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5 Consumption Taxation in a General Equilibrium Model: How Reliable Are Simulation Results?

B. Douglas Bernheim, John Karl Scholz, and John B. Shoven

5.1 Introduction

For years, various economists have argued that the taxation of capital income has a variety of detrimental effects, including the distortion of intertemporal decision making and the reduction of saving and capital accumulation. Many have called upon policymakers to abandon the current system of income taxation and to adopt a consumption (or wage) tax in its place. Recent concerns about the low level of saving in the United States has rekindled interest in the possibility of moving in this direction.

Unfortunately, the effects of consumption taxes are extremely complex and hard to evaluate. On theoretical grounds, the desirability of this alternative is not clear. It has long been recognized that, while consumption taxation reduces intertemporal distortions, it also contributes to the distortion of labor-leisure choices. A priori, there is no particular reason to believe that either effect is quantitatively more important than the other.

It is therefore necessary to evaluate consumption taxes on the basis of models that are somewhat "realistic." This observation has led to the emergence of a large number of papers (e.g., Summers 1981; Auerbach and Kotlikoff 1983; and Fullerton, Shoven, and Whalley 1983) that study various reform proposals in the context of reasonably complex models. These papers share an important feature: the impact of consumption taxation is determined computationally, rather than analytically.

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In general, this literature suggests that our current policy of taxing income is rather costly. Summers (1981) found that a complete shift to consumption taxation might raise steady-state output by as much as 18% and consumption by 16%. Auerbach and Kotlikoff (1983) suggest that the steady-state capital-to-output ratio would more than double. Fullerton, Shoven, and Whalley (1983) studied the imposition of a progressive consumption tax and found that it would result in gains to the economy of roughly 1 percent of the present value of future national income.

Unfortunately, many economists have reservations about these general equilibrium calculations. There is often disagreement about the appropriate values of key parameters. In addition, these computations are usually based upon a large number of parameters, many of which are known with very little precision. It is certainly possible that the cumulative impact of uncertainty concerning these parameters may dwarf the quantitative effects predicted by these models.

This problem has been widely recognized in the general equilibrium literature. There are currently four different ways to deal with uncertainty about key parameters. One option, generally taken in the public finance and international trade literatures, is to conduct sensitivity analysis by varying the key parameters of the model, usually one at a time. At best, such calculations can illustrate policy effects under a few alternative sets of beliefs about the appropriate parameter values. It does not allow one to describe the quantitative importance about uncertainty concerning underlying parameters. A second option is to conduct Monte Carlo simulations. Unfortunately, this is extremely time consuming and expensive, and in practice, it has not been done. A third option is to take a discrete approximation to the underlying distribution of input parameters and *systematically* explore the sensitivity of the model to the choice of parameters. This approach has been developed and discussed in Harrison and Kimbell (1985), Harrison, Jones, Kimbell, and Wigle (1989), Harrison and Vinod (1989). Fourth and finally, for a linear model, if one knows the variance-covariance matrix associated with the underlying parameters, then it is possible to calculate *exact* variances for the model's output. For nonlinear models, one can approximate the variances of outputs through linearization. This approach was first suggested by Pagan and Shannon (1985).

In this paper, we elaborate on the advantages of the Pagan-Shannon approach. We then apply a variant of their methodology to the study of consumption taxation. Our basic objective is to answer the following question: Do we know enough about the underlying parameters in large-scale general equilibrium models to have any confidence about the effects of consumption taxes in these models?

We use the Shoven-Whalley computational general equilibrium model to study the impact of a switch to consumption taxation on four variables: labor supply, output, saving, and a measure of utility. We provide separate results for short-, medium-, and long-run effects, as well as for the overall impact on utility. Our calculations are based upon two different sets of beliefs concerning the precision with which the underlying parameters are known. We refer to these cases as "optimistic" (the uncertainty concerning parameters is small), and "pessimistic" (the uncertainty concerning parameters is large).

Our results are mixed. In the "optimistic" case, one can have a fairly high degree of confidence in many aspects of the basic simulation. Almost all short- and medium-run effects are known with reasonable precision (one can at least rule out the possibility that the effect is zero or of the opposite sign). In contrast, the standard errors of the long-run effects are almost as large as the associated central estimates. Nevertheless, the total (present discounted value) impact of consumption taxation on utility is estimated quite precisely—the calculated welfare gain (roughly \$600 billion) is approximately three times the size of its standard error.

Results based upon pessimistic beliefs are much less encouraging. Short-run effects on saving and utility, as well as medium-run effects on saving and output, are tied down fairly precisely. Unfortunately, one cannot have much confidence in the sign of any other effect. The calculated welfare gain turns out to be approximately 1.5 times its standard deviation. While this lends some support to the case for consumption taxation, it is hardly a ringing endorsement. Our results therefore emphasize the need for more precise econometric estimates of various key parameters used in the Shoven-Whalley model.

This paper is organized as follows. Sections 5.2 and 5.3 review the methodology and the Shoven-Whalley model, respectively. Section 5.4 considers uncertainty in assumed parameter values and proposes optimistic and pessimistic beliefs. Section 5.5 contains simulation and sensitivity results. The paper closes with a brief conclusion.

5.2 Methodological Framework

Computational general equilibrium models employ two types of inputs: a vector of economic parameters (such as price elasticities), henceforth labelled β, and a vector of policy parameters (such as tax rates), henceforth labelled θ. By solving for equilibria, one maps these parameters to outcomes. We will summarize outcomes as a vector of endogenous variables (such as labor supplies, production decisions, utilities, and so forth), henceforth labeled Y. The relationship between inputs and outputs can be summarized as some highly complicated implicit function, $G(\theta, \beta, \gamma)$.

Note the presence of the parameter vector γ in the function $G(\ )$. We will refer to γ as the "calibration" parameters. In practice, the value of γ is not taken from econometric studies, but rather is chosen to replicate base-case data. More formally, for any outcome vector Y and parameters β and θ, let $g(\theta, \beta, Y)$ be defined as the implicit solution to

$$Y = G(\theta, \beta, g(\theta, \beta, Y)).$$

Given some initial data Y_0, a vector of initial policy variables, θ_0, and an estimate $\hat{\beta}$ of the policy parameter vector β, one calibrates the model by setting $\gamma = g(\theta_0, \hat{\beta}, Y_0)$.

Typically, one is interested in the effect of some policy experiment on the equilibrium value of Y. Suppose this policy experiment entails changing the value of θ from its initial value of θ_0 to an alternative value, denoted θ_1. Taking the initial state and policy parameters as given, the change in Y can be written as a function of $\hat{\beta}$:

$$\begin{aligned} (1) \qquad \psi(\beta) &= G(\theta_1, \hat{\beta}, g(\theta_0, \hat{\beta}, Y_0)) - G(\theta_0, \hat{\beta}, g(\theta_0, \hat{\beta}, Y_0)) \\ &= G(\theta_1, \hat{\beta}, g(\theta_0, \hat{\beta}, y_0)) - Y_0. \end{aligned}$$

For a specific model, set of base-case data, and policy experiment, the function Ψ summarizes the entire process of going from economic parameters to conclusions.

The usual research strategy is to obtain $\hat{\beta}$ from econometric studies and then evaluate Ψ at this specific set of point estimates. With a few notable exceptions, uncertainty that is reflected in the estimated standard errors of $\hat{\beta}$ is completely ignored. As a result, computable general equilibrium (CGE) exercises typically provide little information about the degree of confidence that one can have in the results.

In the past, CGE practitioners have eschewed costly Monte Carlo simulations and have attempted to document the robustness of their results in one of two ways. We refer to the first approach, taken in the public finance and international trade literatures, as traditional sensitivity analysis. By varying a few key parameters, it is possible to obtain a general feel for whether central qualitative results depend on the specific point estimate of β. Unfortunately, this approach suffers from a variety of problems. Lacking a formal methodological basis, it is inherently imprecise. Specific information contained in the variances of parameter estimates is simply ignored. The results of standard sensitivity exercises are very difficult to evaluate and summarize: one typically varies one parameter at a time, and there is no basis for aggregating sensitivity over separate parameters. One is forced by expositional and computational considerations to limit the sensitivity analysis to a relatively small set of parameters.

