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CONSUMER DURABLES IN AN INDEX OF CONSUMER PRICES

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The focus of this paper is the proper treatment of durable consumer goods within an index number that is concerned with measuring the changes over time of *a fixed quantity and quality of goods and services* for an appropriately defined population. We take as given, first, the "fixed-base" concept, and second, the definition of the appropriate population (e.g., city wage earners and clerical workers in Cleveland).

Since few goods are literally consumed at the moment of purchase, durability is an elusive concept but we shall limit attention to those commodities whose life is sufficiently long (relative to the consumption horizon of the population) that there is a relatively active market in used commodities of the kind in question. For such commodities there is a real question as to what is meant by a "fixed quantity and quality of goods and services": are these commodities consumption goods or are they assets which produce consumable services?

The oldest axiom of index number construction is that the purpose of use governs the form of the index and therefore it is in principle possible to justify a variety of different procedures. It is not difficult to think of uses for which each of the following three sorts of measures might be useful:

1. An index of the *prices of assets* purchased (or contracted for) by members of the index population.
2. An index of the *current outlays out of income* made by members of the index population.
3. An index of the *user (or opportunity) cost* of consuming the services produced by the assets in question.

For goods of very short durability, the concepts become virtually identical; for goods of substantial durability, but which are typically held for their whole useful lives and which are purchased regularly, the concepts differ, but the three tend to the same result. (This appears to be the case with clothing—while any individual piece has substantial durability, annual expenditures on clothing by the family are relatively stable. Whether it is true for furniture and appliances is not clear to me.) For commodities of long durability that are perforce purchased only intermittently because of the amount of expenditure (or investment) on the individual acquisition is a large fraction of annual income, the differences in the three approaches become substantial. Where in addition the assets are typically not held throughout their full useful lives, the differences become extreme. These conditions are strikingly present with respect to home ownership, and to a somewhat lesser extent with automobile purchase and use. We shall limit attention to these two classes of consumer durables.

Which approach is most nearly appropriate to the CPI? My own view is that it is the third: the user cost of consuming a fixed quantity and quality of services. With respect to housing, this would make the shelter cost of the homeowner congruent with that of the renter (with which it is ultimately combined in the overall index). With respect to automobiles (transportation), it would give proper perspective to the relative importance of this service to shelter, food, apparel, and other services. In this view, then, the index concerns valuation of the cost of using a fixed quantity and quality of services in all cases—although for many services this cost is adequately measured by the purchase prices of the service-producing commodity. Only where such prices prove a poor proxy for user cost is there need to approach the problem indirectly: that this is the case with both housing and automobiles will be shown in this paper.

Present BLS practice is to follow the first approach (asset prices) with respect to automobiles, and a mixture of the first and second approaches with respect to housing. A detailed description of present procedures may be found elsewhere;[1] some discussion of it will follow a development of the user cost approach.

I. The Cost of Using Asset Services

The overall problem may be viewed initially as three separable subproblems:

a. Determining the cost at different periods of time of using a particular asset service from an identifiable asset. This cost for the period dt in the neighborhood of time t we designate as $F_t dt$.

b. Determining the cost of using the asset service from a particular aggregate of assets. The characteristics of the fixed-base index permit visualization of an "average asset" whose cost of service for the period dt in the neighborhood of time t we designate as $\overline{F}_t dt$.

c. Finding appropriate weights that permit combination of the changes in the cost of this asset service with those of other services in an overall index of consumer services. Such weights are determined in the base period and correspond to the fraction of the index population consuming the service in question; we designate them as W_0.

Thus we may describe the desired measure as of the form

$$\frac{\overline{F}_t dt}{\overline{F}_0 dt} W_0 \tag{1}$$

in which year 0 is the base year both for comparison and weighting purposes.

We may note that while equation (1) appears to be very similar to:

$$\frac{P_t}{P_0} W_0^* \tag{2}$$

in which P_t is a price at time t, and W^*_0 is an appropriate weight at time 0, (1) and (2) will not move together unless $\overline{F}_t/\overline{F}_0$ is proportional to P_t/P_0—unless, that is, prices are an adequate index of

[1] See, for a start, *Monthly Labor Review*, November 1955, February 1956, and April 1956.

user costs. It is precisely in the case of durable goods that this may not be the case.

The fixed-base character of the desired index may be interpreted to mean the *use* of a given quantity of a prototype asset for a given period of time. This prototype asset has given physical characteristics, a given age, etc. (It may of course be a synthetic concept corresponding to an appropriate weighted average of actual assets differing in physical characteristics, age, etc.)

The principal components of user cost seem to be:

Depreciation (R)
Interest (I)
Incidental Purchase Costs (J)
Taxes (X)
Maintenance and Repair (MR)
Insurance (G)

Each of these requires careful definition; in principle it is the real cost in dollars of the period in question, whether such costs are reflected in cash outlays or not.

The general form of the required index is

$$\frac{\overline{F}_t dt}{\overline{F}_0 dt} W_0 = \frac{\overline{R}_t + \overline{I}_t + \overline{J}_t + \overline{X}_t + \overline{MR}_t + \overline{G}_t}{\overline{R}_0 + \overline{I}_0 + \overline{J}_0 + \overline{X}_0 + \overline{MR}_0 + \overline{G}_0} dt \cdot W_0 \cdot \qquad (3)$$

It is worth noting, for future reference, that if the relative size of the individual components changes over time, this does not decompose into an aggregate of separate indexes of the components. That is, if:

$$\overline{R}_t : \overline{I}_t : \overline{J}_t : \ . \ . \ . \neq \overline{R}_0 : \overline{I}_0 : \overline{J}_0 : \ . \ . \ . \ , \qquad (4)$$

$$\frac{\overline{F}_t dt}{\overline{F}_0 dt} W_0 \neq \frac{\overline{R}_t}{\overline{R}_0} w_0^R + \frac{\overline{I}_t}{\overline{I}_0} w_0^I + \frac{\overline{J}_t}{\overline{J}_0} w_0^J + \ . \ . \ . \qquad (5)$$

where the w_0 are fixed-component weights. As we will show, there is nothing in the fixed based concept that requires the condition (4) to be an equation.

We turn to a component by component analysis.

A. A NOTE ON NOTATION

While there is nothing formidable in the algebra that follows, a number of different concepts are involved that make the notation complex. In order to be as nearly clear as possible, let me note certain rules of interpretation. (A general glossary of symbols appears as Appendix A.)

We are in all cases concerned with evaluation of costs at a moment in time t; but the costs refer to an interval dt, where dt is of the dimensions of fractions of a year. All magnitudes that are not clearly instantaneous magnitudes are annual rates or amounts unless otherwise indicated. Thus, if i is an interest rate, i_t is the annual rate at t, and $i_t dt$ is the monthly rate if $dt = 1/12$, etc. In our notation dt is not always infinitesimal.

But the assets at time t may be of different ages, and we let the subscript j ($j = t, t-1, t-2, \ . \ . \ .$) indicate the date at which the asset was new. $t-j$ is thus the age of the asset in years. While nothing inherently prevents fractional age, we arbitrarily assume assets have birthdays on a common date. We thus substitute a finite set of age strata for a continuous age distribution.

In a similar way the assets may have been purchased at different dates, and we let the subscript k ($k=t$, $t-1$, $t-2$, . . . j) indicate the date of purchase by the present owner. $t-k$ is thus the years the asset has been owned, again assumed integral. If, but only if, the asset was purchased new, $j=k$.

If P designates price, ${}_tP_j$ indicates the price at t of the asset which was new at year j. If M indicates the size of a mortgage, ${}_tM_{k,j}$ indicates the size of a mortgage at t on an asset acquired in year k, when it was $k-j$ years old. (Its price at k was ${}_kP_j$), etc. In general we shall not identify individual assets, but where it is necessary to do, it will be by superscript. E.g., ${}_tP^i{}_j$ is the price of the i-th asset at time t (the asset was new in year j).

If f designates a frequency distribution, ${}_tf_j$ designates the frequency distribution over j at time t. f_{t-j} (or simply $f_{t,j}$) indicates that the frequency distribution is constant over time, that is, that the fraction of assets having a common pair of values for t and j is a function of $(t-j)$ alone. The difference between ${}_tf_j$ *and* f_{t-j} is that the former is a specific relation at time t, the latter is only a function of $t-j$. ${}_tf_{j,k}$ *and* $f_{t,j,k}$ are bivariate frequency distributions at time t. The latter is constant over time, the former is a changing function of t.

B. DEPRECIATION

1. The Problem: An Intuitive Introduction. To see the problem, consider the following simplified example. Suppose we purchase a new asset at $t=0$ for \$100. Two years later we can sell this used asset for \$128, but an asset identical to ours, brand new at $t=2$, would cost \$200. Suppose the following data:

t Year	Price of our asset ${}_tP_0$	Price of new asset ${}_tP_t$
0	100	100
1	80	100
2	128	200

It is clear that in some way it has cost us \$20 to hold and use the asset during the first year and an additional \$52 during the second year. These are the costs compared to the behavior of a nondepreciating asset. An adequate allowance for depreciation should give us a fund of \$20 at the end of the first year and an aggregate fund of \$72 at the end of the second year. Our problem is to determine how large a contribution to make to this fund during every short time period dt in order that the aggregate amount of the fund is adequate to acquire a new asset in exchange for our used one. Suppose that at $t=1$ we (somehow) had a depreciation fund on hand of the required \$20. Would we have needed to find an additional \$52 during the second year? Not if the fund on hand at the beginning of the period had been properly invested. Indeed, if it had been invested in assets of this type, it would have shared in the asset inflation and been worth \$40 by $t=2$. If we wait until the end of year 2, it would be necessary to contribute \$32 to the fund; but if we set aside each month (week, day, minute) a contribution which was invested (and shared in the inflation), it is clear that the sum of required actual contributions would be less than \$32 in the aggregate.

