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Multilateral Exchange Rate Determination: A Model for the Analysis of the European Monetary System

Giorgio Basevi and Michele Calzolari

13.1 Introduction

The purpose of this paper is to build a theoretical framework that will be as simple as possible and yet adequate for econometric estimation and policy analysis of the process of exchange rate determination in a multicountry and multicurrency world. By using a popular model of exchange rates—extended by Frankel (1979) on the basis of an earlier version proposed by Dornbusch (1976)—we also hope to lay the ground for more satisfactory tests of its theoretical foundations. We do this, first, by making the model multilateral—until now it has been cast in a two-country world—and second, by analyzing rationally formed expectations about future exchange rates on the basis of the model's structural version rather than the reduced-form version used by previous commentators. ¹

Our treatment of rational expectations is, however, limited to the extent that we have not assumed rationally expected processes for the exogenous variables. Thus we cannot present in this paper the analysis of the effects of policy changes and of other exogenous shocks. The characterizing feature of this stage of our research is the analysis of the stability properties of the multilateral exchange rate determination mechanism, whose structure we

This work is a part of a larger project developed at the European Community Commission (the Eurolink Project) and aimed at linking the models of at least four European countries: Germany, France, the United Kingdom, and Italy. The Italian model in the project is being developed at Prometeia Associates in Bologna. We thank C. Corradi, F. Giavazzi, and C. Wyplosz for helpful criticism and suggestions.

1. The same criticism of Frankel's model and tests has recently been made by Driskill and Sheffrin (1981), who also present estimates using the rational expectations hypothesis. Their version of the model, however, is still cast in terms of a two-country world.

have estimated under the assumption of given paths for the exogenous variables.²

In section 13.2 we outline the basic theoretical specification of the model. Section 13.3 presents a variant of the model together with estimates of behavioral functions for the two alternative specifications of the model, while section 13.4 deals with stability analysis of the system.

In section 13.5 we indicate in which direction our research is currently developing.

13.2 The Theoretical Model

Consider any two currencies (countries) i, k chosen from the set of n countries (currencies) that form the elements of a multilateral trade and payments network. Short-term capital movements ensure a covered interest rate relation that need not be at parity as we consider national markets rather than Euromarkets. We thus allow transaction costs and exchange controls to introduce a wedge δ into

(1)
$$[(1 + i_i^s)/(1 + i_k^s)]_t = [(F_{ik}^s/S_{ik}) \delta_{ik}^s]_t,$$

where i^s are short-term interest rates, F^s_{ik} is the forward exchange rate (with the same short-term maturity) between currency i and k, s_{ik} is the spot exchange rate, and δ^s_{ik} is the distortion coefficient due to transaction costs and exchange controls.

Because of exchange rate risk, a spread (δ_{ik}^s) may also open between the forward rate and the expected future spot rate,

$$(2) F_{ik,t}^s = {}_t S_{ik,t+s} \cdot \delta_{ik,t}^s,$$

where $S_{ik,t+s}$ indicates the value of S_{ik} expected at time t for time t + s.

We assume that the nominal quantity of money supplied is demanded according to the following function:

(3)
$$m_{i,t} = \alpha_i p_{i,t} + \beta_i \bar{y}_{i,t} - \gamma_i r^s_{i,t}; i = 1, \ldots, n,$$

where lowercase letters stand for logs (of money, m; prices, p; real income, y), except that $r^s = \log (1 + i^s)$, and where \bar{y} stands for equilibrium real income.

We now consider (1) and (2) for a maturity s = 1, and, taking logs (with log S = e), we write

(4)
$$_{t}e_{ik,t+1} - e_{ik,t} = r_{i,t} - r_{k,t} - w_{ik,i}; i = 1, \ldots, n,$$

where r_i and r_k correspond to 1-period maturity interest rates and $w_{ik} = \log r$

2. We are currently extending the model in the direction of policy analysis. Results based on the estimation of monetary authorities' reaction functions were presented at conferences at the University of Illinois and the University of Louvain. We will publish these results in a conference volume edited by P. De Grauwe and T. Peeters. See Basevi and Calzolari (1982).

 $(\delta_{ik} \cdot \sigma_{ik})$. In line with the asset market approach to exchange rate determination, we assume that the money market is continuously in equilibrium, so that $m = \overline{m}$; further assuming that equilibrium short-term real rates of interest are internationally equal, substitution of equation (3) into equation (4) yields

(5)
$$e_{ik,t+1} - e_{ik,t} = \frac{\alpha_i}{\gamma_i} (p_{i,t} - \overline{p}_{i,t}) - \frac{\alpha_k}{\gamma_k} (p_{k,t} - \overline{p}_{k,t}) + (\overline{\pi}_{i,t} - \overline{\pi}_{k,t}) - w_{ik,t},$$

where $\overline{\pi}$ are equilibrium rates of expected inflation.

We assume that the deviation of real income from equilibrium level is related to the deviation of the real exchange rate (q) from its equilibrium level (\overline{q}) and to the deviation of the dollar price of oil from its trend $(v - \overline{v})$

(6)
$$y_{i,t} - \overline{y}_{i,t} = D_i(L)(q_{i,t} - \overline{q}_{i,t}) + R_i(L)(v_t - \overline{v}_t); i = 1, \ldots, n,$$

where $D(L) = \sum_{j} d_{j}L^{j}$ and R(L), similarly defined, are polynomials in the lag operator L, such that $L^{j}x_{t} = x_{t-j}$.

The real exchange rate q is defined as

(7)
$$q_i = \left(\sum_{j \neq i}^{1} \mu_{ij} e_{ij} - p_i + \sum_{j \neq i}^{1} \mu_{ij} p_j\right); i = 1, ..., n.$$

By assuming that purchasing power parity holds in equilibrium, it follows that $\overline{q}_i = 0$ for all i.