A second approach, taken by Harrison and Kimbell (1985), and Harrison et al. (1989), takes a discrete approximation of the underlying parameters of the model and systematically explores the possible combinations of input parameters. A distinction is drawn between unconditional and conditional systematic sensitivity analysis (USSA and CSSA). With USSA all potential combinations of the discrete approximation of parameters are explored. Under CSSA each parameter is altered assuming all other parameters retain their central values. With even a modest number of parameters and rough approximation of a parameter's distribution, the number of required solutions for USSA becomes prohibitive,[1] consequently, CSSA is generally performed.

Conditional SSA is a major improvement over standard sensitivity analysis. Its primary limitation is that, to conduct sensitivity analysis for alternative beliefs about the distribution of $\hat{\beta}$, one must start from scratch. In most cases, the assumed distribution of $\hat{\beta}$ is somewhat arbitrary. Different individuals may well have different beliefs about the precision of knowledge concerning any economic parameter. When one publishes the CSSA based upon a specific set of beliefs, readers with significantly different beliefs may not be persuaded by the results.

An alternative, seldom-used approach has been proposed by Pagan and Shannon (1985). If one knows the variance-covariance matrix for parameter inputs, then it is possible to calculate *exact* confidence intervals for linear CGE models. This observation suggests that one can calculate approximate standard errors for nonlinear models by linearizing around base-case equilibria.[2]

The practice of approximating standard errors through linearization can be justified formally, as follows. Suppose that we obtain $\hat{\beta}$ from econometric studies. In all but a few cases, only the asymptotic distribution of the estimate will be known. Suppose then that $\hat{\beta}$ is a consistent estimate of the true parameter value, β^*, and that its distribution is asymptotically normal. Let Σ denote the variance-covariance matrix for this asymptotic distribution. Suppose further that the function Ψ has continuous second derivatives (this is usually easy to guarantee—see the literature on regularity, e.g., Kehoe 1983). Then it follows that the distribution of $\Psi(\hat{\beta})$ is asymptotically normal, with variance-covariance matrix $\Psi_\beta \Sigma \Psi'_\beta$ (where the $[i, j]$th element of Ψ_β is the derivative of Ψ_i with respect to β_j, evaluated at β^*).

We can use this result directly to obtain an asymptotic variance-covariance matrix for our estimate of the policy's impact, $\Psi(\hat{\beta})$. Specifically, we calculate Ψ_β numerically at the initial parameter values. Note from equation (1) that we need only compute the derivatives of $G(\theta_1, \hat{\beta}, g(\theta_0, \beta, Y_0))$ with respect to β at $\hat{\beta}$ (because of the calibration, the second term in the formula for Ψ is always equal to Y_0, and its derivative is therefore zero). Accordingly, we may proceed as follows. First, calibrate the model using the base-case equilibrium. Second, find the equilibrium for the "revised" case (i.e., the equilibrium after the policy change). Third, vary a parameter slightly, recalibrate, and, starting from the revised case, recalculate equilibrium. Use the results of the "perturbed" case along with the revised case to compute the derivative of each output variable with respect to that parameter. Repeat for all parameters that are to be treated as uncertain. Finally, construct the variance-covariance matrix for the policy effect by performing the matrix multiplication described at the end of the preceding paragraph.

The procedure described above is justified whenever the parameter estimates β are known to be consistent and asymptotically normal. Unfortunately, as a practical matter this condition is rarely satisfied. On one extreme, CGE practitioners often discover that certain parameters have not been estimated econometrically. In contrast, the literature often contains many attempts to measure other parameters, and the existing estimates are rarely consistent across studies. In such cases, the analyst must exercise "casual Bayesianism," forming subjective beliefs based upon priors, indirect evidence, and judgments about the relative merits of different studies.

When Σ summarizes subjective beliefs rather than the second moments of an asymptotic distribution, this procedure for obtaining an approximate variance-covariance matrix for $\Psi(\hat{\beta})$ requires some reinterpretation. Suppose in particular that we approximate Ψ with a linear function F that is simply the first-order Taylor series expansion of Ψ around $\hat{\beta}$. Given Σ, it is then a simple matter to calculate the variance-covariance matrix for the distribution of $F(\hat{\beta})$. As long as the curvature of Ψ is not too great—or the variances of $\hat{\beta}$ are not too large—this will provide a good approximation to the variance-covariance matrix for the distribution of $\Psi(\hat{\beta})$.

Of course, if the curvature of Ψ is significant—or the variances of $\hat{\beta}$ are sufficiently large—the distributions of $F(\hat{\beta})$ and $\Psi(\hat{\beta})$ will be quite different, and the procedure will provide a rather poor approximation. It is possible to remedy this problem by modifying the Pagan-Shannon approach. Specifically, one would use higher-order Taylor series approximations to Ψ. In many cases, a second-order approximation may suffice. Thus, we write

$$H(\hat{\beta}) = \Psi(\hat{\beta}^*) + \Sigma_i(\hat{\beta}_i - \beta_i^*)\Psi_i(\beta^*) + \sum_i \sum_j (\hat{\beta}_i - \beta^*_i)(\hat{\beta}_j - \beta^*_j)\Psi_{ij}(\beta^*).$$

To evaluate the mean and variance of this expression, one must make some assumptions about the distribution of $\hat{\beta}$. First, we assume that this distribution is normal. This assumption is critical, since the variance of $H(\hat{\beta})$ will typically involve higher-order moments of the distribution of $\hat{\beta}$. Unfortunately, we know very little about these higher order moments, and intuition is a poor guide. However, with the normal distribution all higher-order moments can be expressed as functions of variance. The same kind of argument certainly applies to any two-parameter family of probability distributions. However, to the extent any parameter choices are influenced by econometric estimates, we would favor the normal distribution. While finite sample estimate of the parameters will not typically be normally distributed, asymptotic theory suggests that the normal often provides the best approximation.

Our second assumption is that the $\hat{\beta}_i$s are distributed independently. When $\hat{\beta}$ is taken from econometric estimates, this assumption is frequently justified.

Estimation error is a function of the idiosyncratic shocks in a particular set of data. To the extent shocks are independent across observations, they will also be independent across data sets. Parameter estimates based upon two distinct sets of data will therefore generally be uncorrelated. To illustrate, suppose that we have a single cross-section data set consisting of observations of the economic decisions of distinct households. Suppose these decisions are affected by idiosyncratic, unobservable preference shocks, and that these shocks are independent across households. If we estimate two parameters, say a labor-supply elasticity and an interest elasticity of saving, using the entire data set in both cases, the estimates will be correlated. However, if we randomly divide the data set into two subsamples, and then estimate the labor-supply elasticity with one subsample and the interest elasticity of saving with the other, these estimates will be statistically uncorrelated. Thus, as long as we rely on econometric studies that employ different data, it is arguable that the $\hat{\beta}_i$s are independent.

In practice, different parameters may be estimated with the same data, or with related data (e.g., time-series data for different variables covering the same time period). In addition, if certain techniques of estimation introduce systematic biases, then the use of similar techniques to estimate different parameters may create systematic relationships between the resulting estimates. Unfortunately, very little can be done about this. Short of reestimating all parameters of the model simultaneously, it is impossible to accurately measure correlations between the elements $\hat{\beta}$. At best, one can incorporate ad hoc correlations into subjective beliefs.

Under the assumption that the distribution of the $\hat{\beta}_i$s are independent normal, it is possible to show that

$$E[H(\hat{\beta})] = \Psi(\beta^*) + \frac{1}{2}\sum_i \sigma_i^2 \Psi_{ii}(\beta^*), \tag{2}$$

and

$$V[H(\hat{\beta})] = \sum_i \{\sigma_i^2[\Psi_i(\beta^*)]^2 + \frac{1}{2}\sigma_i^4[\Psi_{ii}(\beta^*)]^2\}, \tag{3}$$

where σ_i^2 denotes the variance of $\hat{\beta}_i$ (i.e., the ith element on the diagonal of Σ). To implement these formulas, we require both first and second derivatives of Ψ, which we evaluate at $\hat{\beta}$. To calculate a numerical second derivative with respect to a parameter, one need only make two small changes in the parameter from the revised case rather than one change as before. Note that it is, in principle, possible to accommodate correlations between the parameters, but that this would necessitate calculating Ψ_{ij} for all i and j. When the set of economic parameters is large, this task would prove onerous.

