terms of a short-run NAIRU, which in turn is a function of the exogenous variable x. What is the result of ignoring the predictive content

10. The complementary problem of including too many variables in the model is in principle less serious, since consistent parameter estimates should assign zero weight to the superfluous variables.

of x? Alternatively, what is the cost of relying on a long-run equilibrium NAIRU (zero in this case) when a short-run informative NAIRU is available?

Thus suppose that the policymaker ignores the presence of x in the basic model (5)–(7). The values of a_3 and b_3 are implicitly set to zero, while the third equation is dropped altogether. Under these conditions, the constrained optimal rule for inflation targeting becomes

$$\tilde{r}_t^* = \frac{1}{1 + \tau_2^{-2}}\left(-\frac{1 + b_1}{b_2}\tilde{u}_t + \frac{1}{1 + \tau_1^{-2}} \cdot \frac{\pi_t - \pi^*}{a_1 b_2}\right).$$

We know, of course, that the value of the objective function has to be higher (i.e., worse) when evaluated at this constrained optimum than when evaluated at the unconstrained optimum r_t^* as in subsection 9.4.1. In fact, we can calculate the difference between the constrained and unconstrained values as

$$\frac{1}{1 + \tau_2^{-2}} \cdot \frac{1}{a_1^2 + \sigma_{a_1}^2}[a_1 a_3(1 + c_3) - (a_1^2 + \sigma_{a_1}^2)b_3]^2 x_t^2.$$

Somewhat surprisingly, uncertainty about b_2 ameliorates the left-out-variable problem.[11] Uncertainty about a_1, in contrast, can make matters worse.

The left-out-variable problem can also increase uncertainty regarding the estimates of the included coefficients, with consequences for the size of the policy response or the reference point for monetary tightening in terms of unemployment. To see this, suppose the inflation equation (5) is estimated by ordinary least squares, leaving out the variable x, after rewriting it in the following form

$$(5') \qquad \pi_t - \pi_{t-1} = -a_1\tilde{u}_{t-1} + \varepsilon_t.$$

One implication of leaving out x, well known from econometrics textbooks, is that the estimate of a_1 may be biased. This occurs unless x and u are contemporaneously uncorrelated.[12] However, even if the two regressors are indeed uncorrelated so that the estimate of a_1 is unbiased, uncertainty in the estimate is greater by the amount

$$\frac{\Sigma_t \pi_t^2}{\Sigma_t \tilde{u}_{t-1}^2} \cdot \frac{R_u^2 - R_c^2}{n},$$

where the numerator of the last term is the difference between the R^2s of the unconstrained and constrained models. Thus excluding the variable x from the model, in addition to producing a policy rule that improperly excludes x, in-

11. The intuition is that as uncertainty about b_2 grows, the optimal response of the policy variable r is reduced so that there is less loss from using the incorrect model.

12. See, e.g., Theil (1971, sec. 11.2). This problem may be bypassed formally by thinking of x as the component of the additional variable that is uncorrelated with u.

creases uncertainty about a_1. One possible consequence is that, for the reasons provided in section 9.4, the policymaker may react to the higher level of σ_{a_1} by adjusting the weight on $\pi_t - \pi^*$ downward and by increasing the absolute size of the NAIRU adjustment ϕ_t.

9.6 Empirical Estimates of Short-Run NAIRU

9.6.1 Empirical Evidence on the Importance of Uncertainty

Although our theoretical framework shows qualitatively the effects of uncertainty on how monetary policy should be conducted, it cannot tell us whether these effects are economically important. To examine this question, we estimate in this section a simple NAIRU gap model for the United States to obtain measures of uncertainty and to assess how these measures affect our view of the optimal reaction of monetary policy to movements in unemployment relative to short-run NAIRU. In order to have in the model a simple lag structure that mimics that of the theoretical model (5)–(7), we start by estimating a model with annual U.S. data over the period 1956–96. The model is

$$\pi_t - \pi_{t-1} = \alpha_0 + -\alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \varepsilon_t, \tag{13}$$

$$u_t = \beta_0 + \beta_1 u_{t-1} + \beta_2 u_{t-2} + \beta_3 r_{t-1} + \beta_4 r_{t-2} + \eta_t, \tag{14}$$

where π is the log change in the CPI from December of year $t-1$ to December of year t, u is the unemployment rate in December of year t, and r is the average monthly three-month Treasury bill rate during year t. Note that α_1 and β_3 correspond to a_1 and b_2 in the theoretical model, and that the key uncertainty ratios τ_1^{-2} and τ_2^{-2} will be based on the former. The results are presented in table 9.1.

