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# 3 Tests of CAPM on an International Portfolio of Bonds and Stocks

Charles M. Engel

## 3.1 Introduction

Portfolio-balance models of international asset markets have enjoyed little success empirically.<sup>1</sup> These studies frequently investigate a very limited menu of assets, and often impose the assumption of a representative investor.<sup>2</sup> This study takes a step toward dealing with those problems by allowing some investor heterogeneity, and by allowing investors to choose from a menu of assets that includes bonds and stocks in a mean-variance optimizing framework.

The model consists of U.S., German, and Japanese residents who can invest in equities and bonds from each of these countries. Investors can be different because they have different degrees of aversion to risk. More important, within each country nominal prices paid by consumers (denominated in the home currency) are assumed to be known with certainty. This is the key assumption in Solnik's (1974) capital asset pricing model (CAPM). Investors in each country are concerned with maximizing a function of the mean and variance of the returns on their portfolios, where the returns are expressed in the currency of the investors' residence. Thus, U.S. investors hold the portfolio that is efficient in terms of the mean and variance of dollar returns, Germans in terms of mark returns, and Japanese in terms of yen returns.

The estimation technique is closely related to the CASE (constrained asset

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1. See Frankel (1988) or Glassman and Riddick (1993) for recent surveys.

2. Although, notably, Frankel (1982) does allow heterogeneity of investors. Recent papers by Thomas and Wickens (1993) and Clare, O'Brien, Smith, and Thomas (1993) test international CAPM with stocks and bonds, but with representative investors.

share efficiency) method introduced by Frankel (1982) and elaborated by Engel, Frankel, Froot, and Rodrigues (1993). The mean-variance optimizing model expresses equilibrium asset returns as a function of asset supplies and the covariance of returns. Hence, there is a constraint relating the mean of returns and the variance of returns. The CASE method estimates the mean-variance model imposing this constraint. The covariance of returns is modeled to follow a multivariate GARCH process.

One of the difficulties in taking such a model to the data is that there is scanty time-series evidence on the portfolio holdings of investors in each country. We do not know, for example, what proportion of Germans' portfolios is held in Japanese equities, or U.S. bonds.<sup>3</sup> We do have data on the total value of equities and bonds from each country held in the market, but not a breakdown of who holds these assets. Section 3.2 shows how we can estimate all the parameters of the equilibrium model using only the data on asset supplies and data that measure the wealth of residents in the United States relative to that of Germans and Japanese. The data used in this paper have been available and have been used in previous studies. The supplies of bonds from each country are constructed as in Frankel (1982). The supply of nominal dollar assets from the United States, for example, increases as the government runs budget deficits. These numbers are adjusted for foreign exchange intervention by central banks, and for issues of Treasury bonds denominated in foreign currencies. The international equity data have been used in Engel and Rodrigues (1993). The value of U.S. equities is represented by the total capitalization on the major stock exchanges as calculated by Morgan Stanley's *Capital International Perspectives*. The shares of wealth are calculated as in Frankel (1982)—the value of financial assets issued in a country, adjusted by the accumulated current account balance of the country.

The Solnik model implies that investors' portfolios differ only in terms of their holdings of bonds. If we had data on portfolios from different countries, we would undoubtedly reject this implication of the Solnik model. However, we might still hope that the equilibrium model was useful in explaining risk premia. In fact, our test of the equilibrium model rejects CAPM relative to an alternative that allows diversity in equity as well as bond holdings. Probably the greatest advantage of the CASE method is that it allows CAPM to be tested against a variety of plausible alternative models based on asset demand functions. Models need only require that asset demands be functions of expected returns and nest CAPM to serve as alternatives. In section 3.6, CAPM is tested against several alternatives. CAPM holds up well against alternative models in which investors' portfolios differ only in their holdings of bonds. But when we build an alternative model based on asset demands which differ across countries in bond and equity shares, CAPM is strongly rejected. While our CAPM model allows investor heterogeneity, apparently it does not allow enough.

3. Tesar and Werner (chap. 4 in this volume) have a limited collection of such data.

There are many severe limitations to the study undertaken here, both theoretical and empirical. While the estimation undertaken here involves some significant advances over previous literature, it still imposes strong restrictions. On the theory side, the model assumes that investors look only one period into the future to maximize a function of the mean and variance of their wealth. It is a partial equilibrium model, in the classification of Dumas (1993). Investors in different countries are assumed to face perfect international capital markets with no informational asymmetries. The data used in the study are crude. The measurement of bonds and equities entails some leaps of faith, and the supplies of other assets—real property, consumer durables, etc.—are not even considered. Furthermore, there is a high degree of aggregation involved in measuring both the supplies of assets and their returns.

Section 3.2 describes the theoretical model, and derives a form of the model that can be estimated. It also contains a brief discussion relating the mean-variance framework to a more general intertemporal approach. Section 3.3 discusses the actual empirical implementation of the model. Section 3.4 presents the results of the estimation, and displays time series of the risk premia implied for the various assets.

The portfolio balance model is an alternative to the popular model of interest parity, in which domestic and foreign assets are considered perfect substitutes. This presents some inherent difficulties of interpretation in the context of our model with heterogeneous investors, which are discussed in section 3.5. These problems are discussed, and some representations of the risk-neutral model are derived to serve as null hypotheses against the CAPM of risk-averse agents.

Section 3.6 presents the test of CAPM against alternative models of asset demand. The concluding section attempts to summarize what this study accomplishes and what would be the most fruitful directions in which to proceed in future research.

### **3.2 The Theoretical Model**

The model estimated in this paper assumes that investors in each country face nominal consumer prices that are fixed in terms of their home currency. While that may not be a description that accords exactly with reality, Engel (1993) shows that this assumption is much more justifiable than the alternative assumption that is usually incorporated in international financial models—that the domestic currency price of any good is equal to the exchange rate times the foreign currency price of that good.

Dumas, in his 1993 survey, refers to this approach as the “Solnik special case,” because Solnik (1974) derives his model of international asset pricing under this assumption. Indeed, the presentation in this section is very similar to Dumas’s presentation of the Solnik model. The models are not identical because of slightly differing assumptions about the distribution of asset returns.

There are six assets—dollar bonds, U.S. equities, deutsche mark bonds, German equities, Japanese bonds, and Japanese equities. Time is discrete.

Table 3.1 lists the variables used in the derivations below.

The own currency returns on bonds between time  $t$  and time  $t + 1$  are assumed to be known with certainty at time  $t$ , but the returns on equities are not in the time  $t$  information set.

U.S. investors are assumed to have a one-period horizon and to maximize a function of the mean and variance of the real value of their wealth. However, since prices are assumed to be fixed in dollar terms for U.S. residents, this is equivalent to maximizing a function of the dollar value of their wealth.

Let  $W_{t+1}^u$  equal dollar wealth of U.S. investors in period  $t + 1$ . At time  $t$ , investors in the United States maximize  $F^{US}(E_t(W_{t+1}^u), V_t(W_{t+1}^u))$ . In this expression,  $E_t$  refers to expectations formed conditional on time  $t$  information.  $V_t$  is the variance conditional on time  $t$  information. We assume the derivative of  $F^{US}$  with respect to its first argument,  $F_1^{US}$ , is greater than zero, and that the derivative of  $F^{US}$  with respect to its second argument,  $F_2^{US}$ , is negative.

Following Frankel and Engel (1984), we can write the result of the maximization problem as

$$(1) \quad \lambda_t^u = \rho_{US}^{-1} \Omega_t^{-1} E_t Z_{t+1}^{US}$$

In equation (1) we have

$$Z_{t+1} = \begin{bmatrix} R_{t+1}^u - (1 + i_{t+1}^u) \\ R_{t+1}^g \frac{S_{t+1}^g}{S_t^g} - (1 + i_{t+1}^u) \\ (1 + i_{t+1}^g) \frac{S_{t+1}^g}{S_t^g} - (1 + i_{t+1}^u) \\ R_{t+1}^j \frac{S_{t+1}^j}{S_t^j} - (1 + i_{t+1}^u) \\ (1 + i_{t+1}^j) \frac{S_{t+1}^j}{S_t^j} - (1 + i_{t+1}^u) \end{bmatrix}$$

$$\Omega_t \equiv V_t(Z_{t+1}) = E_t[(Z_{t+1} - E_t Z_{t+1})(Z_{t+1} - E_t Z_{t+1})'],$$

$$\rho_{US} = -2F_2^{US} W / F_1^{US},$$

and  $\lambda_t^u$  is the column vector that has in the first position the share of wealth invested by U.S. investors in U.S. equities, the share invested in German equities in the second position, the share in mark bonds in the third position, the share in Japanese equities in the fourth position, and the share in Japanese bonds in the fifth position.

We will assume, as in Frankel (1982), that  $\rho_{US}$  (and  $\rho_G$  and  $\rho_J$ , defined later) are constant. These correspond to what Dumas (1993) calls “the market average degree of risk aversion,” and can be considered a taste parameter. The degree of risk aversion can be different across countries.

Table 3.1

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$i_{t+1}^u$	≡	the dollar return on dollar bonds between time $t$ and $t + 1$
$i_{t+1}^g$	≡	the mark return on mark bonds
$i_{t+1}^j$	≡	the yen return on yen bonds
$R_{t+1}^u$	≡	the gross dollar return on U.S. equities
$R_{t+1}^g$	≡	the gross mark return on German equities
$R_{t+1}^j$	≡	the gross yen return on Japanese equities
$S_t^g$	≡	the dollar/mark exchange rate at time $t$
$S_t^j$	≡	the dollar/yen exchange rate
$\mu_t^u$	≡	$W_t^u / (S_t^j W_t^j + S_t^g W_t^g + W_t^u)$ , share of U.S. wealth in total world wealth
$\mu_t^g$	≡	$S_t^g W_t^g / (S_t^j W_t^j + S_t^g W_t^g + W_t^u)$ , share of German wealth in total world wealth
$\mu_t^j$	≡	$S_t^j W_t^j / (S_t^j W_t^j + S_t^g W_t^g + W_t^u)$ , share of Japanese wealth in total world wealth

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Let  $r_{t+1} \equiv \ln(R_{t+1})$ , so that  $R_{t+1} = \exp(r_{t+1})$ . Now, we assume that  $r_{t+1}$  is distributed normally, conditional on the time  $t$  information. So, we have that

$$E_t R_{t+1} = E_t \exp(r_{t+1}) = \exp(E_t r_{t+1} + \sigma_t^u/2),$$

where  $\sigma_t^u = V_t(r_{t+1})$ .