This alternative approach to sensitivity analysis is necessarily approximate. The quality of the approximation depends upon the properties of CGE models and the degree of uncertainty concerning economic parameters. In another as yet unfinished project, we explicitly compare variance-covariance matrices based on first- and second-order expansions to the second moments of distributions generated by Monte Carlo simulations. It is much easier and certainly far less costly to implement these versions of the Pagan-Shannon approach than it is to conduct reliable Monte Carlo simulations. For first-order approximations, one need only calculate as many perturbed cases as parameters. In addition, one computes derivatives locally, so that the perturbed case equilibria are very close to the revised equilibria. By starting the equilibrium algorithm at the revised case, rapid convergence should in general be achieved. This same consideration may also imply that the computational cost of the Pagan-Shannon approach is lower than the corresponding cost for CSSA, even when one must solve for the same number of equilibria (generally, for CSSA, the discretized distribution places weight on widely divergent parameter values). For second-order approximations (assuming independence), one simply requires twice as many local permutations. These requirements are negligible in comparison to the task of performing Monte Carlo analysis. It is worth reiterating that second-order approximations become much more onerous if one wishes to allow for correlations between all parameters. With 100 parameters, one would require 10,000 perturbed cases to compute all of the second derivatives. It is, of course, relatively easy to incorporate a few select correlations without significantly adding to the computational burdens.

The Pagan-Shannon approach is also somewhat more flexible than USSA, CSSA, and Monte Carlo simulations. As mentioned earlier, the value of these alternative approaches is limited by the extent to which different economists agree about Σ. In contrast, variants of the Pagan-Shannon approach permit the researcher to report the vector of first and second derivatives. A reader can then supply his own beliefs about Σ, and, with relatively little effort, compute an approximate variance-covariance matrix for the outputs.

One final caveat is in order. Our discussion has focused on techniques for computing the variances of policy effects given a particular CGE model, data, and policy parameters. We have made no attempt to account for any uncertainty that one might have about the correspondence between the model and the real world. As a result, our estimates of variance reflect uncertainty concerning the impact of policy in the model and not in the actual economy.

5.3 Review of the Ballard-Fullerton-Shoven-Whalley Model

The model we use to investigate the consumption tax is a medium scale CGE model. It is completely documented in Ballard-Fullerton-Shoven-Whalley (BFSW) (1985) and is the same model used previously to evaluate a progressive consumption tax in Fullerton, Shoven, and Whalley (1983). Due to its previous documentation and use, we will only provide a brief description of the model here. If our purpose was simply to produce a new evaluation of the adoption of an expenditure tax, we would have developed a new data set and model structure. Our primary goal in this paper, however, is to assess the impact of uncertainty about parameter values on the certainty of model outcomes. For this purpose, the existing model and data suffice quite well.

While the BFSW model is not as dynamically sophisticated as more recent CGE models, it does have the essential structural features for evaluating a switch to a consumption tax. Within the model, consumers face both an intertemporal consumption decision and a labor-leisure decision. This means that both the intertemporal consumption and the labor-leisure margins are operative and subject to tax distortion. It also implies that there is no a priori presumption within the model that a consumption tax is superior.

The BFSW model has 19 producers and 12 consumer income classes. Consumer behavior is characterized by a nested CES-LES (constant elasticity of substitution/linear expenditure system) utility structure. The outermost nest uses a CES utility function to characterize the consumer's decision between present and future consumption. The parameter of substitution between present and future consumption is calibrated to be consistent with Boskin's (1978) 0.4 estimate of the interest elasticity of saving. The middle nest of the preference structure determines the allocation of the consumer's present consumption between goods and leisure, again, using a CES preference function. The substitution parameter of the preference function is calibrated to be consistent with a composite labor-supply elasticity of 0.128. The innermost nest of the preference structure allocates present consumption between 15 consumer goods according to LES preferences.

Producers use capital and labor to produce their output according to CES value-added production functions. They also use the output of other industries through a fixed-coefficient input-output matrix. The elasticity of substitution between capital and labor used to calibrate the production functions come from studies summarized by Caddy (1976). The 19 producer goods are transformed into the 15 consumer goods through a fixed coefficient transition matrix. The model is calibrated around a benchmark, general equilibrium data set from 1973. This data set defines an equilibrium in transactions terms. Value observations are separated into prices and quantities by assuming that a physical unit of a good or factor is the amount that sells for $1. All benchmark equilibrium prices are $1, and observed values are benchmark quantities.

Through their interaction, utility-maximizing consumers and profit-maximizing producers are assumed to reach a single-period competitive equilibrium where all profits are zero and supply equals demand for each good and factor. Single-period equilibria are sequenced through endogenous saving decisions that augment the capital stock of the economy. An exogenous labor force growth rate is assumed.

We calculate a benchmark balanced growth path that replicates the data, has constant prices, and implies that quantities grow at the labor force growth rate. A simulation is run by altering a tax parameter and calculating a revised sequence of equilibria. The model assumes markets are perfectly competitive with no externalities, quantity constraints, or barriers to factor mobility. Since a complete set of prices and quantities are calculated under different tax policies, we can calculate changes in national income, utility, income changes for consumers, and factor allocations among industries. The model is solved using the factor price revision rule of Kimbell and Harrison (1986).

The income tax system is modeled as a set of linear tax schedules for each of the 12 consumer groups. Each of the 12 income classes has a lump sum tax (or transfer in the bottom income class) that, along with a given marginal tax rate, yields average and marginal tax rates that are consistent with those the income class actually faces. This treatment captures the fact that average and marginal tax rates differ by group, that both are increasing, and that it is the marginal tax rate that causes the distortionary substitution effect of the income tax system.

A significant percentage of saving that occurs in households is channeled through tax deferred savings plans such as private, state, local, and federal pension plans, through individual retirement accounts (IRAs) and Keogh plans, or through the cash value of life insurance. These plans either allow for tax-free contributions and accumulation or taxable contributions with tax-free accumulation and withdrawal. Flow of Funds data indicate that in 1973, roughly 30 percent of all saving occurred through these vehicles and, thus, are taxed on a consumption-tax basis. This suggests that the income tax system is in fact a hybrid tax system that has some features of a consumption-based system. We model this system by allowing households to deduct 30 percent of their saving from the taxable income base. To move from this hybrid to a consumption-based system, we exclude the remaining portion of saving from the tax base. Thus, we act as if all saving occurred through IRAs or qualified savings account. By increasing the saving deduction the tax system moves from a progressive income tax to a progressive consumption tax while maintaining the 1973 structural features of the tax system.

There is a corresponding revenue loss associated with the move to the consumption tax. Since saving is excluded from the tax base, tax rates have to rise to maintain the current levels of revenue. The federal budget is balanced in the model so that in the absence of rate increases, government commodity purchases and transfer payments would be reduced. Since it would be impossible to separate tax effects from the expenditure effects of reducing commodity purchases and transfers, we maintain real government expenditure at a constant level. Therefore, we are doing differential incidence analysis. Given this, we examine three different methods of replacing the tax revenue lost in moving to a consumption tax. The first, lump-sum replacement, replaces the reduction by imposing lump-sum taxes or transfers by altering the zero-income intercept of the linear tax schedule. The second, additive-replacement, raises marginal rates by an equal, additive amount. The third, multiplicative replacement, increases marginal tax rates by a constant, multiplicative factor. The second and third methods allow us to recognize that frequently the replacement schemes necessary to maintain revenues are also distorting. We find that often the *method* of maintaining revenue balance is as important as the policy initiative that is being examined.

5.4 Sources of Uncertainty

In order to run the Ballard-Fullerton-Shoven-Whalley model described in section 5.3 one must supply a large number of parameters, as well as base-case data. Our current objective requires us to obtain variances, as well as point estimates for all parameters. In cases where point estimates have been taken from econometric studies, it should also be possible to obtain formal estimates of variances. Other parameter values are chosen somewhat arbitrarily; in these cases, variances must of necessity be somewhat arbitrary as well, and should be thought of as reflecting subjective beliefs.