These estimates provide some guidelines regarding the importance of uncertainty for monetary policy in this context. First, the adjustments to the unemployment reference point and to the policy reaction as a result of parameter

Table 9.1 **Estimates of Annual U.S. Model, 1956–96**

Coefficient	Estimate	t	$(1 + t^{-2})^{-1}$
α_0	1.67	1.49	
α_1	1.24	4.68	.956
α_2	.98	3.70	
R^2	.366		
β_0	.70	1.31	
β_1	1.00	7.06	
β_2	−.23	−2.18	
β_3	.48	6.30	.975
β_4	−.36	−3.84	
R^2	.833		

Table 9.2 **Implicit Interest Rate Rules**

Rule	Weight on Lagged Interest Rate	Inflation Gap	Unemployment or Output Gap
Unadjusted			
Annual	.77	1.70	−2.56
Quarterly	.94	.47	−.70
With output gap	.94	.47	.35
Uncertainty adjusted			
Annual	.77	1.59	−2.49
Quarterly	.94	.44	−.69
With output gap	.94	.44	.34

uncertainty are not large. The key parameters are estimated with some precision, and the implied multiplicative adjustment factors are both close to one. The Brainard-type adjustment—a 2.5 percent reduction—is particularly small, suggesting that the magnitude of the policy reaction should only be shaded down slightly to reflect parameter uncertainty. However, the unemployment effect adjustment is also less than 5 percent.

These results are confirmed by looking at the implicit optimal policy that corresponds to the two-year-ahead inflation target of the theoretical model in which only inflation is included in the objective function. The rule that results is very similar to the simple Taylor (1993) rule when adjustments are made for the fact that Taylor's rule was defined in terms of quarterly data and an output gap. The annual and quarterly results are presented in table 9.2. If δ is the weight on the lagged interest rate in the annual model, the corresponding quarterly lag is assigned a weight of $\delta^{1/4}$ and the weights on the inflation and unemployment gaps are divided by $1 + \delta^{1/4} + \delta^{2/4} + \delta^{3/4}$. A rule based on the output gap is obtained by applying a simple Okun's law adjustment, dividing the unemployment weight by 2.

The table confirms that the practical significance of parameter uncertainty is quite small. Furthermore, the quarterly results with the output gap are remarkably similar, even numerically, to the parameters suggested by Taylor (1993). The only key difference is that the interest rate is assumed to be much more persistent here, since Taylor did not include a lagged interest rate in the form of his rule.[13]

9.6.2 Empirical Estimates of Short-Run NAIRU

In this subsection, we present estimates of short-run NAIRU. For these purposes, we return to the more general model (2)–(4) and estimate the equations with monthly data from January 1954 to November 1997, using a 12-month-

13. Recent estimates of the Taylor rule by Rudebusch and Svensson (chap. 5 of this volume) and Rotemberg and Woodford (chap. 2), among others, suggest that the persistence parameter is close to one. Fuhrer and Moore (1995) assume that it equals one.

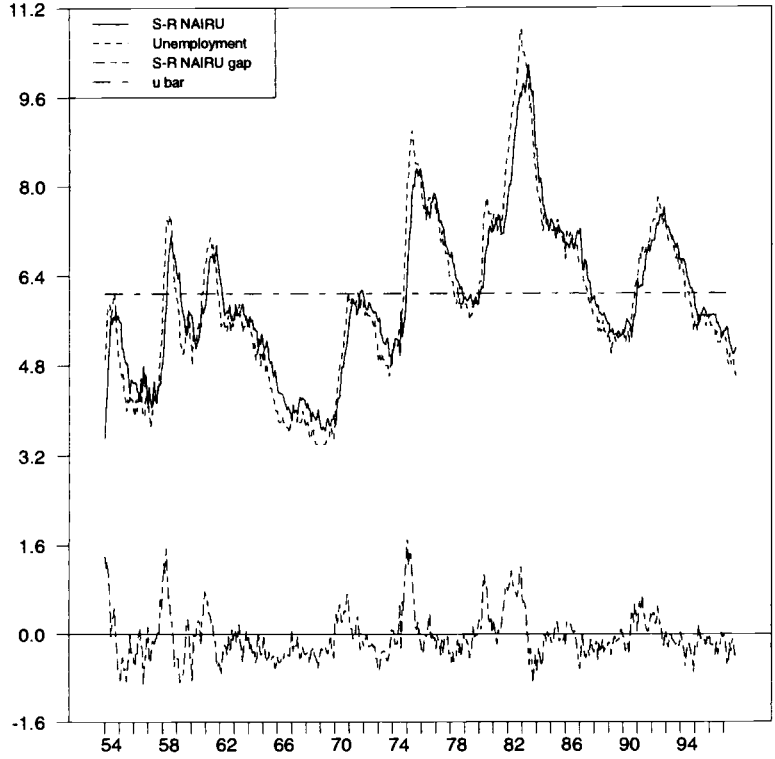

Fig. 9.1 Short-run NAIRU, unemployment, and short-run NAIRU gap, January 1954 to November 1997