Of course, using a depreciating asset during a period of rising asset prices is expensive. During this period the owner-user of the asset is (1) consuming in each moment a service which decreases the value of the asset and should be "costed" in current dollars of the moment of consumption, *and* (2) disinvesting implicitly and perhaps unconsciously. Roughly, the distinction is that the first of these causes the loss of value during the period in question whereas the second leads to the loss of capital appreciation in the periods lying ahead of the disinvestment. The second is of course costly, but it is not a proper cost of *using* the asset service for it might be conceptually avoided in any of several ways (that are equivalent): one is to keep utilizing an asset of constant age by trading in a one day (minute) old asset for a new but otherwise identical one every day (minute); another is to invest in a sinking fund that appreciates at the rate of inflation. Each of these succeeds in keeping the real asset position constant, while measuring the unavoidable cost of using the asset to produce its service.

It seems quite clear to me that maintaining a fixed real asset position is appropriate within the context of the fixed-base index number of the costs of goods and services. But if this is not clear—if some rate of "asset acquisition" belongs in such an index—it is evident that it should be included as a separate category and not be confused with the cost of using asset services.

2. Depreciation of an Individual Asset. Let ${}_tD_j$ = the required size of a depreciation fund at time t for an asset that was new in year j.

$$ {}_tD_j = {}_tP_t - {}_tP_j. \tag{6} $$

At time t we want a fund ${}_tD_j$; at time $t-dt$ we have a fund ${}_{t-dt}D_j$. Assume this fund, and all subsequent additions to it, is actually or conceptually viewed as invested in assets whose prices move with the price of the asset in question. It thus appreciates (if asset prices are rising) at an annual rate r_t defined immediately below. Consider the year as divided into n periods of length dt. Further, let each period dt be divided into m subperiods.

We will define r_t by the condition that

$$ {}_tP_t = {}_{t-dt}P_{t-dt}\left(1+\frac{r_t dt}{m}\right)^m $$

and letting $m \to \infty$

$$ e^{r_t dt} = {}_tP_t / {}_{t-dt}P_{t-dt}. \tag{7} $$

Our problem is to determine the required amount of contributions $({}_tR_j dt)$ to the fund during the period dt so that the fund has the required size at time t. We visualize making m equal contributions, one in each of the subperiods.

$$ {}_tD_j = {}_{t-dt}D_j \cdot e^{r_t dt} + \frac{{}_tR_j dt}{m} (S_{\overline{m}|}), \tag{8} $$

where $S_{\overline{m}|}$ is the amount of an annuity of unit value after m subperiods at an "interest rate" of

$$ \frac{r_t dt}{m} $$

per subperiod.

$$ S_{\overline{m}|} = \frac{m}{r_t dt}\left[\left(1+\frac{r_t dt}{m}\right)^m - 1\right]. \tag{9} $$

Define:

$$ {}_t\alpha_j = {}_tP_j / {}_tP_t, \; all \; t, j, \tag{10} $$

and note that the α's provide the structure of used to new asset prices at the same moment in time. We will assume that ${}_t\alpha_j$ is a constant for a given t and j—in other words, it is the "average" or "normal" used to new asset price ratio at time t, and does not vary among individual assets of the same age at that time. (See the discussion of maintenance and repair, below, for the justification of this assumption.)

Rewriting (6), using (7) and (10), we have:

$$ \begin{aligned} {}_tD_j &= {}_tP_t \; (1 - {}_t\alpha_j) \\ {}_{t-dt}D_j &= {}_{t-dt}P_{t-dt} \; (1 - {}_{t-dt}\alpha_j) = \frac{{}_tP_t}{e^{r_t dt}} \; (1 - {}_{t-dt}\alpha_j). \end{aligned} \tag{11} $$

Substituting (9) and (11) in (8).

$$ {}_tP_t \; (1 - {}_t\alpha_j) = {}_tP_t \; (1 - {}_{t-dt}\alpha_j) + \frac{{}_tR_j dt}{r_t dt}\left[(1+\frac{r_t dt}{m})^m - 1\right]. $$

Letting $m \rightarrow \infty$, and solving for ${}_tR_j dt$,

$$ {}_tR_j dt = {}_tP_t \cdot \left(\frac{r_t dt}{e^{r_t dt} - 1}\right)\left({}_{t-dt}\alpha_j - {}_t\alpha_j\right) \tag{12} $$

which is the charge for depreciation sought.

This is an important result and its interpretation may be clarified by the following remarks. Evidently we have factored the depreciation charge into three parts, the second and third of which are dimensionless coefficients whose significance is explained presently. The first term is the current price of the new asset of the prototype—the replacement cost, if you will. As we have suggested earlier, if prices are rising using a depreciating asset is more expensive than otherwise, and the ${}_tP_t$ term, besides giving magnitude and dimension to the depreciation charge, reflects this fact.

But not all of the change in price level requires a corresponding contribution, for, as we have seen, inflation (to take the rising price case) increases the value of past contributions to the depreciation fund. The second term reflects this influence. If prices are rising, it has a value between 0 and 1; if falling, greater than 1. It is a function of ${}_tP_t$ and ${}_{t-dt}P_{t-dt}$ only.[2] It appears in the expression because we assume that past and present contributions to the depreciation fund change in value with the asset price level.

The justification of this assumption is that it has the effect of separating the cost (if asset prices are rising) or gain (if asset prices are falling) of disinvestment from the cost of using an asset while keeping the total real asset position constant. Thus if the owner of the asset has access to investments that change in value at the rate $r_t dt$, it is the appropriate rate to use whether he chooses actually to invest in them or not.[3] What if he cannot find such investments? Life will cost him more (if prices are rising), but whether this is a proper cost of consuming the asset service is a priori unclear. On the one hand, if there exist nondepreciating assets that appreciate at this rate but, owing to some imperfection in asset markets, the individual owner is denied access to them, he will suffer a (relative) real asset loss over time no matter what assets he holds, depreciating or not. Whether this should be included in his cost of living as the "cost of holding assets" or whether it should be regarded as a change in his income position, it *is* clear that it is not the cost of using the depreciating asset. In this case the use of rdt is still appropriate. On the other hand, if the imperfection in the asset market is such that there exists *no* nondepreciating asset whose price rises at the rate rdt (or more), the excess cost is truly an unavoidable cost of using the asset service.[4] This possibility is subsequently neglected. It does not seem very plausible to me.

The third part of equation (12), ${}_{t-dt}\alpha_j - {}_t\alpha_j$, which will hereafter be designated as ${}_tA_j$, is a determinable function of the age of the asset in question. Since each of the α's reflects the used to new asset price at a moment in time, it may be regarded as the pure effect of age on value. ${}_tA_j$ shows the *joint* effect of the asset growing one period older *and* of any change in the normal used to new price ratio that may have occurred during the period. To see that the latter is a proper cost of using the asset, consider the owner of a car who finds to his dismay that used car prices have collapsed, thus reducing his expected trade-in value. This has been an unanticipated but nonetheless real cost of using the car during the period.

If for a particular type of asset there is a constant age-price structure over time, so that we may replace the ${}_t\alpha_j$ by a_{t-j}, it is often possible

[2] See the following:

$$\frac{rdt}{e^{rdt}-1} = \log_e \frac{{}_tP_t}{{}_{t-dt}P_{t-dt}} \Big/ \left(\frac{{}_tP_t}{{}_{t-dt}P_{t-dt}} - 1\right)$$

[3] Note that we do not credit earnings to the sinking fund. There are of course costs of holding nonearning assets but these are properly treated as implicit (or actual) interest charges and are quite independent of depreciation as such. Depreciation of course can affect interest costs as we shall see below.

[4] If this is the case, if nondepreciating assets rise at the rate $\pi_t < r_t$, we must replace (12) by:

$${}_tR_j dt = {}_tP_t\left(\frac{\pi_t dt}{e^{\pi_t dt}-1}\right)\left[(1-{}_t\alpha_j) - \frac{e^{r_t dt}}{e^{\pi_t dt}}(1-{}_{t-dt}\alpha_j)\right]. \tag{12a}$$

to define A_{t-j} in simple algebraic form. Consider the following cases (assume $dt=1$):

Straight line depreciation over n years:

$$\alpha_{t-j}=\frac{n-(t-j)}{n}$$

$$A_{t-j}=\frac{1}{n}$$

Declining balance, where a constant proportion, a, of the remaining value is deducted each year:

$$\alpha_{t-j}=(1-a)^{t-j}$$

$$A_{t-j}=a(1-a)^{t-j-1}$$

Sum of years' digits, over n years, where

$$Y=\sum_{i=0}^{n} i$$

$$A_{t-j}=\frac{n-(t-j-1)}{Y}$$

If a constant (for a given $t-j$) pattern of age-price ratios is not appropriate, it is necessary to retain the ${}_tA_j$ which is a function of t as well as $t-j$.