We have divided the important model parameters into two sets, henceforth referred to as "group 1" and "group 2." This classification reflects two considerations. First, on the basis of economic reasoning and previous sensitivity analyses, we generally regard group-1 parameters as more important determinants of the effects of consumption taxation. Second, we obtain group-1 parameters from specific econometric studies and, consequently, for these parameters we also have estimated variances. In contrast, no econometric estimates of the variances for group-2 parameters are available. Given these distinctions, it seemed appropriate to present sensitivity calculations for group-1 parameters alone, as well as for all parameters jointly.

Group-1 parameters include the interest elasticity of saving, the labor-supply elasticity, the after-tax rate of return to physical capital, and the elasticities of substitution in production between labor and capital for each of the 19 industries described by the model. Parameter values, along with "optimistic" and "pessimistic" standard errors, are presented in table 5.1a. We discuss these in order.

The first group-1 parameter is the interest elasticity of saving. The key distinction between a consumption tax base and an income tax base is the inclusion of saving, and the main partial equilibrium claim regarding a consumption tax is its alleged neutrality with respect to intertemporal consumption. In a general equilibrium framework, with leisure an untaxed good and other taxes in the model, this appeal to intertemporal neutrality is not theoretically compelling. However, the sensitivity of behavior to the exclusion of saving from the tax base clearly depends on the interest elasticity of saving. This parameter will be the major determinant of the extent of capital deepening that occurs in the long run after the switch to a consumption tax.

Our estimate of the interest elasticity of saving is taken from Boskin (1978). Its standard error can be calculated through a simple transformation of the standard error for Boskin's consumption elasticity. We use this as our "opti-

Table 5.1a Parameter Values and Associated Standard Error for Group-1 Parameters

Parameter	Value	Optimistic Standard Error	Pessimistic Standard Error
Interest elasticity of saving	.4	.109	.6
Labor-supply elasticity	.128	.095	.25
After-tax rate of return to capital	.0384	.0129	.015
Elasticities of substitution between capital and labor			
Agriculture	.6142	.057	.6139
Food	.7117	.0548	.5077
Clothing	.8152	.0416	.6727
Paper	.7682	.0466	.7881
Petroleum refining	.7411	.0792	.5463
Chemicals rubber & plastics	.8284	.0555	.9045
Lumber, clay & glass	.7902	.0375	.6552
Metals, instruments & miscellaneous manufacturing	.8782	.0298	.5480
Transportation equipment & ordinance	.6971	.1304	1.0079
Vehicles	.8207	.1175	1.0167
Others	1.00	.1304	1.0167

mistic" standard error. Under a more pessimistic view, each individual study contains some idiosyncratic bias, and uncertainty concerning this bias is not reflected in the estimated standard error. Thus, the 95% confidence interval around Boskin's point estimate includes both Denison's law (the elasticity equals 0—see Denison 1958), as well as much higher estimates, such as those obtained by Summers (1981). For our pessimistic scenario, we chose a standard error that is intended to subsume uncertainty concerning the idiosyncratic bias in Boskin's study. Given the wide range of prevailing beliefs about the interest elasticity, we take this standard error to be 0.6. Thus, the 95% confidence interval includes elasticities from roughly −0.8 to 1.6.

The second group-1 parameter is the labor-supply elasticity. This is a key parameter in almost all general equilibrium tax policy simulations. Given that leisure is an extremely important untaxed good, the simple stories regarding a consumption tax being first best optimal are destroyed except in very restrictive circumstances that do not hold in the framework of the BFSW model. The labor-leisure decision is important, if only because labor accounts for roughly three-fourths of value added in the economy. There is a large preexisting tax distortion between goods and leisure, and an increase in that tax wedge can potentially cause large incremental welfare losses. The introduction of a consumption tax, via a saving deduction, at least at the time of its introduction, loses revenue. Given that we have assumed period-by-period revenue neutrality, this necessitates raising other taxes, almost certainly exacerbating the existing distortion in the goods-leisure choices.

In the model, the labor-supply elasticity reflects an average response for men and women. We use Mroz's (1985) estimated elasticity of .09 for women (with a weight of one-quarter), and MaCurdy's (1981) estimated elasticity of .14 for men (with a weight of three-quarters). The associated standard errors are .17 and .07, respectively. We obtain a standard error of .095 for the model parameter by applying the one quarter/three quarters weighting to the parameter standard errors. As with our estimate of the saving elasticity, we choose a pessimistic standard error that is intended to reflect, at least in part, the larger range of estimates available in the literature.

The third group-1 parameter is the after-tax rate of return to capital. This is not frequently thought of as a key variable in specifying a general equilibrium tax simulation model. Its importance here is due to the dynamic or intertemporal nature of the effects of an introduction of a consumption tax. While static models look only at the allocation of fixed factor supplies, the issue here is the productivity of the additional capital formation that a saving exclusion will encourage. Certainly, additional saving is more desirable the higher is the base-case rate of return to capital.

Our model, like almost all others, takes as a unit of capital that amount which earned one dollar net of tax in the base-case data or simulation. That is, capital is measured in capital service rental units. However, in the model, household saving results in the acquisition of physical investments or increments to the capital stock. By definition, the base-case after-tax rate of return to capital determines the number of rental units yielded (as a perpetuity) per unit of physical capital acquired. Also, by units definition, these rental units sell for one dollar in the base-case simulation. Their rental price, however, will differ from one dollar once policy alternatives are introduced. The base-case after-tax rate of return to capital thus determines the rate of conversion between capital in stock units and capital in service flow units.

Our estimate of the after-tax return to capital is taken from a paper by Feldstein and Jun (1987). Our optimistic assumption reflects their estimated standard error for this parameter. We have chosen the pessimistic standard error to yield a 95% confidence interval ranging from roughly 1% to 7%.

All other group-1 parameters are elasticities of substitution in production between capital and labor. These elasticities have always been key elements in the applied general equilibrium tax model, at least since Harberger's analysis. They are potentially important in the consumption tax case under examination here, due to the effect of the exclusion of saving from the tax base on relative factor prices. The change in relative factor prices will affect the various output prices differently depending on the factor intensities. All of these effects are made more important by the presence of other factor and partial taxes such as the corporation income tax and the social security payroll tax.

Estimates of these elasticities for 10 industries are taken from a survey paper by Caddy (1976). For each of these industries, Caddy compiles the results of a large number of studies, and provides both the mean estimate and variance of estimates. He reports the statistics separately for analyses that employed time-series data, and for those using cross-section data. He does not report the standard errors of estimates from individual studies. In order to avoid the need for reexamining all of these primary sources, we adopt a simplifying assumption: for each elasticity β, the ith estimate of this parameter is given by $\hat{\beta}_i = \beta + \varepsilon_i$, where the ε_i are distributed identically and independently. Under this assumption, it is appropriate to use the mean estimate as the value of the elasticity. Furthermore, we use the variance of this estimated mean (which is equal to the variance of the estimates divided by the number of studies minus one) for our optimistic beliefs.

These optimistic variances may significantly understate the true degree of uncertainty. In particular, Caddy's study indicates that there is a large systematic difference between results based on time-series and cross-section data. A more appropriate model might be that the ith estimate of the parameter β obtained using the technique k (henceforth denoted $\hat{\beta}_i^k$) is given by

$$\hat{\beta}_i^k = \beta + \mu_k + \varepsilon_i^k.$$

In this equation, μ_k represents the systematic bias inherent in the use of a particular approach to estimation. We assume that the μ_k are distributed identically and independently with mean zero, as are the ε_i^k.