The current rate of price change is assumed to diverge from the equilibrium rate of price inflation $(\overline{\pi}_t)$ because of disequilibrium real income:

(8)
$$p_{i,t+1} - p_{i,t} = G_i(L)(y_i - \overline{y}_i)_t + \overline{\pi}_{i,t}; i = 1, \ldots, n.$$

Substituting (6) into (8), we obtain

(9)
$$p_{i,t+1} - p_{i,t} = H_i(L)q_{i,t} + K_i(L)(v - \overline{v})_t + \overline{\pi}_{i,t};$$

$$i = 1, \ldots, n,$$

with H(L) = G(L) D(L) and K(L) = G(L) R(L).

Taking into account that $e_{ki} = -e_{ik}$, that $e_{kk} = 0$, and that the sum of the weights μ equals unity, it is possible to reduce (g) to a final expression in terms of endogenous (p, e) and of exogenous variables $(\overline{p}, \overline{\pi}, \nu, \overline{\nu},$ and w):

3. The treatment of \overline{p} and $\overline{\pi}$ as exogenous variables is an intermediate step, to be followed by an explicit analysis of the process generating m (or i); the choice depends upon whether we consider the stock of money or the rate of interest as the authorities' control variable. Moreover, w is only provisionally included among the exogenous variables: a full treatment would include a theory of exchange risk, while δ —the foreign exchange control coefficient—should be included among the control variables. As for the short- and long-run price of oil, it can remain exogenous to the model.

(10)
$$p_{i,t+1} - p_{i,t} = H_i(L)e_{ik,t} - H_i(L) \sum_{\substack{j \neq i \\ j \neq k}} p_{ij}e_{jk,t}$$
$$-H_i(L) \left(p_i - \sum_{\substack{j \neq i \\ j \neq k}} p_{ij}p_j\right)t$$
$$+K_i(L)(v - \overline{v})_t + \overline{\pi}_{i,t}.$$

Mathematical convenience and the theory of international monetary systems suggest choosing a specific country as the one whose currency is used as the numeraire of the system. The obvious candidates for this role are the United States and the dollar. While this choice will be explicitly made in section 13.3, we now simply label the nth country (currency) as the reference country (currency). We thus have a system of linear difference equations in n-1 exchange rates (e_{in} ; $i=1,\ldots,n-1$) and in n prices (p_i ; $i=1,\ldots,n$), which, considering equations (5) and (10), can be written in matrix form as

$$(11) x_{t+1} = \Omega x_t + z_t,$$

where the vectors x_{t+1} and x_t of the endogenous variables are defined as

$$x_{t+1} = \begin{bmatrix} t^{e_{1,n;t+1}} \\ \vdots \\ t^{e_{n-1,n;t+1}} \\ p_{1;t+1} \\ \vdots \\ p_{n;t+1} \end{bmatrix}; x_{t} = \begin{bmatrix} e_{1,n;t} \\ \vdots \\ e_{n-1,n;t} \\ \vdots \\ p_{1;t} \\ \vdots \\ \vdots \\ p_{n;t} \end{bmatrix}$$

and the vector z of the exogenous variables⁴ is

$$z_{t} = \begin{bmatrix} \frac{\alpha_{1}}{\gamma_{1}} \overline{p}_{1,t} + \frac{\alpha n}{\gamma n} \overline{p}_{n,t} + \overline{\pi}_{1,t} - \overline{\pi}_{n,t} - w_{1n,t} \\ \vdots \\ \overline{\pi}_{n,t} + K_{n}(L)(v - \overline{v})_{t} \end{bmatrix}.$$

4. As already pointed out, endogenization of exchange risk and of the process generating money supply—and hence equilibrium price levels and inflation rates—would change the content of vector z and consequently that of the matrix Ω .

The square matrix Ω is of dimension 2n-1 and can be partitioned into

$$= \begin{bmatrix} \Omega_{11} & & \Omega_{12} \\ \Omega_{21} & & \Omega_{22} \end{bmatrix},$$

where Ω_{11} is an identity matrix of dimension (n-1)(n-1), Ω_{12} is a matrix of dimension (n-1)n:

$$\Omega_{12} = \begin{bmatrix} \frac{\alpha_1}{\gamma_1} & 0 & \dots & 0 & 0 & -\frac{\alpha_n}{\gamma_n} \\ 0 & \frac{\alpha_2}{\gamma_2} & \dots & \dots & -\frac{\alpha_n}{\gamma_n} \\ \vdots & & & \vdots \\ 0 & \dots & \dots & \frac{\alpha_{n-2}}{\gamma_{n-2}} & 0 & -\frac{\alpha_n}{\gamma_n} \\ 0 & \dots & \dots & \frac{\alpha_{n-1}}{\gamma_{n-1}} - \frac{\alpha_n}{\gamma_n} \end{bmatrix}$$

while the remaining parts of Ω are

$$\Omega_{21} = \begin{bmatrix} H_1(L) & -\mu_{1,n-1}H_1(L) \\ \cdot & \cdot \\ \cdot & \cdot \\ -\mu_{n,1}H_n(L) \cdot \cdot \cdot -\mu_{n,n-1}H_{n-1}(L) \end{bmatrix},$$

 $\Omega_{22}=I-\Omega_{21}^*,$ and Ω_{21}^* is the Ω_{21} matrix augmented by a last column equal to

$$\begin{bmatrix} -\mu_{1,n}H_1(L) & & & \\ & \cdot & & \\ & \cdot & & \\ -\mu_{n-1,n}H_{n-1}(L) & & \\ & H_n(L) & & \end{bmatrix}.$$

13.3 The Estimates

In order to implement the multilateral model of exchange rates just presented we have reduced the "world" to a set of ten countries: the United States, Germany, France, Japan, Canada, the United Kingdom, Italy, the Netherlands, Belgium, and Switzerland. The United States has been chosen as the *n*th country for its size and for the dominant role of its currency in the international monetary system. In view of our empirical aim—which is

to estimate a model to be used for the analysis of the European Monetary System—we also considered Canada, Japan, and Switzerland as an exogenous subset of countries. Moreover, we have provisionally limited the estimates of the structure to five countries: the four large European countries plus the United States.⁵ Thus our system (11) is of dimension 2n - 1 = 9.

