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# Introduction

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This volume is a collection of papers all related in one way or another to the general problem of how to construct a time series of capital in real terms. Capital in real terms is also referred to as "real capital," "aggregate capital stock," "capital in its own units," or just plain "capital," the term I shall employ in this Introduction. Capital in this sense must be distinguished from the "value of capital in current dollars," a related but nonetheless distinct concept. In introducing problems in the measurement of capital it is useful to begin with a case where these problems do not arise at all. This is where all capital goods are constructed from uniform and indestructible blocks, like the blocks children play with, where the quantity of capital per unit of each type of capital good is the number of blocks it contains, and where capital goods can be assembled or disassembled costlessly. The quantity of capital is simply the total number of blocks. Specifically, if there were $n$ distinct types of capital goods, if each type, $i$, of capital good consisted of $P^o{}_i$ blocks, and if there were $K^t{}_i$ units of the $i$ type of capital goods in the economy in the year $t$, the total capital stock $K^t$ in the year $t$ could be measured unambiguously according to the formula

$$K^t = P^o{}_1 K^t{}_1 + P^o{}_2 K^t{}_2 + \dots + P^o{}_n K^t{}_n. \tag{1}$$

The papers can be divided into two distinct groups. The first group, the papers by Young and Musgrave, Coen, Hulten and Wykoff, Engerman and Rosen, Soladay, and to some extent Eisner, start with the working premise that the object in measuring capital is to construct a measure of capital in accordance with equation (1), where the $P^o{}_i$ are

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interpreted not as numbers of blocks, but as prices of capital goods in some chosen base year. The central problem in all these papers is to deal with complexities of the world, notably the diverse patterns of depreciation of capital goods and the changes over time in the nature of capital goods themselves, which are abstracted away in the model where equation (1) is unambiguously defined. Allan Young and John Musgrave discuss the assumptions and methods in the United States Department of Commerce time series of real capital stock in the United States. Other papers in this group can be looked upon as studies of how the series might be improved or modified. Robert Coen, and Charles Hulten and Frank Wykoff, derive alternative ways of measuring depreciation. Stanley Engerman and Sherwin Rosen review two new books with implications for the measurement of capital—a volume by John Kendrick on how to extend the definition of capital, with special emphasis on human capital, and a volume by Robert J. Gordon on new methods of constructing price indexes for capital goods. John Soladay's paper extends the definition of capital to include reserves of oil and gas. Robert Eisner's paper introduces an imputation for capital gains or losses as indicated by, for instance, changes in the value of shares traded on the stock market.

The second group of papers, those by Murray Brown and W. E. Diewert, examine the premise that capital in real terms can be measured in accordance with equation (1). Can time series of quantities of capital goods be combined into a single number that may be interpreted as "the" measure of real capital in the economy as a whole? Can it be said that the capital stock in one industry is greater than the capital stock in another? Does equation (1) provide an adequate representation of real capital? Can a better index number be devised? These papers contain extensive discussions of index number and aggregation problems in capital measurement.

To introduce this volume I shall discuss the papers briefly, not one by one, but in the context of a summary of what I take to be the main problems of capital measurement. I have chosen this format to give the reader a sense of how each paper relates to the other papers in this volume and contributes to the overall problem of capital measurement. I begin by reviewing the purposes of capital measurement, for we cannot evaluate techniques of measuring capital until we know what the measurements are for. Then, following the order of the papers in this book, I consider a series of problems in measuring capital defined in accordance with equation (1). Next I list and compare the different meanings of the term "real capital" in economic analysis. And, finally, there is a brief discussion of index numbers and aggregation.

## The Purposes of Capital Measurement

We can conveniently identify five purposes, though these are not entirely distinct.

1. *The investment function.* We want a measure of capital in real terms to serve as an argument in the investment function

$$(2) \qquad I = f(K,p, \ldots),$$

where $I$ is the amount of investment over some period of time, $K$ is the capital stock at the outset of the period, $p$ is the relative price of capital goods in terms of consumption goods, and the blanks in the function indicate that other factors are also important. This function may be studied on its own or in conjunction with other functions in a large econometric model designed to forecast the progress of the economy.

2. *The consumption function.* As an important component of wealth, real capital appears implicitly as an argument in the consumption function

$$(3) \qquad C = g(Y,W, \ldots),$$

where $C$ is real consumption, $Y$ is real income, and $W$ is real wealth, which includes title to physical assets, financial assets, and whatever extra items are required to take account of liabilities, title to foreign assets, and so on. Once again, the function can be studied by itself or in the context of a large econometric model.

3. *The production function.* Among the many uses of the production function in economic analysis, three should be mentioned here because the role of the time series of capital is different in each case, and these differences might be reflected in the design of the time series themselves. The production function is

$$(4) \qquad Q = f(K,L),$$

where $Q$ is output, $L$ is labor, and $K$ is capital. The first use of the production function is to measure the elasticity of substitution between labor and capital. This elasticity is, of course, essential for predicting the effects upon the distribution of the national income of changes in technology, tax rates, or factor supplies. Typically, when we measure elasticities of substitution, we are exclusively concerned with the shape of the isoquants today. The second use of the production function is to apportion observed economic growth into that which can reasonably be attributed to the replication of factors of production such as were available at the outset of the time period over which growth is observed and that which has to be attributed to technical change between the initial and final year of the time period. In this case, the emphasis of the analysis is upon the technology in the base period. The third use is really a miscellaneous collection. Over the last twenty years or so, the two-sector, two-factor model of the economy has been growing in importance as the basis for much of the analysis in economic history, economic development, public finance, and international trade. We want time series of capital in real terms to enable us to estimate production functions as a way to test theories and quantify their predictions.