Then, note that for small values of  $E_t r_{t+1}$  and  $\sigma_t^u/2$ , we can approximate

$$E_t R_{t+1} = \exp(E_t r_{t+1} + \sigma_t^u/2) \cong 1 + E_t r_{t+1} + \sigma_t^u/2.$$

Using similar approximations, and using lower-case letters to denote the natural logs of the variables in upper cases, we have

$$E_t Z_{t+1} \cong E_t z_{t+1} + D_t, \text{ where}$$

$$z_{t+1} \equiv \begin{bmatrix} i_{t+1}^u - i_{t+1}^u \\ r_{t+1}^g + s_{t+1}^g - s_t^g - i_{t+1}^u \\ i_{t+1}^g + s_{t+1}^g - s_t^g - i_{t+1}^u \\ r_{t+1}^j + s_{t+1}^j - s_t^j - i_{t+1}^u \\ i_{t+1}^j + s_{t+1}^j - s_t^j - i_{t+1}^u \end{bmatrix}$$

$$\Omega_t = V_t(Z_{t+1}) \cong V_t(z_{t+1})$$

and

$D_t = \text{diag}(\Omega_t)/2$ , where  $\text{diag}(\ )$  refers to the diagonal elements of a matrix.

So, we can rewrite equation (1) as

$$(2) \quad \lambda_t^u = \rho_{US}^{-1} \Omega_t^{-1} (E_t z_{t+1} + D_t).$$

Now, assume Germans maximize  $F^G(E_t(W_{t+1}^g), V_t(W_{t+1}^g))$ , where  $W^g$  represents the mark value of wealth held by Germans. After a bit of algebraic manipulation, the vector of asset demands by Germans can be expressed as

$$(3) \quad \lambda_t^g = \rho_G^{-1} \Omega_t^{-1} (E_t z_{t+1} + D_t) + (1 - \rho_G^{-1}) e_3,$$

where  $e_j$  is a vector of length five that has a one in the  $j$ th position and zeros elsewhere.

Japanese investors, who maximize a function of wealth expressed in yen terms, have asset demands given by

$$(4) \quad \lambda_t^j = \rho_j^{-1} \Omega_t^{-1} (E_t z_{t+1} + D_t) + (1 - \rho_j^{-1}) e_3.$$

Note that in the Solnik model, if the degree of risk aversion is the same across investors, they all hold identical shares of equities. Their portfolios differ only in their holdings of bonds. Even if they have different degrees of risk aversion, there is no bias toward domestic equities in the investors' portfolios. This contradicts the evidence we have on international equity holdings (see, for example, Tesar and Werner, chap. 4 in this volume), so this model is not the most useful one for explaining the portfolio holdings of individuals in each country. Still, it may be useful in explaining the aggregate behavior of asset returns.

Then, taking a weighted average, using the wealth shares as weights, we have

$$(5) \quad \lambda_t = (\mu_t^j \rho_j^{-1} + \mu_t^s \rho_{US}^{-1} + \mu_t^g \rho_{US}^{-1}) \Omega_t^{-1} (E_t z_{t+1} + D_t) + \mu_t^s (1 - \rho_G^{-1}) e_3 \\ + \mu_t^g (1 - \rho_j^{-1}) e_3.$$

The vector  $\lambda_t$  contains the aggregate shares of the assets. While we do not have time-series data on the shares for each country, we have data on  $\lambda_t$ , and so it is possible to estimate equation (5). This equation can be interpreted as a relation between the aggregate supplies of the assets and their expected returns and variances.

### 3.2 A Note on the Generality of the Mean-Variance Model

The model that we estimate in this paper is a version of the popular mean-variance optimizing model. This model rests on some assumptions that are not very general. The strongest of the assumptions is that investors' horizons are only one period into the future.

It is interesting to compare our model with that of Campbell (1993), who derives a log-linear approximation for a very general intertemporal asset-pricing model. Campbell assumes that all investors evaluate real returns in the same way—as opposed to our model, in which real returns are different for U.S. investors, Japanese investors, and German investors.

In order to focus on the effects of assuming a one-period horizon, we shall follow Campbell and examine a version of the model in which all consumers evaluate returns in the same real terms. This would be equivalent to assuming that all investors evaluate returns in terms of the same currency, and that nominal goods prices are constant in terms of that currency.

So, we will assume investors evaluate returns in dollars. In that case, we can derive from equation (2) that

$$(6) \quad E_t z_{t+1} = \rho \Omega_t \lambda_t - D_t.$$

Let  $z_i$  represent the excess return on the  $i$ th asset. The expected return can be written

$$E_t z_{i,t+1} = \rho \Omega_{it} \lambda_t - \text{var}_t(z_{i,t+1})/2.$$

In this equation,  $\text{var}_t$  refers to the conditional variance, and  $\Omega_{it}$  is the  $i$ th row of  $\Omega_t$ .

We can write

$$\begin{aligned} \Omega_{it} \lambda_t &= \sum_{j=1}^n \text{Cov}_t(z_{i,t+1}, z_{j,t+1}) \lambda_j = \text{Cov}_t(z_{i,t+1}, \sum_{j=1}^n z_{j,t+1} \lambda_j) \\ &= \text{Cov}_t(z_{i,t+1}, z_{m,t+1}). \end{aligned}$$

$\text{Cov}_t$  refers to the conditional covariance, and  $z_{m,t+1}$ , which is defined to equal  $\sum_{j=1}^n z_{j,t+1} \lambda_j$ , is the excess return on the market portfolio. So, we can write

$$(7) \quad E_t z_{i,t+1} = \rho \text{Cov}_t(z_{i,t+1}, z_{m,t+1}) - \text{var}_t(z_{i,t+1})/2.$$

Compare this to Campbell's equation (25) for the general intertemporal model:

$$(8) \quad E_t z_{i,t+1} = \rho \text{Cov}_t(z_{i,t+1}, z_{m,t+1}) - \text{var}_t(z_{i,t+1})/2 + (\rho - 1)V_{ih,t}$$

where

$$V_{ih,t} \equiv \text{Cov}_t(z_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j z_{m,t+j+1}).$$

$\beta$  is the discount factor for consumers' utility. Campbell's equation is derived assuming that  $\Omega_t$  is constant over time, but Restoy (1992) has shown that equation (8) holds even when variances follow a GARCH process.

Clearly the only difference between the mean-variance model of equation (7) and the intertemporal model is the term  $(\rho - 1)V_{ih,t}$ . This term does not appear in the simple mean-variance model because it involves an evaluation of the distribution of returns more than one period into the future. Extending the empirical model to include the intertemporal term is potentially important, but difficult and left to future research. However, note that Restoy (1992) finds that the mean-variance model is able to "explain the overwhelming majority of the mean and the variability of the equilibrium portfolio weights" in a simulation exercise.<sup>4</sup>

### 3.3 The Empirical Model

The easiest way to understand the CASE method of estimating CAPM is to rewrite equation (5) so that it is expressed as a model that determines expected returns:

4. I would like to thank Geert Bekaert for pointing out an error in this section in the version of the paper presented at the conference.



$$(9) \quad E_t z_{t+1} = -D_t + (\mu_t^j \rho_J^{-1} + \mu_t^g \rho_G^{-1} + \mu_t^u \rho_{US}^{-1})^{-1} [\Omega_t \lambda_t - \mu_t^g (1 - \rho_G^{-1}) \Omega_t e_3 - \mu_t^j (1 - \rho_J^{-1}) \Omega_t e_5].$$

Under rational expectations, the actual value of  $z_{t+1}$  is equal to its expected value plus a random error term:

$$z_{t+1} = E_t z_{t+1} + \varepsilon_{t+1}.$$

The CASE method maximizes the likelihood of the observed  $z_{t+1}$ . Note that when equation (9) is estimated, the system of five equations incorporates cross-equation constraints between the mean and the variance.

There are four versions of the model estimated here:

#### Model 1

This version estimates all of the parameters of equation (9)—the three values of  $\rho$ , and the parameters of the variance matrix,  $\Omega_t$ . It is the most general version of the model estimated. It allows investors across countries to differ not only in the currency of denomination in which they evaluate returns, but also their degree of risk aversion.

#### Model 2

Here we constrain  $\rho$  to be equal across countries. Then, using equation (9), we can write

$$(10) \quad E_t z_{t+1} = -D_t + \rho \Omega_t \lambda_t + \mu_t^g (1 - \rho) \Omega_t e_3 + \mu_t^j (1 - \rho) \Omega_t e_5.$$

#### Model 3

Here we assume  $\mu^i$  is constant over time for each of the three countries. We do not use data on  $\mu^i$ , and instead treat the wealth shares as parameters. Since our measures of wealth shares may be unreliable, this is a simple alternative way of “measuring” the shares of wealth. However, in this case, neither the  $\mu^i$  nor the  $\rho_i$  is identified. We can write equation (24) as

$$(11) \quad E_t z_{t+1} = -D_t + \alpha \Omega_t \lambda_t - \gamma_1 \Omega_t e_3 - \gamma_2 \Omega_t e_5.$$

The parameters to be estimated are  $\alpha$ ,  $\gamma_1$ ,  $\gamma_2$ , and the parameters of  $\Omega_t$ . In the case in which the degree of risk aversion is the same across countries,  $\alpha$  is a measure of the degree of risk aversion.

#### Model 4

The last model we consider abandons the assumption of investor heterogeneity and assumes that all investors are concerned only with dollar returns. So we can use equation (2) to derive the equation determining equilibrium expected returns under these assumptions. We presented this model in section 3.2 as equation (6) and repeat it here for convenience:

$$(12) \quad E_t z_{t+1} = \rho \Omega_t \lambda_t - D_t.$$

The mean-variance optimizing framework yields an equilibrium relation between the expected returns and the variance of returns, such as in equation (9). However, the model is not completely closed. While the relation between means and variances is determined, the level of the returns or the variances is not determined within the model. For example, Harvey (1989) posits that the expected returns are linear functions of data in investors' information set. The equilibrium condition for expected returns would then determine the behavior of the covariance matrix of returns. Our approach takes the opposite tack. We specify a model for the covariance matrix, and then the equilibrium condition determines the expected returns.