An explicit expression of ${}_tA_j$ is:

$${}_tA_j=\frac{{}_{t-dt}P_j\cdot e^{r\,dt}-{}_tP_j}{{}_tP_t}$$

the numerator of which shows the value of the specific asset at the beginning of the period inflated by the rise in asset prices over the period, minus the terminal value; in other words, the loss in value (in constant dollars of t) due to aging.[5]

3. Depreciation of the Asset of Average Age. At time t, suppose there exist in our population ${}_tN$ assets which were identical when new but whose age varies. Suppose we regard them as divided into j strata by year of origin, and assume that all assets of the same age (${}_tN_j$) are fully homogeneous. By definition,

$${}_tN=\sum^{j}{}_tN_i$$

and we define:

$${}_tf_i=\frac{{}_tN_j}{{}_tN},$$

which, as j varies, defines the frequency distribution of assets by age, at time t.

In the previous section we dealt with a particular asset—call it the i-th—and found in equation (12) an expression for ${}_tR^i{}_j dt$ (where the superscript i identifies the particular asset). We need now merely

[5] The reader who suspects that this would be a more convenient form of ${}_tA_j$ than that given in (12) would be correct if (1) we were concerned with a single asset age only and (2) if ${}_tA_j \neq A_{t-j}$. Since the actual problem involves an aggregation of assets of different ages, and since it may frequently be possible to assume ${}_tA_j=A_{t-j}$, the coefficient form proves useful.

to sum over i and j and divide by ${}_tN$ to determine the average depreciation, which we will designate $\overline{R}_t dt$.

$$\overline{R}_t dt = \frac{1}{{}_tN} \sum^{j} \sum^{i} {}_tR^i{}_j dt = \frac{1}{{}_tN} \sum^{j} {}_tN_j {}_tR_j dt = \sum^{j} {}_tR_j \cdot {}_tf_j \cdot dt$$

$$\overline{R}_t dt = {}_tP_t \left(\frac{r_t dt}{e^{r_t dt} - 1} \right) \sum^{j} {}_tA_j {}_tf_j. \tag{13}$$

The ${}_tf_j$ are weights reflecting the age distribution of assets. The fixed-base concept requires "exclusion of changes in quality" and requires a constant age distribution. Hence the ${}_tf_j$ can be replaced by f_{t-j}—a constant set of weights determined in the base year. The summation is thus a weighted average of the ${}_tA_j$ with a fixed set of weights determined once and for all. If the ${}_tA_j$ are constant $(=A_{t-j})$, the whole summation is a constant, determinable in the base period. But even if this is not the case, the data required seem within reach since estimates of the structure of used asset prices for particular types of assets are commonly made in fields where an active used asset market exists.

4. DATA REQUIREMENTS

To compute the depreciation component for a period dt requires: (a) ${}_tP_t$ and ${}_{t-dt}P_{t-dt}$, the prices of a new asset of the prototype at the beginning and end of the period, (b) f_{t-j}, the age distribution of assets in use in the base period, (c) ${}_tA_j$, the structure of used to new asset prices for asset ages included in the f_{t-j}, at both the beginning and end of the period. (If ${}_tA_j = A_{t-j}$ this is a once and for all determination, and it may be possible to find a simple algebraic equivalent.)

Put differently, this requires, for a continuing index, at most ${}_tP_j$ for all relevant t and j, plus the base year age distribution. Precisely the same data are required merely to measure the weighted average asset value.

5. Comparison with Present Practice. The BLS index includes both housing and automobiles but in neither case includes a direct depreciation component, since it rejects the user cost approach.[6] It is possible, however, that the purchase price component (which is the way in which automobiles are included, and is one of the components of the BLS owned-housing index) may serve as a satisfactory proxy for depreciation. That this need not be so is shown in Part II of this paper; the reasons can be quickly seen analytically.

Using our notation, the weighted average purchase price in year t is

$$\overline{P}'_t = \sum^{j} {}_tP_j f'_{t-j} = {}_tP_t \sum^{j} {}_t\alpha_j f'_{t-j} \tag{14}$$

where the f'_{t-j} are the frequency distribution of ages of houses purchased (and may of course differ from f_{t-j}—the age distribution of assets in use).

[6] The apparent use by the BLS of a depreciation factor in the automobile computation (described in *M.L.R.*, November 1955) is only for an intrayear adjustment of prices, and is not material to the comparison we make. It comes about because annual birthdays are assumed and thus one cannot define an exactly three-year-old car in, say, both January and July of the same year. We may note in passing that this intrayear adjustment is made only for used cars, not for new ones, and thus appears to neglect the well-known obsolescence of new cars toward the end of the model year.

Comparison of (13) and (14) makes clear three main sources of difference. First, the f'_{t-j} may differ from f_{t-j}. Second, the ${}_t\alpha_j$ need not be proportional to the ${}_tA_j$.[7] Thirdly, the expression in parentheses in (13) need not equal unity—indeed it will do so only in the (uninteresting) case where prices do not change.[8] It would thus be fortuitous indeed if (14) were an adequate proxy for (13).

Even if $\bar{P}'_t/\bar{P}'_0$ was a good proxy for $\bar{R}_t/\bar{R}_0$, it is worth noting that all would not be well as comparison of (3) and (5) above makes clear, and as subsequent examples will illuminate.

C. INTEREST

1. The Nature of the Problem. We wish to know the interest cost during the period dt of using the asset in question. This cost may be viewed as the sum of (1) the interest payments on the mortgage (or other loan), if any, which will be determined by the size of the mortgage during the period and by the terms of the loan—including the contract rate of interest and the length and pattern of payoff (which in most cases are a function of the credit conditions at the time the loan was initially contracted), and (2) the imputed interest on the owner's equity in the asset, which depends upon the equity during the period dt and appropriate *current* interest rate at which his funds might otherwise be invested.[9]

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[7] Proportional (rather than equal) would be sufficient because it would be easy to inject a constant for level. But they will in general be proportional if, but only if, two conditions are met:

(a) $${}_tA_j = A_{t-j} \qquad \text{(all } t,\ j\text{)} \qquad \textit{and}$$

(b) $$\frac{{}_t\alpha_j}{{}_t\alpha_{j+dt}} \text{ is constant, all } j.$$

Proof: $${}_tA_j = {}_{t-dt}\alpha_j - {}_t\alpha_j$$

for ${}_tA_j = \lambda\,{}_t\alpha_j$, all t, j, where λ is any constant, requires

$$\frac{{}_{t-dt}\alpha_j}{{}_t\alpha_j} = \lambda + 1$$

For this to be true, it requires for all t, j that:

$$\frac{{}_t\alpha_j}{{}_t\alpha_{j+dt}} = \frac{{}_{t+dt}\alpha_j}{{}_{t+dt}\alpha_{j+dt}}$$

which implies both of the stated conditions.

These conditions are met by the constant declining balance special case discussed above.

[8] The limit of $\frac{rdt}{e^{rdt}-1}$ is 1 as $e^{rdt} \to 1$.

[9] A refinement not incorporated in the subsequent analysis is to recognize that some portion of interest (and also taxes) actually paid will be recovered as a tax credit by those who itemize deductions. Thus we might in subsequent equations introduce a number, $0 < \theta \leq 1$, which would represent the fraction not recovered tax-free. E.g., in equation (18), replace the expression in parentheses by $(\theta c^*{}_k - i_t)$.

While it does not seem safe—especially with respect to home ownership—to assume $\theta = 1.0$, determination of its magnitude is so difficult as to make this a nonoperational refinement. Even if we knew the percentage of individuals in the relevant population itemizing deductions, and the marginal tax rate for this group—and these I suspect could be obtained—we would have only a lower bound to θ, since not all of the interest paid would be deductions in excess of the optional 10 percent of adjusted gross income.

The current opportunity cost rate at which the owner can invest we will designate i_t and call the lending rate. Its estimation depends upon what in fact are the lending opportunities (and proclivities) of the particular group who form the index population. Perhaps for the CPI population it may be taken as the current rate on savings deposits, by banks, credit unions, or savings and loan associations.

That this lending rate may differ from the *contract* rate on mortgages, which we will designate c^*_k, is due first to the well-known fact that lending and borrowing rates may differ, and second to the fact that i_t refers to the current rate whereas c^*_k depends upon the (past) conditions at the time the mortgage was negotiated.

Should we limit attention to interest actually paid? While this might be appropriate under a cash outlay approach to the index number problem, it is not appropriate in our user cost approach unless $i_t=0$. It is perfectly apparent that as long as federal deposit insurance is available and banks pay interest on savings deposits, no one is forced to hold idle cash, and thus i_t is not properly regarded as equal to zero.

Whether we should also charge imputed interest on the reserve funds for depreciation is not clear. The general rule is that such imputation is appropriate when, but only when, earning opportunities must be foregone. Since these reserve funds are conceptually invested in nondepreciating assets whose price behavior is similar to that of the asset in use, they are conceptually tied up. The question is whether such investments have earnings (interest or dividends) and if so how such earnings compare to the lending rate. If these investments earn at the lending rate (after allowances for differences in risk), no charge is required. This (easiest) case is assumed in the formal development that follows. If the earnings rate is lower than the lending rate, only the difference is an appropriate charge. If the adjusted earnings exceed the lending rate, an interest credit is earned.[10]

2. Interest on an Individual Asset. The problem of interest is somewhat different from depreciation because it is necessary to pay attention to the age of the mortgage as well as the age of the asset. (We shall assume that mortgages date from the date of acquisition of the asset, and neglect refinancing, etc.) Owner's equity at any time is the difference between the current price of the asset and the remaining size of the mortgage.

