This is an ambitious paper. It documents a novel set of facts about the nature of labor market expansions during the last 15–20 years in the United States, and it develops a significant extension to the Burdett and Mortensen (1998) model to explain them. I am not going to quibble with the facts. Moscarini and Postel-Vinay are correctly cautious in noting that most of their data cover only two recessions, in 1991 and 2001. Both events were mild by historical standards, and the comovement of labor market variables, for example, unemployment and hours, and other economic outcomes, notably labor productivity, was unusual by historical standards. Unfortunately, there is little to do about this except wait until the economy generates more data and possibly look for data from other countries.

I instead focus this discussion on the authors’ theoretical explanation for these facts. I start by explaining the mechanics of their model, the forces driving the firm size distribution and the amount of job-to-job transitions. The main point of this part of my discussion is that one can understand the mechanics without studying firm optimization, which hopefully clarifies how the model works. I then turn to wage determination. While Moscarini and Postel-Vinay assume that firms have the ability to fully commit to a path of wages, they also implicitly assume that following an aggregate shock, firms can reoptimize. Since the solution to the firm’s problem is time inconsistent, I discuss whether reoptimization, rather than the shock, drives the transitional dynamics of wages in the model. Finally, I offer an alternative model of wage determination that both avoids this time inconsistency issue and improves the model’s performance along other dimensions.

I. Mechanics

The mechanics of the model are based on a simple and natural set of assumptions. First, at time $t \in [0, \infty)$, unemployed workers find firms
according to a Poisson process with arrival rate $\lambda_{0,t}$, and employed workers find firms at rate $\lambda_{1,t}$. Second, all workers agree on a ranking of the firms. More precisely, a typical firm is named $x \in [0, 1]$; without loss of generality, firm names are uniformly distributed. An employed worker switches from firm $x$ to a newly found firm $x'$ whenever $x' > x$ and not otherwise.\(^1\) Third, worker-firm relationships end exogenously according to a Poisson process with constant arrival rate $\delta$. This set of assumptions delivers a rich theory of unemployment, the firm size distribution, and job-to-job transitions.

Start with unemployment. According to these assumptions, the fraction of workers who are unemployed evolves according to

$$u_t = \delta(1 - u_t) - \lambda_{0,t} u_t.$$  

(1)

The unemployment rate increases when an employed worker, a fraction $1 - u_t$ of the population, loses her job, at rate $\delta$. It falls when an unemployed worker finds a job, at rate $\lambda_{0,t}$. This is a standard formula in search models, and its quantitative implications are well understood. For example, suppose that $\lambda_{0,t}$ changes from $\lambda_0$ to $\lambda'_0$ at $t = 0$. If $\lambda_{0,t}$ had been constant long enough before $t = 0$ for the unemployment rate to reach its conditional steady state, $u_0 = \delta/(\delta + \lambda_0)$, the subsequent evolution of the unemployment rate satisfies

$$u_t = e^{-(\delta + \lambda'_0)t} \frac{\delta}{\delta + \lambda_0} + [1 - e^{-(\delta + \lambda'_0)t}] \frac{\delta}{\delta + \lambda'_0}.$$  

(2)

Since the unemployment rate is a first-order autocorrelated random variable, it is straightforward to compute the time it takes it to converge halfway to its conditional steady state, $\log 2/(\delta + \lambda'_0)$ periods. With standard parameterizations, the transitional dynamics end trivially fast. For example, in their paper Moscarini and Postel-Vinay set $\delta = 0.025$ and $\lambda'_0 = 0.40$ (compared with $\lambda_0 = 0.37$), all expressed in monthly units. Then the shock reduces the unemployment rate from 6.3% to 5.9% and the halftime of convergence is approximately 1.6 months.