The elements of Ω are combinations of the parameters of equations (3), (4), and (6). Thus, for the five specified countries we estimated the parameters determining money demand, disequilibrium income, and price inflation. The sample is made up of monthly observations from 1971.10 to 1980.12; data sources are given in the statistical appendix.⁶

The data were first used to estimate a version of model (11) modified by the use of current income rather than equilibrium income in the money demand functions. From a formal point of view this modification has the consequence of adding a matrix B to the identity matrix Ω_{11} , with

and of subtracting its augmented version B^* to the matrix Ω_{12} . The modification does not affect the matrices Ω_{21} and Ω_{22} . In other words, only the exchange rate equations and not the price equations are affected by this alternative specification of the money demand function. The set of estimates based on this modified version of the model are presented in tables 13.1, 13.2, and 13.3. The first rows in the tables correspond to ordinary least squares.

In order to keep the system as small as possible to make the stability analysis in the next section feasible, this first set of estimates—while using

^{5.} This choice means that in our empirical use of system (11) the vector z contains, in addition to the variables explicitly written above, the exchange rates and prices of the countries that are left exogenous to the model.

^{6.} In all estimates y is proxied by the index of industrial production and \bar{y} has been constructed by interpolating y on the basis of the following function of time: $\bar{y} = \alpha + \beta t - \gamma t^2$. The same interpolation is used to construct the equilibrium price of oil, \bar{y} .

Table 13.1 Demand for Money: Equation (3) (Monthly Observations, 1971.10–1980.12)

		Constant	Q1	Q12	<i>p</i>	_ y	r	D-W	SE	MDV
United States	OLS	.5148	.0007	.0286	.9409	.3659	-2.4916	.17	.023	6.62
2SLS		(.16)	(800.)	(800.)	(.017)	(.043)	(1.08)	-		
	2SLS	4.7207	.0007	.0288	.9336	.3932	-2.7598	.17	.023	6.62
		(.22)	(.008)	(800.)	(.018)	(.046)	(1.09)			
Germany	OLS	-4.4759	.0096	.0438	1.4426	.8910	-8.9497	.72	.022	6.33
2		(.20)	(800.)	(.007)	(.030)	(.063)	(1.03)			
	2SLS	1.5317	.0091	.0447	1.3907	1.0282	-10.2036	.81	.023	6.33
		(.33)	(800.)	(800.)	(.033)	(.071)	(1.09)			
France	OLS	-1.4572	.0018	.0313	1.2642	.4741	-5.9480	.30	.030	6.63
		(.32)	(.010)	(.010)	(.018)	(.082)	(1.71)			
	2SLS	3.7528	.0018	.0330	1.2418	.6076	-7.3811	.36	.030	6.63
		(.46)	(.010)	(.010)	(.020)	(.099)	(1.82)			
United Kingdom	OLS	1.3612	.0042	.0411	.8587	.9468	-4.6581	.32	.043	9.77
_		(.48)	(.015)	(.015)	(.013)	(.100)	(1.67)	1	Ì	
	2SLS	4.2493	.0378	.0230	.8461	1.1703	-3.0456	.46	.047	9.77
		(.57)	(.016)	(.016)	(.013)	(.122)	(1.78)			
Italy	OLS	4.4458	.0184	.0421	1.2282	.3196	-5.8491	.24	.039	11.71
•		(.28)	(.014)	(.013)	(.018)	(.072)	(1.34)			
	2SLS	9.5919	.0176	.0421	1.2087	.4299	-6.5247	.29	.040	11.71
		(.40)	(.014)	(.013)	(.020)	(.087)	(1.38)		}	

Notes: Standard errors in parentheses. Q1 and Q12 are dummies for January and December. D-W = Durbin-Watson statistic. SE = standard error of the regression. MDV = mean of the dependent variable.

Table 13.2 Income Disequilibrium: Equation (6) (Monthly Observations, 1971.10–1980.12)

		Constant	y_t	q_t	$(v - \overline{v})_t$	D-W	SE	MDV
United States	OLS	-2.6623	1.5547	.2581	1104	.14	.034	4.73
		(.53)	(.11)	(.08)	(.01)			
	2SLS	-2.1006	1.4357	.3900	1100	.16	.035	4.73
		(.58)	(.12)	(.09)	(.01)			
Germany	OLS	-1.9620	1.4402	0084	1214	.21	.033	4.69
		(.48)	(.10)	(.08)	(.02)			
	2SLS	-1.8970	1.4255	0545	1179	.21	.033	4.69
		(.48)	(.11)	(.09)	(.02)			
France	OLS	5248	1.1229	.4584	0725	.45	.032	4.67
		(.37)	(.08)	(.09)	(.01)			
	2SLS	5312	1.1232	.5262	0713	.46	.032	4.67
		(.37)	(.08)	(.10)	(.02)			
Jnited Kingdor	n OLS	-12.0961	3.6267	.0398	1704	.44	.029	4.66
		(1.48)	(.32)	(.03)	(.02)			
	2SLS	-12.0954	3.6265	.0406	1704	.44	.029	4.66
		(1.48)	(.32)	(.03)	(.02)			
taly	OLS	-1.0072	1.2236	.0295	0568	.36	.054	4.71
		(.66)	(.14)	(.097)	(.03)			
	2SLS	-1.0286	1.2281	.0230	0567	.36	.054	4.71
		(.67)	(.15)	(.100)	(.03)			

Notes: Standard errors in parentheses. D-W = Durbin-Watson statistic. SE = standard error of the regression. MDV = mean of the dependent variable.