Since the mean-variance framework does not specify what model of variances is appropriate, we are free to choose among competing models of variances. Bollerslev's (1986) GARCH model appears to describe the behavior of the variances of returns on financial assets remarkably well in a number of settings, so we estimate a version of that model.

Our GARCH model for  $\Omega_t$  follows the positive-definite specification in Engel and Rodrigues (1989):

$$(13) \quad \Omega_t = P'P + G\varepsilon_t\varepsilon_t'G + H\Omega_{t-1}H.$$

In this equation,  $P$  is an upper triangular matrix, and  $G$  and  $H$  are diagonal matrices.

This is an example of a multivariate GARCH(1,1) model: the covariance matrix at time  $t$  depends on one lag of the cross-product matrix of error terms and one lag of the covariance matrix. In general,  $\Omega_t$  could be made to depend on  $m$  lags of  $\varepsilon_t\varepsilon_t'$  and  $n$  lags of  $\Omega_t$ . Furthermore, the dependence on  $\varepsilon_t\varepsilon_t'$  and  $\Omega_{t-1}$  is restrictive. Each element of  $\Omega_t$  could more generally depend independently on each element of  $\varepsilon_t\varepsilon_t'$  and each element of  $\Omega_{t-1}$ . However, such a model would involve an extremely large number of parameters. The model described in equation (27) involves the estimation of twenty-five parameters—fifteen in the  $P$  matrix and five each in the  $G$  and  $H$  matrices.

### 3.4 Results of Estimation

The estimates of the models are presented in tables 3.2–3.5.

The first set of parameters reported in each table are the estimates of the risk aversion parameter. Model 1 allows the degree of risk aversion to be different across countries. The estimates for  $\rho_{US}$ ,  $\rho_G$ , and  $\rho_J$  reported in table 3.2 are not very economically sensible. Two of the estimates are negative. The mean-variance model assumes that higher variance is less desirable, which implies that  $\rho$  should be positive.

Furthermore, we can test the hypothesis that the  $\rho$  coefficients are equal for all investors against the alternative of table 3.2 that they are different. This can

**Table 3.2** GARCH-CAPM Model with Rho Different across Countries (model 1)

Rho (United States, Germany, Japan)				
-1.3565e-07	-3.2690e-07	3.7562e-08		
P'P matrix				
0.00059049	0.00016524	-1.7010e-05	0.00020655	-2.9160e-05
0.00016524	0.00031849	0.00016684	0.00026570	0.00016344
-1.7010e-05	0.00016684	0.00021890	0.00013559	0.00017095
0.00020655	0.00026570	0.00013559	0.00068145	0.00029210
-2.9160e-05	0.00016344	0.00017095	0.00029210	0.00028625
Diagonal elements of G matrix				
-0.024400	0.14830	0.19370	0.42910	0.35240
Diagonal elements of H matrix				
0.84760	0.95300	0.90130	0.82470	0.82200
Log-likelihood value				
2469.47942				

**Table 3.3** GARCH-CAPM Model with Rho Equal across Countries (model 2)

Rho				
4.6540				
P'P matrix				
0.00054260	0.00014092	-2.0086e-05	0.00018704	-3.4940e-05
0.00014092	0.00028210	0.00016825	0.00026609	0.00018164
-2.0086e-05	0.00016825	0.00022854	0.00013989	0.00018902
0.00018704	0.00026609	0.00013989	0.00068100	0.00031419
-3.4940e-05	0.00018164	0.00018902	0.00031419	0.00031561
Diagonal elements of G matrix				
-0.039763	0.087661	0.17620	0.40677	0.37041
Diagonal elements of H matrix				
0.86051	0.96511	0.90048	0.83463	0.80805
Log-likelihood value				
2467.45718				

be easily done with a likelihood ratio test, since table 3.3 estimates the constrained model. The value of the  $\chi^2$  test with two degrees of freedom is 4.056. The 5 percent critical value is 5.91, so we cannot reject the null hypothesis of equal values of  $\rho$  at this level.

In fact, the likelihood value for Model 1 is not as dependent on the actual values of the  $\rho$ s as it is on their relative values. If we let  $\rho$  be different across countries, we are unable to reject some extremely implausible values. For example, we cannot reject  $\rho_{US} = 1414$ ,  $\rho_G = 126$ , and  $\rho_J = 1.6$ .

Based both on the statistical test and the economic plausibility of the estimates, the restricted model—Model 2—is preferred to Model 1. Table 3.3 shows that the estimate of  $\rho$  in Model 2 is 4.65. This is not an unreasonable estimate for the degree of relative risk aversion of investors. It falls within the range usually considered plausible. It is also consistent with the estimates from

**Table 3.4** GARCH-CAPM Model with Wealth Shares Constant (model 3)

Alpha				
4.03400				
Gamma				
-1.11955	0.739053			
P'P matrix				
0.000561370	0.000149152	-2.04214e-05	0.000190815	-3.82150e-05
0.000149152	0.000295907	0.000170993	0.000273723	0.000181881
-2.04214e-05	0.000170993	0.000232242	0.000145623	0.000192640
0.000190815	0.000273723	0.000145623	0.000701762	0.000321168
-3.82150e-05	0.000181881	0.000192640	0.000321168	0.000322825
Diagonal elements of G matrix				
-0.0363077	0.105138	0.179205	0.417467	0.366548
Diagonal elements of H matrix				
0.855632	0.961629	0.898721	0.826812	0.806122
Log-likelihood value				
2467.61284				

**Table 3.5** GARCH-CAPM Model in Dollar Terms (model 4)

Rho				
4.09263				
P'P matrix				
0.000559210	0.000147301	-2.12924e-05	0.000190461	-3.82462e-05
0.000147301	0.000292889	0.000170081	0.000272490	0.000181657
-2.12924e-05	0.000170081	0.000231566	0.000144383	0.000192179
0.000190461	0.000272490	0.000144383	0.000698603	0.000320088
-3.82462e-05	0.000181657	0.000192179	0.000320088	0.000322144
Diagonal elements of G matrix				
-0.0374155	0.101296	0.177600	0.416692	0.367108
Diagonal elements of H matrix				
0.856192	0.962377	0.899183	0.827584	0.806167
Log-likelihood value				
2467.51288				

models 3 and 4. Model 3—which treats the wealth shares as unobserved constants—estimates the degree of risk aversion to equal 4.03. (Recall when reading table 3.4 that the coefficient of risk aversion in Model 3 is the parameter  $\alpha$ .) When we assume all investors consider returns in dollar terms—as in Model 4—the estimate of  $\rho$  is 4.09, as reported in table 3.5.

Inspection of tables 3.1–3.4 shows that the parameters of the variance matrix,  $\Omega_t$ , are not very different across the models. The matrix  $P$  from equation (13) is what was actually estimated by the maximum likelihood procedure, but we report  $P'P$  in the tables because it is more easily interpreted.  $P'P$  is the constant part of  $\Omega_t$ .

The GARCH specification seems to be plausible in this model. Most of the elements of the  $H$  matrix were close to one, which indicates a high degree of persistence in the variance. One way to test GARCH is to perform a likelihood

ratio test relative to a more restrictive model of the variance. Table 3.6 reports the results of testing the GARCH specification against a simple ARCH specification in which the matrix  $H$  in equation (13) is constrained to be zero. This imposes five restrictions on the GARCH model. As table 3.6 indicates, the restricted null hypothesis is rejected at the 1 percent level for each of models 1–4.

Figures 3.1 and 3.2 plot the diagonal elements of the  $\Omega_t$  matrix for Model 2. The time series of the variances for the other models are very similar to the ones for Model 2. In figure 3.1 the variances of the returns on U.S., German, and Japanese equities relative to U.S. bonds are plotted. As can be seen, the variance of U.S. equities is much more stable than the variances for the other equities. In the GARCH model, the 1 – 1 element in both the  $G$  and  $H$  matrices is small in absolute value. This leads to the fact that the variance does not respond much to past shocks, and changes in the variance are not persistent. On the other hand, figure 3.1 shows us that toward the end of the sample the variance of Japanese equities fluctuated a lot and at times got relatively large. Recall that in measuring returns on Japanese and German equities relative to U.S. bonds a correction for exchange rate changes is made, while that is not needed when measuring the return on U.S. equities relative to U.S. bonds.

The variances of returns on German and Japanese bonds relative to the returns on U.S. bonds are plotted in figure 3.2. Interestingly, the variance of Japanese bonds fluctuates much more than the variance of German bonds. The variance is much more unstable near the beginning of the sample period (while the variance of returns on Japanese equities gyrated the most at the end of the sample).

Figures 3.3 and 3.4 plot the point estimates of the risk premia. These risk premia are calculated from the point of view of U.S. investors. The risk premia are the difference between the expected returns from equation (9) and the risk neutral expected return for U.S. investors, which is obtained from equation (6) setting  $\rho$  equal to zero.

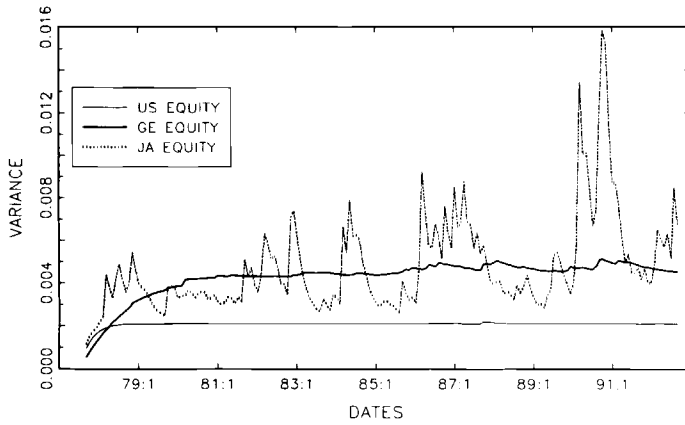
In some cases the risk premia are very large. (The numbers on the graph are the risk premia on a monthly basis. Multiplying them by 1200 gives the risk premia in percentage terms at annual rates.) The risk premia on equities are much larger than the risk premia on bonds. Furthermore, the risk premia vary a great deal over time. Comparing figure 3.3 to figure 3.1, it is clear that the risk premia track the variance of returns, particularly for the Japanese equity markets. The risk premia reached extremely high levels in 1990 on Japanese equities, which reflects the fact that the estimated variance was large in that year. The average risk premium on Japanese equities (in annualized rates of return) is 6.07 percent. For U.S. equities it is 5.01 percent, and 3.36 percent is the average risk premium for German equities.