Now turn to the firm size distribution. Let $L_t(x)$ denote the number of workers employed in a firm of rank less than $x$ at $t$. This evolves as

$$\dot{L}_t(x) = x\lambda_{0,t} u_t - [\delta + (1 - x)\lambda_{1,t}] L_t(x).$$  

(3)

It increases when an unemployed worker finds a job in a firm with rank less than $x$, at rate $x\lambda_{0,t}$. It decreases when a worker employed in such a firm loses her job, at rate $\delta$, or finds a job at a firm with rank above $x$, at rate $(1 - x)\lambda_{1,t}$. It is again straightforward to solve this differential equation. Suppose now that $\lambda_{0,t}$ changes from $\lambda_0$ to $\lambda'_0$ and $\lambda_{1,t}$ changes...
from $\lambda_1$ to $\lambda'_1$ at $t = 0$. Assuming that both parameters had been constant long enough for the distribution of workers across firm sizes to reach a steady state, one can use the previous equations to prove that

$$L_t(x) = x \left\{ \frac{\delta \lambda_0}{(\delta + \lambda_0)(\delta + \lambda_1(1 - x))} e^{-[\delta + \lambda'_1(1 - x)]t} + \lambda'_0 \int_0^t e^{\delta + \lambda'_1(1 - x)(t' - t)} u_{t'} dt' \right\};$$

(4)

it is straightforward to differentiate this equation to get back to equations (2) and (3). There is no longer an exact expression for the halftime of convergence to steady state. However, noting that the unemployment rate rapidly reaches its steady-state value, replace $u_{t'}$ with the conditional steady state $\delta/(\delta + \lambda'_0)$ in equation (4). Then the halftime of convergence to steady state for $L_t(x)$ is $\log 2/[\delta + (1 - x)\lambda'_1]$. While this is reasonably short when $x = 0$, convergence dynamics are slow for $x = 1$. In the calibrated model, $\lambda'_1 = 0.12$, and so the halftime of convergence ranges from 4.8 to 27.7 months, depending on the firm’s rank. In summary, while the transitional dynamics of the unemployment rate is fast enough to be economically uninteresting, the transitional dynamics of the fraction of workers in firms of different size classes is slow.

The objects that Moscarini and Postel-Vinay plot in their figure 14 correspond to the densities $\partial L_t(x)/\partial x$, although I have omitted one important detail: I have assumed that workers are equally likely to contact all firms, whereas in the paper workers are more likely to contact larger firms. To bridge this gap, one has to reinterpret $x$ as the fraction of lower-named firms, weighted by workers’ contact probability. In any case, the results I obtain are qualitatively similar to those reported in the paper. While small (low-$x$) firms grow immediately after the shock, they soon shed workers to big (high-$x$) firms, which grow continuously after the shock. In the short run, employment shifts toward the small firms, but in the long run the positive shock moves employment toward the large firms, the main thesis in this paper.

While the model is qualitatively successful at explaining the firm (or establishment) size distribution and its recent comovement with labor market expansions, I am skeptical that the explanation is correct. According to the model, firm size is entirely determined by firms’ ability to attract and retain workers. All firms in Moscarini and Postel-Vinay’s world would always prefer to have more workers at a given wage, a prediction that is surely counterfactual. The authors suggest that their theory may omit two important limitations on firm size: credit constraints
and the availability of workers with the appropriate human capital. While I agree that there may be a role for these factors in explaining firm size and growth, this discussion seems to miss the most important determinant of firm (and establishment) size: technology. Buera and Kaboski (2008) find that the average manufacturing establishment has 47 workers and the average manufacturing firm has 57. The corresponding numbers for services are 14 and 18. Those authors argue that this reflects the nature of the production technology and the need for service producers to locate close to their customers, not simply an unwillingness to pay high wages. Limits to span of control (Lucas 1978) are also presumably an important determinant of optimal firm size. But if these factors are also important, how should we interpret the consistency between the model and the recent behavior of the firm size distribution? Should the goal be to match the behavior of the entire distribution or just a distribution conditional on some narrowly defined firm characteristics that determine its optimal size? Moscarini and Postel-Vinay argue for the former, whereas my sense is that the model is more appropriate for explaining conditional size distributions.