ahead, 12-month horizon ($j = k = 12$) and 12 lags of both the change in inflation and unemployment.[14] Figure 9.1 shows estimated short-run NAIRU together with the contemporaneous unemployment rate, as well as the short-run NAIRU gap. This figure demonstrates the high variability of short-run NAIRU, in contrast with long-run measures designed to estimate a natural rate as in Gordon (1997) and Staiger et al. (1997a, 1997b). For example, consider a version of our equation (1), which may be used to estimate a constant $\bar{u}$ that is comparable to the long-run measure of those papers:

$$(1') \qquad \Delta\pi_t = \beta(L)(u_t - \bar{u}_t) + \gamma(L)\Delta\pi_{t-1} + \varepsilon_t.$$

When estimated over the same period as equations (2), (3), and (4), the estimate of $\bar{u}$ is 6.1 percent, as shown in figure 9.1.

14. Somewhat surprisingly, extending the horizon to $j + k = 60$ months or even longer does not materially affect the point estimates of short-run NAIRU. Of course, the fit of the equation deteriorates with longer horizons.

Staiger et al. (1997a) have pointed out that such estimates of a constant long-run NAIRU tend to be quite imprecise. Using the delta method in an equation similar to (1′), they obtain an estimate of $\bar{u} = 6.2$ percent, with a standard error of about 0.6. Our estimate of $\bar{u} = 6.1$ has a standard error of 0.43, which is somewhat smaller—perhaps partly because of our larger sample—but is of the same order of magnitude. Estimates of short-run NAIRU n_t are more precise. The standard error of n_t is a time-varying function of the values of the variables in expression (4). Over the sample period, the standard errors range from 0.11 to 0.42, with a mean of 0.20, less than half of the standard error of $\bar{u}$.[15]

Thus short-run NAIRU is estimated with more than twice the precision than standard long-run NAIRU. The practical significance of this result, however, is limited, since we have shown in the theoretical sections that this type of uncertainty plays no role in the determination of the policy rule. Nevertheless, a reduction in the uncertainty may produce a reduction in the value of the cost function, as shown in section 9.5, even if the policy rule remains unaltered.

9.6.3 A Case Study: Recent Signals from a Short-Run NAIRU

Using the estimates of the NAIRU gap from subsection 9.6.2, we now examine the hypothetical results of using the methodology of this paper in the conduct of monetary policy in the United States since June 1992, when the unemployment rate began a prolonged decline. The results will of course be somewhat simplistic, but they may provide some general support for the concepts developed in this paper.

If we refer to one of the policy rules in the theoretical part of the paper, say to equation (9), we note that the appropriate interest rate is determined essentially by two gaps: the difference between unemployment and short-run NAIRU and the difference between actual and target inflation. We present in figure 9.2a the gap between short-run NAIRU and unemployment (signed so that a positive value indicates that monetary policy should be tightened) and the level of inflation (12 previous months) since 1992.

From June 1992 to the end of 1993, declining unemployment brought the NAIRU gap from levels suggesting, if anything, the need for ease to relatively neutral levels. Meanwhile, inflation declined over the period and, in fact, continued to decline into the beginning of 1994. Beginning in 1994, however, the NAIRU gap became positive and remained so until early 1995, suggesting a need for tightening. In addition, inflation stopped declining, remaining around the 3 percent level. These two factors combined are consistent with the monetary tightening undertaken by the Federal Reserve throughout 1994 and into early 1995.

Since then, the NAIRU gap has indicated some pressure to tighten twice, in 1996 and 1997. In the first case, the pressure from the NAIRU gap was accom-

15. All our standard errors are estimated consistently using the Newey-West technique with a 24-lag window (Newey and West 1987).