For individual asset (identifying superscript omitted), the instantaneous annual rate of interest cost at t is

$$ {}_tI_{k,j}=({}_tP_{k,j}-{}_tM_{k,j})i_t+{}_tM_{k,j}c^*_k={}_tP_{k,j}\cdot i_t+{}_tM_{k,j}(c^*_k-i_t) \tag{15}$$

where k is a running subscript indicating the year the mortgage is

---

[10] The effect of our assumption is to simplify equations (15) to (18). For example, to (15) should be added; ${}_tD_t\ (i_t-i^*_t)$ where i^*_t is the adjusted rate of earnings on the depreciation reserve. This refinement, like that in the previous footnote, seems doomed by a sensible attention to operationality.

acquired, M is the amount of the mortgage, and the other terms are as previously defined.[11]

${}_tP_{k,j} = {}_tP_j$ since year of acquisition by present owner does not (we assume) affect the price of an asset of given age at time t.

Equation (15) is the instantaneous interest charge, not that for the period $t-dt$ to t. It would appear that to merely multiply this by dt would introduce a systematic downward bias into our formulation, since the size of a mortgage at the end of the period is less than its effective average size. But this is such a convenient approximation that it is used and the downward bias does not occur after aggregation. This is because while the individual mortgage (the tree) grows older, the age distribution of mortgages (the forest) does not, and thus the average age of mortgage remains constant.

Thus, before aggregating we use

$$I_t dt = \left[{}_tP_j i_t + {}_tM_{k,j}(c^*_k - i_t) \right] dt. \tag{16}$$

3. Interest on the Average Asset. We want (simply) to find:

$$\bar{I}_t dt = \frac{1}{N} \sum^{k} \sum^{j} \sum^{i} I_t dt. \tag{17}$$

To do so we need the bivariate frequency distribution of our N assets by age of asset and age of mortgage.

Let ${}_tf_{k,j}$ represent such a bivariate distribution where

f_{t-k} is the distribution of mortgages by age of mortgage

f_{t-j} is the distribution of assets by age of asset

f_{k-j} is the distribution of ages of assets for a given k

---

[11] The amount of the remaining mortgage is a simple actuarial function of the size of the original mortgage, given c^*_j and the length and type of repayment formula. For a characteristic mortgage calling for n equal payments per year for T years of principal and interest, we may define a remaining mortgage multiplier

$${}_td_k = \frac{{}_tM_{k,j}}{{}_kM_{k,j}} = \frac{a_{\overline{n(T-tk)}|}}{nT} \text{ at } c^*_k$$

(where $\overline{a}|$ stands for the amount of a unit annuity).

Where nT is large, the continuous approximation to this is:

$${}_td_k = \frac{1-e^{-c^*_k[T-(t-k)]}}{1-e^{-c^*_kT}}.$$

If, in addition, the original mortgage is a fixed fraction c_k of purchase price, so that

$$c_k = \frac{{}_kM_{k,j}}{{}_kP_j},$$

$${}_tM_{k,j} = {}_td_k . c_k \; {}_kP_j.$$

Note that we here assume that both ${}_td_k$ and c_k are independent of the age of the house (except as age affects price). This may be wrong—terms of credit may vary with age of houses, but this is a further complication not included at present.

While for computation this is a convenient formula, particularly if terms of credit except interest rate remain stable, we will deal with ${}_tM_k$ only in the algebraic formulation, in order to avoid unnecessary proliferation of terms.

all assumed constant over time.

$$\sum^{k} f_{t-k} = \sum^{j} f_{t-j} = \sum^{k} \sum^{j} f_{t,k,j} = 1$$

We further assume that the N assets are divided into j, k strata and all assets having identical j and k subscripts are homogeneous in all respects.

Performing the operations indicated in (17) we find:

$$\overline{I}_t dt = \left[{}_t\overline{P}_j i_t + \sum^{k} (c^*_k - i_t)\, {}_t\overline{M}_k f_{t-k} \right] dt \qquad (18)$$

$$\text{where } {}_t\overline{P}_j = \sum^{j} {}_tP_j f_{t-j}, \text{ and}$$

$${}_t\overline{M}_k = \sum^{j} {}_kM_j f_{k-j}.$$

This says that the interest charge is the imputed interest on the market value of the asset, plus the excess of contract interest paid over imputed interest on the mortgaged portion. If by any chance ${}_t\overline{M}_k$ is a constant, the summation in (18) reflects a weighted average of past interest rates. If not, it is a somewhat more complicated weighted average.

4. Data Requirements. The data requirements are, in a word, substantial. We require:

(a) The ${}_tP_j$ for all relevant j,

(b) The bivariate asset-mortgage age distribution $f_{t,j,k}$ for the base year,

(c) The contract interest rate c^*_k for all relevant k,

(d) The average size of outstanding mortgage for mortgages of each age. This may be most easily estimated directly or it may be derived by using "average" terms of credit in each relevant past year (including downpayment size and length of payoff period in addition to contract interest rate) as described above.

(e) The *current* lending rate i_t.

Of these, it is (b) and (d) that will be most difficult, but there are any number of simplifying assumptions that would reduce the data requirements, and I leave it to others to investigate whether they are justifiable as approximations.

The greatest simplification of all would be if the differences between c^*_k and i_t are small. If $i_t = c^*_k$ the entire summation of (18) vanishes. I doubt if this is justifiable, but the smaller is $(c^*_k - i_t)/i_t$, the less influence the summation term has in the total $\overline{I}_t$.

A less drastic simplification would be to regard the average size of remaining mortgages as a simple function of, say, ${}_t\overline{P}_j$ (as determined perhaps by a survey in the base year). This assumption would permit pulling the average size of mortgage out of the summation of (18) and multiplying it by a moving average of past contract interest rates, the weights reflecting the rate of acquisition of mortgages (taken perhaps as equal to purchases of assets) in the past years.

The potential error in this assumption is that it neglects changes in the terms of credit other than interest rate, specifically the size of downpayments and the length of mortgage payoff periods. It would not be impossible to "adjust" an average mortgage figure for such changes, or to estimate it in a more sophisticated manner.[12]

In short, if contract rates and other terms of credit vary over time, and if contract rates differ substantially from the imputed lending rate, it is necessary to have some estimate of both size and age distribution of mortgages. Given complete data, an exact determination is possible, using (18). But "sufficiently accurate" estimations may be possible using much less data.

5. Comparison with Present Practice. Interest plays an important role in the BLS index with respect to owned housing, and we shall make the comparison in that context. The concept used is: *interest payments contracted for in the current year for mortgages acquired in that year.* For an individual house, given its price and the credit terms in the current year, it is a straightforward actuarial computation to determine (1) the size of the mortgage and (2) the total payments of principal and interest to be made over T years under the terms of the contract. The difference between these two is the amount of interest contracted on that house. The BLS *concept* is a weighted average of these amounts, weighted by distribution of *purchases* in the base year.[13]

It may be noted initially that this concept fits, if at all, only within the first of the conceptions of an index number, discussed in the first pages of this paper. The "current outlay" approach (not advocated here) would use payments of both principal and interest and would be based on all assets owned, not merely current purchases. The user approach, described above, would include current ineterest payments on mortgages of all ages. Apparently the BLS justification is that this is the cost of purchasing a debt instrument without which the house could not have been acquired. Whether this is an appropriate real measure of that cost is questionable in view of the fact that the average actual life of mortgages is substantially shorter than their contract lives, given the mobility of the American population. Leaving this objection aside, it is not clear why future payments are not subject to discounting to find the present value of the future commitment.

For a mortgage of M dollars, contracted at time t, the

$$\text{Interest Contracted} = M \cdot a^{-1}_{\overline{nT}|} \cdot nT - M$$

(where $a^{-1}_{\overline{nT}|}$ is the present value of an annuity of \$1 for nT periods)

$$= M\left[\frac{c^*_t}{n} \frac{1}{1-(1+c^*_t/n)^{-nT}} - 1\right]$$

[12] One might, for example, try to find an estimation relation of the form:

$$\overline{M}_t = a_1 + a_2\overline{P}_t + a_3\overline{T}_t + a_4\overline{c}_t + a_5\overline{c}^*_t$$

where $\overline{T}_t$, $\overline{c}_t$, $\overline{c}^*_t$ are appropriate weighted moving averages of past terms of credit.

[13] In *practice*, the BLS departs from this concept by taking the terms of credit other than the contract interest rate (c, T) as fixed in the base year. This may be a justifiable simplifying assumption, but it has no inherent logic whatsoever. Our comparison will be with the concept rather than the practice, since we may wish to modify our concept in a similar way to reduce the data requirements, discussed above.

and where n is large, this may be approximated by

$$=M\left[\frac{c^*_t}{1-e^{-c^*_t T}}-1\right]. \tag{19}$$

If we consider mortgages on purchases of different ages, the average amount of interest contracted is

$$\sum^{j}{}_tM_j\left[\frac{c^*_t T}{1-e^{-c^*_t T}}-1\right]f'_{t-j} \tag{20}$$

where the f'_{t-j} are the appropriate purchase weights.