Even if the model abstracts from significant pieces of the explanation for the firm size distribution, individual workers’ job turnover dynamics are at least in part governed by their search for better opportunities, measured by wages, job amenities, match quality, and so on. This the model captures well. Although one worker may prefer firm \( j \) to \( j' \) and another may prefer \( j' \) to \( j \), we can still let \( L_t(x) \) denote the probability that any worker is employed at a firm in the lowest \( x \)th percentile of her own personal firm distribution. Following the posited shock, this satisfies (4), and the rate at which a worker switches jobs conditional on being employed is

\[
J_t = \frac{\lambda_{1,t}}{1 - u_t} \int_0^1 (1 - x) \frac{\partial L_t(x)}{\partial x} dx = \frac{\lambda_{1,t}}{1 - u_t} \int_0^1 L_t(x) dx.
\]

(5)

To understand the first equation, note that an employed worker finds a new firm at rate \( \lambda_{1,t} \) and accepts it with probability \( 1 - x \) if her current firm is at the \( x \)th percentile of her distribution, with density \( \partial L_t(x)/\partial x \). Dividing by the share of workers who are employed gives the rate at which an average unemployed worker switches jobs. The second equation follows from the first using integration by parts.

Using equation (5), I find a path for the job-to-job transition rate that is quantitatively identical to the one in Moscarini and Postel-Vinay’s figure 18. Thus this figure is substantially more general than the specific
assumptions in the paper. The job-to-job transition rate jumps up upon the impact of the shock, continues rising as unemployed workers take bad jobs, but eventually falls back below its immediate postshock value as workers move into better jobs. But although the authors do not stress this point, the result is a quantitative failure. Look at the scale in figure 18. The monthly job-to-job transition rate jumps up from 2.78% to 2.83%, rises slightly, and then falls back to 2.81% in response to a fairly sizable shock. Such movements are quantitatively trivial and would be unnoticeable in their figure 10, which shows the empirical job-to-job transition rate moving by an order of magnitude more.

Given the short time period covered by figure 18, one possible explanation is that the model is correct and the data are anomalous. That is, $J_t$ is barely cyclical. That does not seem to be the case. I obtain a longer measure of the job-to-job transition rate from the public use micro data in the March supplement to the Current Population Survey (CPS). Since 1976, the March supplement has asked workers how many employers they had in the previous year. A coarse measure of the number of job-to-job transitions is $\sum_{i=1}^{\infty} n_i(i - 1)$, where $n_i$ is the number of workers reporting $i$ employers. I multiply this by 52 and divide by the total number of weeks worked in the previous year to obtain the job-to-job transition rate per full year of work. Of course, many workers have a spell of unemployment in between employers, so this should be an upper bound on the job-to-job transition rate. To obtain a lower bound, I look at only people who reported working for 52 weeks. An intermediate estimate follows Blanchard and Diamond (1990) and also examines a question about the number of spells of job search. I assume that a worker who reports $i$ employers and $j < i$ spells of job search had $i - j - 1$ job-to-job transitions during the previous year, and all other workers had zero transitions. Figure 1 plots these three measures against the monthly job-to-job transition rate. Over the common time period, the cyclical behavior of the four series is similar, although the levels are significantly different. Moreover, the annual measure shows a significant decline during the recessions in 1982 and 1991, suggesting that the decline in 2001 was not an anomaly. I conclude that the model’s inability to generate quantitatively significant movement in the job-to-job transition rate is a failure.

II. Wages

The model does generate comparatively large movements in wages (fig. 19 in the paper). To understand the behavior of wages, look back
at the firm’s problem in equations (1)–(4) of the paper. At time 0, each firm commits to a time path of wages $w_t$, common for all its workers. That path determines the value of a job at the firm $V_t$, and so it determines whether a worker who is employed elsewhere moves to the firm when she gets an offer from it and then whether she stays at the firm when she gets an outside offer. That is, wages are used for both recruiting and retaining workers. Of course, the initial stock of workers has already been recruited, and so paying these workers high wages is useful only because it helps to retain them.

Now consider what happens when the firm chooses a time path of wages $\{w_t\}$ at time 0. The firm would like to leave the current stock of workers indifferent about quitting for unemployment by setting $V_0 = U_0$, but then to have a much higher value of $V_t$ at all positive $t$ so that it can recruit new workers and retain its existing workers when they get outside offers. Formally, the firm would like the state variable $V_t$ to jump at date 0. If we ignore the lower bound on wages in equation (4), this could be achieved by a lump-sum payment from the current workers to the firm on that date.