Here, however, we must be on our guard against an elementary fallacy that can crop up in several ways. In the course of this Introduction, I shall present a list of difficulties with the concept of capital. Most of these are well known, and some authors consider them so serious that no reasonably satisfactory time series of capital in real terms can ever be devised. In general, the fallacy is to say that, because capital does not exist, the theories normally formulated by means of a model with capital and labor as the only primary factors of production must be either wrong or useless. The baldest and crudest variant of this fallacy is the assertion that the nonexistence of capital indicates a fundamental contradiction in capitalism itself. If there is no capital, then it cannot have a marginal product. If capital's marginal product is undefined, so too must be the marginal product of labor. Thus the allocation of the national product of labor among factors cannot be determined by economic forces and must be the outcome of political forces, class power, and exploitation. This conclusion may or may not be true, but the argument is surely false in the sense that the conclusion is independent of the existence or nonexistence of an aggregate called real capital. To decide whether and under what conditions wages of labor and returns of capital goods are endogenous to the competitive economy, one should examine not the two-sector model, but the full general equilibrium model as perfected by Arrow, Debreu, and many others. It is clear from these models that the existence or nonexistence of a general equilibrium solution, including an allocation of the national income among factors of production, does not in any way depend upon whether quantities of capital goods can be aggregated into a measure of the total capital stock.

We use the two-sector model in international trade and public finance as a kind of shorthand for the full general equilibrium model with many kinds of capital goods, products, and labor. It would of course be a pity if measures of capital were so unsatisfactory from a theoretical point of view that it would be wrong to estimate production functions at all. But that would not detract from the relevance or usefulness of propositions about conditions under which free trade is best or the burden of tax is shifted onto an untaxed sector of the population.

4. *Budgeting and planning.* Statistics on the size of the capital stock are used in budgeting, planning, and forecasting. The simplest and perhaps still most commonly employed technique in this area is to predict income in the near or intermediate future from actual or expected investment by postulating constant capital-output ratios or constant incremental capital-output ratios. Time series of capital in real terms are used in more complex and subtle ways in budgeting and forcasting to predict the effect of changes in the tax rates or public expenditure upon income and employment.

5. *Connections with the rest of the national accounts.* Real capital has acquired a bad name in that it is alleged to be particularly fraught with theoretical and statistical difficulties. It may be that real capital does not altogether deserve its reputation, for many of these difficulties are present in other elements of the national accounts, especially investment in real terms, depreciation, capital gains, and wealth, all of which are closely connected to real capital itself. In particular, depreciation is the loss of value in the course of the year of that part of the capital stock that was in existence when the year began. Statistics of depreciation can of course be thrown together in some rough and ready way, but an accurate and well-grounded measure can be obtained if and only if we can measure real capital as well.

## The Measurement of Capital in Accordance with Equation (1)

Capital is usually measured by the "perpetual inventory" method[1] in which the time series of the stock of capital is built up step by step from time series of dollar values of investment and prices of capital goods.

To compute a time series of real capital according to the perpetual inventory method, one needs time series of gross investment in current dollars, $I^t{}_i$, where the superscript $t$ refers to the year and the subscript $i$ refers to the type of capital goods, time series of capital goods prices, $P^t{}_i$, and a rule connecting values of new and old capital goods from which one can compute time series of depreciation $D^t{}_i$. Then, for each type of capital goods, the increase in real capital in the year $t$ is

$$(5) \qquad \Delta K^t{}_i = \frac{I^t{}_i - D^t{}_i}{P^t{}_i}$$

and the value of each $K^t{}_i$ in equation (1) can be estimated as

$$(6) \qquad K^t_i = K^o_i + \sum_{\tau=0}^{t-1} \Delta K^\tau_i,$$

where the initial value of capital $K^o_i$ can be computed in a straightforward manner by a variant of the perpetual inventory method itself. The measure of total capital can now be computed by summing up the $K^t_i$ for each year $t$ weighted by the base year prices of capital goods.

This, broadly speaking, is the method of measuring real capital employed by national statistical agencies throughout the world and, in particular, by the Bureau of Economic Analysis of the United States Department of Commerce as described by Young and Musgrave. Two features of their methods are worth emphasizing in view of the discussion of these matters in other papers: their handling of depreciation and their notion of the price of capital goods.

The problem of depreciation is to decide what proportion of capital produced in a given year is deemed to be still available $t$ years later. There are four elements to consider: (1) Part of the capital stock has been *retired*; it is out of the capital stock entirely. (2) Some of the remaining capital stock may have deteriorated; its marginal product in a physical sense is less than when it was new, or it requires more maintenance and repair. (3) The capital stock is *older*; it has fewer years of service left than when it was new. (4) It has become *obsolete*; its marginal value product is less than when it was new because of changing tastes, the availability of more efficient capital goods, or increases in the rents of cooperating factors of production.