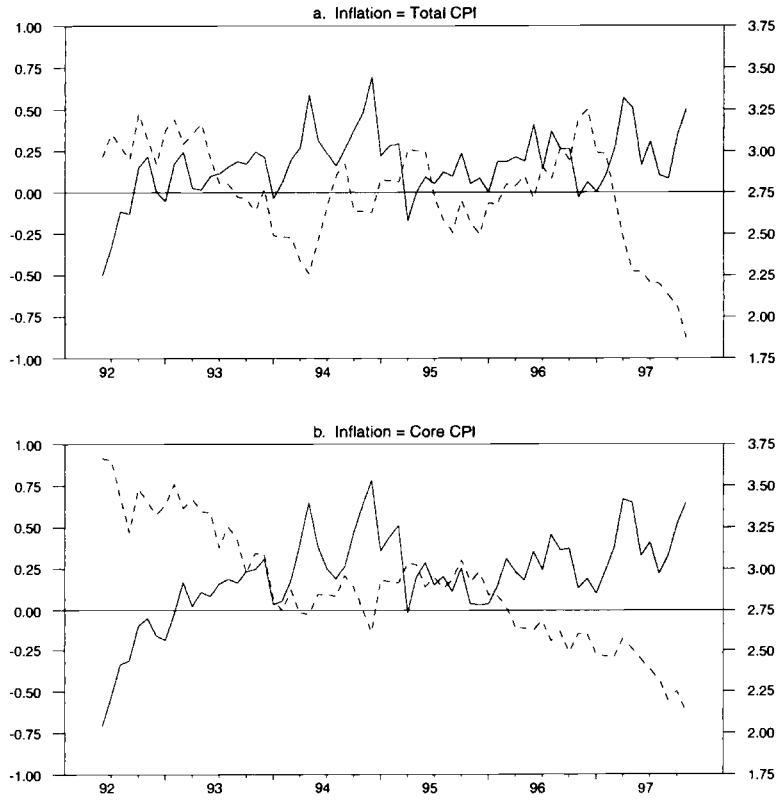

Fig. 9.2 NAIRU gap (*solid line, left-hand scale*) and inflation (*dashed line, right-hand scale*), June 1992 to November 1997

panied by a rise in inflation. Even though inflation subsided toward the end of the year, this episode may seem somewhat inconsistent with the absence of further tightening. Figure 9.2b suggests one reason for this result. Figure 9.2b presents the results of repeating the analysis of figure 9.2a, but using core inflation (excluding food and energy prices) instead of total inflation. Core inflation tends to be a better signal of persistent changes in inflation than total inflation.

Figure 9.2b shows both the level of core inflation as well as the gap between unemployment and short-run NAIRU computed using core inflation in equations (2), (3), and (4). Comparisons of the two panels of figure 9.2 suggests that the effect of using core inflation in the calculation of the NAIRU gap is very slight. But core inflation was falling in 1996, in contrast to the rising total inflation, and this fall may have offset the tightening signals from the NAIRU gap.

In 1997, the pressure arising from the unemployment gap seemed stronger

than in the previous year. Inflation, however, both total and core, moved downward again, offsetting at least partially the signals from the NAIRU gap indicator. Arguably, only during 1994 and early 1995 were there consistent signals for tightening, and this is when the Federal Reserve engaged in most of its monetary tightening.

In order to evaluate the net effect of the unemployment and inflation indicators, it would be helpful to summarize the information in a single measure, as in the policy rules of table 9.2. We would like to do this, not to explain actual policy, but to suggest how the theoretical constructs of this paper could be used in practice. However, this is a problem for two reasons. First, we would have to construct a full optimization model in the context of the monthly equations, which is beyond the scope of the present paper.[16] Second, we would have to know or make an assumption about the target level of inflation. Thus we present only a limited version of a policy rule in which we deal with those problems as follows.

First, we take the weights for the NAIRU and inflation gaps from the annual results of table 9.2 allowing for uncertainty, making allowance also for the monthly frequency of our data. Since the coefficient of the lagged interest rate, $\delta^{1/12} = 0.98$, is very close to one, we further simplify by assuming that the weights are used to calculate a monthly change in the interest rate. We then divide the annual weights by $1 + \delta^{1/12} + \cdots + \delta^{11/12}$ to obtain weights of -0.23 for the NAIRU gap and 0.15 for the inflation gap with total inflation and -0.25 and 0.19, respectively, using core inflation.[17] To deal with the second problem, the fact that the inflation target is unknown, we scale the results so that the policy rule with total inflation is neutral, on average, over the period since June 1992. This assumption is equivalent to an inflation target of 3 percent.

The results are presented as the solid lines in the two panels of figure 9.3. Note that the weighted results are consistent with our earlier discussion of the individual components. In panel 9.3a, which contains the results using the total CPI, the strongest signal for tightening comes during 1994. Note also, however, that there were distinct signals for tightening in 1992–93 and 1996–97, and that there were fairly strong signals for easing at the beginning and toward the end of the sample period. In panel 9.3b, which contains results using the core CPI, there are also strong signals to tighten in 1994, but because the core inflation rate was higher than total CPI in late 1992 and early 1993, there are also strong signals to tighten in this period. In contrast to panel 9.3a, the results with the core CPI do not suggest any need to tighten in 1996.

