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11 Retirement Annuity Design in an Inflationary Climate

Zvi Bodie and James E. Pesando

A pensioner who receives his benefit in the form of a nominal annuity has claim to a stream of payments whose nominal value is certain. The *real* value of this claim, however, will be highly uncertain if there exists substantial uncertainty regarding the future level of prices. Because this appears to be the case, and because pensioners are presumably concerned with the real value of their retirement incomes, the challenge of designing an annuity in an inflationary climate merits increased attention. We believe that at least some individuals may find alternatives to the nominal level-payment annuity better suited to their needs or preferences in an environment of substantial inflation uncertainty.[1] If individuals are to make rational choices, they must first understand the risk return and other characteristics of alternative annuity designs. Our primary objective in this chapter is to clarify these issues.

The first task is to examine the streams of real benefits that are likely to be provided by variable annuities (VAs). Although equity-based VAs appear to have fallen into some disfavor, perhaps because of the inherent volatility of common stocks, recent work by Bodie (1980, 1982) suggests that VAs tied to bills or short-term bonds may produce income streams that are quite stable in real terms. Therefore we examine VAs backed by bills, long-term bonds, common stocks, and a mixed portfolio and compare the results with those for a graduated payment, nominal annuity.

Our second task is to examine more recent annuity designs in which floors or floors-plus-ceilings have been added to the standard VA. The

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Rockefeller Foundation plan, for example, provides cost-of-living adjustments which equal the average prime interest rate for the year less 3% (Heaton 1977). Once granted, these adjustments are never reduced and thus the annuitant—in effect—has a VA subject to a nominal floor.[2] Annuities provided by the Teachers Insurance and Annuity Association (TIAA) also have a guaranteed nominal floor.[3] In recent years, large firms in both Canada and the United States have frequently made ad hoc cost-of-living adjustments to the pensions of retired workers. In Canada, these adjustments have often been financed from pension fund earnings in excess of the plan's valuation rate (Pesando 1981).[4] Once granted, these adjustments are permanent. Moreover, there appears to be a ceiling on these adjustments in that the real value of the initial benefit is never increased even if "excess" fund earnings might so permit. The second part of the chapter thus examines a variable annuity subject to a nominal floor (VAF) and a variable annuity subject to a nominal floor and a real ceiling (VAFC). The former is suggested by the Rockefeller plan, and both may be viewed as an attempt to formalize the apparent practice of many firms in granting cost-of-living adjustments to retired plan members.

The chapter is organized as follows. The performance of a nominal, level-payment annuity is first contrasted with that of a hypothetical purchasing power annuity for the period 1971–80. The latter is formally equivalent to a VA backed by an index bond yielding a certain real return of 0%. Theoretical distributions of the real payments from VAs tied to alternative asset bases are then presented and serve to illustrate the nature of the trade-off between risk and expected real returns. These payments are also contrasted with those provided by a graduated-payment, nominal annuity. The properties of VAFs are then explored, and simulations are conducted to contrast their performance with VAs backed by identical asset portfolios. The same exercise is then repeated for VAFCs. To place the alternative annuity designs in a final perspective, a historical simulation is conducted for the period 1971–80. The final section is a summary and conclusion.

11.1 The Level-Payment, Nominal Annuity

The nominal and real values of the benefits provided by a nominal, level-payment annuity during the period 1971–80 are illustrated in table 11.1. The annuity is purchased at the beginning of 1971 for the sum of $100,000, the annuity is sold at a (nominal) interest rate of 7.5%, and the benefits are payable with certainty for 10 years.[5] The real value of the annual, nominal payment declines by more than 50% during the decade. Further, it is likely that a substantial portion of this decline was unanticipated. If the anticipated rate of inflation embodied in the nominal rate of

Table 11.1 **Illustration of Traditional, Level-Payment Annuity and Purchasing Power Annuity for Period 1971–80**

		Traditional Annuity[a]		Purchasing Power Annuity[b]			
				No Tilting ($RV=0$)		Tilting ($RV=5$)	
Year	Inflation Rate (CPI)	Nominal Value	Real Value	Nominal Value	Real Value	Nominal Value	Real Value
1971	3.3	14,568	14,095	10,336	10,000	12,748	12,334
1972	7.4	14,568	13,630	10,688	10,000	12,555	11,746
1973	8.8	14,568	12,527	11,629	10,000	13,009	11,187
1974	12.2	14,568	11,165	13,047	10,000	13,901	10,654
1975	7.0	14,568	10,434	13,962	10,000	14,167	10,147
1976	4.8	14,568	9,955	14,634	10,000	14,142	9,664
1977	6.8	14,568	9,324	15,624	10,000	14,380	9,204
1978	9.0	14,568	8,551	17,035	10,000	14,932	8,765
1979	13.3	14,568	7,547	19,303	10,000	16,114	8,343
1980	12.4	14,568	6,714	21,698	10,000	17,251	7,951

[a]Assumes that the nominal interest rate is 7.5%, initial capital is $100,000, annuity is payable with certainty for 10 years, and annuity payments are made at the end of the year.

[b]Assumes a real interest rate of zero. If π is the inflation rate in period t, then the nominal annuity payment $B_t = B_{t-1} * (1+\pi_t)/(1+RV)$ where RV is the interest rate used to determine the base annuity payment (B_0); B_0 equals $RV * A/[1-(1+RV)^{-T}]$ where A is the initial capital; and T is the number of years the annuity is payable. For $RV=0$, B_0 equals A/T.

interest was 5% then the annuitant would have expected the real value of his benefit to decline at about 5% per year.[6] Deviations around this rate of decline would then have been unanticipated.

For illustrative purposes, the performance of a PPA for the period 1971–80 is also shown in table 11.1. This annuity is fully linked to the consumer price index and is sold at a certain real return of 0%. Earlier work by Bodie (1980) indicates that the minimum-variance portfolio (in the absence of short selling) consists of one-month Treasury bills hedged with commodity futures and that the expected real return on this portfolio would not exceed 0%. We assume for simplicity that a portfolio could be constructed which would provide a certain real return of 0%. The PPA is analytically equivalent to a VA tied to an index bond which provides this certain real return.

Although the stream of real payments provided by the PPA is certain, there is no requirement that this stream of payments be constant. If RV is the annuity valuation rate used to determine the base value of the annuity payment, and if r is the certain real return on the portfolio, then the real value of the annuity payments will change with certainty at an annual rate equal to $(1 + r)/(1 + RV) - 1$. (See Appendix for details.) With $r = 0$ and $RV = 5$, the real benefit declines with certainty at 4.76% per year, as shown in table 11.1. When r and RV are equal, there is no tilt to the projected stream of real annuity payments. If the real return is uncertain, the previous expression depicts the expected degree of tilting in the real payments stream. If pensioners wish to design a stream of pension payments which is expected to decline in real terms, perhaps due to liquidity constraints or estate motives, this is readily accomplished with vehicles other than the nominal, level-payment annuity. The downward tilt in the real benefit provided by a nominal, level-payment annuity is, of course, equal to the expected rate of inflation.

11.2 Variable Annuities with Alternative Asset Bases

The limitations of fixed-dollar annuities in an inflationary climate prompted life insurance companies in the 1950s to offer equity-based VAs. As emphasized by Bodie (1980), however, an equity-based VA exposes the annuitant to substantial investment risk even if it is assumed that the real return on equities is unaffected by unanticipated changes in the rate of inflation.[7] The purpose of this section is to explore the real income streams provided by VAs with alternative asset bases.

Theoretical distributions are presented in table 11.2 for the real benefits provided by VAs backed by one-month Treasury bills, long-term United States government bonds, common stocks, and a mixed portfolio. (The mechanics of a variable annuity are detailed in the Appendix.) The mixed portfolio is the minimum variance portfolio with the same ex-

Table 11.2 **Variable Annuities with Alternative Asset Bases: Theoretical Distributions**

Portfolio	Expected Real Return[a] (%)	Standard Deviation (%)	Annuity Valuation Rate (%)	Base Annuity Payment[b] ($)	Annuity Payment In Year 5			Annuity Payment in Year 10			Annuity Payment in Year 15		
					Median	Mean	S.D.	Median	Mean	S.D.	Median	Mean	S.D.
Bills	0	1.52	0	6,667	6,667	6,670	227	6,667	6,674	321	6,667	6,678	393
Bills (serial correlation)[c]	0	1.04	0	6,667	6,667	6,677	361	6,667	6,701	679	6,667	6,731	933
Bonds	2.956	7.64	3	8,377	8,377	8,500	1,463	8,377	8,625	2,115	8,377	8,752	2,647
Stocks	7.232	18.61	7.5	11,329	11,329	12,354	5,372	11,329	13,471	8,666	11,329	14,689	12,974
Mixed portfolio[d]	2.956	6.08	3	8,377	8,377	8,455	1,155	8,377	8,533	1,656	8,377	8,613	2,056
Graduated payment,[e] nominal annuity (graduation rate = 8%)	2.956	—	3	8,377	8,377	8,445	1,080	8,377	8,723	2,531	8,377	9,219	4,236

[a]Mean of the logarithm of the real annual wealth relative. Annuity valuation rate in column 4 is the equivalent annual rate.

[b]Initial capital is $100,000, annual payments are made with certainty for 15 years and are reported in constant dollars.

[c]Based on the following autoregression for the annual real return on bills: $r_t = .76r_{t-1} + e_t$ with $\sigma_e = 1.04\%$ per year and $r_0 = 0$.

[d]Mixed portfolio, consisting of bonds (52%), bills (29%), and stocks (19%), minimizes the variance of the annual real return for the given mean.