Even brief comparison will show this is totally different from (18). The magnitude of the expression in brackets may be of interest. If $c^*_t=.05$ and $T=20$, it has a value of .582. In other words, a \$10,000 mortgage acquired under those terms would have an amount of interest contracted of \$5,820. Of course, since the purchase weights are small, this magnitude is subject to a considerable reduction, but, as will become clear in a subsequent example, it is quite fortuitous if the overall interest charge in any given year is congruent with the annual amounts of interest paid or imputed. This is significant because it may distort the influence given to interest in the broader context.

The reason that only the current contract interest rate appears in (20) is that this concept of interest is prospective rather than retrospective, and prospective interest payments on current contracts are based on current rates.

D. INCIDENTAL PURCHASE AND FINANCE EXPENSES

These costs, often described in the housing field as closing costs, should (it may be argued) be amortized over the length of asset ownership, but their significance is sufficiently small that it seems satisfactory to treat them as current expenses in the year incurred. This avoids the need to predict the subsequent duration of ownership of assets currently purchased and also avoids a substantial computation.

We note that, in this view, the amount of such charges is aggregated only over the fraction of assets actually acquired in any year.

The appropriate cost figure may be written directly for the average asset:

$$\bar{J}_t dt=\sum^{j}{}_tJ_jf'_{t-j}dt \tag{21}$$

where the f'_{t-j} are the *purchase* weights such that

$$\sum^{j} f'_{t-j}=\frac{{}_tN_{k=t}}{{}_tN}.$$

The fiction of a constant age distribution of assets implies that

$$\frac{{}_tN_{k=t}}{{}_tN}$$

is a constant.

The data required are at most the ${}_tJ_j$ and the f'_{t-j}. If we can suppose that the ${}_tJ_j$ are at any time a constant fraction a_t of purchase prices, and independent of age except as age affects purchase price, the expression simplifies to

$$\bar{J}_t dt = \bar{P}'_t a_t \frac{{}_t N_{k-t}}{{}_t N} dt \tag{22}$$

in which only $\bar{P}'_t$ and a_t vary over time. (There are of course reasons for supposing that these incidental purchase costs may vary among assets of different ages, but perhaps the differences are sufficiently stable over time that the contrary assumption is workable.)

The procedure implied by (22) corresponds to the BLS practice with respect to housing with the additional assumption that a_t is a constant over time, as determined in the base year. We may note that this is the *only* place where use of purchase weights is deemed appropriate in the *user cost* approach, and here as a simplification that is acceptable only because these incidental purchase costs are in general quite small.

E. TAXES AND ASSESSMENTS

Since we are dealing with an aggregate of assets of constant age distribution, there is no serious problem here. The amount of taxes (X) on a given asset is the assessed valuation times the tax rate. For a collection of assets, we have the current tax rate times the weighted average of assessed valuations

$$\bar{X}_t dt = \left[x_t \sum_j {}_t V_j f_{t-j} \right] dt \tag{23}$$

where x_t is the annual tax rate, and ${}_t V_j$ the assessed valuation.

Equation (23) requires data on assessed valuations of houses of all relevant ages. One simplifying assumption would be to assume that assessed valuations are proportional to prices. A second is to assume that the ratio of assessed valuations of assets of different ages, ${}_t V_j / {}_t V_{j+1}$, is constant over time. This is equivalent to the first assumption if, but only if, ${}_t \alpha_j = \alpha_{t-j}$, for all j.

Supposing the second assumption is adopted, let V_{t-j} be the assessed valuation of an asset of age $(t-j)$ in the base period, and let $\bar{V}_t/\bar{V}_0$ be the average increase in assessed valuation in year t over the base year ($t=0$). Then (23) becomes

$$X dt = \left[x_t \frac{\bar{V}_t}{\bar{V}_0} \sum^{j} V_{t-j} f_{t-j} \right] dt \tag{24}$$

where the summation is a constant.

Equation (24) corresponds to the concept used by the BLS in its owned housing computation. It may be noted that here BLS uses *ownership* weights, not purchase weights.

F. MAINTENANCE AND REPAIR COSTS (MR)

It seems sufficiently accurate to assume MR expenditures are paid for currently and that the real amount of MR is a function only of the age of the asset in question. If so, the only problem is the definition of what is appropriately considered MR. For, given this, we can (given the constant age distribution of assets) estimate the amount of MR in the base year and multiply it by a price index of an appropriate sample of maintenance prices.

But while the definitional issue is the only issue, it is important and it is closely related to the concept of depreciation discussed previously. For some of what may be loosely called MR expenditures may be in lieu of depreciation, and some may be improvements. For some durables, say automobiles, the distinctions may be clear cut; for some, such as houses, they may be blurred.

In principle the appropriate MR expenses on a particular asset are those just required to let the ratio of the price of the used asset to a new asset be at the level indicated by the depreciation ratios—the a's in our earlier notation. That is, depreciation ratios imply some normal or standard condition of used assets, which in turn imply some average amount of MR. Actual expenditures on maintenance and repair items may, in individual cases, depart from this normal amount in either direction, and this will be reflected in a variance in actual prices of used assets of the same age. It is clear that if actual and implicit MR figures differ, it is the latter that is appropriate in computing the base year MR expenditures.

While it is clear that for individual assets actual MR may differ from normal, there may be no problem at the level of aggregation actually used. The BLS reports its concept for housing as "estimated average amount paid . . . in the base year." [14] If depreciation ratios are based on the average asset (and if the "averages" are comparable), average actual expenditure is the appropriate base. Whether this is the case must be determined.

A practical problem is the separation of improvement expenditures from MR. The concept of a "fixed level of living" that underlies the CPI clearly implies the exclusion of those expenditures (e.g., kitchen modernization) that amount to increases in quality, however regular they may be. The shelter component of the CPI now includes such improvements, improperly.[15]

Letting MR_{t-j} be the amount of normal MR on a house of age $(t\text{-}j)$ in the base year, B_t be the price index of MR items in year t,

$$\overline{MR}_t dt = \left[B_t \sum^{j} MR_{t-j} f_{t-j} \right] dt. \qquad (25)$$

The summation is a constant. The value of this constant and B_t are currently determined by the BLS for housing and present no exceptional data requirements.

Aside from the exclusion of improvements, this is the concept currently employed by the BLS, which here again uses home owner (not purchaser) weights.

[14] The original includes the phrase "per index family" whereas we are talking about the average asset holder, but our subsequent fractional weighting of asset holders will ultimately reduce this to the same basis.

[15] See *Monthly Labor Review*, February 1956, p. 193. Quantitatively these expenditures are about 40 percent of the total of maintenance, repair, and improvement.

G. INSURANCE

Several alternative definitions of constant quality are possible. Among these the simplest, which we use, is the notion of insurance as a constant proportion, b, of market value of the asset.[16]

Letting g_t be an index of annual rates for insurance of specified risks,

$$\overline{G}_t dt = g_t . b_t \overline{P}_t dt. \tag{26}$$

This corresponds to the BLS concept in use in housing; once again home owner (not purchaser) weights are used.

II. Comparison With Present CPI Treatment of Automobiles and Houses

Since the BLS procedure is different for automobiles and for housing, it is necessary to compare the user cost approach, advocated in the paper, to each of them separately. This is most effectively done through use of some artificial examples that will highlight the form and nature of the differences in results that occur.

The comparison with BLS procedure in respect to automobiles is a clearcut comparison of the user cost approach with the asset price approach. The BLS procedure with respect to housing is a mixture of the asset price and outlay approaches. For the automobile comparison, the two principal substantive questions are: first, do the indexes move together under the two approaches? And second, are the weights given automobiles for combination with other elements of consumer purchases in an overall index of the same order of magnitude? For the housing comparison, there is the additional question of whether the several components of the housing costs are given similar relative weights in computing the overall housing index. The answers to all of these questions are negative.

The examples below use hypothetical data and are simplified in many ways so that the essential differences become clear. In a number of places we deliberately chose assumptions that will *minimize* differences between the two approaches. In only one respect will the examples seem extreme—we have chosen data which reflect a very rapid and uneven rate of price inflation. The use of inflation rather than deflation is of course arbitrary and inconsequential. The use of extreme price changes facilitates examination of what is really going on in the alternate approaches. It may be argued that if prices are changing very little the differences we develop will be reduced proportionally. But this misses the point: it is only where price changes are significant that index numbers of prices are important and that the proper form of an index is worth debating.

A. AUTOMOBILE EXAMPLES

The automobile index of the CPI consists, in essence, of computing:

$$\frac{\sum_{t}^{j} P_j f'_{t-j}}{\sum_{0}^{j} P_j f'_{t-j}} W^*_0 = \frac{\overline{P}_t'}{\overline{P}_0'} W^*_0 \tag{27}$$

where the overall expenditure weight (W^*_0), like the average prices, reflects purchases in the base year.

[16] This seems satisfactory for housing where the principal insurance is on the dwelling. For automobiles, where liability insurance is involved, the assumption seems more dubious.