Young and Musgrave hold the view that, all things considered, the joint effect of the last three elements of depreciation can best be accounted for by straight-line depreciation; that is to say, if a piece of capital equipment lasts $T$ years and is counted as one unit of capital when new, it should be counted as $(T - t)/T$ units of equipment when it is $t$ years old, for all $t \leq T$. They recognize that this is less than ideal as a measure of depreciation, but they argue that our information about true economic depreciation is so skimpy and imprecise that one cannot do better in practice. Lives of capital equipment are taken from the tax schedule of the Department of the Treasury, with adjustments to approximate actual average economic lives and to account for variability in the lives of the different units of the same kind of capital equipment.

In measuring prices of capital goods, Young and Musgrave follow Denison's lead (1957) in that they avoid on principle treating costless quality change in capital goods as a reduction in price. They state that "deflation of gross fixed investment . . . counts only cost-associated quality change as a change in real capital," and they go on to argue that, viewed in this light, the often-heard criticism of the official price indexes of capital goods—that they overstate the amount of price increase—may be misplaced.

The papers of Coen, Hulten and Wykoff, Engerman and Rosen, Soladay, and Eisner, and of course the comments by Rymes and Faucett can all be thought as developments of themes introduced in the Young and Musgrave paper:

Alternatives to the Perpetual Inventory Method

It is important for us to take a critical and skeptical stance toward the perpetual inventory method, precisely because of its popularity and the apparent ease with which it lets us compile time series of real capital stock. Most of the difficulties with the measurement of capital pertain to the perpetual inventory method to some extent, but two general issues are worth considering now. The first is that the perpetual inventory method is very theoretical in the pejorative sense of the term. At no point in the perpetual inventory method is it necessary to compare quantities of captial goods directly—to make inventories of the capital goods available in the year 0 and the capital goods available in the year *t*, and to decide which inventory constitutes the larger capital stock. This decision is avoided by treating the total stock each year as the sum of the increments in every preceding year. Even the increments are not quantities that may be compared directly from one year to the next. They are ratios of values and prices, and any errors in these data—or more precisely any misjudgment, for there is no unambiguous way of deciding which price index is appropriate—reverberate throughout the time series. A second difficulty with the perpetual inventory method is, as it were, the reverse side of its principal advantage. The perpetual inventory method never fails to yield us a time series of real capital, no matter how long the time series in question or how radically the technology and the nature of capital goods have changed between the first and final years. The perpetual inventory always works as long as there are data on gross investment, depreciation, and price indexes of capital goods. There is no red light that flashes, no internal check that tells us when the whole process becomes absurd. This is, of course, a difficulty with all aggregate time series in real terms—real consumption, real gross national product, and so on. But between real capital and, for instance, real consumption there is a difference in degree, if not in kind. Statistics of real consumption are intended to serve as indicators of the heights of the indifference curves attained in each of the years of the time series, the underlying assumptions being that the indifference curves themselves are stable over time and that the constancy of taste permits us to compare quantities of food, clothing, housing, and so on, from the present day right back to medieval or ancient times. But the continual change in the technology of production brings forth new processes and new machines every year, depriving us of a reference point from which real capital stock can be compared forward and backward in time.

There are several possible alternatives to the perpetual inventory method. The aggregate capital stock might be estimated from book values of companies, insurance records, or direct surveys of capital goods in existence. Faucett suggests in his comment that book values might be preferable to the perpetual inventory method for measuring the industrial composition of the capital stock because book values automatically take account of transfers among industries of secondhand equipment. Survey methods have been employed to measure capital in the Soviet Union. They are said to be very expensive and to involve virtually intractable problems of classifying the myriad types of capital goods employed at different times into standard categories that can play the role of $K_1$, $K_2$, and so forth, in the definition of real capital in equation (1) above.[2] Alternatively, it has been found possible to construct time series of real capital from statistics of fire insurance (see Barna 1957). This would be much cheaper than a survey of all capital goods, but there are of course great problems with the compatability and reliability of the data.

### Alternative Ways of Measuring Depreciation

Rymes and Faucett's principal criticism of the Young and Musgrave paper is that their measure of depreciation fails to reflect the time pattern of the fall in the market value of capital equipment as it ages—fails, that is, to reflect what is commonly called "economic depreciation." The criticism has to do with the conversion from service prices to stock prices and with the reasons a piece of capital might become less productive over time. Faucett and Rymes argue that if, for instance, a capital good yields a constant flow of services over its life, its depreciation ought to be small at first and then progressively larger to reflect the time path of the present value of the capital good. They also argue that all sources of decline in present value should be accounted for—not only physical deterioration of the capital good, but obsolescence owing to increased cost of cooperating factors of production or to competition with new and better machines. Coen's paper and Hulten and Wykoff's paper are attempts to estimate economic depreciation from two quite different sorts of data.

Hulten and Wykoff base their estimates on a United States Treasury sample of prices of new and used structures. Broadly speaking, their findings are that economic depreciation is less than allowed for in the tax code and in the national accounts (so that the measure of the capital stock in real terms is correspondingly larger), and that the time pattern of depreciation that provides the best fit to the data is not straight line but geometric or something even more accelerated, the distinction being that under straight-line depreciation the value of capital declines by a constant amount each year of its life, while under geometric depreciation the value declines by constant proportion. Geometric depreciation is made consistent with a finite lifetime of capital goods by eliminating all remaining value at the terminal date.