16. A model along those lines has been developed for the United States in Clarida et al. (forthcoming). See also the references in that paper.

17. Adjusting fully for coefficient uncertainty would require, in addition to the adjusted weights, an adjustment to short-run NAIRU corresponding to the term ϕ_t defined in subsection 9.4.1. We do not make this adjustment here because our equation for monthly NAIRU is essentially a reduced form and the components are difficult to disentangle, and also because the coefficient of the adjustment factor $(1 + \tau_1^2)^{-1} \leq 0.065$ is empirically small.

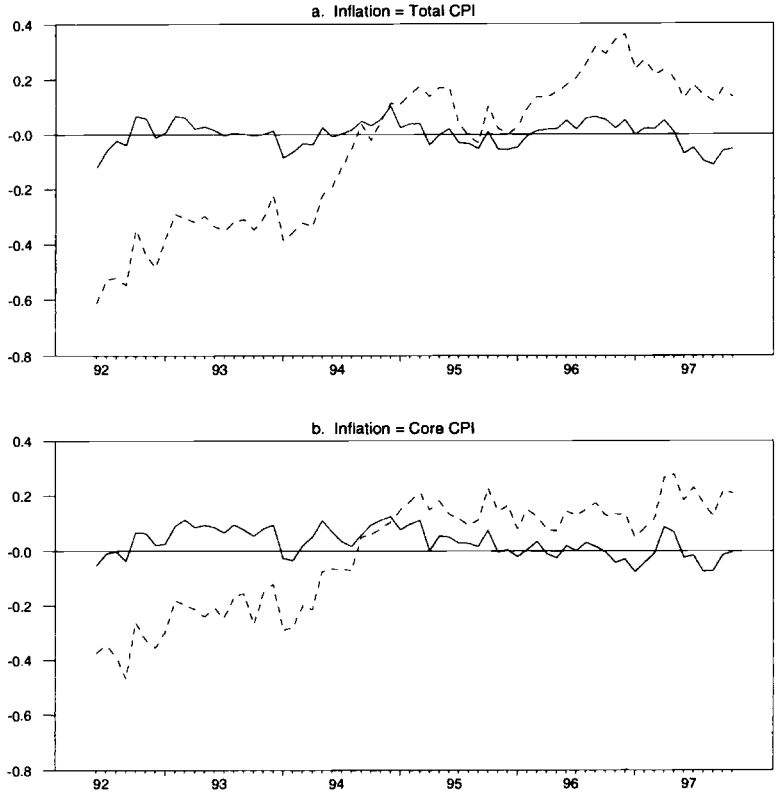

Fig. 9.3 Simple policy rules based on short-run NAIRU (*solid line*) and long-run NAIRU (*dashed line*), June 1992 to November 1997

We may contrast these results with a rule based on the standard unemployment gap—the gap between unemployment and a constant long-run NAIRU. The results are presented as the dashed lines in the two panels of figure 9.3. To obtain weights that are consistent with the assumption of a constant NAIRU, we estimated equations (13) and (14) without the second lag of unemployment, which produces an estimate of NAIRU that is constant. These new weights are −0.35 for the NAIRU gap and 0.34 for the inflation gap using total inflation and −0.36 and 0.37, respectively, using core inflation. Note, however, that if we use the same weights as before, the qualitative results are the same as with these weights.

The results for the long-run NAIRU gap, which are driven by the large steady decline in unemployment over this period, are fairly robust. The main feature of the alternative rule is that it argues for easing throughout the first part of the period, and then for tightening throughout the second part of the period. What this rule misses is that a long-run natural rate is not the best

reference point for unemployment if the goal is to target inflation in the short run.

9.7 Summary and Conclusions

In this paper, we examine how a variant of the NAIRU concept can be usefully employed in the conduct of monetary policy. By thinking of NAIRU in this way, we obtain insights that might be quite useful to monetary policymakers. Because there are quite a few results sprinkled throughout the paper, we list the main ones here.