Although Moscarini and Postel-Vinay do not recognize it, a time 0 lump-sum payment is implicitly part of the solution they provide. When they write the Hamiltonian (9) and the necessary first-order conditions
(10)–(14), they omit one necessary condition. Since the initial value of the state variable $V_0(p)$ is chosen freely, either the costate variable $\nu$ associated with the worker’s value $V$ is equal to zero initially or the no-quitting constraint $V_0(p) \geq U_0$ is binding: $\nu_0(p)[V_0(p) - U_0] = 0$. This condition is violated in their proposed solution, and so the wage path they characterize is not the solution to equations (1)–(4).

Instead, the authors’ analysis implicitly ignores the minimum wage constraint (4) and relaxes the requirement that the value function be continuous. As they show in their companion paper (Moscarini and Postel-Vinay 2008), they can transform the problem in equations (1)–(3) (ignoring [4]) by eliminating wages and turning $V_t(p)$ into the control variable using integration by parts. With this formulation, it is clear that an optimal control $V_t(p)$ need not be continuous, with discontinuities representing lump-sum payments. One can show that the optimum indeed features a lump-sum transfer at time 0 so as to set $V_0(p) = U_0$, with time inconsistency causing no subsequent distortions. That is, if a firm were presented with an unexpected opportunity to reoptimize at some date $t > 0$, it would charge its existing workers a lump-sum fee so as to leave them indifferent about becoming unemployed but would otherwise leave the path of wages unchanged. Thus the (implicit) lump-sum fee takes care of the time inconsistency problem.

In the solution to the problem in equations (1)–(4), the minimum wage constraint (4) binds for some initial period, pushing down workers’ value $V_0(p)$. This distorts the contract offered to new hires and so is more costly than an initial lump-sum transfer; however, the firm would still always attempt to extract value from the initial stock of workers and would try to extract more value if its initial labor force $L_0(p)$ is larger. But this is not the problem that Moscarini and Postel-Vinay solve.

### III. Constant Wage Model

It should be clear that the time inconsistency of wages is a subtle problem. To sidestep it, suppose we relax the assumption that each firm must pay all its workers the same wage, but instead impose that firms resolve the time inconsistency problem by committing to pay each worker a constant wage, independent of the history of shocks. Now the cross-sectional distribution of wages is a state variable and cannot jump following a shock.

To see how this affects the results, I focus on a case in which all firms are identical, so each employed worker produces $y$ units of output but each unemployed worker produces nothing. All workers, employed or
unemployed, contact a firm at an endogenous rate $\lambda_t$, and jobs end exogenously at an exogenous rate $\delta$. The contact rate $\lambda_t$ is determined by firms’ job creation decision: at any point in time, a firm can pay a cost $k$ to contact a worker selected at random from the population. Without knowing the worker’s employment status or wage, the firm offers her a constant wage $w$. If the worker is unemployed, her reservation wage is zero, whereas if she is employed she accepts any job paying more than her current wage. Finally, assume for algebraic simplicity that firms do not discount future profits.

Let $F_t(w)$ denote the distribution of wage offers at time $t$ and $G_t(w)$ denote the fraction of workers who are paid less than $w$ or who are unemployed at $t$. The expected profit from offering a wage $w$ at $t$ is

$$\pi_t(w) = G_t(w)(y-w) \int_t^\infty e^{-\int_t^{t'} \{\delta+\lambda r[1-F_r(w)]\} dt''} dt' - k. \quad (6)$$

The worker accepts the job with probability $G_t(w)$, in which case she generates profit $y-w$ until she leaves. The probability that she is still at the firm at some future date $t'$ is the probability that she has received neither an exogenous separation shock nor a better job offer at any intervening date $t''$. Finally, offering the job costs $k$. In equilibrium, $\pi_t(w) \leq 0$ for all $w$ and $t$. Whenever $\lambda_t > 0$ and $w$ is in the support of $F_t(\cdot)$, $\pi_t(w) = 0$, so firms are willing to offer the appropriate wage. In addition, the fraction of workers paid less than $w$ or unemployed evolves according to

$$\dot{G}_t(w) = \delta[1-G_t(w)] - \lambda_t[1-F_t(w)]G_t(w). \quad (7)$$

It increases when a worker who is employed at a higher wage loses her job and falls whenever an unemployed worker or a worker paid less than $w$ finds a job paying at least $w$.