An important theoretical point emerges from Hulten and Wykoff's analysis—economic depreciation depends upon the tax laws. The value of secondhand equipment declines more or less rapidly with age according to the rate of depreciation for tax purposes; the more rapidly a firm may depreciate a piece of equipment, the more rapidly it declines in value, for part of the value of any piece of equipment is the present value of the remaining depreciation allowances. The existing rate of economic depreciation is therefore different from what it would be if economic depreciation were chosen as the basis for depreciation in the tax code. The relation between tax and economic depreciation is not an infinite regress but more like a set of equations that need to be solved simultaneously.

The dependence of economic depreciation on the tax laws has similar implications for the measurement of capital in accordance with equation (1). Ideally, we would like a measure of real capital to reflect a property of the technology of the economy exclusively. We might like the measure of $K^t$ in equation (1) to play the role of $K$ in the production function, so that any increase in $K^t$ reflects a capacity of the economy to produce more in some sense, regardless of the tax laws or of the tastes of consumers. We now see that the measure of capital constructed by the perpetual inventory method need not have that property, because the size of the capital stock in each category $K^t{}_i$ depends on the rate of depreciation, which in turn depends on the tax laws in force in the year $t$. One might try to get around this problem by estimating $K^t$ "as though" the tax laws remained invariant, by treating base year tax laws analogously to base year prices. Or one might argue that $K^t$ obtained by the perpetual inventory method, though less than ideal, is adequate for some purposes. In fact, Hulten and Wykoff's evidence shows that rates of economic depreciation were virtually constant over the period they studied, despite the substantial changes in the tax laws.

Coen's paper is based on the idea that one can infer the rate of economic depreciation from time series of investment. Firms invest to maintain a proportion between productive capacity and output, but the productive capacity at any moment depends upon the prior rates of deterioration and obsolescence of its capital equipment. Consequently, the time path of deterioration and obsolescence can be inferred by observing which among a variety of possible paths provides the best fit in an equation linking investment to output, productive capacity, and other economic variables. This procedure leads Coen to an exact specification of the relation between economic depreciation and loss of efficiency of capital goods, for it is the latter alone that affects the rate of investment in his model. There is also a discussion of the relation between economic depreciation and the accuracy of expectations about inflation; there are circumstances where past errors in estimating the rate of inflation can affect the rate of depreciation today.

## The Pricing of Capital Goods

The pricing of capital goods may prove to be the Achilles' heel in the measurement of capital in real terms. Equation (1), which is our working definition of real capital, contains the terms $K^t{}_i$. To write such terms is to assume, albeit implicitly, that the nature of each type $i$ of capital goods persists unchanged through time. To measure the output of newly produced capital goods, we divide their value (which can be measured with tolerable accuracy) by a price index. But if our implicit assumption is false, if new capital goods are materially different from old capital goods, then we have no sure basis for choosing a price index; and whatever price index we choose reflects, whether we like it or not, an assumption about the equivalence of new and old types of capital goods. How, to take the prime example of this difficulty, do we construct a price index to convert the Marchant calculator on which the older generation of economists used to run its regressions and the computing facilities now available into amounts of a single type of capital?

There are two main schools of thought on this issue. One school, represented in this volume by Young and Musgrave, would measure real capital on the *supply* side, comparing new and old machines according to their cost of production and thereby excluding costless improvements in capital goods from the measure of the size of the capital stock. The other view, represented in a new book by Robert J. Gordon, reviewed here by Engerman and Rosen, would measure real capital on the *demand* side, comparing new and old machines according to their usefulness as assessed by performance characteristics such as speed, size, and safety of automobiles or number of additions per second of calculators. The difference is empirically important; Gordon's preliminary estimate of the growth rate of real investment in the United States, presented in Engerman and Rosen's table 4.3, is literally twice the rate in the official United States national accounts.[3]

There is no general consensus among economists and statisticians on which concept of the price index is preferable, but there is a recognition on all sides that there are major conceptual and theoretical problems with each. Capital could be measured precisely and unambiguously on the demand side if there were a finite number of performance characteristics in the economy as a whole, if the nature of performance characteristics were invariant over time, if the value of each type of capital good were an invariant function of the amounts of the performance characteristics it contained, and if we could always determine the amounts of the different performance characteristics in any capital good. But the world is not like that. Changes over time in the nature of capital goods cannot be entirely represented as different amounts of invariant characteristics; technical change causes prices of capital goods to rise or fall over time in ways that do not conform to any stable function of amounts of characteristics; prices of characteristics vary greatly over time as characteristics become scarce or plentiful; and, as Denison pointed out long ago, it is difficult to see how machines that embody laborsaving technical change can be compared on a common scale with machines that have no effect upon labor productivity. Capital could be measured precisely and unambiguously on the supply side if the relative prices of machines within any category (such as computers) remained constant over time. We could then say that, for instance, if a Marchant calculator costing $200 in the year when the SR 50 appears on the market counts as two units of capital and if the SR 50 costs $50 in that year, then the SR 50 is always to be counted as one-half a unit of capital regardless of the characteristics of the two machines. But the world is not like that either. Relative prices of capital goods within any category are constantly changing, newly discovered types of capital goods are typically more expensive when they first appear on the market than they become later on when the market is more nearly saturated and when their cost of production has been reduced by further technical change.