One alternative is to estimate this relationship formally to obtain a point estimate and standard error. However, it is clear from inspection of Caddy's numbers that, given the size of the samples, the ε_i^k will essentially average out, while the μ_k will not. That is, if we estimate $\beta + \mu_k$ separately for each k (by taking the average estimated elasticity for time-series and cross-section studies, respectively), the standard errors of these estimates will be very small relative to the differences in the estimates for the two values of k. Accordingly, we obtain a very good approximation by acting as if $\beta_k = \beta + \mu_k$ is estimated without error for each k. We then have, essentially, two observations on β plus noise. It is then appropriate to use the average of these two numbers as our estimated parameter, and to use the variance of this average (which is $[\beta_1 - \beta_2]^2 / 2$) as our estimate of the variance. This calculation forms the basis for our pessimistic scenario.

Estimates of the elasticity of substitution are not available for nine of the 19 industries represented in the model. Following previous practice, we take this elasticity to be unity (i.e., the production function is Cobb-Douglas). It is natural to assume that our uncertainty about these elasticities must be at least as great as those that have been estimated. Accordingly, we take the standard errors of the elasticities for these nine industries to equal the largest standard error for the other 10 industries.

Group-2 parameters include export demand elasticities, the ratio of labor endowment to labor supply, the preference parameters on the LES inner nest of the utility specification, the minimum required purchases in the LES inner nest, the marginal tax rates for the linearized income tax schedules, and the percentage of capital income that is taxable by the individual income tax for each industry. As econometric estimates of standard errors are unavailable for these parameters, we must impose somewhat arbitrary subjective beliefs. Assumed parameter values, along with optimistic standard errors, are given in table 5.1b. While these assumptions are largely self-explanatory, some clarifying comments are in order.

The preference parameters on the LES inner nest of the utility specification must sum to unity. Accordingly, we cannot allow them to vary independently. One alternative is to allow one to be determined as a residual. However, the choice of a residual parameter would be extremely arbitrary, and it would imply a peculiar covariance structure. Instead, we define a new set of parameters ϕ_i, and let the LES parameters β_i be given by

$$\beta_i = \frac{\phi_i}{\sum_i \phi_i}.$$

Initially, we normalize so that the sum of the ϕ_i equals unity. We suppose that we know something about the variances of the ϕ_i, and that these are distributed independently. This implies a more natural covariance structure for the β_i—as one β_i rises, all others decline proportionately. In table 5.1b we have given the standard errors for these LES parameters as a percentage of their assumed values.

We have also parameterized the LES specification by assuming that the minimum required purchase for each consumption category is $2,500. Rather than vary each of these independently, we assume that all minimum purchases equal a common parameter, and we define beliefs over this common parameter.

We follow a similar practice for marginal tax rates. In this case, we assume that each rate is equal to some constant times a common parameter, and we normalize so that the base value of this parameter is unity. A standard error of 0.1 for this parameter therefore signifies that the standard error of each marginal tax rate is 10% of its assumed value, and that all of these tax rates are

Table 5.1b **Parameter Values and Associated Standard Error for Group-2 Parameters**

Parameter	Value	Optimistic Standard Error	Pessimistic Standard Error
Export demand elasticity	−1.4	.28	.56
Supplemental export parameter (v)	−10.0	2.0	4.0
Ratio of labor endowment to labor supply	1.75	.1	.25
LES preference parameters (%)	. . .	10	20
LES minimum purchase parameter	2,500	250	500
Marginal tax rate scaling parameter	1	.1	.2
Scaling parameter for proportion of capital income subject to ITT	1	.1	.2

perfectly correlated. While perfect correlation is probably too strong an assumption, it does seem likely that factors that lead us to under- or overestimate effective marginal tax rates for one class of consumers are likely to do likewise for all other classes. We follow exactly the same practice for the fraction of capital income subject to the individual income tax in each industry.

5.5 Simulation Results

5.5.1 Standard Point Estimates

When the consumption tax is simulated we find there are large returns to moving the 1973 tax system from a hybrid to a consumption based system. As reported in table 5.2, the efficiency gain with additive replacement is $557 billion or roughly 1.1 percent of the present value of future expanded national income. These efficiency gains are calculated as the present discounted value of the sum of equivalent variations for each representative household in the model. Using the expenditure function for each household, we calculate the income changes, at old prices, that would allow each group to obtain the same pattern of instantaneous utility over time in the new tax regime. This instantaneous utility excludes saving (to avoid double counting), and is based on current consumption and leisure.

With additive and multiplicative replacement, the price of capital relative to labor falls immediately after a consumption tax is implemented. Saving increases by 30 percent in the first equilibrium. This savings is used directly for investment. Investment, however, is more labor intensive than other com-

Table 5.2 Dynamic Welfare Effects in Present Value Equivalent Variation over Time (in billions of 1973 dollars)

	Types of Scaling to Preserve Tax Yield		
	Lump-sum	Additive	Multiplicative
Consumption Tax	643 (1.243)	557 (1.076)	540 (1.044)
Time Path for the ratio of the rental price of capital to the wage rate:			
Year:			
0	1.0042	.9678	.9650
10	.9344	.9116	.9103
20	.8901	.8750	.8745
30	.8608	.8504	.8503
40	.8411	.8336	.8336
50	.8275	.8219	.8220

Note: The numbers in parentheses represent the gain as a percentage of the present discounted value of consumption plus leisure in the base sequence. This number is $51.766 trillion for all comparisons and accounts only for the initial population.

ponents of aggregate demand. Therefore, the increase in savings generates an indirect increase in the relative demand for labor and thus, an indirect decrease in the relative price of capital. The time path of prices, given in the bottom of table 5.2, gives an indication of how long the economy takes to resettle into a steady-state growth path. With additive and multiplicative replacement, roughly 45 percent of the total price change occurs in the first 10 years, and 70 percent of the change in 25 years. The corresponding figures for lump-sum replacement are 34 and 64 percent. The patterns of intersectoral change that emerge from the model suggests that industries that are relatively capital intensive, such as real estate and agriculture, prosper over time, as capital deepening occurs.

A somewhat different look at our central case is given in table 5.3. There, the impact of the consumption tax on saving, labor supply, output, and utility are presented from the short- (impact effect), medium- (15 years) and long- (steady state) run perspective. Under this model specification, the consumption tax has very little effect on labor supply. The tax has an ambiguous effect on net wage rates. To the extent labor income is consumed, tax rates rise; to the extent it is saved, tax rates fall. Each of these net effects have corresponding income and substitution effects, the outcome of which is to leave labor supply virtually unchanged.

The impact effect on savings and investment is very strong. However, after the initial increase the rate of saving is steady. Capital prices adjust slowly in the model; therefore, some time has to pass before the increase in investment can be reflected in increased output. In the initial period, consumer demand is reduced by approximately the value of saving. Consequently, aggregate demand and output are roughly constant. After 15 years, output starts reflecting the increased level of capital formation generated by the investment. In the new steady state, the level of production in the economy is roughly 7 percent higher than in the base-case equilibrium. It might seem surprising that, at the

Table 5.3 **Change in Base-Case Quantities of Imposing a Consumption Tax with Additive Replacement and Saving Elasticity of 0.4 (in millions of dollars)**

	Short Run	Medium Run	Long Run
Saving	32,114	46,303	437,258
	(30.5)	(29.2)	(27.2)
Labor supply	6,642	4,428	−23,423
	(.7)	(.4)	(−.2)
Output	17,121	99,592	2,040,690
	(.8)	(3.2)	(6.6)
Utility	−30,425	2,725	681,467
	(−2.8)	(.2)	(4.2)

Note: The percentage change from base-case quantities are given in parentheses. The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state.

same time output is increasing and labor supply is holding steady, utility, which is based on consumption and leisure, fails to increase in the medium run. Output increases in the medium run primarily in response to the investment component of aggregate demand. Consumer demands, and hence consumption, have not yet increased sufficiently to increase utility. In the new steady state, however, sufficient capital deepening occurs so that consumers have more income, consume more, and take the same amount of leisure. This is reflected in higher levels of final period utility.

We also find that, in the long-run, the policy is a Pareto improvement, all income classes are better off. There is far more output in the economy, consumers save more, consume more, and have an equivalent amount of leisure. Though all classes gain from the consumption tax, poorer households are somewhat better off when taxes are replaced in a multiplicative fashion than under additive replacement. The rate increases necessary under a consumption tax are smaller for low-income households under multiplicative replacement and, therefore, consumers are better off.