[e]Uncertainty regarding the real annuity payments stems solely from uncertainty regarding the price level, which is assumed to be log normally distributed. The continuously compounded rate of inflation (π) follows the following autoregressive process: $\pi_t = .77 + .9\pi_{t-1} + U_t$ with $\sigma_U = 2.00\%$ per year and $\pi_0 = 7.7\%$. The steady-state inflation rate of 7.7% is equivalent to the annual graduation rate of 8%. The graduated-payment nominal annuity assumes an expected real return equal to that of bonds (i.e., 2.956%).

pected return as the long-term bond portfolio.[8] The VAs are purchased for $100,000 and benefits are paid with certainty for 15 years. The real returns on bills, bonds, and stocks are assumed to be lognormally distributed with means of zero, 2.956%, and 7.232%, respectively, and standard deviations of 1.52%, 7.64%, and 18.61%. The means are the continuous time equivalents of annual returns of 0%, 3%, and 7.5%. These parameters, together with the covariances necessary to construct the mixed portfolio, are based on historical data for the period 1953–80.[9] The valuation rates used to determine the base level of the annuity payments are the annual equivalents of the continuously compounded real rates of return. Examination of the historical data indicates that real bill returns, but not those on stocks and bonds, are serially correlated.[10] For this reason, the theoretical distribution of real benefit payments is also calculated for a bills-based VA on the assumption that real bill returns are serially correlated.

The assumption that real returns are lognormally distributed implies that annuity payments are lognormally distributed also. Since the valuation rates used to calculate the base values of the benefits are the annual equivalents of the expected real returns on the portfolios, *median* benefit payments show no tendency to rise or to fall over time.[11] Because these payments are lognormally distributed, they exhibit positive skewness and thus the *mean* payments rise steadily over time. The distribution of real benefits provided by a graduated-payment, nominal annuity is also included in table 11.2. For this annuity, all of the uncertainty regarding the real value of the benefit payments stems from price-level uncertainty. Thus, an additional set of assumptions is required. The price level is assumed to be lognormally distributed, and the continuously compounded rate of inflation is assumed to follow the first-order autoregressive process which characterizes the period 1953–80.[12] The degree of graduation is set equal to 8%, which is the (annual) steady-state rate of inflation implied by the autoregression.[13] The purpose of including the graduated, nominal annuity is to emphasize the fact that while its nominal payments are devoid of risk, its real payments are not.

The distributions of real benefit payments reported in table 11.2 mirror the risk-return characteristics of the underlying portfolios. The stream of real benefits provided by the bills-based VA is smaller and more stable than the stream provided by the bonds-based VA, and so on. Recognition of the serial correlation in bill returns produces a riskier stream of benefit payments, especially as the time horizon increases. Even when this serial correlation is acknowledged, however, bills remain the cornerstone of any VA which is intended to limit uncertainty regarding the real value of benefit payments. The importance of diversification is seen in the comparison of the bonds-based VA, with the VA tied to the mixed portfolio with the same expected return. Although the median benefits are identical, the standard deviation of the real benefit payment in the fifteenth year is 22% smaller for the VA tied to the mixed portfolio.

Note, finally, the real benefit stream provided by the graduated-payment, nominal annuity. We assume that the implicit expected real return is 3% (at an annual rate) and is thus equal to the expected real return on long-term government bonds.[14] This assumption is equivalent to assuming that life companies can hedge graduated-payment, nominal annuities by holding an appropriate sequence of long-term bonds and that competitive pressures ensure that this is the implicit real yield at which these annuities are sold. Because of the 3% return assumption, the median benefits are identical to those for the VAs tied to the government bond and mixed portfolios. The standard deviation of the benefits provided by the nominal annuity is less than those for either of the VAs in year 5 but significantly exceeds them by year number 10.

The significant increase in the riskiness of real benefits provided by the nominal annuity as the annuitant ages merits emphasis. *This is a direct reflection of the substantial serial correlation in the inflation rate.* These results, especially as the annuitant ages, illustrate how inappropriate it is to argue that VAs are inferior to nominal annuities because they transfer all of the investment risk to the annuitant. The results also highlight the importance of acknowledging the serial correlation in inflation rates in attempting any assessment of the risk of the real benefits provided by nominal annuities.

11.3 Variable Annuities with Nominal Floors

As noted, the Rockefeller Foundation Plan provides retiring employees with a variable annuity subject to a nominal floor, or VAF. Sun Life Insurance Company of Canada has recently introduced a VAF, tied to Treasury bills, in which excess earnings above 3% are also used to provide permanent benefit enrichments. The nominal floor in each of these cases is equivalent to the plan sponsor's guaranteeing that the fund will earn a nominal rate of at least 3%. If the fund earns less than 3% in a given period, the plan sponsor fully absorbs the loss. (The mechanics of a VAF and the contrast to a standard VA are detailed in the Appendix.)

The pension plans provided by most large firms in the United States (and Canada) are defined-benefit plans. In them the employee typically receives a benefit equal to a given fraction of his average or final earnings for each year of service. Although the promised benefits are nominal, firms—especially in Canada—have typically granted ad hoc cost-of-living adjustments to the pensions of retired employees. Once made, these adjustments tend to be permanent. Thus the nominal value of the pension benefit is never reduced even if the fund performs poorly. This is, of course, what happens explicitly under the Rockefeller Foundation Plan, which functions as an ordinary defined-benefit plan during the preretirement period.

If the source of these adjustments is pension fund earnings in excess of the plan's valuation rate, and if there is no ceiling on the size of the benefit increases, then the plan member effectively owns a VA with a guaranteed nominal floor, or a VAF. Equivalently, he is provided with a traditional VA plus a put option on the nominal investment earnings of the pension assets with a striking price equal to the plan's valuation rate. *The plan's valuation rate becomes the equivalent of the valuation rate used to set the base payment in a standard VA*. If the nominal return on the pension fund is less than this valuation rate, then the nominal benefit is unchanged and the shortfall is absorbed fully by the plan sponsor.

Let A represent the initial amount in the fund, RV the valuation rate, and $\sigma(\tilde{R})$ the measure of the risk of the nominal return that is relevant to option pricing. The value (A_{VAF}) of the VAF is

$$A_{\text{VAF}} = A + \text{put } [A, \text{RV}, \sigma(\tilde{R})]. \quad (1)$$

For a given A, the value of the put option is an increasing function of both RV and $\sigma(\tilde{R})$. If the fund is invested exclusively in the risk-free nominal asset and thus earns the certain nominal return R_f, the value of the put option is zero as long as $RV \leq R_f$. On the other hand, the value of the put option *is* likely to be large if the nominal return on the pension assets is very uncertain, even if RV is well below the expected nominal return on these assets. If the objective of the plan sponsor were to minimize the value of the put option, he would wish to set a low RV and choose an asset base which would effectively make the VAF into a standard VA. If the sole objective of the employee were to maximize the value of the put option, he would—of course—prefer that the funds be invested in the riskiest asset, or common stocks. As the employee presumably cannot sell his VAF, he might nonetheless prefer that the fund not be invested in risky assets if he wishes the real retirement income provided by the VAF to be stable. This point is examined below.

Simulation results (1,000 trials) are presented in table 11.3 for VAFs tied both to bills and to the mixed portfolio described previously. The interest in bills reflects the fact that they represent the cornerstone of any low-risk stream of real annuity payments. Still-active workers may have enough flexibility to vary their consumption-saving and work-leisure decisions; thus they can assume considerable investment risk. This is less likely to be the case for retired workers. The interest in the mixed portfolio stems from the desire to monitor—in effect—the value of the put option when the uncertainty in the return on pension assets is increased. Because the value of this option depends on the nominal return on the pension fund, simulations are performed for both a low-inflation (3%) and a high-inflation (9%) scenario. The nominal return is equal to

Table 11.3 Variable Annuities with Guaranteed Nominal Floors

Portfolio	Expected Real Return[a] (%)	Standard Deviation (%)	Inflation Rate (%)	Annuity Valuation Rate (%)	Base Annuity Payment[b] ($)	Annuity Payment In Year 5			Annuity Payment in Year 10			Annuity Payment in Year 15		
						Median	Mean	S.D.	Median	Mean	S.D.	Median	Mean	S.D.
Bills	0	1.52	3	3	8,377	7,424	7,447	150	6,604	6,619	188	5,867	5,888	204
						(7,219)	(7,223)	(245)	(6,203)	(6,222)	(302)	(5,357)	(5,375)	(313)
Bills	0	1.52	9	3	8,377	7,221	7,334	248	6,219	6,229	312	5,365	5,373	322
						(7,221)	(7,224)	(248)	(6,219)	(6,229)	(312)	(5,365)	(5,373)	(322)
Mixed[c]	2.956	6.08	3	3	8,377	8,851	8,943	928	9,369	9,616	1,434	10,020	10,326	1,976
						(8,366)	(8,448)	(1,180)	(8,459)	(8,595)	(1,687)	(8,512)	(8,724)	(2,139)
Mixed[c]	2.956	6.08	9	3	8,377	8,473	8,564	1,120	8,633	8,738	1,589	8,684	8,918	1,974
						(8,407)	(8,489)	(1,176)	(8,461)	(8,577)	(1,655)	(8,406)	(8,671)	(2,047)
Mixed[c]	2.956	6.08	3	8	11,683	10,787	10,948	747	10,103	10,276	997	9,483	9,629	1,129
						(9,192)	(9,289)	(1,225)	(7,326)	(7,446)	(1,407)	(5,773)	(5,922)	(1,339)
Mixed[c]	2.956	6.08	9	8	11,683	9,586	9,701	1,035	7,960	8,070	1,190	6,549	6,706)	1,299
						(9,226)	(9,276)	(1,263)	(7,219)	(7,380)	(1,390)	(5,708)	(5,879)	(1,370)

Note: Results in parentheses are those for a variable annuity without the nominal floor.

[a]Mean of the logarithm of the real annual wealth relative. The nominal return is the sum of the simulated real return plus the continuous time equivalent of the annual inflation rate noted in the table.