The debate over the choice of a price index of capital goods reminds one of the question posed by Joan Robinson (1953–54) in the opening shot in recent round of debate on capital theory. "In what units," it was asked, "is capital to be measured?" Young and Musgrave's answer is, "In Marchant calculator equivalents where other machines are equated to Marchant calculators according to their cost of production." Gordon's answer is, "In additions per second and other characteristics evaluated at prices in a base year."

## The Scope of Real Capital

Capital may be defined narrowly as produced means of production, or it may be defined broadly to include all or a large part of the factors of production in the economy. Young and Musgrave, adopting the narrower definition, measure capital as the sum at base-year prices of equipment, structures, inventories, and residences, as is the practice in the national accounts of many countries. Kendrick, Soladay, and Eisner are in different ways attempting to account for a wider range of factors of production.

In the volume reviewed here by Engerman and Rosen, Kendrick extends the definition of capital to include land, consumer durables, human capital (the accumulated cost of education treated as investment), and accumulated expenditure on research and development. He does this to provide a test for the hypothesis that growth in real output per head in the United States can be explained by the growth of real capital per head. Clearly, if a large part of capital formation is in human capital, then human capital has to be accounted for in any comparison of growth rates of inputs and outputs. Although the residual that must be attributed to something like aggregate technical change is reduced by Kendrick's extension of the scope of capital, it is still not eliminated.

Soladay adds an imputation for stocks of oil and gas. It is rather queer, when one thinks of it, that the stock of subsoil assets is excluded from the measure of capital, and the depletion of subsoil assets is excluded from the measure of depreciation, though their exclusion is required by the formal definition of capital as produced means of production. Excluding subsoil assets from capital means that Saudi Arabia, despite its oil reserves, must be counted as a capital-poor country. Excluding the wastage of subsoil assets from depreciation means, for instance, that the abandonment of an oil rig is treated as a reduction in the capital stock while the loss of the oil field that led to its abandonment is not. Similarly, a country rapidly using up its oil reserves would be counted as having a net investment in the oil industry if it is devoting resources to drilling new wells to discover what is left of the ever-smaller stock of oil underground. The situation can be rectified by expanding the definition of capital.

The stock of subsoil assets and the corresponding capital consumption allowance might be measured according to assumptions that lie along a continuum. At one extreme, the stock of subsoil assets is looked upon as given at the beginning of time, and all production represents a kind of depreciation. On this assumption, net investment in oil is the expenditure over the year on discovery of oil, drilling, and plant and equipment minus the sum of depreciation of existing facilities and the production of oil evaluated at a shadow price equal to the difference between the world price of oil and the current cost of extraction. At the other extreme, one might identify the stock of capital in oil with the quantity of proved reserves; net investment is positive on this assumption if proved reserves are larger at the end of the year than they were at the beginning. The in-between cases would involve recognizing both proved and unproved reserves as part of the capital stock, but unproved reserves would have a lower shadow price, so that discovery increases the quantity of capital. Soladay chooses the second extreme case, including only proved reserves in the capital stock.

Eisner adds an imputation to the capital stock for accumulated capital gains, permitting him to compare capital stocks among the different sectors of the economy with measures corresponding to their own valuations of their assets at different periods of time. The imputation for capital gains is intrinsically different from the other extensions to the definition of capital and is best discussed in the next section.

## The Definition of Capital

There is widespread agreement that the working definition of real capital in equation (1) is only an approximation, but there is less than full agreement on what the definition is supposed to approximate. There seem to be four main contenders for the definition of capital in real terms: instantaneous productive capacity, long-run productive capacity, cumulated consumption forgone, and real wealth. Because these concepts are logically distinct, it is entirely possible that each of them is preferable to the others for a certain range of purposes. I shall discuss them in turn.

1. *Instantaneous productive capacity.* According to this definition of capital, one lot of capital goods constitutes more capital than another lot of capital goods if more output can be produced this year with the first lot than with the second lot when the production function and the labor force are the same in each case. For simplicity, suppose one good is produced with two kinds of capital and two kinds of labor,

$$Q^t = f^t(K^t{}_1, K^t{}_2, L^t{}_1, L^t{}_2), \tag{7}$$

where the superscript $t$ indicates that the production function represents the technology available in the year $t$, and where $Q^t, K^t{}_1 \ldots$ are quantities of output and input in the year $t$.

Let us choose the year 0 as the base year and arbitrarily set the index of real capital associated with the capital goods employed in that year at 1; we designate the index as $K$, and we say that $K = 1$ for the stocks of capital goods $K^0{}_1$ and $K^0{}_2$. We must now attach a value of $K$ to the mix of capital goods, $K^t{}_1$ and $K^t{}_2$, employed in the year $t$. We can proceed as follows: the basic idea is to choose a definition of capital such that the amount of capital associated with the pair $K^t{}_1$ and $K^t{}_2$ is at least as large as the amount of capital associated with the pair $K^0{}_1$ and $K^0{}_2$ if $K^t{}_1$ and $K^t{}_2$ can replace $K^0{}_1$ and $K^0{}_2$ in the production function without loss of output. Let us say that $K^t{}_1$ and $K^t{}_2$ constitutes an amount of capital $\gamma$, that $K^t = \gamma$, if $K^t{}_1$ and $K^t{}_2$ can be reduced by a factor $\gamma$ and still do the same job as $K^0{}_1$ and $K^0{}_2$. (This is what Diewert calls a Malmquist index.) In other words, the value of $K$ associated with $K^t{}_1$ and $K^t{}_0$ is equal to $\gamma$ if

$$(8) \qquad f^0\left(\frac{K^t{}_1}{\gamma}, \frac{K^t{}_2}{\gamma}, L^0{}_1, L^0{}_2\right) = f_0\left(K^0{}_1, K^0{}_2, L^0{}_1, L^0{}_2\right).$$