- The NAIRU concept that is useful for the conduct of monetary policy differs from the estimate of the natural rate of unemployment, the long-run concept used previously by many researchers. Instead, NAIRU can be viewed as a short-run construct, which is related to past levels of unemployment and inflation as well as other economic variables, that helps forecast future accelerations or decelerations of inflation.
- Short-run NAIRU should be viewed, not as a target for policy, but as an aid in defining the reference point that policymakers can compare to the current rate of unemployment to derive a signal for the appropriate stance of policy. Furthermore, as long as inflation is an element in the policymakers' objective function, the NAIRU gap is not the only signal that should affect the setting of policy instruments: the deviation of inflation from its target level also has an important role in the determination of the appropriate stance of policy.
- The policy rule that comes out of our analysis is a variant of a Taylor rule using an unemployment gap rather than an output gap, but it has one major difference from more standard formulations. The standard Taylor rule implicitly assumes that the reference point to which unemployment should be compared in the unemployment gap term is constant, while in our formulation, the reference point is related to short-run NAIRU, which can have substantial short-run fluctuations over time.
- Uncertainty about the level of NAIRU has no influence on the setting of policy instruments, although it does affect the value of the objective function. This type of uncertainty makes the economy worse off but does not alter policy behavior.
- Uncertainty about the effect of unemployment on inflation leads to an additive adjustment to short-run NAIRU to calculate the reference point for monetary tightening in terms of the level of unemployment. In addition, uncertainty about the unemployment effect on inflation changes the weight on the inflation gap in the policy rule.
- Uncertainty about the effect of the policy variable leads to a scaling down of the reaction of the policy variable, the well-known Brainard (1967) result.
- Uncertainty about model selection can have important effects on the form of the policy rule. In particular, if a constant NAIRU is used—as occurs if

NAIRU is viewed as a long-run concept—so that information about the state of the economy that could be used to forecast inflation is ignored, the performance of the policy rule can be substantially worse. In addition, leaving out relevant variables that help forecast inflation increases the uncertainty about the effect of unemployment on inflation, with the resulting implications described above.

- Although parameter uncertainty has potentially large effects on how policy should be conducted, our empirical results suggest that parameter uncertainty may not be all that important for the setting of policy. We find some evidence of changes in the policy rule resulting from the parameter uncertainty we explored in our theoretical model, but these effects are very modest. They affect the weights in the policy rule by less than 5 percent in both the case of uncertainty about the impact of unemployment and the case of uncertainty about the effect of the policy variable.
- Estimates of short-run NAIRU are highly variable over time. However, there is a fair degree of precision in these estimates.
- Substantial positive NAIRU gap estimates arose throughout 1994 and early 1995 and in parts of 1996 and 1997. However, core inflation was substantially lower in 1996 and 1997 than in 1994. Thus the one period since June 1992 during which there were consistent signals for tightening occurred during 1994 and early 1995, which is when the Fed engaged in most of its monetary tightening.

These results suggest that a short-run NAIRU is indeed a useful concept and that it can be used by policymakers, particularly in deciding how monetary policy should be conducted. However, there are some subtle issues in how the short-run NAIRU concept might be used correctly. First, because our view of NAIRU sees it as a short-run construct, it is dangerous to think of NAIRU as a potential target for unemployment that stays around a particular value, such as 6 percent, for any period of time. Second, deviation of inflation from its target is every bit as important a factor in thinking about setting policy as is the NAIRU gap. Third, uncertainty about parameter values and model selection does have effects on the optimal setting of policy instruments but does not appear to be a barrier to a useful role for the NAIRU concept in policy decisions.

We hope that this paper helps resurrect NAIRU as a useful concept, but only if it is used properly. As we have shown, a short-run NAIRU is a useful construct because it helps tell policymakers what might happen to inflation in the future. Furthermore, the model of this paper suggests that policymakers may want to avoid the impression that an objective of policy is to raise unemployment when it falls below NAIRU or to lower it when it is above NAIRU. To view policy in this way might lead the public to think that policymakers are against low unemployment, an outcome that can reduce support for central bank efforts to control inflation.

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Comment

Robert E. Hall

Reading Estrella and Mishkin's paper made me aware for the first time of the difference in the use of the term NAIRU between academics and practitioners. As an academic out of touch with the parlance of central bankers, I thought that the NAIRU was the unemployment rate such that there is no pressure on inflation from the labor market. In my mind, the NAIRU was effectively another name for the natural rate of unemployment. I learned from this paper that—in circles closer to the real world—the NAIRU is the unemployment rate such that pressure from the labor market is sufficient to offset supply shocks or other transitory sources of inflation. The NAIRU, in those circles, would be the target unemployment rate for a monetary policy that sought to stabilize the inflation rate period by period. Alternatively, as this paper shows, the NAIRU is an element in a monetary policy that rolls with the punches and permits some variation in inflation in the shorter run because there is value in stabilizing real activity as well as inflation.

I will start with some comments on equation (1′), the framework for estimating the natural rate. These comments also apply to the related work by Staiger, Stock, and Watson (1997). The idea of this research is expressed in the following equation:

$$\pi_t = \alpha - \beta u_t + \gamma(L)\pi_{t-1} + \varepsilon_t.$$

My notation is the same as the paper's. The natural rate $\bar{u}$ is defined as the value of u such that the inflation rate stays at the constant level π. It is

$$\bar{u} = \frac{\alpha - [1 - \gamma(1)]\pi}{\beta}. \tag{1}$$

Here $\gamma(1)$ is the sum of the coefficients on lagged inflation in the Phillips curve.