The risk premia on equities are always positive, but in a few time periods the risk premia on the bonds are actually negative. The risk premia on bonds in this model are simply the foreign exchange risk premia. They also show

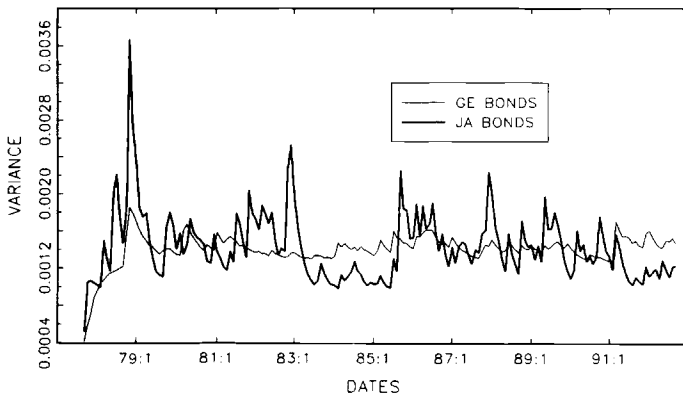
**Table 3.6** Test of Significance of GARCH Coefficients (likelihood ratio tests, 5 degrees of freedom)

Model	Chi-Square Statistic
Model 1	26.740
Model 2	24.174
Model 3	24.039
Model 4	24.050

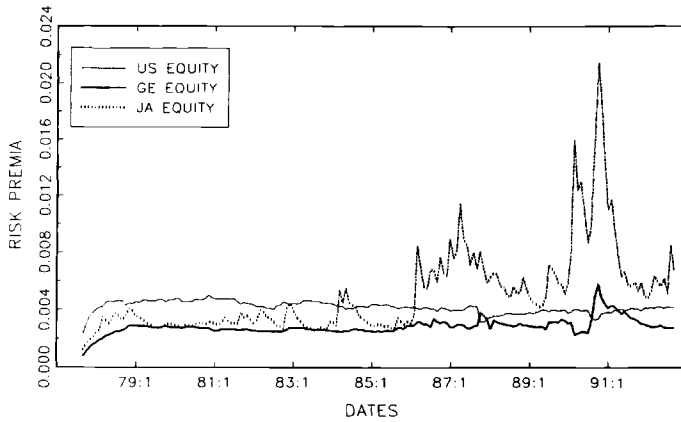
Note: All statistics significant at 1 percent level.



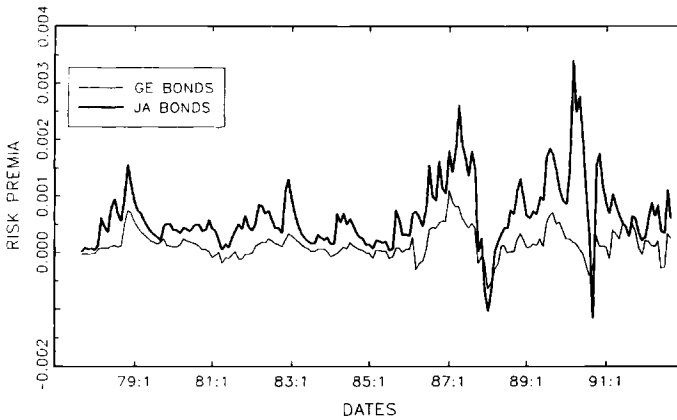
**Fig. 3.1** Variance of equity returns relative to U.S. bonds



**Fig. 3.2** Variance of bond returns relative to U.S. bonds



**Fig. 3.3 Risk premia on equity returns relative to U.S. bonds**



**Fig. 3.4 Risk premia of bond returns relative to U.S. bonds**

much time variation. At times they are fairly large, reaching a maximum of approximately four percentage points on the yen in 1990. Note, however, that the average risk premia—0.18 percent for German bonds and 0.79 percent for Japanese bonds—are an order of magnitude smaller than the equity risk premia.

However, figures 3.3 and 3.4 present only the point estimates of the risk premia, and do not include confidence intervals. The evidence in section 3.5 suggests that these risk premia are only marginally statistically significant.

### 3.5 Tests of the Null Hypothesis of Interest Parity

If investors perceive foreign and domestic assets to be perfect substitutes, then a change in the composition of asset supplies (as opposed to a change in

the total supply of assets) will have no effect on the asset returns. Suppose investors choose their portfolio only on the basis of expected return. In equilibrium, the assets must have the same expected rate of return. Thus, in equilibrium, investors are indifferent to the assets (the assets are perfect substitutes), and the composition of their optimal portfolio is indeterminate. A change in the composition does not affect their welfare, and does not affect their asset demands. Thus, sterilized intervention in foreign exchange markets, which has the effect of changing the composition of the asset supplies, would have no effect on expected returns.

In our model, investors in general are concerned with both the mean and the variance of returns on their portfolios. The case in which they are concerned only with expected returns is the case in which  $\rho$  equals zero. We shall test the null hypothesis that consumers care only about expected return and not risk.

Consider first the version of the model in which all investors have the same degree of risk aversion—Model 2. That is,  $\rho$  is the same across all three countries. Then, the mean-variance equilibrium is given by equation (10). If we constrain  $\rho$  to equal zero in that equation, then we have the null hypothesis of

$$(14) \quad E_t z_{t+1} = -D_t + \mu_i^3 \Omega_t e_3 + \mu_i^5 \Omega_t e_5.$$

Since the version of the model in which  $\rho$  is the same across all countries is a constrained version of the most general mean-variance model, then equation (14) also represents the null hypothesis for the general model (given in equation [9]).

We estimate two other versions of the mean-variance model. Model 3, as mentioned above, treats the shares of wealth as constant but unobserved. The model is given by equation (11). If  $\rho$  is the same for investors in all countries, then  $\alpha = \rho$ . So, the null hypothesis of risk neutrality can be written as

$$(15) \quad E_t z_{t+1} = -D_t + \gamma_1 \Omega_t e_3 + \gamma_2 \Omega_t e_5.$$

The final version of the mean-variance model that we estimate is the one in which all investors evaluate returns in dollar terms—Model 4. Equation (12) shows the equation for equilibrium expected returns in this case. The null hypothesis then, is simply

$$(16) \quad E_t z_{t+1} = -D_t.$$

So, equation (14) is the null hypothesis for Model 1 and Model 2, equation (15) is the null for Model 3, and equation (16) is the null for Model 4.

However, we have finessed a serious issue for the models in which investors assess asset returns in terms of different currencies. If investors are risk neutral, they require that expected returns expressed in terms of their domestic currency be equal. However, if expected returns are equal in dollar terms, then they will not be equal in yen terms or mark terms unless the exchange rates are constant. This is simply a consequence of Siegel's (1972) paradox (see Engel 1984, 1992 for a discussion).



The derivation of equation (9) does not go through when investors in one or more countries are risk neutral. The derivation proceeded by calculating the asset demands, adding these across countries, and equating asset demands to asset supplies. However, when investors are risk neutral, their asset demands are indeterminate. If expected returns on the assets (in terms of their home currency) are different from each other, they would want to take an infinite negative position in assets with lower expected returns and an infinite positive position in assets that have higher expected returns. If all assets have the same expected returns, then they are perfect substitutes, so the investor will not care about the composition of his portfolio. Hence, the derivation that uses the determinate asset demands when  $\rho \neq 0$  does not work when  $\rho = 0$ .

If investors in different countries are risk neutral, then there is no equilibrium in the model presented here. Since it is not possible for expected returns to be equalized in more than one currency, then investors in at least one country would end up taking infinite positions.

So, we will consider three separate null hypotheses for our mean-variance model. One is that U.S. residents are risk neutral, so that expected returns are equalized in dollar terms. The other two null hypotheses are that expected returns are equalized in mark terms and in yen terms. The first of three hypotheses is given by equation (16), which was explicitly the null hypothesis when all investors considered returns in dollar terms. The second two null hypotheses can be expressed as

$$(17) \quad E_t z_{t+1} = -D_t + \Omega_t e_3,$$

and

$$(18) \quad E_t z_{t+1} = -D_t + \Omega_t e_5.$$

Equations (16), (17), and (18) can represent alternative versions of the null hypothesis for models 1 and 3 (expressed in equations [9] and [11]). Model 4—the one in which investors consider returns in dollar terms—admits only equation (16) as a restriction.

The foregoing discussion suggests that the model in which  $\rho$  is restricted to be equal across countries will not have an equilibrium in which  $\rho = 0$ . However, we still will treat equation (14) as the null hypothesis for this model. Note that equation (14) is a weighted average of equations (16), (17), and (18), where the weights are given by the wealth shares. Equation (14) should be considered the limit as  $\rho$  goes to zero across investors. It is approximately correct when  $\rho$  is approximately zero. The same argument can be used to justify equation (15) as a null hypothesis for the model expressed in equation (11).

To sum up:

*Model 1.* The general mean-variance model given by equation (9) will be tested against the null hypotheses of equations (14), (16), (17), and (18).

*Model 2.* The mean-variance model in which  $\rho$  is restricted to be equal across countries, equation (10), will be tested against the null hypothesis of equation (14).

*Model 3.* The version of the mean-variance model in which the wealth shares are treated as constant—equation (11)—will be tested against the null hypotheses of equations (15), (16), (17), and (18).

*Model 4.* The version of the mean-variance model in which investors evaluate assets in dollar terms, given by equation (12), will be tested against the null hypothesis of equation (16).

The results of these tests are reported in table 3.7. The null hypothesis of perfect substitutability of assets is not rejected at the 5 percent level for any model.