This definition of capital has in common with the usual definition of real consumption in the economic theory of index numbers that it is dependent on a functional form and certain base-year values. Real consumption is dependent on the utility function. Real capital is dependent on the production function $f^0$ and on the supplies of labor $L^0{}_1$ and $L^0{}_2$. The definition could be modified in several ways. In particular, we could weaken the requirement that all bundles of capital goods must be combined with precisely $L^0{}_1$ and $L^0{}_2$ by allowing a choice among equal values of labor at base-year prices. I do not think that would affect the essence of any of the problems we discuss here.

This definition of real capital is—so far as I can tell—internally consistent and free from any hint of paradox. But that desirable quality is purchased at no small cost. For most of the purposes of real capital listed above, we would like real capital to be a unique concept. We would like a definition such that if the mix of capital goods $K_1$ and $K_2$ is more real capital than the mix of capital goods $K'_1$ and $K'_2$ within the production function $f^0$ and for supplies of labor $L^0{}_1$ and $L^0{}_2$, then the mix $K_1$ and $K_2$ is more real capital than the mix $K'_1$ and $K'_2$ for all functions $f$ and all supplies of labor $L_1$ and $L_2$. Normally—almost invariably—this is not so. Real capital is a family of concepts, one member for each set of $f$, $L_1$ and $L_2$.

The study of the conditions under which the separate definitions of real capital give rise to the same time series—what unfortunately (for the terminology is off-putting) has come to be known as the problem of existence—constitutes a major part of the papers by Diewert and Brown, both of which are primarily concerned with capital as instantaneous productive capacity as defined here. The conditions under which real capital exists in this sense turn out to be disappointingly restrictive. On one hand, it is sufficient for existence of an aggregate capital stock if the process by which capital goods are produced is such that relative service prices of the different capital goods remain constant over time, for in that case a greater value (again at service prices) of capital goods represents a more productive bundle regardless of the form of the production function, as long as an optimal mix of capital goods is chosen at any given time. This is the Hicks's aggregation theorem; unfortunately, it amounts to saying that there is only one capital good in the system, for many goods with invariant relative prices are just like one good with a variety of uses. On the other hand, capital exists for a particular production function if that function displays "homogeneous weak separability"—that is, if the production function takes the form $f^t(k(K^t_1, K^t_2), L^t_1, L^t_2)$, where the interior function $k$ is homogeneous in degree one with respect to $K^t_1$ and $K^t_2$. If the function $k$ exists, then the value of the function is itself the measure of the aggregate capital stock. Otherwise, the combined productive capacity of $K^t_1$ and $K^t_2$ depends on the mix of $L^t_1$ and $L^t_2$ employed. Similarly, their combined productive capacity depends on which production function they are employed in, unless the interior function $k$ is the same in every function $f$.

These problems of the existence of capital are important in practice because there is always some technical change between the first year and the final year of any time series. Suppose we want a time series of capital beginning in the year $t$ and ending in the year $T$, and suppose that technical change is gradually shifting the production function from $f^t$ to $f^{t+1}$ to $f^{t+2}$ and so on until $f^T$. Which year's production function is to be taken as the basis for constructing the time series? If a measure of capital "exists" in the sense of that word used by index number theorists, then all production functions generate the same time series. If capital almost exists in the sense that the production functions are very similar or that they give rise to very similar time series of real capital, we can be content with a measure of the capital stock based on any one of the set of production functions. But if capital does not exist, then a time series of capital based on the production function and stocks of labor of the year $t$ may well show capital to be increasing from one year to the next when, in fact, the productive capacity of the capital goods available is diminishing.

*Long-run productive capacity*. In our first definition of capital,[4] the quantities of the capital goods $K_1$ and $K_2$ were aggregated according to their capacity to produce output today, but their durability was not taken into account. It made no difference whether the existing stock of $K_1$, for instance, will wear out next year, in two years, or in a hundred years. Only its effectiveness today was considered. A measure of capital as an indicator of long-run productive capacity incorporates both durability and productivity of capital goods. It can be defined analogously to the first measure, except that the production function would need to be generalized to take account of the flow of consumption goods in every future year. A mix of capital goods $K_1$ and $K_2$ would then constitute more real capital than a mix $K'_1$ and $K'_2$ if people are better off with the first mix than they are with the second, where "better off" incorporates potential output tomorrow as well as potential output today.

The empirical measure of capital defined in equation (1) and discussed by Young and Musgrave, Coen, Hulten and Wykoff, Soladay, and Engerman and Rosen seems to approximate a measure of long-run productive capacity because the durability of capital is taken into account and, what amounts to the same thing, because quantities of capital goods are weighted by market prices rather than by service prices. All of the problems of existence and aggregation we encounter in trying to define capital as instantaneous productive capacity carry over into the definition of capital as long-run productive capacity, and there is the additional problem that the mix of captial goods $K_1$ and $K_2$ may count as more or less capital than the mix $K'_1$ and $K'_2$, depending upon the rate of interest. Here once again the issue is not whether capital exists in the special technical sense we are giving to the word "exists"—for in practice capital never exists—but whether it comes close enough to existing for the time series we construct to tell us something useful about the economy.