To characterize the equilibrium, it is useful to define

$$H_t(w) \equiv G_t(w) \int_t^\infty e^{-\int_t^{t'} \{\delta+\lambda r[1-F_r(w)]\} dt''} dt'. \quad (7.1)$$

Time-differentiate this, eliminating $\dot{G}_t(w)$ using equation (7):

$$\dot{H}_t(w) = \delta \frac{H_t(w)}{G_t(w)} - G_t(w). \quad (8)$$

Now since $\pi_t(w) = H_t(w)(y-w) - k$, $H_t(w) \leq k/(y-w)$ in equilibrium and the wage $w$ is offered in equilibrium only if $H_t(w) = k/(y-w)$. 

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Equivalently, the wage \( w \) is offered at \( t \) only if \( t \) is a local maximum of \( H_t(w) \). A necessary condition for this is that \( H_t(w) = 0 \). Then (8) implies that \( w \) is offered in equilibrium only if \( H_t(w) = G_t(w)^2 / \delta \). Combining these expressions for \( H \) gives a necessary condition for \( w \) to be offered at \( t \), \( k/(y - w) = G_t(w)^2 / \delta \) or

\[
G_t(w) = \sqrt{\frac{\delta k}{y - w}} = G^*(w).
\]  

(9)

That is, \( \lambda_t F'_t(w) > 0 \) only if \( G_t(w) = G^*(w) \). Moreover, one can prove that if \( G_t(w) > G^*(w) \), enough firms will offer \( w \) to push \( G_t(w) \) immediately back down to this value—otherwise \( \pi_t(w) > 0 \)—whereas if \( G_t(w) < G^*(w) \), no firm will offer \( w \), \( \lambda_t F'_t(w) = 0 \). Together equations (7) and (9) provide enough conditions to characterize an equilibrium for arbitrary initial conditions.

To understand the implications of these equations, suppose that \( k = 0.1 \) and \( \delta = 0.025 \), and productivity falls from \( y = 1 \) to \( y' = 0.98 \) at \( t = 0 \). Also assume that the economy had reached a steady state by \( t = 0 \). The comparative statics from this shock are fairly uninteresting. The steady-state wage distribution is \( G^*(w) = \sqrt{\delta k / (y - w)} \) on \([0, y - \delta k]\), whereas the unemployment rate is \( \sqrt{\delta k / y} \). The arrival rate of job offers falls from \( \lambda = 0.475 \) to \( \lambda' = 0.470 \), raising the unemployment rate from 5.00% to 5.01%. The mean wage falls from 0.90 to 0.88, in line with the decline in productivity, and the job-to-job transition rate falls from 5.38% to 5.36%.

But this masks some interesting transitional dynamics. One can prove that for all \( t > 0 \) and all \( w > 0 \),

\[
G_t(w) = \min \left\{ \sqrt{\frac{\delta k}{y' - w}} \left( 1 - e^{-\delta t} + e^{-\delta t} \sqrt{\frac{\delta k}{y - w}} \right) \right\}. 
\]  

(10)

That is, the wage distribution at \( t \) is the minimum of the new steady-state distribution with the lower productivity level \( y' \) and the distribution obtained from the decay of the old steady-state distribution as workers lose jobs at rate \( \delta \). Following the adverse shock, there is a brief period when no new jobs are created, ending at \( t = 0.02 \); but then \( \lambda_t \) jumps to its new steady-state value \( \lambda' = 0.470 \). Subsequently, firms create jobs only with wages \( w \in [0, \bar{w}(t)] \), where \( \bar{w}(t) \) satisfies

\[
\sqrt{\frac{\delta k}{y' - \bar{w}(t)}} = 1 - e^{-\delta t} + e^{-\delta t} \sqrt{\frac{\delta k}{y - \bar{w}(t)}}.
\]
rising monotonically over time from 0 at $t = 0.02$ to $y' - k\delta$ as $t \to \infty$. The wage offer distribution at $t$ is

$$F_t(w) = 1 - \frac{\delta[1 - G_t(w)]}{\Lambda'G_t(w)}$$
on $[0, \bar{w}(t)]$. Note that this implies that a mass of firms offer exactly $\bar{w}(t)$ at $t$; however, this does not give rise to any mass points in the cross-sectional wage distribution $G$ because the mass point rises continuously over time.