[b]Initial capital is $100,000, annual payments are made with certainty for 15 years, payments are in constant dollars.

[c]Same as in table 11.2.

the sum of the stochastic real return and the continuous time equivalent of these two inflation rates. Both low (3%) and high (8%) valuation rates are included in the simulations for the VAF tied to the mixed portfolio.

When the inflation rate is 3%, the expected real return of zero on the bills portfolio implies an expected nominal return of 3%. Because the valuation rate is also 3%, the floor frequently binds and thus the put option is frequently exercised. The result is that benefits have a higher median *and* a lower standard deviation than do those provided by the corresponding VA. When the inflation rate rises to 9%, the floor never binds and the result is identical to that for the VA. This result occurs because the combination of the high (expected) nominal return relative to the valuation rate and the low standard deviation of bill returns ensures that the realized nominal return always exceeds the valuation rate. Note also that the expected real return of zero together with the valuation rate of 3% causes the stream of real benefits to be tilted downward. This is most easily seen for the VA, but it occurs for the VAF as well.

When the inflation rate rises from 3% to 9%, the put option is occasionally exercised for the mixed portfolio. This is a direct result of its more uncertain return. When the valuation rate (again, the interest rate used to set the base payment) is raised for a standard VA, the sole effect is to tilt the real payments stream downward relative to what it would otherwise have been. When the valuation rate is raised for a VAF, it has the additional effect of raising the value of the put option. When the valuation rate is raised to 8%, which is typical of the rates now used to value defined-benefit plans in the United States, the striking price of the option rises accordingly. The result is a dramatic rise in the value of the put option in the low-inflation scenario. With an expected nominal return of 3 + 3 = 6%, the nominal return typically falls short of the valuation rate. By the fifteenth year, the median real benefit is 65% greater than that provided by the corresponding VA. In the high-inflation scenario, the effective value of the put option falls sharply as realized nominal returns fall short of the valuation rate with much lower frequency.

It is interesting to note that proponents of the Rockefeller Foundation Plan, which functions like a VAF, emphasize the importance of investing the pension fund reserve for retired employees exclusively in short-term commercial paper. If the nominal interest rate on short-term securities remains high relative to the plan's valuation rate of 3%, the value of the put option which distinguishes the VAF from a traditional VA will be very small. In effect, the Rockefeller Foundation Plan will have been transformed from a defined-benefit plan in the preretirement period to a defined-contribution plan at the date of retirement, with the plan's valuation rate of 3% used to capitalize the nominal benefits due at the date of the employee's retirement. If inflation were to recede and thus short-term interest rates to fall, the value of the put option would increase. Thus the annuitants stand to gain and the plan stands to lose from a reduction in the rate of inflation.

It is also interesting to note the continued emphasis in policy discussions in Canada on investing pension fund reserves held for retired employees exclusively in short-term securities if excess earnings are to be used to provide cost-of-living protection. Since the VAF is virtually identical to a VA when the value of the put option is small, the use of ad hoc adjustments may simply reflect the metamorphosis of defined-benefit into defined-contribution plans as the market response to increased inflation uncertainty (Pesando 1982). Because most large firms had already introduced defined-benefit plans, the use of VAFs—rendered virtually identical to VAs by the combination of low valuation rates and investments concentrated in short-term securities—may be the most convenient way to effect the metamorphosis.[15]

11.4 Variable Annuities with Nominal Floors and (Cumulative) Real Ceilings

In the preceding section, it was assumed that firms which provide ad hoc cost-of-living adjustments could be regarded as providing their employees with VAFs. Although this may well be true for some firms, the reality may also be more complicated. Firms which make ad hoc cost-of-living adjustments may impose a ceiling on such increases and may also bank underwriting losses (when the nominal floor binds) as a first claim on future excess earnings. In citing options for pension reform in Canada, the Task Force (1979) considered an excess interest scheme which contained a cumulative real ceiling. The real value of any enriched pension could not exceed its initial level, and any excess earnings above the amount necessary to preserve fully the real value of the pension would be banked against future investment shortfalls. In addition, any underwriting losses incurred by the plan sponsor by virtue of the guaranteed nominal floor would be banked, would accumulate at a market rate of interest, and would represent a prior claim on future excess earnings. Only after any accumulated losses borne by the plan sponsor were repaid would excess earnings be used to enrich pensions in pay. Significantly, this illustrative scheme was chosen for study after the federal government solicited input from both firms and members of the employee benefits industry.

The most important feature of a VAFC relative to a VAF is its banking provisions. (This is perhaps most easily seen by considering the case in which there is a real floor equal to the initial benefit. In this case, the annuity would be constant in real terms and the banking provisions would mirror the underwriting experience of a plan sponsor who provided a fully indexed pension and held assets other than index bonds in the pension fund.) Nonetheless, it is useful to consider the options inherent in a VAFC without reference to the banking provisions. By virtue of the ceiling on the real value of the pension benefit, the worker has—in effect—sold a call option on investment earnings in excess of those sufficient to provide full cost-of-living protection. Because the nominal return on the plan's assets is the real return plus the inflation rate, this is equivalent to the worker's having sold a call option on real investment earnings in excess of the valuation rate. Let A_{VAFC} represent the value of the variable annuity subject to both a floor and a ceiling; let $\sigma(\tilde{r})$ be the measure of risk of the real return that is relevant to option pricing, and let A_{VAF} and A be as defined in (1). Then

$$(2) \qquad A_{\text{VAFC}} = A + \text{put } [A, RV, \sigma(\tilde{R})] - \text{call } [A, RV, \sigma(\tilde{r})]$$
$$= A_{\text{VAF}} - \text{call } [A, RV, \sigma(\tilde{r})].$$

Unlike a VAF, whose value to the beneficiary is at least as great as that of a standard VA, the value of a VAFC may be greater or less than that of the corresponding VA, depending on the relative values of the put and the call. For a given A, the value of the call option is a decreasing function of the plan's valuation rate and an increasing function of the risk of the real return on the plan's assets. The value of the call option will be zero if the pension fund is invested exclusively in a risk-free real asset and if the risk-free real rate of return $r_f \leqq$ RV. As noted by Bodie (1980), there is no risk-free real asset. A pension fund invested exclusively in bills will, however, earn a real return which is quite stable and which has an expected value of (approximately) zero. The value of this call option will thus be close to zero if (1) the fund holds only bills or their equivalent and (2) the valuation rate is above, say, 3%.[16] In this case, the value of the VAFC will equal that of the VAF. If, in addition, the anticipated rate of inflation is sufficiently high that the nominal bill yield significantly exceeds RV, then the value of the put option contained in both the VAF and the VAFC will equal zero and thus both will be equal in value to the corresponding VA.

Consider first (table 11.4) the distribution of real benefits under a VAFC tied to a bills portfolio when the inflation rate is low. Because the projected stream of real annuity payments is tilted downward (since the expected real return of zero is less then the valuation rate), the ceiling binds rarely and only in the initial years of the annuity payout. Median benefits fall short of those provided by a VAF, primarily due to the banking provisions, but exceed those of a VA. In the high-inflation scenario, the VAFC provides benefits which simply reproduce those of a VA. This result, which was anticipated in the discussion of (2), may be empirically relevant. If so, this might explain the apparent lack of attention that is sometimes accorded to this issue. The Rockefeller Foundation plan, for example, makes no reference as to whether or not a ceiling exists

Table 11.4 Variable Annuities with Guaranteed Nominal Floors and Cumulative Real Ceilings

Portfolio	Expected Real Return[a] (%)	Standard Deviation (%)	Inflation Rate (%)	Annuity Valuation Rate (%)	Base Annuity Payment[b] ($)	Annuity Payment In Year 5			Annuity Payment in Year 10			Annuity Payment in Year 15		
						Median	Mean	S.D.	Median	Mean	S.D.	Median	Mean	S.D.
Bills	0	1.52	3	3	8,377	7,314 (7,219)	7,360 (7,223)	151 (245)	6,348 (6,203)	6,403 (6,222)	185 (302)	5,482 (5,357)	5,543 (5,375)	187 (313)
Bills	0	1.52	9	3	8,377	7,221 (7,221)	7,224 (7,224)	248 (248)	6,219 (6,219)	6,229 (6,229)	312 (312)	5,365 (5,365)	5,373 (5,373)	322 (322)
Mixed[c]	2.956	6.08	3	3	8,377	8,377 (8,366)	8,080 (8,448)	393 (1,180)	8,377 (8,459)	7,900 (8,595)	690 (1,687)	8,377 (8,512)	7,725 (8,724)	973 (2,139)
Mixed[c]	2.956	6.08	9	3	8,377	8,377 (8,400)	7,995 (8,489)	585 (1,176)	8,377 (8,461)	7,865 (8,577)	796 (1,655)	8,377 (8,406)	7,806 (8,671)	971 (2,049)
Mixed[c]	2.956	6.08	3	8	11,683	10,106 (9,182)	10,399 (9,289)	462 (1,226)	8,769 (7,366)	9,063 (7,446)	585 (1,407)	7,564 (5,773)	7,834 (5,922)	561 (1,339)
Mixed[c]	2.956	6.08	9	8	11,683	9,257 (9,226)	9,372 (9,276)	1,129 (1,263)	7,263 (7,219)	7,428 (7,380)	1,420 (1,390)	5,574 (5,708)	5,746) (5,879)	1,470 (1,370)

Note: Figures in parentheses are those for a variable annuity without the floor and ceiling.

[a]Mean of the logarithm of the real annual wealth relative. The nominal return is the sum of the simulated real return plus the continuous time equivalent of the annual inflation rate noted in the table.

[b]Initial capital is $100,000, annual payments are made with certainty for 15 years, payments are in constant dollars.