5.5.2 Standard Sensitivity Analysis

As a point of reference we will describe the conventional sensitivity analysis that is typically done in the Ballard-Fullerton-Shoven-Whalley CGE model. Sensitivity analysis for the consumption tax is generally performed on the interest elasticity of saving, since benefits that occur from a consumption tax are generally thought to be the result of reducing the price of capital. In table 5.4 we vary the interest elasticity of saving from 0.4, the level consistent with Boskin (1978), to 0.0, the level consistent with Denison's law (Denison 1958), and 2.0, a magnitude roughly comparable to those derived in Summers (1981). The magnitude of the results are quite sensitive to the choice of savings elasticity. They range from $416 billion with multiplicative replacement and a 0.0 interest elasticity of saving, to $935 billion with a 2.0 saving elasticity and lump-sum replacement. The last figure is roughly 70% of 1973's national income.

In table 5.4, we also present price ratios of the rental price of capital to the wage rate under different elasticity assumptions. From these results it is clear that the degree of substitution in the economy makes a great deal of difference in how quickly the economy responds to tax changes. With a saving elasticity of 2.0, the transition from steady states occurs quite quickly, despite there being a larger adjustment to make. Sixty-seven percent of the price adjustment occurs in the first five years; after 10 years, 85% of the adjustment has occurred. The adjustment paths are much slower with lower saving elasticities. It takes roughly 15 years for the price of capital to adjust halfway to its steady-state level with a saving elasticity of 0.0, while it is 10 years for the 0.4 saving elasticity.

Table 5.4 **Dynamic Welfare Effects in Present Value Equivalent Variations over Time (in billions of 1973 dollars) with Differing Interest Elasticities**

A. Consumption Tax

Saving Elasticity	Types of Scaling to Preserve Tax Yield		
	Lump-sum	Additive	Multiplicative
.0	521	436	416
	(1.006)	(0.842)	(0.803)
.4	642	557	540
	(1.243)	(1.076)	(1.044)
2.0	935	835	819
	(1.808)	(1.613)	(1.582)

B. Time Path for the Ratio of the Rental Price of Capital to the Wage Rate under Different Saving Elasticities (additive replacement)

Year	Interest Elasticity Saving		
	.0	.4	2.0
0	.9677	.9678	.9681
10	.9284	.9116	.8467
20	.8994	.8750	.8056
30	.8776	.8504	.7884
40	.8611	.8336	.7807
50	.8485	.8219	.7770

Note: The numbers in parentheses represent the gain as a percentage of the present discounted value of consumption plus leisure in the base sequence. This number is $51.766 trillion for all comparisons and accounts only for the initial population.

5.5.3 Formal Sensitivity Results

Following the methodology outlined in section 5.2, we have calculated standard errors for policy effects using both first- and second-order approximations. We provide results for both optimistic and pessimistic beliefs for group-1 parameters alone, as well as for all parameters. Accordingly, we have generated eight sets of results. These are summarized in tables 5.5 through 5.12. Each table indicates the impact of a shift to consumption taxation on four variables (saving, labor supply, output, and utility) in three different time frames (the short, medium, and long runs). In the context of our model, the medium and long runs correspond to about 15 and 100 years, respectively. We also provide an index of the overall impact of consumption taxation on welfare. Each table includes approximate standard errors for all these effects.

We begin with results that reflect uncertainty concerning the values of group-1 parameters alone. Table 5.5 provides standard errors from a first-order approximation under optimistic assumptions. One immediately notes

Table 5.5 **Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using Group-1 Elasticities (in millions of dollars)**

	First-Order Approximation, Optimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	32,114	46,303	437,258
	(2,901)	(6,305)	(397,424)
Labor supply	6,642	4,428	−23,432
	(2,405)	(1,541)	(70,270)
Output	17,121	99,592	2,040,690
	(5,786)	(34,711)	(1,952,181)
Utility	−30,435	2,725	681,467
	(2,443)	(13,428)	(672,048)
	Present discounted value of equivalent variations:		
	556,851		
	(179,582)		

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. Group-1 elasticities include: saving, labor supply, the growth rate, and substitution between labor and capital in each production sector.

Table 5.6 **Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using Group-1 Elasticities (in millions of dollars)**

	Second-Order Approximation, Optimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	32,195	46,416	616,886
	(2,904)	(6,331)	(471,912)
Labor supply	6,640	4,545	−32,643
	(2,407)	(1,561)	(72,134)
Output	17,128	102,788	2,912,719
	(5,790)	(34,872)	(2,307,385)
Utility	−30,536	4,312	978,938
	(2,446)	(13,510)	(793,121)
	Present discounted value of equivalent variations		
	617,385		
	(198,328)		

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. Group-1 elasticities include: saving, labor supply, the growth rate, and substitution parameter between labor and capital in each production sector.

that all of the impact (short-run) effects are estimated very precisely. The same is true for the medium-run effects, with the exception of utility. All precision vanishes in the long run. Nevertheless, the total welfare effect is calculated with a good deal of precision. One can be highly confident about the direction of the effect on total welfare, and, in addition, one can get a fairly good sense for the magnitude of this effect.

Table 5.6 is the second-order counterpart to table 5.5. A quick comparison of these two tables reveals that the use of second-order approximations changes nothing of substance.

While continuing to restrict our attention to group-1 parameters, we move to pessimistic assumptions about parameter variances. Calculations based upon first-order approximations are presented in table 5.7. Relative to table 5.5 (which is the comparable table for optimistic assumptions), precision declines dramatically. One can still be confident about the direction of effects of consumption taxation on saving and utility in the short run and saving in the medium run (the result for output in the medium run is marginal). Once again, uncertainty about parameters essentially implies that nothing is tied down with any precision in the long run. The total impact on welfare is also calculated with a good deal of variance—while one can be fairly confident that consumption taxation is beneficial, our calculations imply that little can be said about the magnitude of this effect.

Table 5.8 is the second-order counterpart to table 5.7. Essentially the same patterns emerge, except that one can have somewhat greater confidence in the

Table 5.7 Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using Group-1 Elasticities (in millions of dollars)

	First-Order Approximation, Pessimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	32,114	46,303	437,258
	(15,951)	(16,357)	(468,240)
Labor supply	6,642	4,428	−23,432
	(8,777)	(5,113)	(180,850)
Output	17,121	99,592	2,040,690
	(21,625)	(61,133)	(2,345,637)
Utility	−30,435	2,725	681,467
	(13,088)	(17,145)	(801,190)
Present discounted value of equivalent variations			
		556,851	
		(293,298)	

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. Group-1 elasticities include: saving, labor supply, the growth rate, and substitution between labor and capital in each production sector.

Table 5.8 Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using Group-1 Elasticities (in millions of dollars)

	Second-Order Approximation, Pessimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	33,736	42,298	669,435
	(16,384)	(16,615)	(581,997)
Labor supply	7,668	6,209	−36,888
	(8,990)	(5,712)	(184,682)
Output	19,963	118,674	3,146,050
	(22,163)	(62,529)	(2,883,600)
Utility	−32,633	17,293	1,051,295
	(13,415)	(19,341)	(985,517)

Present discounted value of equivalent variations
635,487
(317,499)

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. Group-1 elasticities include: saving, labor supply, the growth rate, and substitution between labor and capital in each production sector.

directions of the medium-run impact on output and of the total impact on welfare.

We now turn to results that reflect uncertainty concerning the values of all parameters considered in section 5.4. We discuss these results in the same order as for group-1 parameters. Table 5.9 contains standard errors based upon first-order approximations under optimistic assumptions. Note that, relative to table 5.5 (the counterpart for our group-1 calculations), there are substantial increases in the variance of all policy effects. Nevertheless, this does not alter the set of variables for which the direction of the effect is known with substantial confidence. Table 5.10 provides the second-order counterpart to table 5.9. Not surprisingly, the results are substantively unchanged.

Table 5.11 provides first-order results based on pessimistic assumptions concerning the full set of parameters. Relative to table 5.7 (in which only group-1 parameters are treated as uncertain), precision declines substantially. The impacts on saving in the short run and output in the medium run are no longer known with much confidence. More importantly, the total welfare effect is now less than 1.5 times the size of its standard error.