Table 13.3 Price Functions: Equation (8) (Monthly Observations, 1971.10-1980.12)

		Constant	p_{t-1}	y_{t-1}	$-\overline{y}_{t-1}$	$\overline{\pi}_{t-1}$	Н	SE	MDV	LR
United States	OLS		.9957	.0271	.0290	2.3725	1.24	.0026	.056	4.43
			(.002)	(.005)	(.005)	(.44)				
	2SLS	_	.9950	.0287	.0308	2.5411	1.23	.0026	.056	
			(.002)	(.005)	(.005)	(.47)				
Germany	OLS	_	1	.0	107	.6212	3.17	.0028	.0041	6.28
				0.)	06)	(.04)				
	2SLS	_	1	.0	109	.6202	3.17	.0028	.0041	_
				0.)	07)	(.04)				
rance	OLS	_	1	.0	190	1	2.46	.0027	.0003	11.3
				0.)	06)					
	2SLS	<u> </u>	1		204	1	2.41	.0027	.003	_
				0.)	06)					
United Kingdor	m OLS		1		087	1	2.72	.0078	.0005	8.6
				0.)	•					
	2SLS	_	I		021	i	2.62	.0078	.0005	_
				0.)	•					
Italy	OLS	_	.9965		405	1.2687	4.00	.0057	.13	10.3
			(.002)	0.)		(.06)				
	2SLS		.9964		592	1.2805	3.69	.0058	.13	_
			(.001)	0.)	01)	(.064)				

Notes: Standard error in parentheses. H = Durbin-h statistic. SE = standard error of the regression. MDV = mean of the dependent variable. LR = likelihood ratio to test the linear constraints imposed by the specification.

current rather than equilibrium income in the money demand function—do not allow for lags in endogenous variables; that is, the polynomials D(L) and G(L) are truncated at their first terms d_o and g_o . Possibly as a consequence of this, the estimates in tables 13.1–13.3 denote the presence of first-order serial autocorrelation of residuals, particularly in the income disequilibrium equations (but also in the money demand equations where our theoretical model did not provide for lags).

Even though, as a consequence, their standard errors are underestimated, the coefficients of the money demand functions (table 13.1) all have the theoretically expected sign and are generally of the correct magnitude. Notice, however, that in this first set of estimates we do not constrain the price elasticity of money to equal unity; this homogeneity constraint will, on the other hand, be imposed in the second set of estimates, that is, those using the basic version of the model with \bar{y} in the money demand functions.

In the income equations (table 13.2), the significance of the coefficient of the real exchange rate is particularly low in the case of the United Kingdom and of Italy, while its sign appears contrary to theory, but not significant, in the case of Germany. The price equations reported in table 13.3 are only a subset of our initial estimates. In fact, we have first allowed estimation of a constant in these equations, to account for the fact that the use of the (long-term) nominal interest rate as a proxy for the equilibrium expected inflation rate π introduces into the equations the value of the equilibrium real rate of interest. If this is assumed to be constant (in line with the assumption that $\overline{q} = 0$), its value is estimated by the constant in the price functions. Our initial estimates—not here reported—proved this constant to be insignificantly different from zero for all countries in our sample.

To allow for the possibility that (due to the simultaneity of the model) some of the explanatory variables in each equation are correlated with the error term, we also used two-stage least squares estimation by instrumental variables. We chose as instruments the exogenous variables of the model.⁷ The results are also reported in tables 13.1, 13.2, and 13.3, but they do not show a dramatic change of estimated coefficients.

Because the estimates presented in tables 13.1–13.3 are generally plagued by high autocorrelation of the residuals, we performed an autoregressive transformation of the variables using the Cochrane-Orcutt procedure. The results, not reported here, were unfavorable to the theoretical specification of the model in the sense that some coefficients, particularly in the money demand function, acquired the wrong sign and/or became insignificantly different from zero.

We therefore resumed the basic and simpler model, the one with \bar{y} in the

^{7.} For practical reasons, only those pertaining to the five countries of the model were used. The variables pertaining to third countries—Canada, Japan, the Netherlands, and Belgium—did not significantly change the results when they were included among the instruments.

money demand function as formally presented in section 13.2. We estimated it, allowing for 3-period distributed lags in the real exchange rate coefficient of equation (6) and performing a first-order autoregressive transformation in the variables of all three behavioral equations. In addition, we constrained the coefficient of the price variable in the money demand functions to unity, as theoretical considerations would suggest.⁸

Similarly, we chose to follow the theoretical specification of equations (6) and (8) by imposing unitary coefficients to the \bar{y} variable in equation (6) and to the p_{t-1} variable in equation (8), and also by imposing the same estimate (except for sign) to the coefficient of y and \bar{y} in equation (8).

Tables 13.4–13.6 thus report in the first row of each country OLS estimates of equations (3), (6), and (8) according to the specifications just mentioned. In the case of the United States, poor initial estimates of the interest rate coefficient in their demand for money equation and of the real exchange rate coefficient in the income equation induced us to impose values which seem reasonable on the basis of cross-country comparisons or of results of previous studies.

According to an F-test, the imposed constraint is not rejected by the data in case of the latter coefficient (.005), while the value of the former (-.85) is at the limit of the critical region of acceptance. On the whole, the values of the D-W or h-statistics in tables 13.4-13.6 indicate that much of the problem of autocorrelation of the residuals has been eliminated in this new set of estimates.

To allow for the possibility that (due to the simultaneity of the model) some of the explanatory variables in each equation are correlated with the error term, and to take into account the interequation covariances in the variance-covariance matrix of residuals, we also used three-stage least squares but, because of limited computer storage capacity, we did it by taking the three structural equations together country by country rather than by using the whole 5×3 system of equations. This is equivalent to assuming that the variance-covariance matrix of the residual is block diagonal, which implies that we disregard cross-country effects. The results of three-stage least squares estimations are reported in the second rows of each country in tables 13.4-13.6.