[c]Same as in table 11.2.

on the cost-of-living increases. Because the valuation rate of 3% exceeds the expected real return on a portfolio of short-term commercial paper (or its surrogate, the prime rate), the question of whether or not there is a ceiling may simply not be empirically relevant.

For the mixed portfolio with a valuation rate of 3%, there is no tilt to the projected stream of real benefits provided by the corresponding VA. For the VAFC, unlike the VAF, the benefit payments are similar in both the low- and the high-inflation scenarios. This is, of course, due to the banking provisions. For both scenarios, the ceiling binds frequently (i.e., the call option is exercised), as evidenced by the fact that median benefits remain at the ceiling in all years. Although we do not attempt to evaluate them explicitly, it would appear that the value of the sponsor's call option exceeds the value of the annuitant's put in these two cases. Note that the median and mean benefits are lower than those of the corresponding VA in all years. Furthermore, in contrast to both the VA and the VAF, the mean benefit for the VAFC is well below its own median, reflecting the reverse skewness induced by the truncation of the upper tail of the distribution. The dramatic decline in the standard deviation relative to both the VA and the VAF is also a result of this truncation and therefore reflects, not a reduction in risk from the annuitant's perspective, but the loss of upside potential. Further evidence that in these two cases the value of the VAFC is considerably less than that of the corresponding VA is provided by table 11.5, which shows the distribution of the real accumulation in the "bank" at the end of year 15. When this number turns out to be positive at the end of a simulation run, it means that the years of "excess" earnings from the portfolio were more than enough to compensate for the years of shortfall.

Raising the valuation rate, as noted in the discussion of (2), reduces the value of the call option. When the valuation rate is set at 8%, median benefits do exceed those provided by the VA for all years in the low-inflation scenario, although they remain less than those provided by the VAF. When inflation is high, and thus the permitted real erosion in the value of benefits is also high, the stream of payments provided by the VAFC and the VA are quite similar. The ceiling frequently binds, but the excess funds so banked are then used to enrich nominal benefits in subsequent years.

To sum up, three empirical results merit emphasis. First, if the pension fund is invested exclusively in bills, the VAFC will provide benefits similar to those provided by a standard VA if (1) the inflation rate is high relative to the plan's valuation rate and (2) the valuation rate is, say, 3% or more and thus significantly exceeds the expected real return on bills. In this case, the value of each of the put and the call options is approximately equal to zero. Second, when the expected real return on the plan's assets is equal to the valuation rate, the real benefits provided by a VAFC are

Table 11.5 **Amount in "Bank" at End of Year 15 for VAFCs in Table 11.4**

Portfolio	Expected Real Return (%)	Standard Deviation (%)	Inflation Rate (%)	*RV* (%)	Amount in Bank ($) Median	Mean	Minimum	Maximum	Standard Deviation
Bills	0	1.52	3	3	−1,800	−2,209	−9,421	0	1,787
Bills	0	1.52	9	3	0	0	0	0	0
Mixed	2.956	6.08	3	3	2,591	10,024	−23,458	122,557	16,933
Mixed	2.956	6.08	9	3	3,381	10,898	1,408	127,940	15,459
Mixed	2.956	6.08	3	8	−23,836	−24,112	−61,225	17,841	12,835
Mixed	2.956	6.08	9	8	−63	−770	−13,194	2,988	1,649

likely to be far more stable than those provided by either a VAF or the traditional VA. This result, in essence, reflects the procedure for banking the gains or losses experienced by the plan sponsor. Third, by choosing appropriate combinations of RV and asset allocations, it would appear possible to cancel the values of the put and the call (without setting each equal to zero) and to create a number of VAFCs all having a value equal to A. But the sponsor and annuitant would have to agree on the portfolio's composition, and there would have to be some mechanism for monitoring adherence to it.

As noted, our interest in the VAFC is motivated by the possibility that it may formalize the behavior of at least some firms which make ad hoc cost-of-living adjustments. If so, and if the stream of real payments is smoothed relative to those obtainable from, say, a bills-based VA, then firms must be compensated for their underwriting the attendant investment risk. In principle, this should be reflected in compensating wage differentials. Because the VAFC does not alter the efficient frontier, it will be the basic risk-return trade-offs available in the capital market which dictate the size of these compensating wage differentials.

11.5 Alternative Annuity Designs: Historical Simulations for the Period 1971–80

Historical simulations of the nominal and real benefits provided by VAs, VAFs, and VAFCs for the period 1971–80 are presented in table 11.6 parts A–D. As in table 11.1, the initial capital in 1971 is $100,000 and the payments are made with certainty for 10 years. Of course, VAFs and VAFCs must be underwritten by the plan sponsor (or life insurance company) and their cost may exceed or fall short of the initial capital.[17] Two valuation rates, 0% and 5%, are used in the simulations. The former is the expected real return on the minimum variance portfolio (i.e., bills) while the latter is typical of the rates actually used in the early 1970s to value defined-benefit plans.

Consider first the bill results. When the valuation rate is zero, the real benefit provided by the VA declines from $10,100 in 1971 to $8,794 in 1980—that is, by 13%. This erosion is only modest in view of the substantial unanticipated inflation that appears to have occurred in the 1970s. Because the floor never binds, the VAF produces benefits identical to those produced by the VA. Because the ceiling binds in 1971 and 1972, thus causing excess earnings to be banked for future use, the real benefits provided by the VAFC diverge from those provided by the VA and VAF. With a valuation rate of 5%, the floor binds twice (1971 and 1972) so that the final benefit provided by the VAF exceeds that provided by the VA. Because of the banking feature, which requires that the plan sponsor be compensated for prior underwriting losses, the stream of benefits provided by the VAFC differs from that provided by the VA.

The sharpest contrast among the alternative annuity designs occurs with the riskiest asset base, which is common stocks. Consider only the results when the valuation rate equals 5%. Although this rate is less than the expected real rate of return on common stocks, the real benefits provided by the VA, in fact, decline sharply. This result simply reflects the poor performance of the stock market during the decade. The value of the nominal floor (i.e., the put option) is high as evidenced by the fact that the real benefit provided by the VAF in 1980 is almost twice that provided by the VA. The tendency for the VAFC to stabilize the real stream of benefit payments is readily apparent. In 1973 and 1974, for example, annuitants are partially insulated from the precipitous declines in the stock market. When the stock market recovers in 1975, however, real benefits continue to decline as excess fund earnings are first used to repay the plan sponsors for the net underwriting losses they incurred in the previous years.

The final comparison is between the bond portfolio and the mixed portfolio with the identical expected return. Because of the very adverse performance of the bond market, the real benefit by 1980 is much higher for the VA when it is tied to the mixed portfolio. This ex post result is consistent with the greater ex ante risk of the bond portfolio. The comparisons of the results for the VAs, VAFs, and VAFCs are quite straightforward, and only the continuing tendency for real benefits to be stabilized under the VAFC merits note.

11.6 Summary and Conclusion

Nominal annuities, whether level payment or graduated, expose the annuitant to substantial uncertainty about the real value of his retirement income. This is because he is uncertain of the future level of prices and hence of the future rate of inflation. Standard VAs backed by Treasury bills or their equivalent provide much more stable real retirement incomes, even when consideration is given to the serial correlation in real bill returns. VAs backed by common stocks, long-term government bonds, and a mixed portfolio illustrate the risk-return trade-offs inherent in the alternative portfolios. These should be of interest to plan sponsors who may wish, without increasing their own costs, to provide increased annuity choices to plan members.[18]

The cost of a VAF, which is a VA with a nominal floor, is not known with certainty on the date the annuity is purchased and must be underwritten by a plan sponsor or life company. The plan provided by the Rockefeller Foundation functions, in effect, like a VAF. If the objective

Table 11.6 **Alternative Annuity Designs**

Year	Inflation Rate (%)[a]	Nominal Return	Variable Annuity[b]				Variable Annuity with Nominal Floor				Variable Annuity with Nominal Floor, Real Ceiling			
			$RV=0$		$RV=5$		$RV=0$		$RV=5$		$RV=0$		$RV=5$	
			Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real
A. Simulations for a bills-only portfolio, 1971–80														
1971	3.36	4.39	10,439	10,100	12,876	12,457	10,439	10,100	12,950	12,529	10,336	10,000	12,950	12,529
1972	3.41	3.84	10,840	10,142	12,733	11,913	10,840	10,142	12,950	12,116	10,688	10,000	12,950	12,116
1973	8.80	6.93	11,591	9,967	12,968	11,151	11,591	9,967	13,189	11,341	11,626	9,997	12,950	11,136
1974	12.20	8.00	12,518	9,594	13,337	10,222	12,518	9,594	13,565	10,396	12,556	9,623	13,284	10,181
1975	7.01	5.80	13,245	9,486	13,439	9,625	13,244	9,486	13,669	9,790	13,284	9,514	13,385	9,586
1976	4.81	5.08	13,917	9,510	13,449	9,190	13,917	9,510	13,679	9,347	13,959	9,539	13,396	9,154
1977	6.77	5.12	14,629	9,363	13,465	8,618	14,629	9,363	13,695	8,765	14,673	9,391	13,411	8,583
1978	9.03	7.03	15,657	9,191	13,726	8,057	15,657	9,191	13,959	8,194	15,705	9,219	13,670	8,024
1979	13.31	10.38	17,284	8,954	14,429	7,475	17,284	8,954	14,675	7,602	17,335	8,980	14,371	7,445
1980	12.41	10.40	19,082	8,794	15,170	6,991	19,082	8,794	15,429	7,111	19,138	8,820	15,110	6,964
B. Simulations for a stocks-only portfolio, 1971–80														
1971	3.36	14.31	11,431	11,059	14,098	13,640	11,431	11,059	14,099	13,641	10,336	10,000	13,386	12,950
1972	3.41	18.98	13,601	12,725	15,976	14,947	13,601	12,725	15,976	14,947	10,688	10,000	13,842	12,950
1973	8.80	−14.66	11,607	9,981	12,985	11,166	13,601	11,696	15,976	13,738	11,629	10,000	13,842	11,903
1974	12.20	−26.48	8,533	6,540	9,092	6,968	13,601	10,424	15,976	12,244	11,629	8,913	13,842	10,609
1975	7.01	37.20	11,708	8,385	11,879	8,508	18,660	13,364	20,875	14,951	11,629	8,329	13,842	9,914
1976	4.81	23.84	14,499	9,908	14,012	9,575	23,109	15,791	24,621	16,824	14,359	9,812	13,842	9,459
1977	6.77	−7.18	13,458	8,613	12,386	7,927	23,109	14,790	24,621	15,758	14,359	9,188	13,842	8,859
1978	9.03	6.39	14,318	8,405	12,550	7,367	24,585	14,431	24,947	14,644	14,359	8,429	13,842	8,125
1979	13.31	18.44	16,958	8,785	14,157	7,334	29,119	15,085	28,140	14,578	16,039	8,309	13,842	7,171
1980	12.41	32.01	22,386	10,317	17,797	8,202	38,440	17,715	35,378	16,304	21,173	9,578	13,842	6,379