The user cost index is of the form of equation (3) above. For these examples we use the simplified form:

$$\frac{\overline{F}_t}{\overline{F}_0}=\frac{\overline{R}_t+\overline{I}_t}{\overline{R}_0+\overline{I}_0}W_0. \tag{28}$$

That is, we neglect components other than depreciation and interest and assume $dt=1$. In the examples we also neglect (until the end) the difference between W_0 and W^*_0.

The BLS considers only four age strata of purchases: new cars, and used cars of 3, 4, and 5 years of age. The following distribution of f'_{t-j}, which crudely approximates that in use, is used:

$t-j$	f'_{t-j}
0	0.50
1	.00
2	.00
3	.15
4	.15
5	.20

In order to suppress one element of difference between the approaches, we assume these same weights are the f_{t-j}—the frequency distribution of assets in use by the index population. (There is no reason why the two distributions should be the same. Indeed this particular f'_{t-j} is a virtually impossible distribution of f_{t-j}: what happens to one-year-old cars?) Notice that this distribution gives no clue as to how long individuals operate a given automobile before trading it in, or selling it.

The basic (assumed) price data appear in Table 1. Columns represent the prices of a distribution of cars of different ages at the same time. Rows show the prices at successive times of an automobile built in a specific year.[17]

TABLE 1.—*Prices Relatives* ${}_0P_0=100$

	t			
j	−1	0	1	2
−7	26			
−6	33	26		
−5	41	33	52	
−4	51	41	66	35
−3	64	51	82	50
−2	80	64	102	72
−1	100	80	128	103
0		100	160	147
1			200	210
2				300

[17] I have assumed that the relation of used to new asset prices follows a simple declining balance form, where ${}_ta_j=(1-a)^{t-j}$ with $a=.2$ for the first three columns, and $a=.3$ in the last column. Reference to footnote 7 will show that these assumed values for the first three columns are those that *minimize* the difference between the two approaches.

Table 2, which is derived from these data, permits the direct computation of the depreciation component of our index. The result of this computation, based on equation (13), and the computed values of P'_t are presented in Table 3.

TABLE 2.—*Computed Values, from Table 1*

Age of Asset at t	${}_t A_f$			${}_t A_f f_{(t-f)}$		
	$t=0$	$t=1$	$t=2$	$t=0$	$t=1$	$t=2$
1	.20	.20	.3000	.100	.100	.1500
4	.10	.10	.2700	.015	.015	.0405
5	.08	.08	.2433	.012	.012	.0365
6	.07	.07	.2133	.014	.014	.0427
				.141	.141	.2697

t	${}_t P_t$	e'_t	r_t	$\frac{r_t}{e'_t - 1}$
0	100	1.0	0.00	1.00
1	200	2.0	0.70	0.70
2	300	1.5	0.41	0.82

TABLE 3.—*Depreciation and Purchase Price Indexes*

t	$\overline{P}'_t$	$\frac{\overline{P}'_t}{\overline{P}'_0}100$	$\overline{R}_t$	$\frac{\overline{R}_t}{\overline{R}_0}100$	$\frac{\overline{R}'_t}{\overline{P}'_t}$
0	70.40	100	14.10	100	.20
1	140.80	200	19.74	140	.14
2	186.25	265	66.34	470	.36

Table 3 casts light on the adequacy of purchase prices as a proxy for depreciation. The comparison between years zero and one reflects only the influence of the r factor—the other two sources of difference have been *assumed* away. (See discussion following equation (14) above.) The assumed (relative) collapse of used car prices between year 1 and 2 serves to *reduce* average purchase price (relative to a new car price index) but to *increase* depreciation. This is a perfectly sensible result: A decline in used car prices makes "buying in" cheaper, but having held a depreciating asset more expensive. The last column of Table 3 makes clear that no value of W^*_0 in equation (27) will bring the indexes into alignment. In general, then, purchase prices will not be a reliable proxy for depreciation.

To compute the interest component requires data on interest rates and average effective sizes of mortgage. The data assumed for the latter are given in Table 4.[18]

TABLE 4.—*Size of Mortgages*

Asset Age $t-j$	$t=0$	$t=1$	$t=2$
1	72.0	144.0	189.0
4	28.7	57.4	50.4
5	19.8	39.6	30.0
6	13.0	26.0	17.5
$\Sigma_t M_j f_{t-j}$	45.87	91.75	110.06

Reference to equation (18) makes clear that an index of user interest costs will differ from an index of asset prices even if interest rates and other terms of credit are unchanging over time (but $c^*_k=i_t$) if the ratio of size of remaining mortgage to asset price varies either over time *or* among assets of varying ages. Example 1 in the tables below illustrates this; example 2 lets interest rates vary. The assumed interest data are given in Table 5. (Example 2 again oversimplifies by assuming $f_{t-k}=f_{t-j}$. The problems of the bivariate distribution are deferred to the housing example.)

TABLE 5.—*Assumed Interest Rates*

[Annual, percent]

t	Example 1			Example 2			
	i_t	c^*_k	i_t	c^*_k For mortgages of age			
				1 year	4 years	5 years	6 years
0	4	12	3.0	12	10	10	8
1	4	12	3.5	14	12	10	10
2	4	12	4.0	16	14	12	10

Table 6 summarizes the results of the interest computation.

TABLE 6.—*Interest Component*

t	Example 1			Example 2		
	$\bar{I}_t$	$\frac{\bar{I}_t}{\bar{I}_0}100$	$\frac{\bar{R}_t}{\bar{I}_t}$	$\bar{I}_t$	$\frac{\bar{I}_t}{\bar{I}_0}100$	$\frac{\bar{R}_t}{\bar{I}_t}$
0	6.49	100	2.17	5.99	100	2.35
1	12.98	200	1.52	14.66	245	1.35
2	16.26	251	4.08	20.12	336	3.30

[18] These numbers were found from the basic price data by the following simplifying assumption: Suppose that the average effective size of mortgage during the period $t-1$ to t is a fraction, e_{t-j}, of ${}_tP_j$, with the following value:

$t-j$	e_{t-j}
1	.9
4	.7
5	.6
6	.5

This assumption neglects the influence of changing credit conditions on mortgage sizes. The size-of-mortgage problem is faced head on in the housing comparison in Part II, Section B, below. A more sophisticated approach here would not change any substantive conclusion, although it would change the numerical values.

Notice not only that the interest indexes, *in both examples*, differ from *both* the depreciation component and the purchase prices of assets, but also that the amounts of interest do not bear an even approximately constant relation to the amounts of depreciation. This means that the user cost index must be a ratio of sums, not a sum of ratios. (See nonequalities (4) and (5) above.)

The indexes corresponding to (27) and (28) are presented in Table 7, neglecting the overall weights W^*_0 and W_0. The last column in the table sheds light on these weights. Notice that the amount in the $\bar{P}'_t$ column is greater than in the $\bar{F}_t$ column. The former will be given a weight, W^*_0, proportional to *purchasers* of cars in the base year; the latter a weight, W_0, proportional to *users* in the same year. Whether the amount of car expense[19] to be included in the overall index is larger, smaller, or the same under the two procedures depends upon the ratio of W^*_0 and W_0. But since $\bar{F}_t/\bar{P}'_t$ varies over time, there is no pair of values of W^*_0 and W_0 that will make the procedures equivalent.

TABLE 7.—*Summary of Examples*

t	Amounts: $_0P_0=100$				Indexes: year 0=100				$\bar{F}_t/\bar{P}_t$
	$\bar{P}_t$	$\bar{F}_t$	$\bar{R}_t$	$\bar{I}_t$	Purchase Price	User Cost	Depreciation	Interest	
Example 1:									
0	70.40	20.59	14.10	6.49	100	100	100	100	.29
1	140.80	32.71	19.74	12.98	200	159	140	200	.23
2	186.25	82.60	66.34	16.26	265	401	470	251	.44
Example 2:									
0	70.40	20.09	14.10	5.99	100	100	100	100	.29
1	140.80	34.40	19.74	14.66	200	171	140	245	.24
2	186.25	86.46	66.34	20.12	265	430	470	336	.46

To summarize, our simple example shows several things:

1. The two procedures lead to results that are strikingly different in, first, the magnitude of the size of price changes, and second, the *relative* rankings of years with respect to rate of changes in the index.

2. The several components of the cost of using an asset vary in their relative sizes from year to year so that we must use a ratio of aggregates, not a weighted aggregate of ratios. This point will become critical with respect to housing.

3. The overall weights W^*_0 and W_0 are not only different concepts but there exists no fixed conversion factor that will make them equivalent.

B. HOUSING EXAMPLE

Whereas the BLS treatment of automobiles represented a consistent (if mistaken, according to the user cost approach) use of purchase prices and purchase weights, the home owner index is an amalgam of purchase costs, current user costs—and even future costs.

The simplified representation in equation (29) will highlight certain general problems. We will return to specifics subsequently. Let $\bar{H}_t$=BLS index of home owner costs in year t. (The fraction

[19] Recognizing that we have not included all elements of that expense in our examples. Compare (3) and (28).