Relative to their OLS estimates, the coefficients most affected by the three-stage least squares method are those for the income variable in the money demand function and for the real exchange rate variable in the income disequilibrium function. Unfortunately, the coefficients for the latter do not improve their level of significance, which remains very low. While better estimates could be obtained by extending the lags already present and

^{8.} As a matter of fact this homogeneity constraint would be rejected for some countries on the basis of a *t*-test. We chose, however, to impose it both for theoretical reasons and because in the stability analysis of section 13.4 the results are not significantly affected by the presence or absence of the constraint.

Table 13.4 Demand for Money: Equation (3) (Monthly Observations, 1971.10-1980.12)

		Constant	Time	Q1	Q12	ŷ	r	ρ̂	D-W	SE	MDV
United States	OLS	_	003	.003	.030	.462	85^{a}	.92	2.10	.009	.031
3SLS			(.002)	(.002)	(.022)	(.012)					.001
	3SLS	_	36	.002	.030	1.503	85^{a}	.92	2.08	.008	.031
			(.040)	(.002)	(.002)	(.013)					
3	OLS	_	.25	002	.021	.287	-3.62	.91	2.01	.011	1.707
			(.042)	(.003)	(.003)	(.015)	(1.97)				
	3SLS	-	.05	.001	.022	1.335	-3.61	.93	2.95	.011	1.707
France	OLS	-11.26	28	.006	.038	2.92	-1.91	.84	1.97	.011	1.954
		(3.86)	(.16)	(.003)	(.003)	(.88.)	(1.66)				
	3SLS	6.96	.53	.006	.035	3.07	-1.60	.85	2.08	.011	1.954
		(3.96)	(.23)	(.002)	(.003)	(.92)	(1.78)				
United Kingdom	OLS	<u> </u>	24	.008	.033	1.17	79	.96	1.68	.017	5.068
			(.13)	(.005)	(.005)	(.05)	(1.84)				
	3SLS	-	- .29	.006	.032	2.17	77	.96	1.70	.016	5.068
			(.15)	(.005)	(.005)	(.06)	(1.9)				
Italy	OLS	_	36	.021	.061	1.61	41	.97	1.45	.011	6.973
			(.10)	(.003)	(.003)	(.04)	(1.09)				
	3SLS		54	.024	.062	2.64	50	.98	1.51	.011	6.973
		-	(.26)	(.003)	(.003)	(.11)	(1.1)				

Notes: Standard error in parentheses. $\hat{\rho}$ estimated with Cochrane-Orcutt method. D-W = Durbin-Watson statistic. SE = standard error of the regression. MDV = mean of the dependent variable. ^aValue imposed a priori F(1,109) = 7.6.

Table 13.5 Income Disequilibrium: Equation (6) (Monthly Observations, 1971.10-1980.12)

		Constant	Time	q	q -1	q_{-2}	Σ_q	$(v - \overline{v})_t$	ρ̂	D-W	SE	R^2
United States	OLS	.017	01	.0015	.002	.0015	.005	085	.94	.58	.011	.09
3SLS		(.096)	(.06)				(a)	(.026)				
	3SLS	.012	01	.0015	.002	.0015	.005	115	.92	.61	.011	.11
	(.094)	(.05)				(a)	(.033)					
Germany	OLS	.070	04	.0808	.0538	.0269	.161	105	.87	2.20	.014	
		(.52)	(.03)				(.125)	(.028)				
3SI	3SLS	.064	04	.0243	.0324	.0243	.081	118	.86	2.26	.014	
		(.051)	(.03)				(1.30)	(.035)				
France	OLS	.133	10	.1125	.1500	.1125	.375	076	.76	2.48	.019	.15
		(.039)	(.02)				(.16)	(.025)				
	3SLS	.125	09	.1188	.1584	.1188	.396	101	.76	2.49	.019	
		(.39)	(.2)				(.17)	(.032)				
United Kingdom	OLS	.019	- .01	.0228	.0304	.0228	.076	094	.77	2.12	.018	.13
		(.040)	(.03)				(.07)	(.025)				
	3SLS	.021	01	.0186	.0248	.0186	.062	120	.77	2.13	.018	
		(.030)	(.02)				(.06)	(.031)				
Italy	OLS	.01	01	.0260	.0346	.026	.087	073	.81	2.38	.031	.11
		(.09)	(.06)				(.19)	(.048)				
	3SLS	.017	02	.0513	.0684	.0513	.171	069	.81	2.31	.030	
		(.09)	(.06)				(.21)	(.061)				

Notes: Standard error in parentheses. $\hat{\rho}$ estimated with Cochrane-Orcutt method. D-W = Durbin-Watson statistic. SE = standard error of the regression. R^2 = in terms of changes. "Value imposed a priori; F(1, 109) = 2.996.

Table 13.6 Price Functions: Equation (8) (Monthly Observations, 1971.10-1980.12)

		Constant	Time	$(y-\overline{y})_{t-1}$	$\overline{\pi}_{t-1}$	ρ̂	Н	SE	R^2
United States	OLS	004	(-)	.026	1.546	.15		.003	.42
		(.002)		(.006)	(.249)				
	3SLS	004	(-)	.026	1.558	.15	177	.003	
Germany	OLS	(-)	(-)	.009	.620	.30	.002	.003	.34
38				(800.)	(.054)				
	3SLS	(-)	(-)	(.010)	.618	.30	038	.003	
				(.008)	(.054)				
France	OLS	002	.002	.017	.975	.22	161	.003	.37
		(.002)	(.001)	(.008)	(.320)				
	3SLS	002	.002	.017	.983	.22	121	.003	
		(.002)	(.001)	(.008)	(.317)				
United Kingdo	m OLS	006	004	.023	2.269	.20	.069	.007	.40
		(.006)	(.003)	(.022)	(.597)	ŀ			
	3SLS	006	004	.014	2.237	.20	.066	.007	
		(.006)	(.003)	(.023)	(.588)				
Italy	OLS	.006	(-)	.010	.555	.47	132	.005	.25
		(.004)		(.014)	(.369)				
	3SLS	.006	(-)	.014	.500	.46	120	.005	
		(.004)		(.014)	(.358)				

Notes: Standard errors in parentheses. $\hat{\rho}$ estimated with Cochrane-Orcutt method. H = Durbin's h-statistic. $SE = \text{standard error of the regression. } R^2 = \text{in terms of changes.}$

introducing them in the money and price functions, where they are not present, we have chosen not to follow this avenue. In fact, as a result the size of our Ω matrix would be correspondingly enlarged. As it is, with the lags already present in the q variable of the income equations, the dimension of Ω when the system is transformed into its first-order canonical form is 6n-3. This, with five countries, makes 27, which is a fairly high order for the characteristic equation to be solved numerically in the next section.