*Accumulated consumption forgone.* Both of the preceding definitions of real capital—instantaneous productive capacity and long-run productive capacity—are aggregations of capital goods according to what we can do with them in certain circumstances. One might also measure real capital according to its opportunity cost. Capital in this sense is measured as the amount of consumption forgone in the process of acquiring the stocks of capital goods in existence. Suppose the only consumption good is potatoes, the only capital good is tractors, and tractors last forever (so we need not distinguish between instantaneous and long-run productive capacity). In the year 1 the output of tractors was 100 and the relative price of tractors and potatoes was 20 tons of potatoes per tractor. At the beginning of the year 2, there occurs a technical change in the tractor industry such that the alternative cost of producing tractors falls to half what it was in year 1. The relative price of tractors falls from 20 tons of potatoes to 10 tons of potatoes. Then in the year 2 the output of tractors increases to 200. According to the use definitions discussed above, the output of new capital goods has increased from 100 in the year 1 to 200 in the year 2. According to the opportunity cost definition, based on consumption forgone, the addition to capital is the same in both years because 2,000 tons of potatoes was sacrificed in the process of investment in each year. From a statistical point of view, the main difference between these measures of real capital is that the value of capital goods is deflated by a price index of capital goods in one case and a price index of consumption in the other.

This definition of capital is in a sense the logical conclusion of the attempt to price capital on the supply side according to their cost of production. For cost is only definable with respect to a numeraire, and the only numeraire that presents itself—if we exclude money and if we exclude capital goods themselves (since that is what we want to measure the cost of)—is consumption goods. This definition of capital is that employed by Kendrick in his measures of human capital; the forgone earnings of students and the alternative cost of research and development can be assessed in no other way. A straightforward implication of this definition of capital in real terms is that maintenance and repair should be treated as part of gross investment.

*Real wealth.* Real wealth is the present value, at some given time series of interest rates, of the stream of consumption goods earned by the existing stock of capital goods. Real wealth differs from the other measures of capital in real terms in a number of respects, the most interesting of which from our point of view is that any technical change that enhances the productivity of capital goods increases the quantity of real wealth as well. A given mix of capital goods $K_1$ and $K_2$ should count as the same amount of instantaneous productive capacity or long-run productive capacity at all times, but it represents more real wealth at a time when the current technology has endowed it with a high present and future marginal product than it does at a time when it is less favored by the existing technique.

The statistical implication of this feature of the concept of real wealth is that the measure of capital should include capital gains in addition to the original cost of equipment. It is for this reason that I hesitated to classify Eisner's imputation of capital gains to Young and Musgrave's measure of capital as an attempt to make the measure of real capital conform more closely to the definition of real capital in equation (1). That is not what Eisner is doing at all. Eisner starts out with a conventional estimate of capital in real terms, but he modifies that estimate for the purpose of measuring real wealth, which is in some sense independent of the stocks of capital goods.

Real wealth and accumulated consumption forgone are sometimes called the "forward-looking" and "backward-looking" measures of capital, while instantaneous productive capacity and long-run productive capacity are measures of capital "in their own units." One of the interesting theoretical issues that was touched upon but certainly not resolved at this conference was whether any price index can be constructed to reflect the quantity of capital in its own units, for it is at least arguable that the demand concept of price indexes advocated by Gordon leads inevitably to a measure of real wealth, whereas the supply concept advocated by Young and Musgrave leads to a measure of accumulated consumption forgone, leaving capital in its own units in a sort of theoretical limbo whenever technical change alters the nature of capital goods to a significant extent.

This issue was at least peripheral to the old debates over capital theory between Irving Fisher and Böhm-Bawerk[5] and, later on, between Hayek and Knight,[6] and it crops up again in the more recent controversy over the reswitching of techniques (Samuelson 1966) discussed in Brown's paper.

Ideally, our choice among these concepts of real capital ought to be governed by the purpose of the time series. It would be very convenient if we could go through our list of purposes of real capital and show that one particular concept of capital is preferable to the rest in every case. Unfortunately, this appears not to be so. I think that long-run productive capacity is the most appropriate concept of capital for inclusion in an investment function, because firms assess the need for new capital goods in accordance with their plans for the future and not just in accordance with their capacity to produce today. The concept of capital as wealth may be more appropriate as an argument in the consumption function. On the other hand, instantaneous productive capacity seems to be the appropriate species of capital for estimating production functions because the productivity of capital next year is irrelevant when we are concerned, for instance, to discover the elasticity of substitution between labor and capital today.[7] Views differ on which concept of capital is appropriate for computing the proportion of observed economic growth that can be attributed to technical change. My own view on the matter is that we want a measure of cumulative consumption forgone, because the essense of the problem is to estimate what national income would be today if technical change had not occurred and because change in the relative price of consumption goods and capital goods is one of several forms technical change can take. It is difficult to say which concept of capital is most appropriate for planning and budgeting until we have specified what methods of planning and budgeting are being employed. Presumably, instantaneous productive capacity would be the appropriate concept for the computation of capital-output ratios.