W_0—about .5 in 1952—is factored out of all specific weights in the subsequent representation.)

$$\overline{H}_t = W_0 \Bigg[\frac{\text{Purchase Price}_t}{\text{Purchase Price}_0}\,\text{Purchase Weight}_0 + \tag{29}$$

$$\frac{\text{Incidental Purchase Cost}_t}{\text{Incidental Purchase Cost}_0}\,\text{Purchase Weight}_0 +$$

$$\frac{\text{Interest Commitment}_t}{\text{Interest Commitment}_0}\,\text{Mortgage Acquisition Weight}_0 +$$

$$\frac{\text{Insurance Costs}_t}{\text{Insurance Costs}_0}\,\text{User Weight}_0 +$$

$$\frac{\text{Tax Cost}_t}{\text{Tax Cost}_0}\,\text{User Weight}_0 +$$

$$\frac{\text{Maintenance, Repair \& Improvement}_t}{\text{Maintenance, Repair \& Improvement}_0}\,\text{User Weight}_0\Bigg]$$

Notice particularly the following criticisms:

1. The first three elements in the square brackets apply only to the small fraction of the homeowners who purchase (and/or mortgage) in a specific year. Indeed, it is the fraction in the base year. The last three elements apply to all homeowners. The relative weights given the components may thus be subject to a distortion even if the components are sensible.

2. The limitation to purchasers alone is appropriate only for incidental purchase costs. Purchase price is not an element of user cost at all, unless it is a proxy for depreciation. But if it is that, the limitation to a fraction of homeowners makes it an inherently inadequate proxy for a real user cost. Similarly the interest commitment is largely a stream of future costs over T years from t. That this is not even approximately a satisfactory "proxy" for the true user costs is evident from: (a) the use of a mortgage acquisition rate—limited, like purchase weights, to a fraction of users; (b) the neglect of user cost on past acquisitions, at other interest rates; (c) the neglect of user costs on nonmortgaged assets or on fractions not mortgaged.

3. The index is an aggregate of ratios, aggregating the several ratios with weights from the base year. It thus does not permit changes in the relative importance of components over time, although such changes are fully consistent with an index based upon the use of a constant quantity-quality of houses.

4. The inclusion of improvement in the index seems improper within the fixed base concept of constant quality.

The basic data for this example are specified in Tables 8–11. While we intend to construct an index for years 0, 1, and 2 only, it is necessary to specify data for earlier years because in this example we derive (rather than specify) the size of remaining mortgage in terms of purchase prices and dates, and the terms of credit existing at those dates.

TABLE 8.—*Prices of New Houses* ($_tP_t$)

$_tP_t = 100$

Year	$_tP_t$	Year	$_tP_t$
−21	60	−9	84
−20	62	−8	86
−19	64	−7	88
−18	66	−6	90
−17	68	−5	92
−16	70	−4	94
−15	72	−3	96
−14	74	−2	98
−13	76	−1	100
−12	78	0	100
−11	80	+1	200
−10	82	+2	300

TABLE 9.—*New to Used Prices*

Year	Depreciation Pattern	Depreciation Rate
−21 to +1	Straight Line	2 percent (50 years).
+2	Straight Line	2½ percent (40 years).

TABLE 10.—*Bivariate Distribution of Age of Houses and Age of Mortgages*

[Percent]

Age of Asset (t−j)	Total f_{t-j}	f'_{t-j}	Age of Mortgage (years owned): t−k								
		0	1	2	3	4	5	10	15	20	More
0	5	5.0									
1	5	1.0	4.0								
2	5	1.0	2.0	2							
3	5	1.0	1.0	1	2						
5	20	3.0	3.0	3	3	4.0	4.0				f_{k-j}
10	20	3.0	2.0	2	2	2.0	2.0	7			
20	20	3.0	2.0	2	2	2.0	2.0	3	2	2	
30	10	1.5	1.5	1	1	.5	.5	1	1	1	1
40	10	1.5	1.5	1	1	.5	.5	1	1	1	1
Total	100	20.0	17.0	12	11	9.0	9.0	12	4	4	2
		f_{t-k}									

TABLE 11.—*Terms of Mortgage Credit*

[Interest rates in percent]

Year	Contract rate c^*_k	Length of mortgage T_k	Fraction of sale price c_k	Lending rate i_t	Year	Contract rate c^*_k	Length of mortgage T_k	Fraction sale price c_k	Lending rate i_t
−21	4.0	14	0.75	(1)	−9	4.0	16	.71	(1)
−20	4.0	14	.75	(1)	−8	4.5	17	.70	(1)
−19	4.0	14	.75	(1)	−7	4.5	18	.69	(1)
−18	4.0	14	.75	(1)	−6	5.0	19	.68	(1)
−17	4.0	14	.75	(1)	−5	5.0	20	.67	(1)
−16	4.0	15	.75	(1)	−4	5.0	20	.66	(1)
−15	4.0	15	.75	(1)	−3	5.0	20	.65	(1)
−14	4.0	15	.75	(1)	−2	5.5	20	.64	(1)
−13	4.0	15	.75	(1)	−1	5.5	20	.63	3.5
−12	4.0	15	.75	(1)	0	5.0	20	.62	3.0
−11	4.0	15	.73	(1)	+1	5.5	20	.61	3.5
−10	4.0	15	.72	(1)	+2	6.0	20	.60	4.0

[1] Not required.

Since there is a one-to-one correspondence of terms in equation (29) with terms in the numerator of (3) (if we pretend purchase price is a proxy for depreciation), it will facilitate presentation of the results of the comparison to treat the items sequentially, although, as we have seen in Part I, the individual components are different in the two approaches in some cases, and although relationship (5) is not an equation.

C. PURCHASE COMPONENT (BLS) AND DEPRECIATION

The BLS purchase component is computed from purchase prices in each year weighted by the precentages, f'_{t-j}, in the zero column of Table 10. These weights not only reflect the acquisition pattern but reduce the figures to a "per homeowner" basis. The results of this computation are shown in the first two columns of Table 12. (Had we used weights reflecting the distribution of assets—as we did in the automobile examples—as shown in the total column of Table 10, adjusted for the purchase ratio of 1/5, the results would have been slightly different and are shown in columns 5 and 6 of Table 12.)

TABLE 12

Year	BLS Purchase Component (purchase weights) f'_{t-j}		Depreciation Computation (ownership weights) f_{t-j}		BLS Purchase Component $1/5 f_{t-j}$	
	Amounts (1)	Index (2)	Amounts (3)	Index (4)	Amounts (5)	Index (6)
0	15.68	100	2.00	100	14.28	100
1	31.36	200	2.80	140	28.56	200
2	43.80	279	23.12	1156	38.55	270

Notice not only that the indexes behave very differently but that the expenditure weights in the BLS index are consistently too high. This is due to the fact that we assume that the purchase rate is substantially higher than the reciprocal of the useful life of a house. That is, notwithstanding that houses last 40 to 50 years, we assume that the purchase rate is 20 percent. While the specific numbers are arbitrary, the well-known high mobility of the American population makes these magnitudes seem reasonable.

D. INTEREST

Since the two alternative approaches were compared in Part I, Section C5 above, we shall turn directly to the results of applying (18) and (2) to the assumed data. They are presented in Table 13.

TABLE 13.—*Interest*

Year	BLS: Interest Contracted		User Cost: Interest						Interest Charge*	
			Imputed: $i_{t-t}\overline{P}_t$		Extra Explicit		Total		Price	
	Amount (1)	Index (2)	Amount (3)	Index (4)	Amount (5)	Index (6)	Amount (7)	Index (8)	BLS (9)	User Cost (10)
0	5.66	100	2.142	100	.846	100	2.99	100	7.9	4.2
1	12.42	219	4.998	233	.849	100	5.85	196	8.7	4.1
2	18.83	333	7.710	360	1.020	121	8.73	292	9.8	4.5

*Per average homeowner (percent).

The results of the interest contracted (on a per homeowner basis) calculation are shown in columns 1 and 2; the results of the user cost computation (on the same basis) are shown in columns 3–8. While the overall indexes (columns 2 and 8) are not drastically different under the two approaches, this is due to the numbers chosen rather than to any fundamental similarity of the concepts employed, as attention to the detail will suggest. The *amounts* (columns 1 and 7), which are the implicit weights given interest in total expenditures on housing, are very different. Columns 9 and 10 shed some light on this. Our approach (column 10) reflects current lending rates, modified by a weighted average of past contract rates, and the figures in column 10 are meaningfully related to the underlying interest rates of Table 11. The figures in column 9 bear no such relation—their magnitude is crucially related to length of mortgage and to the mortgage acquisition rate. The latter is chiefly related to the duration of ownership of houses.

E. INCIDENTAL PURCHASE EXPENSES

We assume that these expenses amount to 2 percent of purchase price in year purchased. They appear in Table 14. They are thus *properly* weighted by purchase weights and move with the BLS purchase index, as shown in column 2 of Table 12.

TABLE 14.—*Incidental Purchase Cost*

Year	Amount	Index
0	0.3136	100
1	.6272	200
2	.8760	279

While these amounts are identical in the two procedures, it should be noted that the relative size of incidental purchase expense to depreciation (on the one hand) and to the purchase component (on the other hand) are very different year by year.

F. TAXES, MAINTENANCE-REPAIR, INSURANCE

To simplify drastically, we lump these items together and assume arbitrarily that they amount in any period to 4 percent of the market value of the house at the end of the period. (This will introduce a bias in our index toward conforming with the BLS concept because these amounts then vary exactly with the amounts in column 5 of Table 12—which are closely correlated with the purchase component of the BLS index.) We will use these amounts weighted by the ownership distribution (f_{t-j}) and use identical amounts for the two approaches. The results are shown in Table 15.