All but equation (18) can be rejected as null hypotheses at the 10 percent level when Model 1 is the alternative hypothesis. The  $p$ -value in all cases is close to 0.10, so there is some weak support for Model 1 against the null of risk neutrality.

For Model 2, the  $p$ -value is about 0.12. Since we were unable to reject the null that the coefficient of risk aversion was equal across countries, it is not surprising that models 1 and 2 have about equal strength against the null of risk neutrality.

It is something of a success that the estimated value of  $\rho$  is so close to being significant at the 10 percent level. There are many tests which reject the perfect substitutability, interest parity model. But, none of these tests that reject perfect substitutability are nested in a mean-variance portfolio balance framework. For example, Frankel (1982), who does estimate a mean-variance model, finds that if he restricts his estimate of  $\rho$  to be nonnegative, the maximum likelihood estimate of  $\rho$  is zero. Clearly, then, he would not reject a null hypothesis of  $\rho = 0$  at any level of significance.

Our model performs better than Frankel's because we include both equities and bonds, and because we allow a more general model of  $\Omega_i$ .<sup>5</sup>

Model 3 is unable to reject the null of perfect substitutability at standard levels of significance.

Model 4 rejects perfect substitutability at the 10 percent level. It might seem interesting to test the assumptions underlying Model 4. That is, does Model 4, which assumes that investors assess returns in dollar terms, outperform Model 2, which assumes that investors evaluate returns in their home currency? Unfortunately, Model 4 is not nested in Model 1 or Model 2, so such a test is not possible.

Model 4 is nested in Model 3, the model which treats the wealth shares as constant and unobserved. Comparing equation (11) with equation (12), the restrictions that Model 4 places on Model 3 are that  $\gamma_1 = 0$  and  $\gamma_2 = 0$ . The likelihood-ratio (LR) test statistic for this restriction is distributed  $\chi^2$  with two degrees of freedom. The value of the test statistic is 0.200, which means that the null hypothesis is not rejected. So we cannot reject the hypothesis that all investors evaluate returns in dollar terms. However, equation (11) is not a very strong version of the model in which investors evaluate returns in terms of

5. Frankel assumes  $\Omega_i$  is constant.

**Table 3.7** LR Tests of Null Hypothesis of Perfect Substitutability: Chi-Square Statistics (*p*-value in parentheses)

Model	Null 14	Null 15	Null 16	Null 17	Null 18
Model 1	6.433 (0.092)		6.714 (0.082)	6.719 (0.081)	5.955 (0.114)
Model 2	2.388 (0.122)				
Model 3		1.640 (0.200)	2.981 (0.395)	2.986 (0.394)	2.222 (0.528)
Model 4			2.781 (0.095)		

different currencies. It does not use the data on shares of wealth, and treats those shares as constants. It performs the worst of all the models against the null of perfect substitutability. So we really cannot decisively evaluate the merits of allowing investor heterogeneity.

### 3.6 Tests of CAPM against Alternative Models of the Risk Premia

The CASE method of estimating the CAPM is formulated in such a way that it is natural to compare the asset demand functions from CAPM with more general asset demand functions. Unlike many other tests of CAPM, the alternative models have a natural interpretation and can provide some guidance on the nature of the failure of the mean-variance model if CAPM is rejected.

Any asset demand function that nests the asset demand functions derived above—equations (2), (3), and (4)—can serve as the alternative model to CAPM. That means, practically speaking, that the only requirement is that asset demands depend on expected returns with time-varying coefficients. Thus, in principle, we could use the CASE method to test CAPM against a wide variety of alternatives—models based more directly on intertemporal optimization, models based on noise traders, etc.

In practice, because of limitations on the number of observations of returns and asset supplies, it is useful to consider alternative models that do not have too many parameters. This can be accomplished by considering models which are similar in form to CAPM, but do not impose all of the CAPM restrictions.

Thus, initially, we consider models in which the asset demand equations in the three countries take exactly the form of equations (2), (3), and (4), except that the coefficients on expected returns need not be proportional to the variance of returns. We will test only the version of CAPM in which the degree of risk aversion is assumed to be equal across countries. For that version, we can write the alternative model as

$$(19) \quad \lambda_t^u = A_t^{-1} (E_t z_{t+1} + D_t)$$

$$(20) \quad \lambda_t^g = A_t^{-1} (E_t z_{t+1} + D_t) + a_G e_3$$

$$(21) \quad \lambda_t^i = A_t^{-1} (E_t z_{t+1} + D_t) + a_j e_5.$$

In the alternative model, asset demands are functions of expected returns, but the coefficients,  $A_t^{-1}$ , are not constrained to be proportional to the inverse of the variance of returns. As with the Solnik model, we assume in the alternative that the portfolios of investors in different countries differ only in their holdings of nominal bonds.

Aggregating across countries gives us

$$\lambda_t = A_t^{-1} (E_t z_{t+1} + D_t) + \mu_t^g a_G e_3 + \mu_t^j a_J e_5.$$

This can be rewritten as

$$(22) \quad E_t z_{t+1} = -D_t + A_t \lambda_t - \mu_t^g a_G A_t e_3 - \mu_t^j a_J A_t e_5.$$

The matrix of coefficients,  $A_t$ , is unconstrained. However, for formal hypothesis testing, it is useful if model (10) is nested in model (22). So, we hypothesize that  $A_t$  evolves according to

$$(23) \quad A_t = Q'Q + J\varepsilon_t\varepsilon_t'J + KA_{t-1}K.$$

We will assume that the variance of the error terms in the alternative model follows a GARCH process as in equation (13).

Thus, the CAPM described in equations (10) and (13) imposes the following restrictions on the alternative model described by equations (22), (23), and (13):

$$\begin{aligned} \rho^{1/2}P &= Q \\ \rho^{1/2}G &= J \\ \rho^{1/2}H &= K, \text{ and} \\ \rho^{-1} - 1 &= a_G = a_J. \end{aligned}$$

So, CAPM places twenty-six restrictions on the alternative model.

The alternative model was estimated by maximum likelihood methods. The value of the log of the likelihood is 2481.9026. Thus, comparing this likelihood value with the one given in table 3.3, the test statistic for the CAPM is 28.89. This statistic has a chi-square distribution with twenty-six degrees of freedom. The  $p$ -value for this statistic is 0.316, which means we would not reject CAPM at conventional levels of confidence.

We now consider two generalizations of equation (22). In the first, we posit that the asset demands do not depend simply on the expected excess returns,  $E_t z_{t+1} + D_t$ . Instead, there may be a vector of constant risk premia,  $c$ , so that we replace equation (22) with

$$(24) \quad E_t z_{t+1} = c - D_t + A_t \lambda_t - \mu_t^g a_G A_t e_3 - \mu_t^j a_J A_t e_5.$$

CAPM places thirty-one restrictions on this alternative—those listed above, and the restriction that  $c = 0$ . This test of CAPM is directly analogous to the

tests for significant “pricing errors” in, for example, Gibbons, Ross, and Shanken (1989) and Ferson and Harvey (chap. 2 in this volume).

This model was also estimated by maximum likelihood methods. The value of the log of the likelihood is 2482.6712. The chi-square statistic with thirty-one degrees of freedom is 30.428, which has a  $p$ -value of 0.495. So, again, we would not reject CAPM.

Another alternative is to retain equation (19) to describe asset demand by U.S. residents, but to replace equations (20) and (21) with

$$(25) \quad \lambda_i^g = A_i^{-1}(E_i z_{i+1} + D_i) + a_G$$

$$(26) \quad \lambda_i^j = A_i^{-1}(E_i z_{i+1} + D_i) + a_J.$$

In these equations,  $a_G$  and  $a_J$  are vectors. These equations differ from (20) and (21) by allowing more investor heterogeneity across countries. Each of the portfolio shares may differ between investors across countries—rather than just the bond holdings as in the Solnik model and in the alternative given by equation (22). Thus, aggregating equations (19), (25), and (26), and rewriting in terms of expected returns, we get

$$(27) \quad E_i z_{i+1} = -D_i + A_i \lambda_i - \mu_i^g A_i a_G - \mu_i^j A_i a_J.$$

The CAPM model places thirty-four restrictions on equation (27). The model of equation (27) was estimated using maximum likelihood techniques. When the vectors  $a_G$  and  $a_J$  were left unconstrained, the point estimates of the portfolio shares were implausible. So, the model was estimated constraining the elements of  $a_G$  and  $a_J$  to lie between  $-1$  and  $1$ . This restriction is arbitrary, and is not incorporated in the optimization problems of agents, but it yields somewhat more plausible estimates of the optimal portfolio shares.

The value of the log of the likelihood in this case was 2499.8842. This gives us a chi-square statistic (thirty-four degrees of freedom) of 64.854. The  $p$ -value for this statistic is .0011. We reject CAPM at the 1 percent level.

So, we reject CAPM precisely because the Solnik model does not allow enough diversity across investors in their holdings of equities. However, it would not be correct to conclude that a model that has home-country bias in both equities and bonds outperforms the Solnik model. That is because our estimates of  $a_G$  and  $a_J$  are not consistent with home-country bias.

The vectors  $a_G$  and  $a_J$  represent the constant difference between the shares held by Americans on the one hand, and by Germans and Japanese, respectively, on the other. Our estimate of  $a_G$  shows that Germans would hold the fraction 0.23857 *more* of their portfolio in U.S. equities than Americans. Furthermore, they would hold  $-1.0$  less of German equities, and  $-1.0$  less of Japanese equities than Americans. On the other hand, there would be home bias in bond holdings—they would hold  $1.0$  more of German bonds. They would hold  $-0.31843$  less of Japanese bonds, but they would hold 2.07988 more of U.S. bonds. (Recall that U.S. bonds are the residual asset. So, while

the estimation constrained the elements of  $a_c$  to lie between  $-1$  and  $1$ , the difference between the share of U.S. bonds held by Germans and Americans is not so constrained.)

Likewise, the estimated difference between the Japanese and American portfolio is not indicative of home-country bias in equity holdings. While we do estimate that Japanese hold  $-1.0$  less of U.S. equities than Americans, they also hold less of both German and Japanese equities. The difference between the American and Japanese share of German equities is very small:  $-0.00047$ , and of Japanese equities,  $-0.06301$ . But Japanese are also estimated to hold smaller shares of German bonds and Japanese bonds, the differences being  $-0.20343$  and  $-0.12084$ , respectively. But Japanese are estimated to hold much more of American bonds. The difference in the portfolio shares is  $2.38784$ .