To interpret this equation, we need to go back to the very dawn of the rational expectations era. Prior to Milton Friedman's (1968) American Economic Association presidential address, this equation was seen as describing a long-run trade-off between inflation and unemployment. If $\gamma(1) < 1$, the trade-off is positive. Friedman made a broad argument from first principles that there could not be a long-run trade-off; rather, the unemployment rate would tend to the natural rate, invariant under inflation in the longer run. There followed a brief period of research based on the mistaken belief that Friedman's hypothesis could be tested by estimating $\gamma(L)$ and testing the hypothesis $\gamma(1) = 1$. Then Sargent (1971) straightened the subject out by observing that the coefficients $\gamma(L)$ depend on the nature of monetary policy. If policy makes inflation return to a normal level by offsetting movements away from that level, then

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$\gamma(1) < 1$ even if Friedman is right—as we all now agree—that unemployment is invariant in the long run to the choice of inflation target.

Estrella and Mishkin are unwilling to face up to Sargent's point. Despite the emphasis I gave at the conference to this point, they have not altered this part of the paper and do not cite Sargent, who was present at the conference and agreed that this paper fell into the trap he identified in 1971.

From equation (1), it is apparent that $\bar{u}$ is not identified as a general matter by the equation. There are two possible identifying hypotheses. First, if the long-run rate of inflation, π, is known and is truly constant over time, then the other coefficients in equation (1) can be estimated by standard methods and the equation solved for $\bar{u}$. Second, one could assume that $\gamma(1) = 1$. The authors do the second. I do not believe that this assumption makes sense, especially for data starting after the rationalization of monetary policy in 1979.

We normally view the coefficients $\gamma(L)$ as a description of expectations about inflation. The Phillips curve relates unexpected inflation to the unemployment rate. Under that interpretation, the coefficients $\gamma(L)$ sum to one if inflation does not revert to some mean value but is an integrated process. But one feature of monetary policy agreed upon by every commentator is that policy should always induce mean reversion in inflation. The debates in monetary policy are over the speed of the mean reversion, and also over whether we should induce mean reversion in the price level. In the latter case, mean reversion in the inflation rate occurs automatically. And, at least since 1979, there seems little doubt that policy has tried and succeeded in making inflation mean reverting. Any hint of an upsurge in inflation results in the Fed's stepping on the brake to bring inflation back to target.

A second reason to expect mean reversion in the rate of inflation is that the main source of price disturbances—movements in the price of oil—are temporary. Even without good monetary policy, bursts of inflation are temporary.

Although simple tests of mean reversion of inflation support the hypothesis, I learned at the conference from the master of this craft—James Stock—that one cannot reject the hypothesis of no mean reversion. That gives Estrella and Mishkin some support. But my impression is that the test is not very powerful. Someone like myself, a Bayesian in such matters, can follow his prior and believe that $\gamma(1)$ is well below one. As a result, I do not find the empirical results in this paper, based on the fundamental identifying hypothesis $\gamma(1) = 1$, to be informative.

A second factor also inhibits this type of empirical research. Suppose that the Fed determines policy by minimizing the expected value of the squared deviation of inflation from target. To do so, it sets the current values of variables it controls, such as the unemployment rate, in such a way as to make them uncorrelated with the future inflation deviation. In such a world, a researcher running Phillips curve regressions would find a completely flat rela-

tion between unemployment and inflation. It is well known that optimization can completely conceal structural relationships.

The fact that Estrella and Mishkin's Phillips curve slopes downward and is not completely flat shows that the Fed is not pursuing a policy that conceals the slope. But the problem still lurks in the background. To the extent that there is purposeful policy, the slope of the Phillips curve obtained by regression is biased, and the value of the natural rate is correspondingly biased. In general, I think the paper neglects identification issues in a pretty serious way.

There is a robust estimator of the natural rate available, but it is not discussed in the paper. As Friedman pointed, the unemployment rate fluctuates around the natural rate irrespective of the monetary regime. Hence, the average value of the unemployment rate is a good estimate of the natural rate. No further identifying hypotheses are needed. Two estimates based on this method are 6.08 (0.51) for 1960–96 and 6.05 (0.30) for 1983–96. The standard errors are based on an AR(2) error process. These results are quite similar to Estrella and Mishkin's, though the standard errors are larger. Of course, a finding that the natural rate is over 6 percent only deepens the mystery of low inflation in 1998, with an unemployment rate almost 2 percentage points lower than the natural rate.