## Index Numbers and Aggregation

Once we have decided why we want to measure capital and which among the many possible definitions is appropriate for our purpose, we must set about building a time series of capital with the information at hand. The working assumption in Young and Musgrave's paper is that, as an indicator of long-run productive capacity, capital can be adequately represented by the Laspeyres index of equation (1). Diewert and Brown scrutinize this assumption carefully. They investigate the accuracy of the Laspeyres index as an indicator of the size of the capital stock, they consider alternative index number formulas, and they raise the question whether it is reasonable to postulate an aggregate production function to represent what is in effect the interaction of many production processes in which many capital goods are employed.

Problems in this area can be classified under two main headings. There are index number problems having to do with the measurability of capital with the available data, and there are aggregation problems having to do with the existence of summary measures of the capital stock. As a simple example of the index number problem, suppose we know there is a function $K = g\ (K_1, K_2)$ and we have time series of quantities of capital goods available, $K_1$ and $K_2$, and of prices of capital goods, $P_1$ and $P_2$, where $P_1$ and $P_2$ are proportional to first derivatives of $g$ with respect to $K_1$ and $K_2$; but we do not know the functional form $g$ and we do not have a time series of the values of $K$. The problem is to infer the time series of $K$ from the time series of $K_1$,$K_2$,$P_1$, and $P_2$. As a simple example of the aggregation problem, suppose there exists, not a function $g(K_1,K_2)$, but a pair of production functions $Q^A = f^A(K^A{}_1,K^A{}_2)$ and $Q^B = f^B(K^B{}_1,K^B{}_2)$ for each of two industries, $A$ and $B$, where $Q^A$ and $Q^B$ are outputs, and total supplies of the two capital goods are $K_1 = K^A{}_1 + K^B{}_1$ and $K_2 = K^A{}_2 + K^B{}_2$. The problem is to determine whether and in what circumstances one can derive a function $K = g(K_1,K_2)$ from the production functions $f^A$ and $f^B$, where $g$ has the property that $g(K_1,K_2) = T(Q^A,Q^B)$ and where the function $T$ is the production possibility curve for the economy as a whole.

It is difficult to assess the importance of the aggregation problem. On the one hand, one might argue that all models falsify reality to some extent, that a simple model such as that in which real capital is defined cannot as a rule be derived from richer and more complex models of the economy, and that one must accept the inevitable discrepancy if one is to describe the economy at all. The aggregation problem in capital measurement is not different in principle from the aggregation problem in deriving a community demand curve from the demand curves of the people within the community. On the other hand, it is arguable that if we cannot solve the aggregation problem and if we cannot imagine a variable in a function that our statistics of capital are intended to represent, then we lose all sense of what it is we are supposed to be measuring, we have no basis for choosing among alternative measures of capital, and we do not know what, if anything, the resulting time series of capital tells us about the economy.

Both Brown and Diewert discuss the aggregation problem in detail. They show that aggregation is not normally possible except by stringent and unrealistic restrictions on the form of the production function or on the organization of the market. Diewert also conducts a systematic study of the properties of several alternatives to the Laspeyres index of equation (1). He investigates Fisher's ideal index, the Divisia index, and the Vartia index, which is an approximation to the Divisia index for use on time series data. He considers a class of indexes, which he calls superlative, with the property that they all yield particularly good approximations to the unknown time series $K$ for a wide class of functional forms of $g$. He assesses the usefulness of the different indexes in the measurement of technical change by sector and for the economy as a whole, and he considers some of the problems in incorporating new goods into the index number formulas.

## Notes

1. The classic statement is Goldsmith (1951), followed in 1956 by the three volumes of *A Study of Savings in the United States.*

2. *Measuring the Nation's Wealth* (Joint Economic Committee of the Congress of the United States, 1964), a study directed by J. W. Kendrick. The study contains a great deal of information on many aspects of the measurement of capital. The survey of capital goods in the USSR is discussed in a paper by A. Kaufman.

3. For a useful discussion of this issue, see Griliches (1964), together with comments in the same volume by G. Jaszi, E. Denison, and E. Grove. See also Stigler and Kindahl (1970); Gordon (1971); and the comment on Gordon's paper by J. Popkin and R. Gillingham in the September 1971 issue of *Review of Income and Wealth.*

4. For an early and still very instructive account of the distinction between instantaneous and long-run production capacity, see Griliches (1963).

5. See the section on Böhm-Bawerk entitled "Technical Superiority of Present Goods" in Fisher (1930, pp. 473–85).

6. The controversy is reviewed and the relevant articles by Hayek, Knight, and others are listed in Hayek (1941).

7. In their study of aggregate technical change, Jorgenson and Griliches (1967) have constructed special time series of capital, weighting quantities by rents rather than by capital goods prices to reflect instantaneous productive capacity. Note particularly that the appropriate rate of depreciation on capital as instantaneous productive capacity is different from that on capital as long-run productive capacity, and that it is the latter that is estimated in the studies by Young and Musgrave, Coen, and Hulten and Wykoff. Consider two machines, $A$ and $B$, for which the value of services decline at 10% per year in each year of their lives, and that differ only in that $A$ disintegrates after five years while $B$ disintegrates after twenty. If both machines are two years old, their rates of depreciation as instantaneous productive capacity are the same, but the rate of depreciation of the long-run productive capacity is greater for $A$ than for $B$ because a larger portion of the lifetime services of $A$ is used up in the second year of its life.

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