So, in fact, a general asset demand model that allows for diversity in equity holdings can significantly outperform CAPM. But the failure of CAPM is not due to the well-known problem of home-country bias in equity holdings.

### 3.7 Conclusions

There are three main conclusions to be drawn from this paper.

First, the version of international CAPM presented here performs better than many versions estimated previously. Section 3.5 shows that the model has some weak power in predicting excess returns, whereas almost all previous studies have found that international versions of CAPM have little or no power. The models presented here differ from past models by allowing a broader menu of assets—equities and bonds—and by allowing some investor heterogeneity.

Second, the version of CAPM estimated here—the Solnik model—does not allow for enough investor heterogeneity. Section 3.6 presents a number of tests of CAPM against alternative models of asset demand. The alternative models do not impose the constraint between means of returns and variances of returns that is the hallmark of the CAPM.

Some of these alternative models do not significantly outperform CAPM. Specifically, CAPM cannot be rejected in favor of models which still impose the Solnik result—that portfolios of investors in different countries differ in their bond shares but not their equity shares. But the alternative models need not impose the Solnik result. So, when the alternative model is generalized so that it does not impose the CAPM constraint between means and variances, and does not impose the Solnik result, CAPM is rejected.

The third major conclusion regards the usefulness of the CASE approach to testing the CAPM. In the CASE method, the alternative models are all built up explicitly from asset demand functions. In section 3.6, we considered several different models of asset demands. In each case, we built an equilibrium model

from those asset demand functions that served as an alternative to CAPM. In some of the cases, we were not able to reject CAPM. But when we altered our model of asset demand in a plausible way, we arrived at an equilibrium model which rejects CAPM. The advantage of the CASE approach is that we know very explicitly the economic behavior behind the alternative equilibrium models. When we fail to reject CAPM, we realize that it is not because CAPM is an acceptable model, but because the alternative model is as unacceptable as CAPM. When we reject CAPM, we know precisely the nature of the alternative model that is better able to explain expected asset returns. In this case, we have learned that CAPM must be generalized in a way to allow cross-country investor heterogeneity in equity demand. Perhaps incorporating capital controls or asymmetric information into the CAPM will prove helpful, but this is left for future work.

## Appendix

### **Foreign Exchange Rate**

The foreign exchange rates that were used in calculating rates of return and in converting local currency values into dollar values were taken from the data base at the Federal Reserve Bank of New York. They are the 9 A.M. bid rates from the last day of the month.

### **Equity Data**

The value of outstanding shares in each of the three markets comes from monthly issues of Morgan Stanley's *Capital International Perspectives*. These figures are provided in domestic currency terms. I thank William Schwert for pointing out that these numbers must be interpreted cautiously because they do not correct adequately for cross-holding of shares, a particular problem in Japan.

The return on equities in local currency terms was taken from the same source. The returns are on the index for each country with dividends reinvested.

### **Bond Data**

The construction of the data on bonds closely follows Frankel (1982). For each country, the cumulative foreign exchange rate intervention is computed, on a benchmark of foreign exchange holdings in March 1973. That cumulative foreign exchange intervention is added to outstanding government debt, while foreign government holdings of the currency are subtracted.

**Germany**

$$dmasst = dmdebt + bbint - ndmcb$$

dmdebt = German central government debt excluding social security contributions. Bundesbank Monthly Report, table VII.

dbbint = (DM/\$ exchange rate, IFS line ae)  $\times$  ( $\Delta$ foreign exchange holdings, IFS line 1dd + SDR holdings, IFS line 1bd + Reserve position at IMF, IFS line cd - (SDR Holdings + Reserve position at IMF)<sub>t-1</sub>  $\times$  (\$/SDR)/(\$/SDR)<sub>t-1</sub>, IFS line sa -  $\Delta$ SDR allocations, IFS line 1bd  $\times$  (\$/SDR))

$$bbint - \sum dbbint + 32.324 \times (DM/\$)_{1973:3}$$

ndmcb is derived from the International Monetary Fund (IMF) Annual Report.

**Japan**

$$ynasst = yndebt + bjint - njncb$$

yndebt =  $\sum$  Japanese central government deficit interpolated monthly, Bank of Japan, Economic Statistics Monthly, table 82.

dbjint = (Yen/\$ exchange rate, IFS line ae)  $\times$  ( $\Delta$ foreign exchange holdings, IFS line 1dd + SDR holdings, IFS line 1bd + Reserve position at IMF, IFS line cd - (SDR Holdings + Reserve position at IMF)<sub>t-1</sub>  $\times$  (\$/SDR)/(\$/SDR)<sub>t-1</sub>, IFS line sa -  $\Delta$ SDR allocations, IFS line 1bd  $\times$  (\$/SDR))

$$bjint - \sum dbjint + 18.125 \times (Yen/\$)_{1973:3}$$

njncb is derived from the IMF Annual Report.

**United States**

$$doasst = dodebt + fedint - ndolcb$$

dodebt = Federal debt, month end from Board of Governors Flow of Funds - carter

carter = 1.5952 in 78:12 and 1.3515 in 79:3, the Carter bonds

dfedint =  $\Delta$ foreign exchange holdings, IFS line 1dd + SDR holdings, IFS line 1bd + Reserve position at IMF, IFS line cd - (SDR Holdings + Reserve position at IMF)<sub>t-1</sub> -  $\Delta$ SDR allocations, IFS line 1bd  $\times$  (\$/SDR))

$$fedint = \sum dfedint + 14.366$$

ndolcb is derived from the IMF Annual Report.

**Wealth Data**

Outside wealth is measured by adding government debt and the stock market value to the cumulated current account surplus on Frankel's benchmark wealth.

The monthly current account is interpolated from IFS line 77ad.

**Interest Rates**

Interest rates are one-month Eurocurrency rates obtained from the Bank for International Settlements tape.



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## Comment G. William Schwert

### Introduction

I would like to begin by thanking Charles Engel for clearly listing the major limitations of this paper in his introduction. Moreover, throughout the paper he is clear to explain how his assumptions lead to the model formulations he tests. Overall, this is a paper that is easy to read and understand. Readers who are not familiar with the past work of Jeffrey Frankel on the use of portfolio share information to test the capital asset pricing model (CAPM) will find Engel's explanation and application of this model to be a useful introduction to this line of research.

Engel uses monthly data on rates of return to aggregate portfolios of stocks and bonds for three countries: the United States, Germany, and Japan (I believe the sample period is 1977–92, but it is never explicitly mentioned in the paper). He also uses data on the aggregate value of the stocks and bonds from these countries in some of his tests to impose constraints on his specification of the CAPM (this is the general strategy followed by Frankel 1982 and several subsequent authors).

As a small point, it is worth noting that care must be taken in using the Morgan Stanley *Capital International Perspectives* data on the aggregate value of country stock markets because they do not adjust for interfirm holdings of stock. Particularly in Japan, this problem leads to substantial overestimation of aggregate value (see, for example, French and Poterba 1991).

To model the conditional covariance matrix of the six assets, Engel uses a special form of the multivariate GARCH(1,1) model which assures that the covariance matrix is positive definite. Consistent with much prior work in this area, Engel finds that there is persistence in the conditional variance of returns to stocks and bonds.

Engel does not assume that purchasing power parity (PPP) holds, and he assumes that home country consumption prices are certain, so he uses the Solnik 1974 international CAPM. The unusual prediction from this model is that investors will hold differential amounts of bonds, but the same basket of equities. He relaxes this assumption in some of the empirical tests in section 3.6 by allowing domestic equity shares to differ from conventional CAPM predictions by a constant for Germany and Japan. Not surprisingly, Engel finds that these additional parameters are reliably different from zero (perhaps showing the type of home-country bias that has been found in many other papers).

Engel finds several other results that are not surprising. For example, he is unable to get a precise estimate of the risk aversion parameter  $\rho$ . Also, there is relatively weak power to discriminate among the competing models he consid-

ers in sections 3.5 and 3.6. In particular, there is not much evidence of a positive risk premium. This result is not surprising. As noted by Merton (1980), long time series are needed to derive precise estimates of unconditional expected stock returns because the volatility of stock returns is so large. Using only sixteen years of data (a limitation that is due largely to the desire to have data on all three countries' stock and bond markets) means that it is virtually impossible for Engel to get precise estimates of risk premiums.

## **How Does This Paper Relate to International Finance?**

### **What Makes International Finance Different?**

There are several things that distinguish *international* finance from the broad finance literature. First, there are potentially unusual consumption preferences or opportunities that differ systematically across groups of investors sorted by their home country. Often, regulation, taxes, or transaction costs may cause particular country investors to want to hold unusual portfolios of tradable securities compared with the predictions of typical portfolio models (e.g., the standard CAPM that is the benchmark in this paper).

Constraints on the investment opportunity set facing investors in different countries provide a second reason to focus on international finance. Nontraded assets, such as human capital, are obvious reasons why investors might choose different asset allocations in different countries (see Baxter and Jermann 1993). As discussed in the paper by Campbell and Froot (chap. 6 in this volume), market microstructure differences, such as securities transaction taxes, could also alter asset allocations (see also Schwert and Seguin 1993).

Lacking unusual consumption or investment opportunities that differ systematically across countries, the standard models of finance, such as the CAPM, should be equally as valid across countries as within countries. Therefore, Engel uses as his benchmark model the CAPM with integrated equity markets, but potentially segmented bond markets (because of the failure of PPP). As mentioned above, the data rejects even this form of the CAPM.

### **What Have We Learned about International Finance?**

While Engel's results are suggestive that international constraints or differences might alter asset demands, I am skeptical that this inference is robust. For example, if we could obtain data on the stock holdings of investors in Rochester, New York, San Jose, California, and Seattle, Washington, I suspect that we would find concentrations of Kodak, Xerox, or Bausch & Lomb; silicon valley companies; and Microsoft, Boeing, or Weyerhaeuser, respectively. These examples have no legal barriers to capital flows or sources of exchange rate or political risk that are usually relied on to explain "home-country bias" in international asset-pricing models.