Table 5.12 is the second-order counterpart to table 5.11. Similar patterns appear in this table, except that all three of the effects mentioned in the preceding paragraph appear to be tied down a bit more precisely. In particular, the total welfare effect is now slightly greater than 1.6 times its estimated standard error. One can therefore have a fair degree of confidence about the desirability

Table 5.9 **Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using All Parameters (in millions of dollars)**

	First-Order Approximation, Optimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	32,119	46,303	437,258
	(5,302)	(9,363)	(410,127)
Labor supply	6,641	4,428	−23,432
	(2,634)	(1,787)	(71,709)
Output	17,121	99,592	2,040,690
	(6,331)	(38,731)	(2,012,521)
Utility	−30,435	2,725	681,467
	(4,777)	(13,593)	(690,486)

Present discounted value of equivalent variations

556,851
(216,504)

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. The parameters are listed in the appendix.

Table 5.10 **Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using All Parameters (in millions of dollars)**

	Second-Order Approximations, Optimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	32,291	46,704	627,260
	(5,308)	(9,392)	(482,855)
Labor supply	6,728	4,961	−24,621
	(2,637)	(1,862)	(74,017)
Output	16,872	104,187	2,981,068
	(6,379)	(38,925)	(2,359,806)
Utility	−30,515	4,474	992,724
	(4,795)	(13,682)	(809,107)

Present discounted value of equivalent variations

623,347
(233,142)

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. The parameters are listed in the appendix.

Table 5.11 Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using All Parameters (in millions of dollars)

	First-Order Approximation, Pessimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	32,114	46,303	437,258
	(18,254)	(21,430)	(510,186)
Labor supply	6,642	4,428	−23,432
	(9,077)	(5,443)	(183,182)
Output	17,121	99,592	2,040,690
	(22,321)	(70,145)	(2,541,446)
Utility	−30,435	2,725	681,467
	(15,452)	(17,661)	(861,621)
Present discounted value of equivalent variations			
		556,851	
		(380,177)	

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. The parameters are listed in the appendix.

Table 5.12 Expectations and Standard Errors of the General Equilibrium Output in the Short, Medium, and Long Run Using All Parameters (in millions of dollars)

	Second-Order Approximations, Pessimistic Standard Errors		
	Short Run	Medium Run	Long Run
Saving	35,113	43,429	710,651
	(18,658)	(21,710)	(618,703)
Labor supply	7,961	7,829	−4,698
	(9,293)	(6,285)	(189,973)
Output	18,794	124,148	3,418,549
	(23,042)	(71,801)	(3,059,247)
Utility	−32,572	17,929	1,106,222
	(15,812)	(19,883)	(1,039,031)
Present discounted value of equivalent variations			
		658,847	
		(407,035)	

Note: The short run is the impact effect, the medium run is 15 years, the long run is the new resulting steady state. The numbers in parentheses are approximate standard deviations. The parameters are listed in the appendix.

of consumption taxation, but very little can be said about the magnitude of this effect.

As mentioned in section 5.2, one of the advantages of the Pagan and Shannon (1985) approximation approach to sensitivity analysis is that the results can easily be altered to accommodate alternative sets of beliefs. Accordingly, we present first and second derivatives of the total welfare effect with respect to the full set of parameters in the appendix. The reader can use these derivatives to compute standard errors for the welfare effect under any alternative set of beliefs. To conserve space, we omit derivatives for the other effects discussed above. Tables of these derivatives are available from the authors upon request.

To summarize, we find that, under optimistic beliefs, we know enough about the underlying economic parameters to tie down the short- and medium-run effects of consumption taxation, as well as the total welfare effect, quite precisely. Under pessimistic beliefs, precision is much lower, but one can still be fairly confident that the overall effect on welfare is positive. Our analysis also indicates that differences between first- and second-order approximations are generally small, but in some cases these differences prove to be qualitatively important.

5.6 Conclusion

We have addressed two issues in this paper. First, following the approach of Pagan and Shannon (1985), we have shown that there is a practical way to calculate approximate standard errors for the output of a nonlinearized computational general equilibrium tax model. The method is not demanding computationally and should prove to be a useful methodology for many other applications. Second, we have found that the welfare gains promised by a consumption tax are quite robust to the uncertainty in the underlying parameters of the general equilibrium model. Even in our pessimistic case regarding the uncertainty about key parameters, the existence of a positive welfare enhancement in making the policy switch can be predicted with a reasonable degree of confidence.

Appendix

Numeric First and Second Derivatives of Total Welfare Effect with Respect to Parameters Considered in the Analysis[3]

Parameter	First Derivative	Second Derivative
After-tax rate of return	−13,425,919.3485	716,430,459.7397
Saving elasticity	261,000.5700	−109,256.2500
Labor supply elasticity	384,003.0431	415,114.1869
Substitution elasticity for Cobb-Douglas Industries	102,059.8390	−22,455.2000
Substitution elasticity for agriculture	−13,944.3443	−21,885.2459
Substitution elasticity for food and tobacco	14,157.7873	18,573.8941
Substitution elasticity for textiles	6,808.2220	−9,577.7811
Substitution elasticity for paper and printing	8,702.8299	1,476.8089
Substitution elasticity for petroleum refining	−3,428.8203	−7,913.0752
Substitution elasticity for chemicals and rubber	13,865.8614	24,675.4246
Substitution elasticity for lumber and furniture	14,567.9304	27,237.4619
Substitution elasticity for metals and machinery	38,282.0029	41,636.2457
Substitution elasticity for transportation equipment	2,495.0757	−1,813.0612
Substitution elasticity for motor vehicles	8,131.6317	−608.5663
Elasticity of export demand	1,394.8829	−1,557.9082
External sector closing parameter	3,467.3317	−41.4100
Endowment & time divided by labor hours	25,427.9806	−43,128.0980
Minimum required purchases	−17.3404	−.0101
Marginal tax rates	1,118,829.5500	2,077,134.6000
Proportion of capital income taxable under the individual income tax	443,966.2890	−689,933.2000
LES preference parameter for food	23,627.6617	−8,319,927.6386
LES preference parameter for alcohol	283,606.9259	−1,050,396,021.9478
LES preference parameter for tobacco	284,805.7143	−57,132,653.0575
LES preference parameter for utilities	17,538.7586	170,208,917.9550
LES preference parameter for housing	98,297.2276	−9,439,698.5921
LES preference parameter for furnishings	−58,239.7500	−10,482,142.8570
LES preference parameter for appliances	−42,034.8750	2,946,239,062.4985
LES preference parameter for clothing	−100,611.8889	124,212,208.5048
LES preference parameter for transportation	91,798.2000	−853,319,111.1113
LES preference parameter for motor vehicles	−12,186.5068	−57,081,478.7014
LES preference parameter for services	−91,888.8704	−6,312,787.6848
LES preference parameter for financial services	83,230.6883	−81,242,486.0853
LES preference parameter for reading and recreation	−87,047.8261	151,643,310.2290
LES preference parameter for nonfood, nondurable household consumption	−128,441.5714	890,871,655.3291
LES preference parameter for gasoline and fuels	2,511,688.1429	−15,081,887.7549

Notes

1. As reported in Harrison, Jones, Kimbell, and Wigle (1989), 10 parameters and a discrete approximation characterized by seven values of the parameter would require 282,475,250 model solutions. Harrison and Vinod (1989) demonstrate that it is possible to approximate the USSA result through formal statistical methods.

2. Wigle (1988) used a variant of the Pagan-Shannon approximation on a nonlinear CGE trade model and found this approximation to generate essentially equivalent results to those generated by a sequentially selected unconditional systematic sensitivity analysis of the model investigated.

3. Derivatives are calculated as the change in the revised-case minus base-case simulations generated by a perturbation of the parameter in question. Numbers are given in millions. Complete tables of derivatives are available from the authors upon request. The first 14 derivatives correspond to "group 1."