13.4 Stability Analysis

The theoretical model we have used as the basis of our analysis has been estimated and tested for stability by Frankel and many other authors. In our view one of the main weaknesses of the debate about the model and its empirical verification is due to the fact that it has generally been cast in terms of a two-country world. To justify this assertion, we may consider Frankel's original contribution (Frankel 1979). In it, the deutsche mark to dollar (DM/\$) exchange rate is shown to converge to a stable path determined by purchasing power parity. The speed of convergence depends only on the parameters of the money demand and price functions of the United States and Germany, regardless of the economic structure and events in third countries. Clearly this is not the case when more than two countries are explicitly introduced into the analysis. The roots of the characteristic equation that determine the stability conditions for the system depend, in the general *n*-countries case, upon the parameters of the structural equations of all *n* countries.

Thus, with reference to our system (11), while the vector z drives the endogenous variables along their equilibrium path, the whole structure of the matrix Ω determines whether the system converges again to that path after it is shocked by changes in the exogenous variables.

Although relatively simple as a macroeconomic model, our system is complex enough to require an analytical examination of its stability conditions. We have therefore used the numerical estimates of the structure obtained in the previous section to compute the eigenvalues of matrix Ω . Blanchard and Kahn (1980) have shown that in a linear difference equation system in which a subset of variables is forward looking and the remaining subset is backward looking, uniqueness and stability of solutions are ensured when there are as many roots of the characteristic equation of the system with module larger than one as there are forward-looking variables, and as

^{9.} For criticism and defense of that model, see the exchange comments in the American Economic Review, December 1981.

^{10.} The same criticism has also been made by Driskill and Sheffrin (1981). Notice that tests of the model for different pairs of countries (currencies) have been conducted by many authors. All those that we know of, however, remain pairwise tests, and none of them is built within a multilateral exchange rate framework.

many with module smaller than one as there are backward-looking variables. Our system corresponds to this classification, with n-1 forward-looking variables (the n-1 exchange rates) and n backward-looking variables (the n prices).

Table 13.7, in its part A, shows the eigenvalues of matrix Ω based on the numerical parameters obtained in section 13.3 on the basis of the first set of estimates taken from tables 13.1–13.3. We have chosen for this purpose both OLS and 2SLS estimates. The results show that when all five countries are included in the model the eigenvalues do not conform to the Blanchard-Kahn criterion for uniqueness and stability of the solutions. This seems to be due to the wrong coefficient in the income equation for Germany; in fact, when we take Germany out of the system and reduce its endogenous part to a four-country set (and also when we further reduce it to subsets of three or two countries, always excluding Germany), the eigenvalues conform to the Blanchard-Kahn criterion.

Table 13.7A Numerical Solution for the Roots of the Characteristic Equation of System (11) Based on Results of Tables 13.1–13.3

	OI	_S	2SI	LS
	1.0849	.9769	1.1060	.9826
	1.0645	.9838	1.0862	.9786
N = 5	1.0434	.9942	1.0623	.9959
	1.0330	.9988	1.0420	.9998
	1.0001		1.0008	
	1.0879	.9769	1.1073	.9854
N=4	1.0535	.9955	1.0586	.9966
(excluding	1.0425	.9870	1.0766	.9998
Germany)		.9993		.9998

Table 13.7B Numerical Solution for the Roots of the Characteristic Equation of System (11) Based on Results of Tables 13.4–13.6

	О	LS	3 5	SLS
	1.8400	.9999	1.8404	.9999
	1.8397	.9999	1.8395	.9993
N = 5	1.8397	.9998	1.8391	.9980
	1.8382	.9998	1.8385	.9968
		0260		0270

As the estimates of tables 13.1-13.3 are plagued by first-order auto-correlation of the residuals, the set of roots contained in table 13.7A is not very reliable. We have therefore recomputed the eigenvalues on the basis of the estimates contained in tables 13.4-13.6; while the coefficients in these

tables generally seem not much more significant than the corresponding coefficients of tables 13.1-13.3, they are less weakened by the phenomenon of autocorrelation. We do not report the 18 additional roots introduced by the two lags of q that appear in each country's income equation; in general, these roots introduce a cyclical movement in the adjustment path which was not present in the unlagged version of the model, but their module is always smaller than unity. As for the remaining nine roots, they are reported in table 13.7B.

Contrary to the results of table 13.7A, this new set of roots conforms to the stability criterion even when all the countries in our sample are included. It is, however, alarming to notice that the roots with modules smaller than unity are dangerously close to the edge of instability. We have therefore engaged in a series of sensitivity experiments, by changing the coefficients of the matrix Ω in the neighborhood of their mean values. The results show that the stability of the system is rather robust except when we change the value of the coefficient of the q variable in the income equations. Thus table 13.8, based only on 3SLS estimates, presents three experiments. In the first of them (part A), the coefficient of the real exchange rate in the United States income equation has been increased from .005 to .009. In the second (part B), the coefficients of q for all countries except the United States have been increased by twice their standard errors. Finally, in the third experiment (part C) we have increased by these amounts the q-coefficients of all countries.