C. Simulations for a U.S.-bonds-only portfolio, 1971–80

1971	3.36	13.23	11,322	10,954	13,966	13.512	11,323	10,955	13,966	13,512	10,336	10,000	13,386	12,950
1972	3.41	5.68	11,966	11,195	14,056	13,151	11,966	11,195	14,056	13.151	10,688	10,000	13,842	12,950
1973	8.80	−1.11	11,834	10,176	13,239	11,384	11,966	10,290	14,056	12,087	11,629	10,000	13,842	11,903
1974	12.20	4.35	12,348	9,464	13,156	10,083	12,487	9,570	14,056	10,773	12,721	9,750	13,842	10,609
1975	7.01	9.19	13,484	9,657	13,680	9,798	13,634	9,765	14,617	10,469	13,890	9,948	13,842	9,914
1976	4.81	16.75	15,742	10,757	15,212	10,395	15,918	10,877	16,253	11,106	14,634	10,000	15,041	10,278
1977	6.77	−0.67	15,636	10,007	14,391	9,210	15,918	10,188	16,253	10,402	15,625	10,000	15,041	9,626
1978	9.03	1.03	15,797	9,273	13,847	8,128	16,082	9,440	16,253	9,541	16,965	9,959	15,041	8,829
1979	13.31	−1.22	15,605	8,084	13,026	6,748	16,082	8,331	16,253	8,420	16,965	8,789	15,041	7,792
1980	12.41	−4.45	14,911	6,872	11,854	5,463	16,082	7,412	16,253	7,490	16,965	7,818	15,041	6,932

D. Simulations for a U.S. mixed portfolio,[c] *1971–80*

1971	3.36	10.89	11,089	10,729	13,678	13,233	11,089	10,729	13,677	13,232	10,336	10,000	13,386	12,950
1972	3.41	7.70	11,943	11,174	14,030	13,126	11,943	11,174	14,029	13,125	10,688	10,000	13,842	12,950
1973	8.80	−1.40	11,777	10,127	13,175	11,329	11,943	10,270	14,029	12,064	11,629	10,000	13,842	11,903
1974	12.20	−0.52	11,716	8,979	12,483	9,567	11,943	9,153	14,029	10,752	12,027	9,218	13,842	10,609
1975	7.01	13.59	13,308	9,531	13,503	9,671	13,567	9,717	15,177	10,870	13,661	9,784	13,842	9,914
1976	4.81	14.75	15,271	10,435	14,758	10,085	15,568	10,638	16,587	11,334	14,634	10,000	14,234	9,727
1977	6.77	−0.25	15,233	9,749	14,020	8,973	15,568	9,964	16,587	10,616	15,625	10,000	14,234	9,110
1978	9.03	3.79	15,809	9,280	13,857	8,134	16,157	9,484	16,587	9,737	16,592	9,740	14,234	8,355
1979	13.31	5.89	16,742	8,673	13,975	7,240	17,109	8,863	16,727	8,665	17,570	9,102	14,234	7,374
1980	12.41	6.82	17,882	8,241	14,217	6,552	18,276	8,423	17,017	7,842	18,768	8,649	14,234	6,560

[a]Based on the consumer price index. Inflation and security return data are from Ibbotson and Sinquefeld, *Stocks, Bonds, Bills and Inflation* (Financial Analysts Research Foundation, 1977), updated by the authors.

[b]The initial capital is $100,000; the annuity is payable with certainty for 10 years; annuity payments are made at the end of the year; *RV* is the interest rate used to determine the initial level of the projected stream of annuity payments.

[c]Mixed portfolio consists of bonds (52%), bills (29%), and stocks (19%).

is to provide a stable stream of real benefits, then a VAF must also be linked to a bills portfolio. When nominal interest rates are high and the valuation rate is low, this VAF will produce results virtually identical to those of a bills-based VA. This is the case for the Rockefeller Foundation Plan. In effect, the Rockefeller Foundation Plan functions as a defined-benefit plan in the preretirement period and becomes a defined-contribution plan at the date of the employee's retirement. This metamorphosis of defined-benefit plans into defined-contribution plans appears to have occurred extensively in Canada and may represent a market response to increased inflation uncertainty.

Because the ad hoc adjustments made by firms are never (to our knowledge) more than those necessary to offset fully the impact of inflation, it is likely that the behavior of many firms is more complicated than that suggested by the VAF. We therefore analyze a VA subject to a nominal floor and a real ceiling, in which underwriting losses and gains by the plan sponsor are banked from one period to the next. Under stylized conditions which might well be met in practice, a VAFC tied to bills would closely replicate the benefits provided by a bills-based VA. More generally, due to the interaction of the floor and ceiling with the banking provisions, it is possible for a VAFC to provide substantially more stable real benefits than a VA tied to the same asset base. Because the risk-return trade-offs available in the capital market have not changed, sponsoring firms in these cases would presumably extract compensating wage differentials from their employees *if* mean benefits were unaffected. As noted in the text, however, mean benefits will be reduced if the implicit call option (pertaining to the real ceiling) proves to be more valuable than the implicit put option (pertaining to the nominal floor).

Appendix: Description of Alternative Annuity Designs

Notation

R_t = nominal rate of return earned on the fund in year t.

RV = interest rate used to determine the base value of the annuity payment; also called the annuity valuation rate or valuation rate.

r_t = real rate of return earned on the fund in year t.

B_t = nominal benefit payment received at the end of year t.

B_0 = base value of the benefit, i.e., the value of B_1 if $R_1 = RV$.

b_t = real benefit received at the end of year t.

A_t = nominal value of the amount left in the fund at the end of year t after B_t is paid out.

P_t = Consumer price level at the end of year t with P_0 set equal to one.
T = Number of years the annuity lasts.

Terms of the Annuities

For all annuities the base value of the annuity payment is determined by

$$B_0 = \begin{cases} A_0/T & \text{if } RV = 0, \\ A_0 RV\,[1-(1+RV)^{-T}]^{-1} & \text{if } RV > 0. \end{cases}$$

We assume that benefit payments start at the *end* of the first year, so B_0 is not actually paid out but rather serves as the base value for computing the first year's benefit, B_1.

For the *standard variable annuity* the nominal benefit is

$$B_t = B_{t-1}\frac{(1+R_t)}{(1+RV)}$$

and the real benefit is

$$b_t = b_{t-1}\frac{(1+r_t)}{(1+RV)}$$

or

$$b_t = B_t/P_t.$$

For a *nominal annuity* R_t is nonstochastic, so

$$B_t = B_{t-1}\frac{(1+R)}{(1+RV)}$$

and the rate of graduation in the nominal benefit payments is

$$\frac{(1+R)}{(1+RV)} - 1.$$

Note that if $RV = R$, we have the conventional level-payment nominal annuity.

For a *purchasing power annuity*, r_t is nonstochastic, so

$$b_t = b_{t-1}\frac{(1+r)}{(1+RV)}$$

and the rate of graduation in the real benefit stream is

$$\frac{(1+r)}{(1+RV)} - 1.$$

For the *VAF*, the variable annuity with a nominal floor, the nominal benefit is given by

$$B_t = \begin{cases} B_{t-1}\dfrac{(1+R_t)}{(1+RV)} & \text{if } R_t > RV, \\ B_{t-1} & \text{if } R_t \leq RV, \end{cases}$$

and the real benefit by

$$b_t = B_t / P_t.$$

The *VAFC*, the variable annuity with a nominal floor and real ceiling, is complicated. The benefit calculation follows an iterative procedure that can be seen by a simple flowchart (fig. 11.A.1).

To create an algebraic flowchart, we need some additional notation:

K_t = amount of money in the "bank"; $K_0 = 0$.
X_t = amount of money available to increase the benefit stream.