What this suggests is that tests for "international effects" based on simple

models (e.g., the CAPM) should be calibrated using noninternational data. Given the many sources of evidence that the simple two-period CAPM is not adequate to describe risk-return relations within the set of NYSE-listed stocks (e.g., Fama and French 1993), it is not surprising that it also does not fit the data on portfolio shares.<sup>1</sup> The special implications about optimal portfolio shares from the CAPM used by Frankel (1982) and Engel are likely to be very sensitive to departures from the simple assumptions of the CAPM.

Moreover, if I were to identify a set of countries that are *least* likely to have significant artificial impediments to cross-country trading of financial assets, it would include the United States, Germany, and Japan (probably supplemented by the United Kingdom). Thus, if one were to think about the ideal experiment to identify some form of market segmentation due to international factors, it would not use data from the countries used by Engel.

Thus, while I believe that his empirical results reject the CAPM as an adequate joint description of the means and covariances of country portfolios of stocks, along with the implications for asset holdings, I doubt that this evidence tells us much about the factors that are peculiar to models of international asset pricing.

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1. Fama (1993) discusses the structure of efficient portfolios in the context of a multifactor asset-pricing model.

## Comment Geert Bekaert

In his excellent paper, Engel tests a version of the international CAPM using a technique that builds on the portfolio balance framework of Frankel (1982). The introduction of the paper contains a careful discussion of the limitations that this approach faces. Frankel, in his introduction to this volume, also provides an extensive discussion of the advantages and disadvantages of the approach followed by Engel relative to alternative approaches. To avoid merely reiterating the arguments given there, I will focus on just two issues.

I will start by reflecting on the nondynamic nature of the model and its likely implications for the ability of the model to explain time-variation in expected returns. In doing so, I will provide *empirical* estimates of the variability of risk premiums on equity and foreign exchange returns. The second issue concerns the presence of some form of investor heterogeneity in Engel's model. Since prices are fixed and exchange rate movements are random, the model effectively does not impose purchasing power parity (PPP). Although this "Solnik assumption" is a somewhat ad hoc way of introducing PPP deviations, it remains a fact that PPP is grossly violated in the data. Hence, the model is at least a priori more consistent with a world in which nominal and real exchange rates are very highly correlated than most theoretical models in which the PPP assumption assures that real expected returns are equalized across countries. Below I will attempt to put the importance of PPP deviations in a different perspective by examining empirically the relative importance of PPP deviations versus real interest differentials as determinants of risk premiums in foreign exchange returns.

### Comparison with a Dynamic Model

Engel motivates the nondynamic nature of the model by comparing the expected return equation to a reduced form equation in a recent paper by Campbell (1993), which is reproduced here:

$$(1) \quad E_t z_{i,t+1} = \rho \text{cov}_t(z_{i,t+1}, z_{m,t+1}) - \frac{\text{var}_t(z_{i,t+1})}{2} + (\rho - 1)V_{ih,t}.$$

The first term is the standard covariance with the market portfolio return with  $\rho$  being the coefficient of risk aversion or the price of risk; the second term is a Jensen's inequality term which would be present under risk neutrality as well. Both are present in Engel's framework. The last term is the "dynamic term," which is not present in the model of this paper:

$$(2) \quad V_{ih,t} \equiv \text{cov}_t(z_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^j z_{m,t+1+j}).$$

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A positive correlation between the asset return and news about future returns on the market increases expected returns when agents are more risk averse than log-utility but lowers expected returns for less risk averse ( $\rho < 1$ ) agents.

Numerous studies have shown that equity returns are predictable using various financial variables. The ultimate goal of this paper is to provide an explanation for this observed statistical predictability. What I will try to show below is that the model here is very far from explaining the observed time-variation of expected returns. The analysis in Campbell (1993) suggests that ignoring the dynamic term in (2) might be part of the problem.

Campbell assumes that the return on the market is the first element of a  $K$ -element state vector, which follows a vector autoregressive law of motion (VAR). This implies that the dynamic term follows a factor model with the risks factors being the correlations between the VAR's residuals and the asset return and the risk prices being nonlinear functions of the VAR parameters. Since financial variables with strong predictive power for the market return can be included in the VAR, the dynamic term can potentially account for a major fraction of the predictable variation in returns.

### The Empirical Variability of Risk Premiums

In what follows, I focus on the foreign exchange risk premium. It is well-known that, using the usual logarithmic approximations, this risk premium will be a component of the expected excess return on *any* foreign investment, that is,

$$(3) \quad [E_t(r_{t+1}) - i_t] \approx [E_t(s_{t+1}) - f_t] + [E_t(r_{t+1}^*) - i_t^*],$$

where  $r_{t+1}$  represents the return on Japanese or German equity in dollars,  $i_t$  ( $i_t^*$ ) is the U.S. (foreign) interest rate,  $s_t$  ( $f_t$ ) is the dollar per foreign currency spot (forward) rate, and  $r_{t+1}^*$  is the local currency equity return. The expected excess dollar return has two components: the risk premium in the forward market and the risk premium of Japanese or German equity returns over the Japanese or German nominal interest rate. In Bekaert and Hodrick (1992), we show that both components are predictable using variables such as dividend yields and forward premiums. Hence, explaining risk premiums on foreign equity investments requires an investigation of risk premiums in the forward market.

As is well-known, the puzzle in the foreign exchange market is the *variability* of the risk premium, not its mean. How can we empirically estimate the variability of this risk premium? One way is to project foreign exchange returns linearly on a number of forecasting variables and compute the standard deviation of the fitted value. This approach implicitly assumes that expectations are linear functions of the information set and that the full information set is comprised of the variables used in the projection. These assumptions are less damaging than they seem. First, adding other variables to the information set will only lead to *larger* estimates of the variability of the risk premium. Second, when nonlinear functions are approximated by polynomials, adding the

higher-order terms to the regression can likewise not decrease explanatory power. Hence, our estimate here can be considered a lower bound for the standard deviation of the risk premium.

Let  $Y_t = [\Delta s_t^1, \Delta s_t^2, \Delta s_t^3, fp_{t,n}^1, fp_{t,n}^2, fp_{t,n}^3]'$ , where  $\Delta s_t$  refers to exchange rate changes and  $fp$  is the forward premium (or interest differential) over  $n$  periods for  $n = 4, 13$ . The superscripts 1,2,3 stand for yen, mark, and pound exchange rates, all measured in dollars per foreign currency. I sample  $Y_t$  weekly, run a vector autoregression and use its dynamics to compute the implied risk premium for monthly and quarterly forward contracts as<sup>1</sup>

$$(4) \quad rp_{t,n}^i = E_t \left[ \sum_{j=1}^n \Delta s_{t+j}^i - fp_{t,n}^i \right] \quad i = 1, 2, 3.$$

In figure 3C.1, the implied monthly premiums are plotted for the 1975 to mid-1991 sample period. There is substantial time-variation in expected returns, although the unconditional mean is close to zero. Since the VAR is log-linear, it can be used to infer cross-rate expected returns. These returns are plotted in figure 3C.2. Finally, table 3C.1 reports the standard deviation and autocorrelations of the implied monthly and quarterly risk premiums.

Some striking facts emerge. First, the implied risk premiums are highly variable and very persistent despite the serial uncorrelatedness of actual returns. Second, the variability of the risk premium is of the order of 10 percent a year in all markets, including cross-rates. The well-known forward market puzzle is not a dollar phenomenon! The numbers I report are standard deviations and the unconditional mean is insignificantly different from zero; consequently, about 95 percent of the observations are between  $-20$  percent and  $20$  percent. Is Engel's model explaining this huge variability? From the graphs in Engel's paper, the largest risk premium that occurs, annualized, is smaller than 5 percent. Hence, the variability of the risk premium generated by the model is at least an order of magnitude smaller than what is observed in the data. The graphs also show that the estimated variability of equity premiums is larger, in particular for Japanese equity, but never are the model's risk premiums higher than 25 percent. Using similar methods, empirical estimates of the equity premium variability are of the order of 40 to 60 percent. Hence, looking at this implied moment gives a much more dramatic sense of the failure of the model to account for the data.

I do want to caution that the empirical results reported above are based on statistical estimates using data sets that have been used by numerous researchers, so that there might be problems of data snooping. Also, this time-variation in ex ante returns need not be interpreted as a "risk premium"; it could also be caused by persistent fads or it might partially reflect peso problems (see below).

1. For more details, see Bekaert (1993).

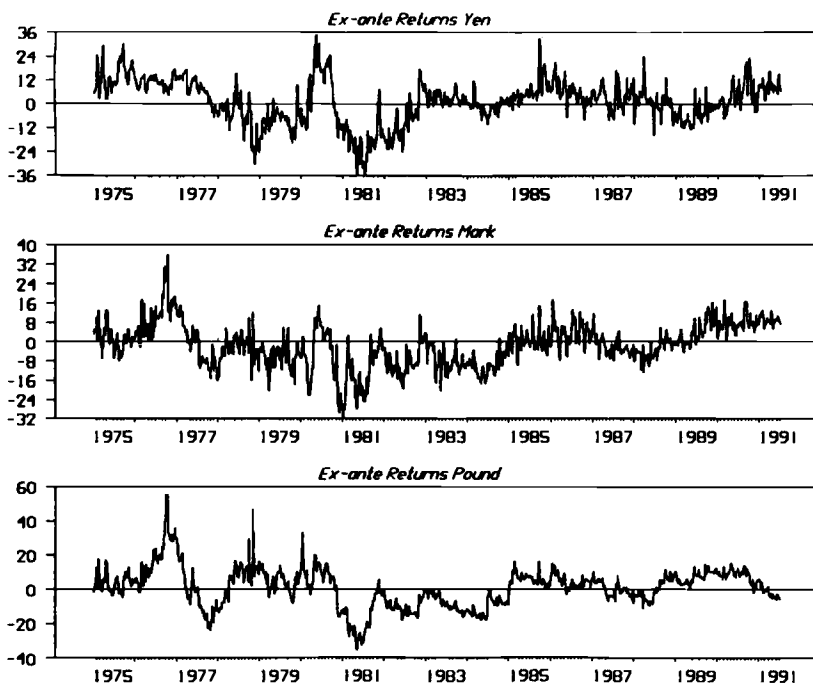


Fig. 3C.1 Ex ante forward market dollar returns

### The Importance of PPP Deviations

Consider the following decomposition of foreign exchange returns:

$$[s_{t+1} - f_t] = [s_{t+1} - s_t + i_t^* - i_t] \equiv [\Delta q_{t+1} + r_{t+1}^* - r_{t+1}],$$

where  $\Delta q_{t+1}$  are (logarithmic) real exchange rate changes,  $r_{t+1}^*$  ( $r_{t+1}$ ) the ex post real interest rate in the foreign country (the United States), defined as the nominal interest rate minus the inflation rate. The first equality follows from covered interest rate parity, the second from subtracting and adding relative inflation rates. Taking expectations, this equation decomposes the risk premium into expected real exchange rate changes and expected real interest differentials. In models based on PPP, real exchange rate changes are set to zero and the risk premium is totally driven by real interest rate differentials.