Table 13.8 Numerical Solutions for the Roots of the Characteristic Equation of System (11) under Alternative Values for the q-Coefficient in Equation (6)

((a)	(1	b)	((c)
1.9404	.9993	1.8436	.9999	1.8436	.9982
1.8396	.9980	1.8404	.9982	1.8404	.9958
1.8391	.9968	1.8389	.9958	1.8389	.9903
1.8386	0324	1.8378	.9903	1.8378	0328
1.0001			0264	1.0001	

It can be seen from the sets of roots thus obtained that the system becomes unstable when the q-coefficient of the United States is increased, whereas it remains stable when only the corresponding coefficients of the other countries are changed.

The robustness of the system's stability with respect to alternative values for the parameters in the money demand functions and in the price functions, together with its sensitivity to different values for the coefficient of the real exchange rate in the income disequilibrium equations of the United States (in the case of the basic version of the model) and for Germany (in

the case of the modified version of the model), suggest that the model's underlying theory should allow for structural changes in equilibrium real exchange rates. In other words, the long-run purchasing power parity condition that is still imposed in our model in the form of $\overline{q}=0$ ought to be relaxed. In view of the fact that the United States and Germany (but also the United Kingdom) are the two countries in our set whose real exchange rate has changed most markedly during the sample period, it is not surprising that the stability of the estimated structure depends so crucially upon those two countries' income sensitivity to their real exchange rate.

13.5 Extensions and Concluding Remarks

In order to perform a detailed analysis of monetary policy in the European Monetary System, our model clearly needs extensions and refinements. Extensions are required in order to include the EMS countries that were left exogenous or absent in our empirical section. More important, theoretical refinement of the model should allow endogenization of the equilibrium price and inflation rates that are here left in the z vector; this must be done by specifying a process for the conduct of monetary policy by the EMS countries and the United States of America. While the minimal assumption is a random walk process, 12 a more relevant approach for our purpose is to specify policy reaction functions for the monetary authorities. These should reflect, in addition to the standard objectives (control of the inflation rate, of unemployment, and of the balance of payments), the effect of the institutional constraints that have ruled exchange rate management of our set of countries during the sample period-for example, the "snake" arrangements-and that still determine monetary and exchange rate policy in the present stage of the EMS. 13

A set of theoretical and econometric problems that arise in this connection are due to the changing role of policy variables between being policy instruments and being policy objectives, and to the switches in institutional regimes that have been taking place through time and across countries because of the evolution of the European exchange rate arrangements and of the varying participation of European countries to them.

While work in these directions is in progress, we hope that the presentation of our model, its estimation, and the analysis of its stability properties may already be a useful contribution to the theory and practice of exchange rate modeling.

^{11.} See Hooper and Morton (1980) for an exchange rate model oriented in this direction.

^{12.} This is indeed the assumption implicitly made by Frankel (1979). See Driskill and Sheffrin (1981).

^{13.} For an attempt to specify and estimate reaction functions along these lines, see Basevi and Calzolari (1982).

Appendix

Most data, excluding short-term interest rates, are taken from the International Financial Statistics tapes distributed by the International Monetary Fund. Money stocks (defined as M2, rows 34 and 35, except for the United Kingdom where M1, row 34, is used for lack of monthly data on row 35) are end of period and not seasonally adjusted. Price indices, long-term interest rates, and indices of industrial production (seasonally adjusted) are monthly averages. Exchange rates are end of period.

Short-term interest rates are taken from Morgan Guaranty Trust of New York, World Financial Markets, table headed "Representative Money-Market Rates," and they are end of period.

The weights μ_{ij} in the definition of q_i are taken from an unpublished study by the staff of the Bank of Italy, which draws upon data periodically distributed in mimeographed form by the Directorate for Economic Affairs of the European Community Commission.

Comment Francesco Papadia

J. M. Keynes wrote that "practical men, who believe themselves to be quite exempt from any intellectual influences are usually the slaves of some defunct economist" (1936, p. 383). In a sense I earn my living, either at the EEC Commission or in the Research Department of the Banca d'Italia, attempting to minimize the lag with which practical policymakers are slaves of economists. I should add, incidentally, that slavery is not that bad if you can choose your own master, and the differences existing in the economic profession assure, in this respect, quite a range of choices to a practical policymaker.

I have introduced this autobiographical note just to illustrate the spirit with which I read the Basevi and Calzolari paper on which I have the pleasure of commenting. In my effort to bridge the gap between academic economists and policymakers, I look at models and their empirical estimates as elements to be inserted into the decision-making machinery. I have to see how, for example, Basevi and Calzolari's paper can be made relevant to such mundane activities as building the next phase of the EMS or managing the exchange rate of the lira. From this perspective, there are two questions I would like to put to the authors.

The paper is the newest offshoot of an old but ever-growing tree. The authors explicitly link their model to the one proposed by Frankel (1979). Frankel was extending, using Frenkel-Bilson material, Dornbusch's (1976) model. Dornbusch, in turn, developed the Fleming-Mundell model of the

sixties. The new branch provided by Basevi and Calzolari usefully extends the model by (1) making it multilateral instead of limiting it to the two-country case; (2) analyzing rational expectations on the basis of the structural version of the model rather than the reduced-form one; (3) introducing, at least in principle, institutional constraints such as those of the EMS. The two questions I would like to put refer, one to the tree, and one to the new branch.

My first question, which relates to the whole class of models to which Basevi and Calzolari's model belongs, has to do with the issue of price stickiness. Dornbusch (1976) underlines the key role played in this class of models by the stickiness of prices in the real goods markets as compared with the instantaneous clearing of asset markets. Frankel (1979), in turn, builds an alternative hypothesis to his real interest differential model utilizing the Chicago (Frenkel-Bilson) hypothesis of perfectly flexible prices.