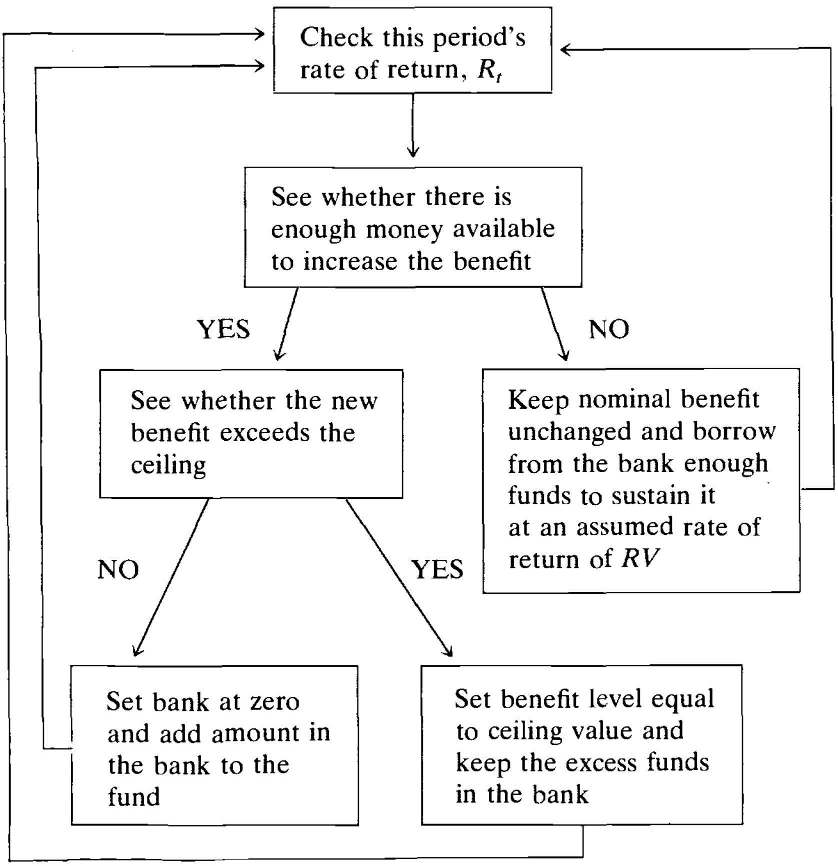

Fig. 11.A.1 Flowchart showing the iterative procedure for calculating benefit stream for a VAFC.

F_t = present value of a \$1 annuity due for $T-t+1$ years at an interest rate of RV.

$\hat{B}_t$ = benefit which would be payable in the absence of the real ceiling. The benefit calculation follows the following iterative procedure (fig. 11.A.2).

In the unnumbered table below we demonstrate how the procedure works for the VAFC based on the stocks only portfolio reported in table

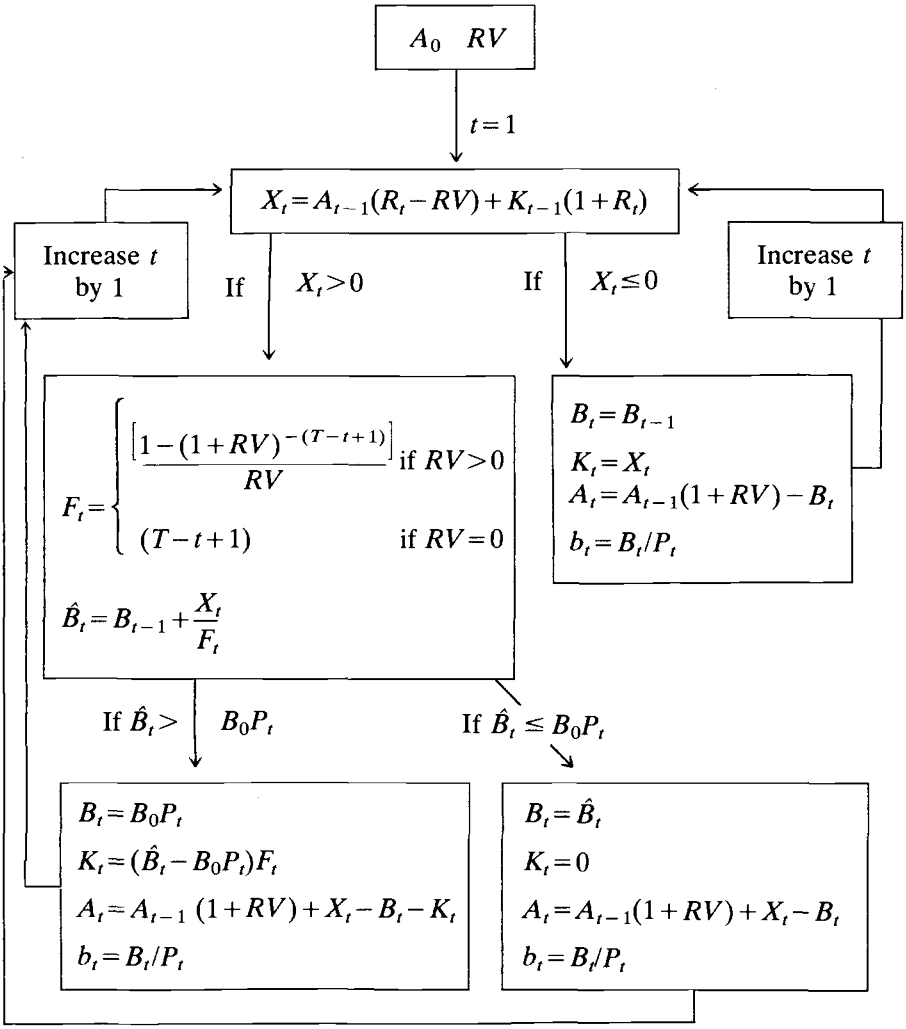

Fig. 11.A.2 Algebraic flowchart showing the iterative procedure for calculating benefit stream for a VAFC.

11.5. In this example, $A_0 = \$100{,}000$ and $RV = .05$. We present the calculation for the first three years only.

	t	R_t	P_t	X_t	F_t	$\hat{B}_t$
	0	.05	1.0000	...	...	...
1971	1	.1431	1.0336	9,310	8.1078	14,099
1972	2	.1898	1.0688	20,180	7.4632	16,090
1973	3	−.1466	1.1629	−3,274	6.7864	...

	$B_0 P_t$	B_t	K_t	A_t	b_t
	...	12,950	0	100,000	12,950
1971	13,386	13,386	5,781	95,142	12,950
1972	13,842	13,842	16,774	89,464	12,950
1973	...	13,842	−3,274	80,095	11,903

Notes

1. For simplicity, the discussion proceeds as if the retiring plan member's sole source of wealth is his claim to a private pension. If he has other sources of wealth, then the risk-return characteristics of his pension benefit must be analyzed in the context of his total portfolio. See M. Feldstein and L. Summers's chapters in this volume for a discussion of the extent to which households may be able to diversify away the inflation risk implicit in nominal pension benefits.

2. A separate provision in the Rockefeller Foundation Plan provides that the cost-of-living adjustment equal at least 4% if the inflation rate as measured by the consumer price index exceeds 4%. Otherwise, however, the floor is that cited in the text (Heaton 1980). There is no reference to a ceiling on the size of the cost-of-living adjustments. Subsequent discussion of the Rockefeller plan ignores the separate floor provision.

3. TIAA, which manages one of the largest pension plans in the United States, offers its members two annuity designs that resemble VAs containing a nominal floor. The older of these, the traditional TIAA annuity, has a guaranteed minimum nominal floor. This floor is embodied in the guaranteed return of 3%. Unlike the Rockefeller plan (which is characteristic of the hybrid annuities examined at length in the text), this nominal floor does not ratchet upward over time. The asset base in the TIAA annuity consists of a portfolio dominated by long-term bonds, mortgages, and other fixed-interest loans. TIAA pays to its beneficiaries a variable benefit which has been smoothed relative to what it would be under a standard VA design by ignoring unrealized capital gains and losses on these dollar-fixed investments. One consequence of this smoothing is that the guaranteed rate cited previously is far less likely to bind. Another consequence is that there can be cross-subsidization of different generations of annuitants. Currently, for example, TIAA is paying a total nominal rate of return of 11% to new retirees, while the risk-free nominal rate of return in the capital markets is well in excess of that. The interest rate used to determine the initial benefit (called the Assumed Interest Rate, or AIR) is also equal to 11%, so the expected nominal benefit stream is level. Recently, TIAA has offered its members an alternative design (called the Graded Benefit Payment Method), which differs in two respects from the older one. First, the expected nominal benefit stream has been given an upward tilt by using an AIR of 4% to

determine the initial benefit level. Second, the guaranteed nominal floor ratchets upward whenever the interest rate declared in each period actually exceeds 4%. Earnings above 4% are credited at the end of the year and—in effect—are used to purchase an additional TIAA annuity with its own guarantees and dividends. (The interest rate used in calculating the increase in the nominal floor is the guaranteed rate of 3%.) It is worth noting that TIAA has been shortening the average maturity of its portfolio in recent years. If this process were to continue, the TIAA graded-payment annuity would come to look more like the Rockefeller plan annuity.

4. Ontario's Select Committee on Pensions (1981) has recommended that the use of excess investment earnings to provide inflation protection be mandated by law. No reference is made to floors and/or ceilings in the proposed scheme, which the analysis in this chapter shows to be of crucial importance.

5. For the purposes at hand, there is no advantage in explicitly incorporating mortality factors into the analysis. Mortality is thus ignored in all of the illustrations presented in this chapter.

6. More precisely, each year's real benefit would be equal to the previous year's benefit divided by 1.05.

7. In fact, real equity returns appear to be negatively correlated with unanticipated inflation, as noted by Bodie (1976) and Pesando and Rea (1977).

8. The mixed portfolio consists of 52% bonds, 29% bills, and 19% stocks. We do not refer to this as an efficient portfolio for two reasons. First, our portfolio proportions are derived from a single-period variance-minimization procedure which ignores the serial correlation in bill returns. Second, the efficiency of an annuity for a particular household can only be determined if we know all of the household's other assets and liabilities.

9. As noted by Bodie (1982), the mean realized real return on bonds is negative during this period. The mean real return on bonds was set equal to an annual rate of 3%, whose continuous time equivalent is 2.956%, while the other parameters were based on the observed means, variances, and covariances.