If the real exchange rate is martingale, that is,  $E_t[\Delta q_{t+1}] = 0$ , the PPP assumption would be a harmless simplifying assumption. Recent evidence in Cumby and Huizinga (1991), however, suggests that real exchange rate changes contain a substantial predictable component. Table 3C.2, reproduced from Bekaert (1994), offers an informal estimate of the relative contributions of expected real exchange rate changes and real interest differentials. To obtain



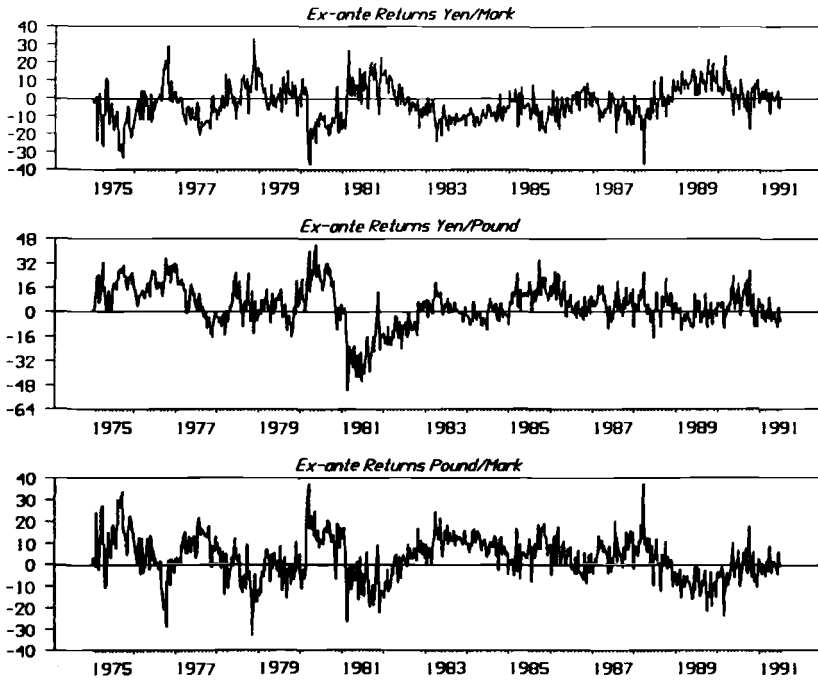


Fig. 3C.2 Ex ante forward market cross-rate returns

Table 3C.1 Properties of the Implied Risk Premiums: Sample Period: January 1975–July 1991 (weekly data)

A: Standard Deviations/Autocorrelations of Implied Risk Premiums						
	$rp_{t,4}^1$	$rp_{t,4}^2$	$rp_{t,4}^3$	$rp_{t,13}^1$	$rp_{t,13}^2$	$rp_{t,13}^3$
$\sigma$	11.125 (2.701)	9.199 (3.007)	11.513 (4.223)	9.923 (3.132)	7.788 (3.794)	10.350 (4.764)
$ac_1$	.887 (.062)	.872 (.090)	.935 (.035)	.959 (.019)	.959 (.030)	.965 (.017)
$ac_2$	.786 (.110)	.775 (.151)	.891 (.052)	.927 (.034)	.926 (.052)	.936 (.030)
$ac_3$	.743 (.125)	.740 (.165)	.851 (.065)	.902 (.044)	.904 (.063)	.908 (.042)
$ac_5$	.681 (.139)	.691 (.177)	.782 (.084)	.859 (.060)	.865 (.081)	.855 (.063)
$ac_{13}$	.528 (.153)	.557 (.194)	.567 (.133)	.720 (.110)	.738 (.134)	.679 (.125)

Table 3C.1 (continued)

B: Standard Deviations of Cross-Rate Risk Premiums						
	$rp_{t,A}^2 - rp_{t,A}^1$	$rp_{t,A}^3 - rp_{t,A}^1$	$rp_{t,A}^3 - rp_{t,A}^2$	$rp_{t,13}^2 - rp_{t,13}^1$	$rp_{t,13}^3 - rp_{t,13}^1$	$rp_{t,13}^3 - rp_{t,13}^2$
$\sigma$	9.773 (2.392)	12.133 (2.738)	7.575 (2.143)	8.138 (2.555)	10.663 (2.999)	5.423 (2.432)

Notes: The standard deviation ( $\sigma$ ) and autocorrelations ( $ac_{\sigma}$ ) of the risk premium (panel A) are computed from the dynamics of a second order VAR. The same methodology is used to compute the standard deviations of the cross-rate risk premiums in panel B. The exchange rate and interests rate data used are described in Bekaert (1993). The returns are annualized by multiplying by 1200 for monthly, by 400 for quarterly data.

Table 3C.2 **Relative Variability of Expected PPP Deviations and Real Interest Differentials for the Dollar/Yen Rate: Sample Period: 1972:02–1989:04 (quarterly data)**

	$\Delta s - fp$	$\Delta q$	$r^* - r$	$(\Delta q, r^* - r)$
$\sigma^2[\cdot]$	696.093	647.063	10.380	19.325
$\sigma^2[E_t[\cdot]]$	122.703	81.059	4.853	18.396

Notes: The symbols  $fb$ ,  $\Delta q$ , and  $r^* - r$  indicate, respectively, the (logarithmic) difference between the future spot and current forward rate, real exchange rate changes, and the real interest differential between the United States and Japan. The data are further described in Bekaert (1993).  $\sigma^2$  denotes the sample variance. The conditional expectation  $E_t[\cdot]$  is computed from the projection of the variable in the column onto a constant, lagged real exchange rate changes, the lagged forward premium, and the lagged inflation differential. The last column contains the sample covariance between (expected) real exchange rate changes and the (expected) real interest differential. All variables are multiplied by 400 to obtain annualized numbers.

estimates of expected values, I simply projected  $fb_{t+1} = s_{t+1} - f_t$ ,  $\Delta q_{t+1}$ , and  $r_{t+1}^* - r_{t+1}$  onto a number of information variables consisting of the lagged real exchange rate, the lagged forward premium, and the lagged inflation differential using quarterly data for the United States and Japan. The table reveals that the predictability of real interest differentials is indeed higher than the predictability of real exchange rates. However, the total variance of real interest differentials is dwarfed by the total variance of real exchange rate changes. According to the table, only 4 percent of the total variance of the risk premium can be fully accounted for by ex ante real interest differentials. Hence, models that rely on PPP should not hope to explain much of the variability of expected foreign exchange returns. Although the table reports results for the yen only, this result is robust to various periods, currencies, and information variables (see Gokey 1993). As a consequence, when Engel, unlike most of the literature, chose not to impose PPP, it was a very wise decision.

### Concluding Comments

First, I want to point out that all the computations here *and* Engel's model crucially rely on the assumption of *rational expectations*. It is conceivable that the estimates of the variance of the risk premiums, reported above, are grossly overestimated because of "peso problems." We do not have large enough samples to experience all of the events on which agents place prior probability, and to have sample moments converge to their population moments. More generally, when the true model of the world is evolving over time, *ex ante* expectations can appear biased *ex post*, although they were formed rationally. Several researchers have pointed out that market participants in foreign exchange markets seem to make systematic forecast errors. This is not surprising in a complex environment where agents, say, anticipate shifts in monetary policy, which do not occur frequently or never materialize. The recent currency turmoil in Europe, for example, has dramatized the importance of monetary policy in shaping the expectations of the major players in financial markets in general and currency markets in particular. Although some research is emerging that tries to address this issue (Engel and Hamilton 1990; Kaminsky 1993; and Evans and Lewis 1993), much more is needed.

Of more direct relevance to this paper, the idea that there are policy regime switches implies that the conditional variance process is likely to be severely misspecified. For instance, Cai (1992) shows that if a shift in the unconditional variance of an interest rate process is allowed for, the estimated GARCH coefficients change dramatically. Since the sample of this paper includes the 1979–82 monetary policy shift in the United States, and money market instruments are part of the assets considered, a different conditional variance specification is likely to substantially affect the maximum likelihood estimates of the model. After all, it is the time-variation of the second moments that largely drives expected returns in this model.

Finally, I want to recall the main result of the paper: the model is rejected only versus an alternative that allows for more diversity across investors in their holdings of equities. In particular, it is the diversity that makes people hold disproportionate amounts of their own equity that has the power to reject the model. This brings us back to the enormously hot topic of "home bias" in asset choice. What might cause this diversity? One obvious candidate is simply location and the differential-transactions cost that comes with buying assets in different markets, including informational costs. It is perhaps no coincidence that with increasingly faster and cheaper access to foreign stocks, partially through the evolution in telecommunications and computers, and with information flowing more frequently and faster throughout the world, portfolios have become more and more "international." Currently, we do not have very good economic models to think about these issues and Engel's paper has once again confirmed that this is where international economists should direct their research efforts.

The internationalization of equity markets might also be related to the dismantling of numerous regulatory barriers to investment over the last decade. Engel's model assumes full market integration, which might be a tenuous assumption, especially in the early part of the sample, when Japanese capital markets were subject to severe regulatory restrictions. Ideally, the model should allow for partial segmentation in the early part of the sample and integrated capital markets for the later part of the sample. Such a model with time-varying degrees of segmentation is developed in Bekaert and Harvey (1993).

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