Dornbusch states his uneasiness in using, as a building block of his model, price stickiness which has no satisfactory theoretical explanation, but he accepts it as an empirical fact. Let me state my uneasiness in having to choose between the Scylla of perfectly flexible good prices and the Charybdis of price stickiness considered as an act of God, a meta-economic fact of life. Of course it would be nice to have a model in which price stickiness was not a parameter but the outcome of an optimizing process and as such changed according to circumstances. In this model, the optimum degree of stickiness would be reached when the marginal cost of an additional unit of flexibility was equalized to the marginal cost of trading at nonequilibrium prices. That flexibility has a cost is obvious if we look at the resources absorbed by the functioning of auction markets and imagine, in addition, the amount of real resources which would be absorbed if, say, salaries were recontracted every 5 minutes. The cost of trading at nonequilibrium prices, in turn, is obviously in terms of misallocation of resources.

The sort of model closest to the one I have hinted at above, of which I am aware, is the so-called surprise supply function proposed by Lucas (1973). In this model the cost of acquiring information is introduced by assuming that no private operator looks at more than one market to estimate the actual rate of inflation. Since, of course, there is no physical constraint to this effect, this limitation must be derived from an economic choice: it does not pay to incur the expenses of looking at more than one market. The cost of trading at nonequilibrium prices, in turn, is implicit in the fact that operators try to minimize it making optimal use of their limited information.

As is well known, in this model the slope of the Phillips curve (where unemployment is replaced by the deviation between actual income and equilibrium income) depends inversely on the informativeness of the price system expressed by the ratio of the variance of relative to aggregate inflation.

As the latter decreases, so does the informativeness of the price signal as a resource allocator: operators attribute a larger and larger share of observed price movements to aggregate rather than to relative inflation, and the steepness of the Phillips curve increases.

This feature is interesting for a practical policymaker because it gives body to the contention that economic policy has to do with relations among optimizing agents and is not a game against nature. Thus, it shows, for instance, that the agent's reaction to an overactive monetary policy which increases, under normal circumstances, the variance of aggregate inflation, will be a reduced sensitivity to demand policy actions. As I understand it, the Lucas hypothesis on the slope of the Phillips curve is immediately relevant for the class of models to which the Basevi and Calzolari paper belongs. In fact, the price adjustment equation (6) of the Basevi-Calzolari paper, similar to that of Dornbusch (1976, appendix), is just a Phillips curve with the addition of a long-term rate of inflation. In a sense the Phillips curve is a "price stickiness" equation. However, while in a model such as Basevi and Calzolari's, the slope of the Phillips curve is constant, an act of God, and is not derived from economic considerations, in the Lucas model it is explained by an admittedly rough optimizing process.

At the end of this long digression comes my question. The policymaker wanting to use a model similar to Basevi and Calzolari's is confronted with a serious delemma. On one hand, if prices are perfectly flexible there is no overshooting of the exchange rates, the relationship between the nominal interest differential and expected changes in the exchange rate is negative, and monetary policy does not influence real output. On the other hand, if prices show some degree of stickiness it is likely that there will be overshooting, the relationship between interest differential and the exchange rate is positive, and the exchange rate is an important channel of transmission of monetary impulses to real activity. Choosing one or the other assumption makes quite a dramatic difference. Of course, as a complement of these undetermined theoretical results, the policymaker is also given some empirical evidence that, with moderate inflation, the price stickiness hypothesis comes out better. However, he is also warned that during a hyperinflation the flexible price model is more appropriate. This, of course, helps but does not really settle the issue. Is a 20% rate of inflation, like Italy's, moderate? What if inflation reaches 40%? Can one count, for policy purposes, on the constancy of the degree of price stickiness, that is, can one rely on its econometric estimates? Is it by mere chance that the degree of price flexibility, as measured in table 13.6 of the paper, is lower in Germany than in the other four countries and the German Phillips curve is correspondingly flatter than in the other countries?

My questions is whether one could not use a model akin to that of Lucas to answer these points, relieve the policymaker of his dilemma, and get

closer to the mythical figure of the "one-handed" economist who cannot present a conclusion with his left hand and its opposite with his right. In other words, what is wrong with the Lucas hypothesis, or some variant thereof, as a theory of price stickiness to be inserted in models of the exchange rate such as the Basevi and Calzolari one?

My second question has to do with the introduction of risk, exchange control, and transaction cost factors in the interest rate parity theory (IRPT) (eq. [1]) and in the expectation theory (ET) of the forward exchange rate (eq. [2]). As I understand it, Basevi and Calzolari assume that transaction costs and exchange controls impinge on the IRPT, while exchange risk impinges on the relationship between forward and expected exchange rates. I have some problems with this distinction. Of course, transaction costs impinge both on the IRPT and on the ET, and they are not trivial even for the latter. I have estimated (Papadia 1981) that the average transaction cost, measured as bid-ask interbank spread, on a forward operation is close to 0.4% for the lira, about 0.2% for the French franc and deutsche mark, and 0.1% for the pound. In addition, one must take into account the fact that the spread changes over time; for instance, on some turbulent days it exceeded 1% on lira-dollar contracts. As far as I know, the most straightforward way (Papadia 1981, pp. 224-25) to take transaction costs into account is to correct the average bid-ask quotes generally used as "forward rates" by adding the spread in the case of expected revaluation and by subtracting it in the case of expected devaluation. This procedure is based on the argument that in the case of expected devaluation one will sell the currency forward until the expected buy spot rate will be equal to today's forward selling rate and vice versa for a revaluation. We can then use the corrected data to make forward/spot rate comparisons.

Exchange controls also impinge on both operations, since foreign exchange transactions are included in both relationships, and one could attempt to measure their effect by the difference between national and international (Euro) rates of interest on similar assets. The issue of exchange risk is notoriously more complicated, and Basevi and Calzolari recognize this. If a satisfactory solution has been found to this problem, I am not aware of it. My question in this respect is whether one could not overlook, in a first approximation, risk factors which are not explicitly considered in the estimation, and try to take into account transaction costs and exchange controls' effects as indicated above. Basevi and Calzolari have substantially improved the estimation method between the version presented at the conference and the present version, and not much can be added here except that I have the suspicion that the residuals of the estimated regressions may not be homoscedastic. In particular, I suspect that the second half of the sample may be noisier than the first. A test to make sure that this is not the case would have been welcome.

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