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Comment Joel Slemrod

Let me begin by stating my conclusion about this paper—that the methodology it proposes and executes represents a major contribution to the tool kit of builders of computable general equilibrium (CGE) models, but that it teaches us little, if anything, about the effect of taxation of saving. I make this pessimistic judgment in spite of the compelling kind of reassurance offered by this and other papers of this sort—that, in the face of swirling controversy among the economics profession about the determinants of saving, it can provide an estimate carried out to six significant digits of the long-run impact of eliminating the taxation of capital income in the United States, a tax change unprecedented in the fundamental economic changes it would generate.

To support my conclusions, I will begin with the trees and then move to the forest. I will first discuss the methodological advancement offered in this paper, and then come back to assess its relevance to understanding saving and how taxation affects it.

Questions about robustness have plagued CGE analysis ever since it has been applied to real policy alternatives and used as a guide to policy formulation. After all, the point estimate of the response of some variable to a policy change depends on the constellation of assumptions about model specification, including parameter values, initial conditions, and the form of the decision rules used by the model's agents.

The principal response among CGE modelers has been to resort to sensitiv-

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ity analyses, with the goal of ascertaining the robustness of key model results to certain aspects of model specification. The most popular approach (undoubtedly because it is the easiest) has been to vary key parameters within a reasonable range and observe whether the principal model results are much affected. Doing this for each parameter separately is what Harrison et al. (1985) refer to as "conditional" sensitivity analysis, conditional because each parameter is perturbed conditional on all other parameters being set only to their point estimate value. Harrison et al. prefer and perform an "unconditional" analysis, in which each parameter is perturbed conditional on all other parameters being perturbed, although they note with concern the large number of simulations that are required for this approach.

Pagan and Shannon (1985) suggest a method similar to the one employed by Bernheim, Scholz, and Shoven, where a covariance matrix of the parameters and a matrix of first and second derivatives of output variables with respect to the parameters are combined to produce estimates of the variance corresponding to model forecasts. They also suggest as an alternative determining the "extreme" bounds for the output vector. The problem is to find the maximal variation in a particular output variable given that the parameters are constrained to be selected from a given confidence interval.

The sensitivity analysis proposed by Bernheim et al. follows from the first suggestion of Pagan and Shannon. The procedure has three steps. First, a covariance matrix for the distribution of key parameters is constructed, using as inputs the reported standard errors of the underlying econometric studies, the good judgment of the authors, and assumptions that the distribution is normal and the parameter estimates are distributed independently. Two distributions are considered: an "optimistic" one where the standard errors are closely related to those estimated in the econometric studies, and a "pessimistic" one where the standard errors are increased to reflect uncertainty about the idiosyncratic bias that any individual econometric study inevitably has.

In the second step certain output variables of interest are selected—saving, labor supply, utility. How these change when a consumption tax is implemented under the base-case parameters is calculated. Then the first and second derivatives of these changes with respect to each parameter are numerically calculated. Finally, the covariance matrix of the parameter estimates is combined with the information about the derivatives of the policy impact with respect to the parameters to obtain an approximate covariance matrix for the estimate of the policy's impact.

Along the way, the authors are faced with several modeling choices. I applaud them for the ingenuity and care that they bring to this task. In almost every case, theirs seems a reasonable way to proceed. The end result is a clear improvement over the kind of unsystematic or systematic but conditional sensitivity analysis that has characterized the great majority of CGE research.

What, though, have they gained versus Monte Carlo simulation or unconditional systematic sensitivity analysis? As they note, much less calculation is required, but with the cost of making assumptions about the behavior of the model out of the neighborhood of the base-case parameters and assuming the independence of parameter estimates. Another advantage is that while the sole output of a Monte Carlo simulation is the distribution of result parameters, this procedure also generates an intermediate output of the first and second derivatives of results with respect to parameters, so that alternative assumptions about the parameters can be inserted to produce different estimates of the covariance matrix of results. Note, though, that these individual parameter derivatives are valuable only because of the independence assumption.

I do have one bone to pick, though, about the relationship between the recalibration procedure employed and the sensitivity analyses. The procedure requires that any set of parameters must be consistent with the base-case equilibrium data. Thus when any given parameter is changed, some other parameter or set of parameters must also be changed to reestablish that consistency. In the paper this recalibration procedure is denoted as the g function. First, note that there is no unique way to specify this function; it is essentially arbitrary. For example, when the parameter of interest is an elasticity of substitution, it is usually a share parameter that will be altered at the same time to restore the consistency of the base-case data set with an equilibrium. Thus when Bernheim et al. speak of a derivative of an output with respect to a particular parameter, in fact they mean the total derivative with respect to at least two parameters, the one of interest and the one (or ones) that had to be changed in the recalibration procedure. Because the model aspects altered in the recalibration procedure vary from one parameter to another, it is not clear exactly how to interpret these derivatives.

At a minimum the authors ought to report, for each parameter whose sensitivity is being studied, what other parameter(s) is changed to restore an equilibrium with base-case data, and by how much it is changed relative to the original parameter. This is important information because, although the parameters of interest are assumed to be independent, there is an implicit assumption of a strong correlation between any given parameter of interest and some unnamed other parameter. A more ambitious approach would be to investigate, for each parameter of interest, alternative recalibration procedures and also doing no recalibration at all (allowing the base-case equilibrium to change when the parameter is changed). A set of such simulations may help to isolate the sensitivity of the model results to a given parameter.

Let me now speak to what this paper tells us about saving and its tax treatment. The main problem with applying this tool kit, improved as it might be, to the problem at hand is recognized by the authors, and here I quote: "We have made no attempt to account for any uncertainty that one might have about the correspondence between the model and the real world. As a result, our estimates of variance reflect uncertainty concerning the impact of policy in the model, and not in the actual economy." Unfortunately, when the subject is saving there is tremendous uncertainty about what the true model is. The model used by Bernheim et al. makes heroic, dare I say incredible, assumptions about absolutely crucial elements of the saving decision. For example, what is the nature of intergenerational transfers, do individuals pierce the corporate veil, are many individuals liquidity constrained, do they consider the balance sheet of the government?

A final example is the subject of an earlier paper by John Shoven and two other coauthors. The conclusion of that paper (Goulder, Shoven, and Whalley 1983) is that the economic effects and welfare implications of switching to a consumption tax depend in a critical way on how the foreign sector is modeled, in particular the degree of international capital mobility that is assumed. In fact the paper concludes that when international capital flows are recognized, the consumption tax is no longer a very attractive policy, causing very substantial welfare losses. It would not significantly increase the U.S. capital stock, and the intertemporal efficiency gains are more than offset by the misallocation of capital between the domestic and foreign economies. Because of the foreign tax credit system, the capital outflows caused by the tax change imply that the U.S. forgoes the gross-of-tax return to capital but only receives the net-of-tax return.

I have mentioned what the model does not contain. Let me take a moment to summarize what it does contain. First of all, let me stress that this is a very complex model. The discussion of it in the paper is by necessity very brief—there is much more to it than this discussion suggests. It is worth emphasizing some of its aspects that are critical to this simulation. Consumers make saving decisions based on a utility function that includes present and future consumption. Come next period, with wealth augmented depending on previous saving decisions, they again decide on the balance between present and future consumption. Consumers are myopic, not liquidity constrained, and have no life-cycle aspect to their decisions. As I have already mentioned, there are no international capital flows allowed, so saving equals investment every period. The welfare measure calculates a present value of the current consumption choice made each period.

Obviously we can argue about each of these assumptions, and others I have not mentioned, at length. My point is that the estimate of the impact of a consumption tax that this model generates is already subject to tremendous uncertainty due to these large number of modeling choices.

Let me summarize. I believe that the authors have succeeded admirably in developing a methodology to assess the sensitivity of CGE-based simulations. I expect that this will be a widely cited paper and a widely used methodology in the CGE field. However, although this is a very valuable addition to the CGE tool kit, I believe this is the wrong set of tools for learning about saving. We are too far from a consensus about so many basic issues that predictions with six significant digits, even when they come with standard errors with six significant digits, are not that helpful. In most cases, the parameters are not yet the key issues in this field.

CGE analysis is more valuable, and has already proven its value, in applications where the basic model structure is less controversial. This points us back in the direction of its original use—analysis of policies whose effects are primarily intersectoral rather than intertemporal.

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