TABLE 15.—*Insurance, Taxes, and Maintenance-Repair*

Year	Amounts	Index
0	2.86	100
1	5.71	200
2	7.71	270

(We have here suppressed certain differences discussed in Part I, Sections E and F, above.)

G. TOTAL HOUSING COST

Tables 16 and 17 present the combined costs under the two procedures. The upper parts of the tables show the amounts of the components and the indexes based thereon; the lower parts reflect the implicit weights of the individual components in the totals. The BLS implicit weights are stable because stability is built in—the wisdom of that assumption is challenged. It should be noted that the relative size of the individual components in the BLS procedure is *critically affected* by the purchase ratio. In our procedure this ratio directly affects incidental purchase costs and to some extent the interest component; but the relative size of the principal components is largely insensitive to the magnitude of the (assumed fixed) purchase ratio.

TABLE 16.—*BLS Concept: Summary*

Year	* Purchase cost	* Incidental purchase cost	* Interest committed	Taxes, etc.	Total	
					Amount	Index
0	15 68	.31	5.66	2.86	24.51	100
1	31.36	.63	12.42	5.71	50.12	204.5
2	43.80	.88	18.83	7.71	71.22	290.6
HORIZONTAL PERCENTAGES						
0	64.0	1.2	23.1	11.7	100.0	
1	62.6	1.2	24.8	11.4	100.0	
2	61.5	1.2	26.4	10.8	99.9	

NOTE.—Starred columns reflect purchase weights, others ownership weights.

TABLE 17.—*User Cost Concept: Summary*

Year	Depreciation	* Incidental purchase cost	Interest	Taxes, etc.	Total	
					Amount	Index
0	2.00	.31	2.99	2.86	8.16	100
1	2.80	.63	5.85	5.71	14.99	183.7
2	23.12	.88	8.73	7.71	40.44	495.6
HORIZONTAL PERCENTAGES						
0	24.5	3.8	36.6	35.0	99.9	
1	18.7	4.2	39.0	38.1	100.0	
2	57.2	2.2	21.6	19.1	100.1	

NOTE.—Starred items reflect purchase weights, others ownership weights.

It may be noted, finally, that not only do the indexes move differently, but that the amounts, which become implicit weights for combining home owner costs with renter costs in a total shelter component, are different in the two approaches. Since rental cost is clearly a user cost, a comparison is valid. Suppose, to get some notion of magnitude, we use the familiar rule of thumb that a long-run average rental of 1 percent per month of market value is appropriate. For the average prices of our housing example this yields *annual* rental amounts of 8.57, 17.14, and 29.13 for years 0, 1, and 2, respectively. These magnitudes are strikingly congruent with the user costs developed and much below the amounts in the BLS index.[20]

III. Conclusion

It has been the primary purpose of this paper to develop the user cost approach to the problem of consumer durables in an index of consumer prices, to describe and characterize the present BLS approach to the two most important durable consumer goods, and to see if the results are significantly different. Both the algebraic analysis of Part I and the examples of Part II indicate that the differences are fundamental, pervasive, and striking. If the user cost approach is accepted in principle, it can hardly be suggested that present procedure is a reasonable approximation to it. At issue is more than just the magnitude of price changes. Additionally there is the behavior of the index over time and the weights given the included durable goods in the overall index.

Is the user cost approach the sensible one? It is clearly *one* sensible one, and in my view perhaps the most sensible one for a general purpose index. But the issue of which is the most useful approach depends crucially upon the purpose for which the index is used, and that is beyond the scope of this paper.

If the user cost approach, here advocated, is adopted, the discussion of present practice can be neglected. If, however, this approach be deemed inappropriate or impractical, a secondary purpose of this paper has been to highlight certain fundamental difficulties in the current procedures. Among the most important of these are:

1. The critical need for the definition of the approach to be used and its implementation. Thus the fundamentally different treatment currently given to automobiles and owner-occupied housing can hardly both be consistent with the same concept. One cannot escape the conclusion that the latter-day introduction of durables into the CPI has been in a series of *ad hoc* steps which have no coherent logic.

2. Should a component-by-component computation such as is currently employed in the treatment of housing be retained, it is essential that attention be directed to two problems. The first concerns the proper determination of the relative weights given to the several components. The second reflects the fact that component weights will not remain constant over time even if the quantity and quality of goods remain constant.

3. The use of a purchase price component creates difficulties of two sorts. The first is whether it is necessary to distinguish among purchases that are (a) net increases in asset holdings by the index population, (b) exchanges among assets of different types, or (c) exchanges of specific assets (e.g., houses or cars) of the same type. The extent of each of these clearly affect the expenditure weights, but equally clearly they are very different in character and bear very differently on the concept of "a fixed quantity and quality of goods and services."

[20] To what extent this is a fortuitous result of the data and assumptions of our hypothetical example is not known. Should a similar result occur in an application to real data, it suggests a very simple shortcut to finding expenditure weights that seems superior to present BLS procedure.

Only the third is even approximately handled by using prices net of trade-ins. The second (and related) difficulty arises from the fact that, e.g., houses are typically not retained for the whole of their useful lives and thus the expenditure weights are critically affected by turnover rates. In particular the interest component of housing seems subject to systematic distortion. In any case such turnover rates must be (as they are not) explicitly introduced into any calculation using asset purchases.

4. The ultimate combination of durables with nondurables in a comprehensive index requires explicit determination of the appropriate relative weights to be given commodities of each kind. Following either a user cost or a current outlay approach, the implicit weights are satisfactory. Using an asset price approach or a mixed approach, the implicit weights are arbitrary and capricious.

Appendix A. Glossary of Symbols Used

Attention is directed to Part I, Section A of the text for an explanation of the principles of subscript notation employed. The following classified glossary omits identifying subscripts in most cases.

TIME: DATING AND DURATION

Time is of the essence of durable goods, and is measured in years from $t=0$, the base date for weighting *and* comparison purposes.

t: a general running subscript identifying years from base date.
j: the year an asset was new.
k: the year an asset was acquired.
dt: fractions of a year.

n: $\frac{1}{dt}$.

m: number of subperiods of n, generally $\rightarrow \infty$.

DOLLAR MAGNITUDES

If unbarred (D) refer to an individual asset; if barred ($\overline{D}$), to an appropriate weighted average collection of assets. See below.

D: Required size of a reserve for depreciation.
F: Total user cost, per year.
G: Insurance cost, per year.
H: Home owner cost, BLS.
I: Interest cost, per year.
J: Incidental purchase cost.
M: Size of loan or mortgage.
MR: Maintenance and repair cost, per year.
P: Price of the asset, at market.
R: Depreciation cost, per year.
V: Assessed valuation.
X: Tax cost, per year.

WEIGHTS

At time t, there are N assets of the type in question owned by the index population which vary according to age of asset, length of ownership by present owner, and in other ways. There are subsets of N which satisfy:

$$N=\sum^{j} N_j=\sum^{k} N_k=\sum^{j}\sum^{k} N_{j,k}$$

${}_tf_j$: the frequency $\frac{N_j}{N}$ at t; also (for all j) the frequency distribution of asset owners over j. (Likewise for ${}_tf_k$ and other subscripts.)

${}_tf'_j$: the frequency distribution of *purchases* in the year t. I.e., the frequency distribution over j for $k=t$.

f_{t-j}, f_{t-k}, f'_{t-j}, etc., indicate constancy of ${}_tf_j$, etc., over time.

W_0: fraction of asset owners to total index population in base year.

$\left.\begin{array}{l} W^*_0: \\ w_0\ : \end{array}\right\}$ base year weights, defined in context.

WEIGHTED AVERAGES

$${}_t\bar{P}_j=\sum^{j} {}_tP_j f_{t-j}$$

$$\bar{P}'_t=\sum^{j} {}_tP_j f'_{t-j}.$$

(There is no second subscript since $k=t$. Thus the summation must be over j only.)

Etc. for other dollar magnitudes.

ANNUAL RATES

i_t: the lending rate of interest, at t. (If i is used as a running superscript, it is without any subscript of its own.)

c^*_k: the contract rate of interest, at k.

r_t: the rate of inflation (+) or deflation (−) of the asset, at t. Defined by:

$$e^{r\,dt}=\frac{{}_tP_t}{{}_{t-dt}P_{t-dt}}$$

π_t: another rate of inflation; like r, but $r\neq\pi$.

x_t: tax rate per dollar of assessed valuation.

DERIVED RATIOS

$${}_t\alpha_j=\frac{{}_tP_j}{{}_tP_t}$$

$${}_tA_j={}_{t-dt}\alpha_j-{}_t\alpha_j$$

$$c_k=\frac{{}_kM_j}{{}_kP_j}$$

$${}_td_k=\frac{{}_tM_{k,j}}{{}_kM_{k,j}}$$

CONSTANTS AND UNSPECIFIED PARAMETERS

$$a,\ a_1,\ a_2,\ \ldots$$

$$b,\ b_1,\ b_2,\ \ldots$$

$$\lambda,\ \theta$$

OTHER

g_t: An index of insurance rates.
B_t: An index of MR prices.
$a_{\overline{n}|}$: Present value of an annuity of 1
$a^{-1}_{\overline{n}|}$: Annuity whose present value is 1
$S_{\overline{n}|}$: Amount of an annuity of 1
$\Bigg\}$ for n periods at a specified rate of interest.

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