The initial period with no job creation leads to an increase in the unemployment rate, raising the cross-sectional wage distribution $G$ uniformly. This has little effect on the incentive to create high-wage jobs since such jobs recruit workers mainly from other firms. However, it does raise the profitability of offering low-wage jobs, and so those are the first to appear. As high-wage workers gradually lose their job as a result of exogenous shocks, higher wage offers become more viable, pushing up $\bar{w}(t)$. Figure 2 shows the decline in the average wage following this one-time adverse shock. Over the course of 10 years, the wage declines by about 2%. The reduction is slow because, by assumption, workers in high-wage jobs do not suffer a wage cut following the shock. However, this is partially offset by firms’ decision to offer only low-wage jobs during the transition following the shock.

![Fig. 2. Change in average wage, constant wage model](image)
I can also compute the job-to-job transition rate. As in the mechanical model, workers switch jobs whenever they find one paying a higher wage, so the job-to-job transition rate is

$$J_t = \frac{\lambda_t}{1 - u_t} \int_0^{\pi(t)} \left[ 1 - F_t(w) \right] dG_t(w).$$

It is again straightforward to compute this, and I show the results in figure 3. Following the adverse shock, the job-to-job transition rate falls from 5.4% briefly to zero as no jobs are created. But although the job-finding rate $\lambda_t$ recovers almost immediately, the job-to-job transition rate stays below its new steady-state value for years because the new jobs being created generally pay lower wages than the existing jobs. Compared with Moscarini and Postel-Vinay’s average job-to-job transition rate in their figure 18, the movements depicted here are enormous.

The response to a positive productivity shock is asymmetric. Immediately following the shock, firms create a positive measure of high-wage jobs, above the old highest wage $y - \delta k$. This implies that a positive fraction of workers switch employers at that instant. The job-to-job transition rate subsequently remains high as firms continue to create high-wage jobs. Eventually firms find it profitable to also create low-wage jobs, in the interval $[0, \bar{w}(t)]$, where $\bar{w}(t)$ is increasing over time. In this case, the economy reaches its new steady state in finite time. In practice, I find that the transitional dynamics are quick. Since $\lambda_t \gg \delta$, it takes much less time to move workers into high-wage jobs than to move them out.
In my view, the constant wage model has some significant advantages over the one in Moscarini and Postel-Vinay’s article. First, it endogenizes firms’ decision to create jobs rather than simply postulating a joint shock to the productivity and the arrival rate of job offers. Second, it delivers significantly larger fluctuations in the job-to-job transition rate and slow movements in wages. However, Moscarini and Postel-Vinay’s model is richer along important dimensions. They allow the job-finding rate to differ for employed and unemployed workers and allow firms to discount future income. Both those extensions to the constant wage model should be feasible at the expense of algebraic complexity. More important, Moscarini and Postel-Vinay allow for firms with heterogeneous productivity; that is tricky to do here because all the jobs would naturally be created by the most productive firms. A technological theory of optimal firm size may again be useful for explaining why this does not happen.

IV. Conclusion

The facts outlined by Moscarini and Postel-Vinay are intriguing. Whether they are important business cycle facts or simply characteristics of two tranquil decades in the United States awaits more data. Their model provides a novel interpretation of those facts. This discussion has tried to explain how the model works by extracting some pieces of it, notably the mechanics of the firm size distribution and the job-to-job transition rate and the time inconsistency of wages. Along the way, I have argued that it may be possible to improve the model’s performance along some important dimensions while making it theoretically more appealing. It should be clear that our understanding of the role of job search in explaining the firm size distribution and the extent of job-to-job transitions is still rudimentary and that the qualitative and quantitative behavior of on-the-job search models depends on some subtle assumptions.

Endnotes

1. While this is a primitive assumption here, in their paper Moscarini and Postel-Vinay focus on a rank-preserving equilibrium in which more productive firms always offer workers a higher value; so workers move whenever they contact a firm that is more productive than their current employer.

2. There is no option value to unemployment since search while employed is as efficient as search while unemployed in this model.
References