10. First-order autoregressions were performed for the logarithms of the real annual wealth relatives of bills, bonds, and stocks. The results are as follows:

$$\text{Bills:}\quad r_t = \underset{(.205)}{-.044} + \underset{(.136)}{.768}\, r_{t-1},\ R^2 = .559,\ \text{SEE} = 1.04\ (\%\ \text{per year});$$

$$\text{Bonds:}\quad r_t = \underset{(1.493)}{-1.619} + \underset{(.207)}{.261}\, r_{t-1},\ R^2 = .056,\ \text{SEE} = 7.64;$$

$$\text{Stocks:}\quad r_t = \underset{(3.849)}{5.847} - \underset{(.201)}{.021}\, r_{t-1},\ R^2 = .0004,\ \text{SEE} = 19.28.$$

Standard errors are in parentheses.

11. The real benefit in year t is given by $b_t = B_0 \Pi_{i=1}^{t}[e^{\tilde{r}_i}/(1+RV)]$, where B_0 is the initial projected annuity payment, $\tilde{r}_i$ is the realization of the stochastic logarithic real return in year i, and RV is the annuity valuation rate. Since b_t is the product of lognormal variates, it is also lognormally distributed: $\log(b_t) = \log(B_0) + \Sigma_{i=1}^{t} \tilde{r}_i - t \log(1 + RV)$. Because we have chosen RV such that $E(r_i) = \log(1 + RV)$, the median value of b_t equals B_0 for all t. By contrast, the mean value of $b_t = B_0 e^{\frac{1}{2}\sigma_t^2}$ where σ_t^2 equals the variance of $\Sigma_{i=1}^{t} \tilde{r}_i$. If there is no serial correlation in the $\tilde{r}_i$ series, $\sigma_t^2 = t\sigma^2$ where σ^2 is the variance of r_i in a single year.

Rea (1981) also discusses the design of a variable annuity which produces a payments stream which is expected to remain constant in real terms.

12. The first-order annual autoregression, based on the consumer price index, is

$$\pi_t = \underset{(.597)}{.794} + \underset{(.117)}{.902}\pi_{t-1}, R^2 = .750,\ \text{SEE} = 2.003\ (\%\ \text{per year}),$$

where $\pi_t = \log(P_t/P_{t-1})$ and P_t is the price level at time t.

13. Assume π_t follows the first-order autoregressive process, $\pi_t = \alpha + \rho\pi_{t-1} + \epsilon_t$, where ϵ_t is distributed $N(0, \sigma)$. Then $\pi^* = \alpha/(1 - \rho)$ is the steady-state rate of inflation. Note that $\log(P_t) = \log(P_0) + \Sigma_{i=1}^{t} \tilde{\pi}_t$ where $\tilde{\pi}_t$ is the realization of the inflation process. Let $P_0 = 1$ and let $\pi_0 = \pi^*$. Then median of $\log(P_t) = t\pi^*$; median of $P_t = e^{t\pi^*}$; and variance of

$$\log(P_t) = \sigma_t^2 = \frac{\sigma^2}{(1-\rho)^2}\left\{t - \rho\frac{(1-\rho^t)}{(1-\rho)}\left[2 - \rho\frac{1-\rho^t)}{(1+\rho)}\right]\right\}.$$

If B_t is the known nominal benefit in period t, then the real benefit $b_t = B_t/P_t$. Thus $\log(b_t) = \log(B_t) - \log(P_t)$. Let $B_t = B_0e^{gt}$, where g is the rate of graduation, and let $\mu_t = (g - \pi^*)t$. Median $b_t = B_t \div$ median $P_t = B_0e^{\mu_t}$. Since B_t is graduated so as to increase at the anticipated inflation rate, $\mu_t = 0$ and median $b_t = B_0$. Since B_t is nonstochastic, variance of $\log(b_t)$ = variance of $\log(P_t) = \sigma_t^2$. Mean $b_t = B_0e^{1/2\sigma_t^2}$ and the variance of $b_t = B_0^2e^{\sigma_t^2}(e^{\sigma_t^2} - 1)$.

14. The continuously compounded nominal interest rate (R) is thus equivalent to an annual rate of 11%, since the (annual) steady-state rate of inflation built into the illustration is 8%.

15. *If* there were no nominal floor on the pension benefits, any decision to channel pension fund reserves exclusively into bills or their equivalent would have an unambiguous interpretation. Workers, who presumably cannot diversify away the inflation risk inherent in nominal pension benefits, are sufficiently risk averse that they will pay the price (i.e., a low expected real return on their pension wealth) of stabilizing their real retirement incomes.

16. Remember that the standard deviation of the continuously compounded real bill return is only 1.52% per annum, so that the expected real return of zero is about two standard deviations less than 3%.

17. The plan sponsor could underwrite VAFs or VAFCs on either a pay-as-you-go or a fully funded basis. This issue is not explored in this chapter.

18. If the sponsor provides a defined-benefit plan, the lump sum necessary to purchase the requisite annuity could be made available to the employee, who could then choose his preferred VA. If the promised pension is purely nominal (and the firm has no tradition of providing ad hoc adjustments), then discounting the promised payments by the risk-free nominal rate R (as well as by mortality) would identify the lump sum to be offered to the employee.

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Among all the institutions in the financial sphere of the economy, probably none is more seriously affected by inflation than private pensions. This impact does not merely reflect the redistribution from creditors to debtors that is generally supposed to accompany inflation. Indeed, redistribution need not occur at all to the extent that inflation is fully anticipated and is accompanied by an offsetting rise in nominal rates. But the pension contract, just like the fully amortized mortgage, stands to be drastically distorted by inflation even if it is largely or fully anticipated, because both contracts were designed in nominal terms for a world of stable prices.

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The typical defined-benefit pension plan was designed to provide the pensioner with a level fixed *real* payment, equal to some fraction of his pay in the neighborhood of his retirement date, reflecting the number of years of service. In the absence of inflation, that intended goal is achieved by awarding to the annuitant a fixed nominal pension, at the appropriate level, for the rest of his life.

But in the presence of inflation this arrangement turns out to produce results which are quite different and less favorable from the point of view of the beneficiary than the intended ones, in three distinct ways: (1) though the stream of benefits starts at the intended level, its purchasing power will thereafter be reduced by the rise in the price level caused by inflation—the higher the inflation over the retirement period, the smaller the aggregate real value of benefits received by the pensioner; (2) the existence of inflation means that the real income stream from the pension received by the annuitant continuously decreases in time, and the beneficiary gets poorer at a rate which is faster the larger the rate of inflation, whether inflation is anticipated or not; and (3) if, as is usually the case, there is uncertainty about the future path of inflation, then the path of real benefits is not only lowered and tilted down, but also becomes uncertain.

Consequence 1 seems to have largely escaped notice, in part perhaps because it would not be expected to arise for a defined-contribution plan. As for the remaining two, it is sometimes suggested that a rational and prudent beneficiary could readily avoid them by saving a portion of his annuity in the early part of his retirement and thus accumulating reserves to be used to support consumption above the annuity in the later part of his life. But this suggestion clearly has very little merit. Given the uncertainty about the future inflation, it would be difficult for the pensioner to make adequate reserves for the future. Furthermore, even if he did know the future inflation, he would still not know the time of his death and therefore the appropriate amount of reserves to make. To put it another way, any reserves he might accumulate would not benefit from the insurance against the risk of life which is an essential feature of a life annuity.

These three problems, particularly problems 2 and 3, could be effectively and easily handled if there existed in the market "indexed" instruments which offered the lender a constant "real rate" over the life of the loan, e.g., by paying a fixed rate on a principal whose nominal value was periodically revalued so as to maintain a constant purchasing power, or, equivalently, by promising a floating nominal rate equal to a constant (the real rate) plus the rate of inflation. Such instruments would enable the pension fund, by investing in them in appropriate amounts and maturities, to provide the annuitant with an indexed nominal annuity which would guarantee a constant real level of benefits based on the fixed real rate promised by the indexed instrument.

Of course that real rate would, presumably, be lower than the going nominal rate for corresponding maturities, to an extent reflecting anticipated inflation over the life of the instruments. This means that to provide a benefit rate at the same initial level as without inflation would require an accumulation higher than that needed at present on the basis of inflation-swollen nominal rates—though presumably no higher than would have been called for in the absence of inflation.

This simple solution is, unfortunately, not available at present since indexed bonds, or similar indexed instruments, cannot be purchased in the market, nor are they likely to be readily available in the foreseeable future. The chapter by Bodie and Pesando makes a valuable contribution by describing and testing a number of alternative arrangements which, even in the absence of indexed assets, could ensure the annuitant an outcome similar to that which would have occurred with stable prices—a constant level payment in real terms through his life. Ideally the result should be achieved with no, or little, change in the magnitude and nature of the risk borne by the provider of the annuity.

Bodie and Pesando focus on a number of solutions, most of which are variants of one basic design—a design which also provided the basis for proposed solutions to the mortgage problem, as set out in the M.I.T. study, "New Mortgage Designs for Stable Housing in an Inflationary Environment."

Focusing for simplicity of exposition on the case in which the annuitant receives an annuity certain of, say, t years, as the authors themselves do, this basic design can be summarized as follows. First, the nominal annuity is recomputed at regular prearranged intervals, say yearly, by applying to the annuitant's endowment remaining at the beginning of the period, the standard annuity formula, for a number of years equal to the remaining life of the annuity, and using a fixed, agreed interest rate (which the authors call the valuation rate, RV). Second, the endowment remaining to the annuitant at any point is invested in some stated instrument, for example, T-bills or long-term bonds or corporate equities, and so on. Third, the endowment remaining at the beginning of any period is computed by taking the endowment of the previous period, crediting to the annuitant the return from the investment in the chosen asset, and subtracting the annuity paid in that period. In the case of a *life* annuity, the procedure would be essentially the same except that in the first step one would rely on the life annuity formula with an interest rate RV.