To summarize in this area, I believe that Estrella and Mishkin neglect identification issues but nonetheless find reasonable values of the natural rate. But I think that they seriously understate the sampling variation in their estimates. I do not believe that we know the natural rate at any particular time with a precision of a few tenths of a percentage point, as they claim.

The paper pays a lot of attention to uncertainty. It uses the framework of quadratic preferences, which is a reasonable starting point. But decision makers with quadratic preferences do not behave in a precautionary way. There are no catastrophes in the way that a quadratic decision maker views alternative random outcomes. For example, a consumer with quadratic preferences does not have marginal utility rising to infinity as consumption approaches zero. Instead, marginal utility is finite at zero consumption. Consequently, quadratic decision makers do not take special precautions to avoid situations like zero consumption or 20 percent inflation. An early next step in the research should be the exploration of behavior under more realistic preferences, such as constant relative risk aversion.

In particular, Brainard's (1967) point that uncertainty about the slopes of structural relations makes decision makers more timid is firmly rooted in quadratic preferences. Decision makers who are anxious to avoid catastrophic outcomes may need to take aggressive action in the presence of slope uncertainties. I believe the special features of Brainard's analysis are inadequately appreciated.

The paper's findings, in table 9.2, of small adjustments for parameter uncertainty need to be accompanied by two warnings. First, as I just noted, they

are special to quadratic preferences; precautionary decision making would be different. Second, I think the paper understates the actual amount of parameter uncertainty. Its lack of concern with identification and corresponding use of least squares almost automatically causes it to understate sampling variation.

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Discussion Summary

Donald Kohn never heard NAIRU discussed as a target at a Federal Open Market Committee meeting, as implied in the paper, but rather as an input into an inflation forecast. Moreover, those inflation forecasts take account of a wider variety of factors affecting prices and therefore already embody the fact that the unemployment rate could differ from its long-run natural rate for quite some time and still be consistent with steady inflation.

Laurence Ball expressed confusion about the empirical results in the paper, especially about figure 9.1. NAIRU seems to be very close to actual unemployment. For example, the unemployment rate in 1982 consistent with stable inflation was about 10 percent. The actual unemployment rate in 1982 was about 10.5 percent, so for the difference between these two rates to matter, it must be multiplied by a huge multiplier. *Bob Hall* agreed that figure 9.1 was very confusing.

Laurence Meyer remarked that tightness in the labor market and supply shocks should not be put into a single variable. Since supply shocks are often transitory, the central bank should not be looking at short-run NAIRU in any case.

James Stock made two comments. First, the issue regarding the sum of the coefficients in the Phillips curve came from the outward drift in the Phillips curve in the 1970s. Empricially, a good specification of inflation is a unit root with a moving average (MA). The MA has with a root of about 0.4, which implies a first AR coefficient of 0.6. *Richard Clarida* objected that with post-1979 data, a standard Dickey-Fuller test does find mean reversion in inflation.

Mishkin noted that with a large MA coefficient, unit root tests are typically affected by a small-sample problem. In this case, a significant Dickey-Fuller test is on the order of 10 rather than 2.5.

Stock's second comment was that in terms of forecasting inflation, many variables have forecast better than unemployment over the last 30 years, such as the natural rate of new claims to unemployment insurance, capacity utilization, housing starts, and adjustments in any of these variables.

Edward Gramlich asked Hall whether the sample mean in his estimation of NAIRU is subject to regime changes. *Hall* replied that this depends on the length of the sample. He mentioned that Milton Friedman avoided taking a stand on the issue of how long it takes for the sample mean of unemployment to respond to a change in policy regime. Regarding the question whether the natural rate of unemployment has declined in the last few years, there is persuasive evidence that it has. Labor macroeconomists find that the duration of unemployment is extremely low. The labor market seems to deal better with routine unemployment, and computer matching starts mattering for the duration of unemployment.

David Longworth mentioned that none of the papers in the conference looks at the changes in the variances of the equations themselves. There are some rules, such as inflation targeting, that would be expected to lower the variance in the Phillips curve.

John Williams noted that the Phillips curve is estimated in the Federal Reserve Board's large-scale macroeconometric model by including expected inflation on the right-hand side and then imposing the condition that the sum of coefficients on all leads and lags of inflation be one.

Michael Woodford remarked that a policy rule based on the difference between NAIRU and current unemployment, as defined by these authors, is equivalent to a policy rule based on inflation and a forecast of future inflation. He asked in that case why a discussion of current economic conditions by policymakers should focus on beliefs about NAIRU rather than on inflation forecasts directly.

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