It is clear that under this plan the annuity actually received by the beneficiary in nominal terms (or, for that matter, in real terms) will vary from period to period—hence the name of "variable annuity," or VA. But it can be shown that the real path of the annuity, over the life of the contract, satisfies one basic recursive relation. This relation, which can be deduced from a formula presented in the appendix to the chapter, can be stated as follows: the real annuity (at the end) of year t is equal to that of the year $t-1$ multiplied by the factor $(1 + i_t - p_t)/(1 + RV)$, where i_t is the nominal rate of return on the chosen investment and p_t is the rate of inflation in year t.

To see the implications of this type of arrangement, suppose first there existed some instrument whose nominal return could be counted on to be always equal to a constant plus the rate of inflation—or equivalently, whose real return was a constant. It is apparent that, by choosing that instrument as the investment vehicle and by choosing for RV the constant real rate, one would be able to guarantee the beneficiary an annuity which would remain constant in real terms at the initial level throughout the duration of the contract—that is, in effect, a fully indexed pension.

Suppose next the endowment were invested in an instrument whose real return in each period was uncertain but which fluctuated around a known expected value. If RV is set equal to this expected return, the result is a variable annuity, which, in real terms, would tend to fluctuate around the initial level, with a variability around that value depending both on the variability and the serial correlation of the real return on the instrument. If, on the other hand, RV was chosen to *exceed* the expected real return, then the annual payment could be expected to fluctuate around a path declining at a rate equal to the difference between RV and the expected real return. Conversely, the choice of RV *below* the expected real return would lead to a path of expected payments rising in time.

Thus, by relying on different possible assets as the investment vehicle and by appropriate choice of RV, one can construct a whole family of variable annuities differing from each other in terms of the expected real average outcome, in terms of the variability of possible outcomes around that level, and in terms of tilt of the real payment path. From finance theory and empirical evidence, one would generally expect that instruments offering a higher expected return, and thus promising a higher expected average level, would also be characterized by greater variability of the outcome path and also greater uncertainty of the average outcome over finite periods. These inferences are supported by the results reported by Bodie and Pesando for a variety of alternative investment vehicles.

It is widely supposed that very short-term loans of high quality, such as short-term Treasury bills, tend to yield fairly stable real returns, year after year, more stable than any other standard instrument. This view receives striking confirmation in Bodie and Pesando's tests. The results reported in table 11.2 suggest that a VA based on the one-month Treasury bill could be expected to provide a remarkably stable annual real payment. Relying on the parameters of the observed distribution of returns in the period 1953–80, the authors find that the standard deviation of the annual payment is but 5% of the mean after 5 years and remains below 15% even after 15 years. Equally impressive are the results of table 11.5, which reports the annual payments that would actually have been realized for the 10-year period from 1971 to 1980. Even in this troubled period the largest deviation from the starting level is just about 10%.

Unfortunately this highly desirable stability obtained with the Treasury bill as the investment vehicle is acquired at the cost of an extremely low real return which, for the period 1953–80, is actually estimated at zero. For the more recent period 1971–80 the real return is even negative in most years and on the average for the period as a whole, and as a result the real annuity drifts down, even though *RV* is taken as zero.

The authors have tested several other investment vehicles and notably bonds, stocks and a mixed portfolio consisting of bonds, stocks, and bills designed to minimize variance for the given mean. Relying on the 1953–80 experience, these other instruments imply considerably higher initial and expected average annual payments, but at a cost of an impressively larger variability of outcomes. In the case of common stock for instance, the expected annual payment is 70% larger than in the case of Treasury bills, but the standard deviation goes up by 15 times! Even with the minimum risk portfolio, the level of expected return goes up by but a quarter, while the standard deviation increases by a factor of 200%–300%.

As shown in table 11.5, the relative attractiveness of these alternative instruments would have been even lower during the seventies, because the average return was in all cases substantially below the historical performance, used in table 11.2. Indeed, the average return was lower than that of T-bills, the only exception being stocks, where the difference was not very large. Thus, in addition to the great variability of annual returns exhibited by these types of VAs, the annuitant would have suffered from a markedly declining overall trend if *RV* had been chosen at a level around the historical return of each asset. In the case of common stock, for instance, it can be seen from the simulation of table 11.5 that even using an *RV* of 5%, somewhat below the historical average of 7%, the annual payment tends to fall by around 40% of its initial level by the end of the period. Incidentally, the results of this particular simulation—a VA backed by common stocks with an *RV* equal to 5%—is of particular interest since it provides a good approximation to the outcome of the only type of VA that was actually in existence during the seventies. This was the VA backed by the portfolio of CREF, for which a value of *RV* of 4% was used—quite close to the 5% assumed in the table. It is apparent that the experience with this instrument was hardly a satisfactory one and that arrangements of this type should not be forced on annuitants without also giving them the option of a less risky alternative such as that consisting of the T-bill-backed VA.

Bodie and Pesando have also explored a few other possible plans which, however, do not strike me as promising. The first is the so-called graduated payment plan in which the *nominal* annuity increases through time at some prearranged rate, intended to match the inflation rate. I have serious objection to graduation for pensions, as I do for mortgages. A graduated annuity is still a nominal contract, which cannot be counted on to eliminate much of the risk of the real outcome when inflation is highly variable. Table 11.2 shows that the graduated payment produces a standard deviation of real outcomes roughly twice as large as that produced by a T-bill-backed VA, even though it is assumed that the graduation exactly coincides with the rate of inflation over the period of the contract. In practice it would be impossible to match the two closely, especially in the case of long contracts. Thus, while the graduation may somewhat reduce the tilt associated with a conventional pension, it might also conceivably increase it if inflation turned out to be sufficiently smaller than the graduation rate.

Finally, Bodie and Pesando have considered the possibility of reducing some of the risk that is borne by the annuitant under a variable annuity by giving him a guarantee that his nominal annuity will never decline, along the lines of the so-called Rockefeller plan. But, as the authors recognize, this guarantee would be nonoperational for the kind of T-bill-backed VA which strikes me as the most feasible arrangement. At least if RV is taken as zero, as would seem appropriate, there is no possibility of the nominal annuity received by the beneficiary ever declining, except if the bill rate itself were negative, which presumably is impossible. Even if RV was taken as 3%, as seems to be the case in the Rockefeller plan, the probability of the nominal rate falling short of 3% seems remote in a period of high inflation. If, on the other hand, inflation should become even lower or somewhat negative, then it is very questionable whether there should be a clause guaranteeing the annuitant against a decline in the nominal payments.

Another possibility considered in this chapter is that of a ceiling on the real level of the annuity with the returns earned by the endowment of the annuitant and not paid to him, returned to the pension fund as a compensation for the risk it is taking in guaranteeing a floor. I have already indicated why I do not regard a floor as a very interesting modification, and there seems to be no justification left for a ceiling either.

As the authors explicitly recognize, the several alternatives explored in this chapter by no means exhaust all possible designs, but I share their view that those examined are by and large the most promising. There is, nonetheless, one more class that deserves brief mention, namely, the class of arrangements that gives the annuitant an option to switch from one instrument to another or from one type of plan to another in the course of his retirement. This could involve switching not only between stock-backed and bill-backed VAs but also between either of these instruments and short-term fixed graduation. For instance, the annuitant could elect to invest in, say, a 3-year nominal instrument choosing simultaneously a lower value for RV with the difference reflecting the expected inflation over the next 3 years. This instrument produces, for the initial 3 years, a sure nominal stream with a fixed graduation at a rate equal to the difference between the nominal rate and RV, and hence related to expected inflation. At the end of 3 years, the annuitant would have an opportunity of choosing another intermediate-term graduation consistent with inflation expected then, but he could also switch to some other form of contract.

Quite generally, I see merits in giving the annuitant as large a choice of instruments and as wide an opportunity to switch between them as is consistent with administrative costs and with his ability to acquire a full understanding of alternatives open to him.

Let me conclude by congratulating Bodie and Pesando for having provided us with an extremely useful analysis which, one may hope, will contribute to significantly advancing the case for a pension design consistent with an inflationary environment.

References

Bodie, Z. 1976. Common stocks as a hedge against inflation. *Journal of Finance* 31 (May): 459–70.

———. 1980. An innovation for stable real retirement income. *Journal of Portfolio Management* 7 (Fall): 5–13.

———. 1982. Investment strategy in an inflationary environment. In *The changing roles of debt and equity in financing U.S. capital formation*, ed. Benjamin M. Friedman. Chicago: University of Chicago Press.

Heaton, H. 1977. Inflation protection for retired employees. *Harvard Business Review* (September/October), pp. 3–4.

———. 1980. Indexing pension payments to inflation without material increases in expected plan costs: The Rockefeller Foundation Plan. Statement to the President's Commission on Pension Policy, Washington, D.C., January 11.

Modigliani, F., and Lessard, D., eds. 1975. *New mortgage designs for stable housing in an inflationary environment.* Federal Reserve Bank of Boston Conference Series, no. 14. Boston: Federal Reserve Bank of Boston.

Ontario Select Committee on Pensions. 1981. *First report.* Toronto: Government of Ontario.

Pesando, J. E. 1981. Employee valuation of pension claims and the impact of indexing initiatives. NBER Working Paper no. 767.

———. 1982. Investment risk, bankruptcy risk and pension reform in Canada. *Journal of Finance* 37 (June): 741–49.

Pesando, J. E., and Rea, S. A., Jr. 1977. *Public and private pensions in Canada: An economic analysis.* Toronto: University of Toronto Press (for the Ontario Economic Council).

Rea, S. A., Jr. 1981. Consumption stabilizing annuities with uncertain inflation. *Journal of Risk and Insurance* 47, no. 4 (December): 596–609.

Task Force on Retirement Income Policy. 1979. *The retirement income system in Canada: Problems and alternative policies for reform. Ottawa: Government Printer.*

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